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TSL-121-2 65

DATE PROCESSED: 7 May 65

PROCESSOR: Clay

Nonparametric Life Test Sampling Plans

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Mimeograph Series No. 29

July 1964

*The research of this author was supported by the Office of Naval Research Contract Nonr-3656(18) with the University of California. Reproduction in whole or in part is permitted for any purpose of the United States Government.

**The work of this author was supported by Army, Navy, and Air Force Contract Nonr-225(53) at Stanford University and also by Contract AF 33 (657)11737 with the Aerospace Research Laboratory at Purdue University.

NONPARAMETRIC LIFE TEST SAMPLING PLANS

1. Introduction

Sampling plans for truncated life tests based on several parametric families of distribution functions have been proposed by Sobel and Tischendorf (1959), Gupta and Groll (1961) and Gupta (1962a). Thus, we have the problem of which family to choose; i.e., exponential, normal, lognormal, gamma, etc. In many cases it is assumed that certain nuisance parameters are known. Unless considerable prior information is available, the choice will necessarily be somewhat arbitrary. We can circumvent this problem by considering a class of distributions restricted only by certain intuitive considerations. If the items in question wear out with age, then it is reasonable to assume that the failure rate function is nondecreasing. For other items such as solid state electronic devices, it seems reasonable to assume that the failure rate function is nonincreasing. By using sharp bounds on such distributions, we can obtain sampling plans which, although more conservative than the parametric plans, offer greater protection since they are valid for a much larger family of distributions.

It has been suggested by Zelen and Dannemiller (1959) that sampling plans should be written in terms of percentiles. Such plans were presented by Gupta and Groll (1961) and Gupta (1962a) for the gamma, normal and log-normal distributions. We present plans based on percentiles under the increasing (decreasing) failure rate assumption as well as plans based on the mean life. A partial discussion of these plans was also presented by Gupta (1962b).

2. Properties of Increasing (Decreasing) Failure Rate Distributions.

Let X be a random variable with distribution F such that $F(x) = 0$ for $x < 0$. If F has density f then $r(t) = f(t)/[1 - F(t)]$ is known as the failure rate function. Note that $r(t) = -\frac{d}{dt} \log [1 - F(t)]$ when a density exists. For this reason, we say that F is IFR (DFR) for increasing (decreasing) failure rate if $\log [1 - F(t)]$ is concave where finite (convex on $[0 , \infty)$). These distributions have been studied by Barlow, Marshall, and Proschan (1963) and Barlow and Marshall (1964). For convenience we summarize some of the fundamental properties of these distributions.

An IFR (DFR) distribution may have at most one point of discontinuity; namely, at the end of its right (left) hand interval of support. The continuous part of an IFR (DFR) distribution is absolutely continuous. Sums of independent IFR random variables themselves have IFR distributions. This is not true for DFR random variables. If F is a mixture of DFR distributions, then it follows that F is DFR. Many of the properties of the IFR (DFR) family follow from comparison with the exponential distribution. This is not surprising since it is the boundary distribution between the two classes. If $U_1 \leq U_2 \leq \cdots U_n$ are the order statistics from an IFR (DFR) distribution F with mean μ_1 then, $E[U_1] \underset{(\leq)}{\geq} \frac{\mu_1}{n}$ while $E[U_n] \underset{(\geq)}{\leq} \mu_1 \sum_1^n \frac{1}{k}$.

To obtain sampling plans based on percentiles we use the fact that $[1 - F(t)]^{1/t}$ is decreasing (increasing) in t when F is IFR (DFR). Let μ_r denote the r^{th} moment of F. To obtain sampling plans based on bounds on moments we use the inequalities for IFR distributions. Now we describe the relevant inequalities.

$$(2.1)\qquad 1 - F(t) \geq \begin{cases} \exp\left[- t/\lambda_r^{1/r}\right] & , t \leq \mu_r^{1/r} \\ 0 & , t > \mu_r^{1/r} \end{cases} \qquad (r \geq 1)$$

$$(2.2)\qquad 1 - F(t) \leq \begin{cases} 1 & , t \leq \mu_r^{1/r} \\ \exp\left[-wt\right] & , t > \mu_r^{1/r} \end{cases} \qquad (r > 0)$$

where w uniquely satisfies

$$\mu_r = r\int_0^t x^{r-1} e^{-wx} dx$$

and

$$\lambda_r = \mu_r/\Gamma(r+1) .$$

If F is DFR, with r^{th} moment μ_r, then

$$(2.3)\qquad 1 - F(t) \leq \begin{cases} \exp\left[- t/\lambda_r^{1/r}\right] & , t \leq r\lambda_r^{1/r} \\ \dfrac{r^r e^{-r} \mu_r}{\Gamma(r+1)t^r} & , t \geq r\lambda_r^{1/r} \end{cases}$$

