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PREDICTION MODELS FOR FIRE SPREAD
FOLLOWING NUCLEAR ATTACKS

Final Report
January 1965

Prepared for

OFFICE OF CIVIL DEFENSE
Office of the Secretary of the Army
Department of the Army
Washington, D.C.
Contract Number OCD-PS-64-48
Subtask 4611C

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by

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ABSTRACT

In Part I a semiempirical approach is used to predict fire spread beyond the area directly ignited by thermal radiation from nuclear weapons. Mathematical models, both stochastic and deterministic, describe the progress of fires in two-dimensional or one-dimensional space. Application of each model to appropriate cases is discussed. Empirical data needed for evaluation of parameters are specified and methods for acquiring these data suggested. Observed data, accumulated over many years in records of past fires, have proved valuable in determining some of the parameters. The remaining parameters require further observed data. At present approximate prediction can be made by use of a specially designed version.

Part II presents the results of a statistical study on observed rate of spread data and discusses a number of specific problems that must be worked out before the method can be used for assessing the fire damage from nuclear attacks.

ACKNOWLEDGEMENT

The authors wish to acknowledge the contributions of Dr. William S. Jewell, Professor of Industrial Engineering, University of California, in the development of the fire front model and for the many helpful discussions the authors have had with him during the course of this study.

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Section 1 INTRODUCTION

PREVIOUS STUDY

Mass fires can produce staggering damage both in wartime and in peacetime. The advancement of nuclear weapons has introduced a powerful means for destroying large areas by direct ignition and subsequent fire spread. The great consequences of mass fires following nuclear attacks have stimulated considerable efforts in investigating various aspects of the behavior of mass fires, despite their highly complex nature.

One of these efforts was a small study performed at URS Corporation (formerly United Research Services) in 1960 under the sponsorship of the U.S. Forest Service (Contract No. 12-11-005-21911).^{*} This study had two major objectives, namely, to develop a simple calculation method for predicting the total burnout area following a nuclear attack on the basis of the U.S. Forest Service 1957 Damage Assessment Study and to explore the possibilities of developing improved models for predicting the total area of spread.

NEEDS FOR IMPROVED MODEL

This study indicated that significant improvements could still be made in fire spread prediction models. It was also concluded

^{*} Jewell, W. S. and A. B. Willoughby, A Study to Analyze and Improve Procedures for Fire Damage Assessment Following Nuclear Attack, BRC 167-1, Parts I and II, Final Report, October 1960 (AD 286 943).

that, in addition to predicting the ultimate burnout area, future studies should attempt to describe the dynamics of fire spread. For gross damage assessment in simulated or actual nuclear attacks, there seems to be a real need for information about the time-dependent behavior of fires, particularly in estimating personnel casualties, since the population distribution will presumably not be static in the early postattack period. Such information is also useful at the local level for planning evacuation routes and for deciding whether fire fighting is feasible in protecting critical installations and shelters. When the use of a model is extended to peacetime forest or urban fires, it is obvious that the knowledge of fire dynamics is even more critical since it permits rational use of manpower and equipment in fire fighting.

Section 2

OBJECTIVES AND SCOPE

DEFINITION OF MAIN OBJECTIVE

The objective of the present study is to develop an improved prediction method describing the extent of fire spread beyond the initial area ignited by nuclear attack and to specify the data required for use with the method and ways of acquiring these data. Thus, no effort is devoted to prediction of the initially ignited area as a function of weapon characteristics, meteorological conditions, and fuel characteristics, a problem which is essential in the assessment of fire damage from nuclear attacks but which has been appropriately considered as a separate task.

SPECIFICATION OF REQUIRED DATA

Since the models are intended for both present and future uses, their implementation must be immediately practical with the existing input data and be capable of subsequent improvement as more and better data become available. Therefore, after models have been selected, the required input data must be indicated and procedures for gathering existing data for immediate implementation and for future acquisition of more extensive data must be suggested.

Section 3

OUTPUT REQUIREMENTS

SPECIFIC APPLICATIONS

The type of fire spread model and the method of implementation must be determined by the desired output information and, consequently, by the particular uses intended for the model. The general purpose of fire spread models in civilian defense is to provide estimates of damage to the nation's resources from the fire effects of a massive nuclear attack. These estimates are needed primarily for preattack planning, although there also exists a requirement for early postattack damage assessment. Pre-attack and postattack applications can be performed at three geographical levels as follows:

Preattack Planning

- National. To help assess the nation's ability to survive massive nuclear attack.
- Regional. To provide a basis for various preattack countermeasures such as stockpiling, protection of critical resources, and development of postattack survival and recovery plans.
- Local. To provide a basis for local preattack thermal countermeasures and development of specific survival plans.

Postattack Indirect Damage Assessment

- National. To help provide early assessment of the nation's overall survival posture.
- Regional. To help select one of several postattack survival plans.

- Local. To provide a basis for specific actions by the population and local civil defense units.

REQUIRED LEVELS OF DETAIL IN THE OUTPUT

It is clear that each type of application calls for a different level of detail in the output information. National uses seem to require only gross information - for example, the fraction of each resource still available after an attack of hundreds of nuclear weapons. Since primary concern is with the average result for a large number of weapons, a great deal of uncertainty in predicting damage resulting from each individual weapon can be tolerated, provided systematic errors are not serious.

For regional uses, a somewhat greater level of detail is needed. For example, to decide where and how much to stockpile certain critical supplies demands reasonably accurate knowledge of the expected damage averaged over areas of a few hundred miles on a side. Local uses would require by far the greatest level of detail. The fire spread for each individual nuclear detonation must be predicted with fairly good accuracy.

In the original planning for the present study, it was assumed that models for national and regional application were most urgently needed. Hence, most effort was concentrated in these types of application, and only preliminary considerations were given to models for local use.

CONVERSION OF FIRE SPREAD INFORMATION TO DAMAGE INFORMATION

For damage assessment purposes, the needed information is the damage by fire or the fraction of resources destroyed rather

than the fire boundaries per se. As with predictions of damage from other nuclear weapons effects, fire damage can be derived from fire spread information by assigning vulnerability factors to various resources. These factors give a measure of the fraction of resources destroyed or the probability of their destruction if the area is passed over by fire.

Section 4

DESCRIPTION OF THE PROBLEM

DIRECTLY IGNITED AREA

Fire behavior in the area directly ignited by nuclear weapons would constitute a useful and interesting study. By combining data on intensity of thermal radiation with a fire spread model especially designed for handling the spread of multiple ignition centers, it would be possible to estimate the size and contour of the directly ignited area, to predict how fast the individual fires grow, merge, and burn out and how weather and fuel affect such a burning process. Although such a study does not constitute the subject matter of the present program, it seems appropriate to have a qualitative picture of what happens in this area shortly after ignition.

The radius of the directly ignited area, i.e., the distance beyond which the thermal energy is insufficient to ignite kindling fuel, depends on three groups of variables: the weapon characteristics (including yield and height of burst), the transmission characteristics of the atmosphere, and the ignition characteristics of fuels.

Although contours of equal thermal energies are, in general, concentric circles with center at ground zero (Fig. 1), the shape of the initially ignited area will not be circular unless the kindling fuels are uniformly distributed, which is not necessarily the case. Furthermore, the ignition radii give no information on the density of ignition centers (number of ignition centers per

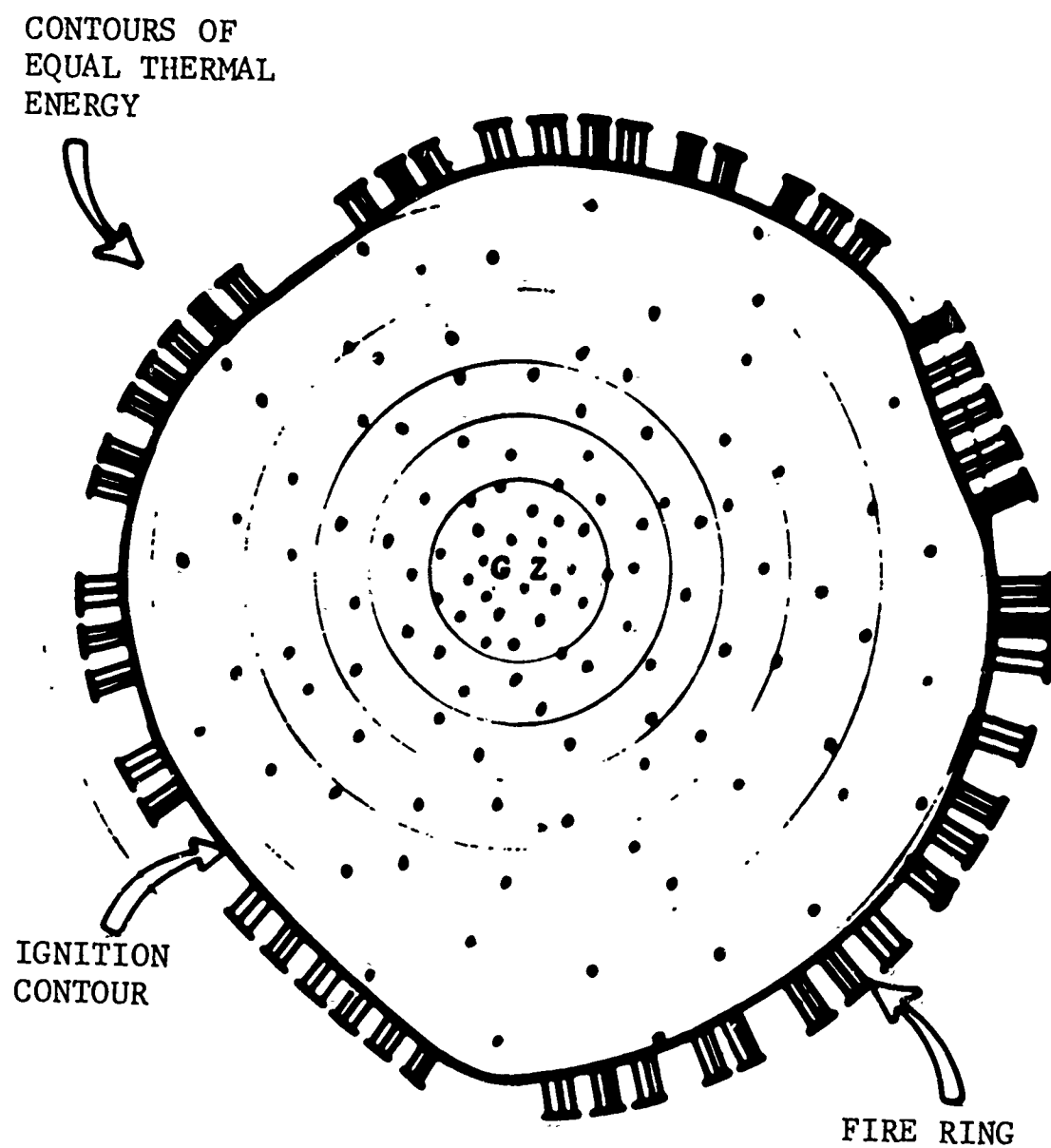


Fig. 1. Initial Fire

unit area) and its variation with distance from ground zero. In general, however, we can expect the density of ignition centers to decrease with increasing distance from ground zero. This is due to the fact that the thermal intensity decreases with increasing distance. Consequently, a decreasing number of kindling fuels actually occurring in the area can be ignited.

The evolution of the initially ignited area can be qualitatively described by three successive stages:

- (1) Provided sufficient fuel is available, some of the ignition centers spread to adjacent fuels and build up to small and sustained fires. The rest go out due to their tenuity or unfavorable surroundings.
- (2) In the next stage, these small fires merge together to form larger fires, which — with further merging — may take mass fire proportions. The time for merging depends on many factors, of which the density of small fires is perhaps the most important. Since this density is higher near the center, the fires in the central portion generally merge and burn out earlier than those near the periphery. The possibility of a fire storm comes into the picture at this stage. Unfortunately, we know little about the factors which produce fire storms and how fire storms modify the evolution of mass fires. We know, however, that there are mass fires without genuine fire storms.
- (3) In the absence of a fire storm, a burnout zone first appears at the central portion and expands toward the periphery. The third stage begins when the burning area is reduced to a more or less continuous ring which keeps moving outward. With a fire storm the behavior is less clear. However, if one assumes that the fire storm does not extinguish the fire, it seems reasonable to again assume a ring fire after the fire storm has died out. It is the spread of this ring that we undertook to study in the present program.

RELATIONS BETWEEN FIRE SPREAD PHENOMENA

Because of the intimate relations between various characteristics of fire spread, the study of burnout area (required for estimating fire damage to resources) cannot be separated from that of other quantities equally fundamental in understanding fire behavior. Such quantities are rate of spread, evolution of heat and flame, size of burning area at a given time, and percentage of burned fuel in a general burnout area.

Rate of spread is naturally derivable from any model that represents fire spread both in time and in space. Heat and flame output, per unit burning fuel, are determined primarily by the type of fuel and possibly by its moisture content. Since higher heat output raises the temperature of exposed fuel more rapidly, the rate of spread is expected to depend on heat output and, hence, on fuel type.

The instantaneous area of the burning fire may play an important role. The ignition rate and the spread rate depend on the total heat flux (plus embers and firebrands), which for a given set of fuel and weather conditions is determined by the area of the burning fire. This total flux will become independent of the burning area only when the fire has reached a dimension such that the heat from the rear portion cannot reach the unignited fuel in front of the fire. Furthermore, a very large burning fire may conceivably modify its environment (create its own weather) and consequently behave in a manner not predictable on the basis of conditions initially specified. Therefore it is important to know how large a burning fire is at a given moment and under what conditions a big fire's behavior is influenced by new weather factors generated by itself.

The percentage of fuel ultimately burned out is determined by the degree of homogeneity of the fuel. Since fuel distribution is an important factor in setting up and applying a fire spread model, it is probably worthwhile to elaborate upon this topic somewhat further.

An area containing a single type of fuel with uniform distribution is either completely burned out or completely unburned. Uniform distribution is frequently encountered in wildland fuel. A large area is normally made up of sections of various sizes, each with a rather uniform and continuous fuel distribution. For urban areas the degree of homogeneity is rather low. With the possible exception of residential tracts, structural materials, sizes, and spacings may vary considerably. Therefore some structures may survive a fire which destroys others.

As output information from the model, the burnout percentage depends on the level of detail of the input data. To illustrate this point, suppose we consider predicting fire spread in a small area, say 20 to 30 square miles, for local damage assessment purpose. This special application requires that fuel characteristics be determined with sufficient geographic details. For example, a set of fuel characteristics might be specified for each unit of one-hundredth of a square mile. This unit would then constitute what might be called the resolution of the model output. Any prediction regarding the burnout event will refer to the unit as a whole, and the burnout percentage within each unit cannot be given.

When very large areas, such as major sections of the country or the whole country itself, are considered, two factors will limit the level of detail in the output information, namely, the

size of the unit area (or cell) and the size of areas for which fuel characteristics can be specified. The maximum number of cells and, hence, the minimum size of the cell is determined by the amount of computation that can be afforded. A similar economical reason will restrict the specification of input data to a single set of fuel conditions for each section (herein called "big square") containing a large number of cells. A big square might be 20 or 25 miles on each side. Thus the model will tell whether a cell has burned out or give the probability of the burnout event but cannot tell what fraction of the cell has burned out. It may give a different answer for each cell in a given big square, but the difference is due to locations of the cells relative to the origin of the fire rather than to variations in fuel characteristics.

FIRE SPREAD VARIABLES

Many factors of different nature combine their action to determine whether a fire will spread, how fast and how far it will spread, and how large the burning area will be at a given moment. These factors are called fire spread variables and may be grouped into three general classes: fuel variables, weather variables, and topography variables. We will now attempt to list in each class all variables that, according to our experience and judgment, may influence fire spread and to qualitatively indicate how this influence is brought about.

Fuel Variables

Fuel Type

Some general fuel types are: grass, brush, and timber for wildland areas, industrial, commercial, and residential structures

for urban areas. Each general type may include a number of varieties. It is rather obvious that fuel types may not be equally ignitable and when burning may have different igniting capability.

Fuel Buildup

This is the thickness of the fuel layer as measured by the number of stories in urban fuel or the fuel height in wildland fuel. Fuels with high buildup probably have longer burning time and put out more heat and more flames per unit burning area. However, the effects of buildup on ignition, spread rate, and long-range spotting are not so obvious.

Fuel Density

Our terminology assigns this term to the fraction of total area occupied by the fuel. Low density has an opposing effect on fire spread because less heat is released per unit burning area. If the fuel distribution is specified stochastically, there exists for any fuel density a finite chance that a fire will spread indefinitely through the area. This chance, however, becomes increasingly small as the density decreases. Therefore, the mean final spread distance decreases with fuel density.

Spatial Distribution of Fuel

The fuel in two areas of the same fuel density may not be distributed in the same way. The fraction of fuel area may spread out and mix uniformly with the nonfuel fraction or, at the other extreme, may form a single continuous section. Obviously the latter mode of distribution greatly enhances the probability of stopping. Fuel distribution may have other types of effects that are less apparent.

Fuel Age

Urban fuels probably become more ignitable after reaching a certain age, particularly if they are not maintained properly. Wildland fuels seem to decrease in overall moisture content as they grow older and, consequently, may become more ignitable. Under fuel age we might include green state and dead state of fuel. The age effect may also operate through fuel fineness.

Fuel Fineness

This factor contributes to ignitability and spread rate. It may promote intense burning and reduce the burning time.

Weather Variables

Moisture Content

Although it certainly has an effect on ignitability, rate of spread, and probability of stopping, this variable is complicated and difficult to define. Rural fuel usually consists of a mixture of light, medium, and heavy components, and each component can be either green or dead. At any time, the moisture content may be different in each case, because the time for fuel moisture to come into equilibrium with the surroundings depends on each component and its state. The finer the fuel, the faster it reaches moisture equilibrium. Thus for very fine fuels, the current relative humidity might be a good measure of the moisture content. Heavy dead fuels dry out slowly, with the result that their moisture content at any time depends on the precipitation and atmospheric moisture for a number of days preceding the time of interest. Also, in larger fuels there is probably a moisture gradient from

the surface to the center. Thus factors such as days since last rain, relative humidity, and temperature history may be important.

Wind

Wind carries the heat into unignited fuel, supplies oxygen to the burning fuel, and promotes direct flame contact and spotting. This variable appears to be simple, easily defined by speed and azimuth. Complications arise, however, because of the possibility of vertical wind gradients and modifications of wind speed and direction by local topography and fuel.

Temperature

The higher the atmospheric temperature, the less heat is required to raise the fuel to the ignition point. However, variations in ordinary atmospheric temperatures do not significantly affect heat requirements for ignition in mass fires. Thus, except for extreme cases perhaps, temperature by itself may play only a minor role in fire spread.

Topography Variables

Slope

Since hot gases tend to go up, the slope of the fuel area favors heat convection either in the direction of unignited fuel or in the opposite direction, depending on whether the fire is spreading up or down slope. Thus slope effect is similar to wind effect in both mechanism and directionality. However, for a given area, wind varies with time, whereas slope effect varies with space only. The slope effect may be complicated or counter-

balanced by concomitant phenomena, such as rolling of burning fuel or changes in spotting conditions.

Altitude

As such, altitude should play no role in fire spread unless it is associated with other variables, such as vertical wind gradients, fuel types, moisture content, etc.

Relief

By this term, we mean the combined effect of the inequalities of land surface, their sizes, and their mode of distribution. With the same total area, the hills can be large and few or small and many. They can also be scattered throughout a region or closely drawn together in one section. It is conceivable that these factors have a bearing on fire spread, though it is difficult to speculate on how important they are and in what direction they operate.

Terms such as "probably," "perhaps," and "may" have been used abundantly in the preceding paragraphs to emphasize the fact that without experimental data, our knowledge on fire spread is highly uncertain and purely qualitative at best. The effect of a variable may be real but indiscernible among other more important ones, or it may remain sensibly constant over the usual range of fire spread conditions. A possible example of constant variable might be the fuel build-up in wildland areas. For a given type of fuel, the variation in buildup is probably not significant. Some variables operate in opposite direction and partially or completely cancel each other's effect. Others are not strictly independent but are frequently associated in their occurrence. For example,

fuel type may be associated with slope; fuel buildup and moisture content with fuel fineness, etc.

Section 5

METHODS OF APPROACH

THREE METHODS OF APPROACH

Possible models for prediction of fire spread can be classified into three general types: purely theoretical, purely empirical, and semiempirical.

Purely theoretical models would be characterized by an attempt to predict fire spread on the basis of physical laws, without resorting to observed data on actual fires. This approach is probably one that we would like to use eventually; however, physical laws needed to describe various phenomena in macroscopic fires are still incomplete or nonexistent. Most of the existing laws have been obtained by calculation and experimentation under highly simplified and rigorously controlled conditions, which are rarely encountered in actual cases. When applied to large-scale phenomena, these laws are likely to yield questionable results. Thus much basic research on macroscopic fires is still to be done before a purely theoretical approach is practical.

A purely empirical approach would be to analyze data from actual fires and obtain a series of tables or graphs showing how a given quantity depends on variation of one or more controlling factors. The given quantity might be the final burnout area, the rate of spread, or some fire behavior that might be of interest in the future. Prediction of potential fires could be made by comparing the fuel characteristics, weather, and topography conditions with these tables or graphs and probably by

doing some interpolation or extrapolation. Because of the complexity of the phenomena and the large number of variables involved, it would be difficult with this type of approach to organize empirical data logically without some theoretical backup. Furthermore the procedures for setting up such a model and for using it are too mechanical to yield much understanding of mass fires.

Selection of the Semiempirical Approach

In view of the present state of the art, a semiempirical approach seems to be the most appropriate and has been adopted in the present study. In this approach, the fire spread process is mathematically represented in terms of a mechanism such as suggested for similar phenomena in epidemiology. The mechanism involves some basic properties which may depend on many variables and which appear in the mathematical description as parameters. Parameters must be provided by a study of records of actual fires or by direct experimentation. Thus the basis for semiempirical prediction of fire spread lies both in the body of experimental data and in the selected mechanism.

Treatment of Fire Spread Variables

Detailed procedures in mathematical representation are largely determined by the stochastic or deterministic character of the variables. At certain scales of time and space, all variables are stochastic, i.e., all undergo random variations of such amplitudes as to significantly affect the process of fire spread. Since a large number of stochastic variables may render the model excessively complicated, it is important to decide which variables

can be treated as deterministic and to use only those stochastic variables required in each particular situation. To help visualize the problem let us qualitatively represent the tendency to fire spread as a function of time T and space X (one dimensional). Tendency to fire spread is an undefined but unambiguous quantity that we shall denote by Z . (See Fig. 2.)

At a given T , time-dependent variables (weather) have the same values throughout a rather large area. Therefore the quantity Z will vary with space only, i.e., with space-dependent variables. Its variations may be represented by a curve such as A. There are three types of fluctuations: small ripples within short distances in the range of 20 - 50 ft, due to lack of perfect uniformity in a given fuel type; wider fluctuations, spanning distances of a quarter of a mile or more, corresponding to changes in general fuel type or slopes (in going from brush to timber, or from residential area to industrial area); and occasional sharp dips to zero in empty spaces.

At a given distance X , the quantity Z varies with time according to some curve such as B. Fluctuations are of two general types: high-frequency fluctuations due to unsteady wind and low-frequency fluctuations due to slow changes in moisture content, temperature, and long-term value of wind velocity. The instantaneous wind velocity is very unstable, even for periods of 10 or 20 min. The effect of this instability on fire spread is depicted on curve B (Fig. 2) by fluctuations of small amplitude. The latter, however, should not be taken too seriously, because most useful quantities in fire spread are not truly instantaneous values but rather

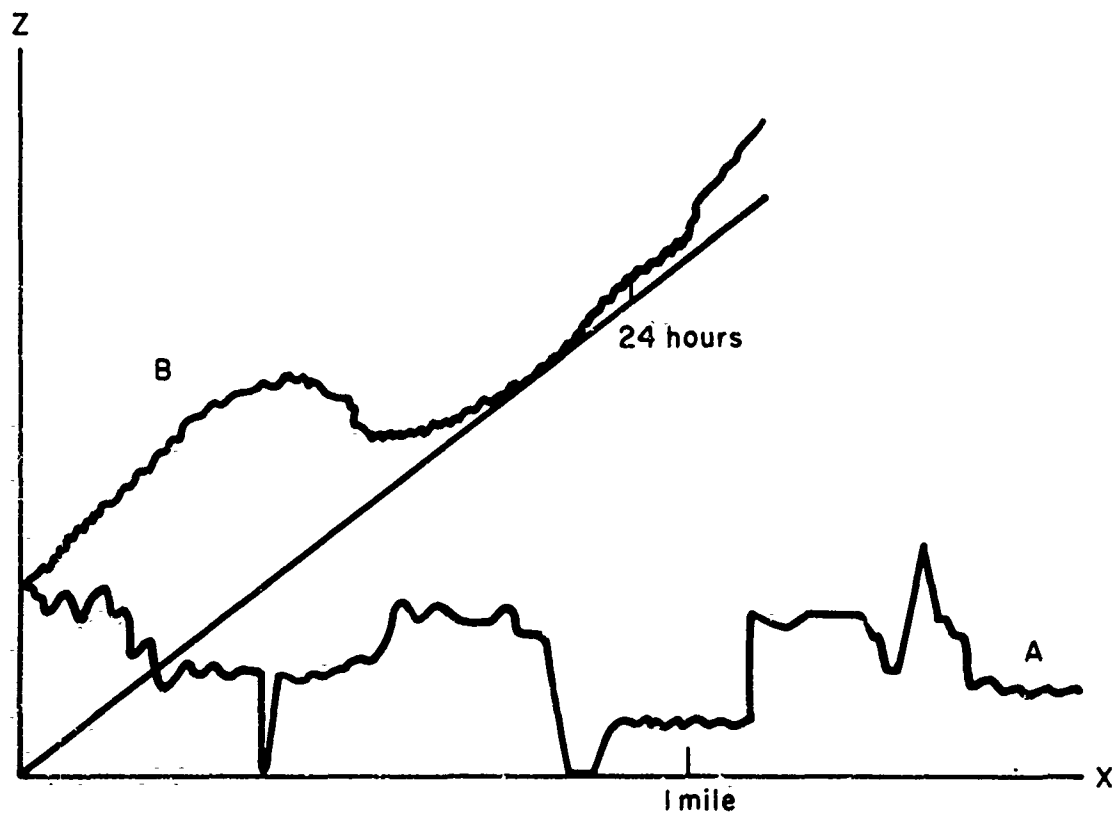


Fig. 2. Probable Effects of Fuel and Weather on Fire Spread

averages over short periods of time. Most other variables in the weather category, including long-term value of wind velocity, change with a much slower tempo. The Z-value resulting from the combined effect of these variables is likely to hold rather steady for intervals of 6 to 12 hr.

It thus appears that in many cases, weather variables can be treated as deterministic quantities. In these cases, their average values would be specified for each time interval (6, 12, or 24 hr).

The X variables, on the contrary, are considerably more random, especially in urban fuel areas. Deterministic treatment of these variables would require specification of any variation in fuel type, density, buildup, or spacing and in empty areas. Whether or not this is practical depends on the level of effort devoted to fuel survey and numerical computation. The level of effort does not seem to be excessive if fire spread prediction is restricted to an area of 25 or 30 square miles. For much larger areas, the effort may not be justified by the detail requirement in the output information. In this case, the probability distributions of space variables must be given and a stochastic model must be used.

Section 6

MATHEMATICAL MODELS

CLASSIFICATION OF MODELS

Figure 3 shows two families of models: fire-front and fuel-state. In each, a number of specific versions have been derived from the parent model. Specific versions serve a variety of purposes: illustrating particular features of the parent model, deriving some general properties which could not be conveniently studied with the parent model, providing more convenience in specific applications.

