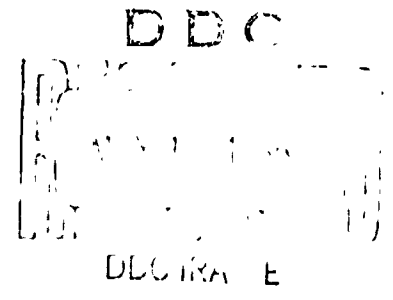ARL 65-7
JANUARY 1965

Aerospace Research Laboratories

PREDICTING EQUILIBRIUM PRESSURES FROM TRANSIENT PRESSURE DATA

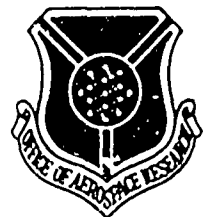DAVID L. BROWN
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OFFICE OF AEROSPACE RESEARCH

United States Air Force

ARL 65-7

PREDICTING EQUILIBRIUM PRESSURES FROM TRANSIENT PRESSURE DATA

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JANUARY 1965

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FOREWORD

This interim technical report was prepared by the University of Cincinnati Hypersonic Aerodynamics Research Staff. This investigation was sponsored by the Aerospace Research Laboratories of the Office of Aerospace Research, U. S. Air Force, under Contract No. AF 33(616)8453 for research on Experimental Aerothermodynamic Investigations, Project No. 7064," Aerothermodynamic Investigations in High Speed Flow", under the technical cognizance of Colonel Andrew Boreske, Jr., Acting Chief, Hypersonic Research Laboratory, of ARL.

This work was carried out during the period February 1964 to August 1964.

ABSTRACT

It was the purpose of this study to investigate the time-lag phenomena associated with wind tunnel pressure measurements, and find a method for predicting the equilibrium pressure from the transient pressure conditions. In order to do this a mathematical model of the actual physical case was formulated. Actual transient pressure data was fitted by least squares to this model and the equilibrium pressure predicted.

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I INTRODUCTION

One of the major problem areas when testing in a hypersonic
blow-down facility of short run duration is obtaining reliable
pressure measurements. Usually at high Mach numbers the pressure
readings are very low (0-1 psia range with the majority of readings
at or below 0.1 psia) which creates two major problem areas. The
first concerns the selection of an accurate, reliable pressure
transducer and the second concerns the time-lag phenomenon created
when using pressure transducers remotely located from the model.

This study was concerned with a method of solving the time-
lag problem which is created because geometric and environmental
conditions necessitate mounting the pressure transducer remotely
from the point where a pressure reading is desired. The physical
model was simulated by connecting a length of tubing from the
orifice to the transducer, thereby restricting the flow which
equalizes the pressure between the transducer and the orifice.

When a time-lag condition exists, the usual technique for
measuring pressure is to wait until the pressure measuring system
reaches equilibrium. In some cases this time period is prohibitive.

It was the purpose of this study to develop a method of extrap-
olating the equilibrium pressure from the transient data. In
order to accomplish this, it was necessary to formulate a mathematical
model of the physical problem. For ease of mathematical manipulation,
this model had to be relatively simple yet still accurately describe
the physical system.

II MATHEMATICAL MODEL

A suitable mathematical model had to be formulated before the equilibrium pressure could be extrapolated from the transient data. The physical case which this model must describe consists of a small internal volume which is connected to a pressure orifice by a length of tubing. (See following diagram).
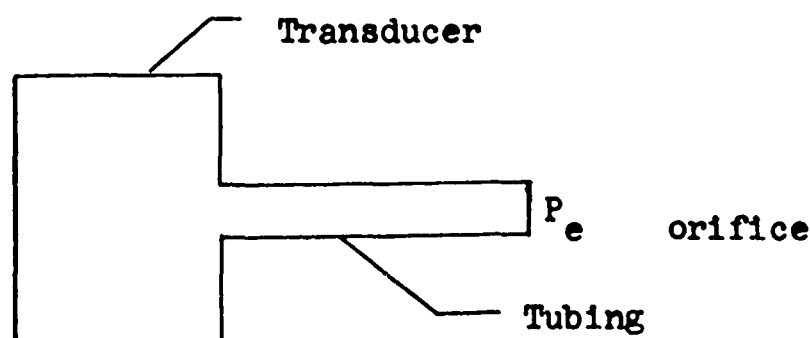
In order for the pressure in the transducer to equal the pressure on the orifice, a certain amount of air must flow through the tube. The physical properties of the gas and the geometric configuration of the tube determines this flow rate.

The mass flow-rate through a circular tube is given by Gordon P. Brown and Albert DiNarda (Reference 1) for the continuum, slip, and free molecular flow ranges as:

$$Q = \frac{\pi\, r^4}{16\ \mu LRT} (P_e^2 - P^2)(1 + 4(\frac{2}{f} - 1) \frac{\lambda_m}{r}) \tag{1}$$

where
  Q = mass flow rate -- slugs/sec

  r = radius of tube -- ft

  μ = viscosity of fluid -- slugs/ft - sec

  L = length of tube -- ft

  R = gas constant, 1716.22 lb_f-ft/slugs $^\circ$R

  f = fraction of molecules diffusely reflected from the walls of tube = 0.8

  $(1-f)$ = fraction of molecules specularly reflected from the walls of tube

  T = temperature -- $^\circ$R

  λ_m = mean free path -- ft

  P_e = equilibrium pressure -- lb_f/ft^2

  P = pressure at transducer -- lb_f/ft^2

  $\dfrac{\lambda_m}{r}$ = Knudsen number

The above equation is based upon the following assumptions:
1. The flow in the tube is isothermal and laminar.
2. No inertial effects are present and the pressure force is balanced only against the viscous force. The flow is always quasisteady.
3. Pressure is constant across any tube cross section.
4. Gas evolution from out gassing, permeation and leakage is neglected.
5. The fluid is Newtonian.

At high pressures the term $4(\frac{2}{f} - 1) \dfrac{\lambda_m}{r}$ approaches zero and the equation reduces to Poiseuille's equation which assumes no slip. As the pressure decreases the above term increases and the slip flow region is defined. At extremely low pressures the term becomes predominant and the equation describes the free molecular range.

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The pressure change, due to mass flow in or out of a fixed volume, is given by:

$$P = \frac{m}{V} RT \tag{2}$$

where

       P = pressure
       m = mass
       V = volume
       R = gas constant
       T = temperature

By differentiation of Eq. (2) with respect to time, holding V and T constant, we obtain:

$$\frac{dP}{dt} = \frac{dm}{dt} \frac{RT}{V}$$

or

$$\frac{dm}{dt} = Q = \frac{V}{RT} \frac{dP}{dt} \tag{3}$$

From Eq. (1) and (3), we obtain:

$$\frac{dP}{dt} = K(P_e^2 - P^2)\left(1 + 4\left(\frac{2}{f} - 1\right)\frac{\lambda_m}{r}\right) \tag{4}$$

where

$$K = \frac{\pi r^4}{16\mu LV}$$

The mean free path, λ_m, is given in Reference 1 as:

$$\lambda_m = \left(\frac{\pi RT}{2}\right)^{1/2} \frac{\mu}{P_m}$$

where P_m equals the mean pressure in the tube, $\left(\frac{P_e + P}{2}\right)$.

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Substituting for the mean free path Eq. (4) becomes:

$$\frac{dP}{dt} = K \left\{ (P_e^2 - P^2) + A(P_e - P) \right\} \qquad (5)$$

where

$$A = \frac{8\mu \ (\frac{2}{f} - 1)(\frac{\pi RT}{2})^{1/2}}{r}$$

However, in this report, for computational reasons, an alternate procedure was used where λ_m was redefined as given in Reference 2:

$$\lambda_m = \frac{1}{2^{1/2} \ \pi N \sigma^2} \ \frac{R^* \ MT_m}{M_o \ P_m}$$

where

T_m = molecular temperature -- $^{\circ}R$

M = molecular weight of air -- lb/mole

M_o = molecular weight at sea level -- lb/mole

$T = \frac{M}{M_o} \ T_m$ = absolute temperature -- $^c R$

N = Avagadro's Number[1] -- 2.73179×10^{26}/lb - mole

σ = effective collision diameter of mean air molecule[1] -- 1.1975×10^{-9} ft

R^* = universal gas constant[1] -- 1545.31 ft lb/lb - mole$^{\circ}R$

P_m = mean pressure $\frac{P_e + P}{2}$ -- lb/ft^2

Substituting into Eq. (4), we obtain:

_____
[1]The above constants are based on the scale $C^{12} = 12.000$.

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$$\frac{dP}{dt} = K\left[(P_e{}^2 - P^2) + A(P_e - P)\right] \tag{6}$$

where

$$A = \frac{8}{r}\left(\frac{2}{f} - 1\right)\ \frac{R^* \ T}{2^{1/2}\ \pi N\sigma^2}$$

Equation (6) is the mathematical model used in the experiment to predict the equilibrium pressure.

It should be noted that A is inversely proportional to the radius, r , and as r decreases A becomes more important in Eq. (6). If the volume of tubing is the same order of magnitude as the volume of the transducer, then the volume term used in constant K should be corrected as indicated in Reference 3. The correction is:

$$V = V_T + 1/2\ V_{Tube}$$

where

V = volume term in constant K

V_T = volume transducer

V_{Tube} = volume tube

This correction is of little importance in this application since K is determined by a curve fitting process.

The mathematical model developed in the above sections is only applicable to single tube systems. In actual wind tunnel testing it is sometimes desirable to construct the pressure tubing in sections of varying diameters. This is usually done to reduce the time-lag, and in some cases to ease model manufacturing difficulties.

