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SEQUENTIAL INFORMATION SEEKING: AN OPTIMAL STRATEGY AND OTHER RESULTS

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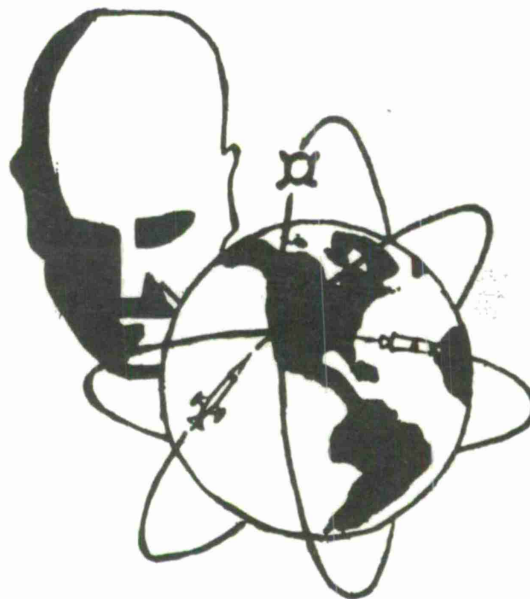David M. Messick

DECISION SCIENCES LABORATORY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
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Project 4690, Task 469003

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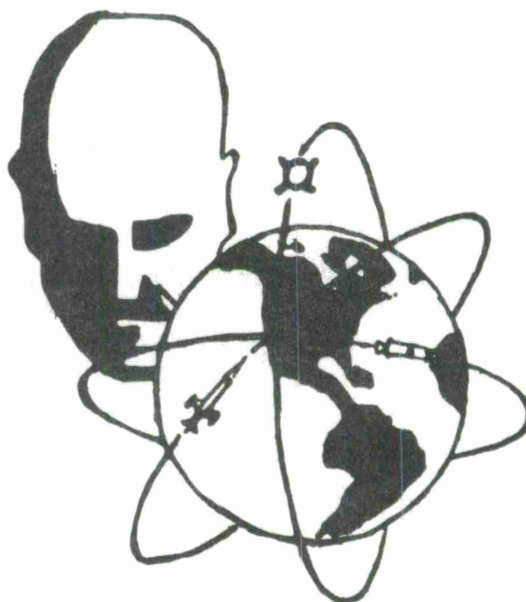
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FOREWORD

This study was conducted under Contract No. AF 19(628)-1610 at the Psychometric Laboratory, University of North Carolina, Chapel Hill, North Carolina. Dr Albert Amon served as principal investigator and Dr Anne Story, as contract monitor.
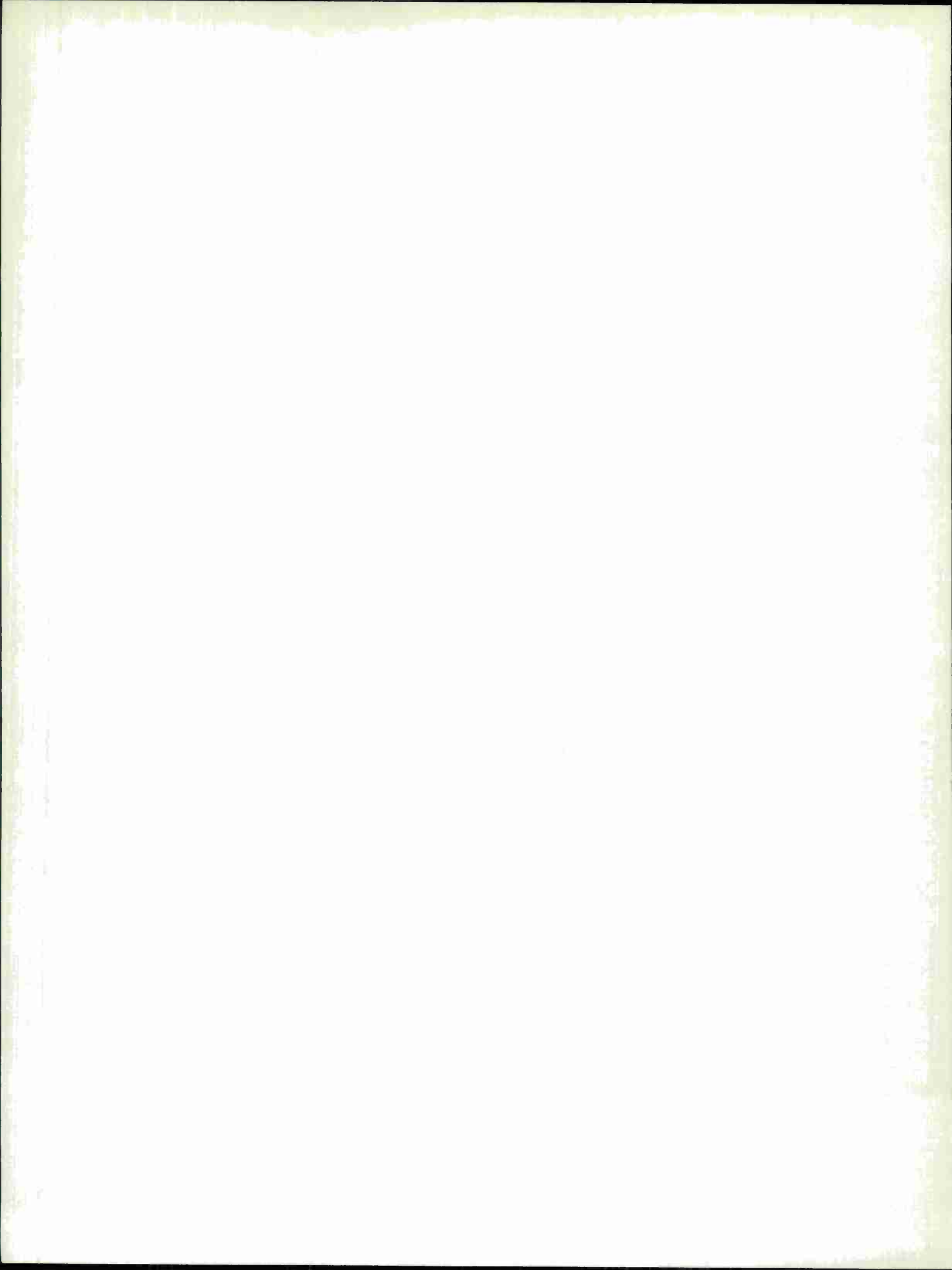
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ABSTRACT