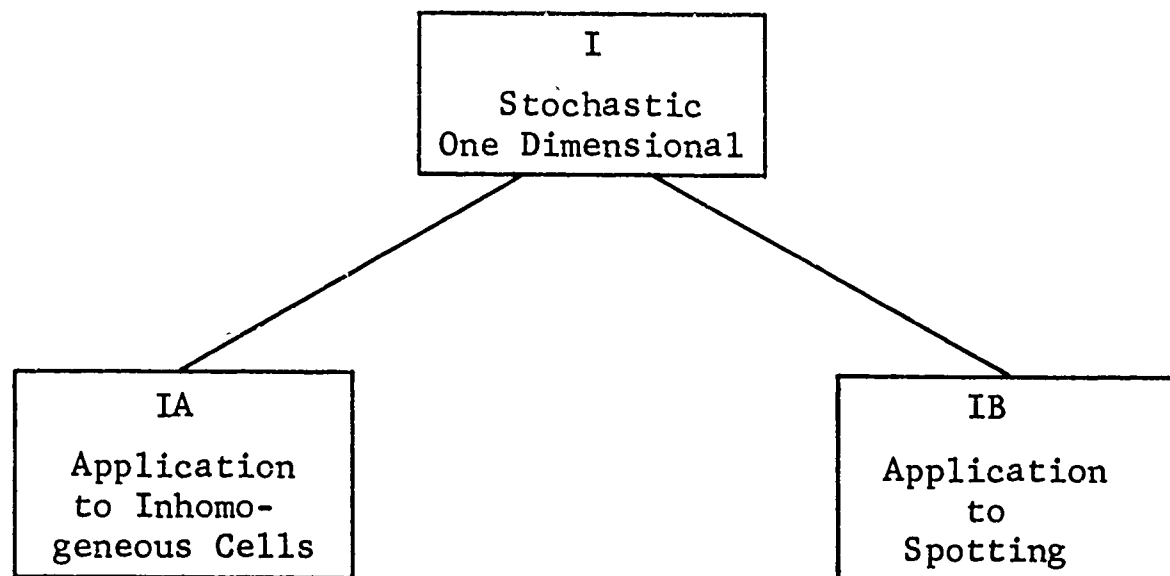
In the following paragraphs, each model or specific version will be formulated in some detail, and the more important results will be explicitly indicated. All mathematical derivations will be presented in Appendixes.

FIRE FRONT MODELS

The fire front model and all its variations treat the fire front (or the entire fire itself) as a special random walker moving along a row of cells or small square areas. In each short time interval dt , the random walker may:

- Die or stop permanently with probability $\mu_f dt$
- Move one cell forward with probability λdt
- Pause or stay where it is with probability $1 - (\mu_f + \lambda)dt$

FIRE FRONT MODELS



FUEL STATE MODELS

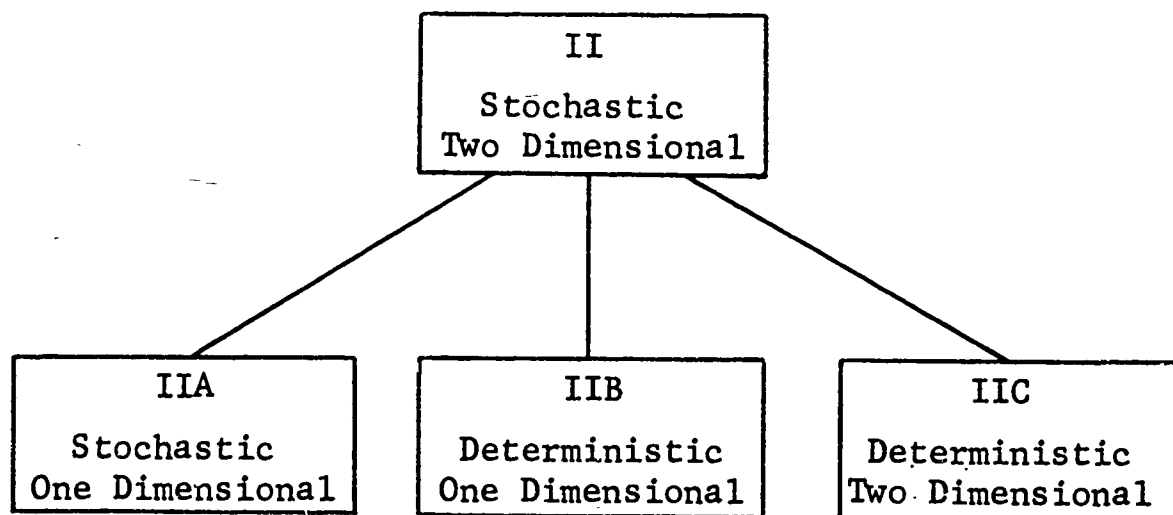


Fig. 3. Two Groups of Models

The parameters μ_f and λ are probability rates or probabilities per unit time for the extinction and spread processes. This model and the following ones describe the fire spread process by using only two or three parameters. Consequently the latter are by necessity complicated functions of all fire spread variables. Their role and significance will be discussed later in this report. For the present, only their general definitions as given above are required.

The mechanism does not allow the fire to move more than one cell during the interval dt . The underlying assumption can be made plausible by choosing appropriate cell dimension and considering only cases in which long-range spotting does not occur. A version to be discussed later is sufficiently general to include the spotting process.

Denoting by $P_n(t)$ and $Q_n(t)$ the probability that at time t the fire front is at cell n or has died at cell n , respectively, the basic equations can be written as follows:

$$\frac{\partial P_0(t)}{\partial t} = -(\lambda + \mu_f)P_0(t) \quad (1)$$

$$\frac{\partial P_n(t)}{\partial t} = \lambda P_{n-1}(t) - (\lambda + \mu_f)P_n(t) \quad n = 1, 2, 3, \dots \quad (2)$$

$$\frac{\partial Q_n(t)}{\partial t} = \mu_f P_n(t) \quad n = 0, 1, 2, \dots \quad (3)$$

with the initial conditions:

$$P_n(0) = \begin{matrix} 1 & n = 0 \\ 0 & n \neq 0 \end{matrix} \quad Q_n(0) = 0 \quad n = 0, 1, 2, \dots \quad (4)$$

The solutions are

$$P_n(t) = \frac{(\lambda t)^n e^{-\theta t}}{n!} \quad (5)$$

$$Q_n(t) = \frac{\mu_f}{\theta} \left(\frac{\lambda}{\theta}\right)^n \left[1 - e^{-\theta t} \left(1 + \theta t + \frac{\theta^2 t^2}{2!} + \dots + \frac{\theta^n t^n}{n!} \right) \right] \quad (6)$$

where

$$\theta = \mu_f + \lambda$$

Variations of $P_n(t)$ and $Q_n(t)$ are illustrated in Fig. 4.

As t approaches infinity, the burnout distances follow a geometric distribution

$$Q_n(\infty) = \frac{\mu_f}{\theta} \left(\frac{\lambda}{\theta}\right)^n \quad (7)$$

The mean ultimate burnout distance is

$$\bar{n}(\infty) = \sum_{n=0}^{\infty} n Q_n(\infty) = \frac{\lambda}{\mu_f} \text{ cell widths} \quad (8)$$

At time t , the mean position of the fire front, given that it is still alive, is given by

$$\bar{n}_p(t) = \frac{\sum n P_n(t)}{\sum P_n(t)} = \lambda t \text{ cell widths} \quad (9)$$

Thus, the mean velocity of the burning front is λ cell widths per unit time. Another important result is that the mean lifetime of the fire is $1/\mu_f$.

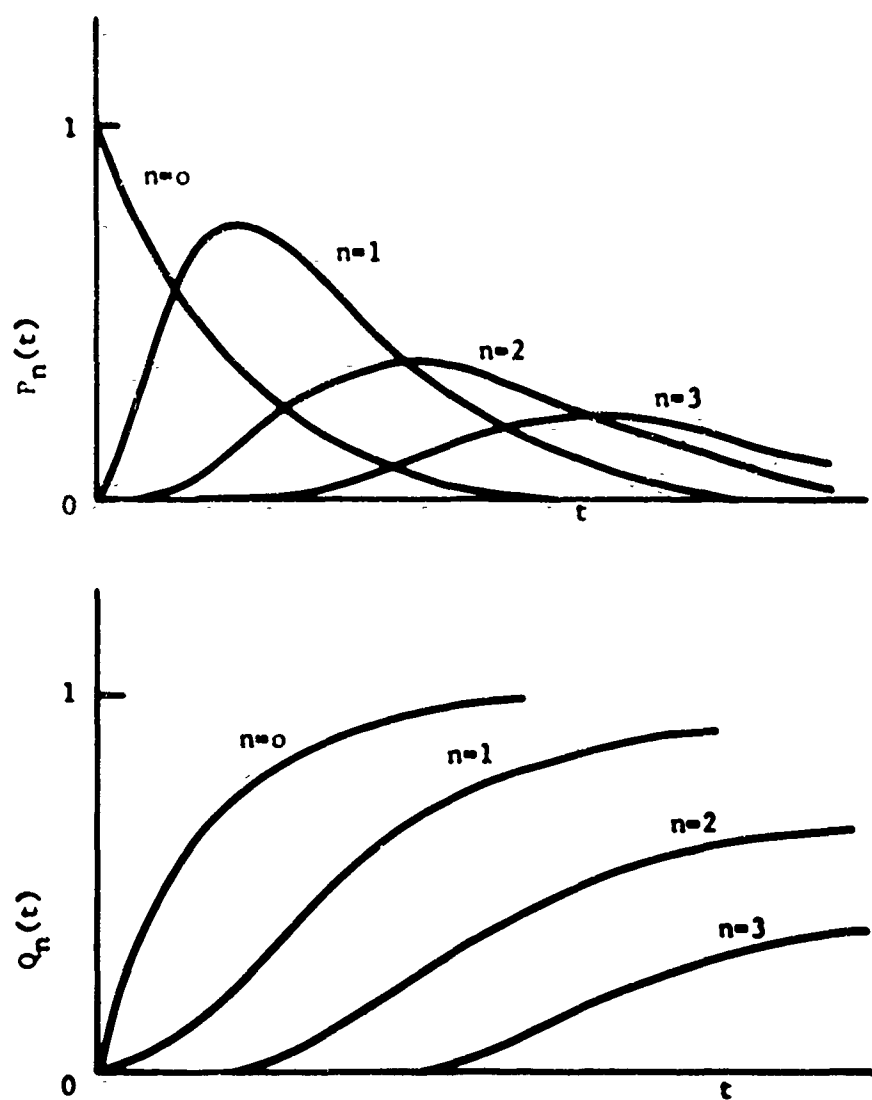


Fig. 4. Variations of $P_n(t)$ and $Q_n(t)$

APPLICATION TO INHOMOGENEOUS CELLS (MODEL IA)

If fuel characteristics are specified for individual cells, the above model can be modified by taking a value μ_n for each cell n and a value $\lambda_{n,n+1}$ for each pair of cells $n, n+1$. The equations become

$$\frac{\partial P_o(t)}{\partial t} = -(\mu_o + \lambda_{o1})P_o(t) \quad (10)$$

$$\frac{\partial P_n(t)}{\partial t} = \lambda_{n-1,n}P_{n-1}(t) - (\lambda_{n,n+1} + \mu_n)P_n(t) \quad (11)$$

$$\frac{\partial Q_n(t)}{\partial t} = \mu_n P_n(t) \quad (12)$$

A closed expression that can be easily obtained is

$$Q_n(\infty) = \frac{\mu_n}{\mu_n + \lambda_{n,n+1}} \prod_{i=0}^{n-1} \frac{\lambda_{i,i+1}}{\mu_i + \lambda_{i,i+1}} \quad (13)$$

Solutions for other quantities are more conveniently performed by computer, although some complicated analytical expressions can be obtained.

If the fuel area contains several uniform regions having different fuel characteristics, the basic model can be applied to each region, and the resulting distributions can be adjusted to match each other at the discontinuities.

APPLICATION TO SPOTTING (MODEL IB)

From any cell n the fire can jump to cells $n+1, n+2, n+3, \dots$ with decreasing probabilities $\lambda_{n,n+1}dt, \lambda_{n,n+2}dt, \lambda_{n,n+3}dt, \dots$. If all cells are stochastically identical, a single μ_f value applies and

$$\begin{aligned}\lambda_{01} &= \lambda_{12} = \lambda_{23} = \dots = \lambda_1 \\ \lambda_{02} &= \lambda_{13} = \lambda_{24} = \dots = \lambda_2 \\ \lambda_{0,i} &= \lambda_{1,i+1} = \lambda_{2,i+2} = \dots = \lambda_i\end{aligned}\tag{14}$$

Calculation of $Q_n(\infty)$ can be performed recursively by using the relation

$$Q_n(\infty) = \frac{\lambda_1 Q_{n-1}(\infty) + \lambda_2 Q_{n-2}(\infty) + \dots + \lambda_n Q_0(\infty)}{\mu_f + \sum_{i=1}^{\infty} \lambda_i}\tag{15}$$

Details on mathematical derivations are presented in Appendix A.

FUEL STATE MODELS

The burning process is evidently a gradual and continuous one. Unless the element of area or cell under consideration is very small (a few square feet), instantaneous and uniform ignition of the entire cell is of rare occurrence. In general the cell is ignited at one or more points and the fire builds up by intra-cell spread. The chance of extra-cell spread (spread outside the cell of origin) becomes increasingly greater, then begins

to decline after the fire has passed its peak intensity. This chance reaches the zero point at some uncertain moment which coincides with or precedes total extinction of the cell. Whether extra-cell spread actually takes place depends obviously on the condition of the surrounding cells as well as on the fire intensity in the cell under consideration.

This continuous train of events is somewhat arbitrarily divided into three distinct stages or states, the unignited state, the flaming state, and the burnout state. The flaming state is defined as one capable of extra-cell spread regardless of whether this type of spread actually takes place or not, and it is left undetermined as to what phase the combustion must reach or what fraction of the cell must burn in order to pass to F-state. The states preceding and following the flaming state are defined as unignited and burnout states, respectively. Thus a cell which has been ignited but still cannot spread fire to its neighbors is by definition in the unignited state.

The concept of fuel states originated in the striking similarity between epidemic breakout and fire spread. Many phenomena in epidemics, such as susceptibility, incubation, infection, removal of victims by isolation or death, dependence of infection on intensity of epidemic, age, state of health, population density, etc., have their obvious counterparts in fire spread.

The fuel state models attempt to predict the state of each cell at various times (deterministic versions) or the time-dependent probabilities of the states of each cell (stochastic versions). Models II, IIA, IIB, and IIC are indicated in Fig. 3 and will be discussed in that order.

Model II

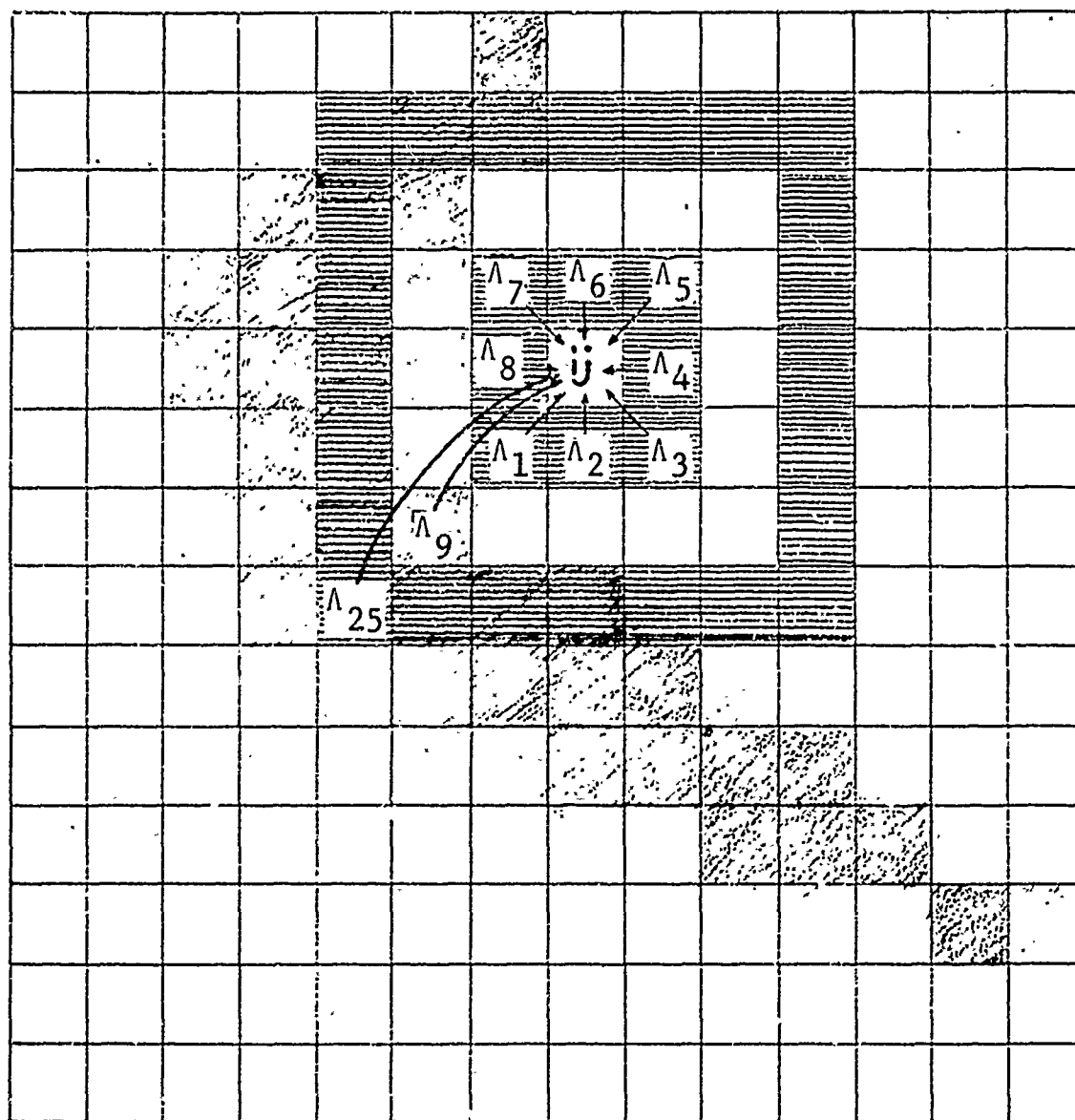
Two-dimensionality, discrete space and probabilistic approach are the main features of this model. The probabilities that at time t cell ij is in the unignited, flaming, and burnout states are denoted in that order by $U_{ij}(t)$, $F_{ij}(t)$, and $B_{ij}(t)$ satisfying the relation:

$$U_{ij}(t) + F_{ij}(t) + B_{ij}(t) = 1 \quad (16)$$

Given that cell ij is in F-state, the probability for transition from F-state to B-state is $\mu_s dt$ for each time increment dt . The decay parameter μ_s depends on some or all fire spread variables. However, for a given set of variable values, it is assumed to be a constant, independent of previous happenings. More extensive discussions will be devoted to this and other parameters in subsequent sections of this report.

If cell ij is in U-state and its neighbors in F-state, the probability that ij is ignited in time dt is denoted by $\Lambda_{ij}dt$. (See Fig. 5.) Suppose, for the moment, that the cell dimension has been chosen so that a burning cell can ignite the immediate neighbor cells but not those lying further away. Then an unignited cell ij can be ignited by one or more of its 8 immediate neighbors. If the chance of ignition by each neighbor separately denoted by Λ_1dt , Λ_2dt , . . . , Λ_8dt , then the chance of ignition by one or more neighbors, i.e., $\Lambda_{ij}dt$, is given by

$$\begin{aligned} \Lambda_{ij}dt &= 1 - (1 - \Lambda_1dt)(1 - \Lambda_2dt) \dots (1 - \Lambda_8dt) \\ &= (\Lambda_1 + \Lambda_2 + \dots + \Lambda_8)dt - (\Lambda_1\Lambda_2 + \Lambda_1\Lambda_3 + \dots)dt^2 + \dots \quad (17) \end{aligned}$$



States: $U_{ij}(t) + F_{ij}(t) + B_{ij}(t) = 1$

<u>Transitions</u>	<u>Probabilities</u>
dies . . .	$\mu_s dt$
gets ignited . . .	$\Lambda_{ij} dt$

Fig. 5. Fuel State Model

Neglecting terms in dt^2 , dt^3 , . . ., we have

$$\Lambda_{ij}dt = (\Lambda_1 + \Lambda_2 + \dots + \Lambda_8)dt \quad (18)$$

For each pair of igniting cell and exposed cell, the spread parameter Λ is the rate of the ignition probability. Its value is determined by the fuel characteristics of the two cells, the weather, and topography variables.

The equations of the model can be written as

$$\frac{\partial F_{ij}(t)}{\partial t} = U_{ij}(t) \sum_{u=1}^8 F_u(t) \Lambda_u - \mu_s F_{ij}(t) \quad u = 1, 2, \dots, 8 \quad (19)$$

$$\frac{\partial U_{ij}(t)}{\partial t} = -U_{ij}(t) \sum_{u=1}^8 F_u(t) \Lambda_u \quad (20)$$

$$\frac{\partial B_{ij}(t)}{\partial t} = \mu_s F_{ij}(t) \quad (21)$$

where the u 's indicate the eight cells surrounding ij .

For initial conditions, one or more cells may be specified as being in F-state and all others in U-state at $t = 0$. For example

$$F_{ij}(0) = 1 \quad \text{for } i = 0 \quad j = 0 \quad (22)$$

$$U_{ij}(0) = 1 \quad \text{for all other } ij \text{ combinations}$$

Solutions for $U_{ij}(t)$, $F_{ij}(t)$, $B_{ij}(t)$ and other useful quantities must be obtained by numerical or machine computation.

However approximate expressions can be obtained if we consider

only the initial period during which $U_{ij}(t)$ has not dropped far below 1. These expressions permit some qualitative study of the system before extensive computation is undertaken. An exact solution for $B_{ij}(\infty)$ is presented in Appendix B.

Detailed Spread Pattern

In the above, we assumed that an unignited cell can be ignited only by its immediate neighbors with spread parameters $\Lambda_1, \Lambda_2, \dots, \Lambda_8$, which may or may not be equal or symmetrical, since they depend on some directional factors such as wind and ground slope. If the cell dimension is small or long-range spotting is important, a cell may be directly ignited by more remote neighbors. In this case the spread pattern should include more than one layer of cells.

The equations are still the same, except the summation is extended to cover all possible ignition sources, for example

$$\frac{\partial F_{ij}(t)}{\partial t} = U_{ij}(t) \sum_{u=1}^{8+16+24} F_u(t) \Lambda_u - \mu_s F_{ij}(t) \quad (23)$$

Linear Version

The equations become somewhat more tractable when the model is applied to a row of cells in which lateral and back spread can be ignored. If fire can spread only from one cell to the next, we can write

$$\frac{\partial F_n(t)}{\partial t} = U_n(t) F_{n-1}(t) \Lambda - \mu_s F_n(t) \quad (24)$$

The following exact solutions can be obtained:

$$B_n(\infty) = 1 - e^{-aB_{n-1}(\infty)} \quad a = \Lambda/\mu_s \quad (25)$$

$$F_0(t) = e^{-\mu_s t} \quad (26)$$

$$F_1(t) = ae^{-a} e^{-\mu_s t} \left[\mu_s t + a(1 - e^{-\mu_s t}) + \frac{a^2}{2.2!} (1 - e^{-2\mu_s t}) + \dots \right] \quad (27)$$

$$B_1(t) = \mu_s \int_0^t F_1(\tau) d\tau \quad (28)$$

Calculation for $F_n(t)$ is most conveniently carried out with the aid of a computer.

The curves for $F_n(t)$ and $B_n(t)$ are somewhat similar to those for $P_n(t)$ and $Q_n(t)$ in the fire front model. (See Fig. 4.)

It will be shown analytically in Appendix B that the mean jump time is $1/\Lambda$ and the mean rate of spread is Λ cell widths per unit time. This result has been confirmed by computer calculation. For this purpose, the mean location $\bar{n}(t)$ of the fire is defined as:

$$\bar{n}(t) = \frac{\sum_n n F_n(t)}{\sum F_n(t)} \quad (29)$$

and the mean velocity is given by

$$\bar{R} = \frac{\bar{n}(t + \Delta t) - \bar{n}(t)}{\Delta t} = \Lambda \text{ cell widths} \quad (30)$$

For any set of conditions (i.e., any value of a), the actual spread distance for $t = \infty$ can be zero, finite, or infinite. There is however a probability for each distance, and the mean spread distance D given by

$$D = \sum_{0}^{\infty} B_n(\infty) \quad (31)$$

is a characteristic of the fuel area and the weather during the fire. In Appendix B it will be shown that D is infinite for $a \geq 1$. When $a < 1$, the mean burnout distance is to a good approximation given by

$$D = \sum_{0}^{\infty} B_n(\infty) \approx 1 + \frac{a \left(1 - \frac{a}{2}\right)}{1 - a} \quad (32)$$

As in the two-dimensional model, a detailed spread pattern can be described by including more than one spread parameter Λ and writing the equation as

$$\frac{\partial F_n(t)}{\partial t} = U_n(t) \left[F_{n-1}(t) \Lambda_1 + F_{n-2}(t) \Lambda_2 + \dots \right] - \mu_s F_n(t) \quad (33)$$

Deterministic Version

When the fuel characteristics and weather conditions can be specified deterministically, as might be in the case in local applications, a deterministic model is desirable. To construct a model of this type we start with the following considerations:

- The ignition rate and, consequently, the spread rate dX/dt of the fire front depends on the total heat flux (including

embers and firebrands) from the burning zone. The total heat flux, in turn, depends on the heat released per unit burning fuel and the width of the fire front.

- When the width of the fire front exceeds a certain value W_0 , the heat flux from the rear cannot reach the unignited fuel at the front. W_0 depends on factors such as wind velocity, fuel density, etc.

The general equation is then

$$\frac{dX}{dt} = \varphi[W(t), k] \quad (34)$$

where X is the position of the leading edge of the fire front, $W(t)$ the width at time t , k a spread constant and a measure of the heat released per unit burning fuel, and φ some function of $W(t)$ and k . The exact form of this should be determined empirically, perhaps with some theoretical backup. However, extensive work in this direction is not justified at this stage, and a linear relation seems to be a fair description of the process. Thus

$$\begin{aligned} \frac{dX}{dt} &= kW(t) & W(t) &\leq W_0 \\ &= kW_0 & W(t) &> W_0 \end{aligned} \quad (35)$$

from which we derive

$$W(t) = \int_{t-\tau}^t \frac{dX(\theta)}{d\theta} d\theta = k \int_{t-\tau}^t W(\theta) d\theta \quad (36)$$

or

$$\frac{dW(t)}{dt} = k[W(t) - W(t - \tau)] \quad (37)$$

where τ is the burning time of the fuel.

The final burnout distance X is given by

$$D = W_0 k \tau \frac{1 - k\tau/2}{1 - k\tau} \quad (38)$$

as derived in Appendix B.

Instantaneous values of $W(t)$ and dX/dt have been obtained by numerical methods and plotted in Fig. 6.

It is interesting to note that the condition for infinite spread in this model, i.e., $k\tau \geq 1$, is equivalent to the condition $\Lambda/\mu_s \geq 1$ in the linear stochastic model. Also for $k\tau < 1$ the final spread distance X is almost exactly equal to the final mean distance in that model. Also k and τ are equivalent to the stochastic Λ and $1/\mu_s$ (the mean burning time).

For $k\tau < 1$, fire front width decreases steadily to zero regardless of whether its initial value $W(0)$ is greater or smaller than W_0 . The velocity however remains constant whenever the front width is greater than W_0 .

For $k\tau = 1$, the front width decreases or increases from its initial value to a steady value, which is W_0 . The velocity also tends to the constant value kW_0 .

For $k\tau > 1$, the front width increases until it reaches a constant value given by $W_0 k\tau$ while the velocity increases to kW_0 and remains unchanged thereafter.

