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CONTENTS

Abstract	ii
Problem Status	ii
Authorization	ii
INTRODUCTION	1
ANTENNA TRANSMISSION PROPERTIES	1
ANTENNA RECEIVING CHARACTERISTICS	2
COUPLED ANTENNAS	2
CIRCULARLY POLARIZED COMPONENTS	4
POLARIZATION EFFICIENCY	6
CONCLUSIONS	9
REFERENCES	9
Appendix - MATRIX TRANSFORMATION	10

ABSTRACT

A solution to the problem of power transfer between two antennas in a communication link can be obtained by analyzing each antenna separately. By using a coordinate system associated with each antenna, an analysis can be performed of its gain and polarization properties along the direction of the link. Based on circularly polarized wave components, the analysis results in relatively simple experimental parameters, and is readily adapted to the establishment of a catalog containing numerous antennas.

PROBLEM STATUS

This is an interim report on one phase of the problem; work on this and other phases is continuing.

AUTHORIZATION

NRL Problem R07-04
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THE POWER TRANSFER BETWEEN ARBITRARILY POLARIZED ANTENNAS FROM EXPERIMENTALLY MEASURABLE PARAMETERS

INTRODUCTION

The power transfer between two antennas in a communication link is a function not only of the antenna gains, but also of polarization and spatial orientation of the two antennas. The details associated with this problem are of particular concern in communicating with satellites or spacecraft and in the more earthly problem of radar tracking a missile-borne transponder. In these situations large and varying axial misalignments between the antennas require repeated computations to determine communication efficiency. Thus the directness of the computations becomes particularly important. The analysis presented lends itself to the establishment of a catalog of parameters on a variety of antennas; later, after the time spatial relationship is known, the computation for power transfer between any pair can be completed.

ANTENNA TRANSMISSION PROPERTIES

The electric field at a point remote from a transmitting antenna may be written in terms of a complex vector, $\mathbf{h}$ (1):

$$\mathbf{E}^t = \frac{Z_0 I_t}{2\lambda s}\,\mathbf{h} \tag{1}$$

where

Z_0 = the free-space impedance,

I_t = the current in the transmitting antenna,

s = distance between the transmitter and the point of observation, and

λ = wavelength.

The vector antenna height $\mathbf{h}$ is defined along a direction to the point of observation and serves to describe the far-field polarization characteristics. The equation neglects a propagation-phase term which need not be considered here.

Choosing the right-hand coordinate system indicated in Fig. 1, the field at P may be written

$$\mathbf{E}^t = \mathbf{a}_x E_x^t + \mathbf{a}_y E_y^t = \left(\frac{Z_0 I_t}{2\lambda s}\right)[\mathbf{a}_x\ \mathbf{a}_y]\begin{bmatrix} h_x \\ h_y \end{bmatrix} \tag{2}$$

where $\mathbf{a}_x$ and $\mathbf{a}_y$ are unit vectors in the directions shown, and h_x and h_y are components of the antenna height along these directions. In general h_x and h_y may be complex, so that their ratio serves to describe the polarization characteristics of the antenna.

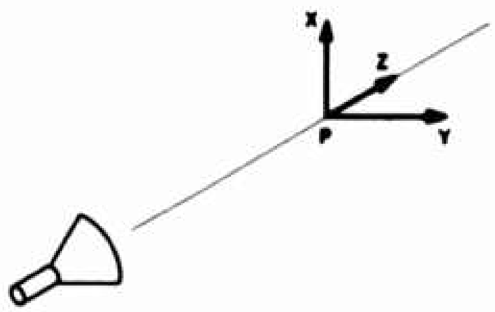

Fig. 1 - Transmitting antenna

ANTENNA RECEIVING CHARACTERISTICS

The induced open-circuit terminal voltage at a receiving antenna (1) is given by:

$$V = \mathbf{h} \cdot \mathbf{E}^r \tag{3}$$

where $\mathbf{E}^r$ is the incident field and $\mathbf{h}$ is the vector height, characteristic of the antenna when used for transmission. In terms of the components a_x, a_y:

$$V = h_x E_x^r + h_y E_y^r , \tag{4a}$$

$$= (h_x h_y) \begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} . \tag{4b}$$

Note that the incident-field amplitudes have been resolved along the component directions a_x, a_y used for transmission.