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In the high pressure region, where slip-flow effects can be neglected, a mathmatical model for a multitube system can very easily be developed. A typical schematic of a multitube system is shown below.
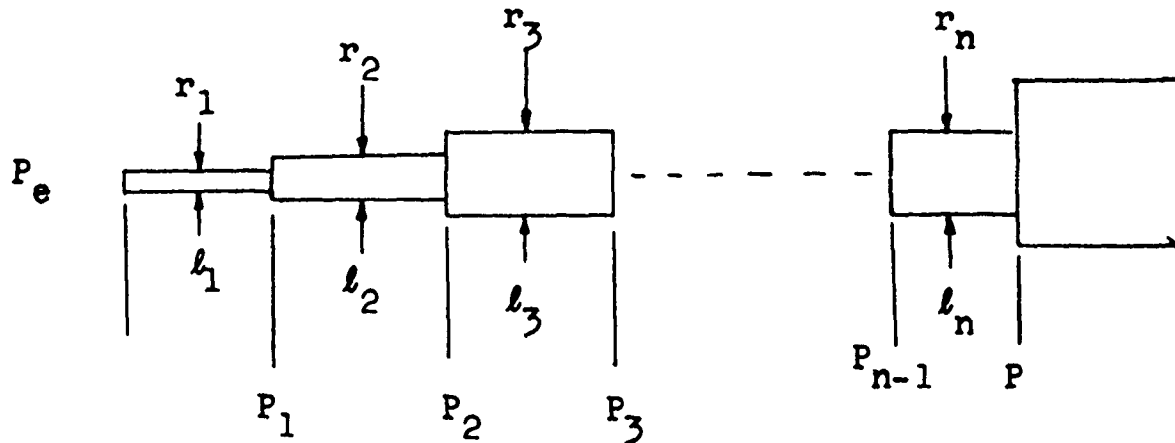
From Eq. (1), neglecting losses due to entrance or joint effects, the mass flow through any tube can be expressed as:

$$Q_1 = K_1 \ (P_e{}^2 - P_1{}^2) \tag{A}$$

$$Q_2 = K_2 \ (P_1{}^2 - P_2{}^2) \tag{B}$$

$$\vdots$$

$$Q_n = K_n \ (P_{n-1}{}^2 - P^2) \tag{C}$$

Since the mass flow through all the tubes is equal, we can write:

$$Q = K_1 \ (P_e{}^2 - P_1{}^2) = \ . \ . \ . \ . \ . \ K_n \ (P_{n-1}{}^2 - P^2)$$

By elimination of the intermediate pressure from Eq.'s (A) through (C) we obtain:

$$Q = \frac{V}{RT}\frac{dP}{dt} = \frac{P_e^2 - P^2}{\sum\limits_{i=1}^{n}\frac{1}{K_1}} = K_e(P_e^2 - P^2) \qquad (6a)$$

where K_e is the effective constant.

The above procedure is analogous to that used to calculate a series resistance network in electricity, where Q would correspond to current, $(P_e^2 - P^2)$ to the E.M.F. and $\frac{1}{K_e}$ to the resistance.

It is evident that Eq. (6a) is parametrically equivalent to Eq. (6) when A is equal to zero. Therefore, since K_e is determined by the curve fitting process, Eq.'s (6a) and (6) can be used in the same manner to predict the equilibrium pressure.

In the slip-flow range the equation for mass flow is non-linear (contains P^2 and P terms) and no simple expression can be developed for a multitube system. Therefore, in this range a single tube system should be used.

Since temperature gradients exist along the tube in most wind tunnels, it is advisable that the mathematical model be suitably modified.

The original model was based upon isothermal flow, however, the theory was modified to include temperature gradients by assuming that the flow is only isothermal across the tube at any station. This assumption seemed reasonable because of the small diameter tubing used and the low velocities of flow in the tubing.

A variation of temperature along the tube has the effect of varying the resistance to the flow along the tube. Therefore, this is analogous to the multitube concept where the resistance was varied by different diameter tubing. As a result, temperature gradients can be handled in a similar manner by using the same

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parametric form of the mathematical model.

CURVE FITTING TECHNIQUE

In the preceeding section a mathematical model of a typical pressure measuring system was derived in differential form (See Eq. 6). For a constant orifice pressure Eq. (6) can be integrated using the boundary condition that $P = P_o$ at $t = t_o$. The integrated form of the equation becomes:

$$t = \frac{1}{K(2 P_e + A)} \ln \frac{(P_e + A + P)(P_e - P_o)}{(P_e - P)(P_e + A + P_o)} + t_o \qquad (7)$$

The above equation describes the pressure response to a step function increase in orifice pressure from an initial value P_o to a final value P_e.

The ideal solution would be to fit a curve which has the form of Eq. (7) to the transient data and determine the parameters K and A and the equilibrium pressure, P_e, from a least squares fit. This method, however, could not be applied since it would involve solving three non-linear transcendental equations to determine K, A and P_e. This procedure would be extremely difficult, thereby making the method impractical.

Equation (6) was fitted by least squares and initially all three parameters K, A and P_e were determined. This method gave very poor and inconsistent results and it became evident that there were too many degrees of freedom in the problem. Thus, the degrees of freedom were reduced from three to two by determining K and P_e by least squares and computing A by theoretical means from the relation given in Eq. (6).

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Since A is a function of temperature, it becomes difficult to
compute an effective value when temperature gradients exist along
the tube. Fortunately, the mathematical model is rather insensi-
tive to this parameter except at low pressures, therefore, only a
rough estimate is needed. For example, in the experimental program
values of A were taken nearly three times the theoretical values.
This would correspond to errors of approximately 1000°R in the
temperature, yet this gave maximum errors of only 0.1 mm HG in the
pressure prediction when the pressure was above 1 mm Hg. These
errors could be further reduced by using more transient data. At
pressures under 1 mm Hg, the errors can become appreciable and
more care must be taken in selecting the value of A (See Figure
1 for variation of A with temperature). When temperature grad-
ients exist, A can be determined from the mean temperature. Since
the model is normally rather insensitive to A, it is only neces-
sary to roughly estimate the temperature gradients.

Using Eq. (6) as the mathematical model and determining
only K and P_e by least squares, excellent results were obtained.

LEAST SQUARES METHOD

When the orifice pressure is constant the transient pressure
data starts at some initial condition (P_o and t_o) and exponen-
tially approaches the equilibrium pressure (See following diagram).
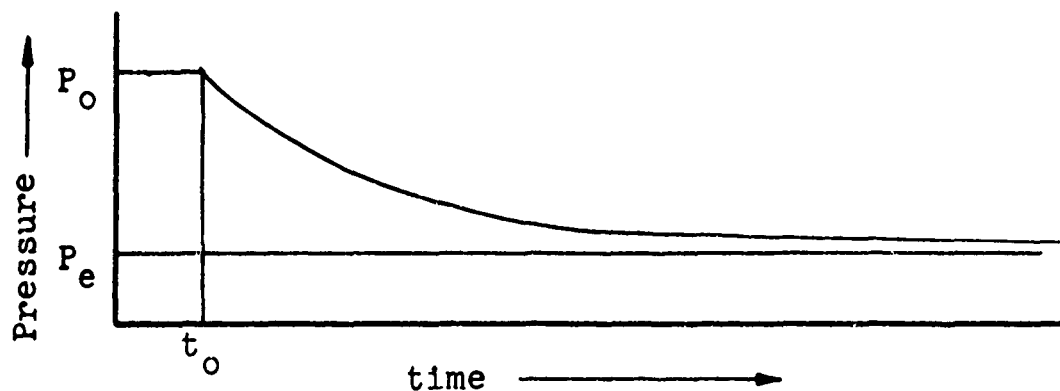
Using Eq. (6) at two points on the transient curve, it is possible to determine the parameters K and P_e. However, this procedure induces large errors and a more logical process would be to use a technique based upon a least squares selection of K and P_e.

The error at any point, i, along the curve is given by:

$$E_i = \left[\frac{dP_i}{dt} - K(P_e^2 + AP_e) + K(P_i^2 + AP_i) \right]^2 \tag{8}$$

where $\frac{dP_i}{dt}$ and P_i are measured from the transient data.

Since the above term is non-linear with respect to P_e, the resulting equations will be non-linear. To linearize the method the following transformations are used:

$$D = K(P_e^2 + AP_e) \tag{9}$$

and

$$C_i = P_i^2 + AP_i \tag{10}$$

where C_i is only a function of the transient data and the computed value of A. Therefore, the equation for the error at point i becomes:

$$E_i = \left[\frac{dP_i}{dt} - D + KC_i \right]^2 \tag{11}$$

Summing the errors terms the total error becomes:

$$ERROR = E_1 + E_2 + E_3 \ldots \ldots E_n$$

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where

$$E_1 = (\frac{dP_1}{dt} - D + KC_1)^2$$

$$E_2 = (\frac{dP_2}{dt} - D + KC_2)^2 \qquad \text{etc.}$$

The next step is to minimize the total error with respect to the parameters D and K.

Therefore,

$$\frac{\partial \text{ ERROR}}{\partial \text{ D}} = 0 \tag{12}$$

and

$$\frac{\partial \text{ ERROR}}{\partial \text{ K}} = 0 \tag{13}$$

determine a set of equations (12 and 13) which can be solved for D and K.

Eq.'s (12) and (13) become:

$$2 \left\{ (\frac{dP_1}{dt} - D + KC_1) + (\frac{dP_2}{dt} - D + KC_2) \right.$$
$$\left. + \ldots (\frac{dP_n}{dt} - D + KC_n) \right\} = 0$$

and

$$2 \left\{ (\frac{dP_1}{dt} - D + KC_1)C_1 + (\frac{dP_2}{dt} - D + KC_2)C_2 \right.$$
$$\left. + \ldots (\frac{dP_n}{dt} - D + KC_n)C_n \right\} = 0$$

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or simplifing

$$\sum_{i=1}^{n} \frac{dP_i}{dt} - n\,D + K \sum_{i=1}^{n} C_i = 0 \tag{14}$$

and

$$\sum_{i=1}^{n} \frac{dP_i}{dt} C_i - D \sum_{i=1}^{n} C_i + K \sum_{i=1}^{n} C_i^{\,2} = 0 \tag{15}$$

Solving Eqs. (14) and (15) for K the following result is obtained:

$$K = \frac{n \sum_{i=1}^{n} \frac{dP_i}{dt} C_i - \sum_{i=1}^{n} \frac{dP_i}{dt} \sum_{i=1}^{n} C_i}{\sum_{i=1}^{n} C_i \sum_{i=1}^{n} C_i - n \sum_{i=1}^{n} C_i^{\,2}} \tag{16}$$

Solving Eq. (14) for D find:

$$D = \frac{K \sum_{i=1}^{n} C_i + \sum_{i=1}^{n} \frac{dP_i}{dt}}{n} \tag{17}$$

The equilibrium pressure P_e can be found by substituting for K and D in the transformation given by Eq. (9). If this is done,

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the relation for P_e becomes:

$$P_e = -\frac{A}{2} + \sqrt{\left(\frac{A}{2}\right)^2 + \frac{D}{K}}$$ (18)

Therefore, using the least squares technique and the mathematical model, a set of equations was developed which can be used to predict the equilibrium pressure from the transient conditions.

