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ABSTRACT

In hypersonic turbulent wakes, mixing between the turbulent inner wake and the outer inviscid wake gives rise to the "intermittency" phenomenon. It is shown that electron-density fluctuations of a turbulent nature, in addition to those caused by "turbulence", will be produced by the intermittency phenomenon. These additional fluctuations depend on the mean electron-density distribution and Townsend's intermittency "δ" function.

Application of the above consideration to turbulent scattering by underdense plasma has been made. In contrast to the conventional theory of scattering, two contributions to the scattering cross section are obtained: The first one arises from the intermittency phenomenon and vanishes if the intermittency is omitted; and the second one is due to fluctuations caused by "turbulence". This second contribution also contains the intermittency effect, and reduces to that given by the conventional turbulent scattering theory when the intermittency is not considered.

Numerical results for the scattering cross section have been obtained by using correlation functions and mean electron-density distribution of the Gaussian form. Based on these results, some characteristics of the scattering cross section such as its aspect-angle and frequency dependence are found to be significantly modified by the intermittency. In addition, the magnitude of the cross section will always be larger if the intermittency effects are considered. This increase can be as high as many orders of magnitude in some cases.

1. INTRODUCTION

Interest in turbulence as a mechanism for scattering of electromagnetic waves probably began when Booker and Gordon[1] suggested it to explain some anomalous phenomena of radio scattering in the troposphre. The original theory developed by Booker and Gordon formed the basis for further theoretical and experimental work[2, 3, 4, 5,] in many related scattering problems. In determining the scattering cross-section, however, assumptions concerning the intensity of the electron density fluctuations and the form of the correlation or spectrum function must be made because of insufficient knowledge about the turbulence. Several different formulas have been proposed for the scattering cross section by different workers, but none of these appear to be entirely satisfactory.[5] Hence, at the present time the theory of turbulent scattering of radio waves in the ionosphere must be considered as still incomplete.

It is also recognized that turbulent scattering may be responsible for radar return signals from the wakes of high-speed reentry objects at certain altitudes. Experimentally, the back-scattering signals from the wake of a reentry object cannot be detected by radar until the object is at an altitude at which the wake apparently becomes turbulent. No significant back-scattering returns are expected from a laminar wake at small aspect angles with respect to the propagation direction of the radar wave (Figure 1).

The problem of turbulent scattering by underdense wakes has been studied in references 6, 7, 8 and others. The theoretical basis of these studies can be traced back to the work of Booker and Gordon,[1] and does not appear to have progressed much beyond Booker's recent work[3] on ionspheric radio propagation. In particular, the same uncertainties about the intensity of the electron-density fluctuations and the correlation or spectrum function are still present. A more detailed discussion of the present status of the theoretical work is given in Reference 8.

It has been shown in Reference 8 that the phenomenon of turbulence intermittency, as generally recognized to be a prominent feature of turbulent wake flows, will introduce additional electron-density flucuations and thus additional contributions to the scattering cross section. The purpose of this paper is to present a more complete analysis of the intermittency effects, and some numerical results for the scattering cross section with and without the intermittency effects. Some significant features of the effects of intermittency will be discussed in some detail. The assumptions used and the limitations of the present analysis will be pointed out.

II. EFFECT OF INTERMITTENCY ON ELECTRON DENSITY FLUCTUATION

The hypersonic wake flows are governed by the processes of mixing between the inner turbulent wake and the outer inviscid wake (see reference 9, for example) so that large gradients of the mean velocity, temperature, mass density of the gas, electron density, etc. in the direction transverse to the wake axis exist. A prominent feature of turbulent flows is thus the phenomenon of flow intermittency. According to Townsend,[10] the cause of the flow intermittency is the production of a convoluted boundary surface between the turbulent and non-turbulent fluid by the large eddies (of fluid motion). Thus, across the wake an intermittency factor is introduced to show statistically the fraction of the time that the flow is found to be turbulent.

Since the hot boundary-layer gas goes into the turbulent core of the wake, electrons produced by thermal ionization will be present in the turbulent core. In the non-turbulent fluid, the electron density is very low and may be taken as zero. (Although this condition appears to be valid for slender bodies, in general enthalpy in the outer inviscid wake is not negligible compared to the inner turbulent wake for hypersonic blunt bodies.[9] The derivation and results to follow can be readily modified to include the electron density distribution in the non-turbulent fluid.) Intermittency of the turbulent flow will, consequently, introduce fluctuations of electron density, in addition to those present in the entirely turbulent fluid.

Let $\delta(\bar{r}, t)$ denote the probability of turbulence at $\bar{r}$, such that δ is zero in non-turbulent fluid and is unity in turbulent fluid.[10] Time average of δ yields $\gamma(\bar{r})$, the intermittency factor at $\bar{r}$. Let $n(\bar{r}, t)$ be the electron density in the turbulent fluid, $N(\bar{r})$ the mean value of n averaged (in time) over both the turbulent and the non-turbulent fluid, and $\bar{n}(\bar{r})$ the mean value of n in the turbulent fluid. Townsend[10] has shown that $N=\gamma\bar{n}$.

The fluctuation of the electron density with respect to N is equal to $n - N$ in the turbulent fluid, and equal to $-N$ in the non-turbulent fluid. On the average (statistically), the density fluctuation is

$$\Delta n = (n - N)\,\delta + (-N)\,(1 - \delta) = n\,\delta - N. \qquad (1)$$

Let $n = \bar{n} + n'$, where n' is the turbulent fluctuation of n (in the conventional sense). Then, the fluctuation of electron density is

$$\Delta n = (\delta - \gamma)\,\bar{n} + n'\delta\,. \qquad (2)$$

Time averaging of the above expression for Δ n will vanish if and only if the time average of $n'\delta$ is zero, implying that there is no correlation between the intermittency function δ controlled by large eddies and the turbulent fluctuations n' produced mostly by small - scale eddies within the turbulent fluid. The assumption of zero correlation between n' and δ appears to be physically reasonable and will be adopted in the present study.

If the fluid is entirely turbulent, i, e., $\delta = \gamma = 1$, Eq. (2) shows that Δ n reduces to n'. When the turbulent flow exhibits the intermittency phenomenon, however, additional fluctuation of electron density will occur. The total fluctuation in electron density consists of two parts: the turbulent fluctuation and that introduced by the intermittency. The appearance of the mean electron density in the fluctuation term should be noted.

Under the assumption that the two functions δ and n' are not correlated, the correlation function for the electron-density fluctuations is given by:

$$< (\Delta n)_1 \ (\Delta n)_2 > \ = (< \delta_1 \ \delta_2 > - \gamma_1 \gamma_2)\,\bar{n}_1 \bar{n}_2 + < \delta_1 \ \delta_2 >< n_1' n_2' >, \qquad (3)$$

where the subscripts 1 and 2 refer to points at $\bar{r}_1$ and $\bar{r}_2$, respectively, in the scattering volume.

Since very little is known about the function $< \delta_1 \ \delta_2 >$, the following approximations will be used. Assume (see Appendix I).

$$<(\delta_1-\gamma_1)(\delta_2-\gamma_2)> \quad = \quad \gamma_o(1-\gamma_o)\ X(\bar{R},\bar{\rho}), \tag{4}$$

where $X(\bar{R}, \bar{\rho})$ is the correlation coefficient for $<(\delta_1-\gamma_1)(\delta_2-\gamma_2)>$ at $\bar{R}$ and $\bar{\rho}$ with

$$\bar{R} = 1/2\,(\bar{r}_1+\bar{r}_2),$$
$$\bar{\rho} = \bar{r}_1-\bar{r}_2,$$

such that $X \to 1$ as $\rho \to 0$ and $X \to 0$ as $\rho \to \infty$, and γ_o denoted the value of γ for $\rho = 0$. It is important to note from the identity

$$<\delta_1\,\delta_2> = \gamma_1\,\gamma_2 + <(\delta_1-\gamma_1)(\delta_2-\gamma_2)> \tag{5}$$

that

$$<\delta_1\,\delta_2> \to \gamma_o \qquad \text{as} \qquad \rho \to 0, \tag{6}$$
$$<\delta_1\,\delta_2> \to \gamma_1\,\gamma_2 \qquad \text{as} \qquad \rho \to \infty \tag{7}$$

The relation (6) can be understood from the fact that the function δ is either zero or one and thus $\delta^2 = \delta$. Eq. (7) simply means that, if the two points are far apart, their δ functions will be independent of each other and $<\delta_1\,\delta_2> = <\delta_1><\delta_2> = \gamma_1\,\gamma_2$.

From the above relations, the correlation function can be written in the form

$$<(\Delta n)_1(\Delta n)_2> \;=\; \gamma_o(1-\gamma_o)\,\bar{n}_1\,\bar{n}_2\;\; X(\bar{R},\bar{\rho}) + \left\{\gamma_1\,\gamma_2 + \gamma_o(1-\gamma_o)\;\; X(\bar{R},\bar{\rho})\right\} <n_1'\,n_2'> . \tag{8}$$

At the present time, there is no rigorous method available for the determination of the functions γ, X and $\langle n_1' n_2' \rangle$. In this study, no attempt will be made to determine these functions theoretically. Instead, simple analytical expressions will be used for these functions, so that the scattering cross sections can be obtained mathematically in closed form in order to faciliate the numerical analysis. These expressions are not exact, but appear to be physically reasonable for a preliminary evaluation of the intermittency effects.

III. SCATTERING CROSS SECTION FOR UNDERDENSE TURBULENT WAKE

The back-scattering cross section for underdense turbulent wakes is (reference 4 or 8)

$$\sigma = r_e^2 \int_V dV_R \int_V \langle \Delta n (\bar{R}) \; \Delta n (\bar{R} + \bar{\rho}) \rangle \, e^{i 2 \bar{k} \cdot \bar{\rho}} \, d V_{\rho} \, , \quad (9)$$

where r_e ($= e^2 / c^2 m_e$) is the classical electron radius.

