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T.&A.M. REPORT NO.270

PHOTOELASTIC STUDY OF THE STRESSES NEAR OPENINGS IN PRESSURE VESSELS



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T&AM Report No. 270

PHOTOELASTIC STUDY OF THE STRESSES NEAR OPENINGS IN PRESSURE VESSELS

by

C. E. Taylor and N. C. Lind

Department of Theoretical and Applied Mechanics University of Illinois Urbana, Illinois

March 1965

Foreword

Beginning somewhat over eight years ago, the PVRC Subcommittee on Reinforced Openings undertook a program aimed at the development of a theory for the reinforcement of openings in pressure vessels and piping under internal pressure. Inasmuch as development of fully analytical methods for calculation of stresses seemed highly problematic at that time, a rather broad scale experimental program was undertaken covering a more or less systematic examination of the principal variables involved in the problem, such as d/D, D/T, and s/S ratios, form of reinforcement, percentage of reinforcement, "length" of reinforcement, and special purpose nozzle configurations. The greater part of this work was accomplished using machined, three dimensional photoelastic models. Altogether, nearly 100 such models have been tested under PVRC auspices. The greater portion of these models have been tested by the University of Illinois, a somewhat lesser number by Westinghouse Research Laboratories, and two models by the University of Waterloo (Canada). Test results from the earlier University of Illinois models were published in Welding Research Bulletin No. 51, but this covered only about 20 percent of the present total. Subsequently, some of the results were summarized in Welding Research Bulletin No. 77, but complete detailed stress profiles were not included except for isolated cases shown to illustrate specific points in the discussion of the data.

With the development of reasonably adequate analytical methods for the analysis of spherical shell problems and seeming partial success in the development of a theoretical treatment for the cylinder-to-cylinder intersection, it became apparent that further experimental work should be held in abeyance and hereafter used only to establish specific points in the light of theoretical results or to investigate problems which cannot be handled analytically.

With the cessation of primary activity in this area, it seemed appropriate that a summary report should be issued covering all of the detailed results of this work. However, the Subcommittee felt that such detailed data may not be of sufficient general interest to warrant publication as a Welding Research Council Bulletin. Decision was, therefore, made to issue the data in the form of "contract" reports with sufficient copies for distribution to research organizations, nuclear design groups, and others having a special interest in such detailed information. As a matter of convenience, the data will be issued in two reports, one by the University of Illinois covering their own work plus that of the University of Waterloo, and the other by Westinghouse Research Laboratories. However, since a number of the Illinois and Westinghouse models constitute an integral series of models, it seemed best that a combined index of the models be provided in order to facilitate examination of the data by those interested.

To those who review or use these data, it must be emphasized that, as in all experimental data, the results from certain models may not be entirely consistent with results from other models, and there may occasionally be considerable inconsistency in data from different slices from the same model. As was indicated in paragraph A. 1.4 of WRC Bulletin No. 77, there is evidence of an over-all scatter band of perhaps as much as 20 percent in the results. Also, as discussed in paragraph A. 7. 2. 4 of Bulletin 77 and in Section 1.6 of WRC Bulletin No. 95, there is seeming evidence of a consistent difference between photoelastic and steel model test results for the inside corner location on the longitudinal axis of a cylinder, with the photoelastic data being lower than the steel model data. Any use of these data for design purposes should, therefore, be accompanied by care and judgment.

It will be noted, upon examination of the data, that most stress quantities are expressed as a S. C. F., related to the calculated stress in the shell as determined from <u>mean diameter formula</u>. In the case of spherical shells, this gives a S. C. F. which is <u>low</u> in relation to commonly used design formulas; the amount of error is negligible in thin shells, but may be significant in thick shells. These factors may be corrected suitably for design purposes by multiplying them by the factor (1.00 + T/Di).

