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SPECIAL PERTURBATION TECHNIQUES APPLICABLE TO SPACETRACK

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-533

JULY 1964

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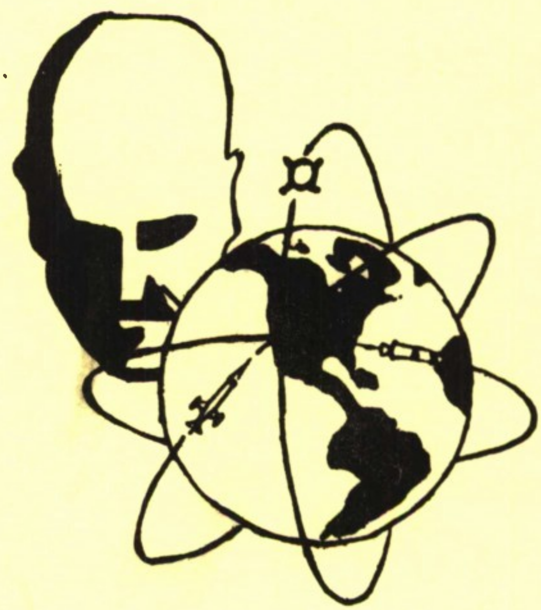
J. Stanfield

496L SYSTEM PROGRAM OFFICE
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts



Project 496L

(Prepared under Contract No. AF 19 (628)-1648 by the System Development Corporation, Santa Monica, California.)

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FOREWORD

This Technical Documentary Report was originally published as System Development Corporation Technical Memorandum TM-LX-145/000/00, dated 13 July 1964. It was prepared under Contract AF 19(628)-1648, System 496L, Spacetrack, for Electronic Systems Division, Air Force Systems Command.

ABSTRACT

This report describes in detail three special techniques which can be specifically applied in the SPACETRACK system to determine the motion of an artificial satellite under the influence of perturbing accelerations. These are Variation of Parameters, Crowell's Method, and Encke's Method.

REVIEW AND APPROVAL

This technical documentary report has been reviewed and is approved.

RICHARD A. JEDLICKA
1st Lt, USAF
Contract Technical Monitor
496L System Program Office
Deputy for Systems Management

TABLE OF CONTENTS

1. INTRODUCTION

The purpose of this report is to describe in some detail three special perturbation techniques which find specific application in the SPACETRACK system for determining the motion of an artificial satellite under the influence of perturbing accelerations. The methods, and a brief comment on each, are:

Variation of Parameters, also commonly called the variation of elements, expresses the perturbations in the orbit as a function of the elements, that is to say, the difference between the elements of the orbit at epoch and those of the osculating orbit at any time (t). Variation of parameters is used in SPWDC and SPIRDEC.

Cowell's method integrates the equations of motion in rectangular coordinates directly, giving the rectangular coordinates of the perturbed body. This method finds application in ESPOD and the final phase of SPIRDEC.

Encke's method differs from Cowell's in that it is the difference between where the body actually is in rectangular coordinates at some instant and where it would have been if no perturbative accelerations were present (two body reference orbit) that is calculated. This method is used in MUNENDC.

The remainder of this report is devoted to a development of these three methods.

