BAYESIAN DECISION THEORY, GAME THEORY AND GROUP PROBLEM SOLVING

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-603

OCTOBER 1964

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DECISION SCIENCES LABORATORY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 4690, Task 469003

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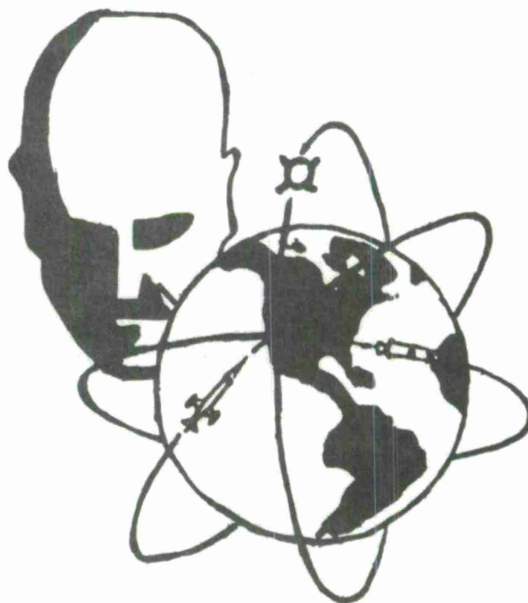
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FOREWORD

This study was conducted under Contract No. AF 19(628)-1610 at the Psychometric Laboratory, University of North Carolina, Chapel Hill, North Carolina. Dr. Albert Amon served as principal investigator and Dr. Anne Story, as contract monitor.

Work was performed under Project 4690 "Information Processing in Command and Control", Task 469003, "Human Information Processing Techniques".

The author wishes to express his gratitude for the aid and encouragement given by Dr. Emir Shuford and Dr. Masanao Toda during the preparation of this report.
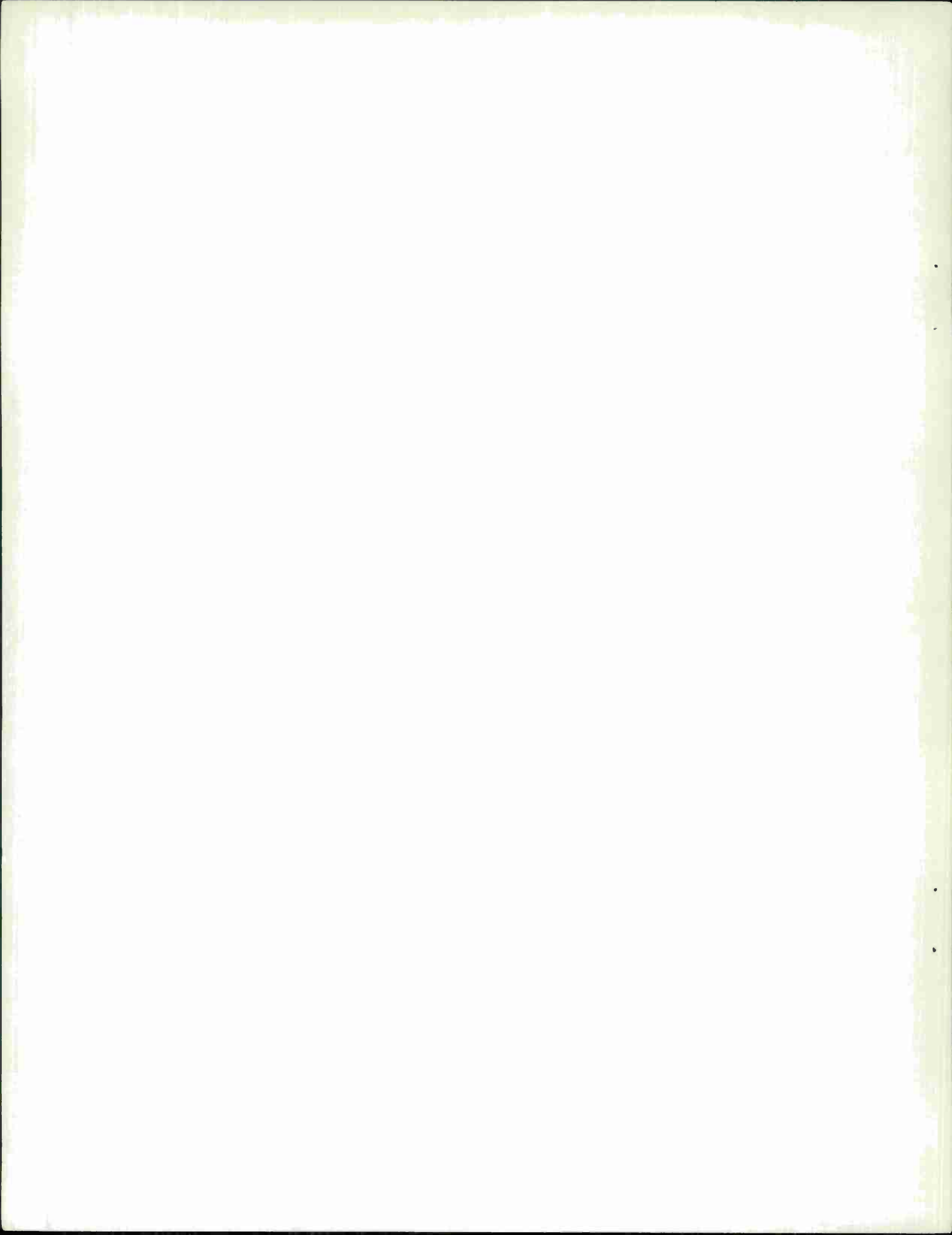
ABSTRACT