The Subcommittee is deeply indebted to Professor Taylor, Professor Lind, and Mr. Leven, as chief investigators, for their conscientious efforts and continued interest in pursuing this work, as well as to numerous students and others who assisted in the tedious work of analyzing the models. We are also indebted to the Bureau of Ships as the primary financial sponsor of the work, to the American Gas Association as a secondary financial sponsor, and to Combustion Engineering, Inc., Atomics International and to the Bureau of Ships for permission to include related data not a part of the PVRC program, but of potential interest to others.

F. S. G. Williams, Chairman

J. L. Mershon, Vice-Chairman

PVRC Subcommittee on Reinforced Openings and External Loadings

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Notes:

- (1) "Basic" Di/T ratio refers to the Di/T ratio of the unreinforced shell; in some cases, reinforcement is provided by an increase in shell thickness, such that the actual D/T ratio may vary.
- (2) Length of reinforcement was sufficient to be considered as essentially a nozzle of uniform thickness.

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		NolA and NolAA wa NAE wa NAE
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di/Di s/S	$\frac{1}{.0.05}$	2 0. 129	$\frac{3}{0.20}$	4 0.385	<u>5</u> 0. 50	<u>6</u> 0.65	$\frac{7}{0.80}$	<u>8</u> 1.00
		Cylindr	ical shell	- Basic Di	 /T Ratio =	12.0		
2.00	-	-	C-3C	-	C-5C	-	C-7C	-
1.50	-	-	-	-	- :	•	-	-
1.00	C-1A	C-2A	C-3A	-	E-4 E-4B E-4E	-	C-7A	C-8A
0.412	-	-	-	-	C-5H	•	C-7H	-
		Cylind	rical shell	- Basic D	I i/T Ratio =	= 4.0		
1.0	-	-	-	-	-	-	-	C-8AW
		Spher	ical shell	- Basic Di	T Ratio =	24.0		
2.00	S-IC	-	S-3C S-3CB	•	S-5C	-		-
1,50	-	N-8D	N-5B	•		•	-	-
1.00	(S-1A (S-1AB	${ N-8E \atop N-8G }$	N-IA N-IAA	{N-9B N-9C	S-5A	•		-
0.67	-		•	•	S-5E		-	-
0.58	-	-	N-3D		•	•	-	-
0.46	-	•	{N-1E N-1EA		•	-	-	-
0.39	S-1G		N-4F	•	-	•		-
		Spher	ical shell	- Basic Di	/T Ratio =	72.0		
1.00	-	S-2AZ	-	-	S-5AZ	-		-
		Spher	rical shell	- Basic D	i/T Ratio =	= 9.0		
1.1		•	-	· ·	S-5AW		•	•

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Diagrammatic Layout of Systematic Series of Models having Uniform Thicknesses of Shell and Nozzle

N	otatio	n

		<u>(()</u>
d _i	Inside diameter of the outlet, in.	t
D _i	Inside diameter of the main vessel, in.	Sketch of slice showing direc-
f	Photoelastic fringe constant, lbs per square inch per fringe per inch of thickness	r, r,
Ι	The stress index, defined as the smaller of the following ratios:	C
	$I = \frac{(\text{maximum principal stress})}{(\text{pressure}) D_i/4T}$	i na
	$I = \frac{(\text{maximum principal stress})}{(\text{pressure}) (d_i + t)/2t}$	
κ1	Ratio of the maximum principal stress to	S.
к2	Ratio of the maximum shear stress to the $(\tau_{nom} = S/2)$	nominal shear stress, τ_{nom} .
к3	Ratio of the maximum octahedral shear st stress, $\tau_{G,nom}$.	ress to the nominal octahedral shear
n n	Observed fringe order when light is passe	ed in n-direction through a slice.
ⁿ r	Observed fringe order when light is passe	ed in r-direction through a subslice.
ⁿ t	Observed fringe order when light is passe	ed in t-direction through a subslice.
(n, r, t)	Coordinate directions for slice	
р	In internal pressure, lb per square inch	
r _i	Corner radius, in.	
r _o	Fillet radius, in.	
S	Hoop stress in the outlet, $psi. s = (press$	sure) ($d_i + t$)/2t.
S	Hoop stress in the main vessel, psi. $S =$	(pressure) (D _i + T)/4T
t	Wall thickness of the outlet, in.	
Т	Wall thickness of the main vessel, in.	