The correlation function for the electron-density fluctuations given by Eq. (8) may be substituted into Eq. (9) to obtain an expression for the scattering cross section in a form more susceptible to physical interpretation. It is convenient to separate the cross section into two parts:

$$\sigma = (\sigma_a + \sigma_b), \quad (10)$$

where

$$\sigma_a = r_e^2 \int_V dV_R \int_V \gamma_o (1 - \gamma_o) \bar{n}_1 \bar{n}_2 \; X(\bar{R}, \bar{\rho}) e^{i 2 \bar{k} \cdot \bar{\rho}} \, d V_{\rho} \, , \quad (11)$$

$$\sigma_b = r_e^2 \int_V d V_R \int_V \left\{ \gamma_1 \gamma_2 + \gamma_o (1 - \gamma_o) \; X(\bar{R}, \bar{\rho}) \right\} \langle n_1' n_2' \rangle e^{i 2 \bar{k} \cdot \bar{\rho}} \, d V_{\rho}, \quad (12)$$

If the effect of flow intermittency is omitted, both γ and X become unity. Then σ_a vanishes, and σ reduces to

$$\sigma = r_e^2 \int_V dV_R \int_V \langle n_1' \; n_2' \rangle \, e^{i 2 \bar{k} \cdot \bar{\rho}} \, dV_{\rho} \, . \quad (13)$$

The correlation function for the electron-density fluctuations $\langle n_1' \; n_2' \rangle$ will be assumed to have the following form

$$\langle n_1' \; n_2' \rangle = \left(\langle n_1'^2 \rangle \langle n_2'^2 \rangle \right)^{\frac{1}{2}} X_n(\bar{R}, \bar{\rho}), \quad (14)$$

where χ_n is the correlation coefficient. In addition, the intensity of the electron-density fluctuations will be expressed in terms of the local mean electron density as follows:

$$\langle n'^2 \rangle = \epsilon \bar{n}^2, \tag{15}$$

where ϵ is a parameter independent of the electron density $\bar{n}$. This expression for the intensity of electron-density fluctuations is often used (Reference 7, for example) with its value of ϵ taken to be a constant (a value of .25 was used in Reference 7). Since no real physical justification has been given to the expression (15) for the intensity of electron-density fluctuations, it should be considered, at most, a working formula. Obviously there is a urgent need for studies in turbulence theory to determine the intensity of electron-density fluctuations. With relations (14) and (15),

$$\gamma_1 \gamma_2 \langle n_1' n_2' \rangle = \epsilon \gamma_1 \gamma_2 \bar{n}_1 \bar{n}_2 \chi_n(\bar{R}, \bar{\rho})$$

$$= \epsilon N_1 N_2 \chi_n(\bar{R}, \bar{\rho}). \tag{16}$$

The correlation coefficient for n'- fluctuations is taken to be

$$\chi_n = \exp\left\{ -\left(\frac{\xi^2}{L^2} + \frac{\eta^2 + \zeta^2}{T^2}\right) \right\}, \tag{17}$$

where ξ, η and ζ are the components of $\bar{\rho}$, and L and T are the correlation lengths assumed to be constant and will be taken to be equal to each other in this analysis. The correlation coefficient for the intermittency is assumed to be

$$\chi = \exp\left\{ -\left(\frac{\xi^2}{\alpha^2 L^2} + \frac{\eta^2 + \zeta^2}{\alpha^2 T^2}\right) \right\} \tag{18}$$

where α^2 is a parameter determining the extent of the intermittency. When $\alpha^2 \to \infty$, no intermittency is present. The Gaussian-type correlation functions are chosen mainly for mathematical simplicity in evaluating the integrals for the scattering cross section. For isotropic turbulent velocity field (or scalar field) the Gaussian-type correlation coefficient is valid in the find period of decay.[11] The correlation coefficient for the intermittency function δ is used here apparently for the first time, and almost nothing about its form is known.

The mean electron-density distribution $\bar{n}$ is chosen to be

$$\bar{n}(\bar{r}) = N_o \exp\left\{-\left(\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2}\right)\right\}, \tag{19}$$

while the intermittency factor γ is assumed to be

$$\gamma = \exp\left(-\frac{s^2}{\alpha^2} \; \frac{y^2 + z^2}{b^2}\right), \tag{20}$$

where s^2 is a constant. It follows that

$$N = N_o \exp\left\{-\left(\frac{x^2}{a^2} + \frac{y^2 + z^2}{\beta^2 \; b^2}\right)\right\} \tag{21}$$

where

$$\beta^2 = \frac{1}{1 + (s^2/\alpha^2)} . \tag{22}$$

An estimate for s^2 has been made by using the experimental results obtained for velocity intermittency in a jet mixing problem.[11] It is found that a value of .25 appears to be a reasonable choice.

It is convenient to introduce a reference cross section obtained from Eq. (13) by substituting the equations (14), (15), (17), and (19) and carrying out the integrations. By putting a = b, the resultant expression, when maximized with respect to L (L = T throughout this analysis) for a given k, has its maximum value of

$$\sigma_{\infty, m} = \frac{e^{-\frac{3}{2}} \pi \epsilon N_o^2 r_e^2}{2(\frac{4}{3})^{\frac{3}{2}} k^6} . \tag{23}$$

The value of L at this maximum is given by

$$\frac{2 b^2}{L^2} = \frac{4}{3} b^2 k^2 - 1 . \tag{24}$$

Since the dependence of the scattering cross section on the wave number k of the incident electromagnetic wave is of particular interest, the following expression obtained from Eqs. (23) and (24)

$$\sigma^* = \frac{e^{-\frac{3}{2}} \pi^3 \epsilon b^6 N_o^2 r_e^2}{2\left(\frac{2 b^2}{L^2} + 1\right)^{\frac{3}{2}}} \tag{25}$$

will be used as the reference cross section.

By using the above relations (14) - (21), the scattering cross sections in Eq. (10) are found to be

$$\frac{\sigma_a}{\sigma^*} = \frac{e^{\frac{3}{2}} \beta^2}{\epsilon} \frac{1-\beta^2}{1+\beta^2} \frac{p^2(1+2t^2)^{\frac{3}{2}}}{\left[1 + 2(pt/\alpha)^2\right]^{\frac{1}{2}} \left[1+2(t/\alpha)^2\right]} \times$$

$$\exp\left\{ - 2t^2 (Lk)^2 \left[\frac{p^2 \cos^2 \theta}{1 + 2(pt/\alpha)^2} + \frac{\sin^2\theta}{1+2(t/\alpha)^2}\right]\right\} \tag{26}$$

$$\frac{\sigma_b}{\sigma^*} = e^{\frac{3}{2}} \beta^2 p^2 (1+2t^2)^{\frac{3}{2}} \left\{ \frac{\exp\left[-2t^2 (Lk)^2 \left(\frac{p^2 \cos^2\theta}{1+2p^2t^2} + \frac{\sin^2\theta}{2t^2+\beta^{-2}} \right) \right]}{\left(1+2p^2t^2\right)^{\frac{1}{2}} \left(2t^2+\beta^{-2}\right)} \right.$$

$$\left. + \frac{1-\beta^2}{1+\beta^2} \frac{\exp\left[-2t^2 (Lk)^2 \left(\frac{p^2\cos^2\theta}{1+2p^2t^2(1+\alpha^{-2})} + \frac{\sin^2\theta}{1+2t^2(1+\alpha^{-2})} \right) \right]}{\left[1+2p^2t^2(1+\alpha^{-2})\right]^{\frac{1}{2}} \left[1+2t^2(1+\alpha^{-2})\right]} \right\}, \quad (27)$$

where $p = a/b$, $t = b/L$, and β^2 can be expressed in terms of α^2 by using Eq. (22).

When $\alpha^2 \to \infty$, $\beta^2 \to 1$, there will be no intermittency effects. Then σ_a vanishes and σ/σ^* becomes

$$\left(\frac{\sigma}{\sigma^*}\right)_\infty = \frac{e^{\frac{3}{2}} (1+2t^2)^{\frac{1}{2}} p^2}{(1+2p^2t^2)^{\frac{1}{2}}} \exp\left\{ -2t^2 (Lk)^2 \left(\frac{p^2\cos^2\theta}{1+2p^2t^2} + \frac{\sin^2\theta}{1+2t^2} \right) \right\} (28$$

Eq. (28) shows no dependence on the aspect angle θ when $p = 1$. Thus, both the intermittency and the non-isotropic distribution of the mean electron-density (for $p \neq 1$) contribute to the aspect-angle dependence of the scattering cross section as can be seen from Eqs. (26) and (27).

IV. NUMERICAL RESULTS

A numerical analysis has been performed in order to evaluate the significance of intermittency. The relative magnitude of σ and σ_{bo} (scattering cross section without intermittency) is of particular interest. Since there are many parameters in the cross sections (26) and (27), only a selected number of calculations considered as of physical interest have been carried out. The range of parameters chosen are as follows:

$\alpha = 1, 2, 5, 10, 15, 10^6$

$\theta = 15^\circ, 30^\circ, 60^\circ, 90^\circ,$

$p = 5, 10, 100$

$t = 1, 2, 3, 5, 10, 15$

The range of Lk is from .1 - 100. For practical purposes, the radar wave length $\lambda (= 2\pi / k)$ has generally the range from about 5 cm (C-band) to 70 cm (UHF). Typical results are shown in Figure 2. To present the results in a more direct and meaningful manner, the following specific values of L and b are used in the graphs and tables given below for radar wave lengths from 3 cm to 100 cm:

L in cm = 1.95, 5.85, 15.

b in cm = 10, 15, 30.

The three values of L are chosen to show how a change in the correlation length L (with altitude, for example, see Reference 7) would modify the scattering cross section. b, the e - folding length in the radial direction for the mean electron distribution, is taken to be larger than or equal to L. The numerical results are presented in tables 1 - 12 and Figures 3 - 14.

Some significant features to be noted are:

(1) By including the intermittency effects, the scattering cross section is found to be always larger. The difference between cross sections with different values of α is generally small for $\lambda > L$, but becomes larger when λ is made smaller compared to L. This can be seen very clearly from Figures 3 - 14 in which the relative magnitudes of cross sections with and without the intermittency effects are shown. In particular, for $\lambda << L$, this ratio can be as large as many orders of magnitude in some cases. The value of 10^6 for α is found to be sufficiently large to yield zero intermittency. At the present time, however, the precise value of α for intermittency is not known. For the jet mixing problem referred to in Reference 11, α is estimated to be between 1 and 2.

(2) There is a significant increase in the magnitude of the cross section as the ratio $p = a/b$ is increased (Tables 1 - 12). The increase is generally larger than one order of magnitude as p is increased from 5 to 100. This indicates that mean electron-density distributions of more non-isotropic form would have larger cross sections.

(3) Aspect-angle dependence of the cross section is also strongly influenced by the ratio p. This is apparent only for larger correlation lengths, e.g., for L = 15 cm, as can be seen from Tables 9 - 12. The action of intermittency, however, is to reduce this aspect-angle dependence.

(4) Another effect of intermittency is on the frequency-dependence of the cross section. As is well known in atmospheric radio propagation study,[12] experimental results show that the frequency-dependence of the cross section predicted by theory generally cannot be confirmed. Evidences are also available from correlating radar return signals from wakes[13] that in the range between C-band and UHF the frequency-dependence is of the form $\sigma \sim k^{-m}$, where m has values between 1.67 and 1.71. Theoretical analysis based on an idealized model for underdense plasmas with homogeneous, locally isotropic turbulence (in the subinertial range) yields the relation $\sigma \sim k^{-11/3}$ (see, for example, p. 72 of Reference 4). There are many obvious reasons for this discrepancy between theoretical and experimental results in the frequency (or wave-number) - dependence of the cross section. The present study indicates that in the range between C-band and UHF, the index m generally has values larger than 1.67 and can be even larger than 11/3 at the small wave-length end. On the other hand, if the intermittency effects are included in the scattering, the values of m will be substantially reduced at the small wave-length end. Uncertainties in both theoretical and experimental work, however, prevent any definite conclusions to be reached at the present time.

V. CONCLUDING REMARKS

The present work has shown that the intermittency phenomenon will introduce electron-density fluctuations of a turbulent nature, in addition to those caused by "turbulence". Application of this idea to turbulent scattering by underdense plasmas has yielded a new expression for the scattering cross section. Numerical analysis based on correlation functions and mean electron-density distributions of Gaussian form shows that the effect of intermittency on the turbulent scattering is significant. The most remarkable feature is that, in general, the scattering cross section always becomes larger, if the intermittency effects are included. This increase in the scattering cross section can reach a magnitude of several orders in some cases. This means that the intermittency is responsible for almost all the turbulent scattering. In addition, the frequency-dependence of the cross section is also modified.

Further numerical analysis is needed to remove the restrictions introduced by the use of Gaussian-type functions for the correlation and mean-electron-density distribution, and by the neglecting of the "wake growth". Therefore, the numerical results in so far as applicability to the scattering by hypersonic wakes is concerned should be considered as preliminary. Obviously, there is a real need for turbulence studies to determine the form of the correlation functions and the distributions of mean electron-density, to understand the phenomenon of intermittency, and to find the intensity of electron-density fluctuations. In view of this fact, a re-evaluation of the intermittency effects should be carried out as soon as more satisfactory formulas for these functions are available. It is quite possible that other significant features of intermittency still remain to be found, and some of the conclusions and results obtained here will be modified. Under the present circumstances, therefore, it is hoped that some appropriate experimental work can be carried out to verify the results obtained in this study. Finally, the idea about the intermittency effect presented in this study may be equally applicable to other atmospheric scattering problems, since mixing and intermittency effects may also be present in the atmosphere.