This paper presents an optimal strategy for sequential sampling from binomial distributions. The strategy presented is general in that it is a "multi-action" rather than a two-action procedure. While the major task is to estimate the proportion, p, of "successes" in a hypothetical, infinite population of binary observations, it is assumed that the decision maker is only concerned with which of a set of mutually exclusive and exhaustive subsets of the unit interval contains p. The derived strategy maximizes the decision-maker's gain without regard to error probabilities.

The important variable in determining a rule for ceasing to look at new data and making a decision is found to be the expected probability of being correct. The criterion involves only the economic aspects of the situation. A "no information" theorem is presented which shows that under some circumstances when a "success" or a "failure" on a given trial are equally probable, the probability of being correct after making the observation is identical to the probability of being correct before the observation was taken. Finally, an appealing derivation of the Beta-binomial probability function is given which suggests a more tractable computational procedure for the distribution and which illuminates its limiting distribution.

iii

PUBLICATION REVIEW AND APPROVAL

This Technical Documentary Report has been reviewed and approved.

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SEQUENTIAL INFORMATION SEEKING: AN OPTIMAL STRATEGY AND OTHER RESULTS

I. Introduction.

It is not uncommon for a person to be faced with the task of having to evaluate a body of information and to make a decision on the basis of his evaluation. A problem which naturally arises is how long to spend collecting or evaluating data before making the decision. This issue is particularly pertinent when data or information can be obtained by the decision maker but only at a cost. The decision maker must determine at what point he will cease looking at new data and make his decision. When a decision maker can at any time choose to observe more data or choose to make a decision, we say that the sampling procedure is sequential.

In the quest of psychologists to understand human behavior in general and human decision making in particular, the study of human information seeking is of prominent importance. The empirical research of Irwin and Smith (1956, 1957); Pruitt (1961); Lanzetta and Kanareff (1962); Edwards (1964); and Messick (1964a); has contributed to understanding of the problem.

Psychologists themselves are engaged in an almost continual process of collecting and evaluating data from empirical research. Wald (1947) has shown that sequential experiments are often more efficient in minimizing required sample size than experiments the size of which are predetermined. Fiske and Jones (1954) have emphasized this argument for psychologists, who as scientists, may benefit from sequential sampling procedures as research tools. For such purposes it is desired to find procedures which have specific properties, e.g., to minimize expected sample sizes holding error probabilities constant (Wald; 1947), to maximize expected utilities holding error probabilities constant (Edwards; 1964), or to reduce uncertainty to a preassigned level (DeGroot, 1962; Lindley, 1956, 1957.