This paper applies the concepts of the "ideal" information processor and rational decision maker to a typical problem in social psychology--that of group problem solving. The structure of the selected task is seen to be that of a nonzero sum game. A strategy is derived and is shown to be the equivalent of the Nash solution to the game. The notion of level of aspiration is discussed and defined within the analysis and two theorems are proved relating level of aspiration to type of group decision strategy employed.

PUBLICATION REVIEW AND APPROVAL

This Technical Documentary Report has been reviewed and is approved.

for HERBERT RUBENSTEIN
Chief, Decision Techniques
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BAYESIAN DECISION THEORY, GAME THEORY, AND GROUP PROBLEM SOLVING

It is the general purpose of this report to attempt to apply concepts and methods of analysis derived from Bayesian decision theory and the theory of non-zero-sum games to the social psychological problem of group decision making and group problem solving. Attention will be restricted to a specific type of task often used in social psychological experiments. It is hoped that the value of this approach will become obvious as the analysis develops.

The task to be described is one of a type which is used not infrequently in studies of group problem solving. The task is such that a problem is presented to a group of individuals. The problem is of such a nature that there is no single deductive solution with which the group members can be certain of their answer being correct. An example of such a task might be one in which the Ss are to esti-mate the number of beans in a jar, or the number of lines in a brief visual display. The group members must communicate and eventually reach a single joint decision which represents the group solution. If they fail to come to an agreement, the payoff to the group is zero. We will consider only tasks in which the group deci-sion is either correct or incorrect. If the group decision is correct, then the group wins a payoff, a; if incorrect, the group wins b, where a>b>0. The payoff, either a or b, is then divided among the group members by a predetermined schedule. It is assumed that the schedule for distributing the payoff is not a function of the group decision rule.

Let us suppose that the group has two members, person 1 and person 2. Fur-ther, let us use as an example of the class of tasks mentioned above, one in which the two members are to estimate the proportion of black balls in an urn containing 100 balls. They receive a sample of n_1 and n_2 balls, respectively, drawn with replacement from the urn. In their samples they observe r_1 and r_2 black balls. Given that they are allowed to communicate, we want to attempt to specify the value they "ought" to choose as their group estimate of p, the proportion of black balls. If the joint estimate, d', is correct, they jointly win a dollars. If it is in-correct, they win b dollars. We will assume that the utilities of both 1 and 2 for money are linear with dollars, at least for the range involved in this task, and also that the persons try to behave so as to maximize their individual expected utilities.

From Bayesian decision theory we will take the assumption that each of the persons can be conceptualized as having a subjective probability distribution over p on the interval 0<p<1. The further assumption will be made that this distribu-tion has the form of a beta distribution with parameters a_1, b_1, where $a_1, b_1 \geq 1$,

i = 1, 2. (We use the subscript i to denote the ith individual.) The beta density function is defined by

$$(1) \qquad f_i(p) = B(a_i,b_i)^{-1} \, p^{a_i-1} \, (1-p)^{b_i-1} \, , \quad \text{where}$$

$$(2) \qquad B(a_i,b_i) = \Gamma(a_i)\Gamma(b_i)/\Gamma(a_i+b_i).$$

If the group member's uncertainty about the value of p can be summarized by a beta distribution with parameters (a_1, b_1) before a sample is drawn from the urn, then we further assume that his uncertainty will be represented by a beta distribution with parameters $(a_1+r_1, b_1+n_1-r_1)$ after observing a sample of n_1 balls, r_1 of which are black. The posterior density function will be given by

$$(3) \qquad f_1(p; r_1, n_1) = B(a_1+r_1, b_1+n_1-r_1)^{-1} \; p^{a_1+r_1-1} \; (1-p)^{b_1+n_1-r_1-1}$$

Finally, we are going to assume that the individual would, if forced to give his estimate without communicating or in any way considering the other person, state as his estimate the mode of his posterior distribution, given by

$$(4) \qquad d_1 = a_1 + r_1 - 1/(a_1 + b_1 + n_1 - 2).$$

So far all we have done is to specify that the Ss in our imaginary experiment will behave like "thou," Savage's (1962) ideal decision maker.