If information on W_0 and the function φ exists, it should be possible to determine the minimum fuel gap required to stop

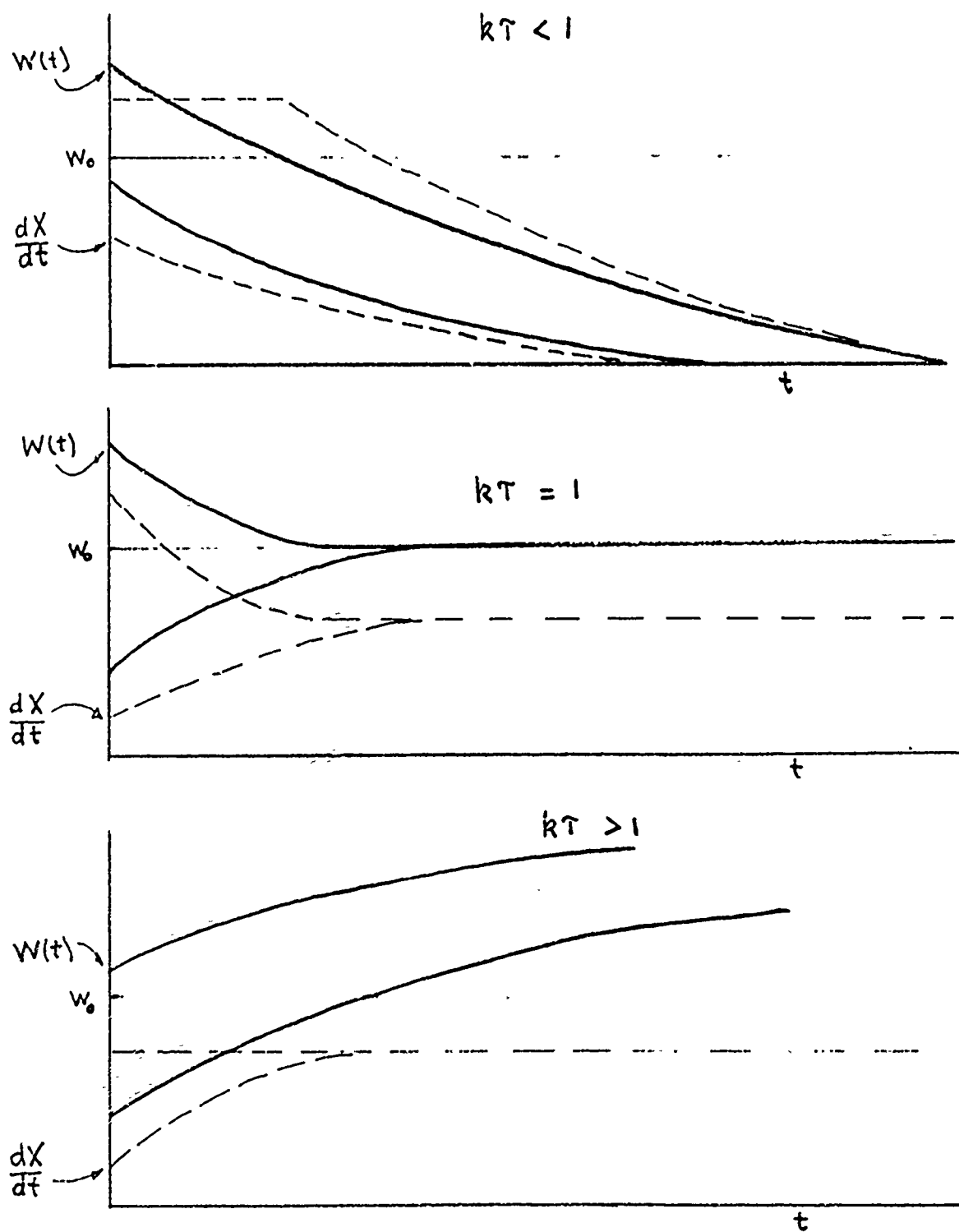


Fig. 6. Variations of $W(t)$ and $\frac{dX}{dt}$

the fire and to investigate the effect of small empty spots (fuel density) on the velocity, the front width, and the final burnout distance.

Model IIC

The problem is divided into two parts: first, prediction of whether fire can spread through the area under consideration; and second, if it does, following its progress in time and space by means of observed data on rate of spread.

Spread Capability

One important result of the fuel state model is that except for a narrow range of conditions, i.e., of Λ/μ_s values, a fire will spread a negligible distance or an infinite distance depending on the particular set of conditions. Thus to a good approximation, spread conditions can be divided into two groups, a spread group and a no-spread group.

In rural fuel weather plays a dominant role in making a fire go or stop because the fuel is normally continuous and the influence of commonly occurring topography is minor.

The spread capability in urban fuel depends on fuel distribution more strongly than on weather conditions. Following are two alternative methods for determining spread conditions in urban areas. It will be seen that for each type of urban fuel, fire spreads indefinitely if the fuel density (fraction of area occupied by fuel) exceeds a certain value.

The following data will be required for each type of urban area: the spacing distribution $\alpha(x)$, i.e., the probability that

an adjacent structure is at a distance between x and $x + dx$, the average structure dimension \bar{s} and the probability $P_{SR}(x)$ that during its burning time a burning structure S ignites a structure R at distance x .

First let us apply the linear stochastic model to an urban area divided into parallel strips, which serve as cells. It is specified that $F_0(0) = 1$. The fraction of structures burning in cell 0 at $t = 0$ is determined by the spacing distribution, size distribution, and by $P_{SR}(x)$ but does not have to be known explicitly. (See p. 6-8.) The same applies to the fraction of structures ignited at any time in other cells.

Let P_{ij} be the probability that cell i , if ignited, will ignite cell j during its burning time. Since each cell acts as a single unit, P_{ij} is also the probability that cell i , if ignited, will ignite each element of cell j . As shown in connection with the linear stochastic model, P_{ij} is related to the parameter a (for the present system) by

$$P_{ij} = 1 - e^{-a} \quad (39)$$

The mean final spread distance is infinite if $a \geq 1$ or $P_{ij} \geq 1 - e^{-1}$. Thus the value of P_{ij} will indicate whether the fire will spread through an urban area.

P_{ij} can be calculated as follows. Each structure in j is exposed to a number of structures in i (the burning cell), with spacings distributed according to $\phi(x)$. For an average pair, the ignition probability must be weighted according to this distribution:

$$P_{SR} = \int_0^{\infty} P_{SR}(x) \phi(x) dx \quad (40)$$

To estimate the average number of structures in i that can be seen from a given structure in j , we line up structures of average size \bar{s} on a half circle of radius $\bar{s}/2 + \bar{x}$ where \bar{x} is the average spacing. Let us call this number ν . Then P_{ij} is given by

$$P_{ij} = 1 - (1 - P_{SR})^\nu \quad (41)$$

The requirement

$$P_{ij} \geq 1 - e^{-1} \quad (42)$$

for infinite spread distance can now be expressed as

$$(1 - P_{SR})^\nu \leq e^{-1}$$

or

$$P_{SR} \geq 1 - e^{-(1/\nu)} \quad (43)$$

The same criterion ($a \geq 1$ or $P_{SR} \geq 1 - e^{-(1/\nu)}$) can also be obtained by an alternative derivation. Fire spread is pictured as proceeding by generations. The original generation is a fire burning at $t = 0$, which may be a straight fire front or the marginal zone of an area fire of considerable dimension. This original generation ignites a group of structures, called first generation, which in turn ignites a second generation and so on. It is assumed that structures in generation n do not participate in the ignition of generation $n+2$.

The structure density of the original generation (number of structures per unit area) is, for convenience, assumed to be the same as the structure density D_0 of the fuel area. The original

generation is separated from the rest of the fuel by a highly irregular boundary. As an average, each burning structure faces ν exposed structures and each exposed structure faces the same number of burning structures. We have derived ν in terms of average spacing \bar{x} and average structure dimension \bar{s} .

The mean probability that a structure of the original generation will ignite an exposed structure is P_{SR} as defined above. The probability p_o that an exposed structure is ignited by one or more structures of the original generation is

$$p_o = 1 - (1 - P_{SR})^\nu \quad (44)$$

which has been also denoted by P_{ij} .

The structures of the first generation are, of course, randomly distributed, and it is rather obvious that their density D_1 is given by

$$D_1 = D_o p_o \quad (45)$$

After the first generation is ignited, each unignited structure faces $\nu D_1/D_o$ burning structures instead of ν and has the ignition probability

$$p_1 = 1 - (1 - P_{SR})^{\nu d_1} \quad (46)$$

where d_1 is D_1/D_o , so that the structure density of the second generation is

$$D_2 = D_o p_1 \quad (47)$$

Putting $(1 - P_{SR})^v = P$, the relative densities of successive generations can be written as

$$\begin{aligned}
 d_0 &= 1 \\
 d_1 &= p_0 = 1 - P \\
 d_2 &= p_1 = 1 - P^{d_1} \\
 d_3 &= p_2 = 1 - P^{d_2} \\
 &\dots \\
 d_n &= p_{n-1} = 1 - P^{d_{n-1}}
 \end{aligned} \tag{48}$$

It should be noted that this model by itself does not say anything about the rate of spread or the spread distance. Possibly the density of burned structures after the end of the fire decreases gradually at increasing distance from the initial fire, with the result that the burnout area has no sharp boundary. However, if we define a mean spread area as one filled with the total number of burned structures, then the mean spread distance is given by $\sum_0^{\infty} d_n$ units of distance.

To investigate the convergence of the series $1 + p_0 + p_1 + p_2 + \dots$, we use the relations

$$P = 1 - p_0 = 1 - (1 - e^{-a}) = e^{-a} \tag{49}$$

and express d_n as

$$d_n = 1 - e^{-ad_{n-1}} \tag{50}$$

which is of the same form as the relation

$$B_n(\infty) = 1 - e^{-aB_{n-1}(\infty)} \tag{51}$$

in the stochastic linear model. It has been shown that the series $\sum B_n$ diverges for $a \geq 1$. The same conclusion applies to the summation $\sum d_n$. For $a \geq 1$, Eq. (49) gives

$$P \leq e^{-1}$$

and by definition of P , we again have

$$(1 - P_{SR})^v \leq e^{-1} \quad (52)$$

$$P_{SR} \geq 1 - e^{-(1/v)} \quad (53)$$

as condition for infinite spread.

One feature of the second model is that it applies equally well to two-dimensional and linear fuel areas. Special cases, such as that of a single structure constituting the original fire, can be readily handled.

Spread Computation

As in Model II, the fuel area is divided into equal-size cells. At any time each cell is in one of five states: susceptible-(ignitable but unignited), immune (unignitable), ignited (burning but still incapable of extra-cell spread), burning (capable of spreading to adjacent cells), and burned out. Usually a susceptible cell is ignited on one edge and some length of time is required for the fire to span the cell dimension and reach adjacent cells. This is the difference between the ignited state and burning state. The travel time across the cell, also called incubation time, is determined by cell size and rate of spread.

The following information is required to implement this model:

- The type of fuel in each cell
- A spread vs no-spread table giving the range of weather conditions for each type of rural fuel and fuel conditions for each type of urban fuel under which the fuel is either susceptible (fire will spread through the fuel indefinitely), or immune (fire will not spread through the fuel)
- A rate of spread table, giving the rate of fire spread for each fuel type as a function of the pertinent weather and topography conditions
- A burning-time table, giving the burning time for each fuel type as a function of pertinent weather variables
- For rural fuel, weather conditions for seven days prior to the assumed starting time and a forecast of future weather conditions (historical data may be used if adequately forecasted information is not available)
- The initial burning conditions (the cells that are initially ignited, burning, or burned out)

The first step in applying the model is to establish the initial state of each cell, which can be accomplished from the fuel and weather data, the spread vs no-spread table, and the specified initial burning conditions.

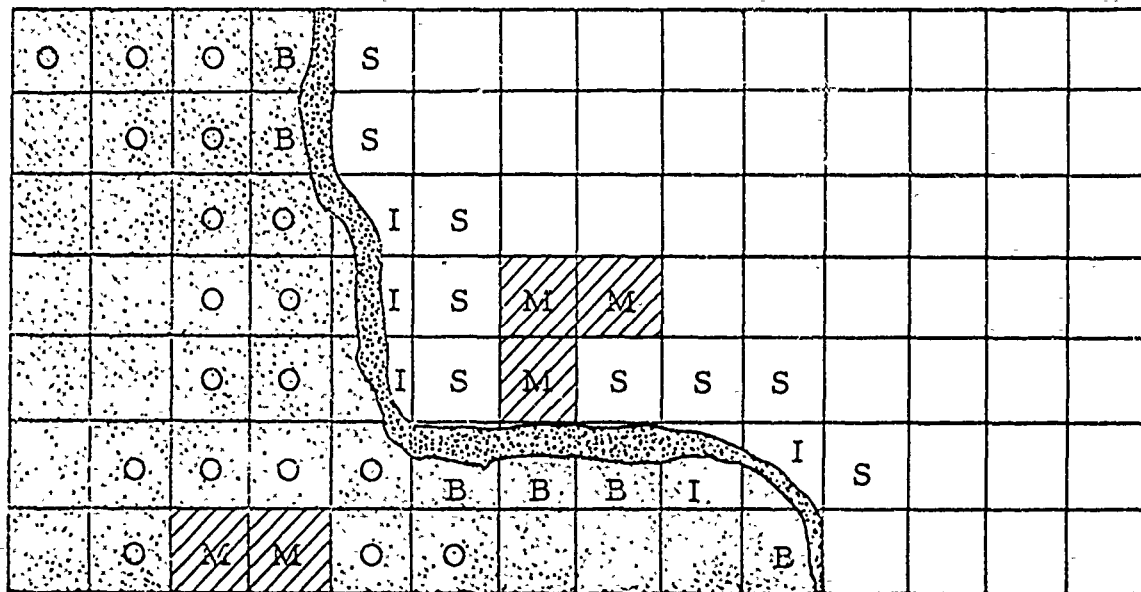
The state of each cell is then redetermined at the end of each successive small time increment Δt until a weather change occurs using the following rules:

- A cell in the immune state will remain in the immune state
- A cell in the susceptible state will instantly change to the ignited state if the cell is immediately adjacent to a cell in the burning state

- A cell in the ignited state will change to the burning state when the time after ignition is equal to or greater than the incubation time
- A cell in the burning state will change to the burned-out state when the time after start of the burning state is equal to or greater than the burning time of the fuel
- A cell in the burned-out state will remain in the burned-out state

Figure 7 illustrates the changes in a fire that can occur in time interval Δt .

When weather conditions change significantly, all cells of rural fuel must be reexamined for possible change of state; also, for both rural and urban fuel, rates of spread and incubation times must be adjusted.

A. Fire front and state of each cell at time t

\bar{B} = burning S = susceptible
 O = burned out M = immune
 I = ignited

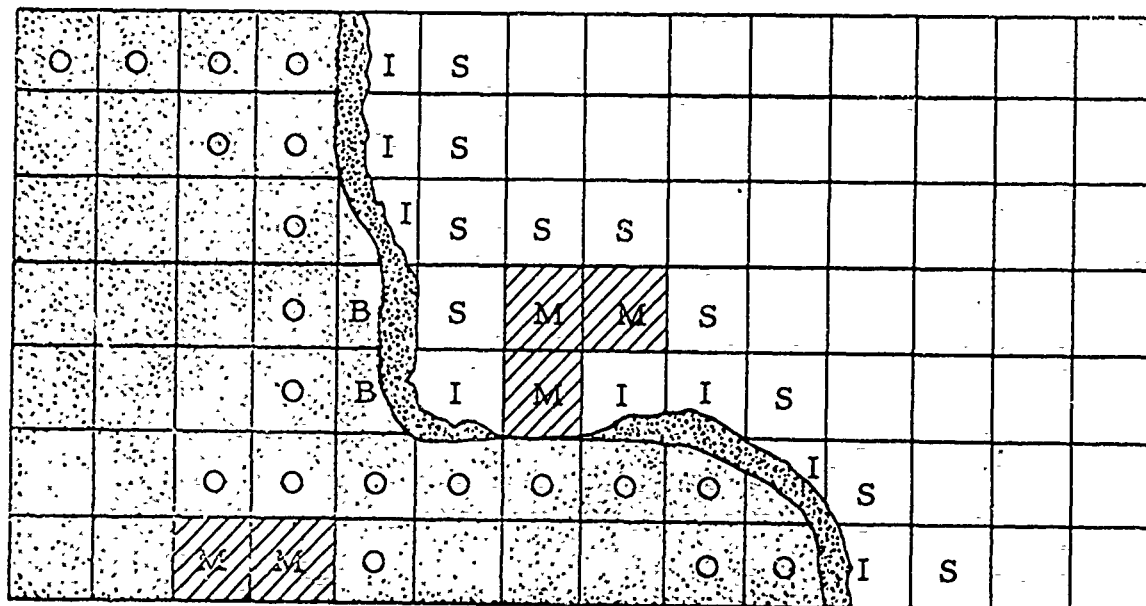
B. Fire front and state of each cell at time $(t + \Delta t)$

Fig. 7. Two-Dimensional Deterministic Model

Section 7

PARAMETERS

IMPLICIT ROLE OF BASIC VARIABLES IN MATHEMATICAL MODELS

All characteristics of a fire are determined by three groups of basic variables, fuel, weather, and topography. However, these variables do not appear explicitly in the mathematical expressions. Instead they are regarded as joint determinants of two or three fundamental properties which enter the equations as parameters and are sufficient to provide complete representation of fire spread phenomena. The derivation of these parameters from the basic variables constitutes a major step in building semiempirical models. In the following paragraphs, we shall discuss the significance of these parameters and the method of determining their values from empirical data.

COMPARISON OF PARAMETERS

We recall that in the fire front model the entire fire is regarded as a random walker which moves along a row of cells and which, during the infinitesimal time dt , has the chance $\mu_f dt$ of dying, the chance λdt of jumping forward, and the chance $1 - (\mu_f + \lambda)dt$ of staying where it is. The probability rate λ represents the dynamic aspect of the fire, its tendency to move forward; hence it is called the spread parameter. The probability rate μ_f on the other hand is a measure of the tendency of the fire to go out and is termed decay parameter. It is intuitively clear

that each of these parameters is determined to various extents by the type, density, and moisture content of the fuel, the velocity of the wind, and the topographic features of the area.

Superficially, the parameters Λ and μ_s in the Models II and IIA are similar to λ and μ_f in Models I, IA, and IB. Fundamental differences exist however between these two pairs of parameters. These differences arise from the fact that II and IIA describe the fire in each cell separately, whereas in I, IA, and IB the whole fire front is an entity which can be in one cell at a time. Although Λ and λ are numerically equivalent, Λ refers to the ignition of cell j by cell i as independent from what happens to the fire in cell i , in contrast with λ , which is the chance that the fire front quits cell i to go to cell j . Similarly μ_s refers to the extinction probability of the fire in one cell and μ_f to the probability of permanent extinction of the whole fire front. It can be expected therefore that, in general, the two parameters have different numerical values.

The deterministic model IIB uses the three parameters k , τ , and W_0 . The first one is a measure of the heat, flames, and firebrands evolved per unit area of burning fuel. Consequently it is the deterministic counterpart of Λ and λ . The parameter τ , the burning time of the fuel, corresponds to $1/\mu_s$. Again it is intuitively clear that both k and τ are functions of the basic fire spread variables. W_0 is the result of a mathematical simplification. The igniting flux per unit burning fuel naturally decreases as the distance from the leading edge increases. The assumption is that the igniting flux can be made uniform within a distance from 0 to W_0 .

The model IIC is a practical method for following the progress of the fire by direct application of data on basic variables. The mathematical framework is that of models II and IIA, including all parameters used in these models.

DETERMINATION OF PARAMETERS

Each parameter combines the effects of several basic variables. One characteristic of semiempirical models is that parameter values are derived from empirical sources, such as observations or measurements of actual fires, and experimentation on reduced-scale fires. In the following paragraphs we shall discuss how existing records of past fires and certain subjective estimates by Forest Service personnel are used to derive some of the parameters. It will appear in this discussion that many important quantities in fire spread have not been made the object of quantitative observation in the past. For parameters which cannot be evaluated immediately because of the lack of pertinent data, we shall briefly indicate procedures for making useful observations on future fires, thus accelerating the build-up of our empirical knowledge on mass fire behavior.

Spread Parameters λ and Λ

It would be very difficult to design experiments or to find observed data which can directly yield the probability rates Λ and λ as such. We have shown analytically, however, that λ and Λ are numerically equal to the mean velocity of the fire, given that it is still alive. If we observe a fire front during a sufficiently large number n of time increments Δt before it dies,

we would see that, on the average, it makes a jump during each of $n\lambda dt$ time increment, that during the other $n(1 - \lambda dt)$ time increments it stays where it is, and that the jumps and stays are randomly distributed. Thus, the rate of motion from cell i to cell j is a random variable, but if the number of cells traveled after a large number of time increments is recorded, then the mean velocity and λ can be computed.

Decay Parameters μ_f and μ_s

It is shown in Appendix A that according to the mechanism used in the fire front model, $1/\mu_f$ is the mean lifetime of the fire. Thus if the mean lifetime of the fire under each set of basic variables is known, μ can be easily derived. Unfortunately the only direct method for determining the mean lifetime, except for a very special case mentioned in Appendix A, is to measure the actual lifetimes of a large number of fires which burn under known weather, fuel, and topography conditions, and unhampered by fire control measures. There are not many real fires satisfying these conditions, and direct experimentation is quite impractical. An indirect method involves calculating the mean lifetime from some other fire spread model. The reciprocal of the mean lifetime thus determined is taken as μ_f . In some particular cases the mean lifetime can be calculated from fuel distribution, as discussed at the end of Appendix A.

Since $1/\mu_s$ is the mean burning time of cells in a given area, the determination of μ_s requires the burning time of each type of cell and their distribution in a given area. If the weather is reasonably constant and all cells are identical with

respect to fuel and topography, the lifetime is the same for all cells, and μ_s is a deterministic parameter. The fuel, in any reasonably large area, however, is likely to have a nonuniform distribution. Some cells are empty, the rest contain fuel at various densities (fraction of total area actually occupied by fuel) and various degrees of buildup (thickness of fuel layer). Furthermore any area is likely to contain more than one type of fuel. Weather and topography are probably not critical factors for burning times. Their small effects, however, fluctuate both in time and in space. These considerations lead to the viewpoint that, for large areas, burning time is predominantly stochastic. In the fuel state models, the treatment of burning time as a stochastic variable is based on the assumption that the cumulative distribution of actual burning times (or of cells according to their burning times) is exponential, i.e., the probability of a cell or the percentage of cells having a burning time equal to or greater than t is $e^{-\mu_s t}$. It should be pointed out that this assumption is not essential to the model. While the exponential distribution is mathematically convenient, other types of distribution, either postulated or experimentally determined, can be used if the computation is carried out by machine. It has been shown that the type of distribution used has no strong effect on the final results as long as the mean burning time remains essentially the same.

Determination of the mean burning time must be carried out in two steps: first, measuring the deterministic burning time of fuel; second, evaluating the fuel distribution among the cells of an extended region.

In the first step, the fuel may be divided into several types such as brush, grass, conifer, timber, hard-wood timber, commercial structures, industrial structures, and residential structures. Each fuel type may be broken down into several sub-types corresponding to structural characteristics and degree of buildup (as defined earlier). The correct classification can be determined only after study of the relative importance of fuel type, structural characteristics, and degree of buildup. The burning time of each fuel subtype is measured, and the effects of weather and topography upon this time are investigated. This measurement should be carried out in a manner consistent with the mechanism employed in the model. Thus the burning time should be the interval during which the cell is capable of igniting an adjacent cell. The mode of cell ignition should approximately duplicate cell ignition that takes place during actual fires. The cell width has been defined as the distance from the ignition source beyond which the probability of normal ignition (by heat and short-range embers, excluding long-range spotting) is negligibly small. For urban fuel a cell is a structure (square cell) or a row of structures (strip cell), and the exact cell width is determined by the fuel density and the average dimension of structure. Cell dimension in the essentially continuous rural fuel is selected in accordance with maximum range of normal ignition which is a characteristic of fuel type.

In the second step, it is seen that whereas burning times can be measured once for all, the spatial distribution of fuel is a characteristic of the area under consideration and, strictly speaking, must be determined in each case by counting the numbers n_1, n_2, n_3, \dots of cells with burning times T_1, T_2, T_3 and

calculating the mean burning time τ from the formula

$$\tau = \frac{\sum n_i T_i}{\sum n_i} \quad (54)$$

For very large areas, cell counting would require excessive work. A more practical method is as follows. A number of typical areas are selected from maps of cities, suburbs, and wildland. The fuel distribution is established in detail, and the mean burning time is calculated for each area type. Mean burning times are tabulated for future use in each model application. To obtain a grand mean burning time, it is required only to estimate the distribution of area types in the total area under consideration.

Deterministic Parameters τ , k , and W_0

In model IIB the parameter τ is the deterministic burning time of the uniform fuel or uniform mixture of fuel, and it can be measured as described earlier in connection with the burning time of various fuel types. The other two parameters, k and W_0 , can be obtained if certain quantities are measured on actual fires.

Rate of spread data, already accumulated in substantial quantity, correspond to an essentially stationary state, i.e., a state in which the width of the fire front and the velocity remain constant. According to the simplified formulation of model IIB, these quantities are given by

$$W(t) = kW_0\tau \text{ and } \frac{dX}{dt} = kW_0 \quad (55)$$

Thus for each area type and each combination of variables the product kW_0 is equal to the constant rate of spread. If the width $kW_0\tau$ is also observed, the two equations can be solved for τ . For cases in which τ has been measured independently, this will provide a good check.

To obtain W_0 and k separately, the rate of spread and front width at the initial stage must also be observed. The data will be used to calculate k from the equation

$$\frac{dX}{dt} = kW(t) \quad (56)$$

Observation of the early stage of fire spread may be extended to fires which go out soon after start. In this case the initial velocity should increase with the initial width $W(0)$ and become constant when $W(0)$ is equal to W_0 . The parameter k is obtained from the instantaneous value of velocity and front width according to the above equation.

It is possible that for a given set of conditions, k is not a true constant. This would be due to inaccuracy of the data and especially to the assumption that velocity is proportional to the front width. Unless the relation between these quantities is established experimentally or theoretically, the value of k must be averaged over a sufficiently large number of observations.

Section 8

APPLICATION OF MODELS

Provided all parameters can be determined, we now have a group of semiempirical models to represent spread characteristics of mass fires and obtain information required for damage prediction. Some of these models are stochastic, some deterministic, and each type includes both linear and two-dimensional varieties. The question now arises as to which of these models is most suitable for a given type of fire.

DETERMINISTIC MODELS VERSUS STOCHASTIC MODELS

We said earlier that weather variables can be treated deterministically because of their relatively slow variations and because of the possibility of specifying average weather conditions which remain applicable for periods of 6 or 12 hours. Furthermore, investigation of fire spread data in rural fuel has shown that the opposing effects of upslopes and downslopes tend to cancel each other to such an extent that the net result over large distances cannot be detected in the best data now available. Thus the choice between stochastic models and deterministic models is essentially determined by fuel variables, i.e., type, density, degree of buildup, and distribution. It seems worthwhile to examine each of these variables for rural fuel and urban fuel separately at various dimension scales.

The main types of rural fuel are brush, conifer timber, hardwood timber, and grass. There is evidence that rate of spread in

each of these types is approximately the same. Their burning times, however, vary considerably. The fuel density or fraction of total area occupied by fuel is normally very high. Variability in buildup or vertical dimension of fuel depends on the number of different fuel types in the area under consideration and, therefore, on the size of the area. Fuel distribution, a function of density and variability in buildup, is generally uniform within relatively large expanses. A rural area tends to be continuously occupied by the fuel except for some patches of empty or almost empty area, e.g., roads, creeks, or rivers.

Urban fuel is essentially discrete. It is concentrated in separate units whose size, structural characteristics, and spacings are highly variable. The density in general increases from residential areas to commercial areas and, except for certain residential tracts, is highly variable locally. Buildup is naturally uniform, at least within a given zone although it may vary over a wide range in commercial areas.