APPENDIX I.

Both forms (4) and (14) for the correlation functions are only approximate ones. But since no method is available to derive the correlation functions, these formulas appear to be the simplest and reasonable ones to use.

Instead of (4), the following expression may be used

$$< (\delta_1 - \gamma_1)(\delta_2 - \gamma_2) > = \left\{ < (\delta_1 - \gamma_1)^2 > < (\delta_2 - \gamma_2)^2 > \right\}^{\frac{1}{2}} X(\bar{R}, \bar{\rho})$$

$$= \left\{ \gamma_1 \gamma_2 (1-\gamma_1)(1-\gamma_2) \right\}^{\frac{1}{2}} X(\bar{R}, \bar{\rho}) \quad . \qquad \text{(A-1)}$$

Thus

$$< \delta_1 \delta_2 > = \gamma_1 \gamma_2 + \left\{ \gamma_1 \gamma_2 (1-\gamma_1)(1-\gamma_2) \right\}^{\frac{1}{2}} X(\bar{R}, \bar{\rho})$$

which shows that

$$\begin{aligned} < \delta_1 \delta_2 > &\rightarrow \gamma_o \quad , \quad \text{as} \quad \rho \rightarrow 0, \\ &\rightarrow \gamma_1 \gamma_2 \quad , \quad \text{as} \quad \rho \rightarrow \infty, \end{aligned}$$

as expected. However, (A - 1) is difficult to handle mathematically, and is replaced by (4) in the present analysis.

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TABLE 1

Data For σ / σ^*

Parameters: $L=1.95cm$, $b=10cm$, $p = 5$

Aspect Angle $\theta = 15^o$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
1.5	2.41×10^{-15}	3.33×10^{-24}	2.86×10^{-28}	2.73×10^{-28}
2	5.18×10^{-9}	4.55×10^{-14}	1.24×10^{-15}	1.22×10^{-15}
3	1.74×10^{-4}	2.01×10^{-6}	1.33×10^{-6}	1.33×10^{-6}
5	1.01×10^{-1}	5.69×10^{-2}	5.62×10^{-2}	5.64×10^{-2}
10	6.16	4.94	5.03	5.05
20	18.51	19.36	15.47	15.55
30	22.71	28.47	19.73	19.14
50	25.23	35.84	30.19	21.30
70	25.97	38.35	40.07	21.93
100	26.37	39.79	48.73	22.28

TABLE 2

Data For σ / σ^*

Parameters: L=1.95 cm, b=10cm, p=5

Aspect Angle $\theta = 30^o$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
1.5	2.55×10^{-15}	3.83×10^{-24}	3.57×10^{-28}	3.39×10^{-28}
2	5.35×10^{-9}	4.94×10^{-14}	1.40×10^{-15}	1.38×10^{-15}
3	1.77×10^{-4}	2.11×10^{-6}	1.41×10^{-6}	1.41×10^{-6}
5	1.03×10^{-1}	5.81×10^{-2}	5.73×10^{-2}	5.76×10^{-2}
10	6.20	4.97	5.05	5.08
20	18.53	19.46	15.49	15.57
30	22.73	28.57	19.93	19.16
50	25.23	35.89	30.98	21.30
70	25.91	38.38	40.87	21.93
100	26.37	39.81	49.27	22.28

TABLE 3

Data For σ/σ^*

Parameters: L=1.95cm, b=10cm, p=5

Aspect Angle $\theta = 60^{\circ}$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
1.5	2.96×10^{-15}	5.62×10^{-24}	6.50×10^{-28}	6.16×10^{-28}
2	5.82×10^{-9}	6.15×10^{-14}	1.96×10^{-15}	1.94×10^{-15}
3	1.84×10^{-4}	2.42×10^{-6}	1.64×10^{-6}	1.63×10^{-6}
5	1.07×10^{-1}	6.14×10^{-2}	6.05×10^{-2}	6.07×10^{-2}
10	6.29	5.05	5.12	5.15
20	18.60	19.75	15.57	15.62
30	22.77	28.83	20.73	19.18
50	25.25	36.03	33.49	21.31
70	25.98	38.46	43.22	21.94
100	26.38	39.85	50.89	22.28

TABLE 4

Data For σ / σ^*

Parameters: L=1.95cm, b=10cm, p=5

Aspect Angle $\theta = 90^{\circ}$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
1.5	3.20×10^{-15}	6.80×10^{-24}	8.78×10^{-28}	8.31×10^{-28}
2	6.07×10^{-9}	6.86×10^{-14}	2.32×10^{-15}	2.29×10^{-15}
3	1.87×10^{-4}	2.59×10^{-6}	1.76×10^{-6}	1.76×10^{-6}
5	1.10×10^{-1}	6.31×10^{-2}	6.22×10^{-2}	6.24×10^{-2}
10	6.35	5.10	5.16	5.18
20	18.64	19.90	15.63	15.65
30	22.79	28.96	21.35	19.20
50	25.26	36.10	34.98	21.32
70	25.98	38.50	44.51	21.94
100	26.38	39.87	51.72	22.28

TABLE 5

Data For σ / σ^*

Parameters: L=5.85cm, b=10cm, p=10

Aspect Angle $\theta = 15^{o}$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
3	6.55×10^{-33}	*	*	*
5	3.56×10^{-12}	2.56×10^{-19}	3.12×10^{-22}	3.04×10^{-22}
10	2.08×10^{-3}	9.48×10^{-5}	7.65×10^{-5}	7.67×10^{-5}
20	2.25	1.68	1.71	1.72
30	13.19	10.66	10.92	10.99
50	33.30	30.96	28.22	28.40
70	43.05	45.80	36.70	36.90
100	49.35	58.66	43.33	42.40

* Number Less Than 1×10^{-37}

TABLE 6

Data For σ / σ^*

Parameters: L=5.85cm, b=10cm, p=10

Aspect Angle $\theta = 30^{\circ}$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
3	1.91×10^{-32}	*	*	*
5	5.24×10^{-12}	6.57×10^{-19}	1.31×10^{-21}	1.27×10^{-21}
10	2.33×10^{-3}	1.34×10^{-4}	1.10×10^{-4}	1.10×10^{-4}
20	2.47	1.84	1.87	1.88
30	13.78	11.15	11.36	11.43
50	33.83	32.02	28.63	28.81
50	43.40	46.96	37.06	37.17
100	49.54	59.54	44.22	42.55

* Number Less Than 1×10^{-37}

TABLE 7

Data For σ / σ^*

Parameters: L=5.85 cm, b=10cm, p=10

Aspect Angle θ = 60°

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
3	3.57×10^{-31}	*	*	*
5	1.50×10^{-11}	8.67×10^{-18}	6.67×10^{-20}	6.27×10^{-20}
10	3.27×10^{-3}	3.48×10^{-4}	2.93×10^{-4}	2.91×10^{-4}
20	3.19	2.37	2.39	2.40
30	15.51	1.28	1.27	12.74
50	35.33	35.56	29.92	29.96
70	44.37	50.52	39.54	37.91
100	50.09	62.11	49.84	42.97

* Number Less Than 1×10^{-37}

TABLE 8

Data For σ / σ^*

Parameters: L=5.85cm, b=10cm, p=10

Aspect Angle $\theta = 90^{\circ}$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
3	1.54×10^{-30}	*	*	*
5	2.54×10^{-11}	3.17×10^{-17}	4.76×10^{-19}	4.41×10^{-19}
10	3.98×10^{-3}	5.66×10^{-4}	4.79×10^{-4}	4.73×10^{-4}
20	3.62	2.70	2.70	2.71
30	16.46	13.91	13.41	13.45
50	36.11	37.80	32.37	30.55
70	44.86	52.57	45.08	38.29
100	50.36	63.49	56.59	43.18

* Number Less Than 1×10^{-37}

TABLE 9

Data For σ / σ^*

Parameters: L=15cm, b=15cm, p=10

Aspect Angle $\theta = 15^{o}$

Radar Wave length λ in cm

λ \ α	1	2	5	$\alpha \rightarrow \infty$ No Intermittency
5	*	*	*	*
10	2.14×10^{-19}	8.36×10^{-31}	1.79×10^{-36}	1.59×10^{-36}
20	3.71×10^{-5}	5.59×10^{-8}	2.26×10^{-8}	2.26×10^{-8}
30	2.02×10^{-2}	3.97×10^{-3}	3.68×10^{-3}	3.70×10^{-3}
50	2.26	1.67	1.71	1.73
70	11.16	8.98	9.31	9.38
100	26.63	22.77	22.88	23.07

* Number Less Than 1×10^{-37}

TABLE 10

Data For $\sigma/\sigma*$

Parameters: L=15cm, b=15cm, p = 10

Aspect Angle $\theta = 30^{o}$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \rightarrow \infty$ No Intermittency
5	*	*	*	*
10	$1.07x10^{-18}$	$3.26x10^{-29}$	$3.72x10^{-34}$	$3.31x10^{-34}$
20	$5.55x10^{-5}$	$1.72x10^{-7}$	$8.66x10^{-8}$	$8.59x10^{-8}$
30	$2.72x10^{-2}$	$7.18x10^{-3}$	$6.68x10^{-3}$	$6.69x10^{-3}$
50	2.79	2.08	2.12	2.14
70	12.50	10.09	10.39	10.46
100	28.19	24.48	24.14	24.33

*Number Less Than $1x10^{-37}$

TABLE 11

Data For $\sigma / \sigma *$

Parameters: L=15cm, b=15cm, p = 10

Aspect Angle $\theta = 60^{o}$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
5	*	*	*	*
10	8.56×10^{-17}	7.27×10^{-25}	8.42×10^{-28}	7.14×10^{-28}
20	1.73×10^{-4}	4.86×10^{-6}	3.39×10^{-6}	3.29×10^{-6}
30	7.98×10^{-2}	3.69×10^{-2}	3.41×10^{-2}	3.38×10^{-2}
50	5.06	3.81	3.82	3.83
70	17.12	14.30	14.02	14.09
100	32.94	31.15	27.99	28.16

* Number Less Than 1×10^{-37}

TABLE 12

Data For $\sigma / \sigma*$

Parameters: L=15cm, b=15cm, p=10

Aspect Angle $\theta = 90^{o}$

Radar Wave Length λ in cm

λ \ α	1	2	5	$\alpha \to \infty$ No Intermittency
5	*	*	*	*
10	7.67×10^{-16}	1.12×10^{-22}	1.28×10^{-24}	1.05×10^{-24}
20	3.41×10^{-4}	2.91×10^{-5}	2.13×10^{-5}	2.04×10^{-5}
30	1.60×10^{-1}	8.45×10^{-2}	7.71×10^{-2}	7.60×10^{-2}
50	6.86	5.33	5.14	5.12
70	20.07	18.09	16.77	16.35
100	35.62	36.57	32.82	30.29