Let us now make the realistic assumption that our Ss come from different environments (which can be experimentally controlled) and have, in general, different pasts regarding p. This is to say that $a_1 \neq a_2$ and/or $b_1 \neq b_2$. They each see a sample drawn from the urn containing 100 balls in which $p \cdot 100$ of the balls are black. They are allowed to communicate and they must state a single joint decision, d', upon which they agree.

We first notice that each S's utility function is a linear function of $f_1(p)$, his subjective probability distribution. This is easily enough shown. The Ss agree to divide the payoff so that 1 gets e and 2 gets k dollars if they are correct (e + k = a); and 1 gets g while 2 gets h dollars if they are incorrect (g + h = b). Then 1's expected utility function is

$$(5) \qquad u_1'(p) = e f_1(p) + g[1-f_1(p)]$$

$$= f_1(p)[e-g] + g.$$

Likewise, 2's expected utility function is

$$(6) \qquad u_2'(p) = f_2(p)[k-h] + h.$$

Assuming that utilities are measured up to a linear transformation we can transform each utility function so that

$$(7) \qquad u_1''(p) = f_1(p)$$

$$(8) \qquad u_2''(p) = f_2(p)$$

To get back to the problem at hand, we would like to find a value, d', which our two idealized Ss will be willing to give as their joint estimate of p. They are starting in the task with different subjective prior distributions and they have seen different samples. Now, inasmuch as we are assuming the cost of communication to be zero, the Ss will exchange messages consisting of the samples they have seen. Thus, each will effectively have a sample of $n_1 + n_2 = n$ balls, of which $r = r_1 + r_2$ are black. After having processed this information and accordingly altered their respective distributions, they discover that their respective estimates do not agree, $d_1 \neq d_2$. Moreover, there is, in general, disagreement as to the probability of any particular p. Thus there will also be disagreement as to the utility of an estimate of any value of d' = p.

The situation at this point can be visualized as two different probability distributions over the inverval (0,1). It is in this form that we can see the correspondence between the decision theoretic formulation and the theory of non-zero-sum games. If we divide the interval (0,1) into small equal subintervals of width .01 and assign to each a probability proportional to the ordinate of the mid-point of the interval, this will give us a good discrete approximation to the continuous function $f_1(p)$. We will continue to refer to the probability of a value of p (and hence the utility, or expected utility of that value) as f(p) inasmuch as the solutions suggested later will be invariant for the functions f(p) under multiplication by a positive constant. Having approximated the continuous distributions by discrete ones we can represent the situation as a two person game as in Figure 1. Since there is a constant zero payoff if the Ss fail to reach an agreement, we will have zeros in all cells outside of the main diagonal. The entries in the main diagonal will be greater than zero although they may be very small.

S 2

	.01	.02	....	1.00
.01	$f_1(.01), f_2(01)$	0,0		0,0
.02	0,0	$f_1(.02), f_2(.02)$		0,0
⋮	⋮	⋮		⋮
1.00	0,0	0,0	......	$f_1(1.00), f_2(1.00)$

S 1

Fig. 1. The group decision problem summarized in the form of a non-zero-sum game. All off diagonal entries are zero.

Our next step is to determine the set of outcomes with which our Ss will concern themselves. It is clear that the only set of decisions, D, the Ss will consider in their communication is

(9) $D = (d' = p; d_1 \leq p \leq d_2)$, $d_1 < d_2$,

where d_1 is defined by Eq. (4). In other words, the Ss will only consider those values of p for their joint decision, d', which fall between the modes of their respective posterior distributions. This is obvious and can be shown quite easily.

The functions $f_1(p)$ and $f_2(p)$ (which we shall henceforth abbreviate as f_1 and f_2) are both monotonically increasing on the interval $(0,d_1)$, assuming as we shall throughout the remainder of this paper that $d_1 < d_2$. Thus $f_1(p) < f_1(d_1)$ and $f_2(p) < f_2(d_1)$ for all $p < d_1$. Likewise both f_1 and f_2 are decreasing monotonically on the interval $(d_2, 1)$. Thus, for $p > d_2$, $f_1(d_2) > f_1(p)$ and $f_2(d_2) > f_2(p)$. Thus, by the assumption of expected utility maximization, values of p less than d_1 and greater than d_2 will not be considered in the decision process.

D has been defined as the set (or interval) of values of p which lie between the modes of the two posterior distributions. This set D is the Paretian optimal set for this decision situation. It is true that there is no value of p, p', not in D such that $f_1(p') \geq f_1(p'')$ and $f_2(p') \geq f_2(p'')$ where p" is any p in D. This follows from the argument above. Moreover, there is no p, p", in D such that $f_1(p'') \geq f_1(p^*)$ and $f_2(p'') \geq f_2(p^*)$ where p* is any p in D, $p^* \neq p''$. This follows from the fact that f_1 is constantly decreasing on the interval (d_1, d_2) while f_2 is constantly increasing. Thus any small increment in f_1 will be accompanied by a decrement in f_2 and vice versa.

