

LINCOLN LABORATORY ANALYSES  
OF PARABOLOIDAL SHELLS  
(LLAPRS)  
USER'S MANUAL

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**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**  
**LINCOLN LABORATORY**

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Enclosed herewith is Sec. IV to be added to your copy of the manual. Please replace pages ii and 49 of your manual with the enclosed, revised pages.

17 November 1965

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## IV. SHALLOW SHELL ANALYSIS OF GRAVITY LOAD

### A. SHALLOW SHELL SOLUTION

A paraboloidal shell of revolution is considered shallow if

$$\left(\frac{dy_3}{dr}\right)^2 = \gamma^2 \ll 1 \quad (IV-1)$$

so that

$$1 + \gamma^2 \approx 1 \quad . \quad (IV-2)$$

If terms involving transverse shears are neglected in the tangential equilibrium equations and terms involving tangential displacements are neglected in the expression for the rotations, a consistent application of (IV-2) leads to the following two simultaneous linear partial differential equations

$$A \nabla^2 \nabla^2 \Phi = -\frac{1}{2f} \nabla^2 w + A(1-\nu) \nabla^2 P \quad (IV-3)$$

$$D \nabla^2 \nabla^2 w = \frac{1}{2f} \nabla^2 \Phi - \frac{1}{f} P + p_3 \quad (IV-4)$$

where

$$\nabla^2 (\cdot) = \frac{\partial^2 (\cdot)}{\partial r^2} + \frac{1}{r} \frac{\partial (\cdot)}{\partial r} + \frac{1}{r^2} \frac{\partial^2 (\cdot)}{\partial \theta^2}$$

and  $P$  is a load potential such that

$$\frac{\partial P}{\partial r} = p_r \quad (IV-5)$$

$$\frac{1}{r} \frac{\partial P}{\partial \theta} = p_\theta \quad . \quad (IV-6)$$

For gravity load

$$P = 2f\rho h \left( \gamma \sin \theta \sin \psi - \frac{\gamma^2}{2} \cos \psi \right) \quad (IV-7)$$

$\Phi$  is a stress function from which the membrane stress resultants are derivable

$$N_r = \left[ \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} \right] - P \quad (IV-8)$$

$$N_\theta = \left[ \frac{\partial^2 \Phi}{\partial r^2} \right] - P \quad (IV-9)$$

$$N_{r\theta} = \left[ \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} - \frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} \right] \quad . \quad (IV-10)$$

The transverse shears and moment resultants are related to the transverse deflection  $w$  by

$$M_r = -D \left[ \frac{\partial^2 w}{\partial r^2} + \nu \left( \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) \right] \quad (IV-11)$$

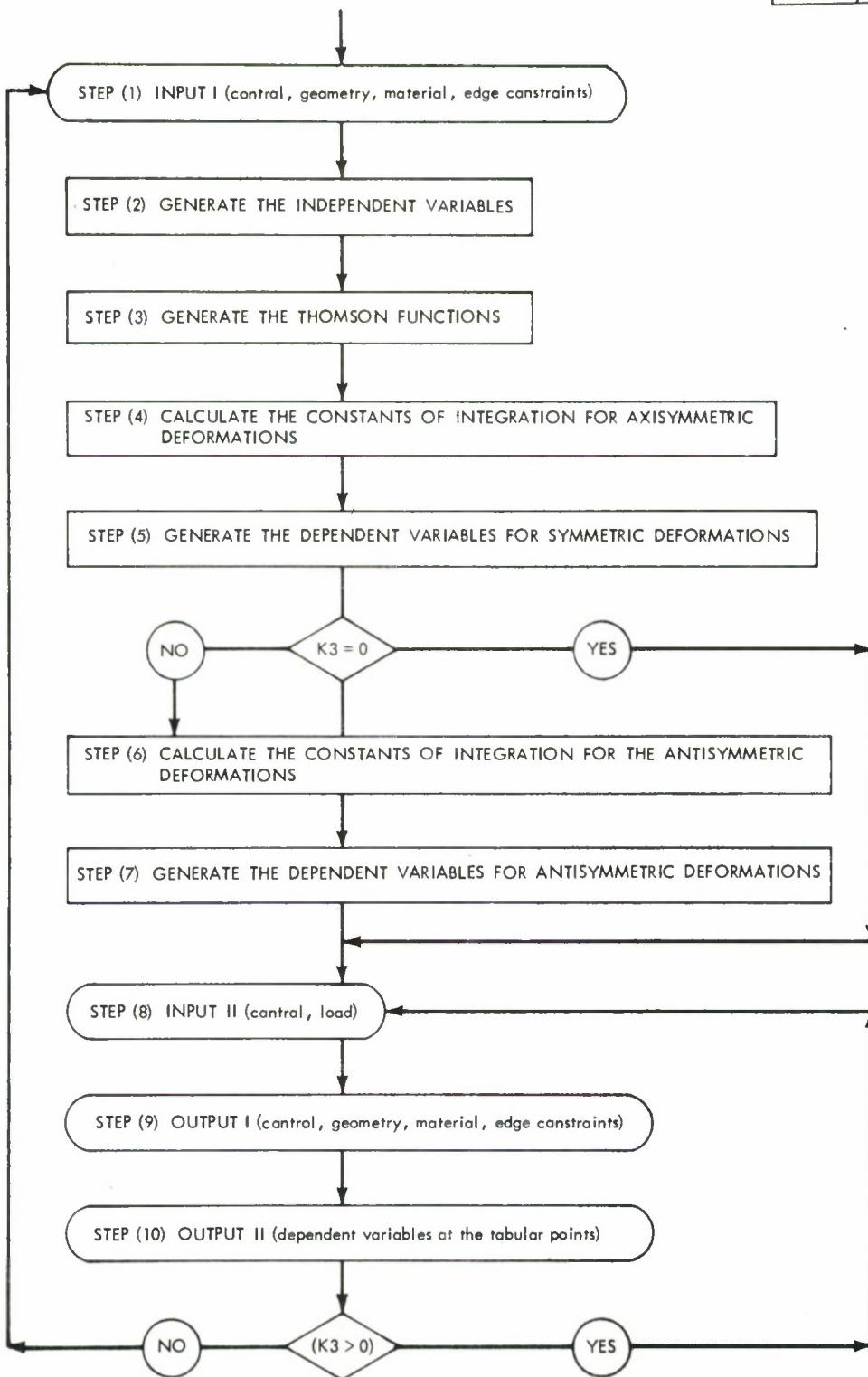


Fig. 13. Master flow chart.

$$M_{\Theta} = -D \left[ \nu \frac{\partial^2 w}{\partial r^2} + \left( \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \Theta^2} \right) \right] \quad (\text{IV-12})$$

$$M_{r\Theta} = -D(1-\nu) \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial w}{\partial \Theta} \right) \quad (\text{IV-13})$$

$$Q_r = -D \frac{\partial}{\partial r} (\nabla^2 w) \quad (\text{IV-14})$$

$$Q_\Theta = -D \frac{1}{r} \frac{\partial}{\partial \Theta} (\nabla^2 w) \quad (\text{IV-15})$$

To be consistent with the shallow shell approximation, the rotations must be approximated as follows [see Eqs. (II-28) and (II-29)]:

$$\omega_r = - \frac{\partial w}{\partial r} \quad (\text{IV-16})$$

$$\omega_\Theta = - \frac{1}{r} \frac{\partial w}{\partial \Theta} \quad (\text{IV-17})$$

Having  $\Phi$  and  $w$ , the two tangential middle-surface-displacement components  $u$  and  $v$  can be obtained by integrating the strain-displacement relations for a shallow shell corresponding to (II-25), (II-26), and (II-27).

The solution to (IV-3) and (IV-4) involves Thomson functions of order zero and their derivatives. While these functions have been tabulated in various sources (most extensively in Ref. 6), none of these tabulations is complete for our purpose. For any particular shell, we may need values of these functions not tabulated in the given reference. Subroutines have been written in double precision arithmetic to generate the values of these functions for any argument (within the limitations of the IBM 7094). Because of the double precision arithmetic and the nature of the computations involved, these subroutines are extremely time consuming. On the other hand, the nature of the Thomson functions is such that they contribute significantly to the complete solution only in the neighborhood of the edges of the shell. In these regions, the magnitude of these functions varies rapidly over a short interval of the independent variable. To get sufficient information about the shell behavior, it is necessary to have smaller increments between the  $\gamma$ -tabular points in these regions. In contrast, the solution in the interior varies slowly over the span of the shell, so that we do not need the same density of  $\gamma$ -tabular points. A waste of computer time can and will be avoided by changing the increment of the  $\gamma$ -tabular points, once we move away from the edge.

## B. DIGITAL COMPUTER PROGRAM

The general scheme of the program for a shallow shell analysis is outlined in the master flow chart (Fig. 13). When it fails to locate additional input at step (1) or step (8), the program exits automatically. The program listing is presented in Table V.

TABLE V  
PROGRAM LISTING

```

*      SHEA,ELAINE  SHALLOW SHELL WITH GRAVITY LOAD
*      XEQ
C      SHEA,FLAINE  SHALLOW SHELL WITH GRAVITY LOAD
C
C
C      DIMENSION R(50), Y(50),BBER(50),BBEI(50),AAKER(50),AAKEI(50),X(2)
1,AAKERP(50),AAKEIP(50),BBRP(50),BBIP(50),SBER(50),SBEI(50),SKER(50)
2),SKEI(50),TBER(50),TBEI(50),TKER(50),TKEI(50),WW(50),W3(50),
3 W(50,10),WP(50,10),V(50,10),U(50,10),CMR(50,10),CMT(50,10),
4CMRT(50,10),CNR(50,10),CNT(50,10),CNRT(50,10),QR(50,10),QT(50,10),
5VV(50),UU(50),Z3(8),DMR(50),DMT(50),DMRT(50),DNR(50),DNT(50),DNRT(650),
6QQR(50),QQT(50),P(50),W1(50),Q(50),EMR(50),EMT(50),ENR(50),ENT
7(50),Q1(50),A(8,8),S(8),AA(4,4),SS(4),ZZ(4),B(6,6),BB(6),B9(6),D(3
8,3),DD(3),D3(3),RG(50)
DIMENSION P2(50),W2(50),FMR(50),FNR(50),Q2(50),FMT(50),FNT(50),Q3
1(50),W4(50),W5(50),GMR(50),GMT(50),GMRT(50),QRR(50),QTT(50),GNR(50
2),GNT(50),GNRT(50),U2(50),V2(50),WT(50),WFLEXT(50),WPT(50),UT(50)
DIMENSION THETA(20),THETR(20),FMT1(12),FMT2(12)
DIMENSION PWB(50),WA5(50),WFLEX(50,10),PWB2(50),WA54(50)
COMMON W,WP,V,U,CMR,CMT,CNRT,CNR,CNT
EQUIVALENCE (BBER,P2),(BBEI,W2),(AAKER,FMR),(AAKEI,FNR),(AAKERP,
1Q2),(AAKEIP,FMT),(BBRP,FNT),(BBIP,Q3),(SBER,W4),(SBEI,W5),(SKER,
2GMR),(SKEI,GMT),(TBER,GMRT),(TBEI,QRR),(TKER,QTT),(TKEI,GNR)
C
C -- (1) INPUT I (CONTROL,GEOMETRY,MATERIAL,EDGE CONSTRAINT)
C
98 READ INPUT TAPE 2,101,M,MU,ML,MC
READ INPUT TAPE 2,101,N,NORM,K1,K2,K3
READ INPUT TAPE 2,102,RHO,E,PRT
READ INPUT TAPE 2,103,R1,R2,R3,R4
READ INPUT TAPE 2,103,THETA3,THETA4,F,H
IF(N)182,182,181
181 READ INPUT TAPE 2,205,FMT1,FMT2
182 WRITE OUTPUT TAPE 3,10
C
C -- (2) GENERATE THE INDEPENDENT VARIABLES
C
F2=2.*F
F1=1./F2
C=SQRTF(12.*((1.-PRT)**2))
C=SQRTF(C)
C=C/SQRTF(F2*H)
PI=3.1415926535
RAD=57.2957795
R5=0.
R6=0.
IF(MC-1)76,77,77
77 R5=F2*H
R5=2.*SQRTF(R5)
EM=MU
DRB=R5/EM
R(M)=R4
DO 1 I=1,MU
J=N-'
1 R(J)=R(J+1)-DRB
IF(ML-1)76,76,78
78 EM=ML

```

TABLE V (Continued)

```

      DRB=R5/EM
      R(1)=R3
      DO 931 I=1,ML
931  R(I+1)=R(I)+DRB
      R6=R5
76   MM=M-ML-MU
      M1=MM-1
      EM1=M1
      DRB=R4-R3-R5-R6
      DRB=DRB/EM1
      MM1=ML+1
      R(MM1)=R3+R6
      J=MM1
      DO 932 I=1,M1
      J=J+1
932  R(J)=R(J-1)+DRB
      IF(NORM)680,680,682
680  DO 681 I=1,M
      RG(I)=R(I)/F2
681  Y(I)=R(I)/F2
      GO TO 684
682  DO 683 I=1,M
      RG(I)=R(I)/F2
683  Y(I)=R(I)
684  G=E*H**3/(12.*(1.-PRT**2))
      DRB=DRB/F2
      PM=PRT-1.
      PP=PRT+1.
      P1=1.-PRT
      T=H/SQRTF(12.*(1.-PRT**2))
      IF(N)84,84,46
46   THETA3=THETA3*PI
      THETA4=THETA4*PI
      N1=N-1
      AN1=N1
      THETA5=THETA4-THETA3
      DELTT=THETA5/AN1
      THETA(1)=THETA3
      DO 47 J=1,N1
      J1=J+1
47   THETA(J1)=THETA(J)+DELT
C
C - - (3) GENERATE THE THOMSON FUNCTIONS
C
84  DO 5 I=1,M
      X=C*R(I)
      X1=1./X
      IF(X-6.0)2,2,3
2    LONG=10
      GO TO 4
3    LONG=16
4    BBER(I)=BER(X,LONG)
      BBEI(I)=BEI(X,LONG)
      BBRP(I)=BRP(X,LONG)
      BBIP(I)=BIP(X,LONG)
      AAER(I)=AKER(X,LONG)
      AAKEI(I)=AKEI(X,LONG)
      AAERP(I)=AKERP(X,LONG)

```

TABLE V (Continued)

```

AAKEIP(I)=AKEIP(X,LONG)
SBER(I)=-BBEI(I)-X1*BBRP(I)
SBEI(I)=BBER(I)-X1*BBIP(I)
SKER(I)=-AAKEI(I)-X1*AAKER(I)
SKEI(I)=AAKER(I)-X1*AAKEIP(I)
TBER(I)=BBEI(I)/X-BBIP(I)+2.*BBRP(I)/X**2
TBEI(I)=BBER(I)-BBER(I)/X+2.*BBIP(I)/X**2
TKER(I)=AAKEI(I)/X-AAKEIP(I)+2.*AAKER(I)/X**2
5 TKEI(I)=AAKER(I)-AAKER(I)/X+2.*AAKEIP(I)/X**2
C
C - - (4) CALCULATE THE CONSTANTS OF INTEGRATION FOR THE AXI-SYMMETRIC
C DEFORMATIONS
C
IF(NORM)32,32,31
31 CNO=1.
CNO1=1.
CNO2=1.
CNO3=1.
CNO8=1.
GO TO 202
32 CNO=F2*RHO*H
CNO1=E/(F2**2*RHO)
CNO2=F2*RHO*H**2
CNO3=RHO*H*SQRTF(F2*H)
CNO8=RHO*F2**2/(E*SQRTF(F2*H))
202 Z=R3
X=C*Z
X1=1./X
X2=C/Z
L=1
MM=1
IF(K1-2)29,28,85
85 IF(K1-3)28,28,34
26 Z=R4
X=C*Z
X1=1./X
X2=C/Z
MM=M
IF(K2-2)29,28,28
C W(R)=0.
28 B(L,1)=BBER(MM)
B(L,2)=BBEI(MM)
B(L,3)=AAKER(MM)
B(L,4)=AAKEI(MM)
B(L,5)=1.
B(L,6)=0.
BB(L)=RHO*P1*Z**2*.5/E
L=L+1
C U(R)=0.
B(L,1)=X1*PP*BBIP(MM)
B(L,2)=-X1*PP*BBRP(MM)
B(L,3)=X1*PP*AAKEIP(MM)
B(L,4)=-X1*PP*AAKER(I/MM)
B(L,5)=1.
B(L,6)=-PP*X1**2
BB(L)=2.*F**2*RHO*PM/E
L=L+1
IF(L-3)203,203,56

```

TABLE V (Continued)

```

203 IF(K1=3)30,29,34
56 IF(K2=3)30,29,34
C   WP(R)=0.
30 B(L,1)=C*BBRP(MM)
B(L,2)=C*BBIP(MM)
B(L,3)=C*AAKERP(MM)
B(L,4)=C*AAKEIP(MM)
B(L,5)=0.
B(L,6)=0.
BB(L)=RHO*P1*Z/E
L=L+1
IF(L=4)26,26,36
C   MR(R)=0.
29 B(L,1)=-(C**2)*BBEI(MM)+PM*X2*BBRP(MM)
B(L,2)=C**2*BBER(MM)+PM*X2*BBIP(MM)
B(L,3)=-(C**2)*AAKEI(MM)+PM*X2*AAKERP(MM)
B(L,4)=C**2*AAKER(MM)+PM*X2*AAKEIP(MM)
B(L,5)=0.
B(L,6)=0.
BB(L)=RHO*H**3/(12.*G)
L=L+1
IF(L=4)35,26,73
73 IF(L=5)35,35,36
C   NR(R)=0.
35 B(L,1)=-X2*BBIP(MM)
B(L,2)=X2*BBRP(MM)
B(L,3)=-X2*AAKEIP(MM)
B(L,4)=X2*AAKERP(MM)
B(L,5)=0.
B(L,6)=1./Z**2
BB(L)=-F*RHO/(E*T)
L=L+1
C   QR(R)=0.
B(L,1)=-C**3*BBIP(MM)
B(L,2)=C**3*BBRP(MM)
B(L,3)=-C**3*AAKEIP(MM)
B(L,4)=C**3*AAKERP(MM)
B(L,5)=0.
B(L,6)=0.
BB(L)=0.
L=L+1
IF(L=4)26,26,36
36 DET=1.
MATR=XSIMEQF(6,6,1,B,BB,DET,B9)
GO TO (6,7,8),MATR
6 B1=B(1,1)
B2=B(2,1)
B3=B(3,1)
B4=B(4,1)
B5=B(5,1)
B6=B(6,1)
GO TO 42
34 X2=1./(C*R2)
X3=C/R2
C   W(R2)=0.
D(1,1)=BBER(M)
D(1,2)=BBEI(M)
D(1,3)=1.

```

TABLE V (Continued)

```

C DD(1)=RHO*P1*R2**2*.5/E
C U(R2)=0.
C D(2,1)=X2*PP*BBIP(M)
C D(2,2)=-X2*PP*BBRP(M)
C D(2,3)=1.
C DD(2)=2.*F**2*RHO*PM/E
C IF(K2=2)37,37,38
37 D(3,1)=C*BBRP(M)
D(3,2)=C*BBIP(M)
D(3,3)=0.
DD(3)=RHO*P1*R2/E
GO TO 43
C MR(R2)=0.
38 D(3,1)=-C**2*BBEI(M)+PM*X3*BBRP(M)
D(3,2)=C**2*BBER(M)+PM*X3*BBIP(M)
D(3,3)=0.
DD(3)=RHO*H**3/(12.*G)
43 DET=1.
MATR=XSIMEQF(3,3,1,D,DD,DET,D3)
GO TO (44,7,8),MATR
44 B1=D(1,1)
B2=D(2,1)
B3=0.
B4=0.
B5=D(3,1)
B6=0.
C
C - - (5) GENERATE THE DEPENDENT VARIABLES FOR THE AXI-SYMMETRIC DEFORMA
C TIONS
C
42 DO 9 I=1,M
X=C*R(I)
X1=1./X
P(I)=B1*BBER(I)+B2*BBEI(I)+B3*AAKER(I)+B4*AAKEI(I)+B5-RHO*P1
1*R(I)**2*.5/E
PWB(I)=(P(I)-B5)*CNO1
P(I)=P(I)*CNO1
W1(I)=B1*C*BBRP(I)+B2*C*BBIP(I)+B3*C*AAKERP(I)+B4*C*AAKEIP(I)-RHO*
1P1*R(I)/E
W1(I)=W1(I)/CNO8
CMR1=B1*(-C**2*BBEI(I)+PM*C/R(I)*BBRP(I))+B2*(C**2*BBER(I)+PM*C/R(I)
1*I)*BBIP(I)
EMR(I)=-G*(CMR1+B3*(-C**2*AAKEI(I)+PM*C/R(I)*AAKERP(I))+B4*(C**2*A
1AKER(I)+PM*C/R(I)*AAKEIP(I)))+RHO*H**3/12.
EMR(I)=EMR(I)/CNO2
ENR(I)=E*T*H*(B2*C/R(I)*BBRP(I)-B1*C/R(I)*BBIP(I)-B3*C/R(I)*AAKEIP
1(I)+B4*C/R(I)*AAKERP(I)+B6/R(I)**2)+F*RHO*H
ENR(I)=ENR(I)/CNO
Q1(I)=-G*(-B1*C**3*BBIP(I)+B2*C**3*BBRP(I)-B3*C**3*AAKEIP(I)+B4*C*
1*3*AAKERP(I))
Q1(I)=Q1(I)/CNO3
CMT1=B1*(-PRT *C**2*BBEI(I)+P1*C/R(I)*BBRP(I))+B2*(PRT *C**2*B
1BER(I)+P1*C/R(I)*BBIP(I))
EMT(I)=-G*(CMT1+B3*(-PRT *C**2*AAKEI(I)+P1*C/R(I)*AAKERP(I))+B4*
1(PRT *C**2*AAKER(I)+P1*C/R(I)*AAKEIP(I)))+RHO*H**3/12.
EMT(I)=EMT(I)/CNO2
ENT1=B1*(-BBER(I)+X1*BBIP(I))-B2*(BBEI(I)+X1*BBRP(I))+B3*(-AAKER(I)
1)+X1*AAKEIP(I))

```

TABLE V (Continued)

```

ENT(I)=E*H*F1*(ENT1-B4*(AAKEI(I)+X1*AAKERP(I))-B6/X**2)+F*RHO*H
ENT(I)=ENT(I)/CNO
U1=B1*X1*BBIP(I)-B2*X1*BBRP(I)+B3*X1*AAKEIP(I)-B4*X1*AAKERP(I)+B5/
1PP-B6*X1**2
Q(I)=R(I)*F1*PP*U1+F*RHO*P1*R(I)/E
Q(I)=Q(I)*CNO1
9 CONTINUE
IF(K3)402,402,45
C
C - - (6) CALCULATE THE CONSTANTS OF INTEGRATION FOR THE ANTI-SYMMETRIC
C DEFORMATIONS
C
45 Z=R3
X=C*Z
X1=1./X
L=1
MM=1
IF(K1-2)54,53,87
48 Z=R4
X=C*Z
X1=1./X
MM=M
IF(K2-2)54,53,53
87 IF(K1-3)53,53,60
C
W(R1)=0.
53 A(L,1)=BBRP(MM)
A(L,2)=BBIP(MM)
A(L,3)=AAKERP(MM)
A(L,4)=AAKEIP(MM)
A(L,5)=X1
A(L,6)=X
A(L,7)=0.
A(L,8)=0.
S(L)=0.
L=L+1
U(R)=0.
A(L,1)=PP*SBEI(MM)
A(L,2)=-PP*SBER(MM)
A(L,3)=PP*SKEI(MM)
A(L,4)=-PP*SKER(MM)
A(L,5)=2.*LOGF(X)
A(L,6)=X**2*.5
A(L,7)=PP*X1**2
A(L,8)=1.
S(L)=.25*X*Z*F*RHO*(1.+5.*PRT)/E
L=L+1
V(R)=0.
A(L,1)=PP*X1*BBIP(MM)
A(L,2)=-PP*X1*BBRP(MM)
A(L,3)=PP*X1*AAKEIP(MM)
A(L,4)=-PP*X1*AAKERP(MM)
A(L,5)=2.*LOGF(X)
A(L,6)=-X**2*.5
A(L,7)=-PP*X1**2
A(L,8)=1.
S(L)=.25*X*Z*F*RHO*(11.+7.*PRT)/E
L=L+1
;F(L-4)57,57,58

```

TABLE V (Continued)

```

57 IF(K1-3)55,54,60
58 IF(K2-3)55,54,60
C   WP(R)=0.
55 A(L,1)=SBER(MM)
A(L,2)=SBEI(MM)
A(L,3)=SKER(MM)
A(L,4)=SKEI(MM)
A(L,5)=-X1**2
A(L,6)=1.
A(L,7)=0.
A(L,8)=0.
S(L)=0.
L=L+1
IF(L-5)48,48,61
C   MR(R)=0.
54 A(L,1)=TBER(MM)+PRT      *X1*(SBER(MM)-X1*BBRP(MM))
A(L,2)=TBEI(MM)+PRT      *X1*(SBEI(MM)-X1*BBIP(MM))
A(L,3)=TKER(MM)+PRT      *X1*(SKER(MM)-X1*AAKER(MM))
A(L,4)=TKEI(MM)+PRT      *X1*(SKEI(MM)-X1*AAKEIP(MM))
A(L,5)=P1*X1**3
A(L,6)=0.
A(L,7)=0.
A(L,8)=0.
S(L)=0.
L=L+1
IF(L-5)62,48,63
63 IF(L-6)62,62,61
C   NR(R)=0.
62 A(L,1)=BBER(MM)-2.*X1*BBIP(MM)
A(L,2)=BBEI(MM)+2.*X1*BBRP(MM)
A(L,3)=AAKER(MM)-2.*X1*AAKEIP(MM)
A(L,4)=AAKEI(MM)+2.*X1*AAKER(MM)
A(L,5)=-1./PP
A(L,6)=0.
A(L,7)=2.*X1**2
A(L,8)=0.
S(L)=-X*Z*RHO*F*.5/E
L=L+1
C   QR(R)+1./ALPHA2*DMRT/DTHETA=0.
A(L,1)=G*C*SBEI(MM)-(G*P1*X1/Z)*(BBEI(MM)+2.*X1*BBRP(MM))
A(L,2)=-G*C*SBER(MM)+(G*P1*X1/Z)*(BBER(MM)-2.*X1*BBIP(MM))
A(L,3)=G*C*SKEI(MM)-(G*P1*X1/Z)*(AAKEI(MM)+2.*X1*AAKER(MM))
A(L,4)=-G*C*SKEI(MM)+(G*P1*X1/Z)*(AAKER(MM)-2.*X1*AAKEIP(MM))
A(L,5)=-PM*G*C*2*F1*X1/(SQRTF(1.+(Z*F1)**2))**3*2.*X1**2
A(L,6)=0.
A(L,7)=0.
A(L,8)=0.
S(L)=0.
L=L+1
C   NRT(R)=0.
A(L,1)=BBER(MM)-2.*X1*BBIP(MM)
A(L,2)=BBEI(MM)+2.*X1*BBRP(MM)
A(L,3)=AAKER(MM)-2.*X1*AAKEIP(MM)
A(L,4)=AAKEI(MM)+2.*X1*AAKER(MM)
A(L,5)=1./PP
A(L,6)=0.
A(L,7)=2.*X1**2
A(L,8)=0.