It thus appears that rural fuel lends itself easily to deterministic treatment, at least within areas of a few square miles. Moderately large areas can also be treated in this manner, provided input data can be collected for each section with different fuel characteristics. For very large areas, a stochastic approach might spare some of the efforts required for data collection and model computation.

For urban fuel, the variability of sizes, structural characteristics, and spacings — all of which are important factors in fire spread — calls for stochastic models except when the fire spread investigation is restricted to a small area.

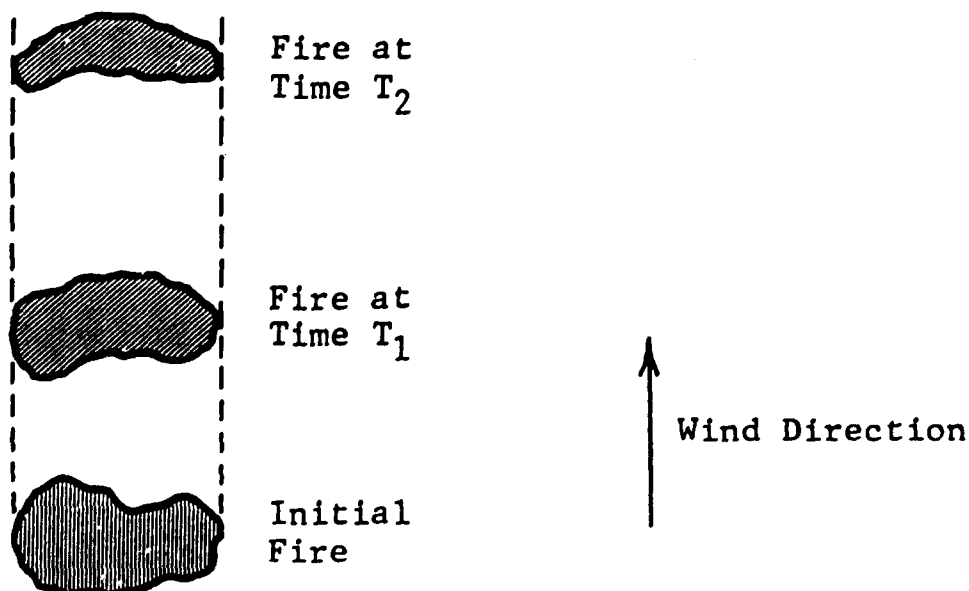
LINEAR MODELS VERSUS TWO-DIMENSIONAL MODELS

Obviously a two-dimensional model can be applied to any fire spread problem regardless of the spread pattern, and situations frequently arise in which such a model is required. In some cases, however, a linear model is sufficient and due to the simple computation it requires, can produce substantial savings in time and effort.

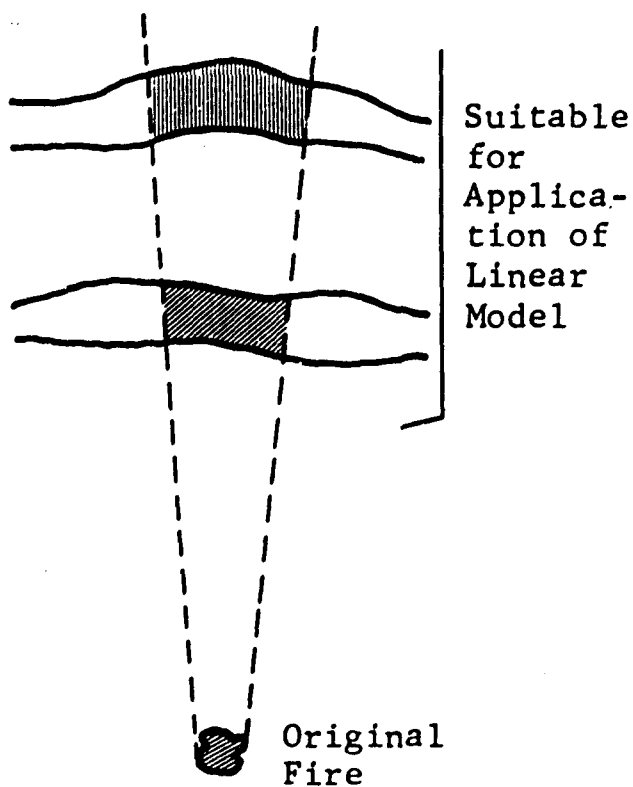
A linear model describes the progress of a fire along the main spread direction (on the assumption that the spread behavior is approximately the same along straight lines perpendicular to the direction of spread). If space is divided into equal cells, the forward dimension of each cell is set by the requirement that fire cannot jump over one or more cells by the normal mode of ignition. The lateral dimension can take any value, provided fuel characteristics (deterministically or stochastically specified) do not vary significantly along this dimension. The assumption of uniform spread throughout the lateral dimension of the cell is applicable in two cases:

- (1) The wind velocity is such that the lateral component of spread velocity is small compared with the forward component. One such case is pictured in Fig. 8A. The size of the initial fire is immaterial in this case.
- (2) The wind velocity is low, the initial fire front is straight or only slightly curved and has a considerable length (Fig. 8B). Under these conditions, a certain portion of the fire front is approximately straight and remains parallel to itself in its forward motion.

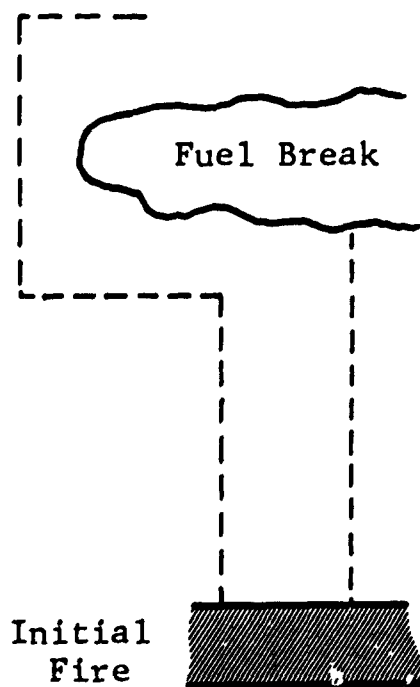
A case requiring special attention in using a linear model is pictured in Fig. 8C. The model is applied to a strip of fuel



A. CONSIDERABLE WIND



B. WEAK WIND AND WIDE FRONT



C. PRESENCE OF MAJOR FUEL BREAK

Fig. 8. Application of Linear Model

area extending from the initial fire to a fuel break (open space). If sufficiently wide, the break should stop the fire (going along this strip) with certainty. However, the possibility for the fire to go around the edge of the fire break and start again on the other side must always be examined.

If the position of the fuel break is stochastic, the entire area is stochastically uniform and can be treated as a general case.

When the situation permits the use of a linear model, the general procedure is as follows:

- (1) From input data, determine whether the fire will stop at a finite distance (Λ/μ_s or $k\tau$ smaller than 1) or spread indefinitely (Λ/μ_s or $k\tau$ equal to or greater than 1).
- (2) In the first case calculate the final spread distance using appropriate formulas and from the final spread distance and average rate of spread calculate the lifetime of the fire.
- (3) In the second case the fire will spread until it encounters a spatial barrier or a weather barrier. A spatial barrier is a completely empty or low-density area whose size and location are specified in the input data. A weather barrier is a set of wind and humidity conditions which inhibits fire spread in the fuel area under consideration. In view of previous discussions on fuel characteristics, urban fires are stopped by spatial barriers more frequently than by weather barriers, and the reverse is true with rural fires.
- (4) If fire spread is expected to be multidirectional (low wind velocity), determine the final burnout contour from spread distances in various directions.

- (5) Position of the fire at a given moment can be found from the rate of spread and time after start.

Thus in many cases linear models only require derivation of parameter values from input data and numerical computation of certain simple quantities such as Λ/μ_s or $k\tau$. Both of these tasks can be accomplished with or without the help of a computer depending on the particular situation.

Table 1 summarizes the characteristics and applicability of various models.

MODEL APPLICATION AS PERMITTED BY EXISTING DATA

As appears in Table 1, only some parameters can be empirically established from existing data. Consequently, some models are not immediately ready for actual application. Thus for models I, IA, and IB, the parameter μ_f would have to be derived from the mean lifetime of the fire, a quantity which cannot be obtained from any observed data. In the case of models II and IIA, the determination of μ_s (reciprocal of mean burning time) would require information on burning time of various types of fuels, which would be combined with spatial distribution of fuel to give the mean burning time. Some estimates of burning time do exist, but their reliability is not considered sufficient for this purpose. Burning time is also a parameter in model IIB.

Until further observed data become available for evaluation of these missing parameters, the problem is how to perform fire spread predictions on the basis of existing data.

Table 1
SYNOPSIS OF MODELS

<u>Model*</u>	<u>Characteristics</u>	<u>Appropriate Fuel Area</u>
I	Fire front mechanism. Stochastic. One dimensional. Parameters are λ and μ_f ; the latter not available; required data difficult to obtain.	Preferably urban
IA	As above with more detail input information on fuel characteristics.	Preferably small or moderate urban areas
IB	As above with provision for long range spotting possibility.	As above
II	Fuel state mechanism. Stochastic. Two dimensional. Parameters are Λ and μ_s . Data for evaluating μ_s not available.	General
IIA	As above but one dimensional.	General, preferably urban
IIB	Deterministic, one dimensional. Existing data for evaluating parameters τ , k , W_0 are incomplete.	General, preferably rural
IIC	Deterministic. Two dimensional. Application uses existing data supplemented by some subjective estimates.	Rural and urban

* See Fig. 3.

The solution is provided by model IIC. It is assumed that for a given area and a given type of weather, fire either spreads indefinitely or does not spread at all, neglecting the rather few cases in which fire spreads to a significant distance, then stop. For rural fuel, personnel of the Forest Service have estimated weather conditions for no-spread. Limited data from past urban fires have provided approximate information on the ignition probability as a function of spacing, from which fuel conditions for no-spread have been derived. If fire is predicted to spread through an area, its position at any time can be determined on the basis of rate of spread data.

Part II

Work described in Part I concerns the development of semi-empirical models for predicting the gross spread behavior of mass fires, the investigation of model parameters, and methods for obtaining their numerical values from observed data.

Part II will present the results of studies on the required input data (rate of spread, weather conditions for spread and for extinction in rural areas, fuel conditions for spread in urban areas, and specification of weather characteristics) and on the vulnerability of resources. Finally, some preliminary considerations are made regarding application of fire spread models on the local scale.

Section 9

ANALYSIS OF RATE-OF-SPREAD DATA IN RURAL FUEL

RAW DATA

The analysis consists in examining how rate of spread depends on fuel type and density, wind velocity and direction, relative humidity, slope and slope distribution. The use of relative humidity instead of moisture content is justified by the commonly observed fact that in natural rural fuel, fire spreads through the finer components, whose moisture content follows the fluctuations of relative humidity rather closely.

A sample of raw data, as supplied by the U.S. Forest Service, is shown in Table 2. Columns 1 and 2 of the table identify the fires and the section of the fires where the measurement was made. Column 3 indicates the time of start and Column 4 the time during which fire spread was measured. Weather variables are stated in Columns 5 to 9, the last three of which refer to relative humidity, stick moisture content, and burning index, respectively. Fuel type is shown in Column 10. Topography variables are given in Columns 11 to 16.* Rate of spread and direction in which spread was measured are indicated in Columns 17 and 18. Column 19 indicates the manner of spreading in the area where the rate of spread was measured. H is a head fire,

* In addition to profile sketches in Column 16, slope angles for each section of a profile were provided separately but not included in this report.

Table 2
SAMPLE FIRE SPREAD DATA

FIRE				WEATHER					10 FUEL	11 12 TOPOGRAPHY						SPREAD		
1	2	3	4	5	6	7	8	9		SLOPE 13 14				15	16	17	18	19
Fire No.	Line No.	Time of Start	Hours of Spread	Wind Vel.	Temp	RH	Stick	BI		UP	DOWN	UP	DOWN	Percent Flat	Sketch	Rate	Angle to Wind	Type
%	Aver. Deg.	%	Aver. Deg.	Percent Flat	Sketch	Rate	Angle to Wind	Type										
77	3F	1200	8	15	92	20	8	20	BG	10	22	60	15	30		.1420	2	H
	3G	1200	8	15	95	19	8	20	BG	20	15	70	15	10		.1300	11	H
	3H	1200	8	15	95	19	8	20	BG			60	16	40		.1000	18	H
	3I	1200	8	15	95	19	8	20	BG			100	20			.0200	38	H
	3J	1200	8	15	95	19	8	20	BG			100	20			.0140	64	H
	3K	1200	8	15	93	18	8	20	BG			100	14			.0100	102	H
	3L	1200	8	15	93	18	8	20	BG			100	10			.0100	103	H
80	4A	1200	6	13	90	14	6	31	G	25	14	45	28	30		.0666	50	H
	4B	1200	6	13	88	15	6	31	G	40	22	60	17			.0700	39	H
81	2A	1030	8 1/2	18	72	14	5	38	GB					100		.3741	12	H
	2B	1030	8 1/2	18	72	14	5	38	TG					100		.4235	7	H
	2C	1030	8 1/2	18	72	14	5	38	TG					100		.4376	1	H
85	1A	1430	7.5	7	81	24	5.5	10	T					100		.0720	55	H
	1B	1430	7.5	7	81	24	5.5	10	T					100		.2580	46	H
	1C	1430	7.5	7	81	24	5.5	10	T					100		.2960	37	H
	1D	1430	7.5	7	81	24	5.5	10	T			100	8			.2080	24	H
	1E	1430	7.5	7	81	24	5.5	10	T					100		.1572	79	H
	2A	2200	8	5	63	43	6.5	5	T					100		.0275	70	F
	2B	2200	8	5	63	43	6.5	5	T					100		.0388	80	F
	2C	2200	8	5	63	43	6.5	5	T					100		.0575	75	F
	2D	2200	8	5	63	43	6.5	5	T					100		.0388	4	F
	2E	2200	8	5	63	43	6.5	5	T	30	8	70	18			.0825	22	F
	2F	2200	8	5	63	43	6.5	5	T	100	14					.0325	45	F
	3A	0600	10	15	78	16	5	28	T					100		.0400	96	F
	3B	0600	10	15	78	16	5	28	T					100		.0780	90	F
	3C	0600	10	15	79	15	5	28	T	20	21	25	35	55		.5500	20	H
	3D	0600	10	15	78	16	5	28	T	55	10	25	12	20		.3260	6	H
	3E	0600	10	15	78	16	5	28	T					100		.2800	6	H
	6A	2100	9	14	52	59	7	9	T					100		.1730	7	H
	6B	2100	9	14	52	59	7	9	T					100		.0910	7	H
	6C	2100	9	14	52	59	7	9	T					100		.1975	4	H
	6D	2100	9	14	52	59	7	9	T					100		.0356	14	H
	6E	2100	9	14	52	59	7	9	T					100		.0333	21	H
86	1A	1200	6	12	78	27	7.5	18	T					100		.7410	6	H
88	1A	1730	6.5	8	86	30	5	12	T	66 2/3	18	33 1/3	36			.0038	7	H

R a rear or backing fire, F a flank fire, and O a circular fire. Detailed explanations of this table has been given in Ref. 1.

ANALYSIS

The first problem examined was the effect of measured time (of spread) on rate of spread. Preliminary analysis had indicated that short measured times are frequently related to high rates of spread. In fact, plots of rate of spread against measured time for various ranges of wind velocity, wind angle, and relative humidity showed that this is true for times shorter than 8 or 9 hr. In this range, the apparent dependence of rate of spread on measured time is possibly due to a tendency to take data on fire location more frequently for fast fires than for slow fires. To minimize possible errors due to this effect, data with measured time less than 9 hr in duration were rejected.

The topography problem was taken up next. In general, upslopes are expected to increase the rate of spread because they favor flame contact and convection heating. In contrast, downslopes are expected to slow down the spread. An exception is when the downslope is steep enough so that burning fragments tend to roll down and spread fire ahead of the fire front. However, in some cases, the fire pockets created by rolling fragments develop slowly and are caught up by the main fire front, so that the ultimate effect of these fragments might not be noticeable.

To test the effect of slopes, pairs of rate measurements on single slopes were selected for which all basic variables except slope were the same for both members of the pair. Thus

any difference in rate of spread could be attributed to difference in slope only and not to any other measured variable. Each pair was marked "R" if the rate of spread was greater for the pair member with either greater positive slope or less negative slope. If the variation in rate of spread was opposite to that of slope, the pair was marked "W." Of a total of 52 pairs, selected from widely different ranges of variables, 24 were marked R and 28 were marked W. This result is taken to indicate that, within the accuracy of these rate of spread measurements, the effect of topography is insignificant. Therefore, slope and slope distribution were ignored in the subsequent treatment of rate-of-spread data.*

After eliminating topography, four variables remain: fuel type, wind velocity (w), wind direction (α) (angle between spread direction and wind direction), and relative humidity (r).

* Another reason for neglecting topography is that in most practical cases, the path of fire spread in non-flat areas during time periods of interest will not be on a single slope, but rather over a number of up and down slopes. Thus, any effect of one type of slope tends to be compensated by that of another type. There are two possible exceptions to this compensating tendency: (1) in the case of a series of small steep ridges, the fire might jump from the top of one rising slope to the next rising slope without burning the falling slope; (2) the fire might remain stationary for some time at the top of a ridge with the result that the overall rate of spread for a combination of upslopes and downslopes is lower than the rate of spread for a flat area. However, such special cases are beyond the scope of our considerations at present.

The use of relative humidity as a variable rather than fuel moisture content is justified by the fact that fire tends to spread through the fine component of the fuel, whose moisture content rather closely follows the variations in relative humidity.

The method used to determine the effect of these variables consisted in obtaining, for each fuel type, a plot of the rate of spread R versus one variable for various ranges of the other two variables. More specifically,

- (1) R vs α for all possible combinations of four r -ranges (0-15, 15-30, 30-45, 45-60%) and six w -ranges (0-5, 5-10, 10-15, 15-20, 20-25, 25-30 mph)
- (2) R vs r for all possible combinations of six w -deg ranges and three α -ranges (0-45, 45-135, 135-180 deg)
- (3) R vs w for all possible combinations of four r -ranges and three α -ranges

Brush was considered first because data for this fuel type are more abundant than for any other. However, there are several subtypes and several mixtures of brush: brush, heavy brush, medium brush, light brush, brush-chamise-chaparral, chamise, chamise-chaparral. To discover possible differences between these subtypes, the R vs α plots were made with each subtype represented by points of different colors or different shapes. These plots showed at a glance that the point shapes and colors would be well blended, if there was a sufficient number of points on the plot. Accordingly, all brush subtypes were considered the same with regard to rate of spread.

Similar plots were also made to show successively that brush and conifer timber, then brush, conifer timber, and mixtures

of both do not differ in rate of spread to the extent that is detectable in these measurements.

For grass and hardwoods, the points are much too scarce to yield any definite result. However, there is some evidence that extrapolation of rate of spread values for other types to grass and hardwood might not cause serious error. We shall assume that this is the case until more data are available for grass and hardwood.

DISCUSSION

There resulted three series of plots in which α , r , and w were taken successively as the running variable for various combinations of ranges of the other two variables. One w -plot is shown in Fig. 9. Each plot was examined in an effort to derive relations between R and the running variable. It was immediately noticed that the amount of scattering is considerable. Although a high percentage of R -values were between zero and 0.10 mph, the number of values above 0.10 was not negligible, and some values were as high as 1.5 mph. Thus, before attempting to derive relations between R and the variables, some consideration of the causes of this scattering seemed necessary.

Low precision in the measurement of R , α , w , r could obviously produce some scattering but did not seem sufficient to account for abnormally high values such as the ones above a few tenths of a mile per hour. The latter were probably due to long-range spotting (across several hundred feet or more). Even under a definite set of conditions, the long-range

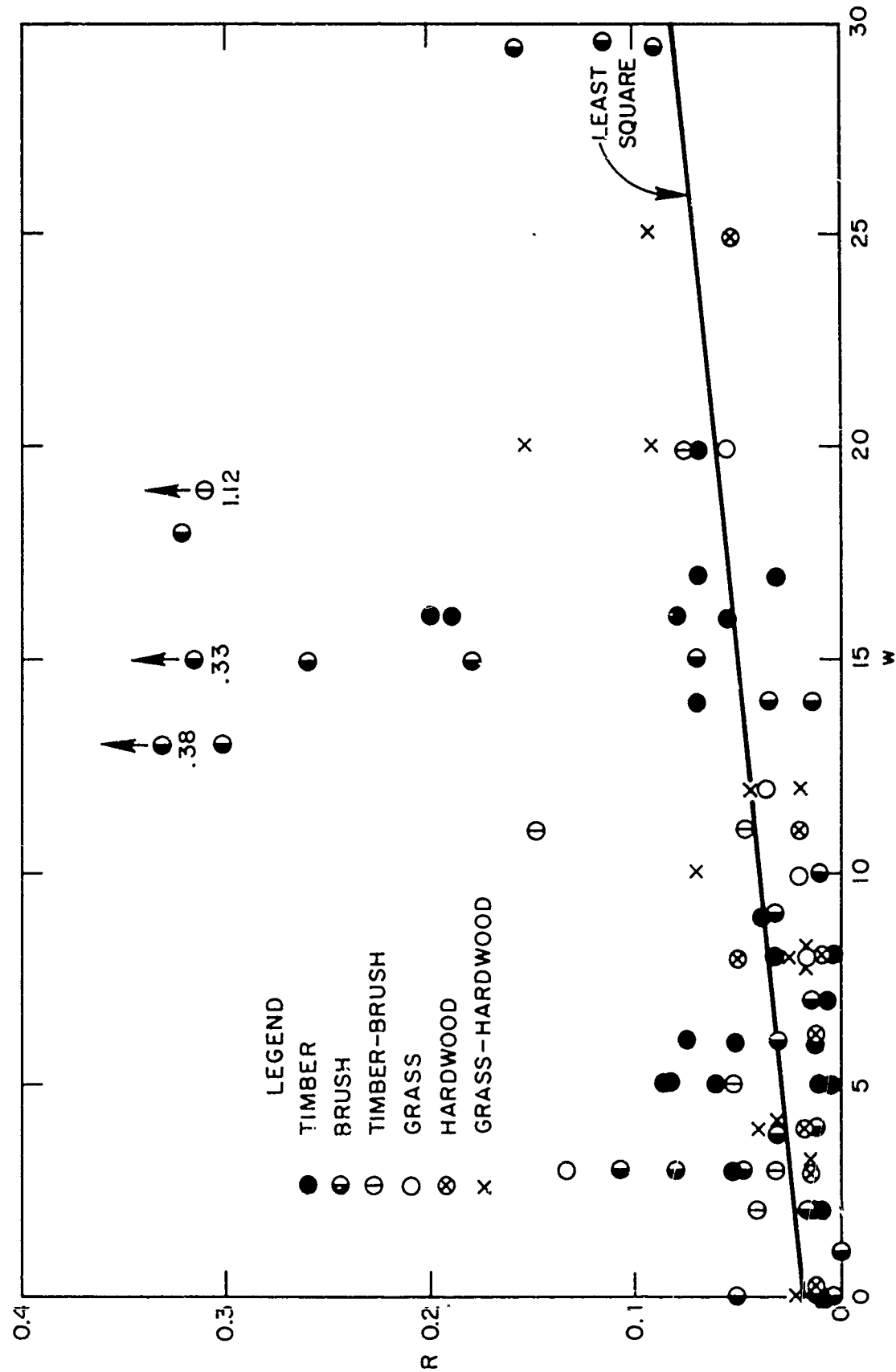


Fig. 9. Rate of Spread Versus Wind Velocity for Various Fuel Types and $r = 30 - 45$ percent $\alpha = 0 - 45$ deg

spotting process is highly random with respect to both jump distance and jump frequency. Since spread by spotting is, in general, much faster than ground spread, the average rate measured over a certain distance can be expected to depend on the spotted portion of that distance. Lack of information on the extent of spotting made it impossible to correct each value for contribution from this mode of spread. However, the following line of reasoning was used in an attempt to derive reasonable and useful results.

Long-range spotting must occur predominantly in the wind direction. Thus, if α -measurements were very accurate, and the natural fluctuations were not too great, rates of spread at wind angles greater than 10-20 deg would be due to normal spread alone. However, due to natural fluctuations and measuring errors in wind angles, pure normal spread could be found only at relatively large α -values. When rates of spread were plotted against α for all r- and w-values and for all fuel types, it was found that

- (a) The number of points higher than 0.10 mph in the α -range from 0 to 45 deg was greater by far than at higher α -ranges.
- (b) In each α -range (0-10 deg, 10-20 deg, etc.) the density of points (number of points per unit area of graph paper) decreased in going from low to high R-values, the rate of decrease being fastest at $R \approx 0.10$.
- (c) Some abnormally high R values (>0.30 mph) were scattered throughout the α -scale, even at its upper end.

Observations in (a) and (b) seemed to support the hypothesis that rate of spread values above 0.10 mph contained significant contributions from spotting, and since spotting occurred only at small wind angles, the result in (c) is due to occasionally large errors in α . This hypothesis is supported by results shown later in this report, indicating that large variations in wind velocity and relative humidity are insufficient to raise the average rate of spread above 0.10 mph, even when the average rate of spread is evaluated by including all values above 0.10 mph.

Thus, in determining relations between rate of spread and basic variables, all points above 0.10 mph were discarded. Analysis of these points would help our understanding of the effect of basic variables on long-range spotting were it not for the insufficient number of such points and the lack of other important information.

Even after removal of all points above 0.10 mph, each plot of R against some variable still showed considerable scattering. However, the general trend qualitatively agreed with the usual views on fire spread. For example, in a plot of R against r (relative humidity), for a certain range of α and w, most spread velocities at low relative humidity are higher than those at high relative humidity, and an upward trend is noticeable in a plot of R against w for a given range of r and a low range of α (See Fig. 9).

Relations between R and the variables were obtained by the usual curve fitting process, and since the excessive scattering made it unfeasible to determine any characteristic other than a general trend, linear relations were assumed and most of the

fitting was done visually. In several cases, however, least-square fitting was also carried out, which showed that visual fitting is adequate for this purpose. For the case of Fig. 9 the resulting least-square equation is

$$R = 0.01805 + 0.002232w$$

The most probable error for the constant 0.01805 is 0.0027 and that for the constant 0.002232 is 0.00025. Regarding the correlation between the observed rate of spread R_o and w , the standard error of estimate S_{R_o} , i.e., the square root of the mean of squared deviations of R_o from values predicted by the least-squares line is 0.021 mph. The correlation coefficient r (defined by $r = \sqrt{1 - (S_{R_o}/\sigma_{R_o})^2}$, where σ_{R_o} is the standard deviation from mean, not from least-square lines) is equal to 0.6. Since the limits of r are 1 for complete correlation and 0 for complete lack of correlation, an r -value of 0.6 indicates that the assumption of linearity is not too unreasonable in view of the data scattering.

Each of the three series of plots with fitted straight lines gave a fitted average value of R for each combination of α -ranges 0-45, 45-135, 135-180 deg, w -ranges 0-5, 5-10, 10-15, 15-20, 20-25, 25-30 mph, and r -ranges 0-15, 15-30, 30-45, 45-60 per cent. Average values thus obtained are shown in Table 3. It is seen that the three ways of plotting lead to three averages which are reasonably close to each other. These averages are averaged once more to give the numbers with bars.