COMPUTATIONAL PROCEDURE

In order to use the relations discussed in the preceeding section, it is necessary to know the value of $\frac{dP}{dt}$ and P at various points along the transient curve. The output of the pressure transducer gives pressure, P, versus time, t, data directly, and the $\frac{dP}{dt}$ data must be computed from the pressure-time curve. Since a differentiation process tends to amplify extraneous noise on the original data, a smoothing technique was used.

The method used was to take n points off the transient data at equal time intervals (See following diagram).
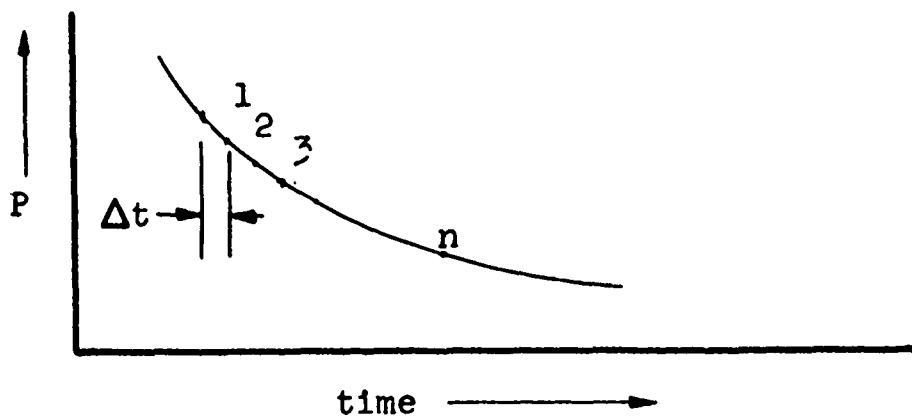
From this data, the slope at any point (i) was computed by the

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following relation

$$\left(\frac{dP}{dt}\right)_i = \frac{P_{(i+m)} - P_{(i-m)}}{2\,m\Delta t}$$

where

Δt = time between successive pressure readings

m = integer (smoothing function)

P_i = pressure reading at point "i"

The product of $m\Delta t$ determines the smoothing ability of the preceeding relation. As this becomes larger both the smoothing process and the potential error in $\frac{dP}{dt}$ become larger. Since the pressure curve is generally a slow varying curve in our case, $m\Delta t$ can be made rather large without introducing serious errors in $\frac{dP}{dt}$.

Since it was necessary to use a large number of points to define the transient region, a computer became almost a necessity in solving this problem. Therefore, the IBM 1620 and 7094 were programmed for this problem (See Figure 2). These programs will be briefly outlined in the following paragraph.

The pressure-time data was read and stored in the computer along with a value for the smoothing function. Using this stored data, the slope at various points along the curve was computed and stored with its corresponding value of C(defined in Eq. 10). From this new data, the summations $\sum_{i=1}^{n} \frac{dP_i}{dt}$, $\sum_{i=1}^{n} C_i$, $\sum_{i=1}^{n} C_i^2$, and $\sum_{i=1}^{n} \frac{dP_i}{dt} C_i$ were determined. The values of K, D and P_e were then determined from Eqs. (16), (17) and (18) using the values of these summations.

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III EXPERIMENTAL PROGRAM

The experimental portion of the time-lag program was divided into the following two basic experiments:

1. Constant Orifice Presure - the pressure on the orifice of a typical pressure measuring system was held constant and the method of analysis developed in the preceeding section was used.

2. Varying Orifice Pressure - in a second series of tests, the pressure on the orifice was slowly varied with time. Equation (6) was then used to predict the orifice pressure based on the pressure-time variation measured by the transducer.

In the first set of experiments a model was constructed which was compatible with the mathematical model developed in the preceeding sections. The model consisted of a long piece of stainless steel tubing of small diameter connected between a reference tank and a 0-1 psia Statham transducer. A valve separated the tubing from the reference tank (See Figures 3-4). The pressure in the system was set at some predetermined value. The valve was closed and the reference tank was readjusted to a new pressure reading. The valve was suddenly opened and the pressure history at the transducer was recorded on a Minneapolis-Honeywell model 1508 Visicorder and later on the Consolidated Systems Corporation Microsadic Data Acquisition System until the system reached equilibrium. From the transient portion of the data the equilibrium pressure was estimated by the method developed in this report.

In the second series of tests the same model was used with the exception that a 0-1 psia Statham transducer was added to measure the variation of pressure in the reference tank. The pressure variation in the reference tank and the transducer was recorded on the Visicorder (see Figures 5-6).

Initially, the tube was calibrated for its value of K by the

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method used in the first set of tests. Once this value was determined, the pressure in the reference tank was varied. From the pressure-time curve at the transducer, the orifice pressure could be predicted from Eq. (6) as follows:

$$P_e = -\frac{A}{2} + \sqrt{\left(\frac{A}{2}\right)^2 + \left(P^2 + AP + \frac{dP/dt}{K}\right)} \qquad (19)$$

The varying orifice pressure experiment was designed primarily to test the feasibility of using the mathematical model developed in this report to predict dynamic pressure variations in a system with considerable time-lag.

TEST MODELS

Two configurations, typical of the installations used in the ARL hypersonic wind tunnel facilities, were tested. One model consisted of a 0.042 I.D. hypodermic tube 8 feet long and the second of 0.038 inch I.D. tube 4 feet long. Both tubes were connected to a 0-1 psia Statham transducer, using a special fitting which effectively increased the internal volume of the transducer. This was done to increase the time-lag of each tube.

The eight foot model was constructed from a piece of surplus tubing. As a result its cross-section varied considerably along its length due to bending. This was done deliberately because actual tubing used in wind tunnel testing is frequently in this condition. The four foot model was an ideal model constructed out of tempered stainless steel tubing with both ends machined to reduce end or entrance effects. This was done to provide a comparison with the more typical case.

DATA REDUCTION

The data from the eight foot tube was taken from the

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Visicorder plots at 0.1 second intervals and punched in the proper format on IBM data cards. Four seconds of data was used to predict P_e. To neglect unsteady effects, this data was taken two seconds after the pressure step was applied. The data cards were then processed on the IBM 1620 and later on the IBM 7094 by the program described in the section on Computational Procedures.

The data from the four foot tube was processed directly by the IBM 7094 by editing the Microsadic tape and rewriting a new tape in Fortran Compatiable language. The new tape was then processed according to the aforementioned computer program.

The variable pressure data was processed in two steps. The calibration step was carried out in the same manner as the constant orifice pressure experiment. Once the calibration data was known, the variable pressure data was reduced as follows. The data was transcribed from the recordings at 0.1 second intervals to IBM cards in a manner analagous to the previously described test. An IBM 1620 was programmed to numerically compute the slope at various points along the curve. From this data Eq. (19) was solved for the orifice pressure.

TEST RESULTS

The test results for the constant orifice pressure experiment are listed in Table I. Also listed in the table is the amount of transient data used in the prediction along with the resulting time-lag (time necessary for the pressure to approach within 1% of the equilibrium pressure). This was done to illustrate the amount of time saved by using the procedure outlined in this report. It should be noted that the time-lag was calculated from Eq. (17) and not measured during the experimental program.

As can be seen from Table I, the results were very good. The maximum error for the eight foot tube was little over 0.1 mm Hg when based on only four seconds of transient data. For the four foot tube (ideal model), which was tested at pressures under

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1 mm of Hg, the maximum error was only 0.013 mm Hg. This is equivalent to the repeatability of the transducer used to measure the pressure.

Some typical results from the variable pressure test are shown in Figures 7 - 9. As can be seen from these figures the predicted pressure is fairly good. However, there is a reasonable amount of scatter (0.2 mm Hg) in the prediction, which was caused by noise on the original data. Since no ultra-smoothing process like least squares can be used, noise constitutes a more serious problem. However, by using the process previously described for smoothing the $\frac{dP}{dt}$ data and by fairing a smooth curve through the predicted data, very good results were obtained.

A second problem area encountered was due to unsteady flow in the tube. When this condition exists, the mathematical model breaks down because the quasi-steady flow assumption is violated. As a result the predicted pressure lags behind the actual orifice pressure.

Any rapidly changing pressure differential across the tube will cause this unsteady flow condition. Therefore, the variable pressure technique is limited to slowly varying pressures. No serious attempt was made in this study to consider these effects, except to determine their possible cause and to propose a semi-empirical correction to the mathematical model.

Initially, no inertial effects were considered in the derivation of Eq. (1). To correct this situation the inertia terms were later included in the derivation (in a manner similar to Reference 3) and the following mathematical model was formulated:

$$\left(\frac{dP}{dt} + B \frac{d^2P}{dt}\right) = K \left[(P_e{}^2 - P^2) + A(P_e - P)\right] \tag{20}$$

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where

$$B = \frac{d^2 \int P_x d_x}{32\mu RT} \cong \frac{d^2(P_e + P)}{64\mu RT}$$

is the inertia coefficient. If B is evaluated for the models tested, the inertia term becomes negligible and Eq. (20) reduces to the mathematical model used in this report. At first this conclusion was quite confusing, but a closer study indicated that the problem must be one of boundary layer growth, not one of inertia. To check this hypothesis, the time necessary for the boundary layer to fill the tube was computed and compared to the time period of unsteady flow as measured from the experimental data. These were equivalent, which would tend to support the above hypothesis.