2. VARIATION OF PARAMETERS METHOD

a. Description

In Figure 1, let A be the position of the satellite at some time t_o. Suppose at this instant all the perturbing forces acting on it ceased to exist. The satellite would then continue to move in an elliptical orbit (AB) with the constant elements $C_1, C_2 \ldots C_6$* of the classical two body problem. Further, if the position and velocity of the satellite is known at t_o, we have the requisite number of equations for determining the six elements of the ellipse. The ellipse AB is called the <u>osculating ellipse</u> at time t_o and the constants $C_1, C_2, \ldots C_6$ are the <u>osculating elements</u> at time t_o. The actual path of the satellite when the effect of the perturbations is taken into account is not along the elliptical path AB but rather along some other path represented by AC in Figure 1. The satellite at the instant t_o has the same coordinates and, by definition, the same velocity components in the unperturbed as in the perturbed orbit. Stated another way, the satellite has the position and is moving instantaneously as it would in purely two-body motion. Obviously one could compute a set of elements to define an osculating orbit at any point of the actual orbit. At time t_1, just subsequent to t_o, there could be defined a new set of osculating elements $C_1^1, C_2^1, \ldots C_6^1$, associated with the corresponding position and velocity of the satellite at point C in its actual path. The satellite's position would have been at B at time t_1, in the absence of all perturbations. It is these differences between the elements, $C_1 - C_1^1$, $C_2 - C_2^1$, etc. that are the perturbations of the elements in the interval $t_1 - t_o$.

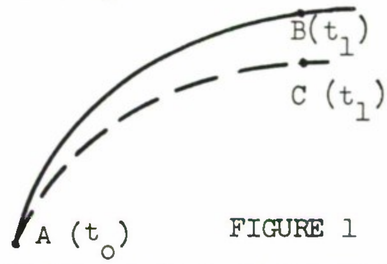

FIGURE 1

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* It should be noted that $C_1, C_2, \ldots C_6$ are identical in the Keplerian case to a, e, i, ω, Ω and T or some combination of these elements.

The major perturbation encountered in dealing with the motion of artificial satellites is the acceleration caused by the oblateness of the earth which is much greater than the perturbing accelerations produced on it by other bodies in the solar system.[1] The effect is that the elements of the Keplerian orbit vary as a function of time. Here the line of nodes and the perigee point move very rapidly under the noncentral force field. When the rates of change of the elements are known, the future orbital characteristics of the satellite can be predicted.

b. Analysis

The analysis known as the variation of parameters begins with the assumption that the cartesian coordinates defining the position of the satellite are known. In vector notation these are

$$\bar{r} = \bar{r}\,(t, C_1, C_2, C_3, C_4, C_5, C_6), \tag{1}$$

where $\bar{r} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$ and $\bar{i}$, $\bar{j}$ and $\bar{k}$ are unit vectors along the X, Y and Z axes respectively. The equations of motion of a satellite of mass M under the central attraction of the earth (mass M_o) and acted upon by a disturbing function R can be written as

$$\ddot{\bar{r}} + \frac{\mu\,\bar{r}}{r^3} = \nabla R \tag{2}$$

where $$\nabla R = \frac{\partial R}{\partial X}\,\bar{i} + \frac{\partial R}{\partial Y}\,\bar{j} + \frac{\partial R}{\partial Z}\,\bar{k} \quad \text{and} \quad \mu = k^2\,(M + M_o). \tag{3}$$

From equation (1) with the C_k's (k = 1, 2, 3...,6) as functions of time,

$$\dot{\bar{r}} = \frac{\partial \bar{r}}{\partial t} + \sum_{k=1}^{6} \frac{\partial \bar{r}}{\partial C_k}\,\dot{C}_k\,. \tag{4}$$

---

1. Additional perturbations associated with earth satellites arise from atmospheric drag, solar radiation, etc.

But $\frac{\partial \bar{r}}{\partial t} = \frac{d \bar{r}}{d t}$ which implies

$$\sum_{k=1}^{6} \frac{\partial \bar{r}}{\partial C_k} \dot{C}_k = 0 \tag{5}$$

Invoking (5) and differentiating equation (4) with respect to t, we obtain

$$\ddot{\bar{r}} = \frac{\partial^2 \bar{r}}{\partial t^2} + \sum_{k=1}^{6} \frac{\partial^2 \bar{r}}{\partial t \, \partial C_k} \dot{C}_k \tag{6}$$

Substituting this result back into equation (2) yields

$$\frac{\partial^2 \bar{r}}{\partial t^2} + \frac{\mu \bar{r}}{r^3} + \sum_{k=1}^{6} \frac{\partial^2 \bar{r}}{\partial t \, \partial C_k} \dot{C}_k = \nabla R \tag{7}$$