Our ideal Ss now find themselves in what has been called a conflict of interest situation and which has been summarized by a non-zero-sum game. It is tempting to say that having specified the set of values or outcomes satisfying the conditions of Paretian optimality we can do no more, and that the actual value of p which is selected will be a result of the relative bargaining abilities of the two persons, on their relative abilities to persuade the other that one knows best what is going to happen, on personality variables, and so on. This argument has been made by Fellner (1949) and has therefore been called the Fellner hypothesis by Siegel and Fouraker (1960). There is no doubt but that these things do make a difference. In fact, Siegel and Fouraker report evidence which suggests that this hypothesis does in fact predict and provide a good fit to some of their data. This is true, however, only when the Ss (in a bargaining situation) have knowledge only of their own payoff function. When both Ss in these experiments had complete knowledge of the other's payoff function as well as their own and when they knew that the other person also had complete information, the data (the negotiated contracts) were fit much more nicely by what the authors call the "marginal intersection hypothesis." It is of some interest to note that the price predicted by the marginal intersection hypothesis is exactly that price which would be the Nash solution to the particular bargaining problem used (see Table 4.3c, p. 58, and Appendix C in Siegel and Fouraker).

It is difficult to envisage just how two problem solvers would go about communicating their respective posterior distributions. For this reason it might be realistic to assume that each S knows only his own utility function. If we do this we may be satisfied by the Fellner hypothesis that choice of p will be a function of the eloquence and persuasiveness of the Ss. This in turn might lead us to neglect the method by which the particular value of p is selected from the Paretian set. However, having a model or a series of models which prescribe selection strategies may lead to a fruitful analysis of this social psychological problem.

Actually, the most interesting aspect of our problem arises with the consideration of selection strategies. We will consider a strategy which does not permit randomization to begin with. We will then discuss the conditions in which our ideal Ss would prefer to flip a coin or use some other randomizing device to choose between two possible values of p. We will conclude by suggesting some psychological interpretations which seem related to the type of analysis being considered.

Let us assume that our ideal S̲s̲ are temporarily without any means of perform-
ing a randomization. They have no coins and no dice and the experimenter believes
that gambling is sinful. They must select one value of p to state as their joint
decision, d'. A method they might consider is to maximize the joint probability that
they are both correct. They decide then to use a selection strategy which will
always guarantee a single value of p. That value of p will be the one which maxi-
mizes the probability that they are both simultaneously correct. They therefore
define a function, g(p), which is proportional to the product of their individual
functions.

(10) $g(p) = K \cdot f_1(p) \cdot f_2(p),$

where K is the normalizing constant.

(11) $g(p) = K \, B(a_1,b_1)^{-1} \, p^{a_1-1} \, (1-p)^{b_1-1} \cdot B(a_2,b_2)^{-1} \, p^{a_2-1}(1-p)^{b_2-1}$

(12) $g(p) = K \cdot B(a_1,b_1)^{-1} \, B(a_2,b_2)^{-1} \, p^{a_1+a_2-2} \, (1-p)^{b_1+b_2-2}$

The joint decision function, g(p), is also a beta distribution, setting
$K = B(a_1+a_2-1, \, b_1+b_2-1)^{-1} \, B(a_1,b_1)B(a_2,b_2)$, with parameters $(a_1+a_2-1, \, b_1+b_2-1)$.
This value of p, $\hat{p}$, is

(13) $\hat{p} = \dfrac{a_1 + a_2 - 2}{a_1 + a_2 + b_1 + b_2 - 4}$.

The solution, $\hat{p}$, derived from this strategy is also the Nash solution to
the bargaining problem implied by the experimental task (see Luce and Raiffa, 1957,
pp. 124-128). If f_1 and f_2 are conceived as utility functions rather than
probability density functions, as was done earlier in this paper, then $\hat{p}$ is seen
to be the outcome which maximizes the product $u_1''(p)u_2''(p)$. Thus, $\hat{p}$, in addition
to the properties discussed in the following paragraph, satisfies the four assump-
tions from which Nash derived his solution (Luce and Raiffa, 1957, pp. 126-127).

This strategy has certain other desirable properties. If, for example,
$a_1 = b_1 = 1$, then $\hat{p} = a_2 - 1/(a_2 + b_2 - 2) = d_2$. Thus if one of the S̲s̲ has no informa-
tion at all concerning p, then the other S̲ will make the decision to the best of
his ability. In general, the greater the magnitude of a_1 and b_1, the more influence
will person i have, where influence is taken to be inversely proportional to the
ratio, $|\hat{p} - d_1|/(d_2 - d_1)$. This solution has another feature which seems to have a
possible "meaning" psychologically speaking. We have been dealing with posterior
distributions. Suppose the S̲s̲ have seen a composite sample (r,n) from our urn.
Then $a_1 = a_1' + r$ and $b_1 = b_1' + n - r$, where the primed parameters are the parameters
of the prior distribution. The same is true for the parameters of 2's posterior
distribution. We can express $\hat{p}$ as a function of the prior distribution and the
observed sample by simple substitution.

$$(14) \qquad \hat{p} = \frac{a_1' + a_2' - 2 + 2r}{a_1' + a_2' + b_1' + b_2' - 4 + 2n} = \frac{a_1' + a_2' + 2(r-1)}{a_1' + a_2' + b_1' + b_2' + 2(n-2)}$$

The interesting feature is that the data variables are weighted twice (and in an n person group they would be similarly multiplied by n). It seems that this decision strategy maintains a roughly constant balance between the prior opinions and the observed data. It does not become a simple weighting of prior opinion if the size of the group is increased.