While these two sources of interest in sequential sampling and information processing are distinct, the modern approach to the study of the "rational man" (called the "ideal observer" in contemporary psychophysics) is tending to bring the two interests together. Becker (1958), for example, studied human sequential sampling from the theoretical point of view of Wald. Edwards (1964) is studying the same behavior from a Bayesian position.

The primary purpose of the present report is to derive an optimal strategy for a special, but not uncommon situation, that of sequential sampling from binomial distributions. The strategy presented here is general in that it is a "multi-action" rather than a two-action procedure. Furthermore, while the major task is to estimate the proportion, p, of "successes" in an hypothetical, infinite population of binary observations, it is assumed that the decision maker is only concerned with which of a set of mutually exclusive and exhaustive subsets of the unit interval contains p. The derived strategy maximizes the decision-maker's gain without regard to error probabilities.

The results of this report have potential value as a general Bayesian research strategy. The immediate need to be filled by them is to provide a formal, rational model of information-seeking which can be used to evaluate the "optimality" of actual human information seeking behavior (Messick; 1964a, 1964b).

II. An optimal Strategy for information seeking.

In deriving optimal strategies for information seeking, attention will be restricted to the case in which the prior probability distribution is of the Beta family, with density given by

(1) $f(p|\alpha,\beta) = B(\alpha,\beta)^{-1} p^{\alpha-1}(1-p)^{\beta-1}$, $\alpha, \beta > 0$,

and in which the data generating process is binomial. The restriction of the prior distribution to the Beta family has been adequately defended by Rapoport (1964) and Lindley (1957). The task of the decision maker is to select one of $\underline{s}$ mutually exclusive and exhaustive subsets of the unit interval which he believes contains $\underline{p}$, the true proportion. We let this decision maker be motivated solely by the desire to maximize his expected gain and we assume that he uses Bayes formula to combine prior opinion with current information to produce a posteriori opinions. Thus, given a particular prior distribution with parameters (α,β), and given that our decision maker has observed $\underline{r}$ "successes" in n Bernoulli trials, then the state of his knowledge concerning $\underline{p}$ will be represented by a Beta distribution with parameters $(\alpha+r, \beta+n-r)$.

We will assume that the economic aspects of the situation have the following form: if the terminal decision is correct (i.e., if $\underline{p}$ is in the selected subset) the decision maker is given A dollars; if the terminal decision is incorrect he is fined L dollars; and each observation made costs C dollars. The expected gain of making a terminal decision after having observed $\underline{r}$ successes in $\underline{n}$ trials is

(2) $E(g|\alpha+r, \beta+n-r) = AP^*(\alpha+r, \beta+n-r) + L(1-P^*(\alpha+r, \beta+n-r)) + nc$

$= (A-L)P^*(\alpha+r, \beta+n-r) - L + nc,$

where c is assumed negative, $L < A$, and where

(3) $P^*(\alpha+r, \beta+n-r) = \max_{i} \int_{I_i} f(p|\alpha+r, \beta+n-r)\, dp;$ $i = 1, 2, \ldots, s.$

$P^*(\alpha+r, \beta+n-r)$ is the maximum probability of being correct, where maximization is with respect to the set of terminal actions or decision categories, $\{I_i\}$.

To avoid notational difficulties let $\alpha_n = \alpha+r$, $\beta_n = \beta+n-r$, $\gamma_n = \alpha_n+\beta_n = \alpha+\beta+n$, and $\delta_n = \alpha_n/\gamma_n$. The question which must now be answered is what is the expected gain of taking another k observations and then making a terminal decision, $k = 1,2, \ldots$. Denote this expected gain $E_k(g|\alpha_n, \beta_n)$. It will be shown that

(4) $E_k(g|\alpha_n, \beta_n) = (A-L) \sum_{t=0}^{k} Pr(t|k, \alpha_n, \beta_n)\, P^*(\alpha_n+t, \beta_n+k-t) + L + c(n+k)$.

In this formula $Pr(t|k, \alpha_n, \beta_n)$ gives the probability of obtaining t successes in the k trials when the prior Beta distribution has parameters (α_n, β_n). This probability is given by the Beta-binomial probability function (see Raiffa and Schlaifer, 1961, 237) defined by

(5) $Pr(t|k, \alpha_n, \beta_n) = \int_{0}^{1} b(t|k,p)\, f(p|\alpha_n, \beta_n)\, dp$

$= \dfrac{(t+\alpha_n-1)!\ (\beta_n+k-t-1)!\ k!\ (\gamma_n-1)!}{t!(k-t)!\ (\alpha_n-1)!\ (\beta_n-1)!\ (\gamma_n+k-1)!}$.