* Number Less Than 1×10^{-37}

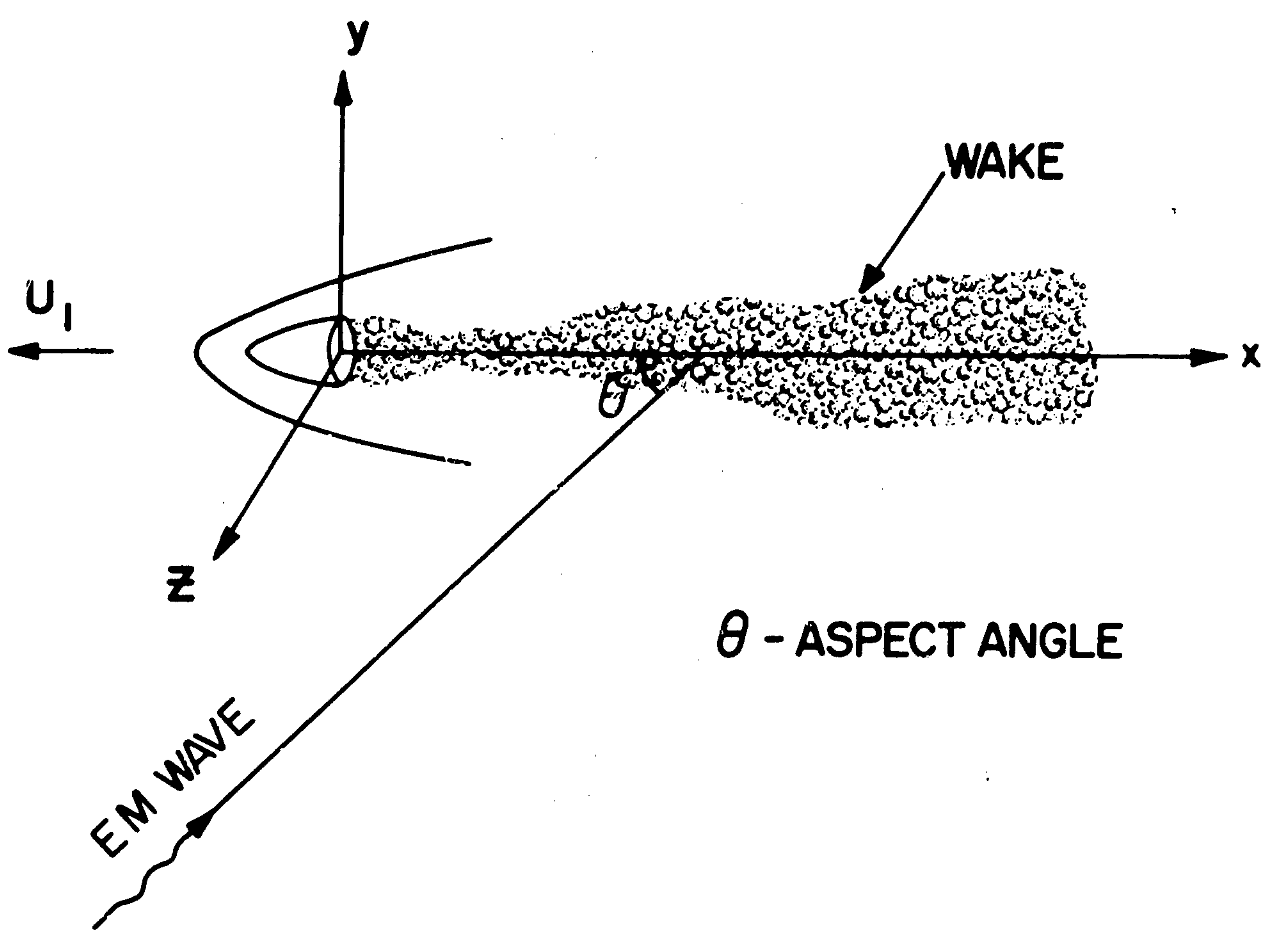
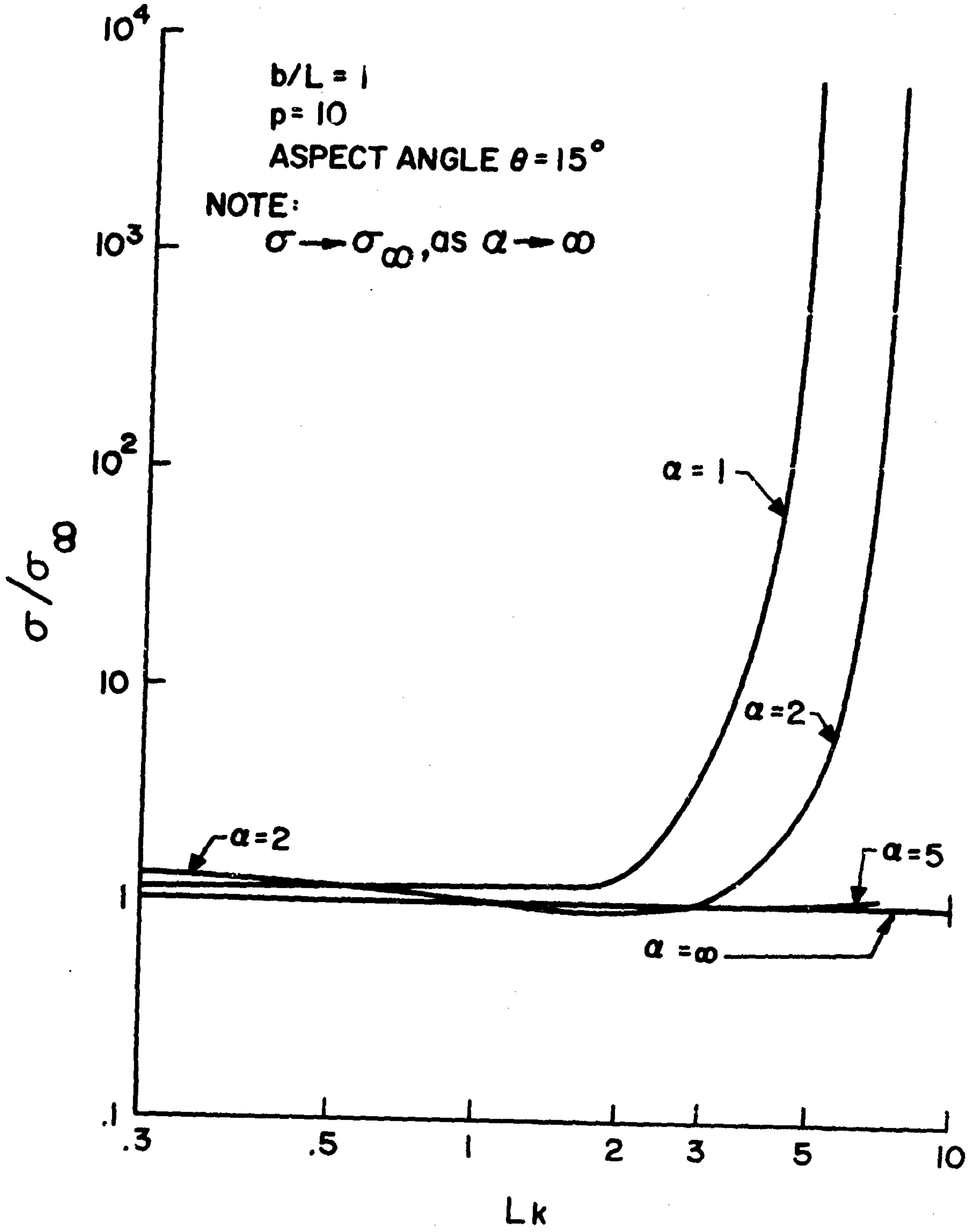
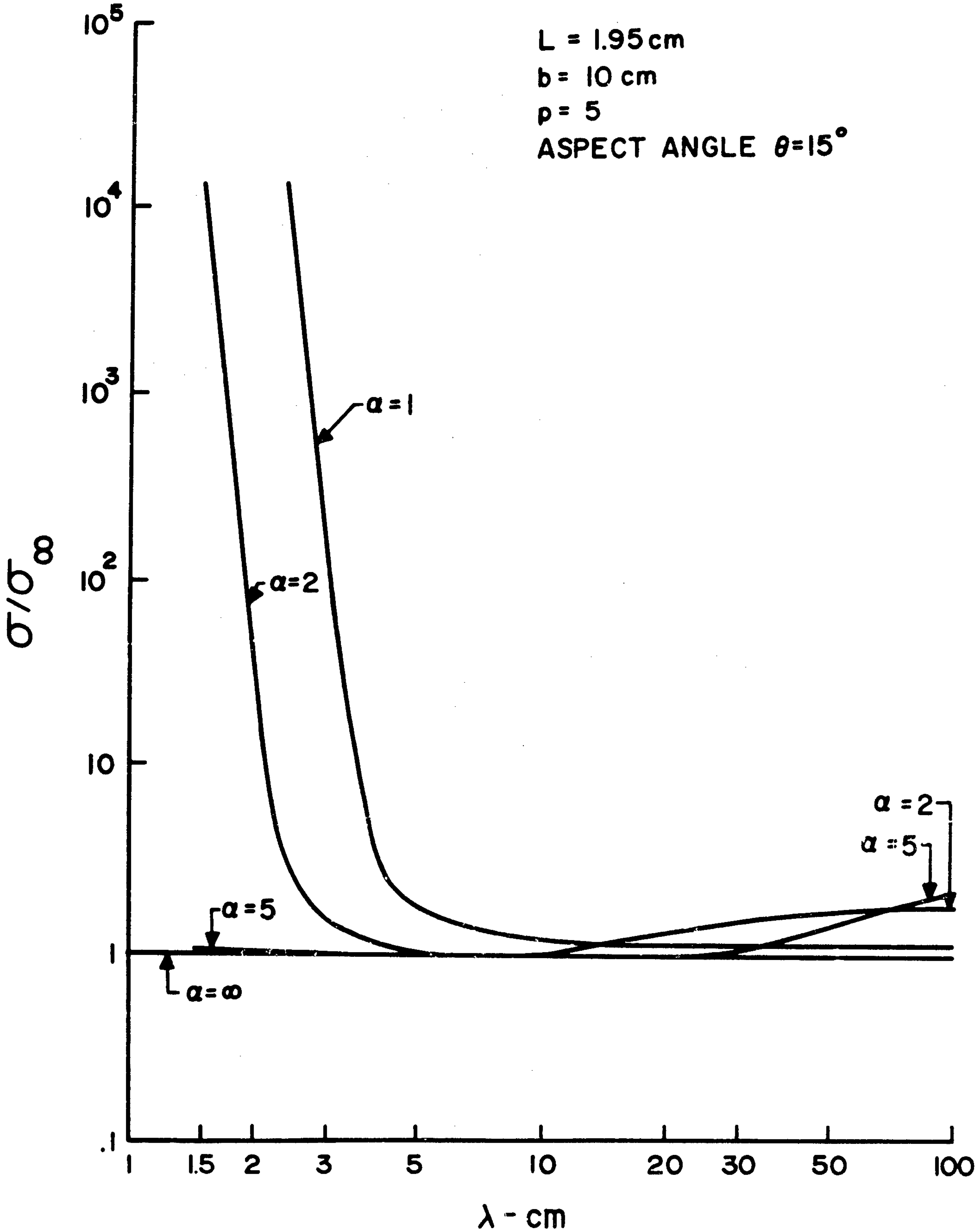


FIGURE 1.



b/L = 1
p= 10
ASPECT ANGLE θ = 15°
NOTE:
σ → σ∞, as α → ∞
α = 1
α = 2
α = 2
α = 5
α = ∞
σ/σ∞
Lk
10⁴
10³
10²
10
1
.1
.3
.5
1
2
3
5
10

FIGURE 2



L = 1.95 cm
b = 10 cm
p = 5
ASPECT ANGLE θ=15°
σ/σ∞
10^5
10^4
10^3
10^2
10
1
.1
α=1
α=2
α=5
α=∞
α=2
α=5
1
1.5
2
3
5
10
20
30
50
100
λ - cm