This selection strategy guarantees each of our ideal Ss an expected utility of $f_1(\hat{p})$ and $f_2(\hat{p})$ respectively. We will now drop the restriction that randomization is impossible and will now attempt to discover when our Ss will prefer to let the choice of p depend on the result of some probabilistic experiment.

A randomization between two values of p, say p_1 and p_2, will be symbolized by $r(x; p_1, p_2)$ where x is the probability that p_1 will be chosen and $1 - x$ is the probability that p_2 will be selected. The expected utilities for a randomization are

$$(15) \qquad u_1[r(x; p_1, p_2)] = x f_1(p_1) + (1-x) f_1(p_2)$$

for S 1, and

$$(16) \qquad u_2[r(x; p_1, p_2)] = x f_2(p_1) + (1-x) f_2(p_2)$$

for S 2.

If a randomization $r(x; p', p'')$ exists such that

$$u_1[r(x; p', p'')] \geq f_1(\hat{p}), \quad \text{and}$$

$$u_2[r(x; p', p'')] \geq f_2(\hat{p}),$$

then the Ss will prefer to use the randomization strategy, rather than the Nash strategy for selecting p. This, of course, follows from the assumptions made concerning the intent of the Ss.

To help in conceptualizing the problem, let us define a function $U(p)$ which is the mapping of every point p into a two dimensional space with $f_1(p)$ on the ordinate and $f_2(p)$ on the abscissa. The domain of U is taken as $d_1 \leq p \leq d_2$. The first derivative of U which gives the slope of the function is found to be

$$(17) \qquad U'(p) = f_1'(p)/f_2'(p),$$

where $f_1'(p)$ is the first derivative of f_1 with respect to p. It is clear that $U'(p)$ will always be negative since f_1' is negative and f_2' is positive over the domain. Furthermore, there is no randomization which will satisfy the conditions for domination given above if $U(p)$ is concave downward which is to say if $U''(p)$ is never positive. If $U''(p)$ is positive on some interval, then a randomization will exist which will dominate some points on $U(p)$. This can be seen graphically in Figures 2 and 3.

It is possible conceptually to find the best randomization, with respect to the points p' and p". To do this we will first assume that $U''(p)$ is greater than zero on some interval in the domain. We will also assume that the sign of $U''(p)$ changes just twice so that there is only one interval on which $U''(p)$ is positive. We want to find the two values of p, p'* and p"*, such that no other randomization and no points on $U(p)$ provide greater expected utilities simultaneously for both 1 and 2. Recall that a randomization between two points in the U-space falls on the straight line joining the two points and that every point on the straight line is a randomization with some value of x. The points p'*, p"*, of course, must be on $U(p)$.

These values of p which provide the best randomization will be those two values which satisfy the following equation.

$$(18) \qquad U'(p'*) = U'(p"*) = \frac{f_1(p'*) - f_1(p"*)}{f_2(p'*) - f_2(p"*)} \; .$$

First, if $U(p)$ is concave downward over the entire domain then $U'(p)$ will be continually decreasing. Thus the first relationship in Eq. (18) can never be true. There will not be two values of p such that $U'(p') = U'(p")$ for p' ≠ p". Eq. (18) says that the best values of p to use in the randomization will be such that the slope of U at p'* is equal to the slope of U at p"* and that this slope is equal to the slope of the straight line joining the two points $U(p'*)$ and $U(p"*)$. It has not yet been proved that only one such pair of values exists, but under the assumptions it seems certain that this is true.

A final matter which is quite intriguing and which will receive attention in this report has to do with the notion of level of aspiration developed by Siegel (1957) and further discussed by Siegel and Fouraker (1960). If outcomes on an achievement scale are ordered in the reverse order of the S's preference for them, and equidistant from adjacent outcomes, so that the least preferred is first, then the level of aspiration as defined by Siegel (1957) is that outcome which is just above the largest interval on the corresponding utility scale. If outcomes are placed equally distant on the abscissa and utility of outcomes plotted on the ordinate, and all the points (x, u(x)) are plotted and all adjacent points joined by straight lines, the level of aspiration will be that value of x which corresponds to the upper point of the line with the maximum slope. Siegel assumes that outcomes below this level of aspiration (l.o.a.) will be accompanied by feelings of displeasure and dissatisfaction, while this outcome and all outcomes more preferred than this one will be accompanied by feelings of satisfaction. Siegel equates these with negative and positive utility, respectively.