X

t _n	Thickness of slice, measured in n-direction	
t _r	Thickness of subslice, measured in r-direction	
t _t	Thickness of subslice, measured in t-direction	
σ _n	Normal stress in n-direction, lbs. per square inch	
σ _{n,i}	Ratio of the maximum stress (or minimum stress, if applicable) in the n direction on the inside surface to S(dimensionless).	
σ _{n, 0}	Ratio of the maximum stress (or minimum stress, if applicable) in the n direction on the outside surface to S (dimensionless).	
σ _r	Normal stress in r-direction, lbs. per square inch.	
σ _t	Normal stress in t-direction, lbs. per square inch.	
σ _{t,i}	Ratio of the maximum stress (or minimum stress, if applicable) in the t direction on the inside surface to S (dimensionless).	
σ _{t, 0}	Ratio of the maximum stress (or minimum stress, if applicable) in the t direction on the outside surface to S (dimensionless).	

Acknowledgements

The research was sponsored by the Bureau of Ships under contracts NObs 72069 and NObs 86112 and by the Pressure Vessel Research Committee of the Welding Research Council. The authors are especially indebted to Mr. F. S. G. Williams and Mr. J. L. Mershon who have made numerous contributions throughout the nine years of the program.

Many graduate and undergraduate students at the University of Illinois have contributed significantly to the research. Special mention is made of the contributions of Mr. T. M. Mulcahy who was active during the last five years of the program and who assisted in preparing the final report.

XI

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PHOTOELASTIC STUDY OF THE STRESSES NEAR OPENINGS IN PRESSURE VESSELS

by C. E. Taylor and N. C. Lind

Introduction

Due to the urgency of gaining information on stresses near openings in pressure vessels, a rather comprehensive experimental program has been conducted over the past nine years. It was anticipated that the results of the research would be valuable for the following three purposes: (1) to evaluate the effect of geometric parameters and methods of local reinforcement, (2) to provide specific information that may be of immediate use to designers and stress analysts for types of pressure vessels most commonly used, and (3) to provide experimental data which will be useful in developing or checking the validity of theoretical solutions for stresses.

The overall program was coordinated by the Reinforced Openings Subcommittee of the Pressure Vessel Research Committee and included similar photoelastic tests conducted by M. M. Leven at the Westinghouse Research Laboratories. In some cases part of the models in an integral series were tested by Leven and part of them were tested by the authors. Consequently, models in both series are included in a combined index given at the beginning of this report even though Leven's results are reported separately in reference (1).

As data became available for each model, complete summary sheets were prepared and were widely distributed to interested persons. A complete set of summary sheets is included in the Appendix. Results of the first few models were given in reference (2). A comprehensive interpretive report based upon information available in 1962 was written by J. L. Mershon (3) who is presently preparing a sequel report which will include analyses of all models in the program. In view of the above listed factors, the scope of this present report will be limited to a description of the experimental techniques, presentation of the results, and a discussion of the probable accuracy.

It should be mentioned that the complete description of stresses near an outlet in a pressure vessel is an extremely complex matter. This is especially true of cylindrical vessels. Even for spherical vessels, which possess axial symmetry, the problem is not simple because more than one area of stress concentration usually occur. Hence the mere listing of the maximum stress or the stress concentration factor for each model would not be sufficient. Curves giving complete stress distributions on outer and inner surfaces are given for each model. Data from these should have a higher probability of satisfying the needs of designers and mathematicians

For some of the models, the stress distribution across the thickness from inside to outside surfaces was computed at key locations. From these results the stress resultant and stress couple may be obtained and compared with solutions derived by shell theory.