COUPLED ANTENNAS

Consider two antennas, A and B, facing along a common axis. Figure 2 illustrates the coordinates of each antenna when employed for transmission and hence for definition of $\mathbf{h}$.

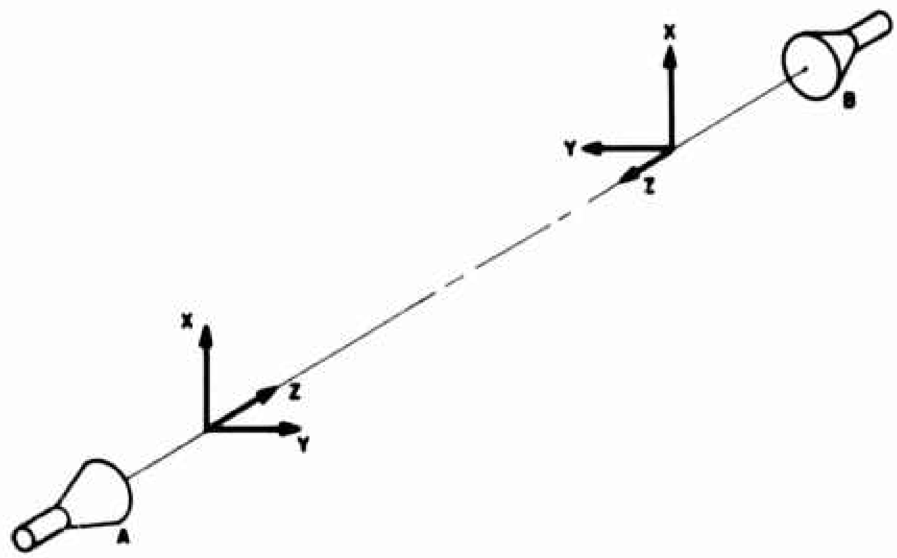

Fig. 2 - Coupled antennas

For each antenna the vector height is determined:

$$\mathbf{h}^A = (\mathbf{a}_x^A \mathbf{a}_y^A) \begin{bmatrix} h_x^A \\ h_y^A \end{bmatrix} \tag{5a}$$

$$\mathbf{h}^B = (\mathbf{a}_x^B \mathbf{a}_y^B) \begin{bmatrix} h_x^B \\ h_y^B \end{bmatrix} . \tag{5b}$$

If antenna A is used for transmission, the transmitted field from Eq. (2) is

$$\mathbf{E}^t = \frac{(Z_0 I_t)}{(2\lambda s)} (\mathbf{a}_x^A \mathbf{a}_y^A) \begin{bmatrix} h_x^A \\ h_y^A \end{bmatrix} . \tag{6}$$

If antenna B is used for reception, its vector height is given by

$$\mathbf{h}^B = (\mathbf{a}_x^B \mathbf{a}_y^B) \begin{bmatrix} h_x^B \\ h_y^B \end{bmatrix} . \tag{7}$$

To evaluate the terminal voltage at B it is necessary to resolve the field, described in terms of $\mathbf{a}_x^A$, $\mathbf{a}_y^A$, along the directions of $\mathbf{a}_x^B$, $\mathbf{a}_y^B$. Since the field transmitted by antenna A is received by antenna B,

$$\mathbf{E} = \mathbf{a}_x^A E_x^t + \mathbf{a}_y^A E_y^t = \mathbf{a}_x^B E_x^r + \mathbf{a}_y^B E_y^r . \tag{8}$$

From Fig. 2, the following relationships hold:

$$\begin{aligned} \mathbf{a}_x^A &= \mathbf{a}_x^B \\ \mathbf{a}_y^A &= -\mathbf{a}_y^B . \end{aligned} \tag{9}$$

Hence Eq. (8) may be written:

$$\mathbf{E} = \mathbf{a}_x^B E_x^t - \mathbf{a}_y^B E_y^t = \mathbf{a}_x^B E_x^r + \mathbf{a}_y^B E_y^r . \tag{10}$$