For the osculating orbit, $\nabla R = 0$ and the C_k's are constants so that

$$\frac{\partial^2 \bar{r}}{\partial t^2} + \frac{\mu \bar{r}}{r^3} = 0 \tag{8}$$

hence from equation (7)

$$\sum_{k=1}^{6} \frac{\partial^2 \bar{r}}{\partial t \, \partial C_k} \dot{C}_k = \nabla R. \tag{9}$$

It is common to rewrite equation (9), making use of the fact that

$$\frac{\partial^2 \bar{r}}{\partial t \, \partial C_k} = \frac{\partial}{\partial C_k} \left(\frac{\partial \bar{r}}{\partial t} \right) = \frac{\partial \dot{\bar{r}}}{\partial C_k} \text{, as}$$

$$\sum_{k=1}^{6} \frac{\partial \dot{\bar{r}}}{\partial C_k} \ddot{C}_k = \nabla R. \tag{10}$$

The time derivatives of the orbital elements can now be found by solving equations (5) and (10) simultaneously for the $\dot{C}_k$. However, the solution of these equations can be performed more readily by a rearrangement which introduced new functions of the parameters C_k called Lagrangian brackets.

If we take the dot product of equation (10) with $\frac{\partial \bar{r}}{\partial C_j}$; and equation (5) with $\frac{\partial \dot{\bar{r}}}{\partial C_j}$ and subtract the two, the resulting six equations may be written as

$$\sum_{k=1}^{6} \left[\frac{\partial \bar{r}}{\partial C_j} \cdot \frac{\partial \dot{\bar{r}}}{\partial C_k} - \frac{\partial \bar{r}}{\partial C_k} \cdot \frac{\partial \dot{\bar{r}}}{\partial C_j} \right] \dot{C}_k = \nabla R \cdot \frac{\partial \bar{r}}{\partial C_j} \quad (j=1, 2, \ldots, 6). \tag{11}$$

The quantity in brackets in equation (11) is Lagrange's bracket and is commonly denoted by

$$\left[C_j, C_k \right] = \frac{\partial (x, \dot{x})}{\partial (C_j, C_k)} + \frac{\partial (y, \dot{y})}{\partial (C_j, C_k)} + \frac{\partial (z, \dot{z})}{\partial (C_j, C_k)} , \tag{12}$$

where $$\frac{\partial (x, \dot{x})}{\partial (C_j, C_k)} \equiv \begin{vmatrix} \frac{\partial x}{\partial C_j} & \frac{\partial x}{\partial C_k} \\ \frac{\partial \dot{x}}{\partial C_j} & \frac{\partial \dot{x}}{\partial C_k} \end{vmatrix} ,$$

with similar expressions for $\frac{\partial (y, \dot{y})}{\partial (C_j, C_k)}$ and $\frac{\partial (z, \dot{z})}{\partial (C_j, C_k)}$.

The right hand side of equation (11) is the partial derivative of R with respect to C_j which is

$$\frac{\partial R}{\partial x} \frac{\partial x}{\partial C_j} + \frac{\partial R}{\partial y} \frac{\partial y}{\partial C_j} + \frac{\partial R}{\partial z} \frac{\partial Z}{\partial C_j} = \frac{\partial R}{\partial C_j} \tag{13}$$

Using equations (12) and (13), equation (11) may be written very simply as

$$\sum_{k=1}^{6} \left[C_j, C_k \right] \dot{C}_k = \frac{\partial R}{\partial C_j} \quad (j=1, 2, \ldots, 6). \tag{14}$$

It is these six equations that are to be solved for $\dot{C}_k$. Equation (14) contains thirty-six Lagrangian brackets but from the definition of these brackets in equation (12) it is noted that

$$\left[C_j, C_j \right] = 0 \qquad \left[C_j, C_k \right] = -\left[C_k, C_j \right] . \tag{15}$$

Therefore, the number of distinct Lagrangian brackets to be evaluated is only fifteen instead of the original thirty-six.