(An intuitively appealing deviation of this probability function is given in Section IV.) Given that t successes have occurred in the k trials the expected gain is given by (2) and is found to be

$$(6) \quad E(g|\alpha_n+t, \beta_n+k-t) = (A-L) P^*(\alpha_n+t, \beta_n+k-t) + L + c(n+k).$$

The expression in (4) results from taking the expectation of (6) with respect to t.

In order to take a sample size that will maximize the expected gain, an additional observation is made if and only if there exists a k such that the difference

$$(7) \quad \Delta_k(\alpha_n, \beta_n) = E_k(g|\alpha_n, \beta_n) - E(g|\alpha_n, \beta_n) > 0 , \quad k = 1, 2, 3, \ldots .$$

Letting

$$(8) \quad \Pi_k(\alpha_n, \beta_n) = \sum_{t=0}^{k} Pr(t|k, \alpha_n, \beta_n) P^*(\alpha_n+t, \beta_n+k-t) - P^*(\alpha_n, \beta_n),$$

then

$$(9) \quad \Delta_k(\alpha_n, \beta_n) = (A-L)\Pi_k(\alpha_n, \beta_n) + ck.$$

Therefore a stopping rule equivalent to (7) is to take another observation if and only if for some k

$$(10) \quad \Pi_k(\alpha_n, \beta_n) > - \frac{ck}{A-L} , \quad k = 1, 2, 3, \ldots .$$

Several interesting and useful properties of this sampling rule may be gleaned from (10). First, since $\Pi_k(\alpha_n, \beta_n)$ is the difference between two probabilities it can range only between 1 and -1. Only positive values of Π_k would lead to additional sampling since the term on the right of the inequality will always be positive under our restrictions. Furthermore it is obvious that values of k greater than $-\frac{A-L}{c}$ need not be considered since

$$k \geq - \frac{A-L}{c} \quad \rightarrow \quad - \frac{ck}{A-L} \geq 1.$$

Since Π_k is bounded above by 1, (10) will never hold for $k \geq - \frac{A-L}{c}$.

Feasible values of k can be further restricted by noting that $P^*(\alpha_n, \beta_n) > 0$. If we let $1-P^*(\alpha_n, \beta_n) = \epsilon_n$, then we note that (10) can be true only for values of k such that

$$(11) \quad k < - \epsilon_n \frac{(A-L)}{c} .$$

Thus only values of k, k = 1, 2, 3, ..., $-\epsilon_n \frac{(A-L)}{c}$ need be tested.

Finally, as n increases without bound $P^*(\alpha_n, \beta_n)$ approaches unity as a result of the fact that $f(p|\alpha_n, \beta_n)$ approaches a point. Therefore, ϵ_n goes to zero and

$$(12) \quad \lim_{n \to \infty} \epsilon_n \frac{(A-L)}{c} = 0.$$

Thus

$$(13) \quad \lim_{n \to \infty} Pr(\text{sampling stops after n observations}) = 1.$$

Exceptions to (13) occur only if (A-L) is infinitely large or if c = 0. These exceptions make good intuitive sense. If A is infinitely large or L infinitely negative, or if observations are free, then sampling might well be expected to continue for an indefinite period.

It will be of practical value to develop approximations to the rule given by (10). Two types of approximation will be suggested but research is needed to determine how good the approximations are. The difficulty with (10) is that in some cases it may be necessary to test π_k for many values of k before a decision can be made as to whether to take another observation. For example, in an experiment performed by Messick (1964)), A = 15, L = 0, and c = .2. The maximum k is thus, $\frac{A-L}{c}$ = 75. Clearly the procedure developed here would not be feasible if π_k had to be tested for all values of k between 1 and 75. One way in which this difficulty could be overcome is to test for k = 1, 2, 3, ..., u, where u is pre-determined by the experimenter on the basis of the degree of accuracy desired. An alternative procedure for approximating (10) would involve selecting more or less arbitrary values of k, perhaps by some random procedure. For example, one might test for k = 1, 2, 6, 16, 23. This latter procedure would be free of any bias which might be involved in testing only small values of k, but it would be more time consuming computationally.