```

TABLE V (Continued)

```

S(L)=1.5*RHO*F*X*Z/E
L=L+1
IF(L=5)48,48,61
61 DET=1.
MATR=XSIMEQF(8,8,1,A,S,DET,Z3)
GO TO (206,7,8),MATR
206 A1=A(1,1)
A2=A(2,1)
A3=A(3,1)
A4=A(4,1)
A5=A(5,1)
A6=A(6,1)
A7=A(7,1)
A8=A(8,1)
GO TO 68
60 IF(K2=2)64,64,65
C WP(R2)=0.
64 AA(1,1)=SBER(M)
AA(1,2)=SBEI(M)
AA(1,3)=1.
AA(1,4)=0.
SS(1)=0.
GO TO 66
C MR(R2)=0.
65 AA(1,1)=TBER(M)+PRT / (C*R2)*(SBER(M)-BBRP(M)/(C*R2))
AA(1,2)=TBEI(M)+PRT / (C*R2)*(SBEI(M)-BBIP(M)/(C*R2))
AA(1,3)=0.
AA(1,4)=0.
SS(1)=0.
C W(R2)=0.
66 AA(2,1)=BBRP(M)
AA(2,2)=BBIP(M)
AA(2,3)=C*R2
AA(2,4)=0.
SS(2)=0.
V(R2)=0.
AA(3,1)=PP*BBIP(M)/(C*R2)
AA(3,2)=-PP*BBRP(M)/(C*R2)
AA(3,3)=-.5*(C*R2)**2
AA(3,4)=1.
SS(3)=-.25*C*R2**2*F*RHO*(11.+7.*PRT)/E
C U(R2)=0.
AA(4,1)=PP*SBEI(M)
AA(4,2)=-PP*SBER(M)
AA(4,3)=.5*(C*R2)**2
AA(4,4)=1.
SS(4)=-.25*C*R2**2*F*RHO*(1.+5.*PRT)/E
DET=1.
MATR=XSIMEQF(4,4,1,AA,SS,DET,Z2)
GO TO (67,7,8),MATR
67 A1=AA(1,1)
A2=AA(2,1)
A3=0.
A4=0.
A5=0.
A6=AA(3,1)
A7=0.
A8=AA(4,1)

```

TABLE V (Continued)

```

      GO TO 68
7 WRITE OUTPUT TAPE 3,120
      GO TO 99
8 WRITE OUTPUT TAPE 3,121
      GO TO 99
C
C - - (7) GENERATE THE DEPENDENT VARIABLES FOR THE ANTI-SYMMETRIC DEFORM
C   ATIONS
C
68 DO 71 I=1,M
  X=R(I)*C
  X1=1./X
  WW(I)=A1*BBRP(I)+A2*BBIP(I)+A3*AAKER(I)+A4*AAKEIP(I)+A5*X1+A6*X
  WA5(I)=(WW(I)-A6*X)*CNO1
  WW(I)=WW(I)*CNO1
  W3(I)=C*(A1*SBER(I)+A2*SBEI(I)+A3*SKER(I)+A4*SKEI(I)-A5*X1**2+A6)
  W3(I)=W3(I)/CNO8
  DMR1=A1*(TBER(I)+PRT *X1*(SBER(I)-X1*BBRP(I)))+A2*(TBEI(I)+PRT
  1 *X1*(SBEI(I)-X1*BBIP(I)))+A3*(TKER(I)+PRT *X1*(SKER(I)-X1*AAK
  2ERP(I)))
  DMR(I)=-G*C**2*(DMR1+A4*(TKEI(I)+PRT *X1*(SKEI(I)-X1*AAKEIP(I))
  1)+A5*(P1*X1**3))
  DMR(I)=DMR(I)/CNO2
  DMT1=A1*(PRT *TBER(I)+X1*(SBER(I)-X1*BBRP(I)))+A2*(PRT *TBEI
  1(I)+X1*(SBEI(I)-X1*BBIP(I)))+A3*(PRT *TKER(I)+X1*(SKER(I)-X1*
  2AAKER(I)))
  DMT(I)=-G*C**2*(DMT1+A4*(PRT *TKEI(I)+X1*(SKEI(I)-X1*AAKEIP(I))
  1-A5*(P1*X1**3))
  DMT(I)=DMT(I)/CNO2
  DMRT1=-A1*(BBER(I)+2.*X1*BBRP(I))+A2*(BBER(I)-2.*X1*BBIP(I))-A3*(
  1AAKEI(I)+2.*X1*AAKER(I))
  DMRT(I)=PM*G*C**2*X1*(DMRT1+A4*(AAKER(I)-2.*X1*AAKEIP(I))-A5*(2.*
  1X1**2))
  DMRT(I)=DMRT(I)/CNO2
  QQR(I)=-G*C**3*(-A1*SBEI(I)+A2*SBER(I)-A3*SKEI(I)+A4*SKER(I))
  QQR(I)=QQR(I)/CNO3
  QQT(I)=-G*C**3*X1*(-A1*BBIP(I)+A2*BBRP(I)-A3*AAKEIP(I)+A4*AAKER
  1(I))
  QQT(I)=QQT(I)/CNO3
  DNR1=A1*(BBER(I)-2.*X1*BBIP(I))+A2*(BBER(I)+2.*X1*BBRP(I))+A3*(AA
  1KER(I)-2.*X1*AAKEIP(I))+A4*(AAKEI(I)+2.*X1*AAKER(I))
  DNR(I)=-E*H*F1*X1*(DNR1-A5/PP+A7*(2.*X1**2))-RHO*H*R(I)/4.
  DNR(I)=DNR(I)/CNO
  DNT1=-A1*TBEI(I)+A2*TBER(I)-A3*TKEI(I)+A4*TKER(I)-A5/PP*X1+A7*(2.
  1*X1**3)
  DNT(I)=E*H*F1*DNT1+5.*RHO*H*R(I)/4.
  DNT(I)=DNT(I)/CNO
  DNRT(I)=E*H*F1*X1*(DNR1+A5/PP+A7*(2.*X1**2))-3.*RHO*H*R(I)/4.
  DNRT(I)=DNRT(I)/CNO
  U1=-PP*(-A1*SBEI(I)+A2*SBER(I)-A3*SKEI(I)+A4*SKER(I))+A5*(2.*LOGF
  1(X))+A6*X**2*.5+A7*(PP*X1**2)+A8
  UU(I)=F1/C*U1-RHO*(1.+5.*PRT)*R(I)**2/(8.*E)
  UU(I)=UU(I)/CNO1
  V1=-PP*X1*(-A1*BBIP(I)+A2*BBRP(I)-A3*AAKEIP(I)+A4*AAKER(I))+A5*(
  12.*LOGF(X))-A6*(.5*X**2)-A7*(PP*X1**2)+A8
  VV(I)=F1/C*V1-RHO*(11.+7.*PRT)*R(I)**2/(8.*E)
  71 VV(I)=VV(I)/CNO1
C

```

TABLE V (Continued)

```

C -- (8) INPUT II (CONTROL,LOAD)
C
C 402 READ INPUT TAPE 2,404,PSI,MGOTO
C
C -- (9) OUTPUT I (CONTROL,GEOMETRY,MATERIAL,EDGE CONSTRAINTS)
C
K3=K3-1
IF(NORM)184,184,183
183 WRITE OUTPUT TAPE 3,118
GO TO 185
184 WRITE OUTPUT TAPE 3,119
185 IF(K1-2)13,14,15
13 WRITE OUTPUT TAPE 3,129
GO TO 16
14 WRITE OUTPUT TAPE 3,130
GO TO 16
15 IF(K1-4)17,22,22
17 WRITE OUTPUT TAPE 3,131
GO TO 16
22 WRITE OUTPUT TAPE 3,136
16 IF(K2-2)18,19,21
18 WRITE OUTPUT TAPE 3,134
GO TO 33
19 WRITE OUTPUT TAPE 3,132
GO TO 33
21 WRITE OUTPUT TAPE 3,133
33 WRITE OUTPUT TAPE 3,128
WRITE OUTPUT TAPE 3,122,RHO,E,H,F,PRT
WRITE OUTPUT TAPE 3,11
TPSI=2.*PSI-1.
PSIDEG=PSI*RAD*PI
PSI=PSI*PI
T3DEG=THETA3*RAD
T4DEG=THETA4*RAD
AR=3./(SQRTF(1.+(R4/F2)**2)**3-SQRTF(1.+(R3/F2)**2)**3)
FM1=0.
FM2=0.
FM4=0.
RM1=0.
RM2=0.
RM4=0.
DO 3000 I=1,M
RM=(P(I)*COSF(PSI))**2+(WW(I)*SINF(PSI))**2/2.
RM1=RM1+RM*4.*RG(I)/SQRTF(1.+RG(I)**2)
FM=(PWB(I)*COSF(PSI))**2+(WA5(I)*SINF(PSI))**2/2.
3000 FM1=FM1+FM*4.*RG(I)/SQRTF(1.+RG(I)**2)
DO 3001 I=2,M1
RM=(P(I)*COSF(PSI))**2+(WW(I)*SINF(PSI))**2/2.
RM2=RM2+RM*4.*RG(I)/SQRTF(1.+RG(I)**2)
FM=(PWB(I)*COSF(PSI))**2+(WA5(I)*SINF(PSI))**2/2.
3001 FM2=FM2+FM*4.*RG(I)/SQRTF(1.+RG(I)**2)
DO 3002 I=2,M1,2
RM=(P(I)*COSF(PSI))**2+(WW(I)*SINF(PSI))**2/2.
RM4=RM4+RM*4.*RG(I)/SQRTF(1.+RG(I)**2)
FM=(PWB(I)*COSF(PSI))**2+(WA5(I)*SINF(PSI))**2/2.
3002 FM4=FM4+FM*4.*RG(I)/SQRTF(1.+RG(I)**2)
FMI=AR*DRB/3.*(FM1+FM2+2.*FM4)
RMS=AR*DRB/3.*(RM1+RM2+2.*RM4)

```

TABLE V (Continued)

```

FMI=SQRTF(FMI)
RMS=SQRTF(RMS)
WRITE OUTPUT TAPE 3,127
WRITE OUTPUT TAPE 3,123,R1,R2,R3,R4+PSIDEV
WRITE OUTPUT TAPE 3,11
WRITE OUTPUT TAPE 3,3003
WRITE OUTPUT TAPE 3,3004,RMS,FMI
WRITE OUTPUT TAPE 3,11
IF(MGOTO-2)186,487,487
487 IF(MGOTO-3)187,187,186
187 WRITE OUTPUT TAPE 3,111
WRITE OUTPUT TAPE 3,112,T3DEG,T4DEG
WRITE OUTPUT TAPE 3,11
186 IF(TPSI)188,189,188
188 WRITE OUTPUT TAPE 3,173
WRITE OUTPUT TAPE 3,174,B1,B2,B3,B4+B5,B6
WRITE OUTPUT TAPE 3,11
IF(PSI)190,190,189
189 WRITE OUTPUT TAPE 3,175
WRITE OUTPUT TAPE 3,176,A1,A2,A3,A4+A5,A6,A7,A8
WRITE OUTPUT TAPE 3,11
190 WRITE OUTPUT TAPE 3,10
C
C - - (10) OUTPUT II (DEPENDENT VARIABLES AT THE TABULAR POINTS)
C
IF(MGOTO-3)450,451,3005
3005 IF(K3)98,98,402
451 IF(TPSI)450,452,450
452 WRITE OUTPUT TAPE 3,140
DO 255 J=1,N
DO 255 I=1,M
W(I,J)=WW(I)*SINF(THETA(J))-P(I)
WFLEX(I,J)=WA5(I)*SINF(THETA(J))-PWB(I)
WP(I,J)=W3(I)*SINF(THETA(J))-W1(I)
U(I,J)=UU(I)*SINF(THETA(J))-Q(I)
255 V(I,J)=VV(I)*COSF(THETA(J))
SKIP=1
GO TO 94
450 IF(PSI)96,96,23
23 IF(TPSI)403,95,403
403 DO 405 I=1,M
P2(I)=P(I)*COSF(PSI)
PWB2(I)=PWB(I)*COSF(PSI)
W2(I)=W1(I)*COSF(PSI)
FMR(I)=EMR(I)*COSF(PSI)
FNR(I)=ENR(I)*COSF(PSI)
Q2(I)=Q1(I)*COSF(PSI)
FMT(I)=EMT(I)*COSF(PSI)
FNT(I)=ENT(I)*COSF(PSI)
Q3(I)=Q(I)*COSF(PSI)
W4(I)=WW(I)*SINF(PSI)
WA54(I)=WA5(I)*SINF(PSI)
W5(I)=W3(I)*SINF(PSI)
GMR(I)=DMR(I)*SINF(PSI)
GMT(I)=DMT(I)*SINF(PSI)
GMRT(I)=DMRT(I)*SINF(PSI)
QRR(I)=QQR(I)*SINF(PSI)
QTT(I)=QQT(I)*SINF(PSI)

```

TABLE V (Continued)

```

GNR(I)=DNR(I)*SINF(PSI)
GNT(I)=DNT(I)*SINF(PSI)
GNRT(I)=DNRT(I)*SINF(PSI)
U2(I)=UU(I)*SINF(PSI)
WT(I)=W4(I)+P2(I)-P(I)
WFLEXT(I)=WA54(I)+PWB2(I)-PWB(I)
WPT(I)=W5(I)+W2(I)-W1(I)
UT(I)=U2(I)+Q3(I)-Q(I)
405 V2(I)=VV(I)*SINF(PSI)
IF(MGOTO-1)96,796,69
69 DO 70 J=1,N
DO 70 I=1,M
W(I,J)=W4(I)*SINF(THETA(J))+P2(I)
WFLEX(I,J)=WA54(I)*SINF(THETA(J))+PWB2(I)
WP(I,J)=W5(I)*SINF(THETA(J))+WZI
CMR(I,J)=GMR(I)*SINF(THETA(J))+FMR(I)
CMT(I,J)=GMT(I)*SINF(THETA(J))+FMT(I)
CMRT(I,J)=GMRT(I)*COSF(THETA(J))
QR(I,J)=QRR(I)*SINF(THETA(J))+Q2(I)
QT(I,J)=QTT(I)*COSF(THETA(J))
CNR(I,J)=GNR(I)*SINF(THETA(J))+FNR(I)
CNT(I,J)=GNT(I)*SINF(THETA(J))+FNT(I)
CNRT(I,J)=GNRT(I)*COSF(THETA(J))
U(I,J)=U2(I)*SINF(THETA(J))+Q3(I)
70 V(I,J)=V2(I)*COSF(THETA(J))
SKIP=0
IF(MGOTO-2)194,94,215
215 DO 217 J=1,N
DO 217 I=1,M
U(I,J)=U(I,J)-Q(I)
W(I,J)=W(I,J)-P(I)
WFLEX(I,J)=WFLEX(I,J)-PWB(I)
217 WP(I,J)=WP(I,J)-W1(I)
WRITE OUTPUT TAPE 3,140
SKIP=1
GO TO 94
96 IF(PSI)230,230,231
231 WRITE OUTPUT TAPE 3,138
230 IF(NORM)240,240,241
240 WRITE OUTPUT TAPE 3,324
GO TO 242
241 WRITE OUTPUT TAPE 3,124
242 IF(PSI)520,520,521
520 WRITE OUTPUT TAPE 3,125,(Y(I),P(I),W1(I),ENR(I),ENT(I),I=1,M)
GO TO 522
521 WRITE OUTPUT TAPE 3,125,(Y(I),P2(I),W2(I),FNR(I),FNT(I),I=1,M)
522 WRITE OUTPUT TAPE 3,10
IF(PSI)232,232,233
233 WRITE OUTPUT TAPE 3,138
232 IF(NORM)243,243,244
243 WRITE OUTPUT TAPE 3,326
GO TO 245
244 WRITE OUTPUT TAPE 3,126
245 IF(PSI)523,523,524
523 WRITE OUTPUT TAPE 3,125,(Y(I),EMR(I),EMT(I),Q1(I),Q(I),I=1,M)
GO TO 525
524 WRITE OUTPUT TAPE 3,125,(Y(I),FMR(I),FMT(I),Q2(I),Q3(I),I=1,M)
525 WRITE OUTPUT TAPE 3,10

```

TABLE V (Continued)

```

    IF(PSI)900,900,901
901 WRITE OUTPUT TAPE 3,138
900 IF(NORM)902,902,903
902 WRITE OUTPUT TAPE 3,380
    GO TO 904
903 WRITE OUTPUT TAPE 3,480
904 IF(PSI)905,905,906
905 WRITE OUTPUT TAPE 3,925,(Y(I),PWB(I),I=1,M)
    GO TO 907
906 WRITE OUTPUT TAPE 3,925,(Y(I),PWB2(I),I=1,M)
907 WRITE OUTPUT TAPE 3,10
    IF(PSI)99,99,95
    95 IF(TPSI)234,235,235
234 WRITE OUTPUT TAPE 3,138
235 IF(NORM)246,246,247
246 WRITE OUTPUT TAPE 3,313
    GO TO 248
247 WRITE OUTPUT TAPE 3,113
248 IF(TPSI)527,526,526
526 WRITE OUTPUT TAPE 3,125,(Y(I),WW(I),W3(I),VV(I),UU(I),I=1,M)
    GO TO 528
527 WRITE OUTPUT TAPE 3,125,(Y(I),W4(I),W5(I),V2(I),U2(I),I=1,M)
528 WRITE OUTPUT TAPE 3,10
    IF(TPSI)236,237,237
236 WRITE OUTPUT TAPE 3,138
237 IF(NORM)249,249,250
249 WRITE OUTPUT TAPE 3,314
    GO TO 251
250 WRITE OUTPUT TAPE 3,114
251 IF(TPSI)530,529,529
529 WRITE OUTPUT TAPE 3,125,(Y(I),DMR(I),DMT(I),DMRT(I),QQR(I),I=1,M)
    GO TO 531
530 WRITE OUTPUT TAPE 3,125,(Y(I),GMR(I),GMT(I),GMRT(I),QRR(I),I=1,M)
531 WRITE OUTPUT TAPE 3,10
    IF(TPSI)238,239,239
238 WRITE OUTPUT TAPE 3,138
239 IF(NORM)252,252,253
252 WRITE OUTPUT TAPE 3,315
    GO TO 254
253 WRITE OUTPUT TAPE 3,115
254 IF(TPSI)533,532,532
532 WRITE OUTPUT TAPE 3,125,(Y(I),DNR(I),DNT(I),DNRT(I),QQT(I),I=1,M)
    GO TO 534
533 WRITE OUTPUT TAPE 3,125,(Y(I),GNR(I),GNT(I),GNRT(I),QTT(I),I=1,M)
534 WRITE OUTPUT TAPE 3,10
    IF(TPSI)908,909,909
908 WRITE OUTPUT TAPE 3,138
909 IF(NORM)910,910,911
910 WRITE OUTPUT TAPE 3,380
    GO TO 912
911 WRITE OUTPUT TAPE 3,480
912 IF(TPSI)914,913,913
913 WRITE OUTPUT TAPE 3,925,(Y(I),WA5(I),I=1,M)
    GO TO 915
914 WRITE OUTPUT TAPE 3,925,(Y(I),WA54(I),I=1,M)
915 WRITE OUTPUT TAPE 3,10
    IF(K3)98,98,402
796 WRITE OUTPUT TAPE 3,140

```

TABLE V (Continued)

```

    IF(NORM)797,797,798
797 WRITE OUTPUT TAPE 3,327
    GO TO 799
798 WRITE OUTPUT TAPE 3,328
799 WRITE OUTPUT TAPE 3,329,(Y(I),WT(I),WFLEXT(I),WPT(I),UT(I),V2(I),
   I=1,M)
    WRITE OUTPUT TAPE 3,10
    GO TO 99
94 DO 160 J=1,N
160 THETR(J)=THETA(J)*RAD
    IF(NORM)601,601,604
601 IF(SKIP)602,602,603
602 WRITE OUTPUT TAPE 3,361
    GO TO 607
603 WRITE OUTPUT TAPE 3,461
    GO TO 607
604 IF(SKIP)605,605,606
605 WRITE OUTPUT TAPE 3,161
    GO TO 607
606 WRITE OUTPUT TAPE 3,261
607 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(W(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)608,608,611
608 IF(SKIP)609,609,610
609 WRITE OUTPUT TAPE 3,362
    GO TO 614
610 WRITE OUTPUT TAPE 3,462
    GO TO 614
611 IF(SKIP)612,612,613
612 WRITE OUTPUT TAPE 3,162
    GO TO 614
613 WRITE OUTPUT TAPE 3,262
614 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(WFLEX(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)615,615,618
615 IF(SKIP)616,616,617
616 WRITE OUTPUT TAPE 3,363
    GO TO 621
617 WRITE OUTPUT TAPE 3,463
    GO TO 621
618 IF(SKIP)619,619,620
619 WRITE OUTPUT TAPE 3,163
    GO TO 621
620 WRITE OUTPUT TAPE 3,263
621 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(WP(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)622,622,625
622 IF(SKIP)623,623,624
623 WRITE OUTPUT TAPE 3,364
    GO TO 628
624 WRITE OUTPUT TAPE 3,464
    GO TO 628

```

TABLE V (Continued)

```
625 IF(SKIP)626,626,627
626 WRITE OUTPUT TAPE 3,164
    GO TO 628
627 WRITE OUTPUT TAPE 3,264
628 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(U(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)629,629,632
629 IF(SKIP)630,630,631
630 WRITE OUTPUT TAPE 3,365
    GO TO 635
631 WRITE OUTPUT TAPE 3,465
    GO TO 635
632 IF(SKIP)633,633,634
633 WRITE OUTPUT TAPE 3,165
    GO TO 635
634 WRITE OUTPUT TAPE 3,265
635 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(V(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(SKIP)599,599,99
599 IF(NORM)267,267,268
267 WRITE OUTPUT TAPE 3,382
    GO TO 269
268 WRITE OUTPUT TAPE 3,582
269 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CMR(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)270,270,271
270 WRITE OUTPUT TAPE 3,366
    GO TO 272
271 WRITE OUTPUT TAPE 3,166
272 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CMT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)273,273,274
273 WRITE OUTPUT TAPE 3,367
    GO TO 275
274 WRITE OUTPUT TAPE 3,167
275 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CMRT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)276,276,277
276 WRITE OUTPUT TAPE 3,368
    GO TO 278
277 WRITE OUTPUT TAPE 3,168
278 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(QR(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)279,279,280
279 WRITE OUTPUT TAPE 3,369
    GO TO 281
```

TABLE V (Continued)

```

280 WRITE OUTPUT TAPE 3,169
281 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(QT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)282,282,283
282 WRITE OUTPUT TAPE 3,370
    GO TO 284
283 WRITE OUTPUT TAPE 3,170
284 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CNR(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)285,285,286
285 WRITE OUTPUT TAPE 3,371
    GO TO 287
286 WRITE OUTPUT TAPE 3,171
287 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CNT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)288,288,289
288 WRITE OUTPUT TAPE 3,372
    GO TO 290
289 WRITE OUTPUT TAPE 3,172
290 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CNRT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
500 IF(MGOTO-3)99,99,215
99 IF(K3)98,98,402
101 FORMAT(7I5)
102 FORMAT(3E20.8)
103 FORMAT(4F15.9)
10 FORMAT(1H1)
11 FORMAT(////)
111 FORMAT(51H           THETA3(DEG.)           THETA4(DEG.)////)
112 FORMAT(2F24.2)
113 FORMAT(1I6H           R(IN.)           W(IN.))
1     OMEGAR           V(IN.)           U(IN.)////)
114 FORMAT(12IH           R(IN.)           MR(IN.-LB./IN.))
1     MTHETA(IN.-LB./IN.)   MRTHTETA(IN.-LB./IN.)   QR(IN./LB.)
2////
115 FORMAT(12IH           R(IN.)           NR(IN./LB.))
1     NTHETA(IN./LB.)      NRTHETA(IN./LB.)      QTHETA(IN./LB.)
2////
118 FORMAT(99H A PARABOLOIDAL SHELL SUBJECTED TO GRAVITY--UN-NORMA
1LIZED RESULTS BY A SHALLOW SHELL ANALYSIS //++)
119 FORMAT(99H A PARABOLOIDAL SHELL SUBJECTED TO GRAVITY--NORMALIZ
1ED RESULTS BY A SHALLOW SHELL ANALYSIS //++)
120 FORMAT(30H UNDERFLOW OR OVERFLOW      //++)
121 FORMAT(30H THE MATRIX IS SINGULAR      //++)
122 FORMAT(5F24.5)
123 FORMAT(4F24.5,F18.2)
124 FORMAT(12IH           R(IN.)           W(IN.))
1     OMEGAR           NR(LB./IN.)      NTHETA(LB./IN.)
1////
125 FORMAT(F24.4,4F24.8)