FIGURE 3

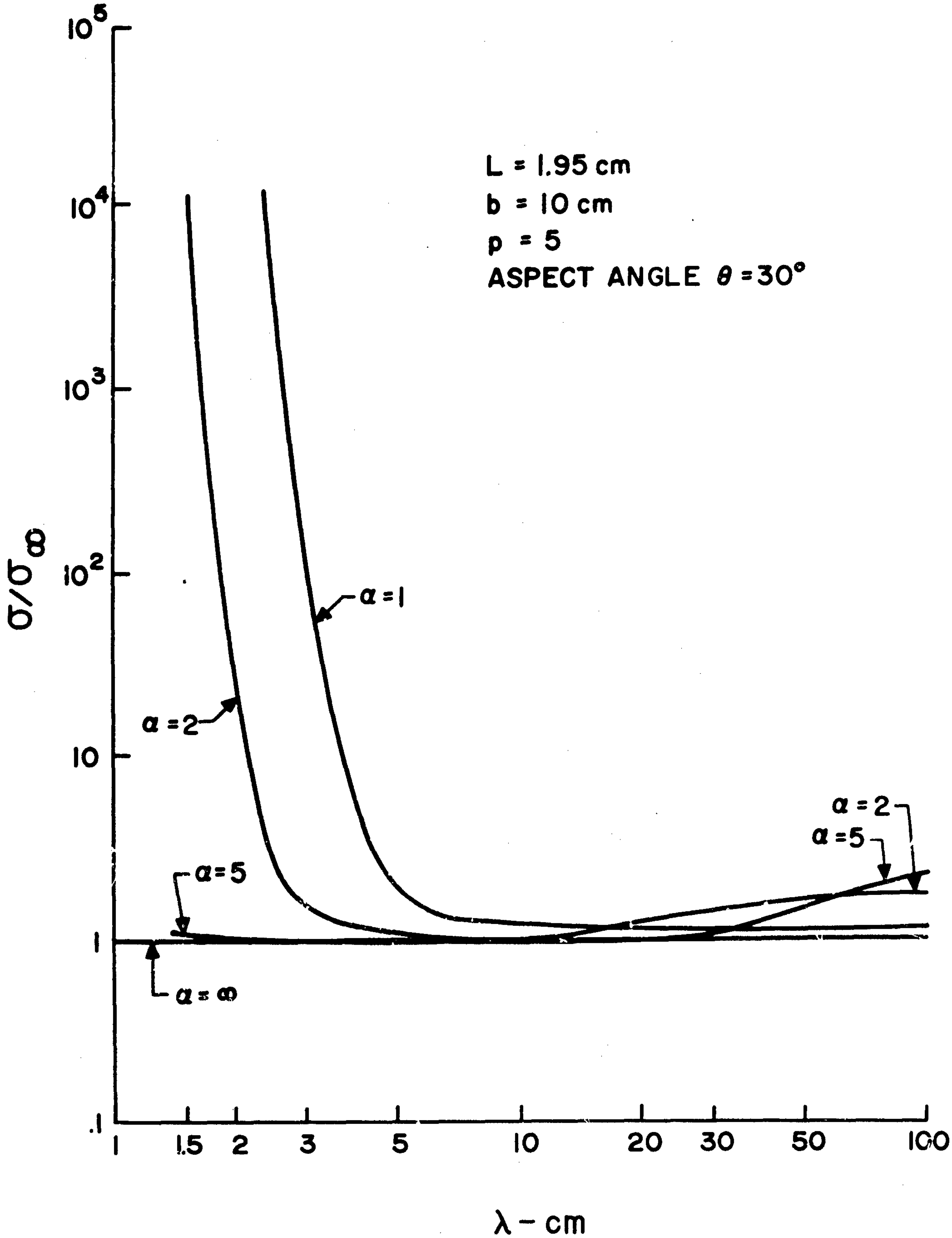

L = 1.95 cm
b = 10 cm
p = 5
ASPECT ANGLE θ = 30°
σ/σ∞
α = 1
α = 2
α = 5
α = ∞
α = 2
α = 5
10^5
10^4
10^3
10^2
10
1
.1
1
1.5
2
3
5
10
20
30
50
100
λ - cm

FIGURE 4

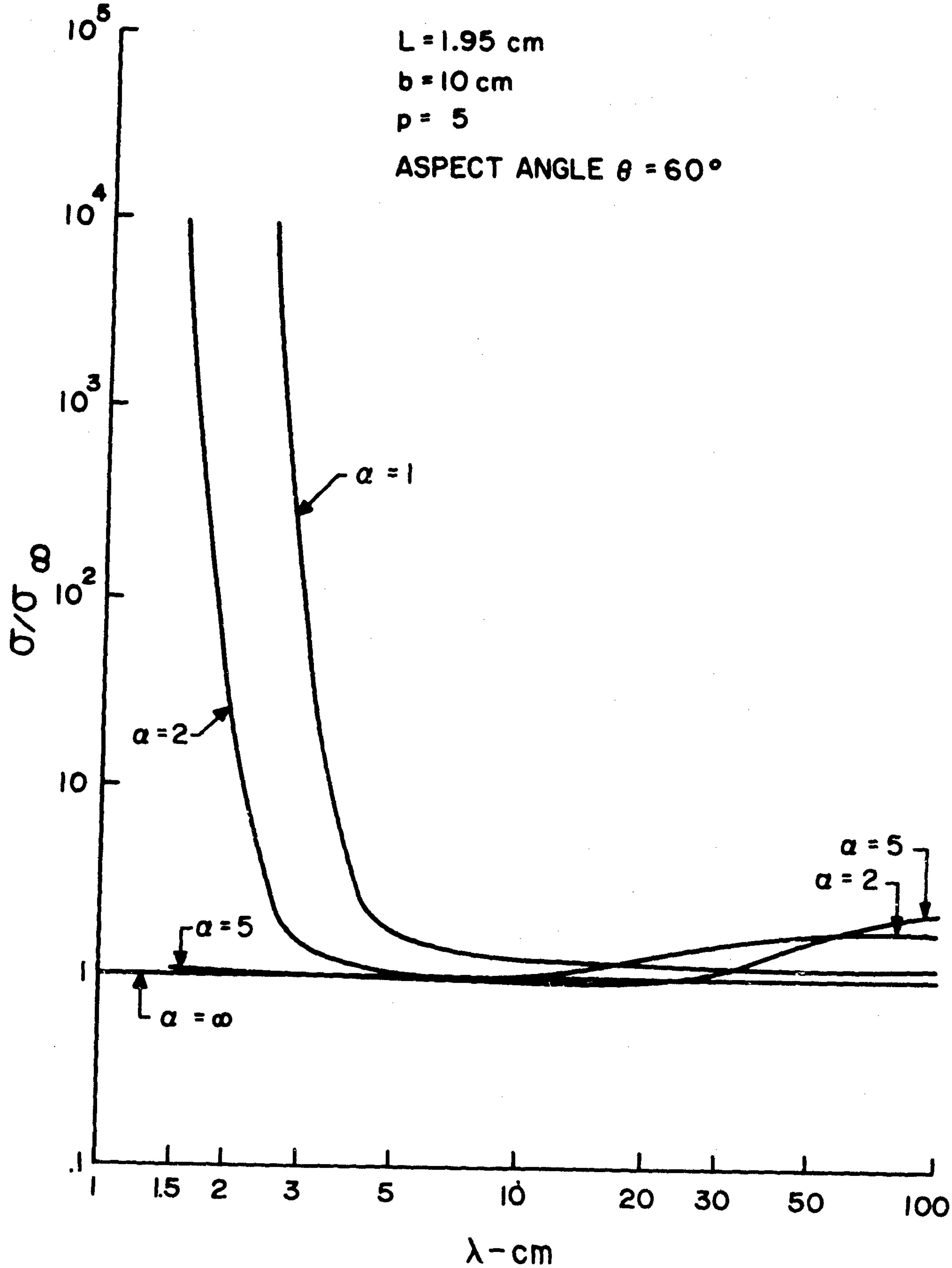
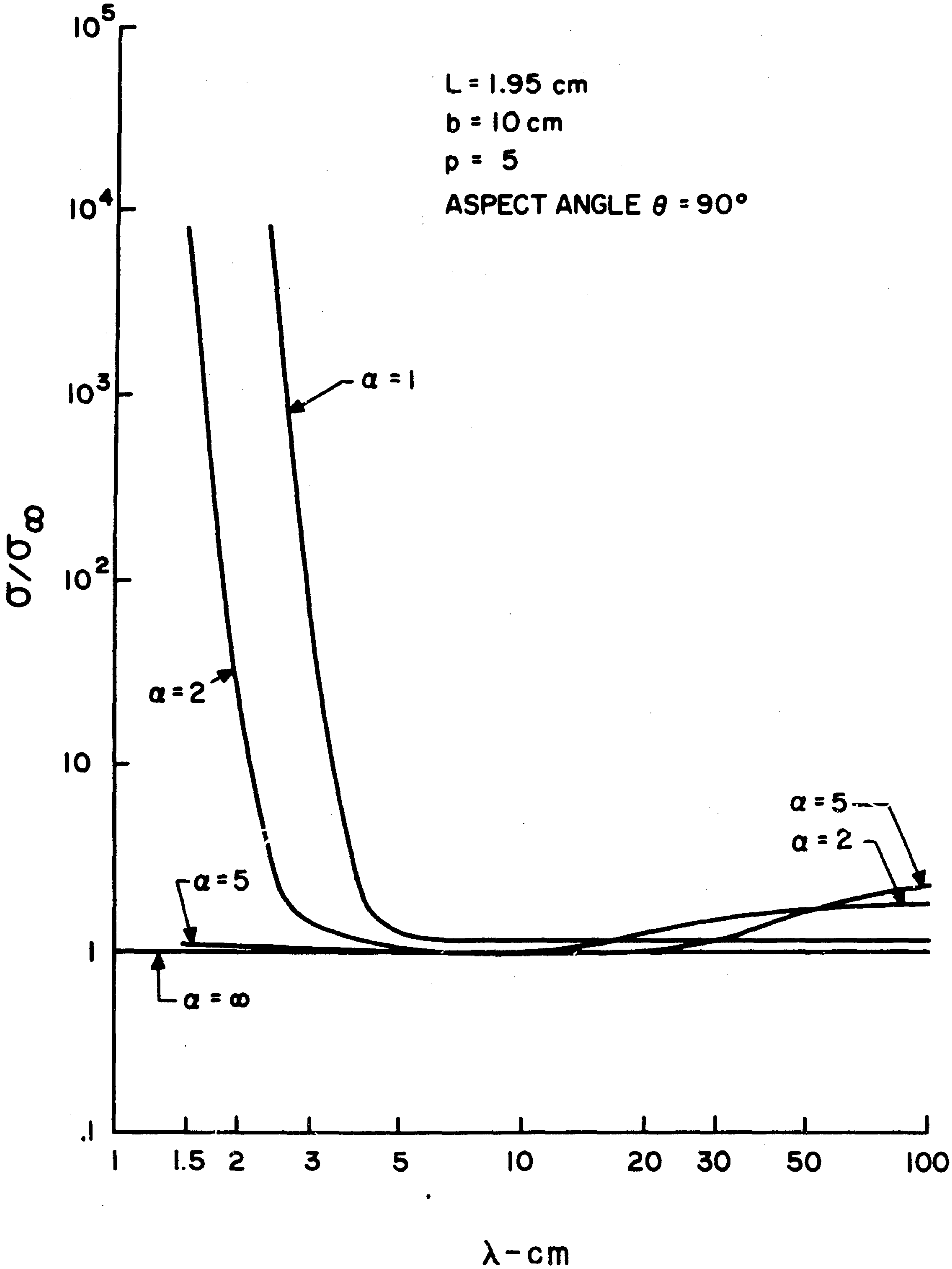