Relationships then exist between the field components transmitted by A and those received by B. From Eq. (10),

$$\begin{aligned} E_x^r &= E_x^t , \\ E_y^r &= -E_y^t . \end{aligned} \tag{11}$$

The preceding may be written in matrix form as

$$\left[E_{x,y}^r\right] = \left[A_{x,y}\right]\left[E_{x,y}^t\right] . \tag{12}$$

in which

$$[A_{x,y}] = \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix} \tag{12b}$$

Combining Eqs. (2), (4), and (12), the terminal voltage at the receiving antenna is

$$V = \frac{(Z_0 I_t)}{(2\lambda s)} \left[h^r_{x,y}\right] \left[A_{x,y}\right] \left[h^t_{x,y}\right]. \tag{13}$$

where

$$\left[h^r_{x,y}\right] = \begin{bmatrix} h_x^r \\ h_y^r \end{bmatrix} = \text{the components of vector height for the receiving antenna}$$

$$\left[h^t_{x,y}\right] = \begin{bmatrix} h_x^t \\ h_y^t \end{bmatrix} = \text{the components of vector height for the transmitting antenna.}$$

With the value for $[A]$ given by Eq. (12b), this voltage becomes

$$V = \frac{Z_0 I_t}{2\lambda s} (h_x^r h_x^t - h_y^r h_y^t) . \tag{14}$$

CIRCULARLY POLARIZED COMPONENTS

Equation (13) was derived by considering linearly polarized field components. Since a field may be resolved along circularly polarized unit vectors, an equivalent expression is possible:

$$V = \frac{Z_0 I_t}{2\lambda s} \left[h^r_{R,L}\right] \left[A_{R,L}\right] \left[h^t_{R,L}\right]. \tag{15}$$

In this equation

$$\left[h^r_{R,L}\right] = \begin{bmatrix} h_R^r \\ h_L^r \end{bmatrix} \tag{16a}$$

and

$$\left[h^t_{R,L}\right] = \begin{bmatrix} h_R^t \\ h_L^t \end{bmatrix}. \tag{16b}$$

Elements h_R and h_L are interpreted as components of the antenna vector height measured along the unit vectors $\mathbf{e}_R$ and $\mathbf{e}_L$ (right and left circularly polarized unit vectors).

Since the original antenna components have been written in a new basis, the matrix $[A_{x,y}]$ must also be transformed. It is shown in the appendix that the required transformation is given by

$$[A_{R,L}] = [\tilde{S}]\,[A_{x,y}]\,[S] \tag{17a}$$

in which

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -j & +j \end{bmatrix} \tag{17b}$$

and $[\tilde{S}]$ denotes the transpose of $[S]$.

Using Eq. (12b),

$$[A_{R,L}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{18}$$

the identity matrix. Equation (15) then becomes

$$V = \frac{Z_0 I_t}{2\lambda s} \left[h^r_{R,L}\right] \left[h^t_{R,L}\right]. \tag{19}$$

In terms of the components in Eq. (16),

$$V = \frac{Z_0 I_t}{2\lambda s} \left(h_R^r h_R^t + h_L^r h_L^t\right). \tag{20}$$

The effect of combining the components differently in Eqs. (14) and (20) may be demonstrated by considering pairs of coupled identical antennas expressed in the two coordinate bases. Table 1 indicates three polarizations, the corresponding antenna representations in terms of a normalized antenna height, and the expected coupling from a pair of identical antennas. It is seen that the representation of a 45-degree linear antenna on a linear basis is identical with the representation of a vertical antenna on a circular basis. The coupling between identical antennas, however, is zero in one case and unity in the other. Similarly, with the exception of a constant, the representation of a 45-degree linear antenna on a circular basis is the same as a right circular antenna expressed in a linear basis. Again a difference in coupling exists. By using the proper value for $[A_{x,y}]$ or $[A_{R,L}]$, the results will be consistent.

Table 1
Coupling Between Identical Antennas

Wave Polarization	Antenna Representation, [h]		Coupling (Identical Ant)
	Linear Basis	Circular Basis	
45-degree linear	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1-j\\1+j\end{bmatrix}$	0
Right circular	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix}$	$\begin{bmatrix}1\\0\end{bmatrix}$	1
Vertical	$\begin{bmatrix}1\\0\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$	1

POLARIZATION EFFICIENCY

A factor may be derived which describes the loss in power transfer between a pair of coupled antennas due solely to their polarization "misalignment." This factor, often called the polarization efficiency, describes the ratio of actual received power to the optimum or polarization matched received power.* For convenience, circularly polarized components will be used.