From these bounds we can obtain the following bound on the q^{th} quantile ζ_q in terms of μ_1 assuming F is DFR. If $q \leq 1 - e^{-1}$, then

$$[-\log(1-q)]\mu_1 \le \zeta_q \le [-\frac{\log(1-q)}{q}]\mu_1 ;$$

if $q \ge 1 - e^{-1}$

$$\mu_1 \le \zeta_q \le [-\frac{\log(1-q)}{q}]\mu_1 .$$

From the fact that $\lambda_r^{1/r}$ is decreasing (increasing) in r when F is IFR (DFR) we see that the coefficient of variation $\sigma/\mu_1 \underset{(\ge)}{\le} 1$. The density and the failure rate are bounded at the origin from above in terms of moments as follows:

$$f(0) \le \frac{\lambda_{i-1}\lambda_{j-1}}{\lambda_{i+j-1}} \qquad i, j = 1, 2, \ldots$$

when F is IFR.

Bounds on densities and failure rates were obtained by Barlow and Marshall [1964]. If F is IFR and $r \ge 1$ then

$$f(t^+) \le r(t^+) \le \frac{[\Gamma(r+1)]^{1/r}}{\mu_r^{1/r} - t} \qquad , \qquad 0 \le t < \mu_r^{1/r}$$

and

$$r(t-) \ge \begin{cases} 0 & , \ t \le \mu_r^{1/r} \\ a & , \ t > \mu_r^{1/r} \end{cases}$$

when a satisfies $\mu_r = r\int_0^t x^{r-1} e^{-ax} dx$.

If F is DFR with density f, then

$$f(t^-) \leq \begin{cases} (te)^{-1} & , \quad t \leq (\lambda_r)^{1/r} \\ \lambda_r^{-1/r} e^{-t/\lambda_r^{1/r}} & , \quad \lambda_r^{1/r} \leq t \leq (r+1)\lambda_r^{1/r} \\ \lambda_r \left(\frac{r+1}{t}\right)^{r+1} e^{-(r+1)} & , \quad t \geq (r+1)\lambda_r^{1/r} \end{cases}$$

Finally, we mention that maximum likelihood estimates of IFR (DFR) distributions have been obtained by Grenander (1956) and Marshall and Proschan (1964).

3. Nonparametric Sampling Plans

Let θ denote a parameter defined by a life distribution F. For example, θ might denote the mean life or the 90^{th} percentile. We would like to establish that, say $\theta \geq \theta_0$ where θ_0 is some specified value. A common practice in life testing is to truncate the experiment at a preassigned time, say t, and note the number of failures (assuming that a failure is well-defined). One object of these experiments is to set a confidence (lower) limit on the mean (quantile) life of the units. If it is desired to establish a mean (quantile) life, with a given probability of at least P^* and if the number of failures in the fixed time does not exceed a given number c, then the experimenter would like to know the minimum sample size necessary to achieve his objective. In following these sampling plans and the associated decision rule the probability of rejecting a lot having a parameter $\theta \leq \theta_0$ is at least equal to P^*. Alternatively, the consumer's risk in adopting the sampling plan (n, c, t, θ_0) does not exceed $1 - P^*$ whatever θ may be. Mathematically, given a number P^* $(0 \leq P^* \leq 1)$, a time t and a value θ_0 of θ and an acceptance number c, we want to find the smallest positive integer n so that if the observed number of failures in time t does not exceed c we can assert with confidence probability P^* that $\theta \geq \theta_0$.