Table 3
AVERAGE RATE OF SPREAD IN EACH RANGE OF VARIABLES

30	15			30			45			60		
	0	15	30	0	15	30	0	15	30	0	15	30
...
0.070	0.046	0.023		0.052	0.030	0.008						
0.059	0.063	0.021		0.067	0.041	0.013						
0.064	0.054	0.022		0.059	0.035	0.010						
...
0.063	0.034	0.004	0.023	0.058	0.038	0.017						
0.057	0.058	0.024	...	0.058	0.038	0.017						
0.060	0.046	0.024	0.023									
...	0.042	0.037	...						
0.042	0.037						
0.069	0.057	0.045	0.019	0.044	0.033	0.018						
0.053	0.053	0.028	0.024	0.043	0.035	0.018						
0.055	0.049	0.036	0.021									
...	0.040	0.032	0.026						
0.040	0.032	0.026	0.026	0.034	0.022	0.010						
0.057	0.044	0.030	0.010	0.041	0.028	0.020						
0.052	0.049	0.032	0.024	0.038	0.027	0.019						
0.050	0.042	0.029	0.020									
...	0.031	0.023	0.018						
0.031	0.023	0.018	0.018	0.030	0.021	0.011						
0.045	0.040	0.034	0.020	0.031	0.024	0.021						
0.050	0.045	0.037	0.024	0.031	0.023	0.017						
0.045	0.036	0.030	0.021									
...	0.023	0.023	0.017						
0.037	0.037	0.037	0.029	0.023	0.023	0.017						
0.039	0.039	0.039	0.026	0.028	0.028	0.028						
0.048	0.040	0.041	0.024	0.023	0.021	0.023						
0.041	0.039	0.039	0.026	0.025	0.024	0.025						
...	0.017	0.017	0.017						
0.017	0.017	0.017	0.020	0.020	0.020	0.020						
0.018	0.018	0.018	0.018	0.019	0.019	0.018						

Notes: In each box, the left column corresponds to $\alpha = 0-45$, the middle to $\alpha = 45-135$, and the right to $\alpha = 135-180$ deg.

Numbers in the first row are from plots of R vs w, those in the second row from plots of R vs α , those in the third row from plots of R vs w, and those in the last row are averages of the three numbers above in the same column.

Conclusions

Examination of Table 3 leads to the following conclusions:

- (1) In each range of relative humidity (r), the rate of spread increases with w rather markedly for α between 0 and 45 deg and less markedly for α between 45 and 135 deg. It generally decreases at increasing w for α between 135 to 180 deg.
- (2) At low wind velocity (w from 0 to 10 mph), the rate of spread decreases with increasing relative humidity, the decrease being more pronounced in the 0-5 mph range than in the 5-10 mph range.
- (3) At higher wind velocity, the inhibiting effect of relative humidity becomes very small and should probably be ignored.
- (4) For w between 0 and 5 mph, the rate of spread does not depend on wind direction.

Conclusion (1) is in accord with our intuition. As the wind becomes stronger, fire spreads faster in the direction of the wind, and the spread pattern becomes more and more asymmetric (provided, of course, that the fuel area is homogeneous). Conclusion 2 is also expected. Conclusion (3) is at first rather surprising. It seems to indicate that, at moderate and high wind velocities, the convection heat is more than sufficient to drive all moisture content off the fine fuel adjacent to the fire front before ignition takes place. Conclusion (4) may indicate that the resolution of w -measurements is no better than 5 mph.

Section 10

ANALYSIS OF RATE-OF-SPREAD DATA IN URBAN FUEL

A sample of data provided by the U.S. Forest Service is shown in Table 4 (taken from Ref. 1). The duration of measurement can be found from the times of start and finish in column 3. Column 4 gives the rate of spread in miles per hour. Wind speeds given in column 6 are also in miles per hour.

Column 10 indicates the fuel type for light wooden structures, for heavy wooden structures, for light stone or concrete structures, and for heavy stone or concrete structures, indicated by the numbers 1, 2, 3, 4, respectively.

The percentage of total area occupied by fuel, termed "built-upness," is given in column 11. We prefer, however, to call this variable "building density" or "fuel density," reserving the term "built-upness" (degree of buildup) for the thickness of the fuel layer. The four ranges of fuel density (in percent), below 20, 20 - 29, 30 - 39, 40 and above, will be denoted by I, II, III and IV, respectively. The number of stories in each building (a measure of "built-upness" is given in column 12.

"V" in column 13 is a value obtained by combining fuel type (T), fuel density (B), and number of stories (S) in a certain manner. This value has not been used in our analysis.

The direction of spread with reference to north is given in most cases in column 12 as angle to wind. The angle between the spread direction and wind direction is found from the spread

Table 4

SAMPLE OF U.S. FOREST SERVICE DATA

1	2	3	4	5	6	7			8	9	10	11	12	13	14	15
Fire		Spread			Weather			Buildings			Slope			Remarks		
Name	Date	Period	Rate	Temp	Wind Speed	Wind Dir.	Rel. Humid.	Dryness	T	B	S	V				
Baltimore, Md.	2/07/04	1050-2100	0.022	58-64	16	WSW	75	recent snow (still melting 2/17/04)	3	40+	6-10	8	Ground Spread, explosion spread blaze			
	2/07/04	2100-2300	0.054	58-60	22	W	86		4	40+	8-10	9	Ground Spread			
	2/07/04	2200-2345	0.031	53	22	NW	88		3	40+	8-10	8	Ground Spread			
	2/07/04	2345-2350	3.175	50	20+	NW	88		3	40+	3	6	Spread by Spotting			
Boston, Mass.	11/9/72	1900-2400	0.030	42	5-9	NNW	60	1.89" rain 11-7-72	3	40+	4	7	Ground Spread (angle to wind = 135° - most nearly with wind)			
	11/9/72	1900-2400	0.042	42	5-9	NNW	60		3	40+	4	7	Ground Spread (angle to wind = 080°)			
	11/9/72	1900-2400	0.025	42	5-9	NNW	60		3	40+	4	7	Ground Spread (angle to wind = 360° - most nearly into wind)			
	11/9/72	1900-2400	0.030	42	5-9	NNW	60		3	40+	4	7	Ground Spread (angle to wind = 290°)			
Berkeley, Calif.	9/17/23	1420-1500	0.397	87	25	NE	27	very dry	1	20-29	2 1/2	6	Spread by Spots			
	9/17/23	1500-1600	0.454	89	28	NE	25	very dry	1	20-29	2 1/2	6	Spread by Spots			
Atlanta, Ga.	5/21/17	1246-1430	0.451	83-86	14-16	S	43-	2 weeks since rain	1	20-29	1	5	Spread by Spots (angle to wind = 000°)			
	5/21/17	1430-1630	0.567	84-86	16	S	35+		1	20-29	1	5	Spread by Spots (angle to wind = 000°)			

direction in column 15 and the wind direction in column 7. It was understood that for cases in which the spread direction is not indicated in column 15, the spread direction is forward, i.e., with the wind.

There is a total of 73 values of rate of spread, of which two have been rejected because the values appear to refer to spread within single structures and another because of unspecified type of spread (ground spread or spotting).

METHODS AND RESULTS

The data were first grouped according to fuel types and fuel densities. The numbers of data points in each group are shown in Fig. 10, with a breakdown into ground spread and spotting. There is only one data point for each type of spread in group IV-4. Consequently, this group was later incorporated into group IV-3.

Two facts were immediately noted. First, a certain relation exists between fuel type and fuel density. Light fuel occurs predominantly in low-density areas and heavy fuel in high-density areas. Thus, care must be taken when attributing some effect to one of these two variables to the exclusion of the other. Second, ground-spread fires tend to occur in high-density or heavy fuel areas more often than in low-density or light fuel areas, whereas the reverse is true for spotting fires. The number of stories naturally tends to follow the fuel type and may well be combined with the latter to form one single variable.

Fuel Type	4						0	X
	3		0 0 X 0 0		0 0 X X 0		0 0 0 0 0 0 0 0 0 0 0 0	X
	2	X X	0 0 X X X		0 0 0 0 X X 0 0 X 0		0 0	
	1	0 0 X X 0 0 X X	0	X X X X X X X X X X	0 0 X 0			
		I	II	III	IV	Fuel Density		

Note: 0 = ground spread
X = spotting

Fig. 10. Number of Data Points in Each Group

The next obvious step was to separate ground-spread data from spotting data and treat them separately, because these two modes of spread involve different mechanisms. By ground spread, the fire reaches an adjacent structure through a relatively short distance. The mechanism is spontaneous ignition or pilot ignition (ignition of preheated fuel by embers). Spotting occurs when a structure located beyond the range of heat and embers is ignited by flying firebrands. It is important to note that sufficiently long spread runs are likely to involve both types of spread. Thus, for most runs reported as ground spread, a small portion of the jumps are probably made by spotting. Similarly, some ground jumps may be expected to occur in relatively long-lasting fires.

Ground Spread

We shall examine ground spread data first. In each group, variables which can have some influence on rate of spread are wind velocity, wind direction, measured time of spread, and fuel dryness. There are only 9 cases of wet conditions, of which 7 are in group IV-3, and 2 in group III-2 (Fig. 11). In addition, there exist 11 cases with unknown dryness (San Francisco fire, April 1906). Two of these are in group III-3, 5 in group III-2, 3 in group III-1, and 1 in group II-2. We shall later find a plausible effect of fuel dryness in group IV-3 which contains 8 cases of dry condition, 7 cases of wet conditions, and no case of unknown dryness. However, the magnitude of this effect in group IV-3 does not seem to interfere with an independent study of the other three variables. From this, we conclude that the effect of fuel dryness in other groups also must be of little

Fuel Type	3		0.215 (4)	0.057 (3)	0.031 (15) wet .029 (7) dry .041 (8)
	2		0.079 (2)	0.038 (7)	0.010 (2)
	1	0.262 (4)	0.121 (1)	0.066 (3)	
		0.26 (4)	0.14 (7)	0.05 (13)	0.03 (17)
		I	II	III	IV
		Fuel Density			

Fig. 11. Average Values for Ground Spread (mph)

consequence in the study of other variables. Furthermore, most data points in each group belong to the same type of dryness; for example, 3 out of 3 data points in group III-1, 5 out of 7 in group III-2, and 2 out of 3 in group III-3 are of unknown dryness. Group II-2 has only 2 points, one of unknown condition, the other of dry condition. However, the change in rate of spread between these 2 data points can be easily accounted for by other effects.

Consequently, it seems justified to neglect fuel dryness at this stage and proceed with the examination of wind direction, wind velocity, and measured time.

Effect of wind direction is a simple problem since of the 41 cases only 5 are backward spread and all others are forward spread. If we assume that for a given wind velocity, the directional effect of wind is the same for both rural and urban fuels, the 5 backward-spread values can be converted to forward spread values by using ratios previously found in data for rural fuel.

The effect of wind velocity was next examined by plotting rate of spread within each group against wind velocity. In the most populated group, IV-3, the 8 dry cases show an upward trend with increasing wind velocity. An approximately fitted straight line can be represented by the equation

$$R = 0.027 + 1.25 \times 10^{-3} w$$

where

R = rate of spread

w = wind velocity

both in miles per hour. This equation can be compared with its equivalent in rural fuel:

$$R = 0.020 + 1.54 \times 10^{-3}$$

for $w > 10$ mph and wind direction between 0 and 45 deg (derived from Table 3).

The 7 wet cases also seem to indicate an upward trend. Unfortunately, all the 7 are grouped in a narrow range of wind velocity and the scattering is such that the rate of increase in rate of spread cannot be reliably estimated. In this case, as in the dry case, the scattering of data points can be reduced somewhat by rejecting points with measured time less than 1 hour (1 such point in the dry case). Some discussion on this effect will appear soon.

All the 3 groups III-1, III-2, and III-3 also show a definite upward trend although with more scattering. For groups with lower fuel densities, the dependence on wind velocity is completely masked by some other effect. We shall attribute this fact and several others to the relative extent of ground spread and spotting which varies with fuel density.

To investigate the remaining variable, i.e., the measured time, it is necessary to keep the effect of wind velocity constant. We have seen that the effect of wind velocity is approximately the same in urban and rural fuel at least in the cases for which this effect can be evaluated. Therefore, this effect can be kept constant in analysis of urban data by converting rates of spread at

any wind velocity to their values at a wind velocity of 10 mph, using appropriate ratios previously found for rural fuel. A velocity of 10 mph was selected as basis for reference because it was approximately the average of wind velocities recorded in the data. Most corrections applied were less than 50 percent.

We are now ready to examine the effect of measured time. Values in each group were plotted against the time during which the spread had been observed. For the largest group, i.e., IV-3, most points are concentrated in the time range from 0-5 hr and are almost uniformly scattered. Outside this time range there is only one point, at 10.2 hr, which is slightly lower than the average of points in the 0-5 hr range. However, this fact was not considered sufficient to indicate a downward trend with increasing time of observation. For other groups, the scattering also tends to be more pronounced at short times of observation, especially when all groups of the same fuel density are combined to obtain enough data points. The scattering at short times will be accounted for in a subsequent section.

The average values for rate of spread in each group are given in Fig. 11. Numbers of values giving the averages are shown in parentheses. Average values shown at the bottom of Fig. 11 are for combined fuel types in each fuel density range. Averaging was done by dividing the sum of distances by the sum of corrected times which were determined from the corrected rate and distance.

We have said that 7 of the 15 values in group IV-3 are for wet conditions. (Roofs and exterior walls were wet due to rain or melting snow.) The average rate of spread for the wet

category is 0.029 mph and that for the dry category is 0.041. The difference seems to indicate a real effect, although no further evidence is available.

Spotting

Data for rate of spread by spotting were next investigated. The 29 data points (including those with measured time less than 1 hr and later discarded) were grouped according to fuel type and building density as before. There was no case of backward spread. The wind velocities range from 4 to 38 mph, with an average of 17.6 mph, whereas in ground spread data, wind velocities vary from 3 to 28 mph and average to 14.9 mph. Thus, the occurrence of spotting does not seem to depend strongly on high wind velocity.

Correction for wind velocity, as has been done with ground spread data, is not possible in this case because data for spotting in rural fuel have not been thoroughly studied, and also, the effect of wind on spotting may not be the same for both types of fuel. Thus, wind velocity and measured time could not be separated.

Rate of spread values were plotted against measured time for each fuel density range and for all fuel types combined. Again, wild fluctuations occur at short measured times (less than 1 hr). However, the average of values in this range are consistently higher than values at longer times. Thus, there seems to be some tendency to report cases of fast jump and to

overlook cases in which the fire spent some time before jumping. To minimize this bias, values with times of measurement less than 1 hr have been discarded (12 such values).

Average rates of spread for each group are given in Fig. 12. Values at the bottom of the figure are averages for all fuel types in each fuel density range.

DISCUSSION

Many results of this analysis can be interpreted in terms of the characteristics of urban fuel and the spread processes in this medium.

A significant characteristic of urban fuel is discontinuity. Each structure is surrounded by an open space of varying dimension, in contrast with the essentially continuous forest fuel. A consequence of this discontinuity is that even ground spread proceeds by jumps. Any jump may or may not take place, and if it does, the time spent before the jump is highly variable, depending on the distances to adjacent structures, the orientation of these structures, and on wind velocity and direction. Spotting is still more stochastic. For an ignition to occur, a firebrand must be still burning on landing and must land on a surface which happens to be ignitable.

Fuel discontinuity also hinders development of full and uniform fire fronts in urban areas. Probability considerations coupled with some data on rate of spread, burning time, and

Fuel Type	3	0.45 (1)	0.23 (1)	0.06 (1)	
	2	0.30 (2)	0.27 (3)		
	1	0.33 (1)	0.38 (7)	0.22 (1)	
		0.33 (1)	0.37 (10)	0.25 (5)	0.06 (1)
		I	II	III	IV
		Fuel Density			

Fig. 12. Average Values for Spotting Spread (mph)

fuel distribution seems to indicate that, except for extreme fuel and weather conditions, a mass fire, at any given time, is made up of a number of small islands (a few structures) burning simultaneously and propagating independently. Most fires recorded in the data were representative of these islands. Since, for a given set of fuel and weather conditions, the rate of spread in nonextreme cases depends on the size of individual islands but not on the size of the overall conflagration, rate of spread values obtained from these moderately "big" fires can be used for very large overall fires as expected during nuclear attacks.

When a spot jump takes place, the fire is considered to have reached the newly ignited structure, although intermediate structures will burn only at a later moment.

Since jumps are essentially random both in occurrence and time, it is easy to understand that measurements on short spread runs involving a small number of jumps are likely to yield widely fluctuating values for rate of spread whereas for long runs, the overall rate of spread is more reproducible. Consequently, the average rate in each group was obtained by dividing total distance by total time rather than by adding all rate values and dividing by the number of values. In this manner, short runs are combined into long runs and fluctuations are expected to be smoothed out.

From the tables of average rates of spread for both ground spread and spotting we note that, within each fuel type, the

rates increase with decreasing fuel density. Within each fuel density range, however, the rates do not show a definite trend as fuel type varies. Consequently, fuel density was considered the more important factor.

The increasing rates of spread when fuel density decreases was interpreted as due to the increasing number of spot jumps compared with the number of ground jumps. To visualize this process, let us consider a case of very high fuel density and a case of very low density. If the density α was near unity, spacing between structures would be practically zero. Therefore, every time a structure burns, at least one adjacent structure would be ignited, i.e., the probability $P_g(\alpha)$ of a ground jump during the burning time would be one. The probability of firebrands being emitted, carried a given distance, and remaining active on landing depends on the type and volume of the burning structure, wind velocity, atmospheric moisture, and so on. For the moment, let us assume that this probability is not zero and denote it by P_s . Ignition by spotting also requires that the firebrand lands on the roofs, so that the probability of a spot jump is a function of α . We assume that this probability is proportional to α and we represent it by αP_s . Thus, in a fuel area of very high density, the fire never misses a ground jump. It proceeds from A to B, then to C, and so on. From time to time it also makes a spot jump from say F to V. If the fire is observed for a sufficiently long time, the average rate of spread can be evaluated by counting the number of ground jumps and the number of spot jumps, multiplying these numbers by their respective distances and dividing by the total time.

As the fuel density α decreases, the probability of spotting is assumed to decrease linearly with α . As to the probability of ground spread $P_g(\alpha)$, evidence suggests that the decrease is faster. On this basis, for very low fuel density, a large percentage of fires would die out soon after start. But those which did spread to some distance would do so predominantly by spotting. Therefore, high rates of spread would be observed.

To describe this process with a mathematical expression, let us consider the following probabilities for each burning structure: the probabilities $P_g(\alpha)$ of making a ground jump and $[1 - P_g(\alpha)]$ of not making it; the probabilities αP_s of making a spot jump; and $[1 - \alpha P_s]$ of not making it. All possible combinations are:

(a)	$P_g(\alpha)$	αP_s	
(b)	$P_g(\alpha)$	$1 - \alpha P_s$	
(c)	$1 - P_g(\alpha)$	αP_s	(5.7)
(d)	$1 - P_g(\alpha)$	$1 - \alpha P_s$	

To evaluate overall rate of spread, spot jumps are counted even when there are simultaneous ground jumps. Therefore, the probability for a count of spotting is (a) + (c) or αP_s , and the probability of a count of ground spread is (b) or $P_g(\alpha)(1 - \alpha P_s)$.

Given that the fire has made a total of n jumps, the number of ground jumps is

$$\frac{n P_g(\alpha)(1 - \alpha P_s)}{P_g(\alpha)(1 - \alpha P_s) + \alpha P_s} \quad (58)$$

and the number of spot jumps is

$$\frac{n\alpha P_s}{P_g(\alpha)(1 - \alpha P_s) + \alpha P_s} \quad (59)$$

The average distance per ground jump is $\sqrt{a/\alpha}$, where a is the average area of the structures. This distance includes the spacing and the average dimension of the structure itself. The distance for spot jumps is highly variable, but we consider a mean value d_s .

The total distance of spread is

$$\frac{n\sqrt{a/\alpha} P_g(\alpha)(1 - \alpha P_s) + n d_s \alpha P_s}{P_g(\alpha)(1 - \alpha P_s) + \alpha P_s} \quad (60)$$

The time per jump is a stochastic variable and depends on fuel density. We assume, however, an average time t_j for both type of jumps. Then the total time is nt_j , and the average rate of spread is given by

$$\bar{R} = \frac{\sqrt{a/\alpha} P_g(\alpha)(1 - \alpha P_s) + d_s \alpha P_s}{[P_g(\alpha)(1 - \alpha P_s) + \alpha P_s] t_j} \quad (61)$$

For very high fuel density, the rate of spread is obtained from this expression by making $P_g(\alpha) = 1$. Unless P_s is zero, the fire proceeds by both ground jumps and spot jumps. For sufficiently low fuel density, $P_g(\alpha) = 0$. The fire then proceeds exclusively by spotting and the rate of spread is simply d_s/t_j . By comparing the formulas for rate of spread in the two

cases it is easy to see that rate of spread can be much greater at low density than at high density.

In addition to the increase in rate of spread with decreasing fuel density, several other results can also be explained by this simple model.

We have seen that more spotting fires were observed in low density areas than in high density areas, whereas for ground-spread fires, the reverse was true.

We also noted that when rates of spread were plotted against wind velocity, the degree of scattering increased rapidly when going from one density range to the next lower range, and for the two lowest density ranges, the scattering completely masked any dependence on wind velocity. This is due to the fact that as density decreases, the relative probability for spot jumps increases. Since the distance per spot jump is, in general, much larger than the distance per ground jump, the average rate of spread for insufficiently long runs is greatly affected by the occurrence or nonoccurrence of spot jumps.

The effect of dryness could not be quantitatively investigated except for group IV-3 in which wet condition seemed to decrease the rate of spread to some extent. If enough data points were available, it would be interesting to see the effect of wet conditions at low fuel densities. Exterior wetness is expected to decrease the rate of ground spread because more heat is required to raise the temperature of exterior walls and because of

a decrease in the amount of pilot ignition due to embers being extinguished. Spotting also becomes less likely if roofs are wet. The effect of wetness on rate of spread is probably more drastic with spotting than with ground spread.

Tables 2 and 3 refer separately to fires which have been labeled as ground-spread or spotting, respectively. Distances covered during each observation are rather short. On the basis of the model just described, both types of spread should occur in any sufficiently long spread run according to their own probabilities. Averages in each table were obtained by adding the distances and dividing by the total time, thus combining many short fires to a long one. If we now combine Tables 2 and 3 in the same way, the resulting data would characterize long, spreading fires and would give better average velocities for fires which spread indefinitely if fuel conditions allow them to spread. The combined averages are 0.28, 0.29, 0.07, and 0.03 for fuel densities I, II, III, and IV (in that order) for all fuel types in each density range.

Some further comments on these final results seem to be relevant. The apparent constancy of the first two figures can be explained by various arguments: it may be a real effect, i.e., the mean rate of spread may level off as fuel density decreases below II; data for density range I may be insufficient, hence the first value is less reliable than others; actual fuel density for range I may not be much less than 20 percent, a value which would put these cases in range II, etc.

As mentioned previously, only the most important factor, namely, fuel density, could be studied quantitatively. The effect of exterior humidity is real but could not be evaluated. The final values for mean rate of spread characterize peacetime fires spreading through significant distance. These fires may have occurred under the more severe fire weather conditions since it is easier to put out fires under poor fire weather conditions. Fires from nuclear attacks will spread under weather conditions that are characteristic of the locality, the year, and the season. If the effect of weather on rate of spread is sufficiently important, the sample of peacetime fires may contain more fast fires than does the population of fires from nuclear attacks, therefore the mean rates of spread given above may be on the high side when used for nuclear attack fires. It is, of course, impossible to make any correction for this fact. However, for conservative prediction of rate of spread for fires from enemy attacks, the uncertainty may not be of great consequence.

Section 11

VARIOUS ESTIMATED DATA

In this section we group various types of information derived from very limited observed data, supplemented in some cases by subjective judgment.

SPREAD PROBABILITY IN URBAN FUEL

The U.S. Strategic Bombing Survey studied the efficiency of 37 linear miles of fire breaks of various widths in Nagoya, strong winds or fire storms. Fire was stopped over 34.8 percent of the total length of breaks in the width range from 65 to 150 ft and 75 percent of the total length for widths ranging from 150 ft upwards (Ref. 2). This seems to indicate that for a spacing of 107 ft (midpoint between 65 and 150 ft), the probability of spread is approximately 0.65, and for some spacing beyond 150 ft, it is 0.25. This spacing is probably not far from the point at 200 ft i.e., halfway between 150 and 250 ft. The latter spacing is considered as the limit for fire spread and is justified only by a feeling acquired through personal experience and training.

These two points and the fact that fire spread is certain when spacing is zero allow us to draw curve A in Fig. 13. It must be emphasized that this curve should be considered as an upper limit of the probability of spread for two reasons: first, fires were not considered stopped in places where burned structures were found on both sides of the fire breaks, although both

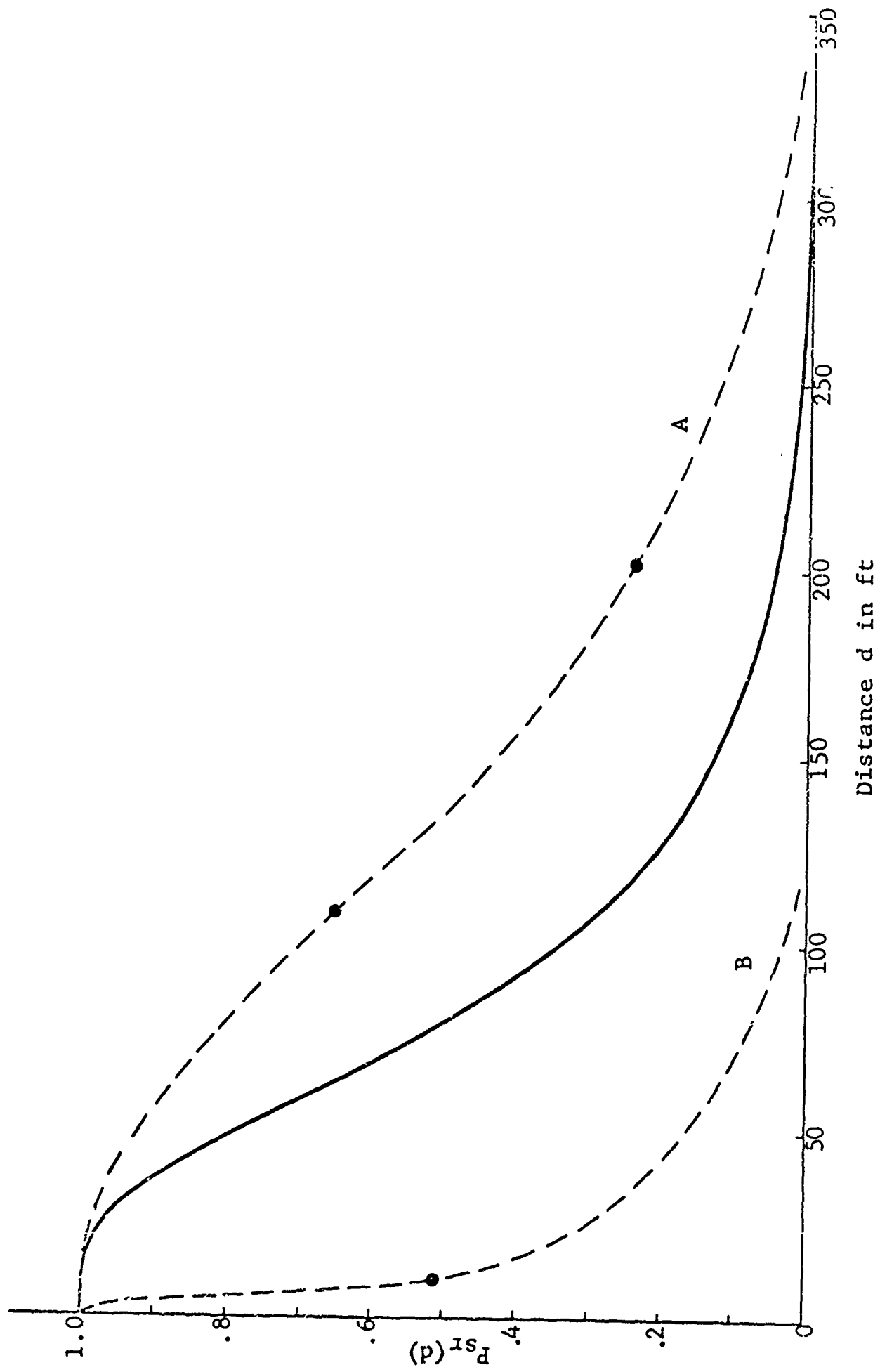


Fig. 13. Probability of Spread Versus Distance

sides of an adequate fire break might have been hit directly by incendiary bombs; second, fire might have jumped across a fire break at one point and spread along some distance on the other side of the break. The stopping efficiency of the break would be underestimated if fire burned on both sides — at points B, C, D, E on one side and at corresponding points B', C', D', E' on the other side — but the fires at these latter points originated from A' instead of B, C, D, E.