Time did not permit a detailed analysis of the boundary layer growth in the tube. Therefore, no analytical solution was formulated. However, a semi-empirical solution was devised which considerably improved the pressure predictions in these unsteady flow regimes.

The second derivative of pressure with respect to time is a measure of the rate of change of mass flow through the tube, and therefore, an indication of the unsteady condition (See Eq. 3). If $\frac{d^2 P}{dt^2}$ equals zero the flow is steady; if $\frac{d^2 P}{dt^2}$ is small the flow is quasi-steady; and if $\frac{d^2 P}{dt^2}$ is large the flow is unsteady. Therefore, the original mathematical model was modified by adding a $\frac{d^2 P}{dt^2}$ term as follows:

$$(\frac{dP}{dt} + F \frac{d^2 P}{dt^2}) = K \left[(P_e^2 - P^2) + A(P_e - P) \right] \qquad (21)$$

where

$$F = \text{the unsteady flow coefficient}$$

The coefficient F was then determined from the experimental
data assuming that it was constant over any small pressure range.
When this was done, the unsteady flow regions were considerably
improved (see Figures 10 and 11). The coefficient F is not actually
constant, but some function of pressure and possibly pressure dif-
ferential across the tube. However, over any small pressure range
(3 to 10 mm Hg and 10 to 20 mm Hg in this study) it appeared to be
fairly constant. At lower pressures it appeared that the variation
was greater, but insufficient data was available to determine its
pressure dependence.

If unsteady flow regimes are of interest, a detailed analytic
study should be conducted to determine a suitable mathematical
model. If it should be impossible to formulate a usable model
from this analysis, then one similar to that proposed in Eq. 21
could be developed both from an analytic and experimental stand-
point.

IV CONCLUSIONS

A method for predicting equilibrium pressures from transient
conditions was developed and tested with very good results. The
tests were concerned only with configurations under consideration
for application in the ARL hypersonic wind tunnel facilities. In
these tests no actual wind tunnel data was evaluated - only simu-
lated data. Several conditions were not simulated by the tests;
temperature gradients along the tube and multitube configurations,
which, however, should not present any problems except at very low
pressures.

When applying this method there are several requirements
which must be followed:

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1. After a pressure disturbance, several seconds of data must be discarded to assure no unsteady effects are present.
2. The pressure on the orifice must be constant during the time the data is taken.
3. Temperature gradients must not appreciably vary during the time when the data is taken. (Effect of temperature on time-lag constant K is shown in Figure 12).

A second procedure for measuring variable pressure was also tested with reasonable results. This procedure, unlike the first, would be more difficult to adapt to wind tunnel testing. Variations of temperature with time would change the time-lag constant, K, which is used in the computation of the orifice pressure (See Figure 12). Since in most wind tunnels this facet would occur, it appears unlikely that this method could be used. One exception would be where the temperature varied slowly and the pressure was periodically constant on the orifice. If this were true, then the tube could be periodically calibrated for K, which would then change only slightly between calibration points.

The second method outlined in this report applied only to slowly varying pressures because the method is based upon a quasi-steady flow assumption. A brief investigation of the unsteady flow region was made and the results are included in the Section on Test Results.

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2. U. S. Standard Atmosphere, 1962, prepared under sponsorship of
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TABLE I - DATA FOR CONSTANT ORIFICE PRESSURE EXPERIMENT

DATA FROM 8-FOOT TUBE

RUN NO.	K TIME-LAG CONSTANT	P_E EQUILIBRIUM PRESSURE mm Hg	P_C PREDICTED PRESSURE mm Hg	TRANSIENT DATA SECS	APPROXIMATE TIME-LAG* SECS
1	.0256	5.91	5.95	4	13
2	.0248	5.95	5.98	4	13
3	.0254	5.92	5.96	4	13
4	.0250	5.83	5.85	4	13
5	.0244	4.81	4.81	4	17
6	.0240	4.76	4.77	4	17
7	.0244	4.73	4.78	4	17
8	.0240	4.79	4.76	4	17
9	.0242	3.92	3.88	4	22
10	.0242	3.68	3.78	4	22
11	.0250	3.78	3.87	4	22
12	.0252	3.70	3.79	4	22
13	.0246	2.90	2.92	4	30
14	.0252	2.80	2.89	4	30
15	.0244	2.84	2.85	4	30
16	.0248	2.83	2.86	4	30
17	.0244	1.98	2.07	4	47
18	.0238	1.89	1.87	4	47
19	.0244	1.85	1.86	4	47
20	.0242	.75	.748	4	105
21	.0236	.58	.51	4	131
22	.0252	1.08	1.16	4	75

DATA FROM 4-FOOT TUBE

RUN NO.	K TIME-LAG CONSTANT	P_E EQUILIBRIUM PRESSURE mm Hg	P_C PREDICTED PRESSURE mm Hg	TRANSIENT DATA SECS	APPROXIMATE TIME-LAG* SECS
1	.1146	.075	.0684	6	75
1**	.1148	.075	.0673	4	75
2	.1158	.065	.0646	4	78
3	.1156	.40	.413	4	34
3**	.1154	.40	.403	2	34
4	.1152	.60	.608	4	26
5	.1152	.080	.074	4	73
6	.1148	.064	.051	4	79

* Time-lag calculated from Eq. 7 for Pressures within 1% of equilibrium
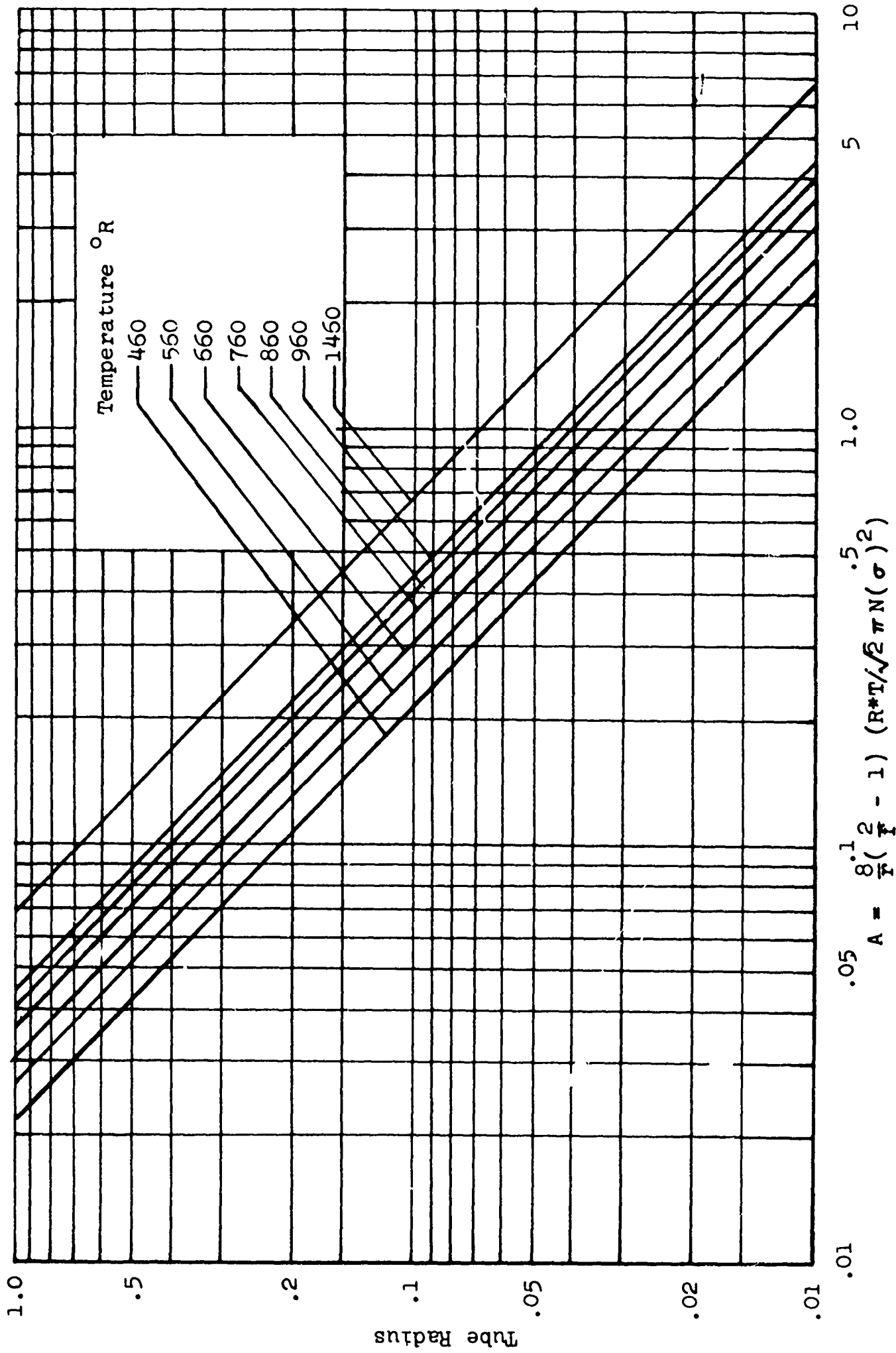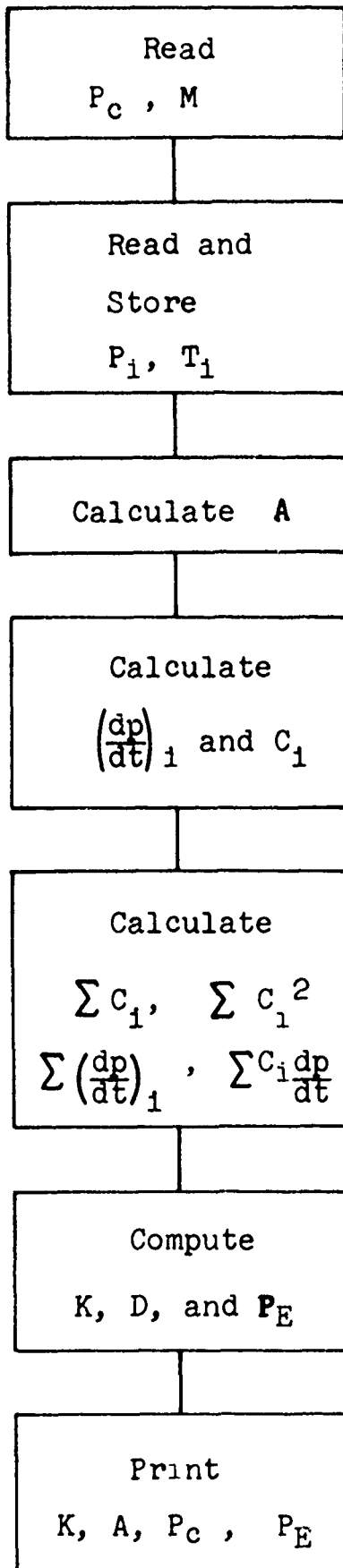
** Same run with different amounts of transient data