It should be pointed out that these life tests can be terminated prior to the time t with rejection as the result. In fact, the termination of the experiment can take place at the smaller of the two times t, $t_{(c+1)}$ where $t_{(c+1)}$ is the time to the $(c+1)^{st}$ failure. Also, the associated decision rule leads to an acceptance only at the

end of time t and only if $t_{(c+1)} > t$. Hence,

$$L(p) = P(\text{Acceptance}) = P(t_{(c+1)} > t)$$

$$(3.1) \qquad = \frac{n!}{(c-1)!(n-c)!}\int_t^\infty F^c(t;\theta)[1 - F(t;\theta)]^{n-c-1}dF(t;\theta)$$

$$= \sum_{i=0}^{c} \binom{n}{i} p^i(1-p)^{n-i}$$

where $p = F(t;\theta)$.

If p is a decreasing function of θ which is true, for instance, if the density $f(t;\theta)$ is TP_2 (Totally positive of order 2) in t and θ, then $F(t;\theta) \leq F(t;\theta_0) \Longleftrightarrow \theta \geq \theta_0$. If $p_0 = F(t;\theta_0)$, then it is clear from the theory of confidence intervals that the smallest c to satisfy

$$(3.2) \qquad \sum_{i=0}^{c} \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq 1 - P^*$$

is such that in adopting the above sampling plans with the associated decision rule, we can assert with probability P^* that $\theta \geq \theta_0$ provided $t_{(c+1)} > t$.

It should be noted that the above discussion assumes a knowledge of $F(t;\theta)$. Now, we consider the case where $F(t;\theta)$ is not explicitly assumed; it is only assumed that we have bounds on $F(t;\theta)$.

Suppose that $p = F(t;\theta) \geq b(t;\theta)$ for $t \geq 0$ where $b(t;\theta)$ is a known function. Suppose further that $b(t;\theta)$ is decreasing in θ. Since $L(p)$ is a decreasing function of p, we have

(3.3) $$L(F(t;\theta)) \leq L(b(t;\theta)) \leq L(b(t;\theta_0))$$

for $\theta \leq \theta_0$. Hence the sampling plans of the above type are obtained by choosing the smallest positive integer n satisfying

(3.4) $$L(b(t;\theta_0)) \leq 1 - P^*$$

where c , θ_0 and t are fixed.

From (3.3) and (3.4) it follows that if the number of failures during time t is less than or equal to c , then we can make the confidence statement that $\theta \geq \theta_0$ with confidence probability P^* .

Note that if $b(t;\theta)$ is a sharp bound on $F(t;\theta)$, then it is nondecreasing in t since $F(t;\theta)$ is nondecreasing in t . If θ is the mean, then $b(t;\theta) = b(t/\theta;1)$ since the mean is a scale parameter for positive random variables and $b(t;\theta)$ is decreasing in θ .

Now we give a few examples where the bound $b(t;\theta)$ is known.

Example 1. (Markov's inequality). Let θ denote the mean. Then

$$F(t;\theta) \geq b(t;\theta) = \begin{cases} 0 & , t < \theta \\ 1 - \theta/t & , t \geq \theta \end{cases}$$

is true for nonnegative random variables. Clearly, $b(t;\theta)$ is decreasing in θ . A sampling plan based on Markov's inequality would afford protection over the class of all distributions on the positive axis. However, the bound is quite wide.

Example 2. (Unimodal density)

If the density f is unimodal with unknown mode and first moment θ and if $F(0-;\theta) = 0$, then from Barlow and Marshall (1963), we have

$$F(t;\theta) \geq b(t;\theta) = \begin{cases} 0 & , \quad 0 \leq t \leq \theta \\ 2 - \frac{2\theta}{t} & , \quad \theta \leq t \leq \frac{3\theta}{2} \\ 1 - \frac{\theta}{2t} & , \quad t \geq \frac{3\theta}{2} \end{cases}$$

Clearly, $b(t;\theta)$ is decreasing in θ and the bound is a slight improvement over Example 1.

Example 3. (PF_2 density).

If $\log f(t;\theta)$ is concave where $f(t;\theta) > 0$, then f is called a PF_2 (Pólya Frequency of Order 2) density. Most of the commonly used densities have this property. If f has mean θ, then

$$F(t;\theta) \geq b(t;\theta) = \begin{cases} 0 & , \quad t \leq \theta \\ 1 - \sup_{m \geq t} (1 - e^{-bt})/(1 - e^{-bm}) & , \quad t \geq \theta , \end{cases}$$

where b is determined by $\theta = \int_0^m xbe^{-bx}dx/(1 - e^{-bm})$.

The above bound is tabulated in a paper by Barlow and Marshall (1963). A sampling plan based on this bound would afford less protection than the IFR sampling plans since f PF_2 implies that F is IFR but not conversely.