Even though we are dealing with continuous utility functions and our utility functions are not over an achievement variable in the sense that Siegel uses the term, it is felt that the notion of l.o.a. might have some meaning in the present context. Let us, by direct analogy to Siegel's definition, define an l.o.a. for each S in our context. For S_1, the l.o.a. is e_1, that value of p corresponding to the inflection point of f_1 at which f_1' is zero and at which f_1'' changes from negative to positive. For S_2, the l.o.a. is e_2, that value of p corresponding to the inflection point of f_2 where f_2' is zero and where f_2'' changes from positive to negative. These points are logically analogous to the definition of l.o.a. for the

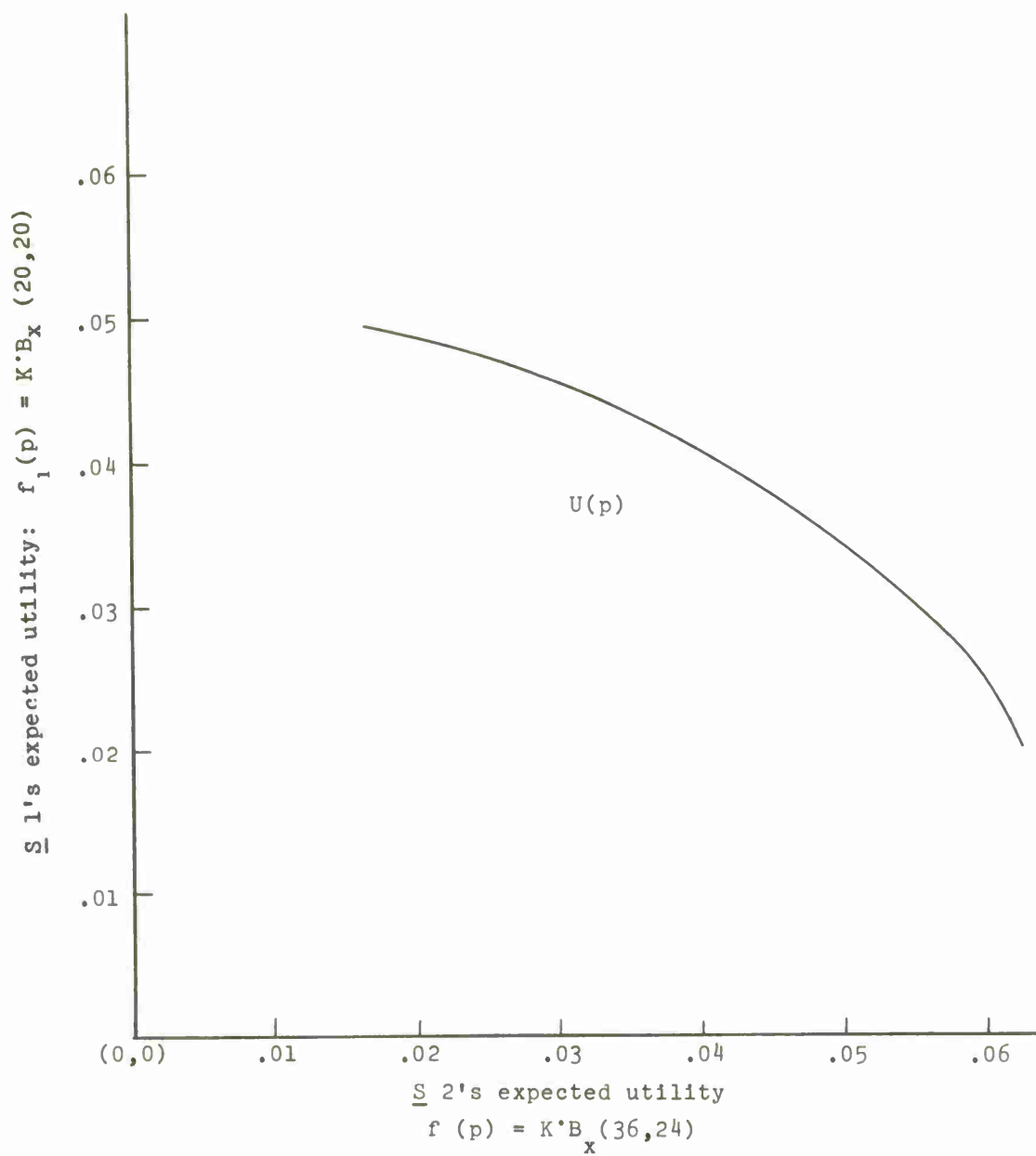Fig. 2. Graph of the function $U(p)$ when $f_1(p)$ is proportional to $B_x(20,20)$ and f_2 is proportional to $B_x(36,24)$.

Clearly since $U(p)$ is concave downward, our ideal $\underline{S}s$ will not use any randomized selection strategy.