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$$A = \frac{8}{r}\cdot\frac{.1}{r}\left(\frac{2}{r} - 1\right)(R*T/\sqrt{2}\pi N(\sigma)^2)^{.5}$$

FIGURE 1 --VARIATION OF "A" WITH TUBE RADIUS AND TEMPERATURE

FIGURE 2 --SCHEMATIC DIAGRAM OF IBM COMPUTER PROGRAM

Read
P_c , M

Read and
Store
P_i, T_i

Calculate A

Calculate
$\left(\dfrac{dp}{dt}\right)_i$ and C_i

Calculate
$\sum C_i$, $\sum C_i^2$
$\sum \left(\dfrac{dp}{dt}\right)_i$, $\sum C_i \dfrac{dp}{dt}$

Compute
K, D, and P_E

Print
K, A, P_c , P_E

WHERE:

P_c = Measured Equilibrium Pressure

P_i = Value of Pressure @ Point i
on Transient Curve

T_i = Time at Point i

$\dfrac{dp}{dt}_i$ = Slope at Point i

M = Smoothing Function

A = Slip Flow Coefficient

K = Time-lag Coefficient

P_E = Calculated Equilibrium Pressure

C_i and D Transformations (see text)
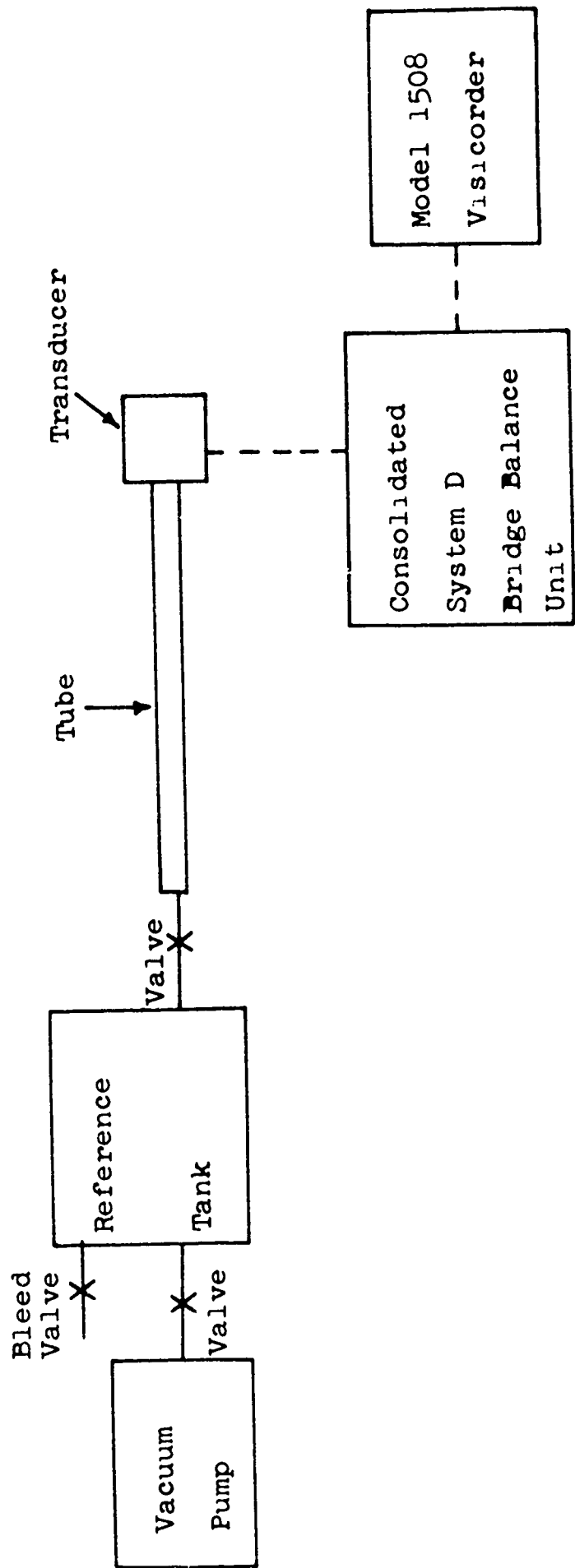
FIGURE 3 --SCHEMATIC DIAGRAM OF TIME-LAG EXPERIMENT IN HIGH PRESSURE RANGE
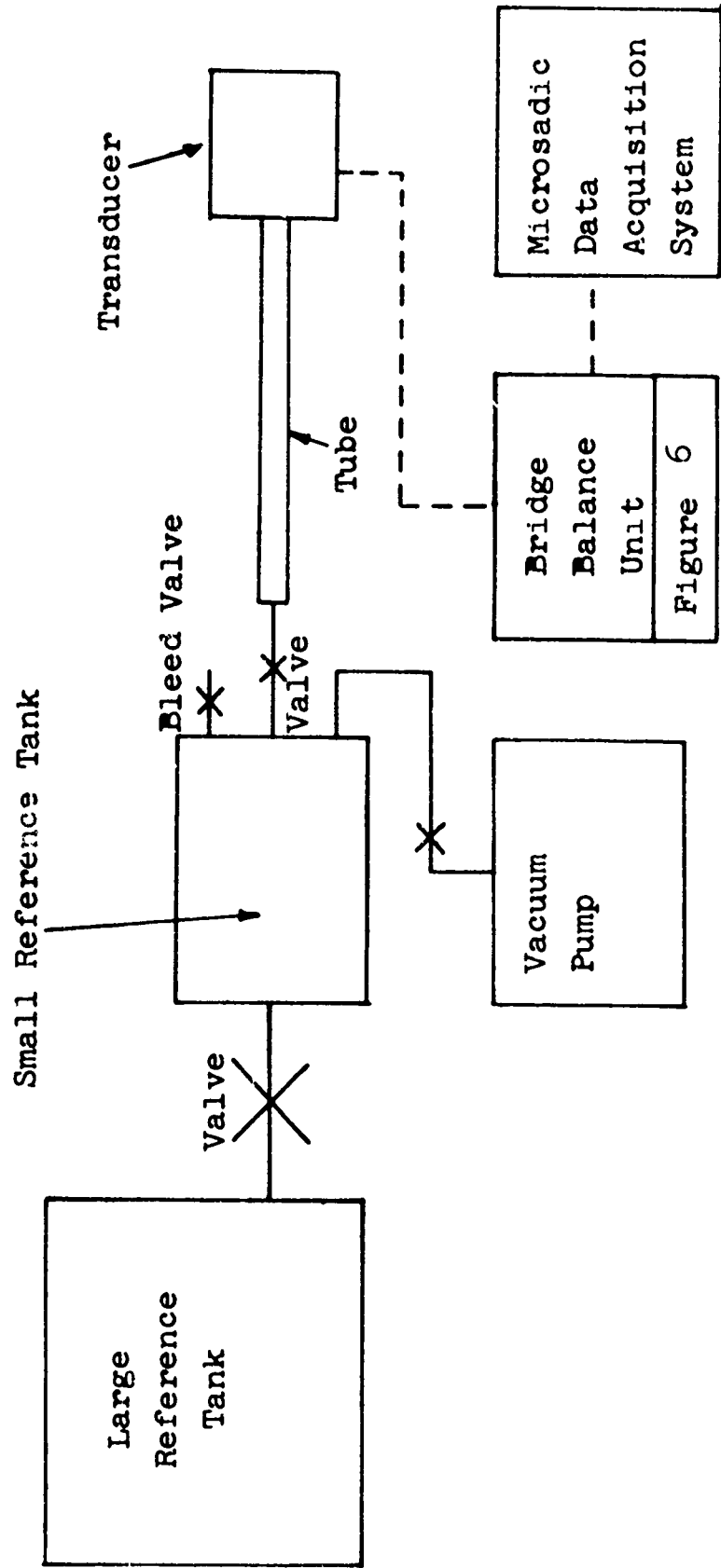
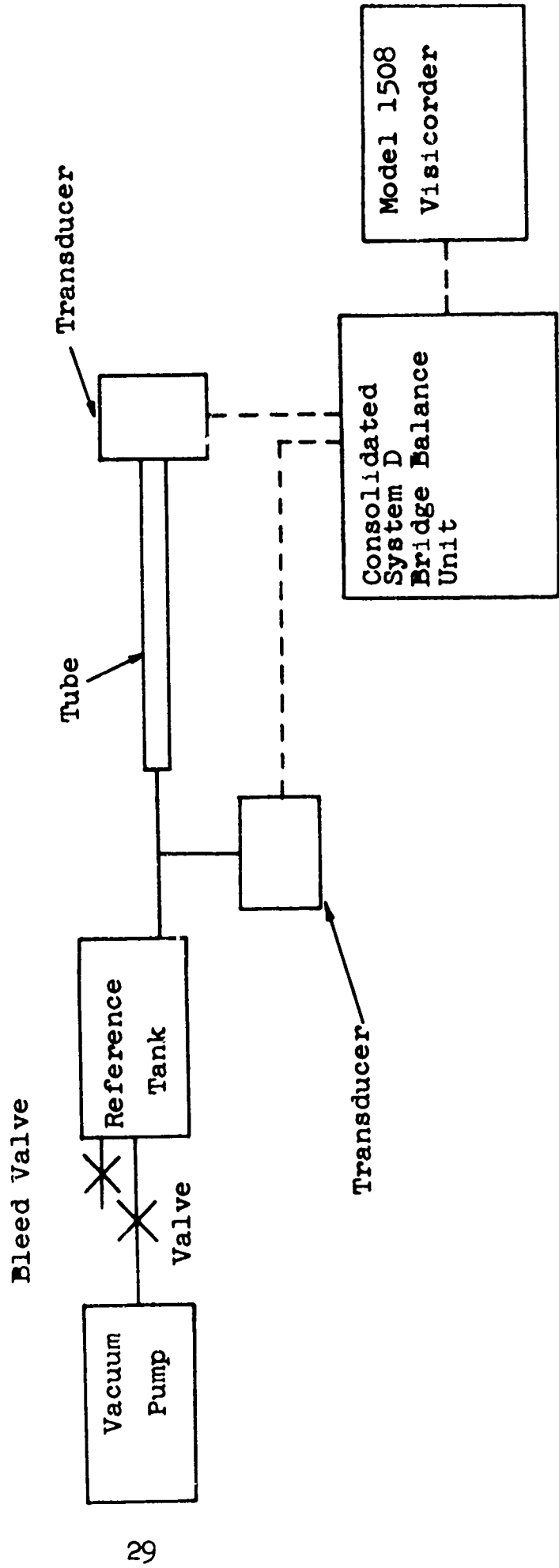FIGURE 4.--SCHEMATIC DIAGRAM OF TIME-LAG EXPERIMENT IN LOW PRESSURE RANGE