Examination of fire spread data in incendiary-attacked German cities has led to the conclusion that under normal fire conditions, a 10-ft space between two brick buildings had about 50 percent chance of preventing fire spread (Ref. 2). The probability curve for this special case (spread between two brick buildings) is perhaps not too far off from curve B in Fig. 13, which is obviously a lower limit of the curve being sought, since brick buildings are generally less vulnerable to fire spread than other types of buildings commonly existing in urban areas.

The solid curve in this figure, drawn between these two limits, represents the best guess and is believed to yield acceptable results when applied to urban fuel in general.

WEATHER CONDITIONS FOR NO-SPREAD IN RURAL FUEL

The list of weather conditions for no-spread in rural fuel and the sets of data following have been provided by the U.S. Forest Service (Ref. 1).

All Fuels. Over 1 in. of snow on the ground at the nearest weather reporting stations.

Grass. Relative humidity above 80 percent.

Brush or Hardwoods. 0.1 in. of precipitation or more within the past 7 days and wind

- 0 - 3 mph; relative humidity 60 percent or higher, or
- 4 - 10 mph; relative humidity 75 percent or higher, or
- 11 - 25 mph; relative humidity 85 percent or higher.

Conifer Timber. (a) 1 day or less since at least 0.25 in. of precipitation and wind

- 0 - 3 mph; relative humidity 50 percent or higher, or
- 4 - 10 mph; relative humidity 75 percent or higher, or
- 11 - 25 mph; relative humidity 85 percent or higher.

(b) Or, 2 - 3 days since at least 0.25 in. of precipitation and wind

- 0 - 3 mph; relative humidity 60 percent or higher, or
- 4 - 10 mph; relative humidity 80 percent or higher, or
- 11 - 25 mph; relative humidity 90 percent or higher.

(c) Or, 4 - 5 days since at least 0.25 in. of precipitation and wind

- 0 - 3 mph; relative humidity 80 percent or higher.

(d) Or, 6 - 7 days since at least 0.25 in. of precipitation and

- 0 - 3 mph; relative humidity 90 percent or higher.

WEATHER CONDITIONS FOR EXTINCTION

Rural Fires

Extinction after total burning time is not considered.

Grass. Measurable precipitation at the three nearest weather stations.

Brush or Hardwoods. 0.1 in. of precipitation or more at the three nearest weather stations.

Conifer Timber. (a) 0.5 in. of precipitation or more at the three nearest weather stations.

(b) Or, 0.25 to 0.5 in. of precipitation at the three nearest weather stations and no-spread conditions for the following two 12-hr periods.

(c) Or, no-spread conditions for eight consecutive 12-hr periods and measurable precipitation at the three nearest weather stations during any two 12-hr periods.

Urban Fires

It is noticed that extinction of urban fires requires rather severe weather conditions.

Light Residential. 1.0 in. of precipitation at the Weather Bureau Station and no-spread conditions for 36 consecutive hr.

Heavy Residential. 1.5 in. of precipitation at the city Weather Bureau Station and no-spread conditions for 72 consecutive hr.

Commercial. 2.0 in. of precipitation at the city Weather Bureau Station and no-spread conditions for seven consecutive days.

City Center or Massive Manufacturing. 2.0 in. of precipitation at the city Weather Bureau Station and no-spread conditions for two consecutive months.

BURNING TIMES

Total burning time is defined as the period during which a large fire might remain stationery yet be capable of resuming active spread if burning conditions changed for the worse. Burning times for various classes of fuel are listed below.

Rural Fuels

<u>Fuel Type</u>	<u>Total Burning Time</u>
Grass	30 min
Light brush	16 hr
Medium brush	36 hr
Heavy brush	72 hr
Timber	7 days

Urban Fuels

<u>Fuel Type</u>	<u>Total Burning Time</u>
Light residential	36 hr
Heavy residential	72 hr
Commercial	7 days
City center and massive mfg.	2 mos.

Section 12

FIRE SPREAD VULNERABILITY OF RESOURCES

The general approach to the problem of fire damage assessment involves, first, estimation of the general burnout of fuel areas and, second, determination of the fire damage to resources by means of fire spread vulnerability factors. This section deals with the second aspect of the problem.

Fire spread vulnerability can be defined in simple terms as follows: Given that the general area is swept by fire, is the resource destroyed or what is its chance of being destroyed? The solution, however, is made difficult by the limited amount of appropriate data. In this study, methods have been selected which can be used for evaluating vulnerabilities on the basis of some existing data and which can be readily improved as more extensive and more accurate data become available.

The vulnerability of resources to fire spread clearly depends on three groups of factors: susceptibility of the resource to fire (normally involving the structural characteristics and contents), the characteristics of the adjacent fuel (as an ignition source), and the spacing between the resource and the adjacent fuel.

It seems advantageous to study first the vulnerability for one type of resource in order to establish the effects of spacing and surrounding fuel on the ignition event. The result is a

measure of that part of vulnerability due to the environment of the resource. Attention is next given to the question of how the vulnerability changes with the structural characteristics of the resource.

The type of resource chosen for studying the environment dependence of vulnerability is the common type of structure in urban areas. These structures are characterized by substantial combustible surface exposed to heat and firebrands. The reason for this choice is that most data on ignition events refer to this type of exposed structures.

With respect to environment or type of fuel in adjacent areas, three situations may be distinguished: (1) the fuel is entirely or predominantly urban; (2) the fuel is entirely or predominantly rural; and (3) the fuel is an approximately equal mixture of rural and urban types. In the following subsections, we shall successively discuss the urban, rural, and mixed environments.

URBAN ENVIRONMENT

The characteristics of surrounding fuel can be specified in three ways:

- A. The size, structural materials of each structure, and its distance to the resource are given.
- B. Only the number of adjacent structures and their distances to the resource are specified.
- C. The only information is that the resource is within an urban area of a given class and given building density.

Case A

When all characteristics of surrounding fuel are completely specified, a deterministic approach is obviously indicated. Two alternative methods are proposed. In one method, the ignition event is predicted on the basis of the intensity of radiated heat, adjusted upward to include the estimated effect of convected heat and embers. The other method uses a formula which gives a plausible measure of total heat and embers evolved from the burning structure, the minimum measure required for ignition being derived empirically or subjectively.

It will appear that either method can be improved by further observed data as they become available. Since it is not certain which one will be first to receive backup data and become more reliable, this report will describe both.

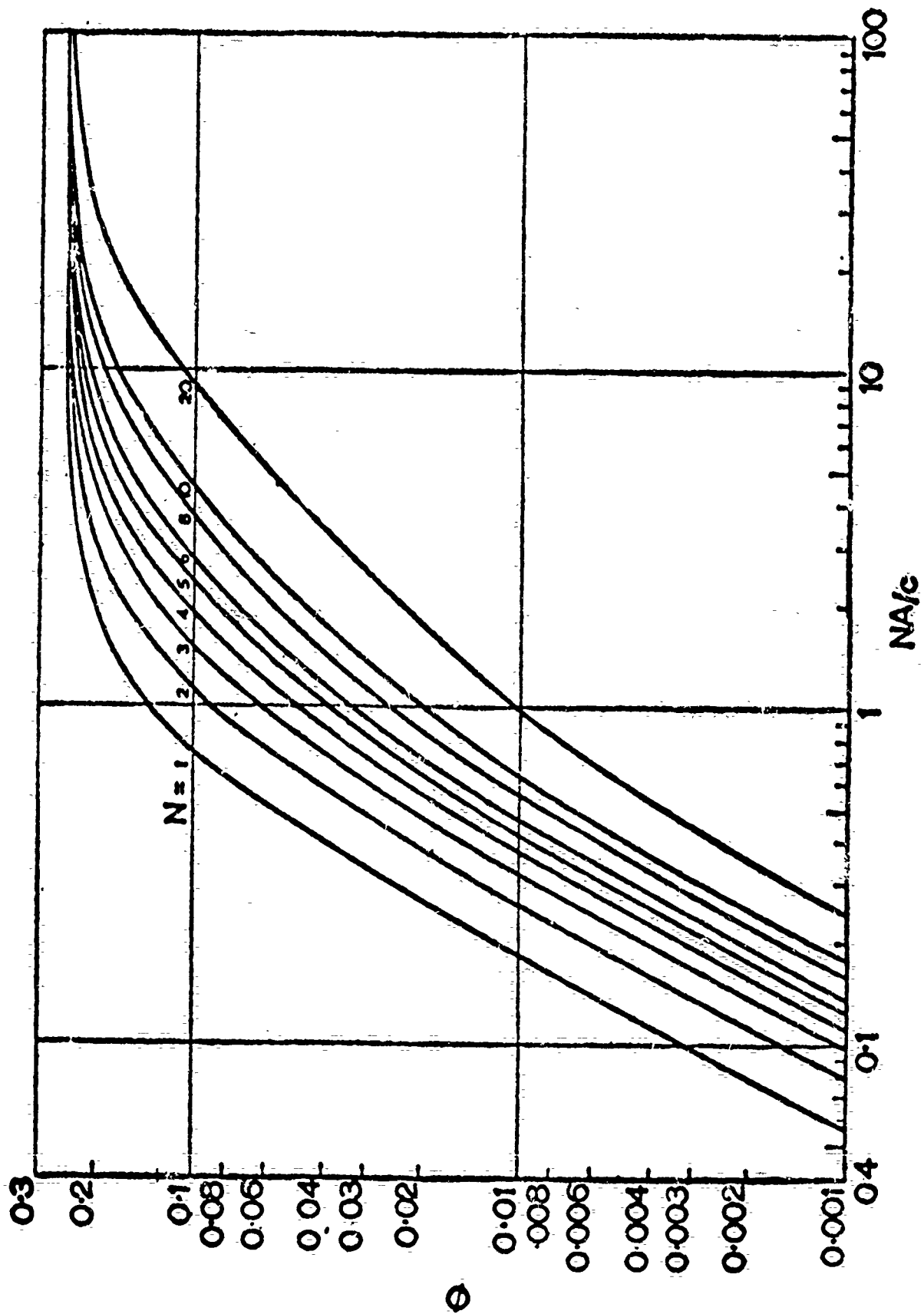
Method of Adjusted Radiation Intensity

The radiant energy as a function of distance from a radiating surface can be measured or calculated if the source characteristics and geometry are well specified. The variation of convected heat with distance presents a more complicated problem and has not been thoroughly investigated. Flame contact and embers (falling at short distance) are predominantly random. Their treatment as deterministic is justified by the following consideration. Since at distances within which these flames and embers are active the fuel is preheated by radiated and convected heat, ignition requires only a little tongue of flame or one or two embers to fall on a piece of fuel. Thus, the

action of flames and embers is primarily to extend the effect of radiated and convected heat to distances at which this heat could not by itself produce ignition. Spotting firebands falling beyond the heat range is a purely stochastic phenomenon. Since nothing is known about spotting except that its probability is usually low compared with that of continuous spread, we leave spotting outside the scope of this study.

Several investigators (Refs. 3, 4, 5) have reported their results on radiated heat. At any distance c on the normal passing through the center of a rectangular radiating surface, the ratio of the intensity I to the intensity I_0 (at zero distance from the source) is given by the so-called configuration factor $\bar{\phi}$, which depends not only on the distance c but also on the shape of the radiating surface, i.e., on the ratio N of the longer dimension to the shorter dimension. In Fig. 14 (from Ref. 5), the factor ϕ , which is one-fourth $\bar{\phi}$, is plotted against NA/c , where A is half the shorter dimension. Thus ϕ is the contribution from each of the four sections of the radiating surface obtained by drawing two lines through the center of the surface, one lengthwise and the other crosswise.

The factors $\bar{\phi}$ and ϕ apply to fully radiating surfaces. When a surface is made up of radiating sections (windows, combustible walls) alternating with nonradiating sections, an exact treatment would require each radiating section to be considered separately and the total effect obtained by summing all contributions. In practice, however, it is much simpler and sufficiently accurate to multiply I_0 by the fraction of total area representing window and combustible area.

Fig. 14. Configuration Factor ϕ for Differently Shaped Radiators

One more correction is needed to account for the fact that the area actually radiating at any time is not necessarily the entire face of the structure. If one horizontal dimension is much larger than the other, the fire may well burn in only one section at a time. Obviously, there is no fixed value for the area of the radiating section, but it is believed that a reasonable maximum value for typical cases is the product of the height and twice the depth (assumed to be the minimum horizontal dimension) of the structure. If the width of the radiating surface is less than twice the depth of the structure or if the radiating surface is along the depth of the structure, no correction is needed.

The intensity I_0 at the radiating surface depends on the fire load or total amount of combustible in the enclosure. For fire loads greater than about 5 lb-ft^{-2} , a common case, the intensity is about $4 \text{ cal-cm}^{-2}\text{-sec}^{-1}$. Smaller fire loads produce intensities of about $2 \text{ cal-cm}^{-2}\text{-sec}^{-1}$.

The minimum heat flux required to ignite wood has been measured (Ref. 5). At $0.8 \text{ cal-cm}^{-2}\text{-sec}^{-1}$ or more, spontaneous ignition occurs within about 2 min. When the heat flux is between 0.8 and $0.3 \text{ cal-cm}^{-2}\text{-sec}^{-1}$, pilot ignition can take place, if after 5 - 10 min of heating, a little flame or ember comes near the heated surface.

There are no data on the amount of convected heat and its variation with distance or on the number of embers given off by a structure of a given size. Consequently, their effects must be taken into account in somewhat arbitrary ways: first,

the intensity at zero distance is increased from 4 to 6 cal-cm⁻²-sec⁻¹ for fire loads equal to or greater than 5 lb-ft⁻² and from 2 to 3 cal-cm⁻²-sec⁻¹ for smaller fire loads. Second, flames and embers are assumed to be always available for pilot ignition at distances for which heat intensity exceeds 0.30 cal-cm⁻²-sec⁻¹, provided the exposed structure has some significant ignitable area. With the further assumption that the flux of convected heat varies with distance in the same manner as the radiated component, it is possible to evaluate the distance c_{\max} for each ignition source at which the heat intensity is equal to the critical value 0.30 cal-cm⁻²-sec⁻¹.

Method of Vulnerability Number

First, a function V_e is selected to represent the igniting capability of the ignition source at various distances. Then, the minimum value this function must have for ignition is empirically determined. For a very small source, an appropriate function is probably

$$V_e = \frac{V}{d^2} \quad (62)$$

where V is the volume of the ignition source (in cubic feet) and d the separation (in feet).

While the inverse square relation is probably correct for very small V , there are two errors when V does not satisfy this requirement. The first error is of a geometrical nature. Since the distance d is measured from the center of the face of the

ignition source to some point P on the normal to the center, the simple formula is correct only for the volume element ΔV_0 exactly at the center of the face. For all other volume elements, the true distance to point P is greater than d by an amount which increases as the volume elements get further away from the center of the face. The correct value of V_e for point P is given by

$$V_e = 4 \int_0^D \int_0^{W/2} \int_0^{H/2} \frac{dx \, dy \, dz}{(d+x)^2 + y^2 + z^2} \quad (63)$$

where D, W, and H are the depth, the width, and the height of the ignition source, respectively, and x, y, z are distances in the direction of D, W, H. Since exact evaluation of this integral is complicated, the following approximate formula is preferred,

$$\begin{aligned} V_e &= \frac{D}{d(d+D)} \tan^{-1} \frac{W}{2d} \tan^{-1} \frac{H}{2d} \\ &= \frac{V}{d^2} \frac{1}{\left(1 + \frac{D}{d}\right)WH} \tan^{-1} \frac{W}{2d} \tan^{-1} \frac{H}{2d} \end{aligned} \quad (64)$$

The second error is due to the fact that heat, embers, and firebrands from one layer of the source may be partially blocked by all layers lying in front. The extent of this effect may vary with each process, i.e., radiation, convection, embers, firebrands, with the fuel arrangement in the source, and with the phase of combustion. A quantitative correction is not possible at the present state of the art. Formula (64), however, will be on the safe side (overestimate) when predicting fire spread vulnerability.

Serious error may result, however, from using Formula (64) if the outside walls (or a significant fraction thereof) are incombustible and do not collapse until the end of the combustion, because in this case all or part of the igniting flux is blocked off. It is reasonable to assume that the fraction of the "igniting flux" that gets through is equal to the fraction of window and combustibile area. Denoting this fraction by α , we write the formula as

$$V_e = \frac{\alpha V}{d^2} \frac{1}{\left(1 + \frac{D}{d}\right) WH} \tan^{-1} \frac{W}{2d} \tan^{-1} \frac{H}{2d} \quad (65)$$

Finally, we must account for the fact that a structure ignited in the normal way frequently burns in only one section at a time, especially when one horizontal dimension is much larger than the other. As in the method of adjusted radiation intensity, we may limit the width W of the radiating face to twice the depth D of the structure, so that the maximum volume of the active ignition source is $2HD^2$.

The minimum V_e value required for ignition cannot be established objectively, because observed data are still very scanty. Hence, only a subjective estimate is available at present. For an ignition source, let us consider a one-story, 50 by 50 ft by 12-ft high structure with 50 percent window area. On the basis of intuition and limited data on urban fires (Ref. 2), one would be inclined to believe that ignition takes place at a distance of 30 ft or less. The corresponding value of V_e is 1.4×10^{-3} .

With either of these two methods, the destruction or survival of the resource is predicted by adding the heat intensities

or the V_e values from all adjacent structures acting on each face of the resource structure and comparing the highest total heat intensity of V_e value on any single face with minimum heat or V_e required for ignition.

Addition of heat intensities or V_e values from adjacent structures is based on the assumption that these structures burn simultaneously. This leads to the question: which ones will burn simultaneously? This depends on the course of the fire front. Suppose one face of the resource is exposed to a parallel row of structures A, B, C, All these structures will burn simultaneously if a fire front approaches these structures in a direction perpendicular to the row. However, this is not necessarily true if the fire spreads along the row of structures. In this case the number of structures burning simultaneously must be determined from the rate of spread and the burning time of fuel. Unless a separate vulnerability determination is made for each of the two cases, it is recommended that the average number of simultaneous burning structures be used.

Case B

In this case the number of adjacent structures and their distances to the resource are specified. Since the structural characteristics are unknown, it is necessary to assume that the distribution of these characteristics, i.e., size, construction materials, number of stories, etc., is typical of most urban areas. Hence the vulnerability problem becomes a stochastic one. Given that all surrounding structures burn (simultaneously or not), we must find the probability that the resource is ignited. Needed is the probability that in a typical urban area a burning structure

ignites another structure at a specified distance. Limited information of this type has been given in the previous section.

The stochastic vulnerability of the resource is the probability that the resource is ignited by one or more surrounding structures. If the spacings are d_1, d_2, d_3, \dots and the corresponding probabilities of ignition are $P(d_1), P(d_2), P(d_3), \dots$, as given in Fig. 13, the vulnerability of the resource is given by

$$1 - [1 - P(d_1)][1 - P(d_2)][1 - P(d_3)] \dots \quad (66)$$

where the subscripts of d extend to all surrounding structures (except those which will certainly not burn in case of general fire).

Case C

Not only structural characteristics but spacings also are left unspecified. In order to apply the data of Fig. 13, we must independently determine the distribution of spacings between adjacent structures by studying many maps of urban areas. This work is best done by dividing urban areas into a number of categories, for example: general residential, residential tract, commercial, industrial, etc. Suppose the spacing distribution is expressed as number (integral) of structures at each spacing range from a given structure and denoted by n_1, n_2, n_3, \dots for the ranges 0-5, 5-10, 10-15, \dots ft, respectively. The probability that the resource is ignited by one or more structures in the i^{th} range is

$$p_i = 1 - [1 - P(i)]^{n_i} \quad (67)$$

where $p(i)$ is the average ignition probability for the i^{th} range, a quantity easily determined from Fig. 13. The vulnerability is then given by

$$1 - \prod_{i=1}^{\infty} (1 - p_i) \quad (68)$$

or

$$1 - \prod_{i=1}^{\infty} [1 - P(i)]^{n_i} \quad (69)$$

RURAL ENVIRONMENT

We are here concerned with an area in which urban fuel is insignificant in comparison with rural fuel. The resource is probably separated from the rural fuel by a zone of open space, parking areas, roads, lawns, etc. The rural fuel surrounding this zone may be continuous or may consist of patches of various sizes separated by open space. Since many types of data for a stochastic treatment in this case are not available, fuel data must be completely specified, for example, by use of a map showing the resource and the surrounding fuel.

As for urban environment above, the heat intensity on each face of the resource structure is calculated by means of either of the two methods described in those paragraphs, and the deterministic vulnerability of the structure is decided on by considering the face which receives the highest intensity.

In applying these methods, one must know the height of the radiating face, i.e., the probable height of flames from burning rural fuel, the dimensions of the source (size of fire burning at

the same time), and the heat intensity at the radiating surface I_0 . Flame height can be estimated by experienced personnel in terms of the height of the fuel. The width and depth of the source must be determined for each of two extreme cases (fire spread perpendicular or parallel to a given face of the resource), as already discussed at the end of Case A.

The intensity of radiated heat I_0 can be reasonably estimated at $2 \text{ cal-cm}^{-2}\text{-sec}^{-1}$ on the basis of experiments on structures (Ref. 5), since for rural fuel the ventilation is complete and the fire load is normally less than 5 lb/ft^2 . For medium brush the fire load is 24 tons per acre or approximately 1 lb/ft^2 (Ref. 1).

MIXED ENVIRONMENT

When comparable proportions of urban and rural fuels exist in the environment, a deterministic approach seems to be the only feasible one, and data specification must be such that this approach can be used.

Either of the two methods of Case A is applied to each structure and each patch of rural fuel, and the overall effect on each face of the resource structure is computed. The computation is likely to be lengthy in some cases. It might be possible to treat all structures irradiating a given face of the resource as a single source and all patches of rural fuel as another single source. Appropriate factors (similar to window factors) must be chosen to account for the fact that only certain sections of the overall source constitute the actual radiating surface.

Section 13
WEATHER DATA FOR FIRE SPREAD PREDICTION

SPECIFICATION OF THE DATE OF ATTACK

Since fire spread is affected by weather and since for a given area the weather varies with the day, the month, and the year, the extent of fire spread predicted clearly depends on the time of attack specified. There are three ways of specifying the time of attack: (1) a specific date in the past is selected; (2) the current date is selected; (3) any date in the future is considered an equally likely date of attack.

ACQUISITION OF WEATHER DATA

Case I

A specific date in the past may be selected for one of three reasons: (a) at that date and during the subsequent interval, weather conditions were typical for the area of interest; (b) the weather had some characteristics desirable for the enemy when planning an attack; (c) information on the fire damage due to a certain type of weather is wanted.

In this case, weather data do not present a major problem. The U.S. Department of Commerce Weather Bureau provides detailed information on all weather factors for most cities and surrounding areas. Weather data for the specified period and specified area can be used directly. Some handling is needed, however, if the

records give more details than can be used in the calculation. For example, in many cases hourly wind velocity and direction are given. The calculation for a large area or a long-lasting fire would be too lengthy if new data for wind velocity and direction were introduced every hour. In such cases the wind data must be reduced to average values for day and night so that calculation need not be interrupted for intervals of approximately 12 hr.

Case II

Selection of the current date as date of attack may be prompted by reasons similar to those mentioned in the previous case. Data on current weather can be obtained from various weather stations. Weather for the immediate future must be predicted on the basis of current weather. The reliability of such prediction depends on the meteorological characteristics of the area and especially on how far into the future the prediction is carried out. At present, long-range forecasts can cover a period of 5 days with reasonable accuracy.

Case III

The date of attack is most likely to be left indeterminate in the future. The task in this case is to estimate the mean fire effect on the assumption that the attack will be completely random in time.

Whereas current weather can serve as a basis for making daily forecasts or forecasts for a future period of a few days, long-range prediction requires a statistical approach based on weather that has been observed over a long period in the past.

One obvious way to evaluate the mean fire effect would be to carry out a fire spread calculation for each day in the past taken as attack day and for a period long enough to cover all types of weather that can occur in the area. The fire effect would then be averaged over all attack days. However, the level of effort involved in this calculation is clearly prohibitive.

A simple approach is to select 2 months with extreme weather by inspection of a period of n years and calculate fire spread for each month according to procedures to be given later in connection with the use of weather data for fire spread prediction. The result would give an upper and a lower limit for fire damage. How satisfactory this prediction is, depends on the requirements of the model user and on how widely these limits are separated.

The third approach is essentially to predict the mean weather for any future year and calculating the fire effect accordingly. This is not rigorously valid because the use of mean weather will yield an exact mean of fire effect only if the latter is linearly related to all weather factors (wind, relative humidity, precipitation, etc.). Nevertheless, the method seems to be satisfactory on two counts: First, analysis of observed data on past fires indicates that relations between weather and certain characteristics of fire spread are not too far from linearity. These relations are between rate of spread on the one hand and wind velocity and relative humidity on the other. Second, the non-linear relation between weather and the rural fires' capability of spread will receive a special treatment, as will be discussed below.

In attempting to determine a mean weather pattern for fire spread prediction, several points must be kept in mind:

- Weather cannot be represented by a single number. As many numbers are required as there are meteorological phenomena which characterize weather. Therefore it is not a simple matter to define a weather pattern that is typical of the area.
- The statistical study of past weather for the purpose of fire spread prediction must center on those phenomena that can significantly affect fire spread. Records of past fires have shown that the general fire behavior is most sensitive to wind velocity and direction, relative humidity, and precipitation.
- For a given area, some correlation may exist between various weather factors. For example low relative humidity may be associated with wind in certain directions. This correlation may be taken into account by studying the seasons separately and possibly by distinguishing day weather and night weather in cases where the weather seems to follow characteristic patterns for days and nights.
- The variability of weather can play an important role. It has been mentioned that for rural fires weather conditions can be divided into three groups. One group makes the fire spread, another group prevents it from spreading, and the third group extinguishes it. Variations within the same group would produce no significant effect in fire spread. However the more often weather conditions pass into the no-spread or the extinction group, the less fire damage should be expected for a random attack.

All these factors make it difficult to define and calculate a mean weather pattern, i.e., a mean combination of all weather factors and their changes. An approximate mean weather pattern can be obtained, however, by selecting from the weather record a single year in which wind velocity, wind direction, relative

humidity, precipitation and the variations of each appear to be most typical. The actual weather in that year is then used to calculate the mean fire effect. Selection of the most typical year can be done in two ways. One is qualitative and simple, the other somewhat more quantitative and more complicated.

The first method of selection requires plotting each day's wind velocity (for each predominant direction) relative humidity, and precipitation on separate graphs and, by visual inspection, selecting a year which seems most typical with respect to all four weather factors and their variations. To include all types of weather patterns that may occur in the area, the years examined must cover one average long trend cycle. In many cases this cycle varies from about 10 to 20 years. This simple method is probably sufficient in an area where the weather pattern does not vary appreciably from year to year.

The second method aims at establishing a measure of "weather normality" for each season of each year. Normal weather is made up of average wind velocity, average relative humidity, average precipitation, and average cycle of each for a period of 10 to 20 years. A measure of weather normality for each season of each year is obtained by combining the deviations of season averages from normal weather. The most normal season, i.e., the season with minimum combined deviations, is selected.