Casting and Fabricating Models

All models were made from an epoxy resin, Araldite 6020. Since the casting and fabrication procedures are described in some detail in a previous paper (4), they will only be summarized here.

To obtain suitable castings 100 parts (by weight) of liquid resin were mixed with 50 parts of phthalic anydride. Depending upon the size of a casting the temperature was raised at from 1/4 to 1° C per hour; the slower rate being used for thicker castings. After a temperature of 160°C was reached, it was maintained for 8-10 days so that the chemical reactions would take place. Castings were cooled slowly at about 1/4 to $1/2^{\circ}$ C per hour in order to avoid thermal stresses.

Where several models were made in approximately the same shape aluminum molds and cores were made so that castings could be made with almost the desired geometry. This procedure saved much resin and shorted the required curing and machining time, but most importantly, yielded castings which developed very little residual stress due to the exothermic reaction during curing.

All surfaces of the models were machined in order to avoid the "rind" on the castings and to obtain accurate control over the final model dimensions. To facilitate the machining of internal and external surfaces, models were made of from three to five pieces (including end caps to maintain the internal pressure). These were cemented together with epoxy cement. Although the photoelastic fringe patterns showed that there was little disturbance caused by the joints, models were planned so that joints occurred away from the areas of greatest interest. The procedure used for machining models in the program is given in reference (4).

Internal pressure loading was used in all cases. This was obtained by means of a high pressure nitrogen tank regulated by a Moore Null-Matic pressure regulator. The "stress freezing" procedure required that the models be placed in an oven and

the temperature raised at a rate of 4° C per hour. When 160° C was reached, the internal pressure loading was applied. After 30 minutes the cooling cycle was commenced at a rate of 2° C per hour. No thermal stresses were expected or detected at that cooling rate. In all tests the nozzles were placed in a vertical position in order to avoid bending stresses caused by the weight of the nozzles.

In most instances the models contained one or more areas sufficiently remote from geometric discontinuities so that the stresses could be computed by the Lame' theory for calibrating the material. The photoelastic material constant was determined directly from these models and any errors in applied pressure were self-compensating. For models in which calibration sections could not be assured, beam specimens were taken from the same casting that were used for the models and subjected to the same stress freezing cycle.

Analysis

After the stress pattern is "frozen" into a model, it may be subsequently sliced for analysis with no effect upon the stress pattern so long as no heat is introduced by the machining processes. Thicknesses of the slices ranged from 1/32" to 3/16", with the thinner slices being used for models with small geometric features such as sharp fillets or thin nozzles.

Due to symmetry all meridianal slices taken from a spherical model (except for the models with hillside nozzles) should be identical. Consequently, three or four slices were analyzed for each spherical model in order to reduce errors, improve the accuracy, and check the reproducibility. A typical photoelastic fringe photograph is shown in Figure 1. The principal stresses may be determined from the fringe pattern by the relationships.

$$\sigma_{t} - \sigma_{r} = \frac{n_{n} f}{t_{n}}$$
(1)

where σ_t and σ_r are defined by Figure 2, f is the photoelastic fringe constant as determined by calibration of the model material, n_n is the observed fringe order, and t_n is thickness of slices or more generally the length of the light path within the model. Subscripts on n and t denote the direction in which the light passes through the slice.

For the outside surface of the model $\sigma_r = 0$ and for inside surface $\sigma_r = -p$. Thus it is possible to determine completely the value of σ_t on outside and inside surfaces by a fringe pattern like the one shown in Figure 1.