From Eq. (1)

$$\mathbf{E}^t = \frac{Z_0 I_t}{2\lambda s}\,\mathbf{h}\,. \tag{1}$$

Using circularly polarized unit vectors, the vector quantities may be written

$$\mathbf{E}^t = \mathbf{a}_R E_R^t + \mathbf{a}_L E_L^t \tag{21}$$

$$\mathbf{h} = \mathbf{a}_R h_R + \mathbf{a}_L h_L\,. \tag{22}$$

The power density in the transmitted wave is given by

$$p = \frac{|E_R|^2 + |E_L|^2}{Z_0} \tag{23}$$

in which the superscripts have been dropped for convenience. If an isotropic antenna is driven by a source of power P_t, the power density at a distance s is given by

$$p_0 = \frac{P_t}{4\pi s^2}\,. \tag{24}$$

Defining the ratio γ of the power densities in preceding equations as the gain of the transmitting antenna:

$$\gamma = G_R + G_L = \frac{p}{p_0}\,. \tag{25}$$

From Eqs. (23), (24),

$$G_R = \frac{4\pi s^2}{P_t Z_0}\,|E_R|^2 \tag{26a}$$

$$G_L = \frac{4\pi s^2}{P_t Z_0}\,|E_L|^2\,. \tag{26b}$$

From Eq. (1),

$$E_R = \frac{Z_0 I_t}{2\lambda s}\,h_R \tag{27a}$$

*This factor is described by Kales (2). The derivation given here is in terms of the antenna heights used earlier.

$$E_L = \frac{Z_0 I_t}{2\lambda s} h_L . \tag{27b}$$

The polarization of a wave may be represented by the ratio of E_R/E_L. In general both E_R and E_L are complex; however, no loss in generality will occur if E_L is assumed real. Thus let

$$E_R = |E_R| \, e^{j\alpha} \tag{28a}$$

$$E_L = |E_L| \tag{28b}$$

in which α is the phase of E_R with respect to E_L. Substituting these values into Eq. (27) and combining with Eq. (26):

$$h_R = \frac{2\lambda s}{Z_0 I_t} \sqrt{\frac{P_t Z_0}{4\pi s^2}} \sqrt{G_R} \, e^{j\alpha} \tag{29a}$$

$$h_L = \frac{2\lambda s}{Z_0 I_t} \sqrt{\frac{P_t Z_0}{4\pi s^2}} \sqrt{G_L} . \tag{29b}$$

Since $P_t = I_t^2 R$, where R is the radiation resistance of the antenna,

$$h_R = \sqrt{\frac{\lambda^2 R}{\pi Z_0}} \sqrt{G_R} \, e^{j\alpha} \tag{30a}$$

$$h_L = \sqrt{\frac{\lambda^2 R}{\pi Z_0}} \sqrt{G_L} . \tag{30b}$$

The preceding may be written in matrix form:

$$[h] = \begin{bmatrix} h_R \\ h_L \end{bmatrix} = \sqrt{\frac{\lambda^2 R}{\pi Z_0}} \begin{bmatrix} \sqrt{G_R} \, e^{j\alpha} \\ \sqrt{G_L} \end{bmatrix} . \tag{31}$$

It is convenient to rewrite this expression so that matrix denotes components of a unit length vector:

$$[h] = \sqrt{\frac{\lambda^2 R}{\pi Z_0}} \sqrt{G_R + G_L} \begin{bmatrix} \sqrt{\frac{G_R}{G_R + G_L}} \, e^{j\alpha} \\ \sqrt{\frac{G_L}{G_R + G_L}} \end{bmatrix}$$

$$= \sqrt{\frac{\lambda^2 R}{\pi Z_0}} \sqrt{\gamma} \, [h'] . \tag{32}$$

Using circular-basis unit vectors, the voltage at the terminals of an antenna described by $[h^r]$, when receiving transmission from an antenna described by $[h^t]$, is given by Eq. (19):