An example of the differential equations representing the variation of orbital elements which results from evaluating the Lagrangian brackets are: (2)

$$\frac{da}{dt} = \frac{2}{na} \frac{\partial R}{\partial M},$$

$$\frac{de}{dt} = \frac{1 - e^2}{na^2 e} \frac{\partial R}{\partial M} - \frac{\sqrt{1 - e^2}}{na^2 e} \frac{\partial R}{\partial \omega},$$

$$\frac{d\omega}{dt} = - \frac{\cos i}{na^2 \sqrt{1 - e^2} \sin i} \frac{\partial R}{\partial i} + \frac{\sqrt{1 - e^2}}{na^2 e} \frac{\partial R}{\partial e},$$

$$\frac{di}{dt} = \frac{\cos i}{na^2 \sqrt{1 - e^2} \sin i} \frac{\partial R}{\partial \omega},$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2 \sqrt{1 - e^2} \sin i} \frac{\partial R}{\partial i},$$

$$\frac{dM}{dt} = n - \frac{1 - e^2}{na^2 e} \frac{\partial R}{\partial e} - \frac{2}{na} \frac{\partial R}{\partial a}.$$

The equations above differ from the differential equations solved for in SPWDC and SPIRDEC which are in terms of an N-M element set. In terms of the N-M element set and considering perturbations due to the earth's bulge, radiation-pressure and drag, the variation of parameter equations become (3)

(2) Kozai, Y., "The Motion of a Close Earth Satellite", The Astronomical Journal, 64, No. 9, Page 369, November 1959.

(3) Aeronutronic, Special Perturbations Weighted Differential Correction Program Document, Technical Documentary Report No. ESD-TDR-63-645, 11 Dec. 63.

$$\frac{dL}{dt} = k_e L' + n \quad ,$$

$$\frac{d\underline{a}}{dt} = k_e \underline{a}' \quad ,$$

$$\frac{d\underline{h}}{dt} = k_e \underline{h}' \quad .$$

In SPWDC, these equations are solved using a Runge-Kutta numeric integration technique to determine the satellite's position and velocity.

3. COWELL'S METHOD

a. Description

Cowell's method of numerical integration makes no explicit use of a conic section as the first approximation to the orbit, but rather, the equations of motion in rectangular coordinates are integrated directly, giving the rectangular coordinates of the disturbed body. The origin is usually taken at the primary body, but this restriction is not necessary since the center of mass of the system or of any of the disturbing bodies may be used. The only restriction is that the motion of all bodies exerting appreciable effects are known relative to the chosen origin at some time. The only practical disadvantage of the method is that the integrals contain many significant figures and change rapidly with time. In consequence, the integration tables are slowly convergent which compels the use of a small tabular interval.

b. Analysis

Consider two point masses, m_a and m_b, with coordinates ξ_a, η_a ζ_a and ξ_b, η_b, ζ_b relative to a Cartesian coordinate system X, Y and Z as illustrated in Figure 2.