In order to evaluate the third principal stress σ_n , it was necessary to make subslices as indicated in Figure 3. If the polarized light were passed through the subslice in the r-direction, the stresses could be computed from the relation

$$\sigma_{n} - \sigma_{t} = \frac{n_{r} f}{t_{r}}$$
(2)

and when the light traveled through the subslice in the t-direction

$$\sigma_n - \sigma_r = \frac{n_t f}{t_t}$$
(3)

The usual procedure was to subslice the model along the internal and external surfaces and to compute the stress σ_n by Eq. 2. This procedure facilitated evaluating σ_n all along the boundaries. However, the fringe order n_r is proportional to the average stress difference $\sigma_n - \sigma_t$ through the thickness t_r . Since there is usually a fairly steep stress gradient in the r-direction, it would be necessary to make the subslice thickness t_r very small to minimize such errors. But small values of t_r would result in very small fringe orders n_r so there were practical lower limits on t_r .

Best results were obtained by using surface subslices together with Eq. 2 to locate areas in the model where σ_n attained peak values. Then a small cube was cut from the subslices in those areas and light was passed in the t-direction through the cubes and maximum values of σ_n could be computed directly by Eq. 3.

The analysis of the cylindrical models was similar to that described above. Two planes of symmetry exist for cylindrical models. The plane which contains the geometric axis of the main cylinder and the axis of the nozzle is called the longitudinal plane and the slice which includes that plane is the longitudinal slice. The plane perpendicular to the axis of the main cylinder and which contains the axis of the nozzle is called the transverse plane. The transverse slice includes that plane. Slices from planes other than the planes of symmetry have to be studied and reported in references 1 and 5. The present report contains only results for planes of symmetry.

The models with hillside nozzles possessed one plane of symmetry and the reported results are limited to that plane. The procedure for analysis was identical to that described earlier for other models.

For three spherical models namely N-1A, S-5C, and N-1EA stresses were calculated along straight lines running from inside to outside surface. The method used was patterned after the three-dimensional shear difference method developed by

Guernsey and Frocht (5). The procedure amounted to a numerical integration of the differential equations of equilibrium to supplement the photoelastic data. Guernsey and Frocht used Cartesian coordinates whereas the present authors employed a set of orthogonal curvilinear coordinates which would take advantage of the symmetry in the models. A short technical note is being prepared to describe completely the equations and procedure used.

Results

Complete results from all of the models are included in the Appendix to this report. Distributions of principal stresses are given for inside and outside surfaces of the model. The plots are expressed in dimensionless form as the ratios $\frac{n}{S}$ and $\frac{t}{S}$ of the described stresses to the nominal stress. For spherical models the nominal stress was defined as

$$S = \frac{p \left(D_{i} + T\right)}{4 T}$$
(4)

and for cylinders

$$S = \frac{p(D_i + T)}{2T}$$
(5)

where p is the internal pressure in lb. per square inch, D_i is the inside diameter of the sphere and T is the wall thickness of the sphere. In general, lower case letters refer to the nozzle and capital letters refer to the main vessel. That is, T would denote the thickness of the main vessel and t would denote the thickness of the nozzle wall.

Peak values of the stresses are also tabulated for each slice analyzed for a model. Weighted averages are given to take into account that occasionally a chip or nonhomogenuity occurred in a given slice and the results from it were not considered as accurate of results from other slices.

The stress concentration factors were computed for each model. These were denoted as K_1 , K_2 , and K_3 and were based upon the maximum normal stress, maximum shearing stress, and maximum octahedral shearing stresses, respectively. The stress concentration factors were evaluated by

$$K_1 = \frac{\text{maximum normal stress}}{S}$$
(6)

$$K_2 = \frac{\text{maximum shearing stress}}{\text{nominal shearing stress}} = \frac{\tau_{\text{max}}}{S/2}$$
(7)

$$K_3 = \frac{\text{maximum octahedral shearing stress}}{\text{nominal octahedral shearing stress}}$$
(8)

The locations on the models where the stress concentrations occurred were indicated on the plots by placing the corresponding symbols K_1 , K_2 , or K_3 .