III. A "No Information" theorem.

Let I_1' be a subset of [0-1] such that $I_1' = [x, 1-x]$, and such that

(14) $P^*(\alpha,\beta) = \int_x^{1-x} f(p|\alpha, \beta)\, dp$, when $\alpha = \beta$; and

(15) $P^*(\alpha+1, \beta) = \int_x^{1-x} f(p|\alpha+1,\beta)\, dp$,   and

(16) $P^*(\alpha, \beta+1) = \int_x^{1-x} f(p|\alpha, \beta+1)\, dp$.

Then,

(17) $P^*(\alpha,\beta) = P^*(\alpha+1, \beta) = P^*(\alpha, \beta+1)$.

The equivalence of $P^*(\alpha+1, \beta)$ and $P^*(\alpha, \beta+1)$ follows from:

(18) $\int_0^x f(p|\alpha,\beta)\, dp = 1 - \int_0^{1-x} f(p|\beta, \alpha)\, dp$.

The second equality in (17) will be assumed and a complete proof of the first will be given.

Writing (14) in full we have

$$P^*(\alpha,\beta) = P^*(\alpha,\beta) = \frac{\Gamma(2\alpha)}{\Gamma(\alpha)^2} \int_x^{1-x} p^{\alpha-1}(1-p)^{\alpha-1}\, dp.$$

However, since the Beta distribution is symmetric when $\alpha=\beta$, this can be written as

(19) $P^*(\alpha,\beta) = 1-2\,\frac{\Gamma(2\alpha)}{\Gamma(\alpha)^2} \int_0^x p^{\alpha-1}(1-p)^{\alpha-1}\, dp$.

Writing (15) in full we have

$$(20) \quad P^*(\alpha+1,\beta) = \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)\Gamma(\alpha)} \int_X^{1-x} p^\alpha(1-p)^{\alpha-1}dp$$

$$= \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)\Gamma(\alpha)} \int_0^{1-x} p^\alpha(1-p)^{\alpha-1}dp - \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)\Gamma(\alpha)} \int_0^X p^\alpha(1-p)^{\alpha-1} dp$$

$$= 1 - \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha)\Gamma(\alpha+1)} \int_0^X p^{\alpha-1}(1-p)^\alpha dp - \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)\Gamma(\alpha)} \int_0^X p^\alpha(1-p)^{\alpha-1}dp$$

To prove (17) we show that (19) minus (20) is identically zero. First notice that

$$\frac{\Gamma(2\alpha+1)}{\Gamma(\alpha)\Gamma(\alpha+1)} = \frac{2\alpha\Gamma(2\alpha)}{\alpha\Gamma(\alpha)^2} = 2\frac{\Gamma(2\alpha)}{\Gamma(\alpha)^2} \quad .$$

Subtracting (20) from (19) we have

$$(21) \quad P^*(\alpha,\beta) - P^*(\alpha+1,\beta) = -2\frac{\Gamma(2\alpha)}{\Gamma(\alpha)^2} \int_0^X [p^{\alpha-1}(1-p)^{\alpha-1} - p^{\alpha-1}(1-p)^\alpha - p^\alpha(1-p)^{\alpha-1}]dp$$

$$= -2\frac{\Gamma(2\alpha)}{\Gamma(\alpha)^2} \int_0^X (p^{\alpha-1}(1-p)^{\alpha-1})(1-(1-p) - p) dp$$

$$= 0.$$

This theorem has the following interpretation. If $\alpha=\beta$, then the mean of the prior distribution is 1/2. This value may be taken as the expected probability of a success on the next trial. If one's best terminal act at this point is to select the decision interval centered at 1/2, and if this same terminal act remains optimal after another observation is taken, regardless of the outcome of the observation, then the probability of being correct before taking the observation is identical to the probability of being correct if the terminal act is selected after the one observation is made. From this point of view, the observation is non-informative.

IV. The Beta-binomial distribution: an intuitive derivation.

The binomial distribution,

$$(22) \quad b(r|n,p) = \binom{n}{r}p^r(1-p)^{n-r} \quad ,$$

gives the probability of obtaining r "successes" in n independent Bernoulli trials when the probability of one success is p, $0< p<1$. In this case, p is constant.

The Beta-binomial probability distribution,

$$(23) \quad \Pr(r|n,\alpha,\beta) = \int_0^1 b(r|n,p)f(p|\alpha,\beta)\, dp$$

$$= \frac{(r+\alpha-1)!\ (\beta+n-r-1)!\ n!\ (\alpha+\beta-1)!}{r!(n-r)!\ (\alpha-1)!\ (\beta-1)!\ (\alpha+\beta+n-1)!} \quad ,$$

gives the probability of obtaining r successes in n trials when p is unknown, but rather treated as a random variable having a distribution of the Beta family with parameters (α,β).