$$V = \frac{Z_0 I_t}{2\lambda s} [h^r][h^t]$$

$$= \frac{Z_0}{2\lambda s} \sqrt{\frac{P_t}{R_t}} [h^r][h^t]. \tag{19}$$

If the antennas are lossless and matched to their loads, then the power dissipated in the receiving-antenna load is $P_r = |V|^2/4R_r$, in which R_r is the radiation resistance of the receiving antenna. Combining Eqs. (19) and (32), the received power is

$$P_r = P_t \left(\frac{\lambda}{4\pi s}\right)^2 \gamma^r \gamma^t |[h'^r][h'^t]|^2 \tag{33}$$

where h' and γ are defined in Eq. (32). In this expression, the polarization efficiency multiplies the familiar expression for coupling between two polarization-matched antennas. Denoting the efficiency by η:

$$\eta = |[h'^r][h'^t]|^2. \tag{34}$$

To evaluate the polarization efficiency, measurements on each antenna may be performed under transmitting conditions. The power gain along the coordinate directions $\mathbf{a}_R$ and $\mathbf{a}_L$ determine G_R and G_L. Since circular components are used, the phase angle α has a particularly simple interpretation. With the wave from the antenna being measured propagating toward an observer, the tilt angle of the major axis of the polarization ellipse, measured clockwise from the vertical, is $\alpha/2$. This angle is illustrated in Fig. 3.

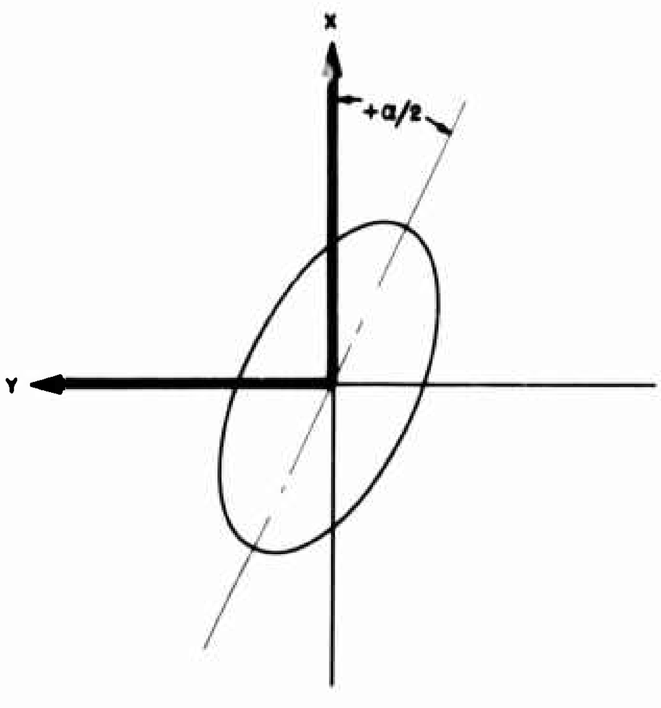

Fig. 3 - Locus of electric-field vector defined for elliptical wave propagating toward observer

CONCLUSIONS

By performing identical measurements on a group of antennas, a catalog of vector-height components may be generated. Since communication between two antennas is not restricted to an "on-axis" condition, the tabulation must consider a range of antenna aspect angles. To determine the power transfer between a coupled pair of antennas over a known line of sight, a simple matrix multiplication is required using the antenna characteristics $[h'^r]$, $[h'^t]$ for the chosen aspect angle. Since circular components are used, this multiplication takes the form:

$$\eta = |h'^r_R h'^t_R + h'^r_L h'^t_L|^2 . \qquad (35)$$

REFERENCES

1. Sinclair, G., "The Transmission and Reception of Elliptically Polarized Waves," Proc. IRE 38:148-151 (Feb. 1950)

2. Rumsey, V.H., Deschamps, G.A., Kales, M.L., and Bohnert, J.I., "Techniques for Handling Elliptically Polarized Waves with Special Reference to Antennas," Proc. IRE 39:533-552 (May 1951)