The accuracy of the results was influenced by three major considerations: (a) errors due to difference of elastic constants for plastic models and steel prototypes, (b) errors due to inaccuracies in the geometry of the models, and (c) errors in measuring the birefringence in the models.

Any rigorous assessment of errors due to Poisson's ratio is difficult, this constant being 0.5 for the models compared to 0.3 for steel. It is generally believed that this difference influences the numerically largest principal stress by only a few per cent while affecting the smaller principal stress up to 10 or 15 per cent. Errors due to the small effective modulus of elasticity (around 5000 psi) for the models would result from the fact that the models deformed more under load than would prototypes. However, for the small internal pressure loadings used in these experiments, it is believed that such errors were very small.

All models tested in this investigation were machined with a high degree of accuracy. It is estimated that any errors due to inaccuracies in model geometry were small. A series of such models should reflect accurately the effects of variation of various geometric parameters on the stresses. Results from photoelastic tests should compare favorably with results of strain gages on machined steel models. Results from steel models fabricated by any other process would show effect of unintended out-of-roundness and in all probability would not agree with the photoelastic results.

The experimental procedure for evaluating the stresses yielded a high degree of accuracy in the determination of the tangential stress σ_t . This could be evaluated directly from a fringe photograph by the use of Eq. (1). It is estimated that the accuracy in the values given for σ_t is ± 5 per cent. Determination of σ_n was another matter. If σ_n were computed by Eq. (2) errors due to the variation in stress through the thickness t_r would be introduced. If Eq. (3) were used to evaluate σ_n , then any error in locating the subslice at the point of maximum stress would lead to

a computed σ_n lower than the true maximum value. For these reasons, it is estimated that the errors in σ_n are between 0 and -15 per cent.

Conclusions

The investigation described in this report is but one phase of a many sided attack on the problem of design of nozzles for pressure vessels. Full benefit will not be realized until the data from this and companion projects are thoroughly interpreted. Data on stress concentration factors for special geometry has already been successfully utilized to estimate stress in vessels of similar shapes. Much care was taken by PVRC in planning the program to make series of related models where the most significant parameters were varied systematically over the practical range of geometries so that stress concentration factors for many proposed new vessels can be accurately estimated by interpolation.

Due to the extreme complexity of three-dimensional nozzle-cylinder intersection, a complete and rigorous three-dimension analytical solution for the problem may not be attained in the near future. The results reported here should be of some immediate use to designers and should also assist in the formulation and evaluation of future analytical solutions.

References

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Fig. 1 Typical Fringe Photographs (a) Cylindrical Model E-4E (b) Spherical Model N-1A (c) Reinforced Spherical Model N-4A and (d) Reinforced Spherical Model N-4D Numerials on Figures Indicate the Fringe Orders



Fig. 2. Sketch of Slice Showing Direction Notation.



Fig. 3. Methods for Subslicing Model. The figure at the left shows surface subslices and light would pass in the r-direction through the subslice as indicated by the arrows. For a model subsliced as shown by the figure on the right, the light would travel in the t-direction. **BLANK PAGE**





SPHERE S-1AB



A-3

A-4



SPHERE N-8F



A - 5



SPHERE N-8G



A-7

SPHERE N-1A



A

D



Stress Distributions Thru the Wall Thickness of Model N-1A

С

x

D



A-9

A-10



SPHERE N-9E



SPHERE N-9A

A-11
A-12



SPHERE N-9B

DIMENSIONS & DIMENSION RATIOS Di t T 1 ľ T/D \$/s 14.333 0.605* 0.473* 0.40 0.40 0.0422 1.02 MEASURED AFTER STRESS FREEZING SLICE 1 2 3 4 S 2.96 2.95 3.01 2.57 2.15 0 2.31 1.86 2.12 1.97 K2 K3 -0.88 -0.77 -0.76 2.96 2.95 3.01 2.96 2.95 3.01 S 2.88 2.62 2.71 2.89 2.83 2.87 MODEL N-9C 0.473