The main steps in the calculation are as follows:

- (1) A season average of wind velocity (for each predominant direction) is obtained for each season of each calendar year. This average is denoted by $\bar{W}_F(i)$, $\bar{W}_S(i)$, $\bar{W}_H(i)$,

and $\bar{W}_W(i)$ for the year i and for spring, summer, autumn, and winter respectively. It is calculated from the formula

$$\bar{W}_F(i) = \frac{1}{90} \sum_i W_F(j) \quad \text{for spring} \quad (70)$$

and from similar formulas for other seasons. Here $W_F(j)$ is the average wind velocity for the day j .

Similar season averages are also obtained for relative humidity and precipitation. They are denoted by $\bar{H}_F(i)$, $\bar{H}_S(i)$, $\bar{H}_H(i)$, $\bar{H}_W(i)$ for relative humidity and by $\bar{P}_F(i)$, $\bar{P}_S(i)$, $\bar{P}_H(i)$, $\bar{P}_W(i)$ for precipitation. Wind velocity is expressed in mph, relative humidity in percent, rainfall in inches, and snowfall in inches of equivalent rainfall.

- (2) The season averages are again averaged over the entire period of n (10 to 20) years according to the formulas

$$\begin{aligned} \bar{W}_F &= \frac{1}{n} \sum_i \bar{W}_F(i) \quad \text{and so on for other seasons} \\ \bar{H}_F &= \frac{1}{n} \sum_i \bar{H}_F(i) \quad \text{and so on for other seasons} \\ \bar{P}_F &= \frac{1}{n} \sum_i \bar{P}_F(i) \quad \text{and so on for other seasons} \end{aligned} \quad (71)$$

- (3) Deviations of seasonal averages from the period average are then computed from the formulas

$$\begin{aligned} D_{WF}(i) &= \bar{W}_F(i) - \bar{W}_F \quad \text{and so on for other seasons} \\ D_{HF}(i) &= \bar{H}_F(i) - \bar{H}_F \quad \text{and so on for other seasons} \\ D_{PF}(i) &= \bar{P}_F(i) - \bar{P}_F \quad \text{and so on for other seasons} \end{aligned} \quad (72)$$

In these equations, the first subscript on D indicates wind, relative humidity, or precipitation and the second subscript indicates the seasons.

- (4) The variability of each weather factor is measured by comparing the average cycle (short range) for each season of each year to the average cycle of the period of n years.

The average cycle for each season is obtained by dividing 90 by the number of cycles counted. One cycle is the number of days during which wind, relative humidity, or precipitation makes a complete uptrend and a complete downtrend, with minor fluctuations lasting one or two days ignored.

Seasonal average cycles are denoted by $\bar{w}_F(i)$, $\bar{h}_F(i)$, $\bar{p}_F(i)$ for wind, relative humidity, precipitation and are given by

$$\begin{aligned}\bar{w}_F(i) &= \frac{90}{c_W} \\ \bar{h}_F(i) &= \frac{90}{c_H} \\ \bar{p}_F(i) &= \frac{90}{c_P}\end{aligned}\tag{73}$$

where c_W , c_H , c_P are the numbers of cycles in wind, relative humidity, and precipitation actually counted.

Seasonal average cycles are then averaged over the period of n year:

$$\begin{aligned}\bar{w}_F &= \frac{1}{n} \sum_i \bar{w}_F(i) \\ \bar{h}_F &= \frac{1}{n} \sum_i \bar{h}_F(i) \\ \bar{p}_F &= \frac{1}{n} \sum_i \bar{p}_F(i)\end{aligned}\tag{74}$$

Deviations of each seasonal average from the grand average for the period are computed from

$$\begin{aligned} D_{wF}(i) &= \bar{w}_F(i) - \bar{w}_F \\ D_{hF}(i) &= \bar{h}_F(i) - \bar{h}_F \\ D_{pF}(i) &= \bar{p}_F(i) - \bar{p}_F \end{aligned} \quad (75)$$

- (5) The extent to which the overall weather of each season in the year i deviates from normal conditions is expressed by the numbers $d_F(i)$, $d_S(i)$, $d_H(i)$, and $d_w(i)$ defined by

$$\begin{aligned} d_F(i) &= D_{wF}(i) + \alpha D_{hF}(i) + \beta D_{pF}(i) + \gamma D_{wF}(i) + \delta D_{hF}(i) \\ &\quad + \epsilon D_{pF}(i) \end{aligned} \quad (76)$$

where α , β , γ , δ , ϵ are normalizing factors such that

$$\bar{w}_F = \alpha \bar{h}_F = \beta \bar{p}_F = \gamma \bar{w}_F = \delta \bar{h}_F = \epsilon \bar{p}_F \quad (77)$$

These normalizing factors imply that wind velocity, relative humidity, precipitation, and their variabilities are given equal importance in making up the overall weather.

The four seasons having minimum $d(i)$ but not necessarily belonging to the same calendar year, will be selected and the actual weather patterns in these seasons will be used to calculate fire spread.

USE OF WEATHER DATA

When a specific date in the past or the current date is specified as attack date, it is simply required to evaluate the fire effect due to the particular weather pattern following the

specified date. With an unspecified attack date, all days of a future period of many years are equally likely to be attack days, and the problem is not to find the fire effect for one particular attack day, but to find the mean fire effect for all possible attack days. By finding a past year that best represents the normal weather of the area, we have avoided the immense problem of calculating the fire effect for each and every day of the entire period and have equivalently changed the unspecified attack date problem into one which specifies the year and season of attack. It is now required to evaluate a mean fire effect, given that the attack occurs on each day of that year with equal probability. This can be accomplished by two simple methods, one for urban fuel and the other for rural fuel.

1. Since the spread/no-spread capability of urban fires is essentially weather-independent, the day of attack and its weather affect the fire spread only to the extent that the rate of spread depends on wind velocity and possibly on precipitation. Therefore the attack can occur on any day, and the mean fire effect is obtained by a single fire spread calculation, provided average wind velocity and average precipitation are used to determine the rate of spread.
2. For rural fuel, the weather of the attack day determines whether the fire will spread or not and how long it will spread. Thus the ultimate result depends strongly on the attack day. However the fire spread calculation can also be independent of the attack days if a mean spread duration is calculated. The mean spread duration is the mean number of consecutive days of spread condition. For the same weather pattern, this mean duration varies with each type of rural fuel (brush, grass, timber). The general procedure for rural fuel is to allow the fire to spread for a number of days equal to the mean spread duration at a rate determined by the average wind velocity and average relative humidity.

For both urban and rural fires, a calculation is made for each of the four seasons. The results may or may not be combined, depending on each particular case.

PROBLEM OF WIND DIRECTION

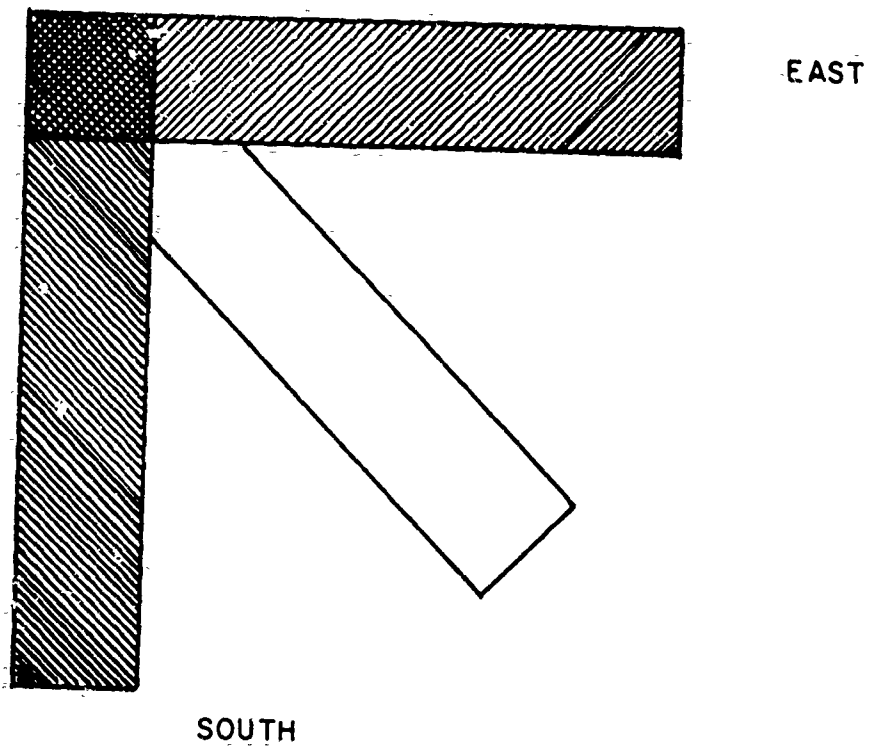
In a given area and during a given season, the wind is likely to blow in several different directions. Averaging wind directions may in some cases lead to meaningless or erroneous results. Suppose the wind is either eastward or southward, and each direction is generally stable for several days. Then the area extending eastward or the one extending southward from the initial fire will burn, depending on the attack day. Averaging the two wind directions would lead to the result that the area extending southeastward burns, which it never does (see Fig. 15A).

Suppose however that the wind in this case changes direction frequently, say, every few hours. The spread pattern can be pictured as in Fig. 15B. It is obvious that the use of an average wind direction would lead to approximately the same result.

These considerations suggest some practical solutions to the problem of wind direction. In studying weather data, some attention must be given to wind directions, their distribution, and the frequency of their changes. If the changes in direction are small (less than 90 deg) or if the wind shifts frequently between two widely divergent directions, the average direction is recorded and used in the calculation.

If the wind changes periodically (every week or more) between two or more predominant directions, the total time spent in each

A



B

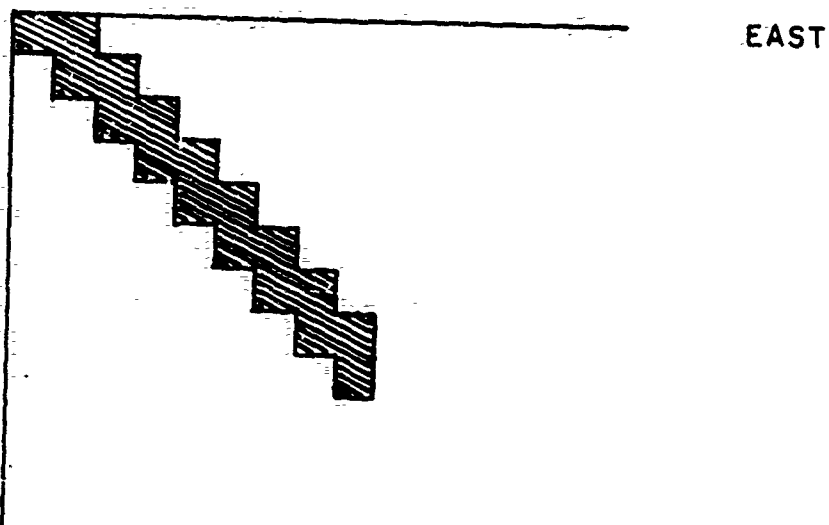


Fig. 15. Effect of Wind Direction on Location of Burnout Area

direction must be recorded. This time is used to evaluate the probability that for random attack the fire will spread in each direction. For the case of Fig. 15A, if the wind blows eastward for a total of 60 days and southward for a total of 30 days, the final result is that the eastward area will burn with a probability 0.66, and the southward area will burn with a probability 0.33.

The same approach can be extended to the case in which the wind changes between two opposite directions. However if these changes are frequent, two different situations must be considered.

- (a) The wind blows in each opposite direction for an average time T_1 less than the burning time of the fuel. The fire will spread in both directions with an overall rate approximately equal to half the rate which corresponds to the actual wind velocity.
- (b) The wind blows in each direction for an average time T_2 greater than the burning time of the fuel. In these cases there will be no significant spread in either direction.

In both (a) and (b), side spread, i.e., at right angle to the wind directions, must be examined. The extent of this spread depends on wind velocity and on urban and rural fuel types.

Section 14

LOCAL APPLICATION OF FIRE SPREAD MODELS

Theoretical models have been formulated in a general manner, largely independent of dimension scale. The size of the area which must be surveyed for fuel characteristics and the effort required for data processing and model computation did not have to be considered.

Methods for large-scale prediction have been obtained by implementing these theoretical models with input data at a level of detail determined by the available resources for the three tasks, data collection, data processing, and model computation. The level of detail in the output information is determined by the level of detail in the input data. The characteristics of the output information for large-scale application have been discussed at some length under output requirements (Section 3).

Application of fire spread models at the local scale involves a higher level of detail in the input data rather than a change in the basic models. Following are general procedures for data collection and model application for cases in which the fuel area is a large or moderate-size city.

DATA COLLECTION

Cities consist mainly of urban fuel. We do not however exclude the possibility of isolated patches of rural fuel large enough to be treated as such. The only required data for these

patches is fuel type — brush, grass, timber, and hardwood. Major fuel gaps are unlikely in an area of this size and the effect of fuel density could not be determined from observed data currently available.

For urban fuel, the following data will be needed: type, fuel density, mean roof area, or spacing distribution.

Urban fuel types of importance at this scale are light residential, heavy residential, commercial, city center, and industrial. Quantitative definitions of the five types must be given before an actual survey can be carried out successfully. What is city center fuel in one city may be the same as commercial fuel in another city. Each fuel type is characterized by a number of factors important in fire spread but which cannot be investigated separately. These factors are: structural materials, fuel loading or amount of fuel in each unit, distribution of number of stories, types of outside walls, and percentage of window area.

Other factors such as mean roof area, fuel density, spacing distribution may also be related to fuel type. They may however vary considerably within each type. This possibility must be examined by selecting a sample of each type and measuring these factors. If variations within each type are sufficiently small, the results of these measurements are permanent data and can be used for any area with known fuel type.

Fuel density is the ratio of total plan area to the total area, which includes normally occurring empty spaces, such as

streets, alleys, lawns, yards, and parking areas but not particularly large open spaces that are specified separately.

The mean roof area can be determined from fuel density if the total number of structures is also counted. It is needed for calculating spread/no-spread requirements in urban fuel. (See Section 4.)

Spacing distribution refers to adjacent structures only. It is expressed as the probability that an adjacent structure is in each range of distance from any structure in the area. A structure is considered as adjacent to another if at least half of it can be seen from the second structure.

The first step in the survey work is to identify sections that can be labeled as one of the five fuel types or as empty space. Large empty spaces that might occur in each section must also be specified.

In the second step, fuel density, mean roof area, and spacing distribution are determined for each section. Convenient methods for these measurements are still to be worked out. Sanborn maps or aerial photos are likely to be needed.

Finally the entire area is divided into grid squares 100 to 300 ft on each side, depending on the particular case. These squares are called cells and will be used to map the course of the fire in time and space.

MODEL APPLICATION

For rural sections, the capability of spread depends on previous and current weather and must be predicted by referring to tables of spread/no-spread weather conditions for each type of fuel. Procedures for following the fire from cell to cell are the same as those given previously for large-scale applications. However, the cell size and time increment can be reduced by factors appropriate to the extent of fuel area and to the level of detail in the input data.

For urban sections, spread capability is predicted by using calculation procedures described previously in connection with model IIC. The course of the fire can be established in the same manner as for rural sections.

Section 15

CONCLUSIONS AND SUGGESTIONS

A semiempirical method for fire spread prediction is now available. With the currently existing data the method gives an approximate prediction of fire spread in rural and urban fuels. The accuracy of prediction, however, can be significantly improved by applying the method according to a special scheme and by acquiring more empirical data, as we shall now suggest.

SPECIAL SCHEME FOR MODEL APPLICATION

The process of collecting fuel, topography, and weather data generally constitutes a major task. It has been suggested that fuel data be determined permanently for each specific type of fuel, for example, light residential or heavy residential, by surveying some area samples containing only one type. Data specification for each fire is done simply by giving the proportion of each type in the total area and certain fuel characteristics that cannot be regarded as inherent in each type.

In general the level of detail of the output information follows that of the input data, which — in turn — depends on the size of the total area. Therefore the area surveyed should not exceed greatly the extent of expected fire spread. It seems good economy to carry out data collection and fire spread calculation simultaneously.

For rural areas, a detailed survey is first made for a distance (extending from the initial fire) that can be expected to be covered by fire before a weather change. Fire spread calculation is made for the section thus surveyed. A second section is surveyed only if fire is expected to reach it during the next weather period. Similarly, for urban areas, the section surveyed each time is determined by the answers to the questions: Can fire spread through it? What distances can the fire travel at the speed predicted by the current weather? By means of this scheme, it is possible to keep the surveyed area at a minimum.

ACQUISITION OF MORE EMPIRICAL DATA

A semiempirical model is as good as its parametric data. In Part II of this report, existing data on rate of spread in rural and urban fuels have been analyzed, and some additional data on weather conditions for spread or no-spread, for extinction in rural fuel, and on spread probability in urban fuel have been estimated. A quick look at the contents of Part II would reveal the need for more empirical data not only for better implementation of this method but for general understanding of fire spread also. The more important ones are briefly indicated below:

- Additional Investigation on Rate of Spread Data in Rural Fuel. A fair amount of data on rate of spread has been obtained. For ground spread the dependence of rate on wind velocity and direction and on relative humidity has been established in some detail. The effect of long-range spotting, however, has not been evaluated. This would require further investigation of the present data. This investigation must probably be backed up by a small amount of new data.

- Additional Data on Rate of Spread in Urban Fuel. Effects of wind and humidity (exterior) on rate of spread cannot be reliably evaluated from present data.
- Burning Time of Rural and Urban Fuels. Some estimates for rural fuel have been given in Part II. More detailed information from direct measurements on both rural and urban fires is needed.
- Spread Probability in Urban Fuel. This is the probability that for a given weather condition and a given type of urban area, fire will spread from one structure to another at a known distance. It is required in Models II and IIA for derivation of the parameter Λ/μ_s , in Model IIC for derivation of spread/no-spread conditions, and in methods for calculating fire spread vulnerability. This probability can be obtained in two ways:
 1. The probability of no-spread can be studied by examining maps of burnout areas in World War II incendiary-attacked cities or in any large-scale urban fire. For a given type of fuel area, the pairs of burned and unburned structures (usually near the burnout contour) with a given spacing are counted and tabulated. It can be shown that the distribution of spacings (a characteristic of the area, not of the fire) must be combined with these data to give the probability of no-spread.
 2. Alternatively and perhaps more basically, the maximum spacing required for ignition to occur is determined for each narrow range of responsible factors. These factors are: type of structural materials, fire load, and size. Data on maximum spacings can be used as such in certain applications, for example, in determining the fire spread vulnerability of resources. They can also be combined with the distributions of type, fire load, and size in a stochastic fuel area to obtain the probability of spread as a function of distance. Data on maximum spacings can be obtained from maps of burnout areas if documentation is sufficiently detailed or from careful observation of real or experimental fires.

Section 16

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Appendix A

FIRE FRONT MODELS

BASIC FIRE FRONT MODEL (MODEL I)

The mechanisms of the model and the definition of terms has been given in Section 6, Part I.

A simple argument on the relation between adjacent probability states leads to the equation

$$P_n(t + dt) = P_{n-1}(t) \lambda dt + P_n(t) [1 - (\lambda + \mu_f) dt] \quad (78)$$

which reduced to

$$\frac{\partial P_n(t)}{\partial t} = \lambda P_{n-1}(t) - \theta P_n(t) \quad \theta = \lambda + \mu_f \quad (79)$$

For $n = 0$, this equation becomes

$$\frac{\partial P_0(t)}{\partial t} = -\theta P_0(t) \quad (80)$$

with solution

$$P_0(t) = e^{-\theta t} \quad (81)$$

For $n \neq 0$, we use the Laplace transform $p_n(s)$ for $P_n(t)$ and write

$$s p_n(s) - P_n(0) = \lambda p_{n-1}(s) - \theta p_n(s) \quad (82)$$

Since for $n \neq 0$, $P_n(0) = 0$, we get the solution

$$p_n(s) = \frac{\lambda^n}{(s + \theta)^{n+1}} \quad (83)$$

or

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\theta t}$$

The expression for $Q_n(t)$ can be easily obtained either by direct integration of $P_n(t)$ or through Laplace transformation.

The probability that at time t the fire front is still alive in some cell is given by

$$P_0(t) + P_1(t) + P_2(t) + \dots \text{ad inf.}$$

or

$$e^{-\theta t} \left[1 + \lambda t + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^3}{3!} + \dots \right] \quad (84)$$

Since this probability is equal to $e^{-\mu_f t}$, the mean lifetime τ of the fire is given by

$$\tau = \int_0^{\infty} t \left(\frac{d e^{-\mu_f t}}{dt} \right) dt = \frac{1}{\mu_f} \quad (85)$$

Thus the mean ultimate burnout distance $\bar{n}_q(\infty)$, found to be λ/μ_f , is the product of the mean velocity and mean lifetime.

APPLICATION TO INHOMOGENEOUS CELLS (MODEL IA)

By Laplace transform, the equation

$$\frac{\partial P_n(t)}{\partial t} = \lambda_{n-1,n} P_{n-1}(t) - (\lambda_{n,n+1} + \mu_n) P_n(t) \quad (86)$$

becomes

$$s p_n(s) = \lambda_{n-1,n} p_{n-1}(s) - (\lambda_{n,n+1} + \mu_n) p_n(s) \quad (87)$$

or

$$p_n(s) = \frac{\lambda_{n-1,n}}{s + \lambda_{n,n+1} + \mu_n} p_{n-1}(s) \quad (88)$$

whose solution is

$$p_n(s) = p_0(s) \prod_{i=1}^n \frac{\lambda_{i-1,i}}{s + \lambda_{i,i+1} + \mu_i} \quad (89)$$

and since

$$p_0(s) = \frac{1}{s + \lambda_{01} + \mu_0} \quad (90)$$

we have

$$p_n(s) = \frac{1}{s + \lambda_{01} + \mu_0} \prod_{i=1}^n \frac{\lambda_{i-1,i}}{s + \lambda_{i,i+1} + \mu_i} = \frac{\prod_{i=1}^n \lambda_{i-1,i}}{\prod_{i=0}^n (s + \lambda_{i,i+1} + \mu_i)} \quad (91)$$

The Laplace transform of $Q_n(t)$ is

$$q_n(s) = \frac{\mu_n p_n(s)}{s} \quad (92)$$

Therefore

$$q_n(s) = \frac{\mu_n}{s} \frac{\prod_{i=1}^n \lambda_{i-1,i}}{\prod_{i=0}^n (s + \lambda_{i,i+1} + \mu_i)} \quad (93)$$

Terms in $Q_n(t)$ corresponding to factors $\prod (s + \lambda_{i,i+1} + \mu_i)$ involve e^{-ct} , where c is a constant, and therefore vanish at $t = \infty$.

Retaining only nonvanishing terms, we have

$$Q_n(\infty) = \frac{\mu_n \prod_{i=1}^n \lambda_{i-1,i}}{\prod_{i=0}^n (\lambda_{i,i+1} + \mu_i)} = \frac{\mu_n}{\lambda_{n,n+1} + \mu_n} \prod_{i=0}^{n-1} \frac{\lambda_{i,i+1}}{\lambda_{i,i+1} + \mu_i} \quad (94)$$

APPLICATION TO SPOTTING (MODEL IB)

Starting from the basic equation

$$\begin{aligned} \frac{\partial P_n(t)}{\partial t} = & \lambda_1 P_{n-1}(t) + \lambda_2 P_{n-2}(t) + \dots + \lambda_n P_0(t) \\ & - (\mu_f + \lambda_1 + \lambda_2 + \dots) P_n(t) \end{aligned} \quad (95)$$

we obtain by Laplace transformation

$$\begin{aligned} s p_n(s) - P_n(0) = & \lambda_1 p_{n-1}(s) + \lambda_2 p_{n-2}(s) + \dots + \lambda_n p_0(s) \\ & - \left(\mu_f + \sum_{i=1}^{\infty} \lambda_i \right) p_n(s) \end{aligned} \quad (96)$$

where

$$P_n(0) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

For $n = 0$

$$P_0(t) = e^{-\left(\mu_f + \sum \lambda_i\right)t} \quad \text{and} \quad p_0(s) = \frac{1}{s + \mu_f + \sum \lambda_i} \quad (97)$$

For $n \neq 0$

$$p_n(s) = \frac{\lambda_1 p_{n-1}(s) + \lambda_2 p_{n-2}(s) + \dots + \lambda_n p_0(s)}{s + \mu_f + \sum \lambda_i} \quad (98)$$

or

$$p_{n+1}(s) = \frac{\lambda_1 p_n(s) + \lambda_2 p_{n-1}(s) + \dots + \lambda_{n+1} p_0(s)}{s + \mu_f + \sum \lambda_i}$$

Defining the generating function $\hat{p}(z)$ and $\hat{\lambda}(z)$ as

$$\hat{p}(z) = p_0(s) + p_1(s)z + p_2(s)z^2 + \dots + p_n(s)z^n \quad (99)$$

and

$$\hat{\lambda}(z) = 0 + \lambda_1 z + \lambda_2 z^2 + \dots + \lambda_n z^n \quad (100)$$

Eq. (98) can be written

$$\hat{p}(z) = \frac{1}{s + \mu_f + \sum \lambda_i - \hat{\lambda}(z)} \quad (101)$$

Since

$$Q_n(t) = \mu_f \int_0^t P_n(\theta) d\theta \quad (102)$$

we can write

$$q_n(s) = \frac{\mu_f p_n(s)}{s} \quad (103)$$

and

$$q_n(s) = \frac{\lambda_1 q_{n-1}(s) + \lambda_2 q_{n-2}(s) + \dots + \lambda_n q_0(s)}{s + \mu_f + \sum \lambda_i} \quad (104)$$

$$q_0(s) = \frac{\mu_f}{sS} \quad \text{where} \quad S = s + \mu_f + \sum \lambda_i$$

$$q_1(s) = \frac{\mu_f \lambda_1}{sS^2} \quad (105)$$

$$q_2(s) = \frac{\mu_f \lambda_2}{sS^2} + \frac{\mu_f \lambda_1^2}{sS^3}$$

Factors in S will lead to terms in $Q_n(t)$ involving negative exponentials in t , hence vanishing at $t = \infty$.

$$Q_0(\infty) = \frac{\mu_f}{\mu_f + \sum \lambda_i} \quad (106)$$

$$Q_1(\infty) = \frac{\mu_f \lambda_1}{(\mu_f + \sum \lambda_i)^2} = \frac{\lambda_1 Q_0(\infty)}{\mu_f + \sum \lambda_i} \quad (107)$$

$$Q_2(\infty) = \frac{\mu_f \lambda_2}{(\mu_f + \sum \lambda_i)^2} + \frac{\mu_f \lambda_1^2}{(\mu_f + \sum \lambda_i)^3} = \frac{\lambda_1 Q_1(\infty) + \lambda_2 Q_0(\infty)}{\mu_f + \sum \lambda_i} \quad (108)$$

$$Q_n(\infty) = \frac{\lambda_1 Q_{n-1}(\infty) + \lambda_2 Q_{n-2}(\infty) + \dots + \lambda_n Q_0(\infty)}{\mu_f + \sum \lambda_i} \quad (109)$$

It can be easily seen that the generating function $\hat{Q}(z)$ of $Q_n(\infty)$ is given by

$$\hat{Q}(z) = \frac{\mu_f}{\mu_f + \sum \lambda_i - \hat{\lambda}(z)} \quad (110)$$

although this relation is probably not as useful as the relation for $Q_n(\infty)$.

SPECIAL CALCULATION OF MEAN LIFETIME OF FIRES

To obtain the parameter μ_f , we must know the mean lifetime of the fire, i.e., we must observe the lifetimes or the spread distances and velocities of many actual fires burning under the same or similar fuel and weather conditions. Such information does not exist in records of past fires and is not likely to become available in the future. In one special case, however, the mean lifetime can be calculated directly, as we shall now discuss.

Suppose the weather is steady and corresponds to spread condition for the type of fuel under consideration. The fire spreads until it encounters a fuel break that it cannot jump. A fuel break here is a cell void of fuel or containing protected fuel. We call such a cell empty and any other cell full. The empty cells are randomly distributed. Let e be the fraction of empty cells and $1 - e$ the fraction of full cells or "fuel density" of the area.