Sampling Plans for the IFR and DFR Distributions

Now we discuss the derivation of sampling plans for the IFR and DFR distributions for the following two cases:

IFR Distributions

Case (i) $\theta = \mu_r$

In this case, from (2.2), we have

$$b(t;\mu_r) = \begin{cases} 0 & , \quad t < \mu_r^{1/r} \\ 1 - e^{-wt} & , \quad t > \mu_r^{1/r} \end{cases} \tag{3.5}$$

where w uniquely satisfies

$$\mu_r = r \int_0^t x^{r-1} e^{-wx} dx \ . \tag{3.6}$$

Clearly $b(t;\mu_r)$ is a decreasing function of μ_r. Hence the required n to insure $\mu_r > \mu_r^0$ is given by the smallest positive integer which satisfies

$$\sum_{j=0}^{c} \binom{n}{j} [b(t;\mu_r)]^j [1 - b(t;\mu_r)]^{n-j} \leq 1 - P^* \ . \tag{3.7}$$

It should be noted that the solution of (3.7) depends on the fact that $t^r > \mu_r^0$. Of special importance is the case where $r = 1$. In this case (3.6) reduces to

$$\mu_1 w = 1 - e^{-wt} \tag{3.8}$$

Hence, using (3.8) to solve for w and then using (3.7) with $b(t;\mu_1^0) = b(t/\mu_1^0;1) = 1 - e^{-wt}$, sampling plans to insure a specified mean life have been computed and are given in Table 1. This table also gives the actual confidence level attained.

Case (ii) $\theta = \zeta_q$, the q^{th} percentile.

In this case

$$(3.9) \qquad b(t;\zeta_q) = \begin{cases} 0 & , \quad t < \zeta_q \\ 1 - (1 - q)^{t/\zeta_q} & , \quad t \geq \zeta_q \end{cases},$$

which is again decreasing in ζ_q and clearly depends only on q and the ratio t/ζ_q. Hence, to insure a specified quantile life ζ_q^0, we substitute from (3.9) the value of $b(t;\zeta_q^0)$ in the general equation (3.4) and solve for the smallest n. Table II at the end of this paper gives minimum sample sizes for the life testing situation where $t > \zeta_q^0$.

DFR Distributions

Case (i) $\theta = \mu_r$.

In this case, letting $\lambda_r = \mu_r/\Gamma(r + 1)$, we have

$$(3.10) \qquad b(t;\mu_r) = \begin{cases} 1 - e^{-t/\lambda_r^{1/r}} & , \quad t \leq r\lambda_r^{1/r} \\ 1 - r^r\lambda_r(et)^{-r} & , \quad t \geq r\lambda_r^{1/r} \end{cases},$$

and one can construct sampling plans for the cases where $t \leq r\,(\lambda_r^0)^{1/r}$ and $t \geq r(\lambda_r^0)^{1/r}$. For the special case $r = 1$, and $t < \mu_1^0$, it is

easy to check that the sampling plans based on the bounds in (3.10) reduce to the sampling plans for the negative exponential with $\lambda_1^{\circ} = \mu_1^{\circ} = \theta_{\circ}$. These tables have already been given by Sobel and Tischendorf (1959) for $t < \lambda_1^{\circ} = \mu_1^{\circ}$.

Case (ii) $\theta = \zeta_q$.

In this case the bound is

$$b(t;\zeta_q) = \begin{cases} 1 - (1 - q)^{t/\zeta_q} , & t \leq \zeta_q \\ q , & t > \zeta_q \end{cases} \qquad (3.11)$$

To use the non-trivial bound, we require $t < \zeta_q^{\circ}$. The sampling plans are obtained as in case (ii) of IFR distributions discussed above. Table III at the end of this paper gives these sampling plans for $\lambda = t/\zeta_q^{\circ} \leq 1$.

OC Function and the Producer's Risk

The OC function represents the probability of acceptance as a function of θ . The sampling plans and the decision rule guarantee that this probability is less than or equal to $1 - P^*$ for all $\theta \leq \theta_{\circ}$.