```

TABLE V (Continued)

126 FORMAT(121H	R(IN.)	MR(IN.-LB./IN.)	
1   MTHETA(IN.-LB./IN.)	QR(LB./IN.)	U(IN.)	
1////)			
127 FORMAT(116H	R1(IN.)	R2(IN.)	
1   R3(IN.)	R4(IN.)	PSI(DEG.)////)	
128 FORMAT(120H	WEIGHT DENSITY(LB./IN.3)	YOUNGS MODULUS(LB./IN.	
12)   THICKNESS(IN.)	FOCAL LENGTH(IN.)	POISSONS RATIO/	
2///)			
129 FORMAT(51H	THE SHELL IS FREE AT R1	////)	
130 FORMAT(51H	THE SHELL IS CLAMPED AT R1	////)	
131 FORMAT(51H	THE SHELL IS SIMPLY SUPPORTED AT R1	////)	
132 FORMAT(51H	THE SHELL IS CLAMPED AT R2	////)	
133 FORMAT(51H	THE SHELL IS SIMPLY SUPPORTED AT R2	////)	
134 FORMAT(51H	THE SHELL IS FREE AT R2	////)	
136 FORMAT(51H	THE SHELL IS CLOSED AT THE APEX	////)	
138 FORMAT(51H	THE PORTION OF THE RESULTS INDEPENDENT OF THETA////)		
140 FORMAT(95H	THE DISTORTION OF THE SHELL GIVEN BELOW IS MEASURED REL		
ATIVE TO THAT OF THE FACE-UP POSITION   ////)			
173 FORMAT(7X,2HB1,13X,2HB2,13X,2HB3,13X,2HB4,13X,2HB5,13X,2HB6,////)			
174 FORMAT(6E15.6)			
175 FORMAT (7X,2HA1,13X,2HA2,13X,2HA3,13X,2HA4,13X,2HA5,13X,2HA6,13X,			
12HA7,13X,2HA8,////)			
176 FORMAT(8E15.6)			
205 FORMAT(12A6)			
313 FORMAT(116H	GAMMA	W*	
1   OMEGAR*	V*	U*   ////)	
314 FORMAT(116H	GAMMA	MR*	
1   MTHETA*	MRTHETA*	QR*   ////)	
315 FORMAT(116H	GAMMA	NR*	
1   NTHETA*	NRTHETA*	QTHETA*////)	
324 FORMAT(116H	GAMMA	W*	
1   OMEGAR*	NR*	NTHETA*////)	
326 FORMAT(116H	GAMMA	MR*	
1   MTHETA*	QR*	U*   ////)	
327 FORMAT(121H	GAMMA	W TILDE*	WFLEX
1TILDE*   OMEGAR TILDE*	U TILDE*	V TILDE*	
2///)			
328 FORMAT(121H	R(IN.)	W TILDE(IN.)	WFLEX TI
1LDE(IN.)   OMEGAR TILDE(IN.)	U TILDE(IN.)	V TILDE(IN.)	
2///)			
329 FORMAT(F20.4,F20.8)			
380 FORMAT(51H	GAMMA	WFLEX*   ////)	
404 FORMAT(F15.9,I15)			
480 FORMAT(51H	R(IN.)	WFLEX(IN.) //++)	
925 FORMAT(F24.4,F24.8)			
3003 FORMAT(51H	EPSILON	EPSILONFLEX //++)	
3004 FORMAT(2F24.8)			
161 FORMAT(50H	----DISPLACEMENT W(IN.)----	////)	
162 FORMAT(50H	----DISPLACEMENT WFLEX(IN.)	////)	
163 FORMAT(50H	----DISPLACEMENT OMEGAR----	////)	
164 FORMAT(50H	----DISPLACEMENT U(IN.)----	////)	
165 FORMAT(50H	----DISPLACEMENT V(IN.)----	////)	
582 FORMAT(60H	----BENDING MOMENT MR(IN.-LB./IN.)----		
1   //++)			
166 FORMAT(60H	----BENDING MOMENT MTHETA(IN.-LB./IN.)---		
1-   //++)			
167 FORMAT(60H	----BENDING MOMENT MRTHETA(IN.-LB./IN.)--		
1--   //++)			

TABLE V (Continued)

168 FORMAT(50H	----TRANSVERSE SHEAR QR(LB./IN.)-----//
169 FORMAT(60H	----TRANSVERSE SHEAR QTHETA(LB./IN.)----
1       //)	
170 FORMAT(60H	----STRESS RESULTANT NR(LB./IN.)----
1       //)	
171 FORMAT(60H	----STRESS RESULTANT NTHETA(LB./IN.)----
1       //)	
172 FORMAT(60H	----STRESS RESULTANT NRTHETA(LB./IN.)----
1       //)	
261 FORMAT(50H	----DISPLACEMENT W TILDE(IN.)---- //
262 FORMAT(50H	----DISPLACEMENT WFLEX TILDE(IN.)---- //
263 FORMAT(50H	----DISPLACEMENT OMEGAR TILDE---- //
264 FORMAT(50H	----DISPLACEMENT U TILDE(IN.)---- //
265 FORMAT(50H	----DISPLACEMENT V TILDE(IN.)---- //
361 FORMAT(75H	----NORMALIZED DISPLACEMENT W*(DIMENSIONL
1ESS)----	//)
362 FORMAT(75H	----NORMALIZED DISPLACEMENT WFLEX*(DIMEN
1SIONLESS)---	//)
363 FORMAT(75H	----NORMALIZED DISPLACEMENT OMEGAR*(DIMEN
1SIONLESS)	//)
364 FORMAT(75H	----NORMALIZED DISPLACEMENT U*(DIMENSIONL
1ESS)----	//)
365 FORMAT(75H	----NORMALIZED DISPLACEMENT V*(DIMENSIONL
1ESS)----	//)
382 FORMAT(75H	----NORMALIZED BENDING MOMENT MR*(DIMENSI
1ONLESS)---	//)
366 FORMAT(75H	----NORMALIZED BENDING MOMENT MTHETA*(DIM
1ENSIONLESS)----	//)
367 FORMAT(75H	----NORMALIZED BENDING MOMENT MRTHETA*(DI
1MENSIONLESS)----	//)
368 FORMAT(75H	----NORMALIZED TRANSVERSE SHEAR QR*(DIMEN
1ISIONLESS)----	//)
369 FORMAT(75H	----NORMALIZED TRANSVERSE SHEAR QTHETA*(D
1IMENSIONLESS)----	//)
370 FORMAT(75H	----NORMALIZED STRESS RESULTANT NR*(DIMEN
1ISIONLESS)----	//)
371 FORMAT(75H	----NORMALIZED STRESS RESULTANT NTHETA*(D
1IMENSIONLESS)----	//)
372 FORMAT(75H	----NORMALIZED STRESS RESULTANT NRTHETA*(
1DIMENSIONLESS)----	//)
461 FORMAT(75H	----NORMALIZED DISPLACEMENT W TILDE*(DIMENSIO
1NLESS)----	//)
462 FORMAT(75H	----NORMALIZED DISPLACEMENT WFLEX TILDE*(DIM
1ENSIONLESS)----	//)
463 FORMAT(75H	----NORMALIZED DISPLACEMENT OMEGAR TILDE*(DIM
1ENSIONLESS)----	//)
464 FORMAT(75H	----NORMALIZED DISPLACEMENT U TILDE*(DIMENSIO
1NLESS)----	//)
465 FORMAT(75H	----NORMALIZED DISPLACEMENT V TILDE*(DIMENSIO
1NLESS)----	//)
END	

\*     LABEL  
 CFCTRL  
 FUNCTION FCTRL(INTGER)

TABLE V (Continued)

```

D      FCTRL=1.0
D      IF (INTGER-1) 2,2,3
D      3 DO 4 N=2,INTGER
D      AN=N
D      4 FCTRL=FCTRL*AN
D      2 RETURN
D      END

*
*      LABEL
CBER
FUNCTION BER (X,LONG)
D      PI=3.141592653589793
D      IF(X-17.0)5,5,8
D      5 BER=1.0
DO 16 K=1,LONG
D      AK=K
D      TOPBER= (X/2.0)**(2.0*AK)
D      TERBER=((TOPBER/FCTRL(2*K))**2)*((-1.0)**K)
D      16 BER=BER+TERBER
GO TO 40
D      8 COEF1=SQRTF(PI/(2.*X))*EXP(-((X/SQRTF(2.)))
D      COEF2=EXP((X/SQRTF(2.))/SQRTF(2.*PI*X))
D      ALPHA=X/SQRTF(2.)-PI/8.
D      BETA=X/SQRTF(2.)+PI/8.
D      FIRST=-SINF(BETA)
D      SECOND=COSF(ALPHA)
D      PROD=1.
D      BER=COEF2*SECOND
DO 53 K=1,LONG
D      AK=K
D      CCPSS=COSF(PI*AK/4.)*COSF(ALPHA)+SINF(PI*AK/4.)*SINF(ALPHA)
D      PROD=PROD*(2.*AK-1.)**2
D      SCFM=PROD/FCTRL(K)
D      SCFM=SCFM/(8.*X)**(AK/2.)
D      SCFM=SCFM/(8.*X)**(AK/2.)
D      SUM2=(SCFM*CCPSS)*COEF2
IF(X-27.0)60,60,53
D      60 SCFM=SCFM*(-1.)**K
D      SCMCS=-SINF(PI*AK/4.)*COSF(BETA)-COSF(PI*AK/4.)*SINF(BETA)
D      BER=BER-COEF1*FIRST /PI
D      SUM1=      (SCFM*SCMCS)*COEF1/PI
D      53 BER=BER+SUM2-SUM1
40 RETURN
END

*
*      LABEL
CBEI
FUNCTION BEI (X,LONG)
D      PI=3.141592653589793
D      IF(X-17.0)5,5,8
D      5 BEI=0.0
DO 20 K=1,LONG

```

TABLE V (Continued)

```

D      AK=K
D      TPABEI=(X/2.0)**(2.0*AK)
D      TPBBei=(X/2.0)**(2.0*AK-2.0)
D      TERBEI=(TPABEI/FCTRL(2*K-1))*(TPBBei/FCTRL(2*K-1))*((-1.0)**(K-1))
D 20 BEI=BEI+TERBEI
    GO TO 40
D 8 COEF1=SQRTF(PI/(2.*X))*EXP(-((X/SQRTF(2.)))
D COEF2=EXP(X/SQRTF(2.))/SQRTF(2.*PI*X)
D ALPHA=X/SQRTF(2.)-PI/8.
D BETA=X/SQRTF(2.)+PI/8.
D FIRST=COSF(BETA)
D SECOND=SINF(ALPHA)
D PROD=1.
D BEI=COEF2*SECOND
DO 51 K=1, LONG
D AK=K
D CSMSC=COSF(PI*AK/4.)*SINF(ALPHA)-SINF(PI*AK/4.)*COSF(ALPHA)
D PROD=PROD*(2.*AK-1.)*2
D SCFM=PROD/FCTRL(K)
D SCFM=SCFM/(8.*X)**(AK/2.)
D SCFM=SCFM/(8.*X)**(AK/2.)
D SUM2=(SCFM*CSMSC)*COEF2
IF(X-27.0)60,60,51
D 60 SCFM=SCFM*(-1.)*2
D CCMSS=COSF(PI*AK/4.)*COSF(BETA)-SINF(PI*AK/4.)*SINF(BETA)
D SUM1=(SCFM*CCMSS)*COEF1/PI
D BEI=BEI+COEF1*FIRST/PI
D 51 BEI=BEI+SUM1+SUM2
40 RETURN
END

```

```

*      LABEL
CAKER
FUNCTION AKER (X, LONG)
D PI=3.141592653589793
D GL2MIG=.1159315156584124
IF(X-8.89)5,5,8
D 5 SAME=GL2MIG-LOGF(X)
D PI4TH= PI/4.0
D AKER=SAME
DO 25 K=1, LONG
D AK=K
D TOPBER=(X/2.0)**(2.0*AK)
D TERBER=((TOPBER/FCTRL(2*K))**2)*((-1.0)**K)
D TPABEI=(X/2.0)**(2.0*AK)
D TPBBei=(X/2.0)**(2.0*AK-2.0)
D TERBEI=(TPABEI/FCTRL(2*K-1))*(TPBBei/FCTRL(2*K-1))*((-1.0)**(K-1))
D TERM1=SAME*TERBER
D TERM2=PI4TH*TERBEI
I=K-1
IF (I) 7,6,7
D 6 SUM1=1.5
GO TO 11
D 7 PART=0.0
57 DO 10 L=I,K

```

TABLE V (Continued)

```

D      AL=L
D  10 PART=PART+(1.0/(AK+AL))
D 110 SUM1=SUM1+PART
D  11 TOP3=((X/2.0)**(2.0*AK))
D 24 TERM3=((TOP3/FCTRL(2*K))**2)*SUM1*((-1.0)**K)
D  25 AKER=AKER+TERM1+TERM2+TERM3
      GO TO 40
D  8 COEFL=SQRTF(PI/(2.*X))*EXP(-((X/SQRTF(2.)))
D  ALPHA=X/SQRTF(2.)-PI/8.
D  BETA=X/SQRTF(2.)+PI/8.
D  FIRST=COSF(BETA)
D  PROD=1.
D  AKER=COEFL*FIRST
DO 50 K=1,LONG
D  AK=K
D  PROD=PROD*((2.*AK-1.)**2)
D  CCMSS=COSF(PI*AK/4.)*COSF(BETA)-SINF(PI*AK/4.)*SINF(BETA)
D  SCFM=PROD/FCTRL(K)
D  SCFM=SCFM/(8.*X)**(AK/2.)
D  SCFM=SCFM/(8.*X)**(AK/2.)*((-1.)**K)
D  SUM1=(SCFM*CCMSS)*COEFL
D  50 AKER=AKER+SUM1
40 RETURN
END

```

```

*      LABEL
CAKEI
      FUNCTION AKEI (X,LONG)
D  GL2MIG=.1159315156584124
D  PI=3.141592653589793
D  IF(X-8.89)5,5,8
D  5 SAME=GL2MIG-LOGF(X)
D  PI4TH=PI/4.0
D  AKEI=-PI4TH
DO 25 K=1,LONG
D  AK=K
D  TOPBER= (X/2.0)**(2.0*AK)
D  TERBER=((TOPBER/FCTRL(2*K))**2)*((-1.0)**K)
D  TPABEI=(X/2.0)**(2.0*AK)
D  TPBDEI=(X/2.0)**(2.0*AK-2.0)
D  TERBEI=(TPABEI/FCTRL(2*K-1))*(TPBDEI/FCTRL(2*K-1))*((-1.0)**(K-1))
D  TERM1=SAME*TERBEI
D  TERM2=PI4TH*TERBER
I=K-1
IF (I) 86,86,88
D  86 SUM2=1.0
      GO TO 80
D  88 PART=0.0
DO 94 L=I,K
D  AL=L
D  94 PART=PART+(1.0/(AK+AL-1.0))
D  SUM2=SUM2+PART
D  80 TERM3=SUM2*((-1.0)**(K-1))*(((X/2.0)**(2.0*AK))/FCTRL(2*K-1))*(((X
D  Z/2.0)**(2.0*AK-2.0))/FCTRL(2*K-1))
D  25 AKEI=AKEI+TERM1-TERM2+TERM3

```

TABLE V (Continued)

```

      GO TO 40
D   8 COEF1=SQRTF(PI/(2.*X))*EXP(-X/SQRTF(2.))
D   ALPHA=X/SQRTF(2.)-PI/8.
D   BETA=X/SQRTF(2.)+PI/8.
D   FIRST=-SINF(BETA)
D   PROD=1.
D   AKEI=COEF1*FIRST
DO 52 K=1,LONG
D   AK=K
D   SCMCs=-SINF(PI*AK/4.)*COSF(BETA)-COSF(PI*AK/4.)*SINF(BETA)
D   PROD=PROD*((2.*AK-1.)**2)
D   SCFM=PROD/FCTRL(K)
D   SCFM=SCFM/(8.*X)**(AK/2.)
D   SCFM=SCFM/(8.*X)**(AK/2.)*((-1.)**K)
D   SUM1=(SCFM*SCMCs)*COEF1
D 52 AKEI=AKEI+SUM1
40 RETURN
END

*
*      LABEL
CBRP
      FUNCTION BRP (X,LONG)
D   PI=3.141592653589793
D   IF(X-17.0)5,5,8
D   5 BRP=0.0
DO 65 K=1,LONG
D   AK=K
D   PARTA=((X/2.0)**(2.0*AK))/FCTRL(2*K-1)
D   PARTB=((X/2.0)**(2.0*AK-1.0))/FCTRL(2*K)
D   TERBRP=PARTA*PARTB*(-1.0)**K
D   65 BRP=BRP+TERBRP
GO TO 40
D   8 COEF1=-(SQRTF(PI/(2.*X))*EXP(-X/SQRTF(2.)))
D   COEF2=EXP(-X/SQRTF(2.))/SQRTF(2.*PI*X)
D   ALPHA=X/SQRTF(2.)-PI/8.
D   BETA=X/SQRTF(2.)+PI/8.
D   FIRST=-SINF(ALPHA)
D   SECOND=COSF(BETA)
D   BRP=COEF2*SECOND
D   PROD=1.0
DO 55 K=1,LONG
D   AK=K
D   SSMCC=-SINF(PI*AK/4.)*SINF(BETA)-COSF(PI*AK/4.)*COSF(BETA)
D   J=K-1
D   AJ=J
D   IF(J)70,70,71
D   70 PROD=1.0
GO TO 72
D   71 PROD=PROD*(2.*AJ-1.)**2
D   72 SCFM=PROD*(4.*AK**2-1.)/FCTRL(K)
D   SCFM=SCFM/(8.*X)**(AK/2.)
D   SCFM=SCFM/(8.*X)**(AK/2.)
D   SUM2=SCFM*SSMCC*COEF2
IF(X-27.0)60,60,55
D   60 SCFM=SCFM*(-1.)**K

```

TABLE V (Continued)

```

D      SCPCS=SINF(PI*AK/4.)*COSF(ALPHA)+COSF(PI*AK/4.)*SINF(ALPHA)
D      SUM1=SCFM*SCPCS*FIRST/PI
D      BRP=BRP-COEF1*FIRST/PI
D  55  BRP=BRP-SUM1+SUM2
40  RETURN
END

*
*      LABEL
CBIP   FUNCTION BIP (X,LONG)
D      PI=3.141592653589753
D      IF(X-17.0)5,5,8
D  5  BIP=0.0
DO 68 K=1,LONG
D      AK=K
IF (K-1) 66,66,67
D  66  PARTC=(1.0/(X/2.0))/FCTRL(2*K-1)
D      PARTD=((X/2.0)**(2.0*AK))
GO TO 74
D  67  PARTC=((X/2.0)**(2.0*AK-3.0))/FCTRL(2*K-1)
D  73  PARTD=((X/2.0)**(2.0*AK))/FCTRL(2*K-2)
D  74  TERBIP=PARTC*PARTD*((-1.0)**(K-1))
D  68  BIP=BIP+TERBIP
GO TO 40
D  8  COEF1=-(SQRTF(PI/(2.*X))*EXP(-X/SQRTF(2.)))
D      COEF2=EXP(X/SQRTF(2.))/SQRTF(2.*PI*X)
D      ALPHA=X/SQRTF(2.)-PI/8.
D      BETA=X/SQRTF(2.)+PI/8.
D      FIRST=COSF(ALPHA)
D      SECOND=SINF(BETA)
D      BIP=COEF2*SECOND
D      PROD=1.0
DO 57 K=1,LONG
D      AK=K
SCMCS=SINF(PI*AK/4.)*COSF(BETA)-COSF(PI*AK/4.)*SINF(BETA)
D      J=K-1
D      AJ=J
IF(J)70,70,71
D  70  PROD=1.0
GO TO 72
D  71  PROD=PROD*(2.*AJ-1.)**2
D  72  SCFM=PROD*(4.*AK**2-1.)/FCTRL(K)
D      SCFM=SCFM/(8.*X)**(AK/2.)
D      SCFM=SCFM/(8.*X)**(AK/2.)
D      SUM2=SCFM*SCMCS*COEF2
IF(X-27.0)60,60,57
D  60  SCFM=SCFM*(-1.)**K
SSMCC=SINF(PI*AK/4.)*SINF(ALPHA)-COSF(PI*AK/4.)*COSF(ALPHA)
D      SUM1=SCFM*SSMCC*COEF1/PI
D      BIP=BIP+FIRST*COEF1/PI
D  57  BIP=BIP+SUM1+SUM2
40  RETURN
END

```

TABLE V (Continued)

```

*      LABEL
CAKERP
      FUNCTION AKERP(X,LONG)
D      GL2MIG=.1159315156584124
D      PI=3.141592653589793
      IF(X-8.89)5,5,8
D      5 SAME=GL2MIG-LOGF(X)
D      PI4TH=PI/4.0
D      CONST=1.0/X
D      AKERP=-CONST
      DO 25 K=1,LONG
      AK=K
      TOPBER= (X/2.0)**(2.0*AK)
      TERBER=((TOPBER/FCTRL(2*K))**2)*((-1.0)**K)
      PARTA=((X/2.0)**(2.0*AK))/FCTRL(2*K-1)
      PARTB=((X/2.0)**(2.0*AK-1.0))/FCTRL(2*K)
      TERBRP=PARTA*PARTB*(-1.0)**K
      IF (K-1) 66,66,67
D      66 PARTC=(1.0/(X/2.0))/FCTRL(2*K-1)
D      PARTD=((X/2.0)**(2.0*AK))
      GO TO 74
D      67 PARTC=((X/2.0)**(2.0*AK-3.0))/FCTRL(2*K-1)
D      73 PARTD=((X/2.0)**(2.0*AK))/FCTRL(2*K-2)
D      74 TERBIP=PARTC*PARTD*(-1.0)**(K-1)
      TERM1=SAME*TERBRP
      TERM2=CONST*TERBER
      TERM3=PI4TH*TERBIP
      I=K-1
      IF (I) 7,6,7
D      6 SUM1=1.5
      GO TO 11
D      7 PART=0.0
      57 DO 10 L=I,K
      AL=L
      10 PART=PART+(1.0/(AK+AL))
      110 SUM1=SUM1+PART
      11 TERM4=SUM1*(2.0*AK)*(((X/2.0)**(2.0*AK))/FCTRL(2*K))*(((X/2.0)**(2
      Z.0*AK-1.0))/FCTRL(2*K))*((-1.0)**K)
      25 AKERP=AKERP+TERM1-TERM2+TERM3+TERM4
      GO TO 40
D      8 COEF1=-(SQRTF(PI/(2.*X))*EXP(-X/SQRTF(2.)))
D      ALPHA=X/SQRTF(2.0)-PI/8.
D      BETA=X/SQRTF(2.0)+PI/8.
D      FIRST=COSF(ALPHA)
D      AKERP=COEF1*FIRST
D      PROD=1.0
      DO 54 K=1,LONG
      AK=K
      SSMCC=SINF(PI*AK/4.0)*SINF(ALPHA)-COSF(PI*AK/4.0)*COSF(ALPHA)
      J=K-1
      AJ=J
      IF(J)70,70,71
D      70 PROD=1.0
      GO TO 72
D      71 PROD=PROD*(2.*AJ-1.0)**2
D      72 SCFM=PROD*(4.*AK**2-1.0)/FCTRL(K)

```

TABLE V (Continued)

```

D      SCFM=SCFM/(8.*X)**(AK/2.)
D      SCFM=SCFM/(8.*X)**(AK/2.)*(-1.)*K
D      SUM1=SCFM*SSMCC*COEF1
D  54 AKERP=AKERP+SUM1
D  40 RETURN
D      END

*
*      LABEL
CAKEIP
FUNCTION AKEIP(X,LONG)
D      GL2MIG=.1159315156584124
D      PI=3.141592653589793
D      IF(X-15.5)5,5,8
D  5 SAME=GL2MIG-LOGF(X)
D      CONST=1.0/X
D      PI4TH=PI/4.0
D      AKEIP=0.0
DO 25 K=1,LONG
D      AK=K
D      TPABEI=(X/2.0)**(2.0*AK)
D      TPBBI=(X/2.0)**(2.0*AK-2.0)
D      TERBEI=(TPABEI/FCTRL(2*K-1))*(TPBBI/FCTRL(2*K-1))*((-1.0)**(K-1))
D      PARTA=(X/2.0)**(2.0*AK))/FCTRL(2*K-1)
D      PARTB=(X/2.0)**(2.0*AK-1.0))/FCTRL(2*K)
D      TERBRP=PARTA*PARTB*((-1.0)**K)
IF (K-1) 66,66,67
D  66 PARTC=(1.0/(X/2.0))/FCTRL(2*K-1)
D      PARTD=(X/2.0)**(2.0*AK))
GO TO 74
D  67 PARTC=(X/2.0)**(2.0*AK-3.0))/FCTRL(2*K-1)
D  72 PARTD=(X/2.0)**(2.0*AK))/FCTRL(2*K-2)
D  74 TERBIP=PARTC*PARTD*((-1.0)**(K-1))
D      TERM1=SAME*TERBIP
D      TERM2=CONST*TERBEI
D      TERM3=PI4TH*TERBRP
I=K-1
IF (I) 86,86,88
D  86 SUM2=1.0
GO TO 80
D  88 PART=0.0
DO 94 L=I,K
D      AL=L
D      PART=PART+(1.0/(AK+AL-1.0))
D      SUM2=SUM2+PART
D  80 TERM4=SUM2*(2.0*AK-1.0)*((-1.0)**(K-1))*(((X/2.0)**(2.0*AK))/FCTRL
D      Z(2*K-1))*(((X/2.0)**(2.0*AK-3.0))/FCTRL(2*K-1))
D  25 AKEIP=AKEIP+TERM1-TERM2-TERM3+TERM4
GO TO 40
D  8 COEF1=-(SQRTF(PI/(2.*X))*EXP(-X/SQRTF(2.)))
D      ALPHA=X/SQRTF(2.)-PI/8.
D      BETA=X/SQRTF(2.)+PI/8.
D      FIRST=-SINF(ALPHA)
D      PROD=1.0
D      AKEIP=COEF1*FIRST
DO 56 K=1,LONG

```

TABLE V (Continued)

```
D    AK=K
D    SCPCS=SINF(PI*AK/4.)*COSF(ALPHA)+COSF(PI*AK/4.)*SINF(ALPHA)
D    AJ=J
D    J=K-1
D    IF(J)70,70,71
D 70 PROD=1.0
D    GO TO 72
D 71 PROD=PROD*(2.*AJ-1.)**2
D 72 SCFM=PROD*(4.*AK**2-1.)/FCTRL(K)
D    SCFM=SCFM/(8.*X)**(AK/2.)
D    SCFM=SCFM/(8.*X)**(AK/2.)*(-1.)**K
D    SUM1=SCFM*SCPCS*COEF1
D 56 AKEIP=AKEIP+SUM1
40 RETURN
END
```

## 1. Input

The input to our program as indicated by step (1) and step (8) in the master flow chart consists of a number of records prestored on machine tape A2.