To derive the Beta-binomial distribution, use will be made of the fact that the first moment of the Beta-distribution, which may be interpreted as the expected probability of a success, is given by

$$(24) \quad \delta = \frac{\alpha}{\alpha+\beta} \quad ,$$

and therefore the expected probability of a "non-success" is

$$(25) \quad 1-\delta = \frac{\beta}{\alpha+\beta} \quad .$$

If n observations are taken, r of which are successes, then the posterior Beta distribution has parameters $(\alpha+r,\ \beta+n-r)$. The mean of this posterior distribution is

$$(26) \quad \delta_n = \frac{\alpha+r}{\alpha+\beta+n} \quad , \text{ and}$$

$$(27) \quad 1-\delta_n = \frac{\beta+n-r}{\alpha+\beta+n}. \quad .$$

We will only be concerned with the special case in which n = 1. Given (α,β) we wish to find the probability of obtaining r successes in n trials.

To clarify the argument, consider the grid in Figure 1. A person begins at the origin with coordinates (β,α). If a success occurs our decision maker moves up one step. If a non-success occurs, he moves to the right one step. From any point on the grid, the probability of moving up a step is the ratio of the ordinate of the point to the sum of the coordinates and the probability of going to the right is the ratio of the abcissa to the sum of the coordinates.

We first note that the sum of the coordinates of any point is $\alpha+\beta+n$, where n is the number of steps (trials) required to reach the point from the origin. Second, the numerator of the probability of moving one step to the right does not depend on the ordinate and the numerator of the probability of moving one step up does not depend on the abcissa. Thus the numerator of the probability of moving from (β,α) to $(\beta+1,\alpha)$ is β, which is the same as the numberator of the probability of moving from $(\beta,\ \alpha+4)$ to $(\beta+1,\ \alpha+4)$. Finally we note that the probability of going to any specified point in the grid from the origin via a specified path is the product of the probabilities of each of the component moves in the path.
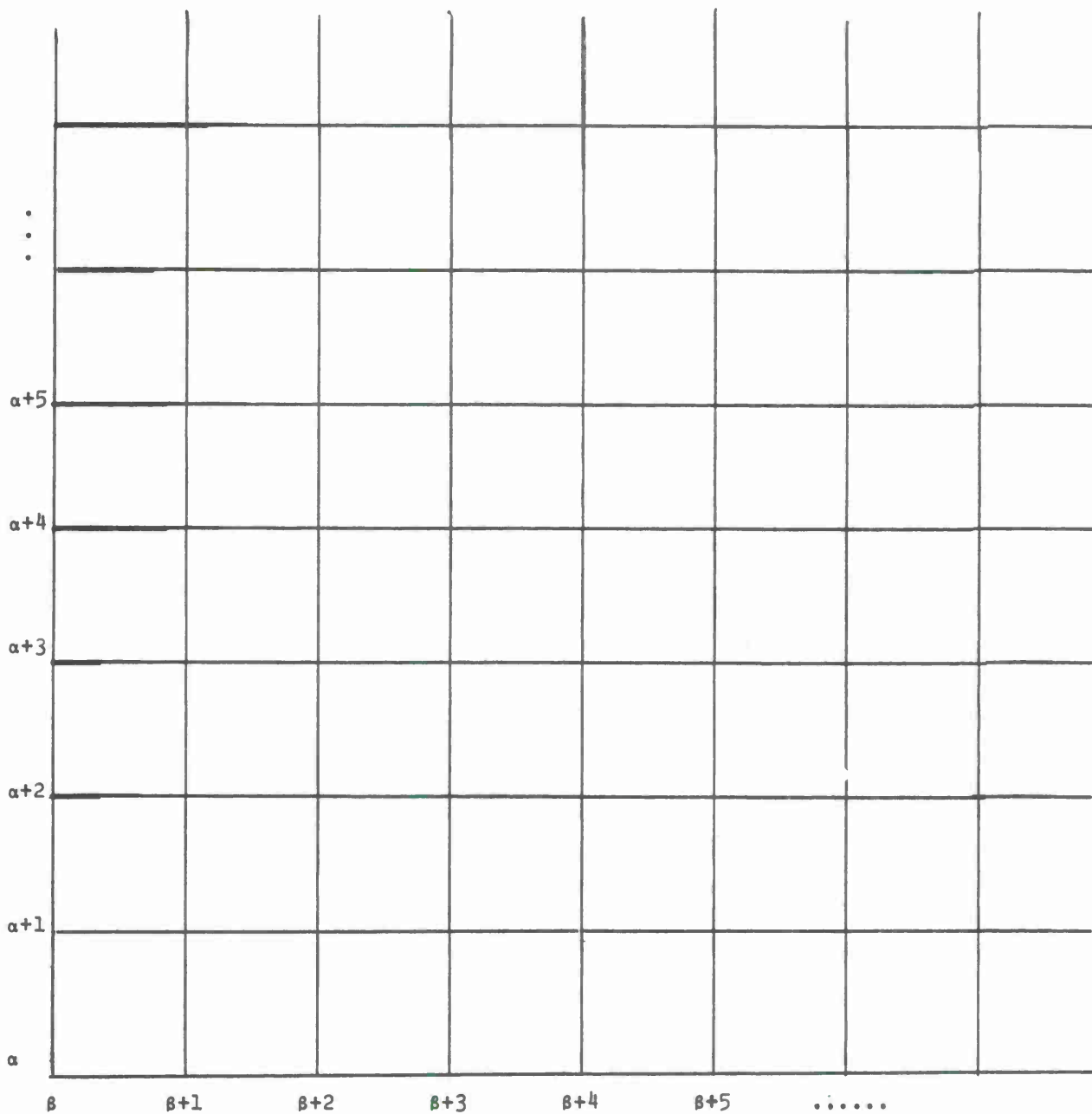
Figure 1: Grid representing sequential sampling procedure. The origin is (β, α).