Suppose there exists a function $B(t;\theta)$ such that $F(t;\theta) \leq B(t;\theta)$ for all t and all θ , then we have

$$P\{\text{Acceptance}\} \geq L(B(t;\theta)) . \qquad (3.12)$$

In order to insure that the producer's risk will not exceed a number α $(0 < \alpha < 1)$, whenever $\theta \geq \theta_1$, one needs to satisfy the

inequality

$$(3.13) \qquad L(B(t;\theta)) \geq L(B(t;\theta_1)) \geq 1 - \alpha$$

which will be satisfied if $B(t;\theta)$ is decreasing in θ. One may use the inequality (3.13) to determine c such that for fixed θ_1, θ_0 and P^*, the sampling plan and the acceptance rule will insure that the producer's risk will not exceed α. Alternatively for fixed n, t, θ_0, c and P^*, one may want to know the minimum value θ_1 of θ such that for all $\theta > \theta_1$, the probability of acceptance $\geq 1 - \alpha$.

Now we describe the evaluation of the OC function for the IFR and DFR distributions.

IFR Distributions

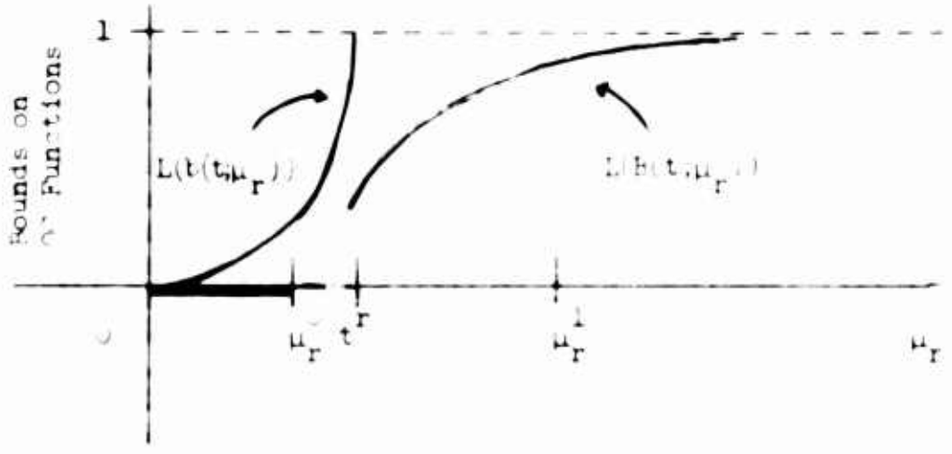
Case (i) $\theta = \mu_r$

$$(3.14) \qquad B(t;\mu_r) = \begin{cases} 1 - e^{-t/(\lambda_r)^{1/r}} & , \quad t \leq \mu_r^{1/r} \\ 1 & , \quad t > \mu_r^{1/r} \end{cases}$$

Thus, for $\mu_r > \mu_r^1 \geq t^r$, we have

$$(3.15) \qquad L(F(t;\mu_r)) \geq L(B(t;\mu_r)) \geq L(B(t;\mu_r^1))$$

which gives a lower bound on the OC function. This bound is graphed below.



Case (ii) $\theta = \zeta$

In this case

$$B(t;\zeta_q) = \begin{cases} 1 - (1-q)^{t/\zeta_q}, & t \le \zeta_q \\ 1, & t \ge \zeta_q \end{cases} \tag{3.16}$$

and the lower and upper bounds can be obtained similarly by using $b(t;\zeta_q)$ and $B(t;\zeta_q)$ in the appropriate intervals.

Finally, it should be noted that sampling plans which protect the producer can be constructed for test time less than the goal using the bound $B(t;\theta)$ in the IFR case.

It is clear from the bounds on OC functions for the IFR case that we have to test beyond the mean (quantile) life goal to protect the consumer whereas we can protect the producer by testing for a time less than the mean (quantile) life goal.

DFR Distributions

Case (i) $\theta = \mu_r$

In this case there is no nontrivial lower bound for the OC

function L since there is no nontrivial upper bound on F. However, there exists a nontrivial upper bound on the L function both for $\mu_r^{\circ} < t$ and $\mu_r^{\circ} \geq t$. This lower bound is obtained by using (3.16) in place of F in the L function.

Case (ii) $\omega = \zeta_q$

Using the upper bound

$$B(t;\zeta_q) = \begin{cases} q & , \quad t \leq \zeta_q \\ 1 - (1-q)^{t/\zeta_q} & , \quad t \geq \zeta_q \end{cases} , \tag{3.17}$$

one can construct a lower bound on L for all t.