### Record 1 Control Parameters I (4I5)

This card contains four non-negative fixed-point variables.

M	MU	ML	MC
---	----	----	----

These parameters control the number and spacing of the  $\gamma$ -tabular points between R3 and R4 (to be defined later under Record 4).

- M ( $\leq 50$ ) Total number of  $\gamma$ -tabular points between R3 and R4.
- MU Number of (evenly spaced) intervals in the upper edge zone  
(a region  $2(2fh)^{1/2}$  from R4).
- ML Number of (evenly spaced) intervals in the lower edge zone  
(a region  $2(2fh)^{1/2}$  from R3).
- MC A control parameter to be set equal to 0, 1, or 2.  
MC = 0  $\gamma$ -tabular points are evenly spaced.  
MC = 1 Tabular points are spaced differently (usually denser) only in the upper edge zone. In particular, there will be MU + 1 tabular points in this edge zone. This option is used primarily when the shell is closed at the apex so that no edge effect appears in the lower edge zone.  
MC = 2 Tabular points are spaced differently (usually denser) in both edge zones. There are MU + 1 and ML + 1 tabular points in the upper and lower edge zones respectively.

### Record 2 Control Parameters II (5I5)

This card contains five non-negative fixed-point variables

N	NORM	K1	K2	K3
---	------	----	----	----

- N ( $\leq 10$ ) Number of equally spaced  $\Theta$ -tabular points between THETA3 and THETA4 (to be defined later under Record 5) along any fixed latitude of the shell. If only the superscripted quantities in (II-85) and (II-86) are desired, N must be set equal to zero and Records 6 and 7 omitted.
- NORM Control parameter specifying whether normalized results are desired.  
NORM = 0 Normalized results.  
NORM > 0 Un-normalized results.

K1	Control parameter specifying the edge constraints at R1 (to be defined later under Record 4). K1 = 1     Shell is free at R1. K1 = 2     Shell is clamped at R1. K1 = 3     Shell is simply supported at R1. K1 = 4     Shell is closed at the apex.
K2	Control parameter specifying the edge constraints at R2 (to be defined later under Record 4). K2 = 1     Shell is free at R2. K2 = 2     Shell is clamped at R2. K2 = 3     Shell is simply supported at R2.
K3	Different combinations of PS1 and MGOTO (see Record 8) may be used for each structure. If more than one such combination is to be run for the same structure, only Record 8 needs to be changed. K3 then specifies the total number of such combinations for the same structure. Set K3 = 0, if the only run wanted is for $\psi = 0$ .

#### Record 3    Material Parameters (3E20.8)

This card contains three E-type floating-point variables.

RHO	E	PRT
-----	---	-----

RHO	Volume weight density of the shell (lb/in. <sup>3</sup> ).
E	Young's modulus (lb/in. <sup>2</sup> ).
PRT	Poisson's ratio.

#### Record 4    Geometrical Parameters I (4F15.9).

This card contains four F-type floating-point variables.

R1	R2	R3	R4
----	----	----	----

R1 (in.)	Value of r at the lower edge of the shell. If the shell is closed at the apex, then R1 = 0.
R2 (in.)	Value of r at the upper edge of the shell.
R3 (in.)	Smallest value of r at which stresses and displacements are to be calculated (smallest $\gamma$ -tabular point). Generally, this is the same as R1. However, if the shell is closed at the apex, we set R3 > 0 to avoid possible complications arising from calculating the desired quantities at the apex.
R4 (in.)	Largest value of r at which the output is to be given (i.e., the largest $\gamma$ -tabular point). Generally, this is the same as R2.

Record 5 Geometrical Parameters II (4F15.9)

This card contains four F-type floating-point variables.

THETA3	THETA4	F	H
--------	--------	---	---

THETA3 Smallest value of  $\Theta$  in fractions of  $\pi$  at which output is desired (the smallest  $\Theta$ -tabular point).

THETA4 Largest  $\Theta$ -tabular point in fractions of  $\pi$  at which output is desired.

F Focal length of the paraboloidal surface (in.).

H Shell thickness (in.).

Record 6 Variable Format Statement I(12A6)

This card provides a format statement for the set of  $\Theta$ -tabular points, for example,

19H	R(IN.)/THETA(DEG.),	NF11.2
-----	---------------------	--------

Record 7 Variable Format Statement II (12A6)

This card provides a format statement for the dependent variables in the output, for example,

F19.6,	NF11.6
--------	--------

Record 8 Control Parameters III (F15.9, I5)

This card contains one F-type floating-point variable and one fixed-point variable.

PSI	MGOTO
-----	-------

PSI The pointing angle  $\psi$  in fractions of  $\pi$ .

MGOTO Control parameter specifying the type of output desired.

MGOTO = 0 Stresses and displacements for symmetric or antisymmetric deformations or the portion of the results independent of  $\Theta$  are given in a concise form.

MGOTO = 1 Only the zero-superscripted tilde quantities (and the corresponding  $\tilde{w}_{flex}^0$  displacement) are given.

MGOTO = 2 Usual set of structural quantities are given.

MGOTO = 3 Only the tilde displacement quantities (and the corresponding  $\tilde{w}_{flex}$  displacement) are given.

MGOTO = 4 Only the root-mean-square of the phase error\* is given.

\*To be discussed in a subsequent chapter.

## 2. Output

The output to the shallow shell program is arranged in a manner very similar to that of the membrane analysis. The first part of the output reproduces the input to the program. It states the problem, the edge constraints, the various geometrical and material parameters and whether the numerical results have been normalized. The second part of the output gives the value of the physical quantities for the net of tabular points. If  $\psi = 0$ , the behavior of the shell is axisymmetric. For this case, we have  $v = N_{r\Theta} = Q_\Theta = M_{r\Theta} = 0$ , while  $u, w, \omega_r, N_r, N_\Theta, M_r, M_\Theta$ , and  $Q_r$  are all functions of  $\gamma$  only. The output in this instance is given in a concise form as in the membrane program. For  $0 < \psi \leq 0.5$ , we have several options of output which are very similar to those of the membrane program:

- (1) List only the zero-superscripted quantities for the set of  $\gamma$ -tabular points with the displacement quantities being those relative to the undeformed shell (set MGOTO = 0).
- (2) List only the zero-superscripted displacement quantities  $\hat{u}$ ,  $\hat{v}$ , and  $\hat{w}$  relative to those of the face-up position (set MGOTO = 1).
- (3) List all the actual stresses and displacements for the entire net of tabular points with the displacement quantities being those relative to the undeformed shell (set MGOTO = 2).
- (4) List only the actual displacements for the net of tabular points relative to those of the face-up position (set MGOTO = 3).
- (5) List only the root-mean-square of the phase error\* (set MGOTO = 4).

## 3. Operational Information

The program is coded in FORTRAN language for a 32K IBM 7090 (and 7094) and is compiled by a FORTRAN II compiler. It requires 9 FORTRAN functions, which calculate the values of the Thomson functions and their first derivatives using double-precision arithmetic as described in IBM Bulletin J28-6114 "32K 709/7090 FORTRAN: Double-Precision and Complex Arithmetic." All other routines used appear as library routines on the Lincoln Laboratory library tape.

Some of the subscripted variables and their analytical counterparts are presented in Table VI.

It should be understood that if normalized results are requested, affected expressions in the third column of Table VI should be replaced by the corresponding starred quantities.

## C. EXAMPLES

### 1. Axisymmetric Deformations

Consider a shell in the face-up position ( $\psi = 0^\circ$ ) which is closed at the apex and simply supported at the upper edge  $r_2$  with

---

\*To be discussed in a subsequent chapter.

TABLE VI  
SOME SUBSCRIPTED VARIABLES  
AND THEIR ANALYTICAL COUNTERPARTS

Subscripted Variable	Dimension	Analytical Expression
W	50 × 10	w
WP	50 × 10	$\frac{\partial w}{\partial r}$
V	50 × 10	v
U	50 × 10	u
CMR	50 × 10	M <sub>r</sub>
CMT	50 × 10	M <sub>θ</sub>
CMRT	50 × 10	M <sub>rθ</sub>
CNR	50 × 10	N <sub>r</sub>
CNT	50 × 10	N <sub>θ</sub>
CNRT	50 × 10	N <sub>rθ</sub>
QR	50 × 10	Q <sub>r</sub>
QT	50 × 10	Q <sub>θ</sub>
WFLEX	50 × 10	w <sub>flex</sub>
P	50	w <sup>s</sup>
W1	50	( $\frac{\partial w}{\partial r}$ ) <sup>s</sup>
Q	50	u <sup>s</sup>
EMR	50	M <sub>r</sub> <sup>s</sup>
EMT	50	M <sub>θ</sub> <sup>s</sup>
ENR	50	N <sub>r</sub> <sup>s</sup>
ENT	50	N <sub>θ</sub> <sup>s</sup>
Q1	50	Q <sub>r</sub> <sup>s</sup>
PWB	50	w <sub>flex</sub> <sup>s</sup>
WW	50	w <sup>a</sup>
W3	50	( $\frac{\partial w}{\partial r}$ ) <sup>a</sup>
VV	50	v <sup>a</sup>
UU	50	u <sup>a</sup>
DMR	50	M <sub>r</sub> <sup>a</sup>

TABLE VI (Continued)

Subscripted Variable	Dimension	Analytical Expression
DMT	50	$M_{\Theta}^a$
DMRT	50	$M_{r\Theta}^a$
DNR	50	$N_r^a$
DNT	50	$N_{\Theta}^a$
DNRT	50	$N_{r\Theta}^a$
QQR	50	$Q_r^a$
QQT	50	$Q_{\Theta}^a$
WA5	50	$w_{flex}^a$
WT	50	$\tilde{w}$
WPT	50	$\frac{\partial \tilde{w}}{\partial r}$
UT	50	$\tilde{u}$
V2	50	$\tilde{v}$
WFLEXT	50	$\tilde{w}_{flex}$
R	50	r
THETA	20	$\Theta$
BBER	50	ber
BBEI	50	bei
AAKER	50	ker
AAKEI	50	kei
BBRP	50	ber'
BBIP	50	bei'
AAKERP	50	ker'
AAKEIP	50	kei'
SBER	50	ber"
SBEI	50	bei"
SKER	50	ker"
SKEI	50	kei"
TBER	50	ber'''
TBEI	50	bei'''
TKER	50	ker'''
TKEI	50	kei'''

$\rho = 0.1 \text{ lb/in.}^3$   
 $E = 10^7 \text{ lb/in.}^2$   
 $\nu = 0.3$   
 $r_1 = 0 \text{ in.}$   
 $r_2 = 300 \text{ in.}$   
 $f = 450 \text{ in.}$   
 $h = 1 \text{ in.}$

Since the deformation is axisymmetric, we set  $N = 0$  and omit Records 6 and 7. For the purpose of illustration, we consider twelve  $\gamma$ -tabular points with a finer grid in the upper edge zone ( $MC = 1$ ). As the shell is closed at the apex, we avoid calculating the limiting value of the output at the apex by setting  $R3 > 0$ . The results will not be normalized, so we get the actual physical quantities. Output will be the zero-superscripted quantities for the set of  $\gamma$ -tabular points, where the displacement quantities are those relative to the undeformed shell.

### INPUT

Record 1 Control Parameters I (415)

M	M	M	M
M	U	L	C
12	.6	0	1
0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0
1 2 3 4 5 6 7 8 9 10 11 12 13 14	15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30	31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46	47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62
1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6
7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8
9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9	9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9	9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9	9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
1 2 3 4 5 6 7 8 9 10 11 12 13 14	15 16 17 18 19 20 21 22 23 24 25 26 27 28 29	30 31 32 33 34 35 36 37 38 39 40 41 42 43 44	45 46 47 48 49 50 51 52 53 54 55 56 57 58 59
111111111111	111111111111	111111111111	111111111111







TABLE VII  
COMPUTER OUTPUT (AXISYMMETRIC DEFORMATIONS)

A PARABOLOIDAL SHELL SUBJECTED TO GRAVITY - - UN-NORMALIZED RESULTS BY A SHALLOW SHELL ANALYSIS

THE SHELL IS CLOSED AT THE APEX

THE SHELL IS SIMPLY SUPPORTED AT R2

WEIGHT OENSATY (LBS./IN.3)	YOUNG'S MODULUS (LBS./IN.2)	THICKNESS (IN.)	FOCAL LENGTH (IN.)	POISSONS RATIO
0.10000	10000000.00000	1.00000	450.00000	0.30000
R1 (IN.)	R2 (IN.)	R3 (IN.)	R4 (IN.)	PSI (OEG.)
0.	300.00000	45.00000	300.00000	0.
EPSILON	EPSILON OMFLX			
0.00574002	0.00205270			
0.1	0.2	0.3	0.4	0.5
0.91666E-07	-0.758887E-08	0.	0.	-0.299815E-02

TABLE VII (Continued)

R(IN.)	W(IN.)	OMEGAR	W(L8./IN.)	N(THETA(L8./IN.)
40.0000	-0.00300372	-0.00000026	44.99040995	45.00002670
80.0000	-0.00300108	-0.00000057	45.00001921	45.00002630
120.0000	-0.00300130	-0.00000072	45.00002352	45.00002252
160.0000	-0.00300160	-0.00000080	45.00002683	45.00002623
200.0000	-0.00300198	-0.00000040	45.00002720	45.00001631
240.0000	-0.00300228	-0.00000053	45.00002697	45.00001631
250.0000	-0.00300248	-0.00000050	45.00002680	47.37836547
260.0000	-0.00300267	-0.00000058	45.00002699	45.00001720
270.0000	-0.00300287	-0.00000081	45.00002720	45.00001730
280.0000	-0.00300306	-0.00000082	45.00002740	45.00001730
290.0000	-0.00300325	-0.00000082	45.00002760	45.00001730
300.0000	-0.00300343	-0.00000082	45.00002780	45.00001730
				45.00001631
R(IN.)	W(RIN.=L8./IN.)	N(THETA(RIN.=L8./IN.)	Q(RIN.)	U(RIN.)
40.0000	0.000000762	3.000054930	0.32021731	-0.000000724
80.0000	0.00002081	0.00051767	-0.00021790	-0.0000001451
120.0000	0.00001077	0.00053754	-0.00026682	-0.0000002170
160.0000	0.00001010	0.00092526	0.00105726	-0.0000002877
200.0000	0.00001045	0.00098863	0.00307810	-0.0000003500
240.0000	0.00001045	0.0012955520	-0.00050854	-0.000000475
250.0000	-1.00000048	-0.000000269	-0.0000005497	-0.0000005504
260.0000	-1.00000019	-0.000000198	-0.0000005910	-0.0000005910
270.0000	-2.00000039	-1.00000075478	-0.0000005907	-0.0000005907
280.0000	-3.00000032	-1.000000265966	-0.0000005340	-0.0000005340
290.0000	-3.00000015	-1.0000002389700	0.0000003497	0.0000003497
300.0000	-2.00000061	-0.0000000805	0.0000001642	0.0000001642
	-0.00000002	-0.0000000565	-0.00000002856	-0.00000002856
R(IN.)	W(FLEXIN.)			
40.0000	-0.000000557			
80.0000	-0.0000002202			
120.0000	-0.0000005015			
160.0000	-0.0000001604			
200.0000	-0.0000001616			
240.0000	2.00000000	-0.0000004669		
250.0000	2.00000000	-0.0000004527		
260.0000	2.00000000	-0.0000003468		
270.0000	2.00000000	-0.0000003430		
280.0000	2.00000000	0.0000004816		
290.0000	2.00000000	0.00000016754		
300.0000	2.00000000	0.00000029815		

TABLE VIII  
COMPUTER OUTPUT (UNSYMMETRIC DEFORMATIONS)

A PARABOLOIDAL SHELL SUBJECTED TO GRAVITY - NORMALIZED RESULTS BY A SHALLOW SHELL ANALYSIS

THE SHELL IS CLAMPED AT R1

THE SHELL IS FREE AT R2

WEIGHT DENSITY(lb./IN.3)	YOUNG'S MODULUS(lb./IN.2)	THICKNESS(IN.)	FOCAL LENGTH(IN.)	POISSONS RATIO
0.10000	1000000.00000	1.00000	500.00000	0.30000
R1(IN.)	R2(IN.)	R3(IN.)	R4(IN.)	PSI(DEG.)
100.00000	300.00000	100.00000	300.00000	30.00
EPISTOLON	EPISTOLONFLEX			
0.02560503	1.64701623			
THETA3(DEG.)	THETA4(DEG.)			
-90.00	90.00			
B1	B2	B3	B4	B5
-0.527099E-09	-0.799029E-10	0.418463E 01	0.050382E 01	-0.455703E-01
A1	A2	A3	A4	A5
-0.765727E-09	-0.490867E-09	-0.323470E-01	-0.289259E 01	0.336206E-01
A6	A7	A8		
0.352988E-02	0.192405E 01	-0.964529E 01		

TABLE VIII (Continued)

THE DISTORTION OF THE SHELL, GIVEN BELOW IS MEASURED RELATIVE TO THAT OF THE FACE-UP POSITION						
----NORMALIZED DISPLACEMENT W TILDE(DIMENSIONLESS)----						
GAMMA/THETA(DEC.)	-90.00	-60.00	-30.00	-0.	30.00	60.00
0.1000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.1500	1.826221	1.658665	1.209895	0.575569	-0.09758	-0.50728
0.2000	2.52249	2.268985	1.573825	0.625557	-0.35591	-1.220878
0.2500	3.031448	2.787484	1.822198	0.612748	-0.566683	-1.041949
0.3000	3.561562	3.166764	2.088154	0.614746	-0.586653	-1.91772
----NORMALIZED DISPLACEMENT W TILDE(DIMENSIONLESS)----						
GAMMA/THETA(DEC.)	-90.00	-60.00	-30.00	-0.	30.00	60.00
0.1000	-1.625186	-1.459178	-1.117816	-0.610526	-0.103236	0.268126
0.1500	-0.386174	-0.269838	-0.178566	-0.018651	0.199723	0.26259
0.2000	-0.117191	-0.089745	-0.052080	0.013031	0.078183	0.12487
0.2500	-0.115527	-0.089751	-0.050652	0.002222	0.061060	0.10978
0.3000	-0.092163	-0.071717	-0.044241	0.012228	0.052661	0.09142
----NORMALIZED DISPLACEMENT OMEGAR TILDE(DIMENSIONLESS)----						
GAMMA/THETA(DEC.)	-90.00	-60.00	-30.00	-0.	30.00	60.00
0.1000	-0.000021	-0.000021	-0.000021	-0.000021	-0.000021	-0.000021
0.1500	1.885566	1.970226	1.568792	1.250119	0.95245	0.117011
0.2000	0.205508	0.162073	0.130606	0.108697	0.080799	0.39067
0.2500	0.335796	0.291021	0.168894	0.081591	0.0165512	0.28742
0.3000	0.343544	0.238932	0.177849	0.010553	-0.155942	-0.277825

TABLE VIII (Continued)

---NORMALIZED DISPLACEMENT U TILDE@DIMENSIONLESS---

$\text{GAMMA/THETA1(DEC.)}$	-98.00	-68.00	-38.00	-8.	38.00	68.00	98.00
0.1800	-0.000000	-0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.1500	0.76307	0.070276	0.053797	0.031291	0.058183	-0.007694	-0.013725
0.2000	0.20685	0.18653	0.15932	0.17319	0.066226	-0.059727	-0.059727
0.2500	0.35345	0.320861	0.211567	0.160959	0.012988	-2.186482	-0.15366
0.3000	0.523242	0.472295	0.333168	0.142955	0.047173	-0.166368	-0.237312

---NORMALIZED DISPLACEMENT V TILDE@DIMENSIONLESS---

$\text{GAMMA/THETA1(DEC.)}$	-98.00	-68.00	-38.00	-8.	38.00	68.00	98.00
0.1800	-8.	0.000000	0.000000	0.000000	0.000000	0.000000	0.
0.1500	-0.	0.053118	0.079497	0.108627	0.094874	0.053118	0.
0.2000	-0.	0.18510	0.184481	0.213828	0.184881	0.18510	0.
0.2500	-0.	0.168862	0.203817	0.327728	0.283817	0.168862	0.
0.3000	-0.	0.220297	0.397155	0.556595	0.397155	0.220297	0.









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13. ABSTRACT

The primary design requirement of a high-performance antenna is that the reflecting surface remain paraboloidal. For an antenna housed in a radome, strength considerations play a minor design role. Therefore, the antenna must have adequate structural stiffness accompanied by minimum weight. The basic structural components of the antenna are paraboloidal panels, which, when joined together, form a surface of revolution. Such a structural configuration, if properly fabricated, can be considered as a shell. Shell structures derive many of their attractive features from their two-dimensional surface nature, which brings with it geometrical complications to the strain-deflection relations and the equilibrium equations. Although the deflections of trusses, beams, and space frameworks are well understood, well documented, and easily obtained, this is not true for shells. In fact, shell behavior is currently the major topic of study in the structural mechanics field. The available solutions for even simple loadings of simple shells are of such a form that numerical results are not easily obtained. For these reasons, Lincoln Laboratory has been actively studying the deformations of paraboloidal shells. This user's manual will describe the capability, potentiality, and idiosyncrasies of the various LLAPS computer programs which are products of the above study.

14. KEY WORDS

paraboloidal shells

antenna design

computer program

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LINCOLN LABORATORY ANALYSES  
OF PARABOLOIDAL SHELLS  
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LINCOLN LABORATORY ANALYSES  
OF PARABOLOIDAL SHELLS  
(LLAPS)

USER'S MANUAL

I. INTRODUCTION

The quest for more and more precise radars in the ultra-high frequency régime has imposed stringent requirements on the structural behavior of large antennas. The tolerance requirements for an antenna surface at 10,000 Mcps, for instance, are generally set at about  $1/16$  of the operating wavelength. At a frequency of 10,000 Mcps, this is  $3/16$  of a centimeter or 0.074 inch. Such a tolerance on the permissible distortions of a structure which may be a hundred feet or more in over-all size, and which assumes different orientations with respect to the direction of gravity, requires an extremely high degree of sophistication in analysis, design, and construction.

Structures such as bridges, buildings, and flight vehicles are designed mainly by strength considerations, although flight vehicles must also have a certain minimum stiffness in order to avoid aero-elastic difficulties. Machine tools must possess great stiffness, but machine tools are generally compact and weight limitations are relatively unimportant. On the other hand, the primary design requirement of a high-performance antenna is that the reflecting surface remain paraboloidal and, in the case of an antenna housed in a radome, strength considerations play a minor role in the design. The antenna must therefore have adequate structural stiffness but, since the predominant loads are its own dead weight, the structural stiffness should be accompanied by minimum weight; that is, the antenna design should maximize the ratio of structural stiffness to weight.

The basic structural components of the antenna are paraboloidal panels which, when joined together, form a surface of revolution. Such a structural configuration, if properly fabricated, can be considered as a shell. The calculation of stresses for the strength design of a shell can be achieved with a fair degree of confidence, since confidence in the integrity of a structure can be obtained by increasing the factor of safety, i. e., by putting more material into the structure. Such a course of action may be self-defeating in an antenna which has only to resist its own dead weight. Moreover, the determination of the shape of the antenna surface must be precise, and cannot be approached with the same philosophy which is attendant to a strength design, i. e., hidden under a factor of safety.