L = 1.95 cm
b = 10 cm
p = 5
ASPECT ANGLE θ = 60°
σ/σ∞
λ-cm
α = 1
α = 2
α = 5
α = ∞
α = 5
α = 2
10^5
10^4
10^3
10^2
10
1
.1
1
1.5
2
3
5
10
20
30
50
100

FIGURE 5



L = 1.95 cm
b = 10 cm
p = 5
ASPECT ANGLE θ = 90°
α = 1
α = 2
α = 5
α = ∞
α = 5
α = 2
σ/σ∞
λ - cm
10⁵
10⁴
10³
10²
10
1
.1
1
1.5
2
3
5
10
20
30
50
100

FIGURE 6

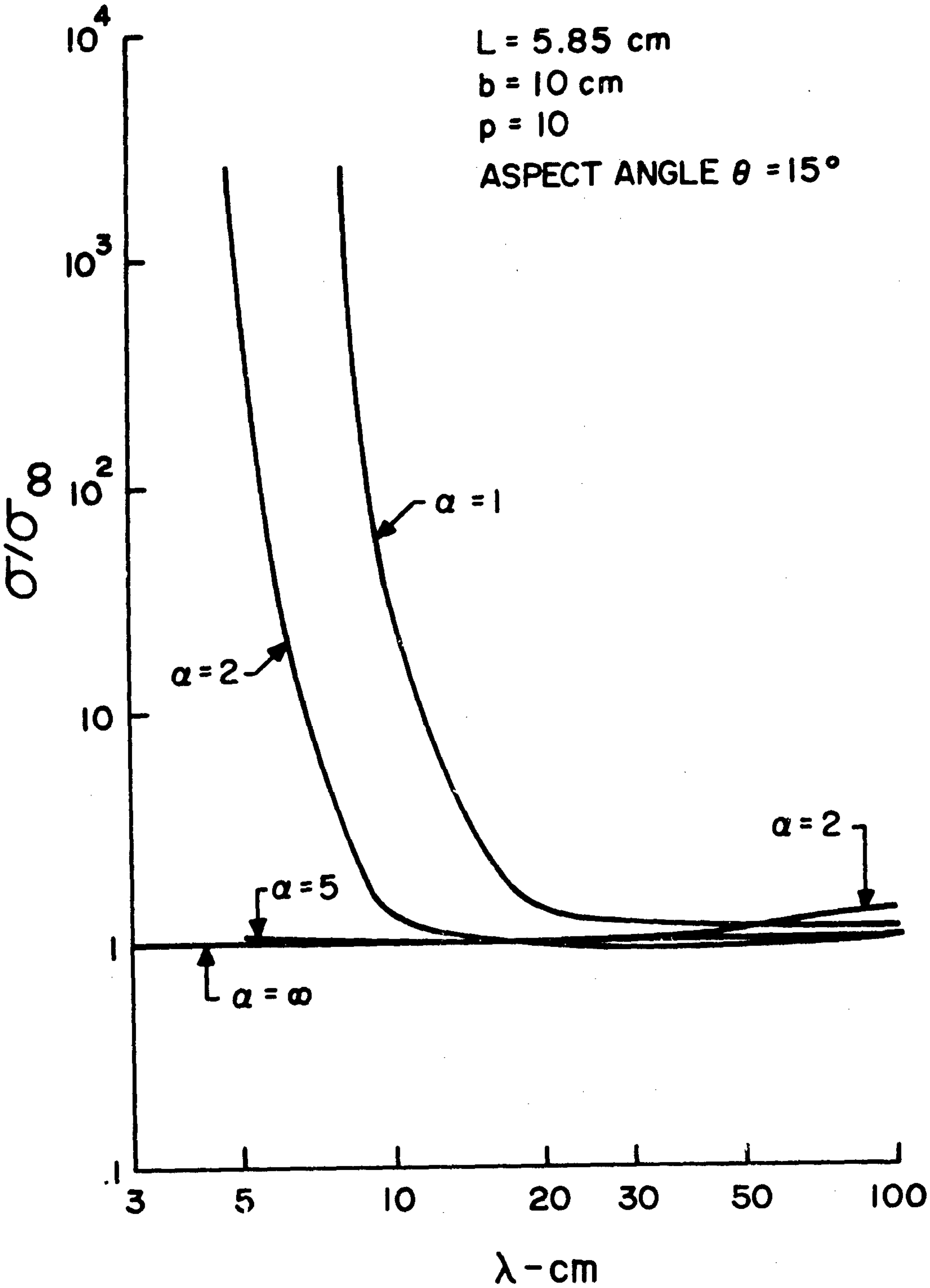
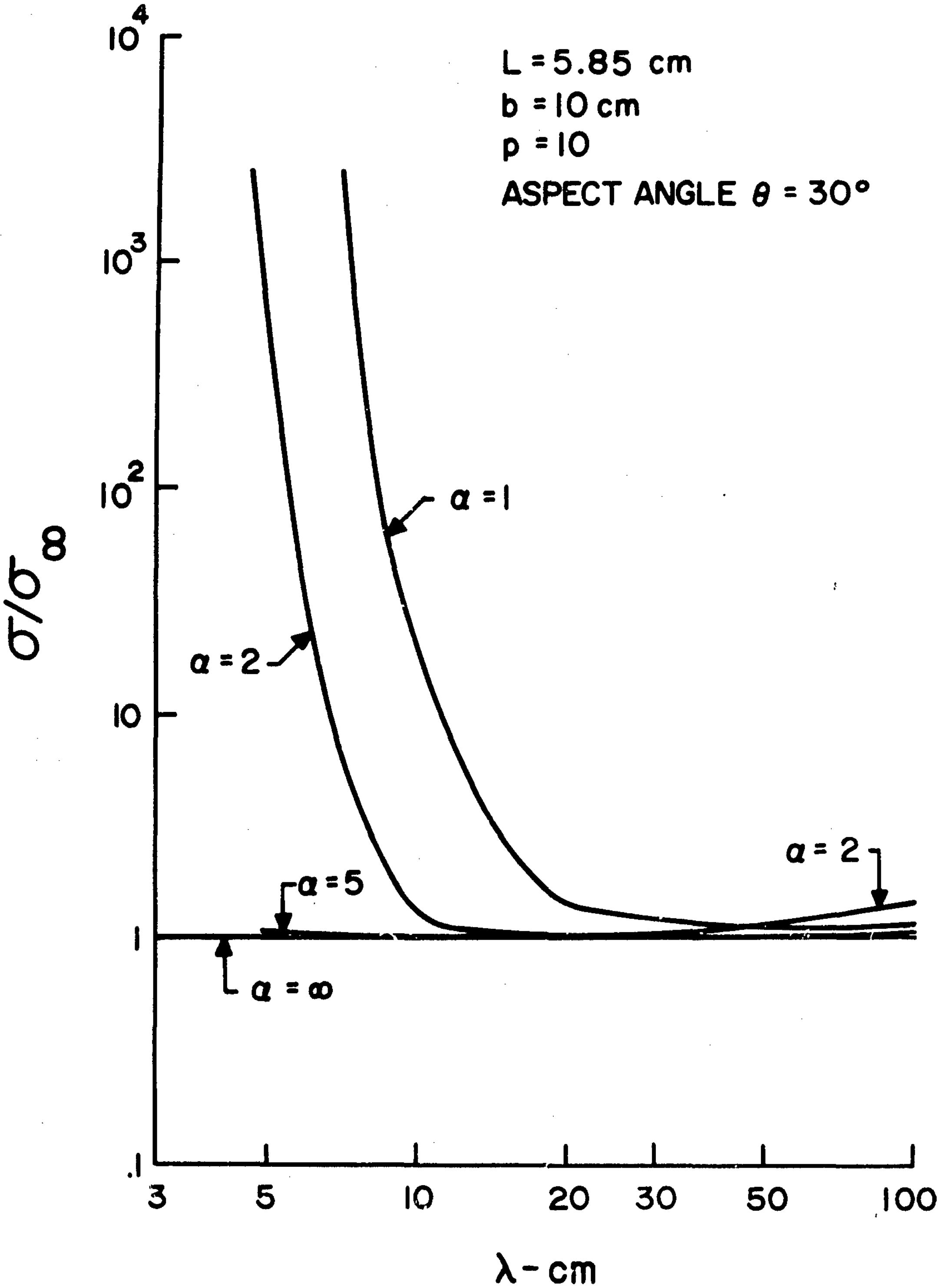

L = 5.85 cm
b = 10 cm
p = 10
ASPECT ANGLE θ = 15°
σ/σ8
10^4
10^3
10^2
10
1
.1
α = 1
α = 2
α = 2
α = 5
α = ∞
3
5
10
20
30
50
100
λ - cm

FIGURE 7



L = 5.85 cm
b = 10 cm
p = 10
ASPECT ANGLE θ = 30°
σ/σ8
10⁴
10³
10²
10
1
.1
α = 1
α = 2
α = 2
α = 5
α = ∞
3
5
10
20
30
50
100
λ - cm

FIGURE 8

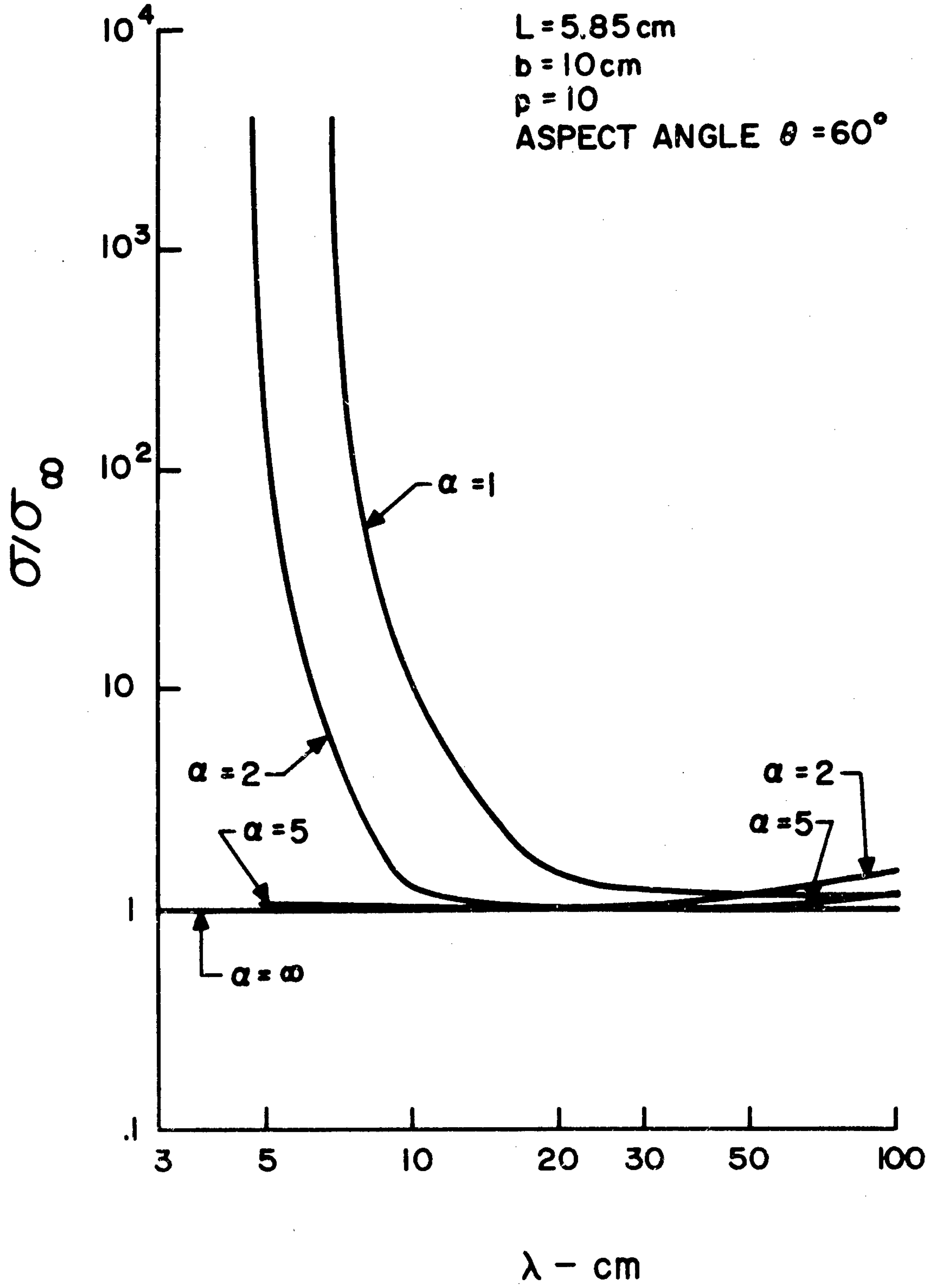

L = 5.85 cm
b = 10 cm
p = 10
ASPECT ANGLE θ = 60°
10^4
10^3
10^2
10
1
.1
σ/σ_∞
α = 1
α = 2
α = 5
α = ∞
α = 2
α = 5
3
5
10
20
30
50
100
λ - cm