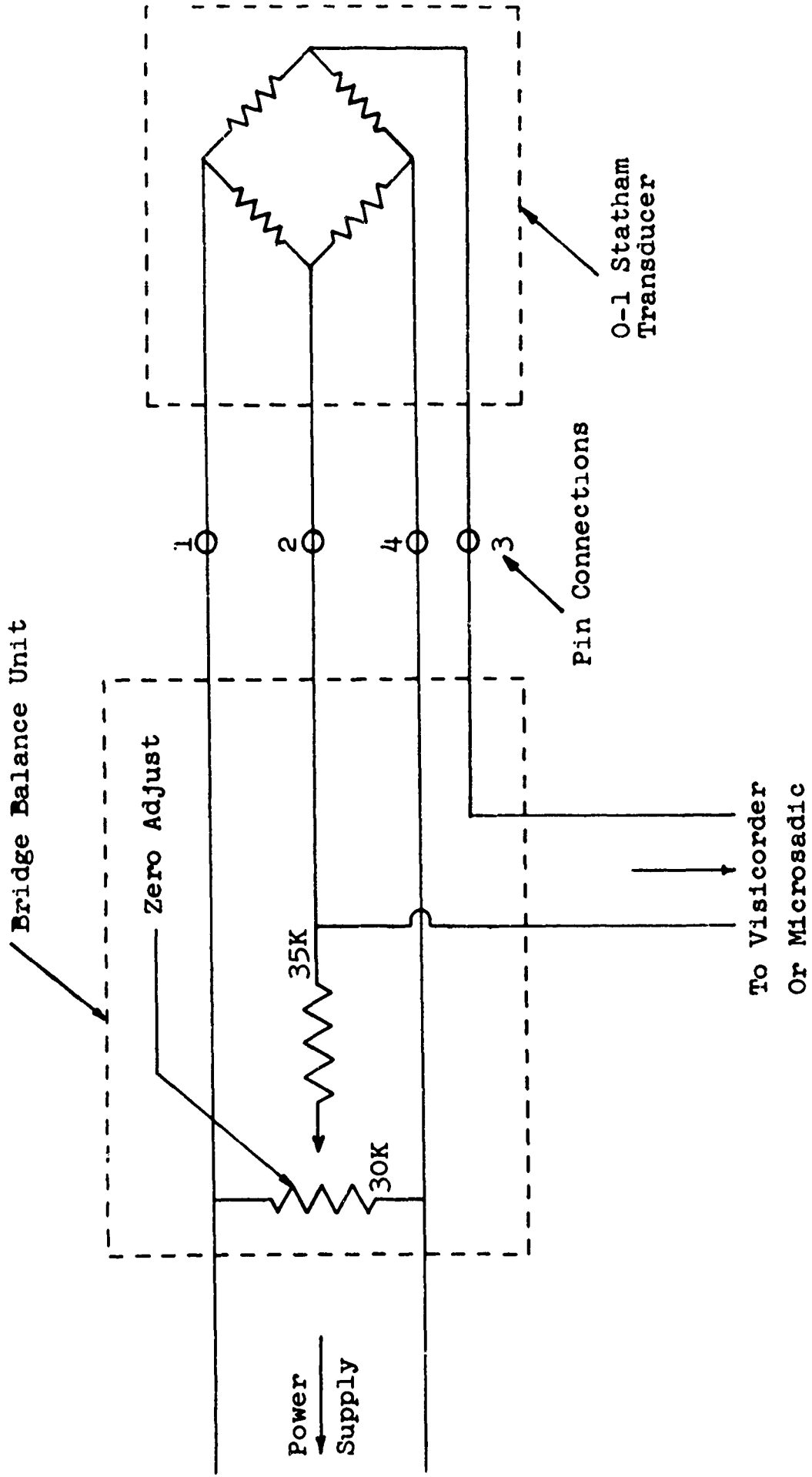FIGURE 5--SCHEMATIC DIAGRAM OF VARIABLE PRESSURE EXPERIMENT
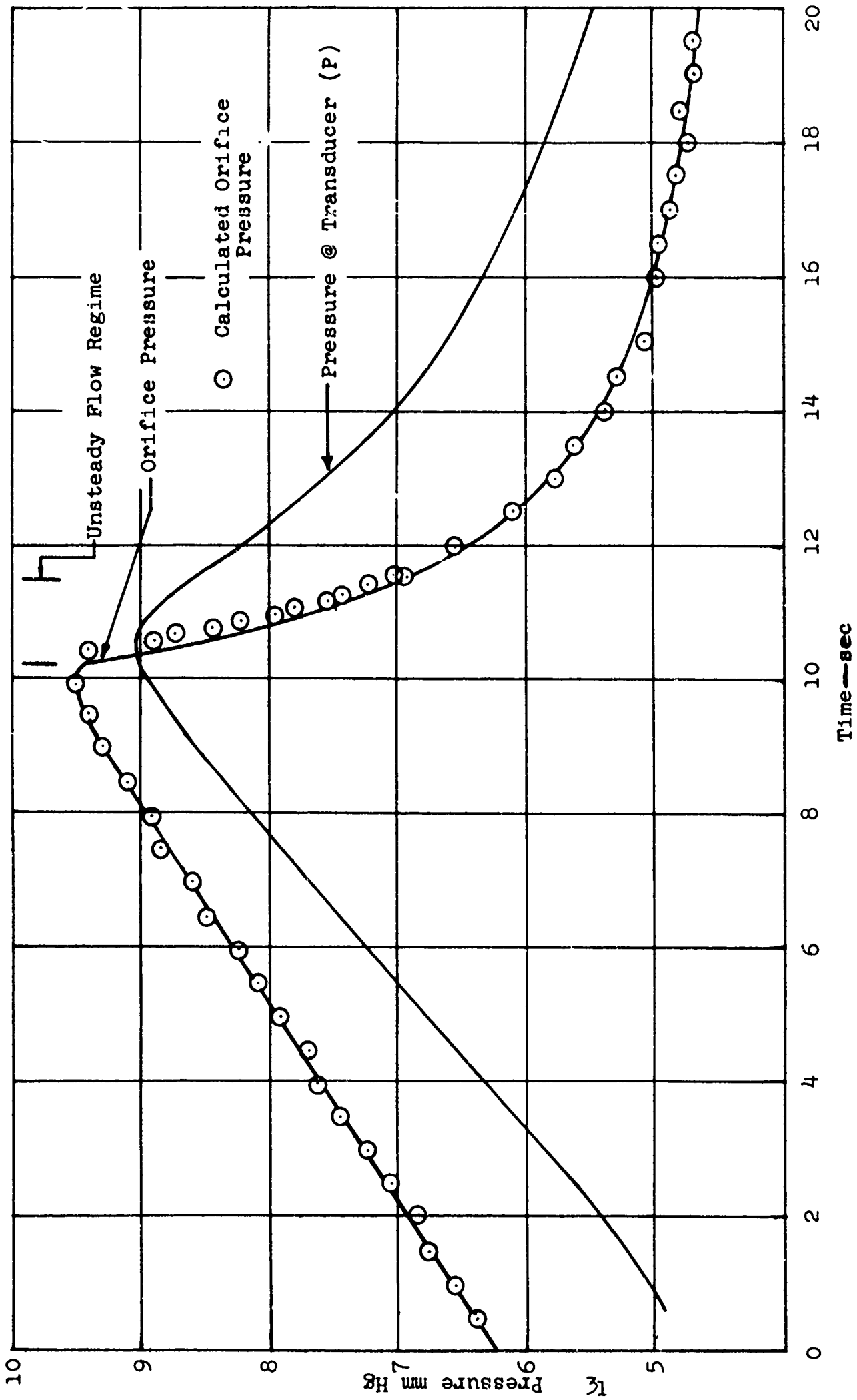
FIGURE 6—SCHEMATIC DIAGRAM OF BRIDGE BALANCING UNIT