Shell structures derive many of their attractive features from their two-dimensional surface nature. This two-dimensional nature brings with it geometrical complications to the strain-deflection relations and the equilibrium equations. Although the deflections of trusses, beams, and space frameworks are well understood, well documented, and easily obtained, the exact opposite is true for shells, as evidenced by the fact that shell behavior is currently the major topic of study in the field of structural mechanics. The available solutions for even simple

loadings of simple shells are of such a form that numerical results are not easily obtained. For these reasons, Lincoln Laboratory has been actively studying the deformations of paraboloidal shells.<sup>1-5</sup>

This user's manual will describe the capability, potentiality, and idiosyncrasies of the various LLAPS (Lincoln Laboratory Analyses of Paraboloidal Shells) computer programs which are products of the above study. These programs are all directed at the deflection problem of antennas although the force and moment resultants are also available. This manual is open ended, and additions will be made as new developments are completed.

Accepted for the Air Force  
Stanley J. Wisniewski  
Lt Colonel, USAF  
Chief, Lincoln Laboratory Office

## II. FORMULATION OF PROBLEM

### A. GEOMETRY OF SHELL

The middle surface of the antenna is a paraboloid of revolution whose geometry is described by the following figures and formulas. A point  $o$  on the middle surface is located by rectangular Cartesian coordinates  $y_1, y_2, y_3$  or by circular cylindrical coordinates  $r, \theta, y_3$  (Fig. 1).

$$y_1 = r \cos \theta \quad (II-1)$$

$$y_2 = r \sin \theta \quad (II-2)$$

$$y_3 = \frac{r^2}{4f} \quad (II-3)$$

where  $f$  is the focal length of the parabola (Fig. 2). Let

$$\gamma = \frac{r}{2f} \quad . \quad (II-4)$$

Then the slope of the parabola (and therefore the slope of a meridian of the paraboloidal surface) is

$$\frac{dy_3}{dr} = \frac{r}{2f} = \gamma \quad . \quad (II-5)$$

An element of arc length along a meridian (see Fig. 3) is

$$ds_r = 2f\sqrt{1 + \gamma^2} dy \quad . \quad (II-6)$$

An element of arc length along a latitude (see Fig. 3) is

$$ds_\theta = 2f\gamma d\theta \quad . \quad (II-7)$$

An element of surface area on the middle surface is

$$dA = 4f^2\gamma\sqrt{1 + \gamma^2} dy d\theta \quad . \quad (II-8)$$

A point  $p$  in the shell is located by orthogonal coordinates  $r, \theta, \xi$  (see Fig. 3). [ $\xi$  is perpendicular to the middle surface (see Fig. 3) and is positive inward,  $h$  is the thickness of shell.]

Let  $\overline{ed}$  be of unit length, then the directional cosines of  $\xi$ , that is, the orientation of  $\xi$  (see Fig. 4), are given by

$$\overline{ec} = \frac{\gamma \cos \theta}{\sqrt{1 + \gamma^2}} \quad (\text{Note it is in the negative } y_1 \text{ direction.}) \quad (II-9)$$

$$\overline{eb} = \frac{\gamma \sin \theta}{\sqrt{1 + \gamma^2}} \quad (\text{Note it is in the negative } y_2 \text{ direction.}) \quad (II-10)$$

and

$$\overline{ad} = \frac{1}{\sqrt{1 + \gamma^2}} \quad . \quad (II-11)$$

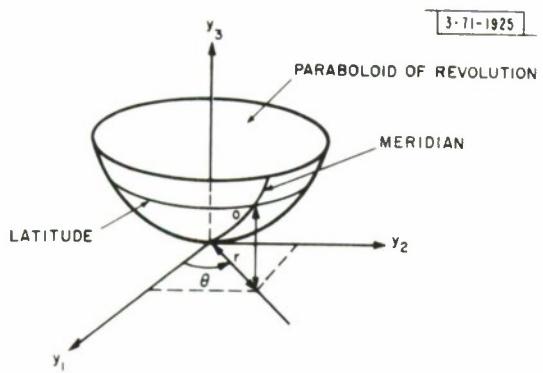


Fig. 1. Paraboloid of revolution.

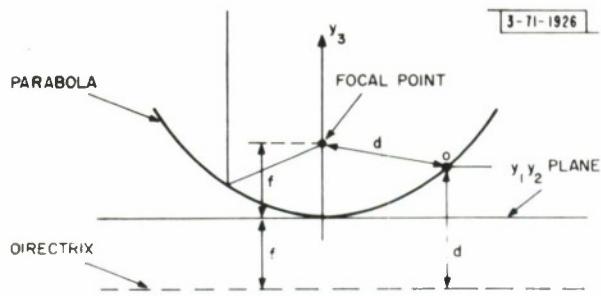


Fig. 2. Focal length of parabola.

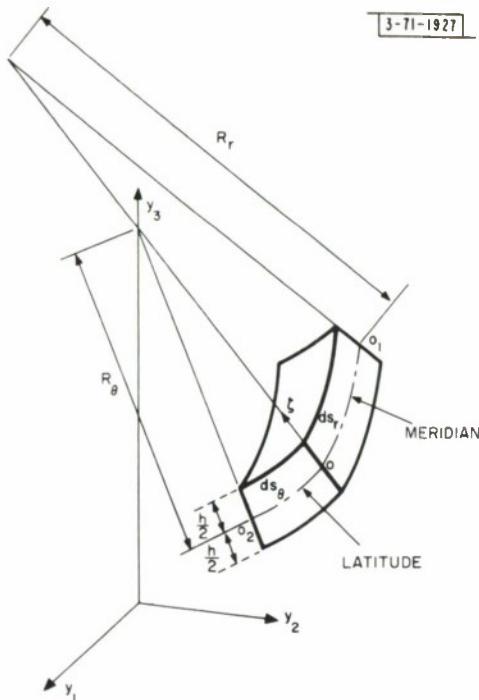


Fig. 3. Element of shell.

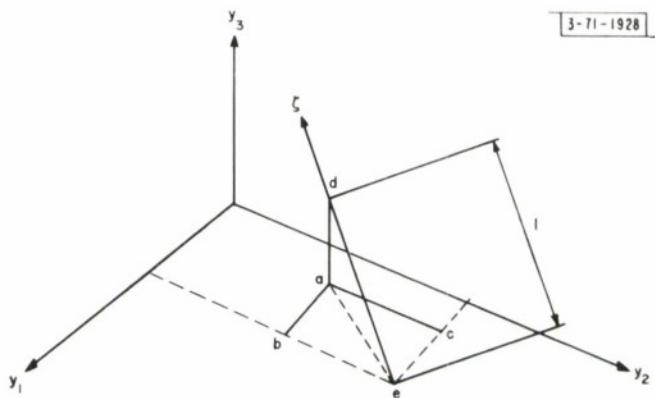


Fig. 4. Directional cosines.

The principal radius of curvature along a meridian (see Fig. 3) is

$$R_r = 2f(1 + \gamma^2)^{3/2} \quad (\text{II-12})$$

and the principal radius of curvature along a latitude (see Fig. 3) is

$$R_\Theta = 2f \sqrt{1 + \gamma^2} \quad (\text{II-13})$$

The boundaries or edges of shell are described by (see Fig. 5)

$$r = r_1 = \text{the radius of the inner edge} \quad (\text{II-14})$$

$$r = r_2 = \text{the radius of the outer edge} \quad (\text{II-15})$$

The corresponding value of  $\gamma$  at the edges will be denoted by  $\gamma_1$  and  $\gamma_2$ , respectively. The directional cosines of a unit vector tangent to a meridian (see Fig. 6) are

$$y_1 \text{ component } \frac{\cos \theta}{\sqrt{1 + \gamma^2}} \quad (\text{II-16})$$

$$y_2 \text{ component } \frac{\sin \theta}{\sqrt{1 + \gamma^2}} \quad (\text{II-17})$$

$$y_3 \text{ component } \frac{\gamma}{\sqrt{1 + \gamma^2}} \quad (\text{II-18})$$

The directional cosines of a unit vector tangent to a latitude (see Fig. 6) are

$$y_1 \text{ component } -\sin \theta \quad (\text{II-19})$$

$$y_2 \text{ component } \cos \theta \quad (\text{II-20})$$

$$y_3 \text{ component } 0 \quad (\text{II-21})$$

Finally, we define a pointing angle  $\psi$  to be the angle between the axis of revolution of the paraboloidal surface and the direction of gravity (see Fig. 7).

## B. STRUCTURAL PARAMETERS OF SHELL

The parameters which describe the structural behavior of the shell are the middle-surface displacements, strains, rotations, curvatures, stresses, force resultants, transverse shear resultants, and moment resultants. These will now be defined.

Let

$$u = \text{displacement of point } o \text{ along tangent to meridian at point } o \\ (\text{see Fig. 8}) \quad (\text{II-22})$$

$$v = \text{displacement of point } o \text{ along tangent to latitude at point } o \\ (\text{see Fig. 8}) \quad (\text{II-23})$$

$$w = \text{displacement of point } o \text{ along normal at point } o \text{ (see} \\ \text{Fig. 8).} \quad (\text{II-24})$$

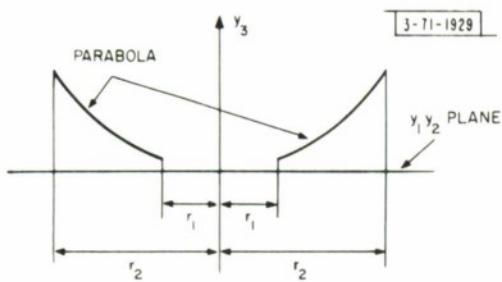


Fig. 5. Edges of shell.

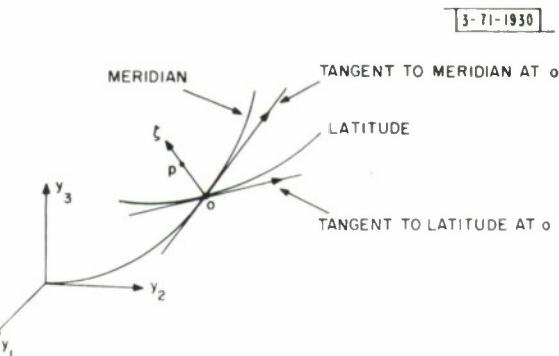


Fig. 6. Base vector and midsurface normal.

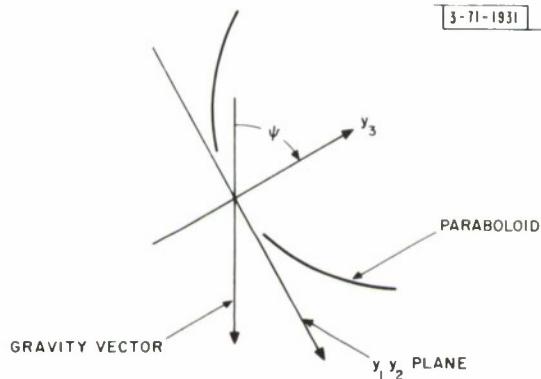


Fig. 7. Pointing angle.

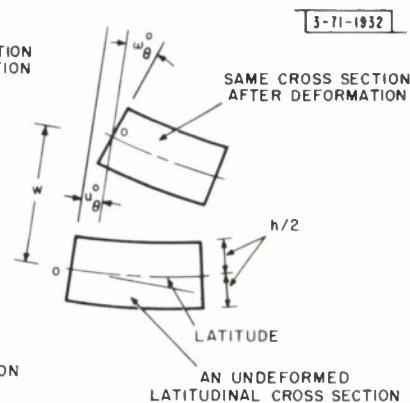


Fig. 8. Deformation of shell element.

Then the middle surface strains and rotations are given by the formulas listed below:

strain along meridian

$$\epsilon_r = \frac{1}{2f\sqrt{1+\gamma^2}} \frac{\partial u}{\partial r} - \frac{w}{2f(1+\gamma^2)^{3/2}} \quad (\text{II-25})$$

strain along latitude

$$\epsilon_\Theta = \frac{1}{2f\gamma} \frac{\partial v}{\partial \Theta} + \frac{u}{2f\gamma\sqrt{1+\gamma^2}} - \frac{w}{2f\sqrt{1+\gamma^2}} \quad (\text{II-26})$$

shearing strain

$$\epsilon_{r\Theta} = \frac{1}{2f\sqrt{1+\gamma^2}} \frac{\partial v}{\partial \gamma} - \frac{v}{2f\gamma\sqrt{1+\gamma^2}} + \frac{1}{2f\gamma} \frac{\partial u}{\partial \Theta} \quad (\text{II-27})$$

rotation of  $\overline{op}$  in meridional direction (see Fig. 8)

$$\omega_r = -\frac{u}{2f(1+\gamma^2)^{3/2}} - \frac{1}{2f\sqrt{1+\gamma^2}} \frac{\partial w}{\partial \gamma} \quad (\text{II-28})$$

rotation of  $\overline{op}$  in latitude direction (see Fig. 8)

$$\omega_\Theta = -\frac{v}{2f\sqrt{1+\gamma^2}} - \frac{1}{2f\gamma} \frac{\partial w}{\partial \Theta} \quad (\text{II-29})$$

change in curvature along meridian

$$\kappa_r = \frac{1}{2f\sqrt{1+\gamma^2}} \frac{\partial \omega_r}{\partial \gamma} \quad (\text{II-30})$$

change in curvature along latitude

$$\kappa_\Theta = \frac{\omega_r}{2f\gamma\sqrt{1+\gamma^2}} + \frac{1}{2f\gamma} \frac{\partial \omega_\Theta}{\partial \Theta} \quad (\text{II-31})$$

twist

$$\kappa_{r\Theta} = \frac{1}{2} \left\{ \frac{1}{2f\sqrt{1+\gamma^2}} \frac{\partial \omega_\Theta}{\partial \gamma} - \frac{\omega_\Theta}{2f\gamma\sqrt{1+\gamma^2}} + \frac{1}{2f\gamma} \frac{\partial \omega_r}{\partial \Theta} \right\} \quad (\text{II-32})$$

The stresses at point p in the shell are defined by means of Fig. 9. It is more convenient to define a system of force resultants and moment resultants acting at the middle surface of the shell which are equipollent to the stresses integrated over the thickness (Figs. 10 and 11). Let

$$N_r = \text{force resultant in meridional direction} = \int_{-h/2}^{h/2} \sigma_r d\xi \quad (\text{II-33})$$

$$N_\Theta = \text{force resultant in latitude direction} = \int_{-h/2}^{h/2} \sigma_\Theta d\xi \quad (\text{II-34})$$

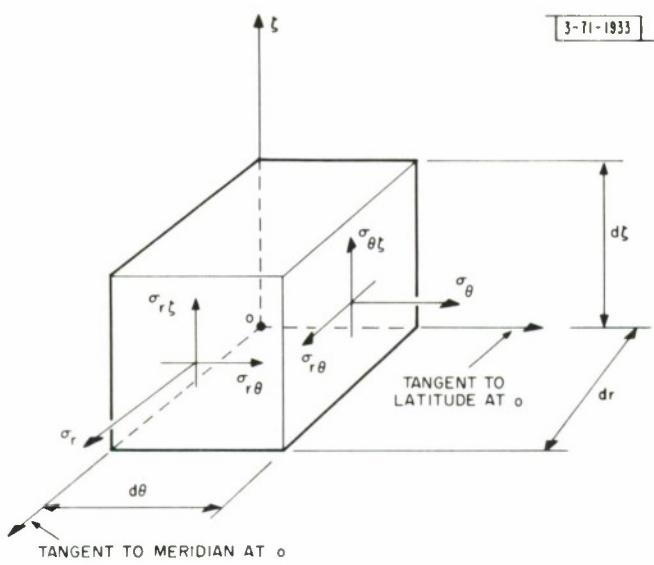


Fig. 9. Stresses in shell element.

Fig. 10. Stress resultants.

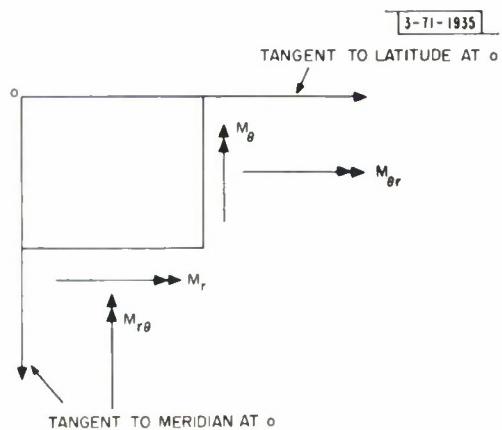
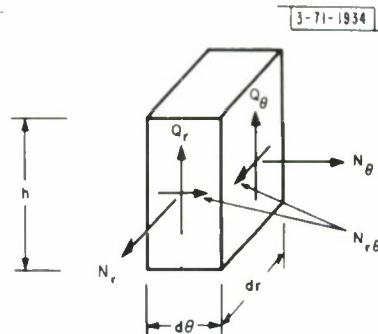


Fig. 11. Moment resultants. (The double-headed arrows represent moments. Use right-hand rule.)

$$N_{r\Theta} = N_{\Theta r} = \text{shear force resultant} = \int_{-h/2}^{h/2} \sigma_{r\Theta} d\xi \quad (\text{II-35})$$

$$Q_r = \text{transverse shear resultant on } r \text{ face} = \int_{-h/2}^{h/2} \sigma_{r\xi} d\xi \quad (\text{II-36})$$

$$Q_\Theta = \text{transverse shear resultant on } \Theta \text{ face} = \int_{-h/2}^{h/2} \sigma_{\Theta\xi} d\xi \quad (\text{II-37})$$

$M_r$  = bending moment about tangent to latitude at o

$$= \int_{-h/2}^{h/2} \sigma_r \xi d\xi \quad (\text{II-38})$$

$M_\Theta$  = bending moment about tangent to meridian at o

$$= \int_{-h/2}^{h/2} \sigma_\Theta \xi d\xi \quad (\text{II-39})$$

$$M_{r\Theta} = M_{\Theta r} = \text{twisting moment} = \int_{-h/2}^{h/2} \sigma_{r\Theta} \xi d\xi \quad (\text{II-40})$$

Then an element of volume bounded by surfaces  $r$ ,  $r + dr$ ,  $\Theta$ ,  $\Theta + d\Theta$ , and  $\pm h/2$  (see Fig. 3) satisfies the following equations of force and moment equilibrium.

$$\gamma \frac{\partial N_r}{\partial \gamma} + \sqrt{1 + \gamma^2} \frac{\partial N_{r\Theta}}{\partial \Theta} + N_r - N_\Theta - \frac{\gamma}{1 + \gamma^2} Q_r + 2f\gamma \sqrt{1 + \gamma^2} p_r = 0 \quad (\text{II-41})$$

$$\gamma \frac{\partial N_{r\Theta}}{\partial \gamma} + \sqrt{1 + \gamma^2} \frac{\partial N_\Theta}{\partial \Theta} + 2N_{r\Theta} - \gamma Q_\Theta + 2f\gamma \sqrt{1 + \gamma^2} p_\Theta = 0 \quad (\text{II-42})$$

$$\gamma \frac{\partial Q_r}{\partial \gamma} + \sqrt{1 + \gamma^2} \frac{\partial Q_\Theta}{\partial \Theta} + \frac{\gamma}{1 + \gamma^2} N_r + \gamma N_\Theta + Q_r + 2f\gamma \sqrt{1 + \gamma^2} p_n = 0 \quad (\text{II-43})$$

$$\gamma \frac{\partial M_r}{\partial \gamma} + \sqrt{1 + \gamma^2} \frac{\partial M_{r\Theta}}{\partial \Theta} + M_r - M_\Theta - 2f\gamma \sqrt{1 + \gamma^2} Q_r = 0 \quad (\text{II-44})$$

$$\gamma \frac{\partial M_{r\Theta}}{\partial \gamma} + \sqrt{1 + \gamma^2} \frac{\partial M_\Theta}{\partial \Theta} + 2M_{r\Theta} - 2f\gamma \sqrt{1 + \gamma^2} Q_\Theta = 0 \quad (\text{II-45})$$

where  $p_r$  and  $p_\Theta$  are the components of the applied surface loads along the tangent to meridian and to a latitude, respectively;  $p_n$  is the component along the normal to the middle surface. For a shell subjected to gravity load with an arbitrary pointing angle  $\psi$ ,

$$p_r = \frac{\rho}{\sqrt{1 + \gamma^2}} [\sin \Theta \sin \psi - \gamma \cos \psi] \quad (\text{II-46})$$

$$p_\Theta = \rho \cos \Theta \sin \psi \quad (\text{II-47})$$

and

$$p_n = -\frac{\rho}{\sqrt{1 + \gamma^2}} [\gamma \sin \Theta \sin \psi - \cos \psi] \quad (\text{II-48})$$

where  $\rho$  is the surface weight density of the shell ( $\text{lb/in.}^2$ ) and is related to the volume weight density  $\rho_o$  by

$$\rho = \int_{-h/2}^{h/2} \rho_o d\xi . \quad (\text{II-49})$$

For a homogeneous shell, we have  $\rho = \rho_o h$ .

For an isotropic shell, the stress resultants and the moment resultants are related to the shell middle-surface strains and curvature changes as follows:

$$N_r = C(\epsilon_r + \nu_m \epsilon_\theta - \Delta \bar{T}) \quad (\text{II-50})$$

$$N_\theta = C(\epsilon_\theta + \nu_m \epsilon_r - \Delta \bar{T}) \quad (\text{II-51})$$

$$N_{r\theta} = C(1 - \nu_m) \epsilon_{r\theta} \quad (\text{II-52})$$

$$M_r = D(\kappa_r + \nu_b \kappa_\theta - \Delta \tilde{T}) \quad (\text{II-53})$$

$$M_\theta = D(\kappa_\theta + \nu_b \kappa_r - \Delta \tilde{T}) \quad (\text{II-54})$$

$$M_{r\theta} = D(1 - \nu_b) \kappa_{r\theta} . \quad (\text{II-55})$$

The extensional stiffness  $C$ , the bending stiffness  $D$ , the effective stretching and bending Poisson's ratios  $\nu_m$  and  $\nu_b$  are given in terms of the Young's modulus  $E$  and Poisson's ratio  $\nu$  by

$$C = \int_{-h/2}^{h/2} \frac{E}{1 - \nu^2} d\xi \quad (\text{II-56})$$

$$\nu_m = \frac{1}{C} \int_{-h/2}^{h/2} \frac{\nu E}{1 - \nu^2} d\xi \quad (\text{II-57})$$

$$D = \int_{-h/2}^{h/2} \frac{E}{1 - \nu^2} \xi^2 d\xi \quad (\text{II-58})$$

$$\nu_b = \frac{1}{D} \int_{-h/2}^{h/2} \frac{\nu E}{1 - \nu^2} \xi^2 d\xi . \quad (\text{II-59})$$

The membrane and bending thermal strains  $\Delta \bar{T}$  and  $\Delta \tilde{T}$ , are given in terms of  $E$ ,  $\nu$ , the change in temperature  $\Delta T$ , and the coefficient of thermal expansion of the shell  $\alpha$  by

$$\Delta \bar{T} = \frac{1}{C} \int_{-h/2}^{h/2} \frac{\alpha E \Delta T}{1 - \nu} d\xi \quad (\text{II-60})$$

$$\Delta \tilde{T} = \frac{1}{D} \int_{-h/2}^{h/2} \frac{\alpha E \Delta T}{1 - \nu} \xi d\xi . \quad (\text{II-61})$$

If the shell is homogeneous in the thickness direction, then

$$\nu_m = \nu_b = \nu \quad (\text{II-62})$$

$$C = \frac{Eh}{1 - \nu^2} \quad (II-63)$$

$$D = \frac{Eh^3}{12(1 - \nu^2)} \quad (II-64)$$

$$\Delta \bar{T} = \frac{\alpha(1 + \nu)}{h} \int_{-h/2}^{h/2} \Delta T d\xi \quad (II-65)$$

$$\Delta \tilde{T} = \frac{12(1 + \nu)}{h} \alpha \int_{-h/2}^{h/2} \Delta T \xi d\xi \quad . \quad (II-66)$$

We shall have occasion in the subsequent development to refer to a parameter defined by

$$\lambda^2 = \frac{r_2^2}{\sqrt{R^2 AD}} \quad (II-67)$$

where  $R$  is the representative magnitude of the principal radii of curvature. For a paraboloidal shell of revolution, we have  $R = 2f$ . If, in addition, the shell is isotropic and homogeneous, we have

$$\lambda = \frac{r_2}{\sqrt{2fh}} = \sqrt[4]{12(1 - \nu^2)} = O\left(\frac{r_2}{\sqrt{2fh}}\right) \quad . \quad (II-68)$$

Equations (II-25) through (II-32), (II-41) through (II-45), and (II-50) through (II-55) form a set of nineteen equations for the nineteen structural parameters ( $N_r, N_\Theta, N_{r\Theta}, Q_r, Q_\Theta, M_r, M_\Theta, M_{r\Theta}, \epsilon_r, \epsilon_\Theta, \epsilon_{r\Theta}, \omega_r, \omega_\Theta, \kappa_r, \kappa_\Theta, \kappa_{r\Theta}, u, v$ , and  $w$ ). They are the shell equations for a paraboloidal shell of revolution.