FIGURE 9

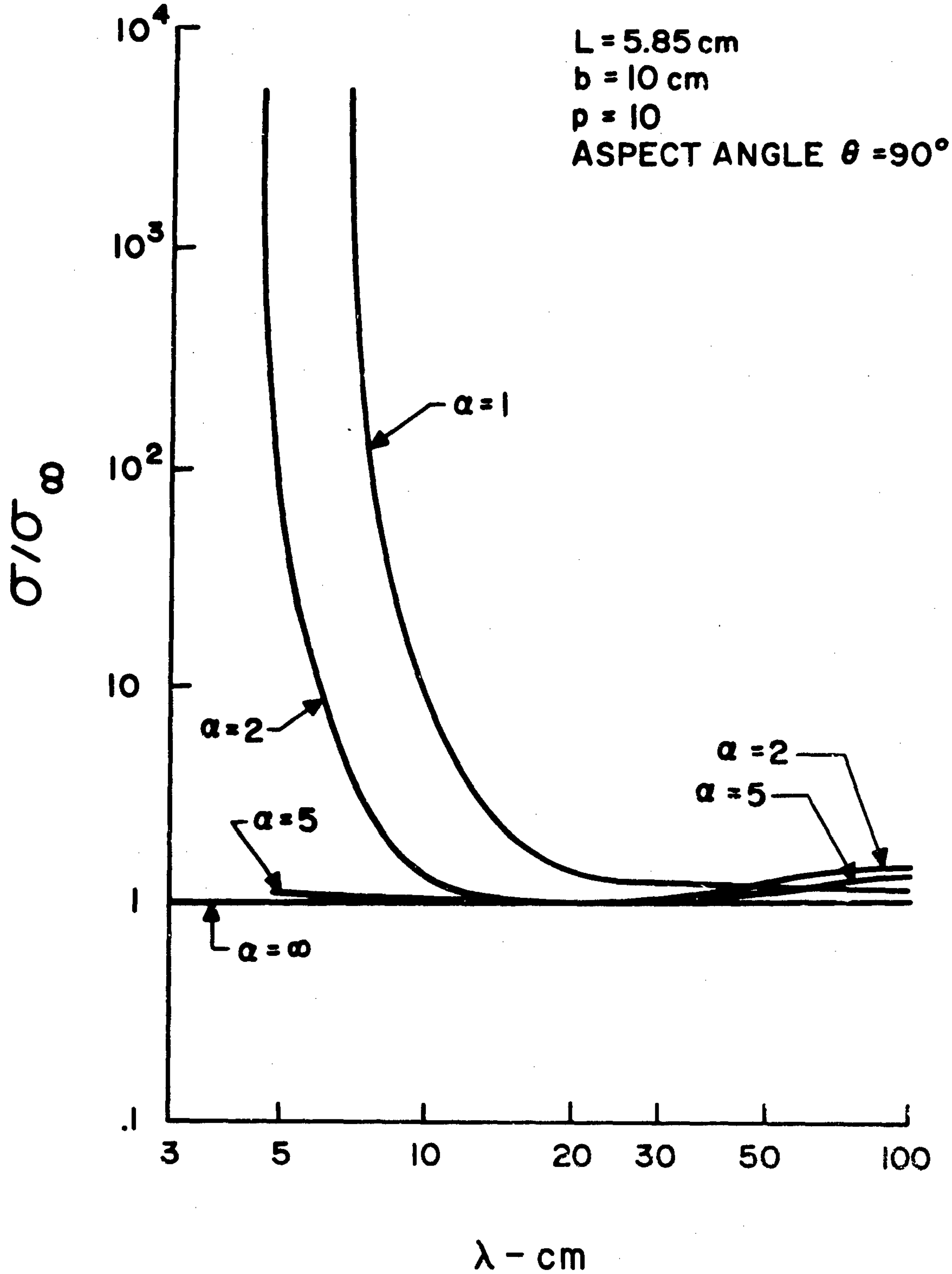
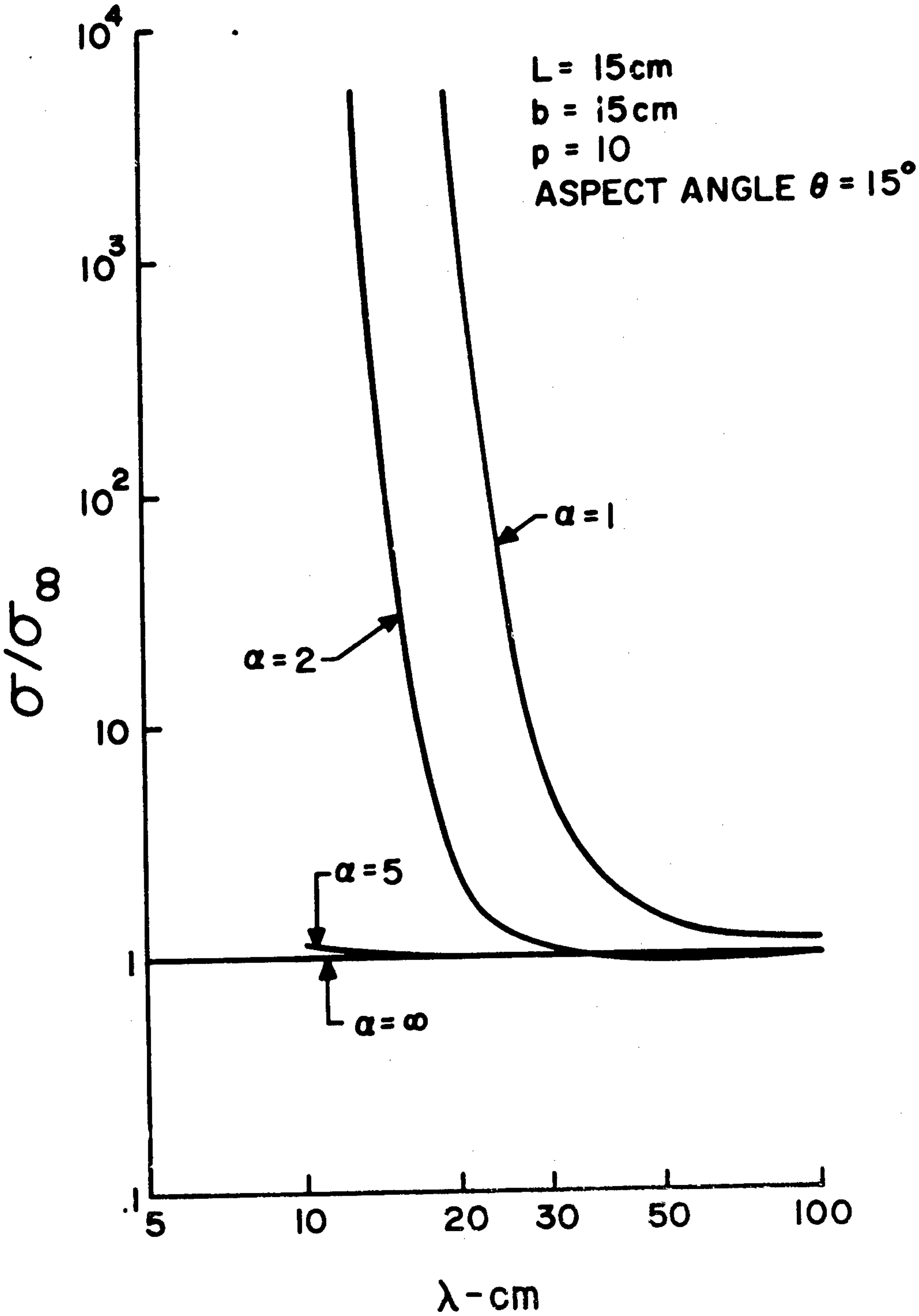

L = 5.85 cm
b = 10 cm
p = 10
ASPECT ANGLE θ = 90°
10^4
10^3
10^2
10
1
.1
σ/σ_∞
α = 1
α = 2
α = 5
α = ∞
α = 2
α = 5
3
5
10
20
30
50
100
λ - cm

FIGURE 10



L = 15cm
b = 15cm
p = 10
ASPECT ANGLE θ = 15°
10^4
10^3
10^2
10
1
.1
σ/σ_8
α = 1
α = 2
α = 5
α = ∞
5
10
20
30
50
100
λ-cm