There are two ways of specifying the fuel density: (1) the fraction e of empty cells is given for an unlimited distance (or area). A consequence of this specification is that the probability of having a continuous series of full cells is finite even for very long series; (2) the fraction e has been measured over a given finite distance. In this case the largest possible spread distance is the total number of cells minus the number of empty cells. The probability of spreading to a larger distance is zero.

We shall now calculate the mean spread distance in each case and obtain the mean lifetime by dividing the mean distance by λ , the mean velocity.

- (1) With the exception of cell 0, which has been ignited initially, all other cells have the probability e of being empty and $1 - e$ of being full. Thus the probability of spreading to zero cell is e and the probability of spreading to one or more cells is $1 - e$ (here denoted by f). The probability of spreading to one cell only is fe and so on. The mean spread distance D_m in terms of cell width is

$$D_m = \frac{fe + 2fe^2 + 3fe^3 + \dots}{e + fe + fe^2 + \dots} = \frac{f}{1 - f} = \frac{1 - e}{e} \quad (111)$$

- (2) In a sequence of N cells, v cells are empty and are randomly distributed, with $v/N = e$.

The number of different ways a group of ν cells can be selected from N cells is

$$\binom{N}{\nu} = \frac{N!}{\nu! (N - \nu)!} \quad (112)$$

For the fire to spread to cell n and then stop, cells $1, 2, \dots, n$ must be full and cell $n + 1$ must be the first cell to be empty. The remaining $\nu - 1$ empty cells are distributed randomly among a total of $N - n - 1$ cells. Out of $N - n - 1$ cells, a group of $\nu - 1$ cells can be selected in

$$\binom{N - n - 1}{\nu - 1} = \frac{(N - n - 1)!}{(\nu - 1)! (N - n - \nu)!} \quad (113)$$

different ways with the obvious condition $n \leq N - \nu$.

Therefore the probability of spreading to n and only n cells is

$$\frac{(N - n - 1)!}{(\nu - 1)! (N - n - \nu)!} \cdot \frac{\nu! (N - \nu)!}{N!} = \frac{(N - n - 1)! (N - \nu)! \nu}{(N - n - \nu)! N!} \quad (114)$$

This probability can also be expressed as

$$e \frac{F(F - 1)(F - 2) \dots (F - n + 1)}{(N - 1)(N - 2) \dots (N - n)} \quad (115)$$

where F represents $N - \nu$, the total number of full cells.

It is easily seen that for a given ν/N , if N and F tend to infinity, we obtain the same expression as before, i.e., $f^n e$.

The probability distribution for $n = 0, 1, 2, \dots$ can be found from this formula and can be used to calculate the mean spread distance D_m .

Appendix B

FUEL STATE MODELS

BASIC FUEL STATE MODEL (MODEL II)

The mechanism of the model and the definitions of $U_{ij}(t)$, $F_{ij}(t)$, $B_{ij}(t)$ have been given in Section 6.

The probability that an unignited cell ij is ignited by at least one of its eight neighbors during time dt is

$$\Lambda_{ij} dt \approx (\Lambda_1 + \Lambda_2 + \dots + \Lambda_8) dt \quad (116)$$

The three equations of the model are

$$\frac{\partial U_{ij}(t)}{\partial t} = - U_{ij}(t) \sum_{u=1}^8 F_u(t) \Lambda_u \quad (117)$$

$$\frac{\partial F_{ij}(t)}{\partial t} = U_{ij}(t) \sum_{u=1}^8 F_u(t) \Lambda_u - \mu_s F_{ij}(t) \quad (118)$$

$$\frac{\partial B_{ij}(t)}{\partial t} = \mu_s F_{ij}(t) \quad (119)$$

where ij is any cell outside the initial fire and the u 's indicate the eight cells surrounding ij .

An expression for $B_{ij}(\infty)$ can be obtained as follows:

Define the quantities $F_{ij}^*(t)$ and $B_{ij}^*(t)$ as

$$F_{ij}^*(t) = \sum F_u(t) \Lambda_u \quad (120)$$

$$B_{ij}^*(t) = \sum B_u(t) \Lambda_u \quad (121)$$

Equation (119) can be written

$$\frac{\partial B_u(t)}{\partial t} = \mu_s F_u(t) \quad (122)$$

Multiplying both sides by Λ_u and summing we get

$$\frac{\partial B_{ij}^*(t)}{\partial t} = \mu_s F_{ij}^*(t) \quad (123)$$

and division of Eq. (117) by Eq. (123) gives

$$\frac{\partial U_{ij}(t)}{\partial B_{ij}^*(t)} = -\frac{1}{\mu_s} U_{ij}(t) \quad (124)$$

which integrates to

$$\ln U_{ij}(t) \Big|_0^t = -\frac{1}{\mu_s} B_{ij}^*(t) \Big|_0^t \quad (125)$$

$B_{ij}^*(0)$ is obviously zero and since cell ij is not part of the initial fire, $U_{ij}(0)$ is 1. Therefore

$$U_{ij}(t) = e^{-\frac{1}{\mu_s} B_{ij}^*(t)} \quad (126)$$

By use of Eq. (119) and the relation

$$U_{ij}(t) + F_{ij}(t) + B_{ij}(t) = 1 \quad (127)$$

$U_{ij}(t)$ can be expressed in terms of $B_{ij}(t)$, and Eq. (126) becomes

$$\frac{1}{\mu_s} \frac{\partial B_{ij}(t)}{\partial t} = 1 - B_{ij}(t) - e^{-\frac{1}{\mu_s} B_{ij}^*(t)} \quad (128)$$

As t tends to ∞ , $B_{ij}(t)$ and $B_{ij}^*(t)$ approach their limits which are equal to or less than 1 and $\partial B_{ij}(t)/\partial t$ approaches zero; therefore

$$B_{ij}(\infty) = 1 - e^{-\frac{1}{\mu_s} B_{ij}^*(\infty)}$$

or

$$B_{ij}(\infty) = 1 - e^{-\frac{1}{\mu_s} \sum B_u(\infty) \Lambda_u} \quad (129)$$

LINEAR STOCHASTIC VERSION (MODEL IIA)

Solution for $B_n(\infty)$

The assumed mechanism can be described by the equations

$$\frac{\partial U_n(t)}{\partial t} = -U_n(t) F_{n-1} \Lambda \quad (130)$$

$$\frac{\partial F_n(t)}{\partial t} = U_n(t) F_{n-1} \Lambda - \mu_s F_n(t) \quad (131)$$

$$\frac{\partial B_n(t)}{\partial t} = \mu_s F_n(t) \quad (132)$$

with

$$F_n(0) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

By changing n to $n - 1$ in Eq. (132) and multiplying by Λ we have

$$\frac{\partial B_{n-1}(t) \Lambda}{\partial t} = \mu_s \Lambda F_{n-1}(t) \quad (133)$$

and dividing Eq. (130) by Eq. (133),

$$\frac{\partial U_n(t)}{\partial B_{n-1}(t) \Lambda} = -\frac{1}{\mu_s} U_n(t) \quad (134)$$

which gives

$$U_n(t) = e^{-a B_{n-1}(t)} \quad a = \frac{\Lambda}{\mu} \quad (135)$$

Upon substitution for $U_n(t)$ we have from 127 and 132

$$\frac{1}{\mu_s} \frac{\partial B_n(t)}{\partial t} = 1 - B_n(t) - e^{-a B_{n-1}(t)} \quad (136)$$

and for $t = \infty$

$$B_n(\infty) = 1 - e^{-a B_{n-1}(\infty)} \quad (137)$$

Solution for $F_1(t)$ and $B_1(t)$

Explicit expressions for $F_1(t)$ and $B_1(t)$ can be obtained as follows:

According to Eq. (130),

$$\frac{\partial U_1(t)}{U_1(t)} = -\Lambda e^{-\mu_s t} dt \quad (138)$$

which gives by integration

$$U_1(t) = e^{-a} \cdot e^{ae^{-\mu_s t}} \quad (139)$$

Substituting for $U_1(t)$ in Eq. (131) we get successively

$$\begin{aligned} \frac{\partial F_1(t)}{\partial t} &= \Lambda e^{-a} e^{ae^{-\mu_s t}} e^{-\mu_s t} - \mu_s F_1(t) \\ F_1(t) &= \Lambda e^{-a} e^{-\mu_s t} \int_0^t e^{ae^{-\mu_s t}} dt + k e^{-\mu_s t} \end{aligned} \quad (140)$$

where k is a constant of integration. Letting

$$x = e^{-\mu_s t} \quad \text{and} \quad dx = -\mu_s x dt \quad (141)$$

and integrating, we obtain

$$\begin{aligned} F_1(t) &= a e^{-a} e^{-\mu_s t} \left[\mu_s t + a \left(1 - e^{-\mu_s t} \right) + \frac{a^2}{2 \cdot 2!} \left(1 - e^{-2\mu_s t} \right) \right. \\ &\quad \left. + \frac{a^3}{3 \cdot 3!} \left(1 - e^{-3\mu_s t} \right) + \dots \right] \end{aligned} \quad (142)$$

which can be integrated term by term to give $B_1(t)$ according to Eq. (132).

Mean Lifetime of Individual Cell

Consider the probability $F_n(t)$ that cell n is burning at time t . At time $t + dt$, this probability is given by

$$F_n(t + dt) = F_n(t) (1 - \mu_s dt) \quad (143)$$

or

$$dF_n(t) = -\mu_s F_n(t) dt$$

If n is 0, i.e., the initially ignited cell, $F_0(0) = 1$ and

$$F_0(t) = e^{-\mu_s t} \quad (144)$$

For any other cell n , let θ denote the time after ignition, then

$$F_n(\theta) = e^{-\mu_s \theta} \quad (145)$$

The probability that cell n goes out between θ and $\theta + d\theta$ is $F_n(\theta)\mu_s d\theta$. The mean lifetime τ is therefore given by

$$\tau = \int_0^\infty \theta e^{-\mu_s \theta} \mu_s d\theta = \frac{1}{\mu_s} \quad (146)$$

Mean Rate of Spread

Once ignited, a cell has, for each time increment dt , a probability Λdt of igniting the next cell and a probability $\mu_s dt$ of dying. Therefore the time for jumping from one cell to the next is a stochastic variable. It is required to calculate the mean jump time given that the fire has made a large number of jumps.

Let $P(t)$ be the probability that the jump takes place before time t , then

$$P(t + dt) = P(t) + [1 - P(t)] \Lambda dt \quad (147)$$

The term $\mu_s dt$ does not appear in this equation because it is known that none of the cells involved died before the jump. The solution is

$$P(t) = 1 - e^{-\Lambda t} \quad (148)$$

The probability that the jump occurs between t and $t + dt$ is

$$dP(t) = \Lambda e^{-\Lambda t} dt \quad (149)$$

Hence the mean jump time t_j is

$$t_j = \int_0^{\infty} t \Lambda e^{-\Lambda t} dt = \frac{1}{\Lambda} \quad (150)$$

and the mean rate of spread \bar{R} is given by

$$\bar{R} = \frac{1}{t_j} = \Lambda \quad (151)$$

Final Mean Spread Distance

The recursive formula for burnout probabilities is

$$B_{n+1} = 1 - e^{-a B_n} \quad a = \frac{\Lambda}{\mu_s} \quad (152)$$

For sufficiently large a , B_n approaches a limit at which $B_n = B_{n+1} = B_{n+2} = \dots$. The limit is the solution of the equation

$$x = 1 - e^{-ax}$$

or

$$\frac{z}{a} = 1 - e^{-z} \quad (153)$$

where

$$z = ax$$

The solution $z = 0$ or $x = 0$ always exists. In addition, a finite solution exists if

$$\frac{1}{a} \leq 1 \quad \text{or} \quad a \leq 1$$

It is evident that when the limit of B_n is finite the sum $\sum_{n=1}^{\infty} B_n$ is infinite. We now show that when the limit of B_n is 0 or when $a < 1$, the sum is finite. Equation (152) above can be written as

$$B_{n+1} = a B_n - \frac{a^2 B_n^2}{2!} + \frac{a^3 B_n^3}{3!} - \dots \quad (154)$$

The ratio B_{n+1}/B_n is

$$\frac{B_{n+1}}{B_n} = a - \frac{a^2 B_n}{2!} + \frac{a^3 B_n^2}{3!} - \dots \quad (155)$$

Since B_n tends to zero as n increases, the ratio tends to a . Therefore if a is less than 1, the ratio is also less than 1 and by the ratio test the series B_n converges.

Burning Time and Ignition Probability

We have been dealing with the probability Λdt that ignition occurs in stochastic fuel during the time increment dt given that the ignition source is still burning. The parameter Λ is also called probability of ignition per unit time. Evidently the longer the fuel burns, the greater the ultimate probability of ignition. The relation between the stochastic burning time and ignition probability is derived as follows:

Let $P(t)$ be the probability that ignition has occurred before time t . The probability that ignition occurs between t and $t + dt$ is

$$dP(t) = e^{-\mu_s t} [1 - P(t)] \Lambda dt \quad (156)$$

where the factor $e^{-\mu_s t}$ is the probability that the ignition source is still burning at time t .

Integration from 0 to t gives

$$P(t) = 1 - e^{-\left(\Lambda/\mu_s\right) \left(1 - e^{-\mu_s t}\right)} \quad (157)$$

Since stochastic burning time may extend to some high value, the probability that ignition ultimately occurs is

$$P(\infty) = 1 - e^{-\left(\Lambda/\mu_s\right)} = 1 - e^{-a} \quad (158)$$

It is this probability that we usually derive from data on past fires.

LINEAR DETERMINISTIC VERSION (MODEL IIB)

The basic equation representing the simplified mechanism of the model is

$$\frac{dX}{dt} = \begin{cases} kW(t) & \text{for } W(t) < W_0 \\ kW_0 & \text{for } W(t) \geq W_0 \end{cases} \quad (159)$$

where X , $W(t)$, W_0 have been defined in Section 6.

The width of the burning fire is the distance traveled from the time a fuel element is ignited to the time that same element goes out. Therefore

$$W(t) = \int_{t-\tau}^t \left(\frac{dX}{d\theta} \right) d\theta = k \int_{t-\tau}^t W(\theta) d\theta \quad (160)$$

or

$$\frac{dW(t)}{dt} = k [W(t) - W(t - \tau)] \quad (161)$$

It is convenient to assume that $W(0)$ has been chosen to be equal to W_0 .

The initial fire (assumed to be ignited simultaneously throughout the region $W(0)$) will keep burning until $t = \tau$, the burning time of fuel. At that time it suddenly goes out. During the period $0 - \tau$, the width grows at the constant rate $W_0 k$, so that at time $t = \tau$, after the initial fire goes out, the fire width becomes $W_0 k \tau$.

If $W_0 k \tau$ is equal to or greater than W_0 (i.e., $k \tau \geq 1$), the velocity remains constant at $W_0 k$ and from Eq. (160) the front width remains constant at $W_0 k \tau$.

For the case $W_0 k \tau < W_0$, the value of $W(t)$ can be best calculated numerically for each successive time increment Δt .

The final burnout distance is readily derived in the following manner.

Divide τ into ν equal intervals of length $d = \tau/\nu$. Let X_i be the distance covered during the i^{th} time interval. From the above discussion

$$X_1 = X_2 = \dots = X_\nu = X_0$$

where X_0 is $W_0 k \tau / \nu$. $X_{\nu+1}$, $X_{\nu+2}$, . . . can be expressed according to Eq. (161). Thus

$$X_1 = X_0$$

$$X_2 = X_0$$

$$\dots$$

$$X_\nu = X_0$$

(162)

$$X_{\nu+1} = dk (X_\nu + X_{\nu-1} + \dots + X_1)$$

$$X_{\nu+2} = dk (X_{\nu+1} + X_\nu + \dots + X_2)$$

$$\dots$$

$$X_i = dk (X_{i-1} + X_{i-2} + \dots + X_{i-\nu})$$

Summing each column we get

$$\sum_1^\infty X_i = \nu X_0 + dk \left[\sum_\nu^\infty X_i + \sum_{\nu-1}^\infty X_i + \dots + \sum_1^\infty X_i \right] \quad (163)$$

To the right hand side we add and subtract the quantity

$$dk \left[\sum_1^{\nu-1} X_i + \sum_1^{\nu-2} X_i + \dots + \sum_1^1 X_i \right] \quad (164)$$

which is equal to

$$dk X_0 \frac{\nu(\nu-1)}{2}$$

Equation (163) now becomes

$$\sum_1^\infty X_i = \nu X_0 - dk X_0 \frac{\nu(\nu-1)}{2} + dk \nu \sum_1^\infty X_i \quad (165)$$

Denoting the final distance $\sum_1^\infty X_i$ by D , we find upon substitution of τ/ν for d and $W_0 k\tau/\nu$ for X_0

$$D = \frac{W_0 k\tau}{1 - k\tau} \left[1 - \frac{k\tau}{2} \frac{\nu-1}{\nu} \right] \quad (166)$$

As ν tends to infinity, i.e., the time interval d becomes infinitesimal, the last factor in the bracket becomes 1, and the final expression for D is

$$D = \frac{W_o k\tau}{1 - k\tau} \left(1 - \frac{k\tau}{2} \right) \quad (167)$$

Summary Report
of
PREDICTION MODELS FOR FIRE SPREAD
FOLLOWING NUCLEAR ATTACKS

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FOREWORD

This document summarizes the information contained in URS 641-6, Prediction Models for Fire Spread Following Nuclear Attack, January 1965, which is bound separately.

Summary Report
of
PREDICTION MODELS FOR FIRE SPREAD
FOLLOWING NUCLEAR ATTACKS

OBJECTIVE

The main objective of the study is to develop an improved method for predicting the extent of fire spread beyond the area directly ignited by nuclear explosion and to specify the data required for use with the method and ways of acquiring these data. Since the ultimate purpose is assessment of fire damage to resources, the method also includes general procedures for deriving damage information from fire spread information.

The study has been aimed at a method that is both applicable with the currently existing data and capable of subsequent improvement as more and better data become available.

OUTPUT REQUIREMENTS

Estimates of fire damage to resources are needed primarily for preattack planning and possibly for early postattack applications and can be performed at three geographical levels: national, regional, and local.

Each type of application calls for a different level of detail in the output information. National application seems to require only gross information, for example, the average loss

from a large number of weapons. For regional use, a somewhat greater level of detail is needed. Local use would require the highest level of detail.

DESCRIPTION OF THE PROBLEM

Mass fires are characterized by rate of spread, heat and flame intensity, size of fire burning at a given time, final burnout area, and percentage of burned fuel. Due to the intimate relations between these characteristics, the final burnout area (required for estimating fire damage) cannot be easily singled out as a subject of investigation. Thus the prediction method is likely to involve a representation of the fire in time and in space and require data on rate of spread, burning time of fuel, igniting capability, and ignition susceptibility.

Factors that can influence fires are called fire spread variables and are grouped into three general classes: Fuel variables, fire spread variables, and topography variables.

Fuel variables include fuel types, fuel build-up or thickness of the fuel layer, fuel density or fraction of total area occupied by the fuel, spatial distribution of fuel, fuel age, and fuel fineness.

Weather variables are moisture content of fuel, wind velocity and direction, and temperature.

The topography group of variables consists of slope, relief, and possibly altitude.

The effect of a variable may be real but indiscernible among other more important ones, or it may remain sensibly constant over the usual range of fire spread conditions.

METHODS OF APPROACH

A purely theoretical approach, relying extensively on physical laws, is considered to be impractical in the present stage of knowledge. A purely empirical method could be used to predict fire spread by locating, in the body of data on past fires, a set of conditions which matches the future situation and by using the past to predict the future. However, without some theoretical back-up, it would be difficult to untangle the numerous factors and efficiently organize the empirical data. Thus a semi-empirical approach seems to be most appropriate. In this approach a mathematical model describes various characteristics of fire spread in terms of some parameters to be evaluated from empirical data.

Whether the model (and its output information) is deterministic or stochastic depends largely on how fuel, weather, and topography conditions are specified. Although for most cases they are all specified in a stochastic manner, simplicity is gained by treating weather conditions as deterministic.

MATHEMATICAL MODELS

Two families of mathematical models have been developed: fire-front and fuel-state. In each a number of specific versions

have been derived from the parent model. Specific versions serve a variety of purposes: illustrating particular features of the parent model, deriving some general properties which could not be conveniently studied with the parent model, providing more convenience in specific applications. Table 1 lists various models with their main characteristics and applications.

Table 1
SYNOPSIS OF MODELS

<u>Model</u>	<u>Characteristics</u>	<u>Appropriate Fuel</u>
<u>FIRE FRONT MODELS</u>		
I	Stochastic. One-dimensional. Parameters are λ and μ_f ; the latter not available; required data difficult to obtain.	Preferably urban
IA	Same as I but with more detailed input information on fuel characteristics.	Preferably small or moderate urban
IB	Same as I but with provision for long-range spotting possibility.	As above
<u>FUEL STATE MODELS</u>		
II	Stochastic. Two-dimensional. Parameters are Λ and μ_s . Data for evaluating μ_s not available.	General
IIA	Same as II but one-dimensional.	General, preferably urban
IIB	Same as II but deterministic and one-dimensional. Existing data for evaluating parameters τ , k , W_0 are incomplete.	General, preferably rural
IIC	Same as II but deterministic. Application uses existing data supplemented by some subjective estimates.	Rural and urban

Fire Front Models

The entire fire area or fire front is pictured as a random walker moving along a strip of fuel area divided into small square sections called cells. At any moment, one of three possible events may take place: either the fire front dies (is burned out) or it moves one cell forward or it stays where it is. The occurrence of these events is governed by their relative probabilities, which are determined by fire spread variables, fuel density, distribution and type, wind velocity, relative humidity, . . . combined into two main parameters λ and μ_f . These two parameters are defined as the probability rates (or probability per unit time) of spreading and of dying, so that within the time increment dt , the probability of spreading to the next cell is λdt and that of stopping permanently is $\mu_f dt$. It can be shown that λ is numerically equal to the mean rate of spread in cells per unit time and $1/\mu_f$ is equal to the mean lifetime of the fire as a whole. These interpretations provide convenient methods for deriving λ and μ_f from observed data, provided such data exist. Mathematical relations based on this mechanism give the probability of finding the fire still burning and that of finding the fire burned out at a given location and a given time. In addition, the mean rate of spread in terms of cells per hour is readily calculated.

With slight modifications, the mechanism can be applied to a strip of fuel area in which the cells differ significantly with respect to fuel characteristics or to cases in which, by the process of long-range spotting, the fire can jump over one or more cells.

Fuel State Models

These models stochastically or deterministically describe the events in each cell of the fuel area. At any time, a given cell is in one of three states: the unignited state, including the early phase in which the cell, though ignited, is still incapable of igniting other cells; the flaming state, in which the burning intensity is sufficient to ignite other cells; and the burnout state. Whether and when a cell passes from one state to the next depends on the particular set of fuel, weather, and topography variables. The mathematical formulation however does not involve these variables directly. All transitions and derived quantities are expressed in terms of two parameters, λ and μ_s , which are defined in a manner similar to λ and μ_f described above. They can be analytically related to the burning time of the cell (not of the entire fire) and to the mean rate of spread in cell widths per unit time.

The same mechanism gives rise to four models, each requiring a slightly different method of implementation and serving a specific purpose.

In the linear version, an unignited cell may pass to the burning state with probability λdt per time increment dt , provided the preceding cell is in F state. The two-dimensional version computes this transition probability by taking into account the possibility of ignition from more than one neighboring cell. Either version can handle the situation in which the fire can jump over one or more cells, provided λ values can be determined according to jump distances.

For the two-dimensional version, no analytical solution can be obtained except for the probability of any cell being burned out at time equal to infinity, from which the mean number of burnout cells or the mean spread area is readily computed. The linear version, however, gives several interesting analytical results. The rate of spread is Λ cell widths per unit time (as in the linear model). The mean burning time of cells is $1/\mu_s$ time units. The mean final spread distance is infinite if Λ/μ_s is equal to or greater than 1. For Λ/μ_s less than 1, this distance is finite and can be calculated.

The fuel state model includes a deterministic version which is readily applicable to normally continuous fuel in wildland areas. The rate of spread is assumed to be proportional to the total heat flux (with embers and firebrands) emerging from the burning zone. It is further assumed that the heat supply per unit burning area is independent of distance from the front. A cut-off is made, however, when the width of the fire exceeds a certain value W_0 . There are three parameters: W_0 , the burning time τ of the fuel, and a spread constant k .

As in the stochastic model, the final spread distance is infinite for $k\tau > 1$ and finite for $k\tau < 1$, in which case it is easily calculated. The model also gives the rate of spread and fire width as a function of time.

Finally, a practical method is set up for applying the fuel state model with parametric data that are currently available or can be reasonably estimated. It consists of two steps. First, it predicts whether fire can spread through the area

under the specified conditions. For rural fuels, spread capability is largely determined by weather, since fuel distribution is normally continuous. For urban fuels, the controlling factor is fuel density. Weather conditions for spread in rural fuels and fuel conditions for spread in urban fuel are estimated from some limited data.

Second, if fire spread is predicted, its progress is followed from cell to cell and from one time increment to the next by means of rate-of-spread data until weather conditions or fuel conditions change.

PARAMETERS

Each parameter is a complicated quantity representing the combined effects of several basic variables. Its evaluation, therefore, requires extensive empirical data. The parameter μ_f could be derived from data on the mean lifetime of the fire. Unfortunately no such data can be found. Evaluation of μ_s requires data on burning time of fuel. Some estimates do exist but are not considered sufficiently accurate for the purpose. Λ and λ are directly given by the mean rate of spread. Information on rate of spread exists in fair amounts for rural fuel but is rather limited for urban fuel.

RATE OF SPREAD DATA

Statistical analysis of rate of spread data provided by the Forest Service gives relationships between rate of spread and basic variables.

For rural fires, rate of spread increases rather markedly with wind velocity for forward wind, less markedly for lateral wind, and generally decreases with increasing wind velocity for back wind. For wind velocity from 0 to 10 mph, rate of spread expectedly decreases with increasing relative humidity. However, it becomes independent of relative humidity as wind velocity increases beyond 10 mph. A table in Part II gives rate of spread values for various combinations of wind velocity, wind direction, and relative humidity.

For urban fires, effects of exterior humidity and wind cannot be evaluated quantitatively. Strongest influence seems to come from fuel density and fuel type. An important result, at first startling, is that observed rate of spread increases rapidly as fuel density decreases. The explanation is that for fires which do not go out in low-density fuel, the relative tendency for ground spread decreases much faster than the tendency for spotting.

VARIOUS ESTIMATED DATA

The probability of fire spread from one structure to another in a random fuel area is estimated from very limited data. The approximate curve obtained shows that at distances up to about 20 ft, fire spread is certain in every case, and at 300 ft it hardly ever takes place. Until more detailed information is available -- for example, one curve for each type of fuel area -- this curve is needed for estimating fire spread vulnerabilities of resources and fuel requirements for spread or no-spread.

More or less subjective estimates on weather conditions for no-spread in rural fuel, weather conditions for fire extinction, and burning times of fuel are presented.

FIRE SPREAD VULNERABILITY OF RESOURCES

General procedures are given for evaluating the chance of fire destruction for resources located in areas swept by fire. Distinction is made of three types of environments: urban, rural, and mixed. The procedures also depend on the specified details of the environment.

WEATHER DATA FOR FIRE SPREAD PREDICTION

A method is worked out for obtaining a set of weather conditions that is consistent with the meteorological characteristics of the area under consideration and with the fact that the attack may take place at any time in the future. Depending on the available effort, the weather is predicted on a statistical basis, or an average weather pattern is selected by visual inspection of weather records, or a worst and a best weather patterns are selected to obtain an upper and lower limits of fire damage.

LOCAL APPLICATION OF FIRE SPREAD MODELS

A brief discussion is devoted to the use of the fire spread models for cities of large or moderate size and to the problem of data collection at this geographical scale.

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CONCLUSIONS AND SUGGESTIONS

Major accomplishments of this program are summarized, and suggestions for obtaining certain important empirical data are given.