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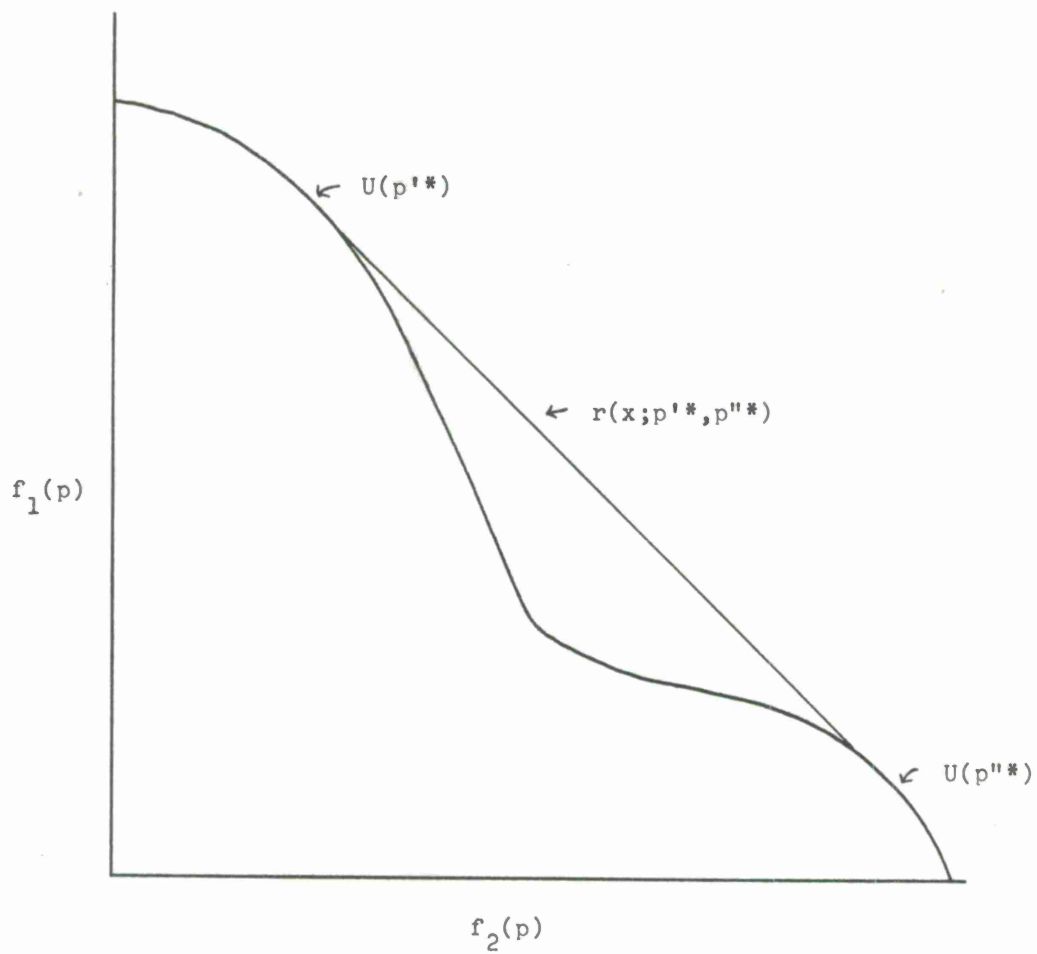
Fig. 3. Hypothetical U(p) showing the relationship between the sense of concavity of U(p), the points U(p'*) and U(p"*), and the randomization r(x;p'*,p"*).

discrete achievement variables with which Siegel was concerned. Moreover, by putting this concept of l.o.a. to use we can prove two theorems regarding the behavior of our ideal $\underline{Ss}$.

Theorem 1: If $e_1 < e_2$, $\underline{Ss}$ will not choose their d' such that $e_1 < d' < e_2$. They will use rather a randomized selection strategy between two points, p' and p'', such that $p' < e_1$ and $e_1 < p''$.

To prove this theorem it is sufficient to prove that U(p) is concave upward between e_1 and e_2. This will be true if U"(p) is positive on this interval. By Eq. (14) we know the first derivative of U(p) is

$$\text{(14)} \qquad U'(p) = f_1'/f_2' \; ,$$

where primed functions are derivatives with respect to p. Therefore,

$$\text{(19)} \qquad U''(p) = \frac{d}{df_2}\left(\frac{f_1'}{f_2'}\right) = \frac{f_2' f_1'' - f_1' f_2''}{(f_2')^3} \quad .$$

The denominator in the last term of the equality will always be positive since f' is positive in the domain of U. The sign of U"(p) will thus be determined by the sign of the numerator. We know that f' is always positive and f' is always negative in the domain of U. We also know the following relationships by definition.