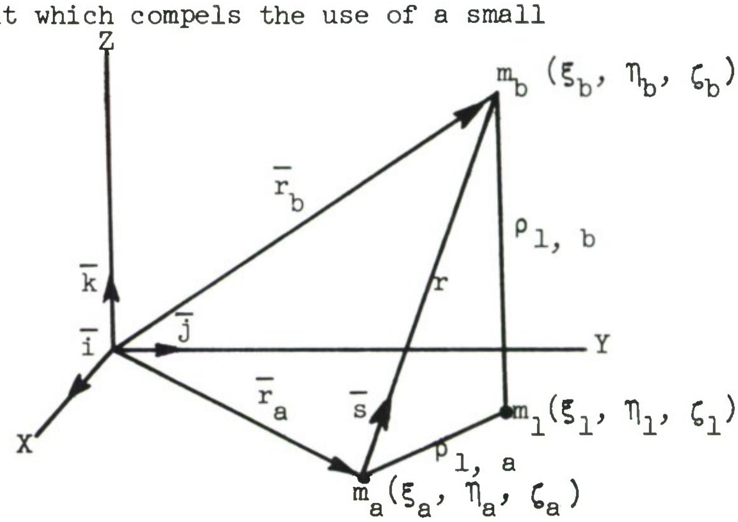

FIGURE 2

Let $\bar{r}_a$ be a vector from the origin of the coordinate system to m_a and $\bar{r}_b$ be a vector to m_b.

In addition, let $\bar{s}$ be a unit vector in the direction $\overline{m_a m_b}$ and $\bar{i}$, $\bar{j}$, and $\bar{k}$ unit vectors from the origin along X, Y and Z.

From Newton's law, the gravitational attraction between m_a and m_b can be expressed by:

$$F = \frac{k^2 m_a\, m_b}{r^2} \tag{16}$$

where r is the distance between m_a and m_b and k is a constant of proportionality depending on the units of mass, time and distance chosen. The force on m_a due to m_b is

$$\bar{F}_a = m_a \ddot{\bar{r}}_a = \frac{k^2 m_a m_b}{r^2} \bar{s} \quad , \tag{17}$$

and similarly, that on m_b due to m_a is

$$\bar{F}_b = m_b \ddot{\bar{r}}_b = \frac{k^2 m_a m_b}{r^2} \bar{s} \tag{18}$$

To determine the components of the force acting on m_a in the X, Y, Z directions, i.e., the ξ-, η-, and ζ- components, it is necessary to obtain the dot product of $\bar{F}$ with the unit vectors $\bar{i}$, $\bar{j}$ and $\bar{k}$. Thus, the ξ- component of the force on m_a is

$$F_{a\xi} = \bar{F}_a \cdot \bar{i} = m_a \ddot{\bar{r}}_a \cdot \bar{i} = m_a \ddot{\xi}_a = k^2 \frac{m_a m_b}{r^2} \cos (\bar{F}_a, \bar{i}), \text{ or,}$$

$$m_a \ddot{\xi}_a = k^2 m_a m_b \frac{(\xi_b - \xi_a)}{r^3} \quad , \tag{19}$$

where $r = \left[(\xi_a - \xi_b)^2 + (\eta_a - \eta_b)^2 + (\zeta_a - \zeta_b)^2 \right]^{\frac{1}{2}}.$

Similarly, the ξ- component of the force on m_b due to m_a is

$$m_b \ddot{\xi}_b = k^2 m_b m_a \frac{(\xi_a - \xi_b)}{r^3} \tag{20}$$

Similar expressions can be written for the η- and ζ- components.

If additional point masses m_1, m_2, m_3, ... are introduced into this system and denoting any one of these point masses by m_j, equations similar to (19) and (20) expressing the total accelerations of m_a and m_b may be obtained by summing all these attractions.

In figure 2, m_1, represents one such mass whose distance from m_a and m_b is $\rho_{1,a}$ and $\rho_{1,b}$ respectively.
The expression then for the ξ- components would be

$$m_a \ddot{\xi}_a = k^2 m_a m_b \frac{(\xi_b - \xi_a)}{r^3} + \sum_j k^2 m_a m_j \frac{(\xi_j - \xi_a)}{\rho^3_{j,a}} \tag{21}$$

$$m_b \ddot{\xi}_b = k^2 m_b m_a \frac{(\xi_a - \xi_b)}{r^3} + \sum_j k^2 m_b m_j \frac{(\xi_j - \xi_b)}{\rho^3_{j,b}} \tag{22}$$

where

$$\rho_{j,a} = \left[(\xi_a - \xi_j)^2 + (\eta_a - \eta_j)^2 + (\zeta_a - \zeta_j)^2\right]^{\frac{1}{2}}, \tag{23}$$

$$\rho_{j,b} = \left[(\xi_b - \xi_j)^2 + (\eta_b - \eta_j)^2 + (\zeta_b - \zeta_j)^2\right]^{\frac{1}{2}} \tag{24}$$