Appendix

MATRIX TRANSFORMATION

For a plane wave traveling away from an observer, the field in a plane at a fixed point along the propagating direction may be written

$$E = E_x \mathbf{a}_x + E_y \mathbf{a}_y = E_R \mathbf{a}_R + E_L \mathbf{a}_L \tag{A1}$$

where $\mathbf{a}_x$ and $\mathbf{a}_y$ are unit real vectors along the x and y coordinate axes (Fig. 1) and $\mathbf{a}_R$ and $\mathbf{a}_L$ are orthogonal circularly polarized complex vectors. Using the IRE definition of a right circular wave as rotating clockwise when traveling away from an observer, a pair of unit vectors are*

$$\begin{aligned} \mathbf{a}_R &= \frac{1}{\sqrt{2}} (\mathbf{a}_x - j\mathbf{a}_y) \\ \mathbf{a}_L &= \frac{1}{\sqrt{2}} (\mathbf{a}_x + j\mathbf{a}_y) . \end{aligned} \tag{A2}$$

The components of the field $\mathbf{E}$ along these unit vector directions are given by*

$$\begin{aligned} E_R &= \mathbf{E} \cdot \mathbf{a}_R^* \\ E_L &= \mathbf{E} \cdot \mathbf{a}_L^* . \end{aligned} \tag{A3}$$

Combining Eqs. (A3) with Eqs. (A1) and (A2), a relationship may be written between the transmitted linear and equivalent circular components:

$$\begin{bmatrix} E_R^t \\ E_L^t \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & +j \\ 1 & -j \end{bmatrix} \begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix} . \tag{A4}$$

The superscript t denotes the outgoing transmitted wave. An observer receiving this wave sees a reversal in the rotational directions of the components with respect to his own right-hand coordinate system. Thus for an observer at B in Fig. 2, using Eq. (9), the unit vectors are

$$\begin{aligned} \mathbf{a}_R^r &= \frac{1}{\sqrt{2}} (\mathbf{a}_x + j\mathbf{a}_y) \\ \mathbf{a}_L^r &= \frac{1}{\sqrt{2}} (\mathbf{a}_x - j\mathbf{a}_y) . \end{aligned} \tag{A5}$$

Proceeding as before, the relationship between the received linear and equivalent circular component is:

*M. L. Kales, "Part III - Elliptically Polarized Waves and Antennas," Proc. IRE 39:544-549 (1951).

$$\begin{bmatrix} E_R^r \\ E_L^r \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ 1 & +j \end{bmatrix} \begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} . \tag{A6}$$

Comparing Eqs.(A4) and (A6), it is seen that

$$\left[E^t_{R,L}\right] = [U] \left[E^t_{x,y}\right] \tag{A7}$$

$$\left[E^r_{R,L}\right] = [U^*] \left[E^r_{x,y}\right] \tag{A8}$$

in which

$$[U] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & +j \\ 1 & -j \end{bmatrix} .$$

From Eq. (12), and with reference to Fig. 2, the relationship between the field transmitted by A and that received by B is

$$\left[E^r_{x,y}\right] = \left[A_{x,y}\right] \left[E^t_{x,y}\right] \tag{12a}$$

in which

$$[A_{x,y}] = \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix} . \tag{12b}$$

Combining this equation with Eq. (A8),

$$\left[E^r_{R,L}\right] = [U^*] \left[A_{x,y}\right] \left[E^t_{x,y}\right] .$$

Substituting the value of $\left[E^t_{x,y}\right]$ from Eq. (A7):

$$\left[E^r_{R,L}\right] = [U^*] \left[A_{x,y}\right] \left[U^{-1}\right] \left[E^t_{R,L}\right] .$$

If $[S] = [U^{-1}]$, then since $[U]$ is unitary, $[U^{-1}] = [\tilde{U}^*]$ and the preceding becomes

$$\left[E^r_{R,L}\right] = \left[\tilde{S}\right] \left[A_{x,y}\right] \left[S\right] \left[E^t_{R,L}\right] .$$

If an equation similar to Eq. (12) is written for the circular base components,

$$\left[E^r_{R,L}\right] = \left[A_{R,L}\right] \left[E^t_{R,L}\right] . \tag{A9}$$

Thus it is seen that

$$[A_{R,L}] = [\tilde{S}] [A_{x,y}] [S] , \tag{A10}$$

where

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -j & +j \end{bmatrix} . \tag{A11}$$

* * *