SPHERE S-5A

A - 15



SPHERE S-1C

A - 16



ST HERE ST



SPHERE S-3CB



SPHERE S-5C



Stress Distributions Thru the Wall Thickness of Model S-5C

A - 19

DIMENSIONS & DIMENSION RATIOS \$/s d_{/D} d Di t 1 ľ T T/ D Measured 1.803 14.000 0.858* 0.153* 0.289 0.289 0.0612 1.48 0.129 MEASURED AFTER STRESS FREEZING DATA FROM ANALYSIS SLICE Maximum 10 2.53 weighted 2 3 1 4 Minimum average <u>5</u> On,o σ 2. 53 2.53 2.44 2.52 σ. 1.52 1.53 1.36 1.51 σ, $\sigma_{n,i}$ 2.14 2.16 1.94 2.10 K, K σ. -0.23 -0.22 -0.23 -0.23 K, 2. 53 132 2.53 2.44 2.52 K, σ 2. 53 2.53 2.52 2.44 K, <u>σ</u> 5 2.30 2.32 2.13 2.28 σ I 023 1.72 1.73 1.67 1.69 MODEL N-8D MINIMUM VALUES ARE UNDERLINED .803 0.153 0.289 R 0.289 R 7.000

SPHERE N-8D





SPHERE S-1G





SPHERE N-1E



SPHERE N-1EA



Stress Distributions Thru the Wall Thickness of Model N-1EA

.5

0

A

σ,

X

в



SPHERE N-4F





SPHERE S-2AZ



SPHERE S-5AZ



SPHERE S-5AW





SPHERE N-8C



SPHERE N-8A



SPHERE N-8B





SPHERE CE-2



0 S

5

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D	ATA	FROM	AN/	LYSIS	
Maximum	SLICE				
or Minimum	weighted average	1	2	3	4
O _{n,0}	1.96	1. 95	1.99	1.89	
$O_{t,0}$	1.74	1. 61	1.77	1.74	
$O_{n,i}$	1.81	1.80	1.78	1.83	
$O_{t,i}$	<u>-0.38</u>	<u>-0. 26</u>	- <u>0. 39</u>	<u>-0.40</u>	
K,	1.96	1. 95	1.99	1.89	
K,	2.04	2.04	2.02	2.05	
K ₃	2.01	2.00	1.98	2.01	
1	2.00	1.98	2.05	1.93	

MINIMUM VALUES ARE UNDERLINED



SPHERE N-3B

96

0.38

N-38

TUTUT

σ



SPHERE N-1B



SPHERE N-1C

A - 39

A-40



3-40

DIMENSIONS & **DIMENSION RATIOS** d/Di T/D 5/5 Di T t 1 d ľ 0.0404 0.976 0.201 2.875 14.333 0. 579* 0.248* 0.933 0.933 Measured MEASURED AFTER STRESS FREEZING DATA FROM ANALYSIS SLICE Maximum 10 weighted 2 3 1 4 5 Minimum average **O**,,,, 1.74 1.75 1.75 OT,. 1.54 1.52 1.54 0 $\sigma_{n,i}$ 1.58 1.58 1.58 TURINIUS . O. -0.11 -0.08 -0.11 K, 1.75 1.75 1.74 K3 K2 1.75 1.75 1.74 K, S 1.70 1.68 1.70 I 1.79 1.79 1.79 -N-28(modified) MINIMUM VALUES ARE UNDERLINED 100% Local Reinforcement 2.875 0.248 30° 0.598 3.14 0.933 R .933 7.167 R • SPHERE N-2B MOD.