Again, similar expressions can be written for the η- and ζ- components. Let the origin of coordinates be taken at m_a which is equivalent to the linear transformation

$$\xi_b - \xi_a = x; \ \xi_j - \xi_a = x_j. \tag{25}$$

It then follows directly from (25) that

$$\xi_j - \xi_b = x_j - x . \tag{26}$$

Finally, put

$$r_j^2 = x_j^2 + y_j^2 + z_j^2; \ \rho_j = \left[(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2\right]^{\frac{1}{2}} \tag{27}$$

Divide equation (21) by m_a and equation (22) by m_b and then subtract (21) from (22). The result is the equation of motion of m_b relative to m_a and can be expressed as

$$\ddot{x} = - k^2 (m_a + m_b) \frac{x}{r^3} - \sum_j k^2 m_j \frac{x_j}{r_j^3} + \sum_j k^2 m_j \frac{x_j - x}{\rho_j^3} \qquad (28)$$

A more familiar expression for equation (28) is

$$\ddot{x} = - k^2 (m_a + m_b) \frac{x}{r^3} + \sum_j k^2 m_j \left(\frac{x_j - x}{\rho_j^3} - \frac{x_j}{r_j^3} \right) \qquad (29)$$

This equation and similar equations in $\ddot{y}$ and $\ddot{z}$ are the fundamental equations in Cowell's method. Other accelerations can be combined in the right hand side of equation (29) such as drag, zonal harmonics of the earth, radiation pressure, etc.

4. ENCKE'S METHOD

a. Description

Encke's method differs from Cowell's in that the coordinates of the disturbed body are not obtained directly but rather from the difference between the position the body would have in an osculating orbit referenced to some time t, called the epoch of osculation, and its true position that is calculated. The departures from the osculating orbit are the perturbations. The advantage of this method is that for times near the epoch of osculation, the perturbations are small and can be expressed by a few significant figures permitting a larger tabular interval than with Cowell's method. Against this is the disadvantage that each step of Encke's method takes longer and the perturbations increase with time requiring an occasional redetermination of the osculating orbit.

b. Analysis

Let m_a and m_b be two point masses with x_o, y_o, z_o the coordinates of m_b relative to m_a where m_b is moving under the attraction of m_a alone. The equations of motion are known to be

$$\ddot{x}_o = - k^2 (m_b + m_a) \frac{x_o}{r_o^3} , \tag{30}$$

$$\ddot{y}_o = - k^2 (m_b + m_a) \frac{y_o}{r_o^3} , \tag{31}$$

$$\ddot{z}_o = - k^2 (m_b + m_a) \frac{z_o}{r_o^3} , \tag{32}$$

where

$$r_o = (x_o^2 + y_o^2 + z_o^2)^{\frac{1}{2}}$$

Let α, β, γ represent the perturbations produced by the presence of additional masses m_j such that the true position (x, y, and z) of m_b at any time can be represented by

$$x = x_o + \alpha, \; y = y_o + \beta, \; z = z_o + \gamma, \tag{33}$$

and the actual equations of motion are

$$\ddot{x} = - k^2 (m_b + m_a) \frac{x}{r^3} + \sum_j k^2 m_j \left(\frac{x_j - x}{\rho_j^3} - \frac{x_j}{r_j^3} \right) \tag{34}$$

with similar expressions for $\ddot{y}$ and $\ddot{z}$.[1] Note that

$$r = (x^2 + y^2 + z^2)^{\frac{1}{2}}.$$

Subtracting (30) from (34) yields

$$\ddot{x}-\ddot{x}_o = \ddot{\alpha} = k^2 (m_b + m_a)\left(\frac{x_o}{r_o^3} - \frac{x}{r^3}\right) + \sum_j k^2 m_j \left(\frac{x_j - x}{\rho_j^3} - \frac{x_j}{r_j^3}\right) \tag{35}$$

again with similar expressions for $\ddot{\beta}$ and $\ddot{\gamma}$.