Description of the Tables and Some Illustrative Examples

Table I. This table gives the sample sizes necessary to establish that the true unknown mean μ of an IFR distribution is at least μ° at a nominal confidence level P^*. The actual level attained is also tabulated.

Illustration 1. Suppose an experimenter wants to choose the sample size corresponding to $c = 2$, $P^* = .75$, $\lambda = t/\mu^{\circ} = 1.0$, then this required n from Table 1 is equal to 5. For this sampling plan the actual probability level is equal to .7523.

Table II. This table gives the sample sizes necessary to establish that the true unknown q^{th} quantitle of an IFR distribution is at least ζ_q° at a nominal confidence level P^*. The actual level attained is also tabulated.

Illustration 2. Assuming an IFR distribution and given $c = 2$,

$P^* = .75$, $\lambda = t/\zeta_q^0 = 1.6$, $q = .2$, we find from Table II that the required minimum sample size is 13. For this sampling plan the actual probability attained if the decision rule accepts is equal to .7981.

Application to Drug Screening

Suppose a drug which is known to be non-toxic is administered to patients suffering from a specified cancer to determine its effectiveness. The drug will be considered effective if, say 20% of the patients respond to treatment after T weeks (T = 6, say). The treatment will continue t weeks (t = 12, say) unless more than c patients respond before this time--in which case the drug will be declared effective at the end of the t_{c+1} weeks. Let F(t) denote the probability of a response by time t and $F(T) = p$. Since we do not want to discard a potentially useful drug, we set

$$P[\text{rejecting drug} \mid p \geq .20] = \sum_{j=0}^{c} \binom{n}{j} [F(t)]^j [1 - F(t)]^{n-j} \leq \alpha .$$

Now $F(T) \geq .20$ if and only if $\zeta_{.20} \leq T$ where $\zeta_{.20}$ is the 20th percentile. If we assume an IFR distribution time; i.e., $\frac{f(t)}{1 - F(t)}$ increasing in t , then this problem is identical with case (ii) considered under IFR sampling plans.

Illustration. We want to determine the number of patients to put on trial to establish with 90 per cent confidence that, $F(T) \geq .20$; i.e., the response rate after T weeks is at least 20 per cent. If $\frac{t}{T} = 2$, $c = 3$, $Q = .20$ and $P^* = .90$, we find from Table II that $n = 17$ and the true confidence level is .91.

Table III. This table gives the sample sizes necessary to establish that the true unknown c^{th} quantile of a IFR distribution is at least ζ_q^0 at a nominal confidence level P^*. Here $\lambda = t/\zeta_q^0 \leq 1$. Also, the actual confidence attained is tabulated.

Approximations for Sample Sizes

The sample sizes necessary to establish a quantile for the IFR and DFR distributions can be approximated as follows. If $1 - (1 - q)^{\lambda}$ is small, then

$$(3.18) \qquad n \doteq \left[\frac{y_{c+1,P^*}}{1 - (1 - q)^{\lambda}} \right] + 1$$

where y_{c+1,P^*} is the P^* percentage point at a standardized gamma variable with shape parameter $r = c + 1$ or one-half times the P^* percentage point of a χ^2 with $(2c + 2)$ degrees of freedom. This approximation was discussed by Gupta (1962a) where tables of y_{c+1,P^*} are also given.

If $p = 1 - q$ is close to .5, then another approximation for n is

$$(3.19) \qquad n \doteq \left[\frac{(2c + 1 + pz_p^{2*})\left(1 + \sqrt{1 - (2c + 1)^2/(2c + 1 + pz_p^*)^2}\right)}{2(1 - p)} \right] + 1$$

where z_p^* is the P^* percentage point of the $N(0, 1)$ random variable. For example, if $q = .1$, $\lambda = 1$, $c = 1$, $P^* = .75$, then $y_{c+1,P^*} = 2.6926$ and from (3.18), $n \doteq 27$. The exact answer from Table II is 27. As an example of the second approximation, let $\lambda = 1$, $p = 1 - q = .5$,

$P^* = .75$, $c = 2$, then the approximate answer from (3.19) is 7 which coincides with the exact value.

Acknowledgment: The authors wish to thank Mrs. Shirley Ebberly, formerly of Stanford University, for programming and carrying out the computations and Phyllis De Fabio for typing assistance.

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