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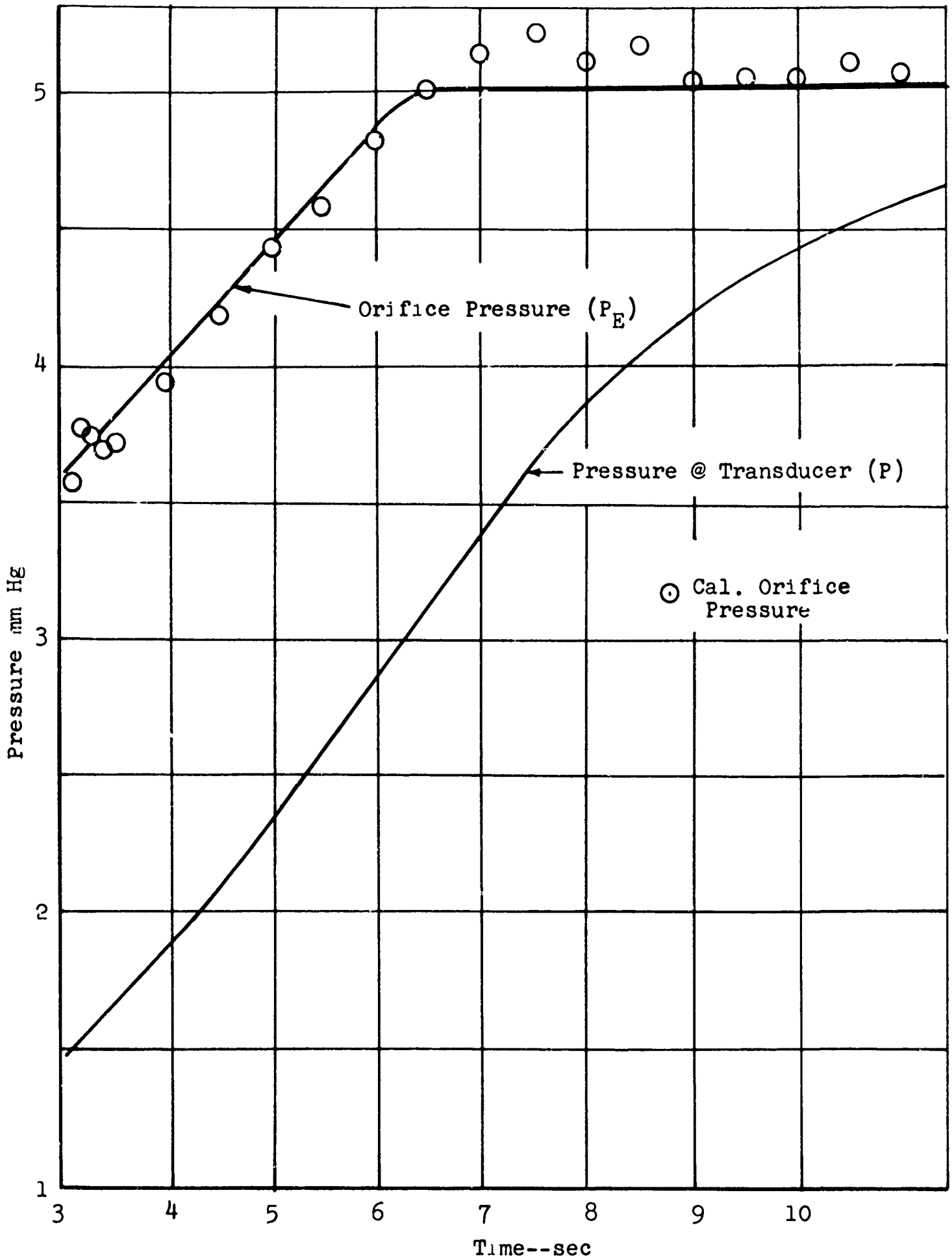FIGURE 7--TYPICAL VARIABLE PRESSURE TECHNIQUE PREDICTIONS

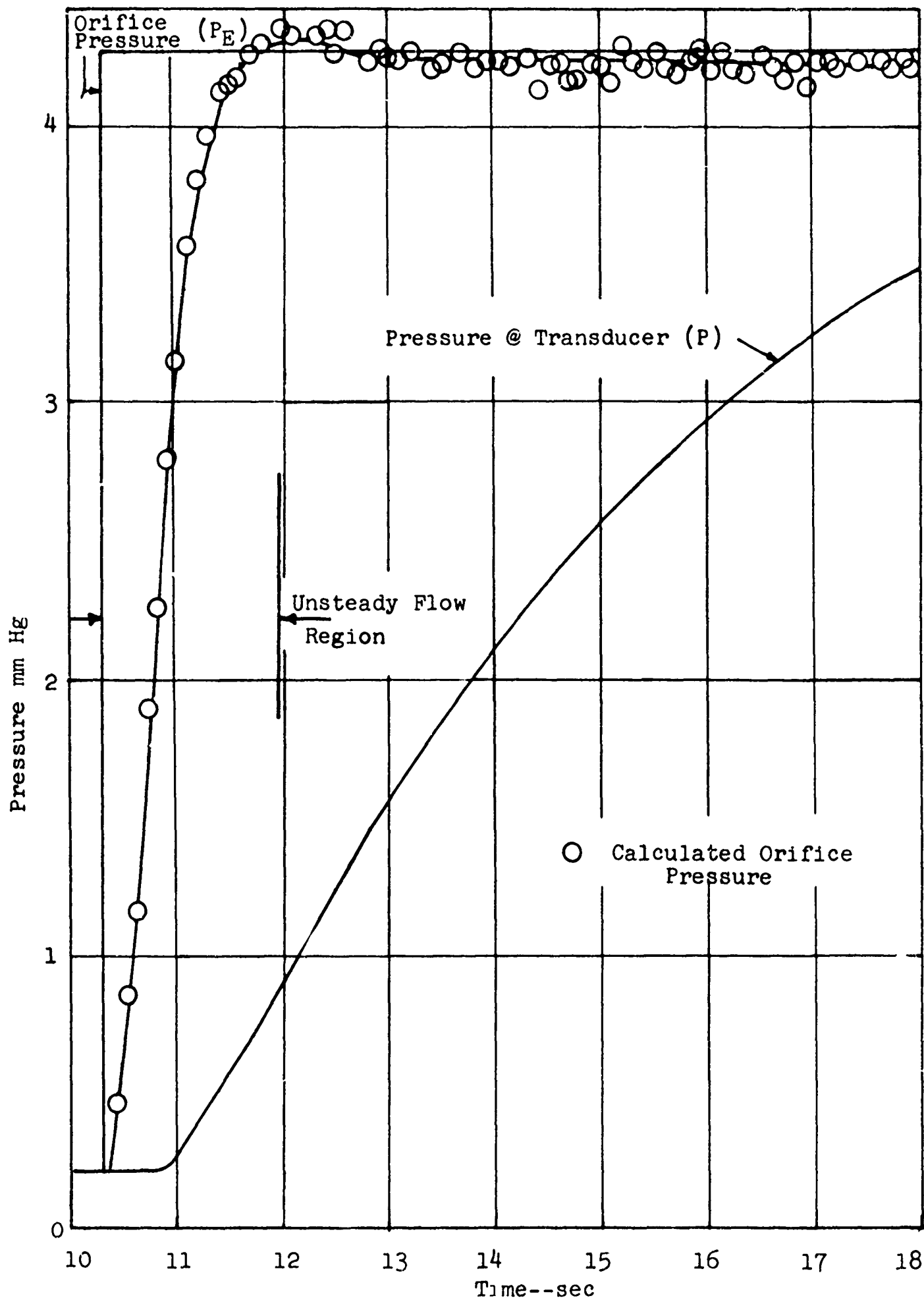FIGURE 8 --TYPICAL VARIABLE PRESSURE TECHNIQUE PREDICTIONS

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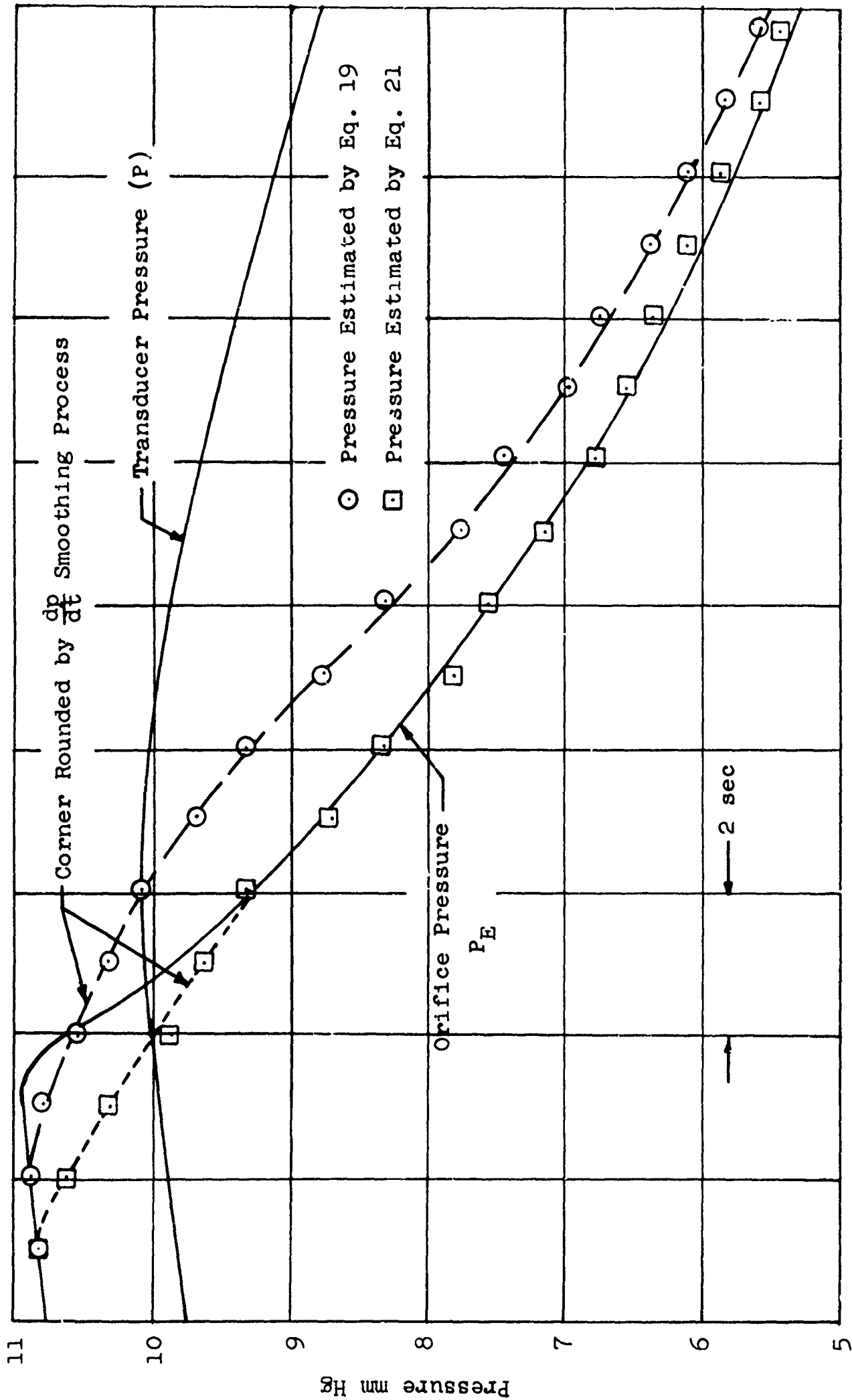FIGURE 9 --TYPICAL VARIABLE PRESSURE TECHNIQUE PREDICTIONS