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SPHERE N-4E

DIMENSIONS & DIMENSION RATIOS Di d, 1 T/D \$/5 T P_i 1 d_i/D Measured 2.808 14.040 . 581* .246* - - -. 042 ... 0.987 . 200 **MEASURED AFTER STRESS FREEZING** DATA FROM ANALYSIS SLICE Maximum 10 weighted 2 3 1 4 Minimum average 0++2.24 0-210 $\sigma_{n,o}$ 2.10 2.11 2.08 2.09 2.10 σ, $\sigma_{t,0}$ 2.24 2.19 2.22 2.38 2.23 $\sigma_{n,i}$ 1.63 1.64 1.68 1.72 1.68 $\sigma_{t,i}$ -. 12 -. 14 -. 10 -.13 -.<u>10</u> K; 2.24 2.19 2.22 2.38 2.23 K₂ 2.24 2.19 2.22 2.38 2.23 σ, K₃ 2.18 2.15 2.15 2.25 2.18 2.29 2.22 2.25 2.45 2.26 MODEL N-48 MINIMUM VALUES ARE UNDERLINED 80% Local Reinforcement 2.000 DIA 0.244 0.008 R 0.712 0.000 8 0.000 8

A-43

SPHERE N-4G



SPHERE N-5A







SPHERE N-4D



SPHERE N-7A


SPHERE N-6A



N-6D

A-49



SPHERE N-6E



MS-1

A-51



MS-2



CYLINDER C-1A





CYLINDER C-2A











CYLINDER C-7A

A-60



CYLINDER C-8A



CYLINDER C-3C

A-62



1 m m

CYLINDER C-7C



A-64



CYLINDER C-5H

DIMENSIONS & DIMENSION RATIOS d/D \$/5 1, Di t ľ d D T .083 . 412 . 800 . 5625 Measured •1.022 . 5625 •.464 5.569 4.453 MEASURED AFTER STRESS FREEZING DATA FROM ANALYSIS TRANS. LONG. Maximum 10 weighted. average weighted average 3 5 7 Minimum $\sigma_{n,e}$ -.<u>10</u> -.<u>15</u> -.<u>05</u> 1.20 1.29 1.25 $O_{t,0}$ -.38 -.36 -.39 1.98 2.10 2.10 $\overline{O_{n,i}}$ 2.73 2.72 2.73 - - -- - -- - --.<u>20</u> -.<u>20</u> .<u>15</u>.<u>16</u> -. 16 $\sigma_{t,i}$. <u>13</u> 1.98 2.10 2.10 K₁ 2.73 2.72 2.73 K₂ 2.88 2.88 2.88 1.98 2.10 2.10 MODEL C-TH K₃ 3.16 3.18 3.14 1.99 2.11 2.11 Q+2.10 MINIMUM VALUES ARE UNDERLINED ٩, 9 K, K4K3 11 11 LOD 11 5 Q- - 20 MODEL C-TH VERSE LONGITUDINAL

CYLINDER C-7H

A-66















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DIMENSIONS & DIMENSION RATIOS d/D Di t d, T/D \$/5 T ľ P. Measured 4.50 9.00 0.763* 0.374* 0.420 0.420 0.0817 1.08 0.50 MEASURED AFTER STRESS FREEZING DATA FROM ANALYSIS Maximum OBTUSE ACUTE 10 SLICE SLICE Minimum $O_{n,o}$ 3.5 3.0 $\sigma_{t,0}$ 3.3 2.2 $\sigma_{n,i}$ 3.9 7.2 Å . $\sigma_{t,i}$ 2.8 2.3 K₁ 3.9 7.2 K₂ 4.1 7.35 K₃ 4.2 8.24 OBTUSE SLICE MINIMUM VALUES ARE UNDERLINED Model UW-1 was tested at the University of Waterloo ŧ 0 763 04204 04204 04208 4508 450R OBTUSE SLICE ACUTE SLICE ş ACUTE SLICE

A-72



CYLINDER F

A-74



CYLINDER P-4A



CYLINDER P-4D

A-76



E-6