The components of stress at a point  $o$  are related to the stress and moment resultants by the following formulas.

$$\sigma_r = \frac{N_r}{h} + \frac{12M_r}{h^3} \xi \quad (II-69)$$

$$\sigma_\Theta = \frac{N_\Theta}{h} + \frac{12M_\Theta}{h^3} \xi \quad (II-70)$$

$$\sigma_{r\Theta} = \frac{N_{r\Theta}}{h} + \frac{12M_{r\Theta}}{h^3} \xi \quad (II-71)$$

$$\sigma_{r\xi} = \frac{3Q_r}{2h} \left(1 - \frac{4\xi^2}{h^2}\right) \quad (II-72)$$

$$\sigma_{\Theta\xi} = \frac{3Q_\Theta}{2h} \left(1 - \frac{4\xi^2}{h^2}\right) \quad . \quad (II-73)$$

### C. EDGES OF SHELL

Along a  $\gamma = \text{constant}$  edge, we may prescribe an appropriate combination of:

- (1) Any one of the four quantities

(a) Normal displacement  $w$

(b) Axial displacement  $u_v$

$$u_v = w \cos \varphi + u \sin \varphi \quad (II-74)$$

(c) Effective transverse shear  $Q_r + (1/2f\gamma) (\partial M_{r\theta}/\partial \Theta)$

(d) Effective axial resultant  $V$

$$V = \left( Q_r + \frac{1}{2f\gamma} \frac{\partial M_{r\theta}}{\partial \Theta} \right) \cos \varphi + N_r \sin \varphi \quad . \quad (II-75)$$

(2) Any one of the four quantities

(a) Meridional displacement  $u$

(b) Radial displacement  $u_h$

$$u_h = -w \sin \varphi + u \cos \varphi \quad (II-76)$$

(c) Meridional stress resultant  $N_r$

(d) Effective radial resultant  $H$

$$H = - \left( Q_r + \frac{1}{2f\gamma} \frac{\partial M_{r\theta}}{\partial \Theta} \right) \sin \varphi + N_r \cos \varphi \quad (II-77)$$

(3) Either the circumferential displacement  $v$  or the effective shear resultant  $N_{r\theta} + (M_{r\theta}/R_\theta)$ , and

(4) Either the moment  $M_r$  or the rotation  $\omega_r$ , where

$$\cos \varphi = \frac{1}{\sqrt{1 + \gamma^2}} \quad (II-78)$$

$$\sin \varphi = \frac{\gamma}{\sqrt{1 + \gamma^2}} \quad . \quad (II-79)$$

A combination of prescribed conditions is appropriate if the structure is in global static equilibrium under these conditions.

The standard idealized edges supports are merely some special combinations of the above general set. For a simply supported edge, we have

$$w = u = v = M_r = 0 \quad . \quad (II-80)$$

For a clamped edge, we have

$$w = \omega_r = u = v = 0 \quad . \quad (II-81)$$

For a free edge, we have

$$N_r = N_{r\theta} + \frac{M_{r\theta}}{R_\theta} = M_r = Q_r + \frac{1}{2f\gamma} \frac{\partial M_{r\theta}}{\partial \Theta} = 0 \quad . \quad (II-82)$$

If the shell is closed at the apex, we require that the stresses and displacements be finite at  $\gamma = 0$ .

#### D. SOLUTION TO SHELL EQUATIONS FOR GRAVITY LOAD

For a shell under gravity load and without edge loads and temperature gradients, the solution to our shell equations must take the following form.<sup>4</sup>

$$(N_r^0, N_\Theta^0, Q_r^0, M_r^0, M_\Theta^0, u^0, w^0, \omega_r^0) = (N_r^S, N_\Theta^S, Q_r^S, M_r^S, M_\Theta^S, u^S, w^S, \omega_r^S) \cos \psi \\ + (N_r^a, N_\Theta^a, Q_r^a, M_r^a, M_\Theta^a, u^a, w^a, \omega_r^a) \sin \Theta \sin \psi \quad (II-83)$$

$$(N_{r\Theta}^0, M_{r\Theta}^0, Q_{r\Theta}^0, v^0) = (N_{r\Theta}^a, M_{r\Theta}^a, Q_{r\Theta}^a, v^a) \cos \Theta \sin \psi \quad (II-84)$$

In the subsequent development, we will not be concerned with the other seven structural quantities which can be obtained by appropriate combinations of those previously given. It should be said at this point that, although the form of the solution to our shell equations has been obtained with relative ease, the exact solution to these same equations does not seem likely. The computer programs presented herein represent several approaches to an approximate solution. These different approaches will be described under the appropriate programs and more thoroughly discussed in Refs. 1-5. In the remaining portion of this section, we shall discuss some general features of the output of these programs in connection with the shell behavior from the designer's point of view.

The superscripted quantities appearing in Eqs. (II-83) and (II-84) are independent of  $\psi$  and  $\Theta$ . Their dependence on  $\gamma$  is generally complicated. It is clear from the same equations that these are the key quantities to our problem. Once they are determined, the physical quantities appearing on the left-hand sides of Eqs. (II-83) and (II-84) can be obtained for any value of  $\psi$  and  $\Theta$  by straightforward calculations. The computer programs discussed herein are designed mainly to calculate these superscripted quantities for a given set of values of  $\gamma$  which will be referred to as the  $\gamma$ -tabular points. However, since  $\psi$  is really a load parameter, its effect on the stresses and deformations of the shell is generally of prime interest to the designers of larger antennas. Therefore, the programs for gravity load give generally the following quantities as its output.

$$(N_r^0, N_\Theta^0, \dots, w^0, \omega_r^0) = (N_r^S, N_\Theta^S, \dots, w^S, \omega_r^S) \cos \psi \\ + (N_r^a, N_\Theta^a, \dots, w^a, \omega_r^a) \sin \psi \quad (II-85)$$

$$(N_{r\Theta}^0, M_{r\Theta}^0, Q_{r\Theta}^0, v^0) = (N_{r\Theta}^a, \dots, v^a) \sin \psi \quad (II-86)$$

The various physical quantities themselves will also be calculated upon request. To this end, we must prescribe, in addition to the  $\gamma$ -tabular points, a set of values for  $\Theta$  which will be referred to as the  $\Theta$ -tabular points. Together with the former, they form a net of points each describing a particular point on the middle surface of the shell. The programs calculate the stress and displacement of the shell for this net of tabular points. In all cases, the unit for length is inch and the unit for force is pound. While the shell is closed in the circumferential direction so that the range of  $\Theta$  is  $(0, 2\pi)$ , it is clearly sufficient to calculate for any particular latitude (i.e., for any fixed value of  $\gamma$ ) the values of the physical quantities for  $0 \leq \Theta \leq \pi$ . This rather trivial observation may save a great deal of computing time if a large number of runs is desired.

It is often desirable to present numerical results in a form which is independent of the various geometrical and material parameters associated with the shell. In general, this is not possible in shell analysis. Thus, we can only hope to find normalizing factors which yield dimensionless quantities of the same order of magnitude for shells with different geometry and different materials. To this end, we let

$$(u^*, v^*, w^*) = \frac{C(1-\nu^2)}{4f^2\rho} (u, v, w) \quad (\text{II-87})$$

$$(N_r^*, N_\Theta^*, N_{r\Theta}^*) = \frac{1}{2f\rho} (N_r, N_\Theta, N_{r\Theta}) \quad (\text{II-88})$$

$$\left( M_r^*, M_\Theta^*, \frac{M_{r\Theta}^*}{mk_o} \right) = \frac{(mk_o)^2}{4f^2\rho} (M_r, M_\Theta, M_{r\Theta}) \quad (\text{II-89})$$

$$\left( Q_r^*, \frac{Q_\Theta^*}{mk_o} \right) = \frac{mk_o}{2f\rho} (Q_r, Q_\Theta) \quad (\text{II-90})$$

$$\omega_r^* = \frac{C(1-\nu^2)}{2f\rho(mk_o)} \omega_r \quad (\text{II-91})$$

$$k_o^4 = \frac{4f^2 C}{D} \quad (\text{II-92})$$

$$m^4 = (1-\nu^2) \quad . \quad (\text{II-93})$$

Although the starred quantities are not invariant with respect to the various geometrical and material properties of the shell (for instance, they obviously depend on Poisson's ratio  $\nu$ ), their magnitude does not vary appreciably even with appreciable changes in the relevant geometrical and material parameters. Both the starred (normalized) quantities and the unstarred (un-normalized) quantities may be obtained from our computer programs.

It can be seen from an examination of the solution that a portion of the displacements corresponds to a rigid body translation and/or rigid body rotation about the  $y^1$  axis. For example, in Group Report 71G-1-II (Ref. 2), the constant  $C_2$  in Eq. (6.4.21) for  $w$  (membrane analysis) is seen to represent a translation of the shell parallel to the  $y^3$  axis. Since this is a rigid body motion,  $C_2$  does not appear in any of the equations for the force resultants. Similarly, in the asymptotic analysis, the constant  $C_2$  [Eqs. (8.9.1) and (8.9.2), Group Report 71G-1, Part IV (Ref. 4)] denotes a rigid body translation parallel to the  $y^3$  axis for symmetric loads. For anti-symmetric loads, the constants  $C_3$  and  $C_4$  [Eqs. (8.10.1) to (8.10.3), Group Report 71G-1, Part IV (Ref. 4)] represent a rigid body rotation about the  $y^1$  axis and a rigid body translation parallel to the  $y^2$  axis, respectively. The rigid body displacements do not affect the shape of the paraboloidal surface and hence can be interpreted as a change in the position of the focal point. We will denote the deflections due to rigid body translations and/or rotation by  $\hat{u}$ ,  $\hat{v}$ , and  $\hat{w}$ . Then, the actual flexibility, i.e., distortions, will be given by

$$u_{\text{flex}} = u - \hat{u} \quad (\text{II-94})$$

$$v_{\text{flex}} = v - \hat{v} \quad (\text{II-95})$$

$$w_{\text{flex}} = w - \hat{w} \quad . \quad (\text{II-96})$$

From the previous discussion, it is clear that these quantities should be of considerable interest to the designers of large antennas. Our computer program will display these as part of the output.

Thus far, it has been assumed that the shell has its prescribed shape before the introduction of the applied loads. Once loaded, the distortion of the shell is taken with respect to this prescribed (paraboloidal) shape. The antenna designer may also be interested in the distortions of the shell relative to its shape when the axis of the shell coincides with gravity vector. This is usually the attitude of the shell during erection and the designer may elect to construct the shell to a prescribed shape while it is in this attitude. Thus, we are often interested in

$$\tilde{w} = w^S(\cos \psi - 1) + w^A \sin \Theta \sin \psi \quad (\text{II-97})$$

$$\tilde{u} = u^S(\cos \psi - 1) + u^A \sin \Theta \sin \psi \quad (\text{II-98})$$

$$\tilde{v} = v^A \cos \Theta \sin \psi \quad (\text{II-99})$$

where the tilde quantities are the components of displacement relative to the face-up (i.e.,  $\psi = 0$ ) position. The corresponding quantities  $\tilde{u}_{\text{flex}}$ ,  $\tilde{v}_{\text{flex}}$ ,  $\tilde{w}_{\text{flex}}$ ,  $\tilde{u}^o$ ,  $\tilde{v}^o$ ,  $\tilde{w}^o$ ,  $\tilde{u}^o_{\text{flex}}$ ,  $\tilde{v}^o_{\text{flex}}$ , and  $\tilde{w}^o_{\text{flex}}$  are defined in the obvious way.

### III. MEMBRANE ANALYSIS OF GRAVITY LOAD

#### A. MEMBRANE SOLUTION

Under conducive circumstances, the interior of the shell behaves like a membrane. Away from the edges of the structure, a good approximation of the behavior of the shell can be obtained by the so-called membrane (or momentless) theory of shell. Such a theory assumes that the shell has no bending stiffness, and quantities associated with the bending action of the shell vanish identically, i.e.,

$$M_r = M_\Theta = M_{r\Theta} = Q_r = Q_\Theta = 0 \quad . \quad (\text{III-1})$$

Consequently, Eqs. (II-41) to (II-43) become

$$\gamma \frac{\partial N_r}{\partial \gamma} + \sqrt{1 + \gamma^2} \frac{\partial N_{r\Theta}}{\partial \Theta} + N_r - N_\Theta + 2f\gamma \sqrt{1 + \gamma^2} p_r = 0 \quad (\text{III-2})$$

$$\gamma \frac{\partial N_{r\Theta}}{\partial \gamma} + \sqrt{1 + \gamma^2} \frac{\partial N_\Theta}{\partial \Theta} + 2N_{r\Theta} + 2f\gamma \sqrt{1 + \gamma^2} p_\Theta = 0 \quad (\text{III-3})$$

$$\frac{N_r}{1 + \gamma^2} + N_\Theta + 2f \sqrt{1 + \gamma^2} p_n = 0 \quad (\text{III-4})$$

and Eqs. (II-44) and (II-45) are identically satisfied.

Equations (III-2) to (III-4) contain only three unknowns and can therefore be solved for the stress resultants  $N_r$ ,  $N_\Theta$ , and  $N_{r\Theta}$ . In other words, the membrane problem is statically determinate. The solution for  $N_r$ ,  $N_\Theta$ , and  $N_{r\Theta}$  can then be inserted in the left-hand side of Eqs. (II-25) to (II-27) by way of Eqs. (II-50) to (II-52) with the temperature terms omitted. The three strain-displacement relations also contain only three unknowns and can therefore be solved for  $u$ ,  $v$ , and  $w$ .

Note that associated with the momentless assumption is a reduction of the order of our system of differential (shell) equations from eight to four. Thus, only half the conditions at each edge (Sec. II-C) can be satisfied. For a free edge, the last two conditions [Eq. (II-82)] are satisfied identically; therefore, we are left with the necessary two boundary conditions. In the case of a supported edge, it has been shown<sup>5</sup> that the appropriate boundary conditions are

$$u = 0 \quad (\text{III-5})$$

and

$$v = 0 \quad . \quad (\text{III-6})$$

Since the shell can not be acted upon by transverse forces or bending moments, it seems reasonable that it may not be constrained in the normal direction.

The exact analytical solution for the membrane stress resultants as well as the middle-surface displacement components in accordance with the momentless theory have been obtained and tabulated in Ref. 2. It can be seen from these analytical solutions that the corresponding normalizing factors given in Sec. II-D are nearly the true scale factors for these quantities. For a fixed value of  $\nu$ , one set of normalized results generated from our computer program is valid for the entire class of geometrically similar shells, i.e., shells with the same  $\gamma_1$  and  $\gamma_2$ .

The analytic expressions for the various physical quantities determined by the momentless theory of shell are formed by complicated combinations of elementary functions. To evaluate

them for a large number of tabular points even by a desk calculator is a formidable task. A computer program is written to make this information easily accessible to the designers of large antennas and to others who might be interested. The program generates the tabular points, computes the constants of integration, and calculates the stress resultants and displacements of the middle surface.

## B. DIGITAL COMPUTER PROGRAM

The general scheme of the computer program for a membrane analysis is outlined in Fig. 12, the master flow chart. At the end of a complete run, the program returns to step (1). When it

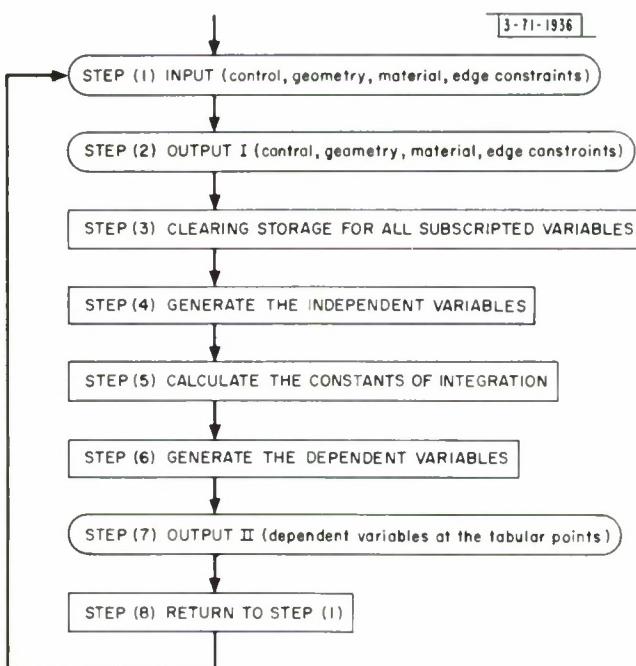


Fig. 12. Master flow chart.

fails to locate additional input, the program exits automatically. This is a special feature of the Lincoln Laboratory Express Runs. Slight modifications of the ending will be necessary if the program is to be run differently or if some indication of a successful run is desired. The same Express Runs also require that the input be prestored on machine tape A2 and that the output be written on A3. In complying with this restriction, the program reads all its input from A2 and writes all its output on A3. For the purpose of the Express Runs, the object deck must be preceded by two cards each with an asterisk in column one. The first of these is an identification card while the second contains the work XEQ occupying columns seven, eight, and nine (Sample Card 1). Following the object deck is another card with an asterisk in column one and the word DATA occupying columns seven, eight, nine, and ten (Sample Card 2). Then comes the input to the program to be prestored on A2. The program listing will be presented in Table I.



TABLE I  
PROGRAM LISTING

```

*      SHEA-WAN      MEMBRANE SHELL WITH GRAVITY LOAD      9/3/64
*      XEQ
C      WAN, FRED      MEMBRANE SHELL WITH GRAVITY LOAD      9/3/64
      DIMENSION GAMMA(150),THETA(10),ENR(150,10),ENT(150,10),ENRT(150,10
      1),U(150,10),V(150,10),W(150,10),EN1(150),EN2(150),U1(150),W1(150),
      2R(150),A(150),B(150),C(150),G(150),P(150),Q(150),D(3,3),Y(3),Z(3),
      3GAMMR(150),S(150),FMT1(12),FMT2(12)
      DIMENSION UFLEXA(150),VFLEXA(150),WFLEXA(150),UFLEXS(150),WFLEXS
      1(150),UFLEX(150,10),VFLEX(150,10),WFLEX(150,10)
      COMMON ENR,ENT,ENRT,V,W,U
      EQUIVALENCE (GAMMA(1),R(1)),(GAMMR(1),S(1))

C      - - (1) INPUT
C
      10 READ INPUT TAPE 2,101,M,N,KE,NORM,KD,KP
         READ INPUT TAPE 2,105,R1,R2,R3,R4
         READ INPUT TAPE 2,105,F,PR,PSI
         X1=1.+PR
         X6=1.-PR
         X7=X1*X6
         IF(KP-1)320,321,322
      320 READ INPUT TAPE 2,102,RHO,H,E
         RHO=RHO*H
         CC=E*H/X7
         GO TO 323
      321 READ INPUT TAPE 2,102,RHO,CC
         GO TO 323
      322 READ INPUT TAPE 2,102,RHS,RHC,T,H,E
         RHO=2.*T*RHS+(H-T)*RHC
         CC=2.*T*E/X7
      323 IF(N)7,7,8
         8 READ INPUT TAPE 2,105,THETA3,THETA4
         READ INPUT TAPE 2,205,FMT1,FMT2
C      - - (2) OUTPUT I
C
         7 WRITE OUTPUT TAPE 3,107
         IF(KP-1)67,68,69
      67 WRITE OUTPUT TAPE 3,160
         GO TO 73
      68 WRITE OUTPUT TAPE 3,161
         GO TO 73
      69 WRITE OUTPUT TAPE 3,162
      73 IF(NORM)74,74,75
      74 WRITE OUTPUT TAPE 3,139
         GO TO 302
      75 WRITE OUTPUT TAPE 3,140
      302 IF(KE-1)93,92,50
      93 WRITE OUTPUT TAPE 3,115
         GO TO 91
      92 WRITE OUTPUT TAPE 3,116
         GO TO 91
      50 WRITE OUTPUT TAPE 3,118
      91 IF(KP-1)76,77,78
      76 WRITE OUTPUT TAPE 3,112
         WRITE OUTPUT TAPE 3,111,RHO,E,H,F,PR
         GO TO 79
      77 WRITE OUTPUT TAPE 3,163

```

TABLE I (Continued)

```

      WRITE OUTPUT TAPE 3,111,CC,RHO,F,PR
      GO TO 79
78  WRITE OUTPUT TAPE 3,164
      WRITE OUTPUT TAPE 3,111,E,H,T,RHS
      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,165
      WRITE OUTPUT TAPE 3,111,RHC,RHO,F,PR
79  WRITE OUTPUT TAPE 3,114
      PI =3.14159265
      RAD=57.2957795
      PSIDEGR=PSI*PI*RAD
      WRITE OUTPUT TAPE 3,113
      WRITE OUTPUT TAPE 3,108,R1,R2,R3,R4,PSIDEGR
      WRITE OUTPUT TAPE 3,114
      IF(N)1,1,9
9   T3DEG=THETA3*PI*RAD
      T4DEG=THETA4*PI*RAD
      WRITE OUTPUT TAPE 3,117
      WRITE OUTPUT TAPE 3,119,T3DEG,T4DEG
1   WRITE OUTPUT TAPE 3,114

C
C - - (3) CLEARING STORAGE FOR SUBSCRIPTED VARIABLES
C
      DO 51 I=1,150
      EN1(I)=0.
      EN2(I)=0.
      U1(I)=0.
      W1(I)=0.
      A(I)=0.
      B(I)=0.
      C(I)=0.
      G(I)=0.
      P(I)=0.
      Q(I)=0.
      GAMMR(I)=0.
51  GAMMA(I)=0.
      DO 52 J=1,10
      THETA(J)=0.
      DO 52 I=1,150
      U(I,J)=0.
      W(I,J)=0.
      V(I,J)=0.
      ENR(I,J)=0.
      ENT(I,J)=0.
52  ENRT(I,J)=0.

C
C - - (4) GENERATE INDEPENDENT VARIABLES
C
      F2=2.*F
      UR=.00000005
      TWOPI=2.*PI
      R1=R1/F2
      R2=R2/F2
      R3=R3/F2
      R4=R4/F2
      M1=M-1
      EM1=M1
      DELTR=R4-R3

```

TABLE I (Continued)

```

DELTR=DELTR/EM1
GAMMA(1)=R3
DO 30 I=1,M1
I1=I+1
EI=I
30 GAMMA(I1)=R3+DELTR*EI
IF(NORM)630,630,632
630 DO 631 I=1,M
631 GAMMR(I)=GAMMA(I)
GO TO 634
632 DO 633 I=1,M
633 GAMMR(I)=GAMMA(I)*F2
634 IF(N)5,5,40
40 THETA3=THETA3*PI
THETA4=THETA4*PI
N1=N-1
AN1=N1
THETA5=THETA4-THETA3
DELT=THETA5/AN1
THETA(1)=THETA3
DO 2 I=1,N1
I1=I+1
2 THETA(I1)=THETA(I)+DELT
IF(THETA4-TWOPI) 5,6,6
6 THETA(N)=THETA3
C
C - - (5) CALCULATE THE CONSTANTS OF INTEGRATION
C
5 CS=F2*RHO
CD=F2**2*RHO/(X7*CC)
PHI=PI*PSI
CPHI=COSF(PHI)
SPHI=SINF(PHI)
IF(PSI)38,31,32
38 WRITE OUTPUT TAPE 3,143
GO TO 10
32 CSS=CS*CPHI/3.
CDS=CD*CPHI/3.
CSA=2.*CS*SPHI
CDA=CD*SPHI
31 X2=.5*(3.-PR)
X3=.25
X4=-2.*(.1.-PR)
X5=PR
IF(KE-1)95,96,98
98 A1=-1.
C1=-1./3.
C2=-1./15.
ARG1=1.+R2**2
ARG1=SQRTF(ARG1)
X0=-(1.+PR)*A1
A3=-ARG1*(A1*R2+X0/R2)-X0*R2*LOGF(1.+ARG1)+(X0-X1)*R2*LOGF(R2)-X2*
1R2**3-X3*R2**5+X1/R2
A3=A3/R2
GO TO 94
96 A1=1.+R1**2
A1=SQRTF(A1)
A1=-A1**3

```

TABLE I (Continued)

```

ARG1=1.+R2**2
ARG1=SQRTF(ARG1)
X0=-(1.+PR)*A1
A3=-ARG1*(A1*R2+X0/R2)-X0*R2*LOGF(1.+ARG1)+(X0-X1)*R2*LOGF(R2)-X2*
1R2**3-X3*R2**5+X1/R2
A3=A3/R2
ARG1=1.+R1**2
ARG1=SQRTF(ARG1)
C1=-ARG1**3/3.
C2=-(.5*C1*R1**2+ARG1**5/15.)
94 ARG1=SQRTF(1.+R2**2)
ARG2=ARG1-1.
ARG3=ARG2/R2
H1=ARG1*(R2 *.5-X1/R2 )
H2=X6*.5*R2 *LOGF(ARG3)+X1*ARG1*(1.-.5* R2 **2)/R2 **3
H3=R2 **5-(40.+29.*PR )*R2**3+X6*2.*R2*LOGF(R2)+X1*(4.-20.*R2**2
1)/R2**3
H3=H3/60.
G1=-X1*(ARG1+LOGF(ARG3))+ARG1**3/6.
G2=.25*X6*(R2 **2*LOGF(ARG3)+1.+ARG1)-.5*X1*ARG1**3/R2 **2
G3=R2 **6-1.5*(40.+29.*PR )*R2 **4-3.*X6*R2 **2+LOGF(R2 )*(1
16.*X6*R2 **2-X1*120.)-X1* 12./R2 **2
G3=G3/360.
TERM1=-H2+2.*X1*ARG1/R2**3
TERM2=H1+ X1*ARG1/R2
TERM3=-H3+2.*X1*(1.-4.*R2**2)*ARG1**4/(15.*R2**3)
C3=(TERM3+C2*TERM1-TERM2*C1)/R2
C4=-(.5*C3*R2**2+C2*G2+C1*G1+G3)
GO TO 54
95 ARG11=SQRTF(1.+R1**2)
ARG12=SQRTF(1.+R2**2)
ARG3=(1.+ARG11)/R1
ARG4=(1.+ARG12)/R2
T1=R1*ARG11-X1*(R1*LOGF(ARG3)+ARG11/R1)
T2=R2*ARG12-X1*(R2*LOGF(ARG4)+ARG12/R2)
T3=-X3*R1**5-X2*R1**3-(R1*LOGF(R1)-1./R1)*X1
T4=-X3*R2**5-X2*R2**3-(R2*LOGF(R2)-1./R2)*X1
E1=R2*T3-T4*R1
E2=T3*T2-T4*T1
DENOM=T1*R2-T2*R1
A1=E1/DENOM
A3=-E2/DENOM
ARG21=ARG11-1.
ARG22=ARG12-1.
ARG31=ARG21/R1
ARG32=ARG22/R2
H11=ARG11*(R1*.5-X1/R1)
H12=ARG12*(R2*.5-X1/R2)
H22=X6*.5*R2*LOGF(ARG32)+X1*ARG12*(1.-.5*R2**2)/R2**3
H21=X6*.5*R1*LOGF(ARG31)+X1*ARG11*(1.-.5*R1**2)/R1**3
H31=R1**5-(40.+29.*PR )*R1**3+X6*2.*R1*LOGF(R1)+X1*(4.-20.*R1**2
12)/R1**3
H32=R2**5-(40.+29.*PR )*R2**3+X6*2.*R2*LOGF(R2)+X1*(4.-20.*R2**2
12)/R2**3
H31=H31/60.
H32=H32/60.
G11=-X1*(ARG11+LOGF(ARG31))+ARG11**3/6.
G12=-X1*(ARG12+LOGF(ARG32))+ARG12**3/6.