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FIGURE 10--TYPICAL UNSTEADY FLOW REGION
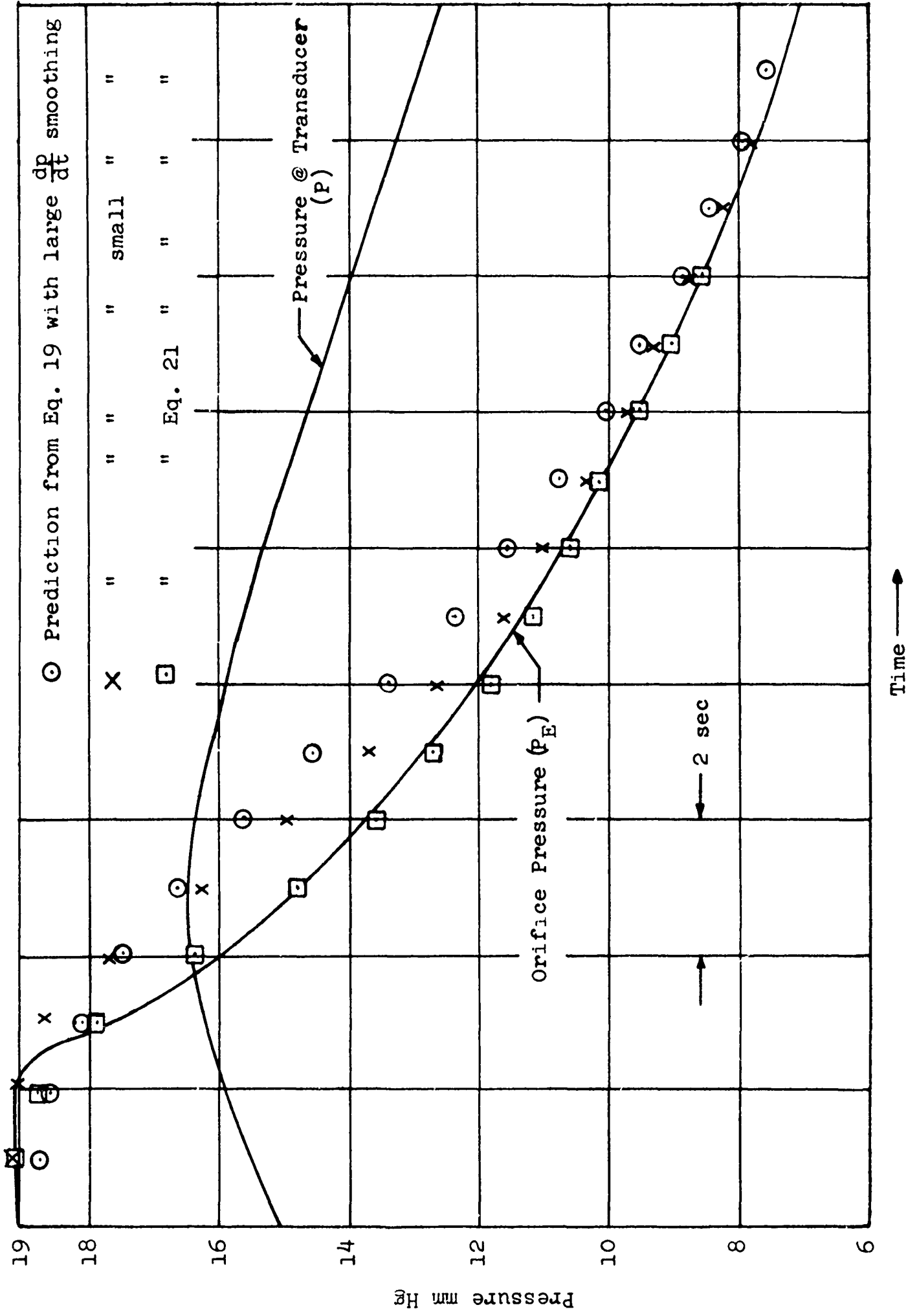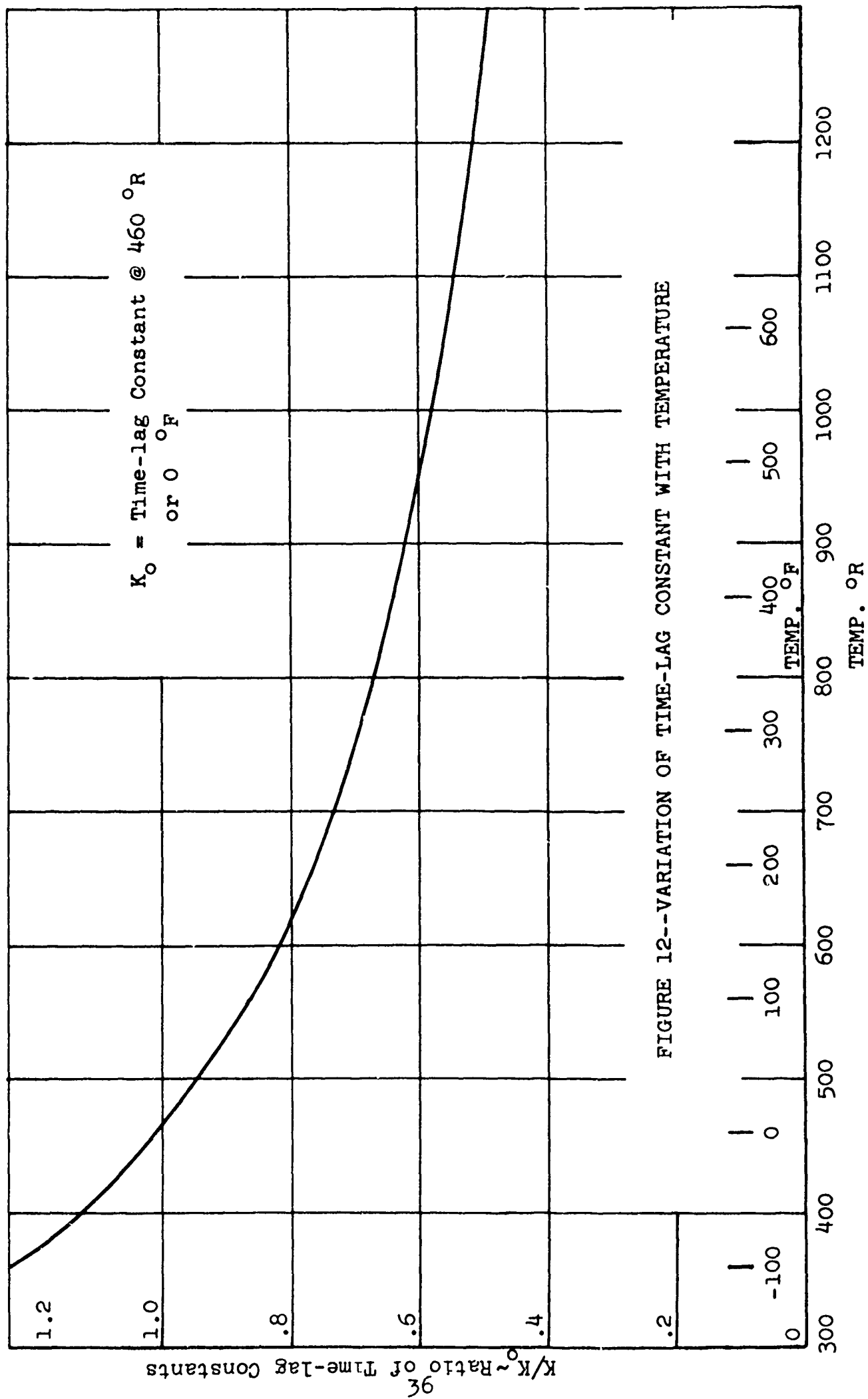
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FIGURE 11.--TYPICAL UNSTEADY FLOW REGION

Time ──►

Pressure mm Hg

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FIGURE 12--VARIATION OF TIME-LAG CONSTANT WITH TEMPERATURE

K_o = Time-lag Constant @ 460 °R or 0 °F

TEMP. °F

TEMP. °R

K/K_o~Ratio of Time-lag Constants

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	2b GROUP

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INTERIM February 1964 to August 1964

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13. ABSTRACT

It was the purpose of this study to investigate the time-lag phenomena associated with wind tunnel pressure measurements and to find a method for predicting the equilibrium pressure from the transient pressure conditions. In order to do this a mathematical model of the actual physical case was formulated. Actual transient pressure data was fitted by least squares to this model and the equilibrium pressure predicted.

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Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
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