The major result to establish is that <u>all paths</u> to a <u>point</u> <u>from the origin</u> <u>have the same probability</u>. To prove this, first consider the numerator to the probability of moving from (β,α) to $(\beta+s, \alpha+r)$. By the argument given in the above paragraph, the numerator of the probability of going s steps to the right and r steps up is given by $\beta(\beta+1) \ldots (\beta+s-1)\alpha(\alpha+1) \ldots (\alpha+r-1)$ regardless of the order in which the steps occur. This number may be written as $\dfrac{(\beta+s-1)!}{(\beta-1)!} \dfrac{(\alpha+r-1)!}{(\alpha-1)!}$.

The denominator of the probability of moving in either direction from a point is the sum of the coordinates of the point, and, as stated above, this number depends only on the number of steps required to reach the point from the origin. Since the denominator of the product equals the product of the denominators of the step by step probabilities, it will be equal to $(\alpha+\beta)(\alpha+\beta+1) \ldots (\alpha+\beta+s+r-1)$, which can be written $\dfrac{(\alpha+\beta+s+r-1)!}{(\alpha+\beta-1)!}$. Therefore, the probability of going from (β,α) to $(\beta+s, \alpha+r)$ by any specified path to

(28) Pr[any single path from (β,α) to $(\beta+s, \alpha+r)$] $=$

$$\frac{(\beta+s-1)!}{(\beta-1)!} \frac{(\alpha+r-1)!}{(\alpha-1)!} \frac{(\alpha+\beta-1)!}{(\alpha+\beta+s+r-1)!}$$

Finally, the number of paths from (β,α) to $(\beta+s, \alpha+r)$ is

$$\binom{s+r}{r} = \binom{s+r}{s} = \frac{(s+r)!}{r!\,s!}$$

and the probability of going from (β,α) to $(\beta+s, \alpha+r)$ regardless of the path is

$$\Pr(r \mid r+s, \alpha, \beta) = \frac{(s+r)!}{r!\,s!} \frac{(\beta+s-1)!}{(\beta-1)!} \frac{(\alpha+r-1)!}{(\alpha-1)!} \frac{(\alpha+\beta-1)!}{(\alpha+\beta+s+r-1)!}$$

Setting $n = s+r$ and $s = n-r$, the above expression is the Beta-binomial distribution given in (23).

This derivation of the Beta-binomial distribution is of practical value for two reasons. First, it provides a simpler computational procedure for finding probabilities than that in (23), which entails evaluating the factorials. Second, it becomes apparent from this point of view that the limiting distribution of the Beta-binomial is the binomial. As α, β increase, the ratio $\dfrac{\alpha}{\alpha+\beta}$ approaches p (see Raiffa and Schlaiffer, 1961). Therefore

(29) $\displaystyle \lim_{\alpha, \beta \to \infty} \frac{\alpha(\alpha+1) \ldots (\alpha+r-1)\,\beta(\beta+1) \ldots (\beta+s-1)}{(\alpha+\beta)(\alpha+\beta+1) \ldots (\alpha+\beta+s+r-1)} = p^r(1-p)^s = p^r(1-p)^{n-r}$.