```

TABLE I (Continued)

```

G21=.25*X6*(R1**2*LOGF(ARG31)+1.+ARG11)-.5*X1*ARG11**3/R1**2
G22=.25*X6*(R2**2*LOGF(ARG32)+1.+ARG12)-.5*X1*ARG12**3/R2**2
G31=R1**6-1.5*(40.+29.*PR )*R1**4-3.*X6*R1**2+LOGF(R1)*(6.*X6*R
11**2-X1*120.)-X1*12./R1**2
G32=R2**6-1.5*(40.+29.*PR )*R2**4-3.*X6*R2**2+LOGF(R2)*(6.*X6*R
12**2-X1*120.)-X1*12./R2**2
G31=G31/360.
G32=G32/360.
D(1,1)=G11-G12
D(1,2)=G21-G22
D(1,3)=.5*(R1**2-R2**2)
D(2,1)=H11+X1*ARG11/R1
D(2,2)=H21-2.*X1*ARG11/R1**3
D(2,3)=R1
D(3,1)=H12+X1*ARG12/R2
D(3,2)=H22-2.*X1*ARG12/R2**3
D(3,3)=R2
Z(1)=G32-G31
Z(2)=-H31+2.*X1*ARG11**4*(1.-4.*R1**2)/(15.*R1**3)
Z(3)=-H32+2.*X1*ARG12**4*(1.-4.*R2**2)/(15.*R2**3)
DET=1.
L=XSIMEQF(3,3,1,D,Z,DET,Y)
GO TO (70,71,72),L
72 WRITE OUTPUT TAPE 3,103
GO TO 10
71 WRITE OUTPUT TAPE 3,104
GO TO 10
70 C1=D(1,1)
C2=D(2,1)
C3=D(3,1)
C4=-(G31+C1*G11+C2*G21+C3*.5*R1**2)
X0=-X1*A1
C
C - - (6) GENERATE THE DEPENDENT VARIABLES
C
54 IF(PSI)38,35,39
39 DO 4 I=1,M
  ARG3=1.+GAMMA(I)**2
  ARG4=SQRTF(ARG3)
  ARG1=ARG3-1.
  ARG2=ARG1/GAMMA(I)
  BEBA=.5*C1*ARG4/R(I)+(C2*ARG4/R(I)**3)+(ARG3**3/(15.*R(I)**3))
  BEBA=-BEBA
  BEBC=(-.5*C1/R(I))+(C2/R(I)**3)+(ARG4**3*(1.-4.*R(I)**2)/(15.*R(I
  1)**3))
  BEBB=.5*R(I)+((C2/R(I)**3)+(.5*C1/R(I))+ARG4*ARG3**2/(15.*R(I)**3)
  1)/ARG4
  A(I)=CSA*BEBA
  B(I)=CSA*BEBB
  C(I)=CSA*BFBC
  ARG1=SQRTF(1.+R(I)**2)
  ARG2=ARG1-1.
  ARG3=ARG2/R(I)
  G1=-X1*(ARG1+LOGF(ARG3))+ARG1**3/6.
  G2=.25*X6*(R(I)**2*LOGF(ARG3)+1.+ARG1)-.5*X1*ARG1**3/R(I)**2
  G3=R(I)**6-1.5*(40.+29.*PR )*R(I)**4-3.*X6*R(I)**2+LOGF(R(I))*(6.*X6*R
  16.*X6*R(I)**2-X1*120.)-X1*12./R(I)**2
  G3=G3/360.

```

TABLE I (Continued)

```

VFLEXA(I)=2.*CDA*(C1*G1+C2*G2+G3)
G(I)=2.*(C4+.5*C3*R(I)**2+C2*G2+C1*G1+G3)*CDA
H1=ARG1*(R(I)*.5-X1/R(I))
H2=X6*.5*R(I)*LOGF(ARG3)+X1*ARG1*(1.-.5* R(I)**2)/R(I)**3
H3=R(I)**5-(40.+29.*PR )*R(I)**3+X6*2.*R(I)*LOGF(R(I))+X1*(4.-2
10.*R(I)**2)/R(I)**3
H3=H3/60.
G4=-X1+(G1-R(I)*H1)/ARG1
G5=2.*X1*ARG1+R(I)**2*(G2-R(I)*H2)
G5=G5/(ARG1*R(I)**2)
G6=(G3-R(I)*H3)+2.*X1*(1.-4.*R(I)**2)*ARG1**4/(15.*R(I)**2)
G6=G6/ARG1
G7=(G4-ARG1*G1-.5*X1-.5*PR *R(I)**2)/R(I)
G8=(G5-ARG1*G2-PR -X1/R(I)**2)/R(I)
G9=-.5*R(I)*(2.+R(I)**2)/ARG1
G10=-R(I)/ARG1
G11=ARG1*(15.*R(I)**4+2.*ARG1**4+2.*PR *ARG1**6)/(30.*R(I)**2)
G11=(G6-ARG1*G3-G11)/R(I)
P(I)=2.*CDA*(C1*G7 +C2*G8 +C3*G9 +C4*G10+G11)
Q(I)=2.*CDA*(C1*G4+C2*G5+G6+(C4-.5*C3*R(I)**2)/ARG1)
WFLEXA(I)=2.*CDA*(C1*G7+C2*G8+G11)
UFLEXA(I)=2.*CDA*(C1*G4+C2*G5+G6)

4 CONTINUE
UB1=ABSF(Q(1))
UB2=ABSF(Q(M))
IF(UB1-UR)47,47,53
47 Q(1)=0.
53 IF(UB2-UB)58,58,35
58 Q(M)=0.
35 DO 11 I=1,M
ARG1=1.+GAMMA(I)**2
ARG1=SQRTF(ARG1)
ARG2=1./GAMMA(I)
ARG2=ARG2**2
EN1(I)= CS * ARG1*ARG2*(ARG1**3+A1)/3.
EN2(I)=CS *(3.-ARG2*(ARG1**3+A1)/ARG1)/3.
ARG1=1./GAMMA(I)
ARG1=ARG1**2
ARG2=1.+GAMMA(I)**2
ARG2=SQRTF(ARG2)
ARG21=1./ARG2
ARG3=LOGF(GAMMA(I))
ARG4=ARG1/GAMMA(I)
ARG5=ARG4*ARG1
U1(I)=CD * ((X0*GAMMA(I)*LOGF(1.+ARG2)+(X1-X0)*GAMMA(I)*LOGF(GAM
1MA(I))+A3*GAMMA(I)+X2*GAMMA(I)**3+X3*GAMMA(I)**5-X1/GAMMA(I))*ARG2
21+A1*GAMMA(I)+X0/GAMMA(I))/3.
UFLEXS(I)=U1(I)-CD*A3*GAMMA(I)*ARG21/3.
W1(I)=CD * ((X0*LOGF(1.+ARG2)+(X1-X0)*LOGF(GAMMA(I))+X2*GAMMA(I)
1**2+X3*GAMMA(I)**4+A3-X1/GAMMA(I)**2)*ARG21+(X4+X5*GAMMA(I)**2+X1/
2GAMMA(I)**2)*ARG2-X0)/3.
WFLEXS(I)=W1(I)-CD*A3*ARG21/3.

11 CONTINUE
UB1=ABSF(U1(1))
UB2=ABSF(U1(M))
IF(UB1-UB)59,59,62
59 U1(1)=0.
62 IF(UB2-UB)63,63,80

```

TABLE I (Continued)

```

63 U1(M)=0.
C
C - - (7) OUTPUT II
C
80 IF(N)225,225,226
225 IF(PSI)38,226,227
226 WRITE OUTPUT TAPE 3,229
    WRITE OUTPUT TAPE 3,231,A1,A3
    WRITE OUTPUT TAPE 3,114
    IF(N)228,228,227
227 WRITE OUTPUT TAPE 3,230
    WRITE OUTPUT TAPE 3,231,C1,C2,C3,C4
228 WRITE OUTPUT TAPE 3,107
    IF(PSI)38,33,34
33 IF(NORM)36,36,61
36 DO 48 I=1,M
    EN1(I)=EN1(I)/CS
    EN2(I)=EN2(I)/CS
    UFLEXS(I)=UFLEXS(I)/CD
    WFLEXS(I)=WFLEXS(I)/CD
    W1(I)=W1(I)/CD
48 U1(I)=U1(I)/CD
61 WRITE OUTPUT TAPE 3,109
    IF(NORM)13,13,14
13 WRITE OUTPUT TAPE 3,310
    GO TO 15
14 WRITE OUTPUT TAPE 3,110
15 WRITE OUTPUT TAPE 3,811,(GAMMR(I),U1(I),W1(I),EN1(I),EN2(I),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)184,84,85
84 WRITE OUTPUT TAPE 3,348
    GO TO 86
85 WRITE OUTPUT TAPE 3,148
86 WRITE OUTPUT TAPE 3,144,(GAMMR(I),UFLEXS(I),WFLEXS(I),I=1,M)
    WRITE OUTPUT TAPE 3,107
    GO TO 10
34 IF(N)57,57,56
57 DO 60 I=1,M
    A(I)=A(I)+EN1(I)*CPHI
    B(I)=B(I)+EN2(I)*CPHI
    WFLEXA(I)=WFLEXA(I)+WFLEXS(I)*CPHI
    UFLEXA(I)=UFLEXA(I)+UFLEXS(I)*CPHI
    P(I)=P(I)+W1(I)*CPHI
    Q(I)=Q(I)+U1(I)*CPHI
    CNORM=1.
    IF(NORM)44,44,45
44 DO 46 I=1,M
    A(I)=A(I)/CS
    B(I)=B(I)/CS
    C(I)=C(I)/CS
    G(I)=G(I)/CD
    P(I)=P(I)/CD
    Q(I)=Q(I)/CD
    UFLEXA(I)=UFLEXA(I)/CD
    VFLEXA(I)=VFLEXA(I)/CD
    WFLEXA(I)=WFLEXA(I)/CD
46 CONTINUE
    CNORM=CD

```

TABLE I (Continued)

```

45 IF(NORM)16,16,17
16 WRITE OUTPUT TAPE 3,303
   GO TO 18
17 WRITE OUTPUT TAPE 3,103
18 WRITE OUTPUT TAPE 3,106,(S(I),A(I),B(I),C(I),I=1,M)
   WRITE OUTPUT TAPE 3,107
   IF(KD-1)42,42,43
42 IF(NORM)19,19,20
19 WRITE OUTPUT TAPE 3,304
   GO TO 21
20 WRITE OUTPUT TAPE 3,104
21 WRITE OUTPUT TAPE 3,812,(S(I),Q(I),G(I),P(I),I=1,M)
   WRITE OUTPUT TAPE 3,107
   IF(NORM)87,87,88
87 WRITE OUTPUT TAPE 3,349
   GO TO 89
88 WRITE OUTPUT TAPE 3,149
89 WRITE OUTPUT TAPE 3,106,(S(I),UFLEXA(I),VFLEXA(I),WFLEXA(I),I=1,M)
   WRITE OUTPUT TAPE 3,107
   IF(KD-1)10,43,43
43 DO 49 I=1,M
   P(I)=P(1)-W1(I) /CNORM
   Q(I)=Q(I)-U1(I) /CNORM
   WFLEXA(I)=WFLEXA(I)-WFLEXS(I)/CNORM
   UFLEXA(I)=UFLEXA(I)-UFLEXS(I)/CNORM
49 CONTINUE
   WRITE OUTPUT TAPE 3,142
   IF(NORM)22,22,23
22 WRITE OUTPUT TAPE 3,439
   GO TO 24
23 WRITE OUTPUT TAPE 3,239
24 WRITE OUTPUT TAPE 3,812,(S(I),Q(I),G(I),P(I),I=1,M)
   WRITE OUTPUT TAPE 3,107
   IF(NORM)200,200,201
200 WRITE OUTPUT TAPE 3,440
   GO TO 202
201 WRITE OUTPUT TAPE 3,240
202 WRITE OUTPUT TAPE 3,106,(S(I),UFLEXA(I),VFLEXA(I),WFLEXA(I),I=1,M)
   GO TO 10
56 DO 3 J=1,N
   DO 3 I=1,M
   V(I,J)=G(I)      *COSF(THETA(J))
   W(I,J)=      SINF(THETA(J))*P(I)          +W1(I)*CPHI
   U(I,J)=      SINF(THETA(J))*Q(I)          +U1(I)*CPHI
   VFLEX(I,J)=VFLEXA(I)*COSF(THETA(J))
   WFLEX(I,J)=WFLEXA(I)*SINF(THETA(J))+WFLEXS(I)*CPHI
   UFLEX(I,J)=UFLEXA(I)*SINF(THETA(J))+UFLEXS(I)*CPHI
   ENR(I,J)=A(I)      *SINF(THETA(J))          +EN1(I)*CPHI
   ENT(I,J)=B(I)      *SINF(THETA(J))          +EN2(I)*CPHI
   ENRT(I,J)=C(I)*COSF(THETA(J))
3 CONTINUE
DO 12 J=1,N
12 THETA(J)=THETA(J)*RAD
SKIP=0
CNORM=1.
IF(NORM)64,64,65
64 DO 66 I=1,M
   DO 66 J=1,N

```

TABLE I (Continued)

```

W(I,J)=W(I,J)/CD
U(I,J)=U(I,J)/CD
V(I,J)=V(I,J)/CD
WFLEX(I,J)=WFLEX(I,J)/CD
UFLEX(I,J)=UFLEX(I,J)/CD
VFLEX(I,J)=VFLEX(I,J)/CD
ENR(I,J)=ENR(I,J)/CS
ENT(I,J)=ENT(I,J)/CS
ENRT(I,J)=ENRT(I,J)/CS
66 CONTINUE
CNORM=CD
WRITE OUTPUT TAPE 3,330
GO TO 25
65 WRITE OUTPUT TAPE 3,130
25 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
WRITE OUTPUT TAPE 3,114
WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(ENR(I,J),J=1,N),I=1,M)
WRITE OUTPUT TAPE 3,107
IF(NORM)26,26,27
26 WRITE OUTPUT TAPE 3,331
GO TO 28
27 WRITE OUTPUT TAPE 3,131
28 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
WRITE OUTPUT TAPE 3,114
WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(ENT(I,J),J=1,N),I=1,M)
WRITE OUTPUT TAPE 3,107
IF(NORM)29,29,37
29 WRITE OUTPUT TAPE 3,332
GO TO 41
37 WRITE OUTPUT TAPE 3,132
41 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
WRITE OUTPUT TAPE 3,114
WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(ENRT(I,J),J=1,N),I=1,M)
WRITE OUTPUT TAPE 3,107
IF(KD-1)82,82,83
82 IF(NORM)501,501,504
501 IF(SKIP)502,502,503
502 WRITE OUTPUT TAPE 3,333
GO TO 507
503 WRITE OUTPUT TAPE 3,433
GO TO 507
504 IF(SKIP)505,505,506
505 WRITE OUTPUT TAPE 3,133
GO TO 507
506 WRITE OUTPUT TAPE 3,233
507 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
WRITE OUTPUT TAPE 3,114
WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(U(I,J),J=1,N),I=1,M)
WRITE OUTPUT TAPE 3,107
IF(NORM)508,508,511
508 IF(SKIP)509,509,510
509 WRITE OUTPUT TAPE 3,334
GO TO 514
510 WRITE OUTPUT TAPE 3,434
GO TO 514
511 IF(SKIP)512,512,513
512 WRITE OUTPUT TAPE 3,134
GO TO 514

```

TABLE I (Continued)

```
513 WRITE OUTPUT TAPE 3,234
514 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
    WRITE OUTPUT TAPE 3,114
    WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(V(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)515,515,518
515 IF(SKIP)516,516,517
516 WRITE OUTPUT TAPE 3,335
    GO TO 521
517 WRITE OUTPUT TAPE 3,435
    GO TO 521
518 IF(SKIP)519,519,520
519 WRITE OUTPUT TAPE 3,135
    GO TO 521
520 WRITE OUTPUT TAPE 3,235
521 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
    WRITE OUTPUT TAPE 3,114
    WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(W(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)522,522,525
522 IF(SKIP)523,523,524
523 WRITE OUTPUT TAPE 3,350
    GO TO 528
524 WRITE OUTPUT TAPE 3,436
    GO TO 528
525 IF(SKIP)526,526,527
526 WRITE OUTPUT TAPE 3,150
    GO TO 528
527 WRITE OUTPUT TAPE 3,236
528 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
    WRITE OUTPUT TAPE 3,114
    WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(UFLEX(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)529,529,532
529 IF(SKIP)530,530,531
530 WRITE OUTPUT TAPE 3,351
    GO TO 535
531 WRITE OUTPUT TAPE 3,437
    GO TO 535
532 IF(SKIP)533,533,534
533 WRITE OUTPUT TAPE 3,151
    GO TO 535
534 WRITE OUTPUT TAPE 3,237
535 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
    WRITE OUTPUT TAPE 3,114
    WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(VFLEX(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)536,536,539
536 IF(SKIP)537,537,538
537 WRITE OUTPUT TAPE 3,352
    GO TO 542
538 WRITE OUTPUT TAPE 3,438
    GO TO 542
539 IF(SKIP)540,540,541
540 WRITE OUTPUT TAPE 3,152
    GO TO 542
541 WRITE OUTPUT TAPE 3,238
542 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
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TABLE I (Continued)

```

      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(WFLEX(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,107
      IF(SKIP)215,215,10
215 IF(KD-1)10,83,83
   83 DO 224 J=1,N
      DO 223 I=1,M
         UFLEX(I,J)=UFLEX(I,J)-U1(I)/CNORM
         WFLEX(I,J)=WFLEX(I,J)-W1(I)/CNORM
         U(I,J)=U(I,J)-U1(I)/CNORM
223 W(I,J)=W(I,J)-W1(I)/CNORM
224 CONTINUE
      WRITE OUTPUT TAPE 3,142
      SKIP=1
      GO TO 82
C
C - - (8) RETURN TO (1)
C
101 FORMAT(8I5)
102 FORMAT(5E14.8)
103 FORMAT(116H      R(IN.)          NR(LB./IN.)
1           NTHETA(LB./IN.)      NRTHETA(LB./IN.)    /////
104 FORMAT (104H      R(IN.)          U(IN.)
1           V(IN.)          W(IN.)    /////
105 FORMAT (4F15.9)
106 FORMAT(F25.4,3F25.8)
107 FORMAT (1H1)
108 FORMAT(4F24.5,F18.2)
109 FORMAT (75H THE LOADING AS WELL AS THE DEFORMATION OF THE SHELL IS
      1S AXISYMMETRIC    /////
110 FORMAT(122H      R(IN.)          U(IN.)
1           W(IN.)          NR(LB./IN.)      NTHETA(LB./IN.
2)///)
111 FORMAT(5F24.5)
112 FORMAT(120H      WEIGHT DENSITY(LB./IN.2)  YOUNGS MODULUS(LB./IN.
12)      THICKNESS(IN.)      FOCAL LENGTH(IN.)  POISONS RATIO/
2///)
113 FORMAT(116H      R1(IN.)          R2(IN.)
1           R3(IN.)          R4(IN.)          PSI(DEG.)/////
114 FORMAT (////)
115 FORMAT ( 50H THE SHELL IS FIXED TANGENTIALLY AT BOTH EDGES    /////
116 FORMAT ( 70H THE SHELL IS FIXED TANGENTIALLY AT R2 AND IS FREE A
      1T R1    /////
117 FORMAT(51H      THETA3(DEG.)      THETA4(DEG.)/////
118 FORMAT(75H THE SHELL IS CLOSED AT THE APEX AND IS FIXED TANGENTI
      1ALLY AT R2    /////
119 FORMAT(2F24.2)
130 FORMAT(50H      -----STRESS RESULTANT NR(LB./IN.)----- /////
131 FORMAT(53H      -----STRESS RESULTANT NTHETA(LB./IN.)-----///
1     //)
132 FORMAT(54H      -----STRESS RESULTANT NRTHETA(LB./IN.)-----///
1     //)
133 FORMAT(50H      -----DISPLACEMENT U(IN.)----  /////
134 FORMAT(50H      -----DISPLACEMENT V(IN.)----  /////
135 FORMAT(50H      -----DISPLACEMENT W(IN.)----  /////
139 FORMAT(51H ---NORMALIZED RESULTS BY MEMBRANE ANALYSIS    /////
140 FORMAT(51H ---UN-NORMALIZED RESULTS BY MEMBRANE ANALYSIS  /////
142 FORMAT(95H THE DISTORTION OF THE SHELL GIVEN BELOW IS MEASURED REL

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TABLE I (Continued)

1ATIVE TO THAT OF THE FACE-UP POSITION    ////)  
 143 FORMAT(55H THE PROGRAM DOES NOT ACCEPT A NEGATIVE POINTING ANGLE  
 1////)  
 144 FORMAT(F24.4,2F24.8)  
 148 FORMAT(116H    R(IN.)                                  UFLEX(IN.)  
   1    ///)  
 149 FORMAT(116H    R(IN.)                                  UFLEX(IN.)  
   1    WFLEX(IN.)                                  ///)  
 150 FORMAT(50H    ----DISPLACEMENT UFLEX(IN.)----                  ///)  
 151 FORMAT(50H    ----DISPLACEMENT VFLEX(IN.)----                          ///)  
 152 FORMAT(50H    ----DISPLACEMENT WFLEX(IN.)----                          ///)  
 160 FORMAT(56H A HOMOGENEOUS PARABOLOIDAL SHELL SUBJECTED TO GRAVITY  
 1////)  
 161 FORMAT(56H A LAMINAR PARABOLOIDAL SHELL SUBJECTED TO GRAVITY  
 1////)  
 162 FORMAT(56H A SANDWICH PARABOLOIDAL SHELL SUBJECTED TO GRAVITY  
 1////)  
 163 FORMAT(116H STRETCHING STIFFNESS(LB/IN\*\*2) WEIGHT DENSITY(LB/IN\*  
   1\*2) FOCAL LENGTH(IN.) POISSONS RATIO                  ///)  
 164 FORMAT(116H YOUNGS MODULUS(LB/IN\*\*2)                          CORE THICKNESS(IN.)  
   1    SKIN THICKNESS(IN.)                                  RHO OF SKIN(LB/IN\*\*3)                  ///)  
 165 FORMAT(116H    RHO OF CORE(LB/IN\*\*3)                                  WEIGHT DENSITY(LB/IN\*  
   1\*2) FOCAL LENGTH(IN.)                                POISSONS RATIO                                  ///)  
 205 FORMAT(12A6)  
 229 FORMAT(19X,2HA1,23X,2HA2,////)  
 230 FORMAT(19X,2HC1,23X,2HC2,23X,2HC3,23X,2HC4,////)  
 231 FORMAT(4F25.6)  
 233 FORMAT(50H    ----DISPLACEMENT U TILDE(IN.)----                  ///)  
 234 FORMAT(50H    ----DISPLACEMENT V TILDE(IN.)----                  ///)  
 235 FORMAT(50H    ----DISPLACEMENT W TILDE(IN.)----                  ///)  
 236 FORMAT(50H    ----DISPLACEMENT UFLEX TILDE(IN.)----                  ///)  
 237 FORMAT(50H    ----DISPLACEMENT VFLEX TILDE(IN.)----                  ///)  
 238 FORMAT(50H    ----DISPLACEMENT WFLEX TILDE(IN.)----                  ///)  
 239 FORMAT(116H    R(IN.)    U TILDE(IN.)  
   1    V TILDE(IN.)                                        W TILDE(IN.)                                  ///)  
 240 FORMAT(116H    R(IN.)    UFLEX TILDE(IN.)  
   1    UFLEX TILDE(IN.)    ///)  
 303 FORMAT(116H    GAMMA    NR\*  
   1    NTHETA\*    NRTHETA\*    ///)  
 304 FORMAT(104H    GAMMA    U\*  
   1    V\*    W\*    ///)  
 310 FORMAT(121H    GAMMA    U\*  
   1    W\*    NR\*    NTHETA\*  
 2////)  
 330 FORMAT(75H    ----NORMALIZED STRESS RESULTANT NR\*(DIMENSION  
   1LESS)----    ///)  
 331 FORMAT(75H    ----NORMALIZED STRESS RESULTANT NTHETA\*(DIMEN  
   1SIONLESS)----                                        ///)  
 332 FORMAT(75H    ----NORMALIZED STRESS RESULTANT NRTHETA\*(DIME  
   1NSIONLESS)----                                        ///)  
 333 FORMAT(60H    ----NORMALIZED DISPLACEMENT U\*(DIMENSIONLESS)  
   1----//----    ///)  
 334 FORMAT(60H    ----NORMALIZED DISPLACEMENT V\*(DIMENSIONLESS)  
   1----//----    ///)  
 335 FORMAT(60H    ----NORMALIZED DISPLACEMENT W\*(DIMENSIONLESS)  
   1----//----    ///)  
 348 FORMAT(116H    GAMMA    UFLEX\*  
   1    WFLEX\*    ///)

TABLE I (Continued)

349 FORMAT(116H	GAMMA	UFLEX*
1          VFLEX*	WFLEX*	////)
350 FORMAT(65H	----NORMALIZED DISPLACEMENT	UFLEX*(DIMENSIONL
1FSS----  ////)		
351 FORMAT(65H	----NORMALIZED DISPLACEMENT	VFLEX*(DIMENSIONL
1FSS----  ////)		
352 FORMAT(65H	----NORMALIZED DISPLACEMENT	WFLEX*(DIMENSIONL
1ESS----  ////)		
433 FORMAT(75H	----NORMALIZED DISPLACEMENT	U TILDE*(DIMENSIO
1LESS)---	////)	
434 FORMAT(75H	----NORMALIZED DISPLACEMENT	V TILDE*(DIMENSIO
1LESS)---	////)	
435 FORMAT(75H	----NORMALIZED DISPLACEMENT	W TILDE*(DIMENSIO
1LESS)---	////)	
436 FORMAT(75H	----NORMALIZED DISPLACEMENT	UFLEX TILDE* (DIM
1EENSIONLESS)---	////)	
437 FORMAT(75H	----NORMALIZED DISPLACEMENT	VFLEX TILDE* (DIM
1EENSIONLESS)---	////)	
438 FORMAT(75H	----NORMALIZED DISPLACEMENT	WFLEX TILDE* (DIM
1EENSIONLESS)---	////)	
439 FORMAT(116H	GAMMA	U TILDE*
1          V TILDE*	W TILDE*	////)
440 FORMAT(116H	GAMMA	UFLEX TILDE*
1          VFLEX TILDE*	WFLEX TILDE*	////)
811 FORMAT(F24.4,E24.6,3F24.8)		
812 FORMAT(F25.4,E25.6,2F25.8)		
*          END		
*          DATA		

## 1. Input

The input to our program as indicated by step (1) in Fig. 12 consists of a number of records prestored on tape A2.