Equation (35) could be solved for α, β, γ by direct integration by calculating $\frac{x_o}{r_o^3}$ for each step by the laws of elliptic motion and the term $\frac{x}{r^3}$ at each step by extrapolating α and adding it to x_o to give x, etc., but this approach would not be convenient in practice for, since α is a small quantity, $\frac{x_o}{r_o^3}$ is nearly equal $\frac{x}{r^3}$, and these two terms would have to be calculated to many more significant figures than are needed in their difference. Encke developed the following transformation to overcome this difficulty. Treating only the equation for $\ddot{\alpha}$, since those for $\ddot{\beta}$ and $\ddot{\gamma}$ are exactly similar,

$$\frac{x_o}{r_o^3} - \frac{x}{r^3} = \frac{1}{r_o^3} \left(x_o - \frac{x r_o^3}{r^3}\right) = \frac{1}{r_o^3} \left[\left(1 - \frac{r_o^3}{r^3}\right)x - \alpha\right] \tag{36}$$

1. Refer to equation (29)

But

$$r^2 = x^2 + y^2 + z^2 = (x_o + \alpha)^2 + (y_o + \beta)^2 + (z_o + \gamma)^2 \qquad (37)$$

$$= r_o^2 + 2x_o\alpha + 2y_o\beta + 2z_o\gamma + \alpha^2 + \beta^2 + \gamma^2$$

Dividing (37) by r_o^2 yields

$$\frac{r^2}{r_o^2} = 1 + 2\left[\frac{(x_o + \frac{1}{2}\alpha)\alpha + (y_o + \frac{1}{2}\beta)\beta + (z_o + \frac{1}{2}\gamma)\gamma}{r_o^2}\right] \qquad (38)$$

and putting

$$q = \frac{(x_o + \frac{1}{2}\alpha)\alpha + (y_o + \frac{1}{2}\beta)\beta + (z_o + \frac{1}{2}\gamma)\gamma}{r_o^2}, \qquad (39$$

equation (38) may be rewritten as

$$\frac{r^2}{r_o^2} = 1 + 2\,q. \qquad (40)$$

From this,

$$\frac{r_o^3}{r^3} = (1 + 2\,q)^{-3/2} \qquad (41)$$

$$1 - \frac{r_o^3}{r^3} = 1 - (1 + 2\,q)^{-3/2} \qquad (42)$$

If we now define a function f by

$$f = \frac{1 - (1 + 2\,q)^{-3/2}}{q}$$

equation (35) can be rewritten in the form

$$\ddot{\alpha} = \frac{k^2 (m_b + m_a)}{r_o^3} (f\,q\,x - \alpha) + \sum_j k^2 m_j \left(\frac{x_j - x}{\rho_j^3} - \frac{x_j}{r_j^3} \right) \qquad (44)$$

Similar equations for $\ddot{\beta}$ and $\ddot{\gamma}$ can be expressed; it is these equations that are solved in Encke's method. Again, as in the discussion of Cowell's method, additional perturbing accelerations can be added to the right half of equation (44).

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13. ABSTRACT

This report describes in detail three special techniques which can be specifically applied in the SPACETRACK system to determine the motion of an artificial satellite under the influence of perturbing accelerations. These are Variation of Parameters, Crowell's Method, and Encke's Method.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Space Surveillance Systems Satellites (Artificial) Tracking Orbit Equations Mathematical Analysis Variation of Parameters Method Cowell's Method Encke's Method						

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