Summary

In this report an optimal strategy is presented for sequential information seeking. The important variable in determining a stopping rule is found to be the expected probability of being correct. The criterion involves only the economic aspects of the situation. A "no information" theorem is presented which shows that under some circumstances when a "success" or a "failure" on a given trial are equally probable, the probability of being correct after making the observation is identical to the probability of being correct before the observation was taken. Finally, an appealing derivation of the Beta-binomial probability function was given which suggests a more tractable computational procedure for the distribution and which illuminates its limiting distribution.

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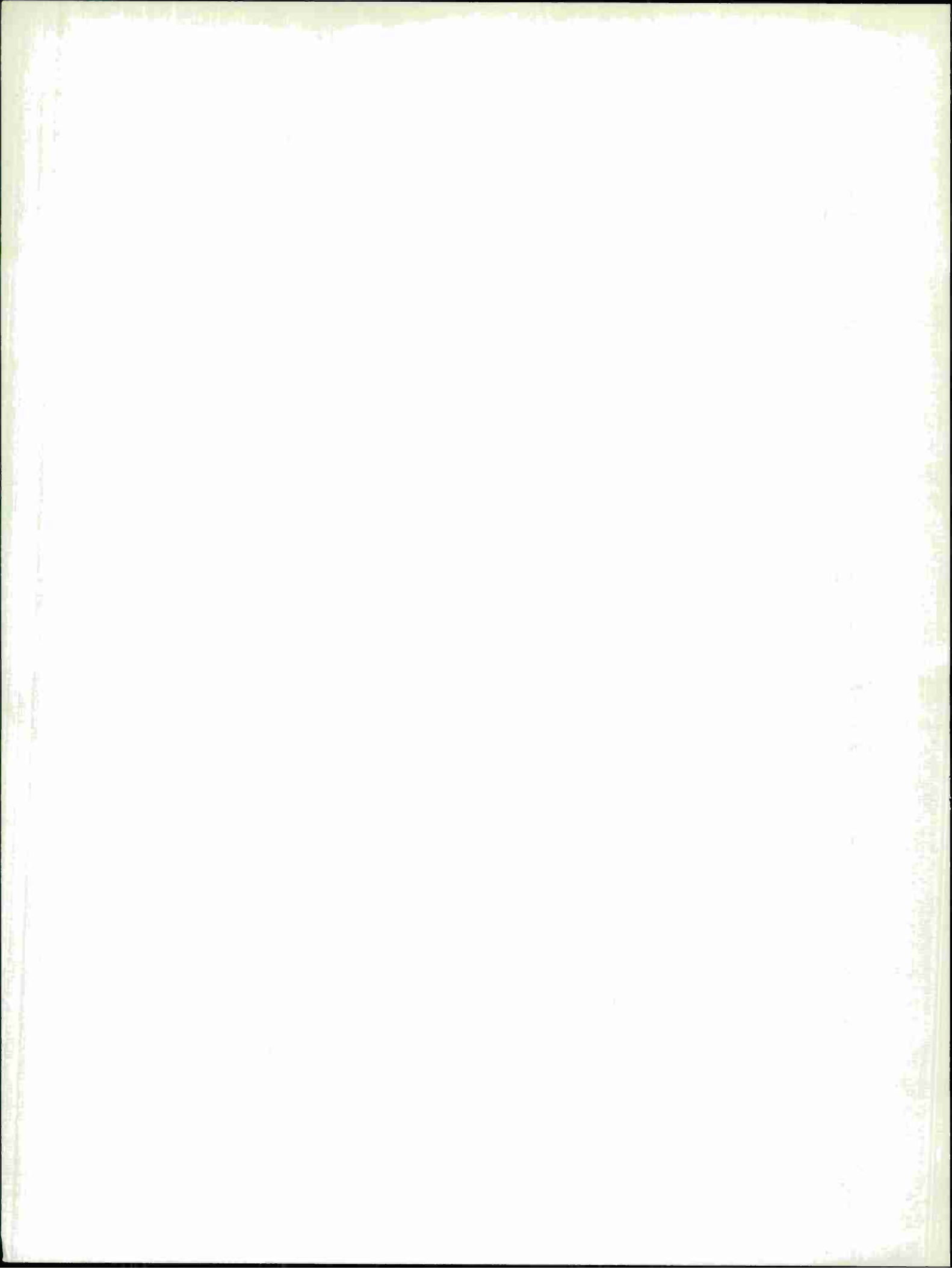
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13. ABSTRACT

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