### Record 1 Control Parameters (6I5)

This line contains six non-negative fixed-point variables (integers).

M	N	KE	NORM	KD	KP
---	---	----	------	----	----

M ( $\leq 150$ ) Number of (equally spaced)  $\gamma$  - tabular points between R3 and R4 (R3 and R4 are to be defined later) for any fixed value of  $\Theta$ .

N ( $\leq 10$ ) Number of (equally spaced)  $\Theta$  - tabular points between THETA3 and THETA4 (THETA3 and THETA4 are to be defined later) for any fixed value of  $\gamma$ . If only those quantities with superscript zero are desired [see Eqs. (11-85) and (11-86)], set N equal to zero.

KE Control parameter specifying the edge conditions at R1 and R2 (R1 and R2 will be defined later. They are not to be confused with  $r_1$  and  $r_2$  which are the lower and upper edge of the shell).

KE < 1 Shell is fixed tangentially at both edges.

KE = 1 Shell is fixed tangentially at R2 and is free at R1.

KE > 1 Shell is closed at the apex and is supported tangentially at R2.

NORM Control parameter specifying whether normalized results are desired.

NORM = 0 Normalized results.

NORM > 0 Un-normalized results.

KD Control parameter specifying the type of distortion to be given in the output.

KD < 1  $u$ ,  $v$ ,  $w$ , and the corresponding  $u_{flex}$ ,  $v_{flex}$ ,  $w_{flex}$  are given in the output.

KD > 1  $\tilde{u}$ ,  $\tilde{v}$ ,  $\tilde{w}$ , and the corresponding  $\tilde{u}_{flex}$ ,  $\tilde{v}_{flex}$ ,  $\tilde{w}_{flex}$  are given in the output.

KD = 1  $u$ ,  $v$ ,  $w$ ,  $\tilde{u}$ ,  $\tilde{v}$ ,  $\tilde{w}$ , as well as the corresponding six flex quantities.

If N = 0, a superscript zero should be added to these displacement quantities.

KP Control parameter specifying the type of shell under consideration.  
Set

KP < 1 if the shell is homogeneous,

KP > 1 if the shell is a sandwiched construction with a very soft core,

KP = 1 if the shell is laminar in that its material properties vary across the thickness in a manner other than previously mentioned.

## Record 2 Geometrical Parameters (4F15.9)

This line contains four non-negative F-type floating point variables.

R1	R2	R3	R4
----	----	----	----

R1 (in.) Value of  $r$  at the free edge of the shell. If the shell is closed at the apex, then  $R1 = 0$ . If the shell is fixed tangentially at both edges, then set R1 equal to the value of  $r$  at the lower edge.

R2 (in.) Value of  $r$  at the tangentially fixed edge. If both edges of the shell are fixed tangentially, set R2 equal to the value of  $r$  at the upper edge.

R3 (in.) Smallest value of  $r$  at which the stresses and distortion of the shell are to be given in the output. Generally, this is the same as  $r_1$ . However, if the shell is closed at the apex, it is usually wise to avoid calculating the limiting value of the desired output at the apex by setting  $R3 > 0$ .

R4 (in.) Largest value of  $r$  at which the stresses and distortion of the shell are to be given in the output. Generally, this is the same as  $r_2$ .

## Record 3 Geometrical and Material Parameters (3F15.9)

This line contains three non-negative F-type floating point variables.

F	PR	PSI
---	----	-----

F Focal length of the paraboloidal surface (in.).

PR Poisson's ratio.

PSI Pointing angle  $\psi$  in fractions of  $\pi$  (e.g., if  $\psi = 45^\circ$ , then psi = 0.25).

## Record 4 Geometrical and Material Parameters (5E14.8)

KP < 1 This line contains three E-type floating point variables.

RHO	H	E
-----	---	---

RHO Volume weight density of shell ( $\text{lb/in.}^3$ ).

H Shell thickness (in.).

E Young's modulus ( $\text{lb/in.}^2$ ).

KP > 1 This line contains five E-type floating point variables.

RHS	RHC	T	H	E
-----	-----	---	---	---

RHS Volume weight density of skin ( $\text{lb/in.}^3$ ).

RHC Volume weight density of core ( $\text{lb/in.}^3$ ).

T Thickness of skin (in.).

H Thickness of core (in.).

E Young's modulus of skin ( $\text{lb/in.}^2$ ).

KP = 1 This line contains two E-type floating point variables.

RHO	CC
-----	----

RHO Surface weight density of shell (lb/in.<sup>2</sup>).

CC Extensional stiffness of shell.

#### Record 5 Geometrical Parameters (2F15.9)

This line contains two F-type floating point variables.

THETA3	THETA4
--------	--------

THETA3 Smallest value of  $\Theta$  in fractions of  $\pi$  at which the stresses and distortion of the shell are to be given in the output.

THETA4 Largest value of  $\Theta$  in fractions of  $\pi$  at which the stresses and distortion of the shell are to be given in the output.

#### Record 6 Variable Format Statement 1

This line provides a format statement for listing the set of  $\Theta$ -tabular points (see also Sec. III-B-2), for example,

(19H R(IN.)/THETA(DEG.), NF11.2)
----------------------------------

#### Record 7 Variable Format Statement 11

This line provides a format statement for the values of each physical quantity at different positions (see also Sec. III-B-2), for example,

(F19.4, NF11.6)
-----------------

Remark:— If  $N = 0$  or if  $PS1 = 0$ , then the input must consist of only the first four records in the preceding list. Additional records will be treated as a new set of input upon return. For the purpose of an Express Run, each of these records is punched out on one card. A complete set of input will then consist of seven cards (or four cards if  $N = 0$  or if  $\psi = 0$ ). It is to be placed after the DATA card which in turn follows the object deck. Additional runs can be made by merely placing additional sets of input after the first set.

If some restriction on the input is violated, the program returns to step (1) to take in a new set of data. It may or may not give a statement pointing out the source of trouble.

## 2. Output

The first part of the output, given by step (2) in Fig. 12, reproduces for the record the input to the program. It states the problem, the edge conditions, the various geometrical and material parameters and whether the numerical results have been normalized.

The second part of the output, given by step (7) in Fig. 12, gives the values of the six physical quantities,  $N_r$ ,  $N_\Theta$ ,  $N_{r\Theta}$ ,  $u$ ,  $v$ , and  $w$  at the tabular points. If  $\psi = 0$ , the behavior of the shell is axisymmetric. Thus,  $v \equiv N_{r\Theta} \equiv 0$ , and  $N_r$ ,  $N_\Theta$ ,  $u$ , and  $w$  are functions of  $\gamma$  only. The output in this case is given in one block of five columns of numbers. The first column lists the set of  $\gamma$ -tabular points. The next four columns list the values of  $u$ ,  $w$ ,  $N_r$ , and  $N_\Theta$  in that order, corresponding to these values of  $\gamma$ .

If  $\psi \neq 0$ , the stress resultants and the displacements generally depend on both  $\gamma$  and  $\Theta$ . The output for each of these quantities is arranged in a rectangular block so that each column of data

corresponds to the values of the quantity for fixed value of  $\Theta$  and for different values of  $\gamma$ , and each row corresponds to the values of the quantity for a fixed  $\gamma$  and for different values of  $\Theta$ . Inasmuch as  $N$  is not constant, the number of columns may vary for different runs. Therefore, if  $N > 0$ , two variable format statements are needed for the output. The first of these is used to list all possible values of  $\Theta$  as the first record of the block. The second is used repeatedly to list records of  $(N + 1)$  numbers. The first number of each record is a value of  $\gamma$  while the remaining numbers are the  $N$  values of the physical quantity in question corresponding to the  $N$  positions given by this value of  $r$  and the  $N \Theta$ -tabular points listed in the first record. Examples given in a later section will clarify this discussion. Having listed all the stress and displacement quantities, the next block of data gives the maximum values (in modulus) of  $u_{flex}^0$ ,  $v_{flex}^0$ , and  $w_{flex}^0$  or  $\tilde{u}_{flex}^0$ ,  $\tilde{v}_{flex}^0$ , and  $\tilde{w}_{flex}^0$  for the set of tabular  $\gamma$ 's.

If  $N = 0$ , so that only the zero-superscripted quantities are requested, this second part of the output will be arranged in three blocks of data each consisting of four columns. The first of these columns in each block lists the set of  $\gamma$ -tabular points. The remaining three columns of the first block list  $N_r^0$ ,  $N_\Theta^0$ , and  $N_{r\Theta}^0$ , those of the second block list  $u^0$ ,  $v^0$ , and  $w^0$ , and those of the third list  $u_{flex}^0$ ,  $v_{flex}^0$ , and  $w_{flex}^0$  for the given set of tabular points.

The above discussion applies only when  $KD < 1$ . For  $KD > 1$ ,  $\tilde{u}^0$ ,  $\tilde{v}^0$ , and  $\tilde{w}^0$  and the corresponding  $\tilde{u}_{flex}^0$ ,  $\tilde{v}_{flex}^0$ , and  $\tilde{w}_{flex}^0$  will take the place of  $u^0$ ,  $v^0$ , and  $w^0$  and the corresponding  $u_{flex}^0$ ,  $v_{flex}^0$ , and  $w_{flex}^0$ . If  $KD = 1$ , additional blocks of data will appear for the obvious reason. Keeping in mind the discussion in the earlier sections, the output in this case is self-explanatory.

The numerical results in the output are generally printed in the F-type floating point format. There are a few exceptions due to the size of the numbers involved. In these exceptional cases, the E-type floating point format is used.

### 3. Operational Information

The main program is coded in FORTRAN language for a 32K IBM 7090 (and IBM 7094) and is compiled by the FORTRAN II compiler. It requires no routines other than those appearing as library routines on the Lincoln Laboratory library tape. It should be cautioned that these may be different from the routines under the same name used elsewhere. The correspondence between logical tape units and machine tape units is also different at Lincoln Laboratory. Approximately 11,000 storage locations have not been touched by the program. Neither sense switch nor sense light is used. Some of the subscripted variables and their analytical counterparts are presented in Table II.

It should be understood that if normalized results are requested, the stress and displacement expressions in the third column of Table II should be replaced by the corresponding starred quantities.

## C. EXAMPLES

### 1. Axisymmetric Deformations

Consider a sandwich shell supported at the lower edge  $r_1$  in a face-up position ( $\psi = 0$ ) with

$$\begin{aligned} rhc &= 0.01 \text{ lb/in.}^3 \\ rhs &= 0.1 \text{ lb/in.}^3 \\ E &= 10^7 \text{ lb/in.}^2 \\ \nu &= 0.3 \end{aligned}$$

TABLE II  
SOME SUBSCRIPTED VARIABLES  
AND THEIR ANALYTICAL COUNTERPARTS

Subscripted Variable	Dimensionality	Analytical Expression
U	150 × 10	u
V	150 × 10	v
W	150 × 10	w
UFLEX	150 × 10	$u_{flex}$
VFLEX	150 × 10	$v_{flex}$
WFLEX	150 × 10	$w_{flex}$
ENR	150 × 10	$N_r$
ENT	150 × 10	$N_\Theta$
ENRT	150 × 10	$N_{r\Theta}$
U1	150	$u^s$
W1	150	$w^s$
G	150	$v^a$
P	150	$w^a$
Q	150	$u^a$
EN1	150	$N_r^s$
EN2	150	$N_\Theta^s$
A	150	$N_r^a$
B	150	$N_\Theta^a$
C	150	$N_{r\Theta}^a$
UFLEXA	150	$u_{flex}^a$
VFLEXA	150	$v_{flex}^a$
WFLEXA	150	$w_{flex}^a$
UFLEXS	150	$u_{flex}^s$
VFLEXS	150	$v_{flex}^s$
WFLEXS	150	$w_{flex}^s$
GAMMR	150	r
GAMMA	150	$\gamma$
THETA	150	$\Theta$







TABLE III  
COMPUTER OUTPUT (AXISYMMETRIC DEFORMATIONS)

A SANDWICH PARABOLOIDAL SHELL SUBJECTED TO GRAVITY

---UN-NORMALIZED RESULTS BY MEMBRANE ANALYSIS

THE SHELL IS FIXED TANGENTIALLY AT R2 AND IS FREE AT R1

YOUNG'S MODULUS(LB/IN<sup>2</sup>) CORE THICKNESS(IN.) SKIN THICKNESS(IN.) RHD OF SKIN(LB/IN<sup>2</sup>)

1802000.00000 0.50000 0.10000 0.10000

RHD OF CORE(LB/IN<sup>2</sup>) WEIGHT DENSITY(LB/IN<sup>2</sup>) FOCAL LENGTH(IN.) POISSON'S RATIO

0.01022 0.02400 500.00000 0.30000

R1(IN.) R2(IN.) R3(IN.) R4(IN.) PS1(NF0.1)

300.00000 100.00000 100.00000 300.00000 2.

THE LOADING AS WELL AS THE DEFORMATION OF THE SHELL IS AXISYMMETRIC

R1(IN.) U1(IN.) W1(IN.) NR(LB./IN.) NTHETA(LB./IN.)

100.0000	0.	-0.07614445	-98.85535336	121.87658787
150.0000	-0.594946E-02	-0.07597278	-37.41095829	60.58773375
200.0000	-0.106249E-01	-0.07553648	-15.78601956	39.17886400
250.0000	-0.147280E-01	-0.07489289	-5.64614444	29.31481658
300.0000	-0.184626E-01	-0.07407248	0.	23.99999928

R1(IN.) UFLFX(IN.) WELEX(IN.)

100.0000	0.00755791	-0.00056531
150.0000	0.00531790	-0.0005453
200.0000	0.00426735	-0.00105540
250.0000	0.00369412	-0.00120465
300.0000	0.00336318	-0.00131774









TABLE IV COMPUTER OUTPUT (UNSYMMETRIC DEFORMATIONS)					
A HOMOGENEUS PARABOLOIDAL SHELL SUBJECTED TO GRAVITY					
---NORMALIZED RESULTS BY MEMBRANE ANALYSIS					
THE SHELL IS FIXED TANGENTIALLY AT BOTH EDGES					
WEIGHT DENSITY(lb./in. <sup>2</sup> )	YOUNG'S MODULUS(lb./in. <sup>2</sup> )	THICKNESS(in.)	FOCAL LENGTH(in.)	POISSON'S RATIO	
0.0010J	1000000.00000	0.01000	500.00000	0.30000	
		R1(in.)	R3(in.)	R4(in.)	Psi(deg.)
		100.00000	300.00000	100.00000	30.00
		THETA3(deg.)	THETA4(deg.)		
		-90.00J	90.00		

TABLE IV (Continued)

----NORMALIZED STRESS RESULTANT NR(DIMENSIONLESS)----						
GAMMA/THETA(DEG.)	-90.0.	-60.00	-30.02	-0.	30.02	60.00
0.100	0.287559	0.304694	0.351534	0.415518	0.479583	0.526343
0.150	0.255226	0.365669	0.391181	0.431036	0.468892	0.496684
0.200	0.294514	0.428773	0.417654	0.460713	0.463773	0.486654
0.250	0.424292	0.427715	0.477666	0.449839	0.462612	0.471962
0.300	0.453388	0.451637	0.455848	0.459789	0.464369	0.467788
0.350	-	-	-	-	-	-
0.400	-	-	-	-	-	-
0.450	-	-	-	-	-	-
0.500	-	-	-	-	-	-
0.550	-	-	-	-	-	-
0.600	-	-	-	-	-	-
0.650	-	-	-	-	-	-
0.700	-	-	-	-	-	-
0.750	-	-	-	-	-	-
0.800	-	-	-	-	-	-
0.850	-	-	-	-	-	-
0.900	-	-	-	-	-	-
0.950	-	-	-	-	-	-
1.000	-	-	-	-	-	-

----NORMALIZED STRESS RESULTANT NR(THETA*(DIMENSIONLESS))----						
GAMMA/THETA(DEG.)	-90.0.	-60.00	-30.02	-0.	30.02	60.00
0.100	0.531323	0.521447	0.452972	0.454621	0.416276	0.388195
0.150	0.443519	0.443647	0.444996	0.444447	0.444511	0.377919
0.200	0.396638	0.396365	0.414435	0.442262	0.478890	0.445429
0.250	0.341691	0.352177	0.392217	0.442649	0.493126	0.494660
0.300	0.321776	0.321776	0.375550	0.444274	0.414999	0.536847
0.350	-	-	-	-	-	0.566773
0.400	-	-	-	-	-	-
0.450	-	-	-	-	-	-
0.500	-	-	-	-	-	-
0.550	-	-	-	-	-	-
0.600	-	-	-	-	-	-
0.650	-	-	-	-	-	-
0.700	-	-	-	-	-	-
0.750	-	-	-	-	-	-
0.800	-	-	-	-	-	-
0.850	-	-	-	-	-	-
0.900	-	-	-	-	-	-
0.950	-	-	-	-	-	-
1.000	-	-	-	-	-	-

----NORMALIZED STRESS RESULTANT NR(THETA*(DIMENSIONLESS))----						
GAMMA/THETA(DEG.)	-90.0.	-60.00	-30.02	-0.	30.02	60.00
0.100	0.052359	0.050689	0.104718	0.096689	0.052359	0.052359
0.150	0.018912	0.032275	0.037085	0.032757	0.018912	0.018912
0.200	-0.017564	-0.084442	-0.025129	-0.026442	-0.025564	-0.025564
0.250	-0.019423	-0.033641	-0.038866	-0.033641	-0.019423	-0.019423
0.300	-0.03497	-0.059258	-0.06194	-0.059858	-0.03497	-0.03497
0.350	-	-	-	-	-	-
0.400	-	-	-	-	-	-
0.450	-	-	-	-	-	-
0.500	-	-	-	-	-	-
0.550	-	-	-	-	-	-
0.600	-	-	-	-	-	-
0.650	-	-	-	-	-	-
0.700	-	-	-	-	-	-
0.750	-	-	-	-	-	-
0.800	-	-	-	-	-	-
0.850	-	-	-	-	-	-
0.900	-	-	-	-	-	-
0.950	-	-	-	-	-	-
1.000	-	-	-	-	-	-

TABLE IV (Continued)

THE DISTORTION OF THE SHELL GIVEN HELD IN IS MEASURED RELATIVE TO THAT OF THE FACE-UP POSITION

----NORMALIZED DISPLACEMENT U TILDE(DIMENSIONLESS)----

GAMMA/THETA(DEC.)	-90.00	-60.00	-30.00	-0.	30.00	60.00	90.00
0.100	-0.00000	-0.00000	-0.00000	-0.	0.00000	0.00000	0.
0.150	-0.00022	-0.00022	-0.00022	-0.	0.00022	0.00022	0.
0.200	-0.00035	-0.00035	-0.00035	-0.	0.00035	0.00035	0.
0.250	-0.00056	-0.00056	-0.00056	-0.	0.00056	0.00056	0.
0.300	-0.	-0.	-0.	-0.	0.	0.	0.

----NORMALIZED DISPLACEMENT V TILDE(DIMENSIONLESS)----

GAMMA/THETA(DEC.)	-90.00	-60.00	-30.00	-0.	30.00	60.00	90.00
0.100	-0.00000	-0.00000	-0.00000	-0.	0.00000	0.00000	0.
0.150	0.00000	0.00000	0.00000	0.	0.00000	0.00000	0.
0.200	0.00002	0.00002	0.00002	0.	0.00002	0.00002	0.
0.250	0.00002	0.00002	0.00002	0.	0.00002	0.00002	0.
0.300	-0.00002	-0.00002	-0.00002	-0.	0.00002	0.00002	0.

----NORMALIZED DISPLACEMENT W TILDE(DIMENSIONLESS)----

GAMMA/THETA(DEC.)	-90.00	-60.00	-30.00	-0.	30.00	60.00	90.00
0.100	-0.00000	-0.00000	-0.00000	-0.	0.00000	0.00000	0.
0.150	0.00000	0.00000	0.00000	0.	0.00000	0.00000	0.
0.200	0.00000	0.00000	0.00000	0.	0.00000	0.00000	0.
0.250	0.00000	0.00000	0.00000	0.	0.00000	0.00000	0.
0.300	-0.00000	-0.00000	-0.00000	-0.	0.00000	0.00000	0.

TABLE IV (Continued)

----NORMALIZED DISPLACEMENT UFLX TILDE* (DIMENSIONLESS)----						
GAMMA/THETA(CEG.)		-90.0°	-62.0°	-30.0°	30.0°	60.0°
0.100	-0.012328	-2.021889	0.013410	0.010864	0.034309	0.045616
0.150	-0.044671	0.022421	0.021807	0.020382	0.044156	0.056742
0.200	0.012259	2.015644	0.024892	0.037525	0.050157	0.061138
0.250	0.031658	0.037476	0.00951	0.046244	0.051338	0.062798
0.300	0.064636	0.065174	0.059555	0.054474	0.049393	0.056031
						0.044313
						0.045674
						0.044313
----NORMALIZED DISPLACEMENT VFLX TILDE* (DIMENSIONLESS)----						
GAMMA/THETA(CEG.)		-90.0°	-62.0°	-30.0°	30.0°	60.0°
0.100	0.040200	0.026730	0.035985	0.041462	0.03595	0.02730
0.150	0.020320	0.02390	0.019173	0.056782	0.049173	0.028390
0.200	0.000202	0.031692	0.000703	0.06774	0.058003	0.030892
0.250	0.000200	0.030162	0.006699	0.076324	0.066699	0.030162
0.300	0.000200	0.041557	0.01908	0.063115	0.071982	0.041557
						0.030162
						0.030162
						0.030162
----NORMALIZED DISPLACEMENT WFLX TILDE* (DIMENSIONLESS)----						
GAMMA/THETA(CEG.)		-90.0°	-62.0°	-30.0°	30.0°	60.0°
0.100	0.232115	0.233176	0.235975	0.239935	0.243895	0.246795
0.150	0.384954	0.361871	0.369733	0.384512	0.367292	0.315153
0.200	0.478900	0.46395	0.367514	0.236538	0.25261	0.096860
0.250	0.62551	0.55182	0.48298	0.34243	0.49988	-0.022415
0.300	0.69824	0.636256	0.452801	0.231062	0.22277	-0.134466
						-0.174132
						-0.236816

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13. ABSTRACT  <p>The primary design requirement of a high-performance antenna is that the reflecting surface remain paraboloidal. For an antenna housed in a radome, strength considerations play a minor design role. Therefore, the antenna must have adequate structural stiffness accompanied by minimum weight. The basic structural components of the antenna are paraboloidal panels, which, when joined together, form a surface of revolution. Such a structural configuration, if properly fabricated, can be considered as a shell. Shell structures derive many of their attractive features from their two-dimensional surface nature, which brings with it geometrical complications to the strain-deflection relations and the equilibrium equations. Although the deflections of trusses, beams, and space frameworks are well understood, well documented, and easily obtained, this is not true for shells. In fact, shell behavior is currently the major topic of study in the structural mechanics field. The available solutions for even simple loadings of simple shells are of such a form that numerical results are not easily obtained. For these reasons, Lincoln Laboratory has been actively studying the deformations of paraboloidal shells. This user's manual will describe the capability, potentiality, and idiosyncrasies of the various LLAPS computer programs which are products of the above study.</p>		
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