$$\text{(20)} \qquad f_1'' < 0; \quad p < e_1$$

$$\text{(21)} \qquad f_1'' > 0; \quad p > e_1$$

$$\text{(22)} \qquad f_2'' > 0; \quad p < e_2$$

$$\text{(23)} \qquad f_2'' < 0; \quad p > e_2$$

$$\text{(24)} \qquad f_1'' = 0; \quad p = e_1$$

$$\text{(25)} \qquad f_2'' = 0; \quad p = e_2$$

For $e_1 < p < e_2$, $f_1'' > 0$ and $f_2'' > 0$. Thus the first term in the numerator of Eq. (19) will be positive and the second term will be negative. The numerator will therefore always be positive. For $p = e_1$, $f_1'' = 0$ and the first term of the numerator vanishes, leaving -1 times the negative second term. Again the numerator will be positive. For $p = e_2$, $f_2'' = 0$ and the second term in the numerator of Eq. (19) vanishes leaving the first term, which is positive, to determine the sign of U"(p). Thus the theorem is proved. At a later time, we hope to be able to state what the values of p' and p" will be. They will be such as to satisfy Eq. (18). In a similar manner, we can prove the following theorem.

Theorem 2: If $e_1 \geq d_2$ and $e_2 \leq d_1$, the Ss will never use a randomized selection strategy.

To prove this theorem, we prove that $U''(p)$ is always negative on the entire domain of U under these conditions. It is clear that f_1'' is always negative in the domain and that f_2'' is always negative as well. Thus the first term in the numerator of Eq. (19) will be negative and the second term will be positive in the domain of U. Thus the numerator will always be negative. If $e_1 = d_2$ then the first term in the numerator will vanish leaving a negative second term. (Actually, at this point $U''(p)$ is indeterminant because the denominator is zero. $U''(p)$ remains negative as this limit is approached, however, so the proof is not invalidated.) Likewise, at $e_2 = d_1$, $U''(p)$ is zero, but as this limit is approached $U''(p)$ is negative.

In terms of our interpretation of l.o.a., Theorem 1 says that if there is no possible decision which is simultaneously above both Ss' l.o.a., then the Ss will always choose a pair of decisions such that one is above S_1's l.o.a. and that the other is above S_2's l.o.a. and use some sort of randomization to choose between these decisions. The randomization could be done by flipping a coin, tossing dice, or in many cases, by submitting the problem to arbitration. To the extent that the results of an external arbitration are not known beforehand with certainty, the arbitration method of problem solving can be seen as a meaningful randomization.

Theorem 2 tells us that if all values of p in D (defined by Eq. (9)) are at once above both Ss' levels of aspiration, then the Ss might well be content with the Nash strategy, $d' = \hat{p}$ (defined in Eq. (13)). In no case will the Ss use a randomization.

Although the two theorems which have been derived here specify the behavior of our conceptual subjects in only two of the several possible states which can obtain regarding l.o.a. and d_1 and d_2, it is important to point out that the theorems are not specific only to the experimental task discussed. The theorems are general to any family of prior (and posterior) distributions having a single mode and no more than two points of inflection. Thus the theorems hold for normal and gamma distributions as well as the beta distributions used in the example.

The aim of this report has been modest. It has, first and foremost, attempted to show that the use of normative mathematical models is not alien to social psychology. We have shown that meaningful predictions can be obtained from such an analysis. This type of model also provides a precise and explicit framework against which the effects of other variables not explicitly included in the model can be evaluated. This paper has barely opened the door to the possible applications of formal decision theory to social psychological problems. What are the consequences, for example, of increasing the size of the group? What happens when we place restrictions on the communication facilities? What would be the result of using a linear group payoff function instead of an all-or-none function? More will have to be known of the relationship between the Nash strategy and the level of aspiration. How might we predict the formation of bargaining coalitions in groups of more than two Ss? These are just a few of the meaningful and important questions which may eventually be answered by a combined approach of decision theory and game theory.

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DOCUMENT CONTROL DATA - R&D
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Psychometric Laboratory, University of North Carolina Chapel Hill, North Carolina	Unclassified
	2b. GROUP n/a

3. REPORT TITLE

Bayesian Decision Theory, Game Theory and Group Problem Solving.

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
None

5. AUTHOR(S) *(Last name, first name, initial)*

Messick, David M.

6. REPORT DATE October 1964	7a. TOTAL NO. OF PAGES 14	7b. NO. OF REFS 5

8a. CONTRACT OR GRANT NO. AF 19 (628)-1610	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 4690	35
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	ESD-TDR-64-603

10. AVAILABILITY/LIMITATION NOTICES
Qualified requesters may obtain from DDC
OTS release authorized

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Decision Sciences Laboratory, Electronic Systems Division, L. G. Hanscom Field, Bedford Massachusetts

13. ABSTRACT

    This paper applies the concepts of the "ideal" information pro-cessor and rational decision maker to a typical problem in social psychology--that of group problem solving. The structure of the selected task is seen to be that of a nonzero sum game. A strategy is derived and is shown to be the equivalent of the Nash solution to the game. The notion of level of aspiration is discussed and defined within the analysis and two theorems are proved relating level of aspiration to type of group decision strategy employed.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Decision Making Game Theory Analysis Experimentation Mathematical Models						

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