FIGURE 11

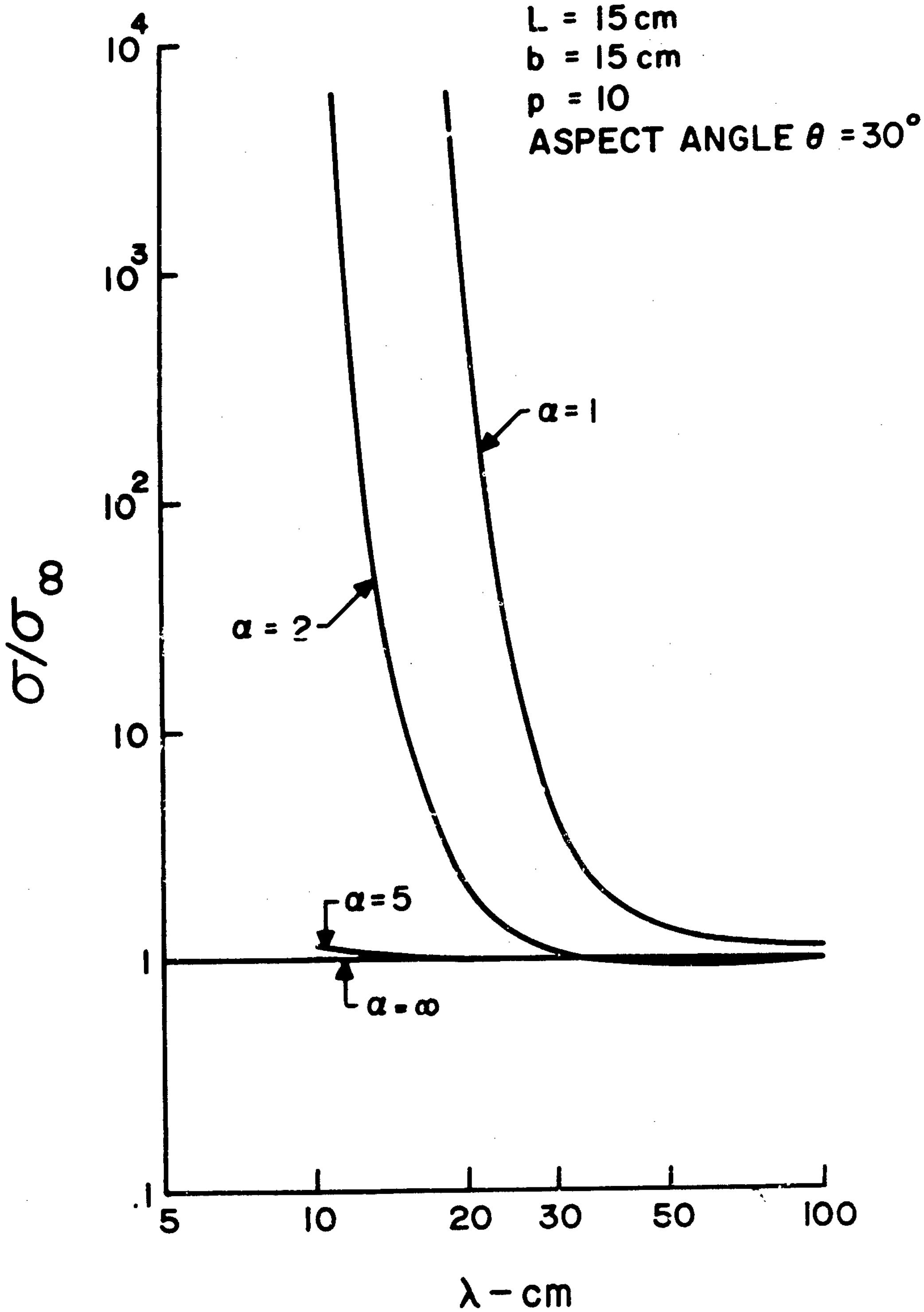
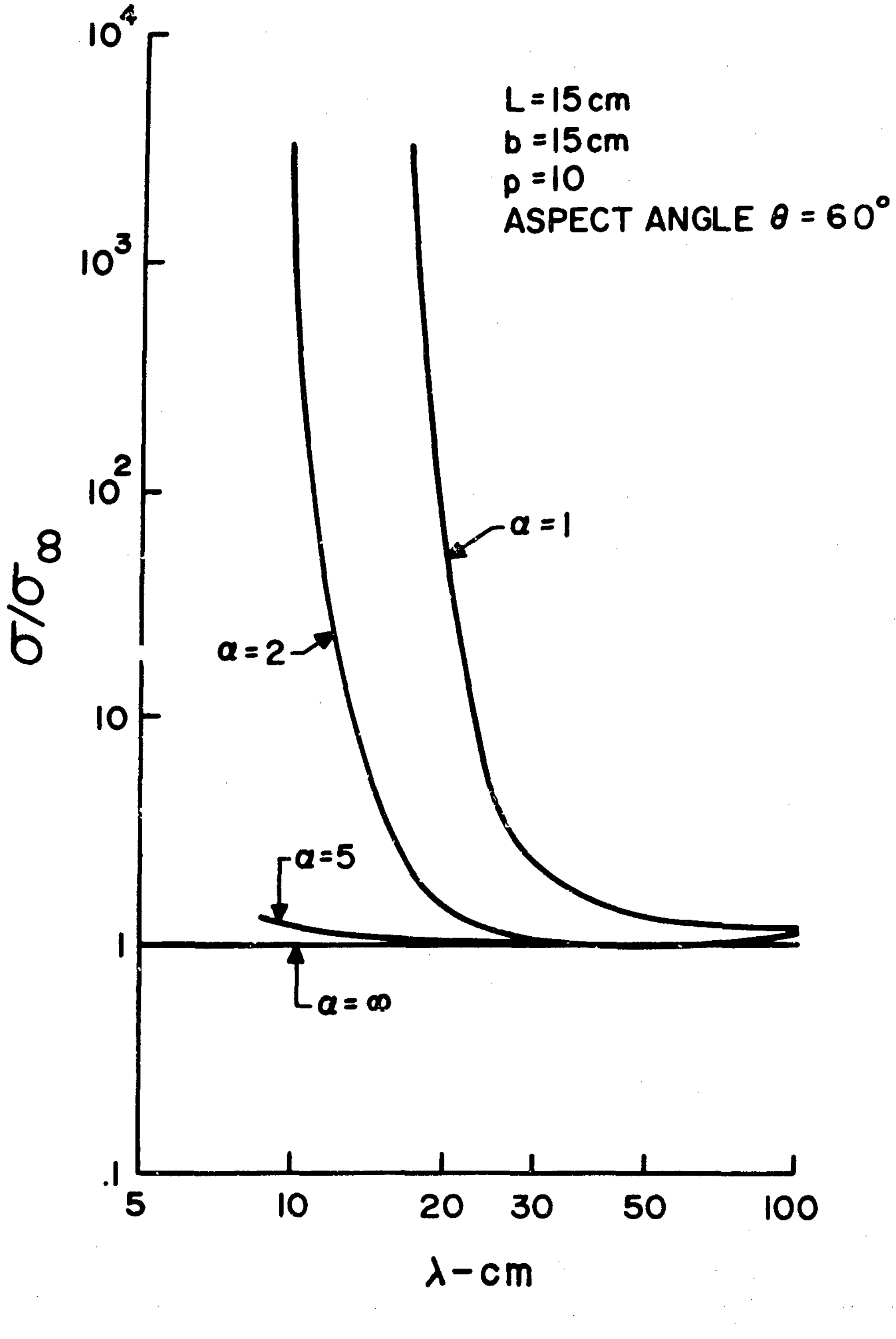
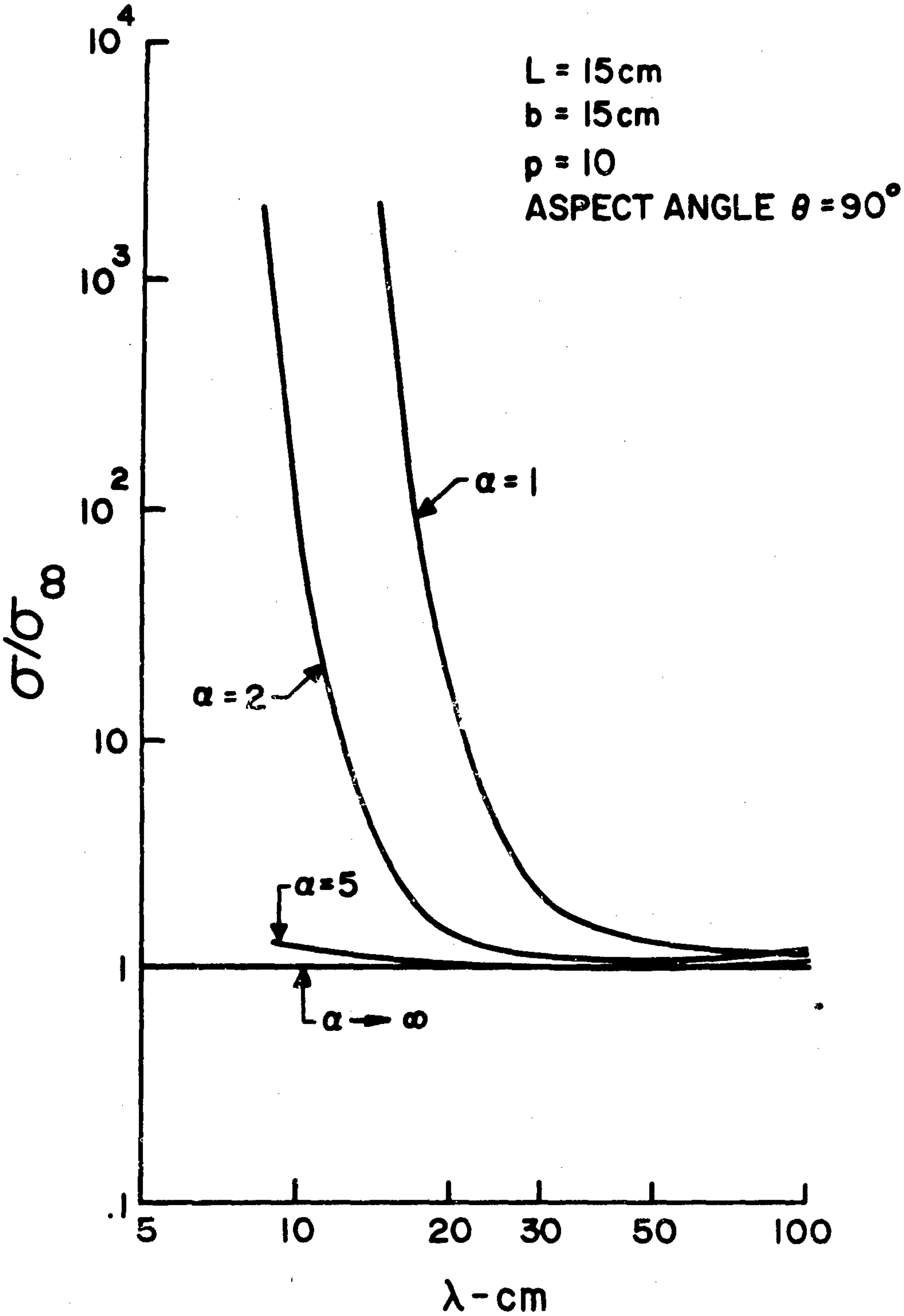

L = 15 cm
b = 15 cm
p = 10
ASPECT ANGLE θ = 30°
10^4
10^3
10^2
10
1
.1
α = 1
α = 2
α = 5
α = ∞
σ/σ8
5
10
20
30
50
100
λ - cm

FIGURE 12



L = 15 cm
b = 15 cm
p = 10
ASPECT ANGLE θ = 60°
10⁴
10³
10²
10
1
.1
σ/σ∞
α = 1
α = 2
α = 5
α = ∞
5
10
20
30
50
100
λ - cm

FIGURE 13



L = 15cm
b = 15cm
p = 10
ASPECT ANGLE θ = 90°
10^4
10^3
10^2
10
1
.1
σ/σ8
α = 1
α = 2
α = 5
α → ∞
5
10
20
30
50
100
λ - cm

FIGURE 14

SPACE SCIENCES LABORATORY
MISSILE AND SPACE DIVISION

GENERAL ELECTRIC

TECHNICAL INFORMATION SERIES

AUTHOR	SUBJECT CLASSIFICATION	NO.
K. T. Yen	Hypersonic Turbulent Wakes	R64SD72
		DATE Nov. 1964
TITLE Effect of Turbulence Intermittency on the Scattering of Electromagnetic Waves by Underdense Plasmas		G. E. CLASS I
		GOV. CLASS None
REPRODUCIBLE COPY FILED AT MSD LIBRARY, DOCUMENTS LIBRARY UNIT, VALLEY FORGE SPACE TECHNOLOGY CENTER, KING OF PRUSSIA, PA.		NO. PAGES 45

SUMMARY In hypersonic turbulent wakes, mixing between the turbulent inner wake and the outer inviscid wake gives rise to the "intermittency" phenomenon. It is shown that electron-density fluctuations of a turbulent nature, in addition to those caused by "turbulence", will be produced by the intermittency phenomenon. These additional fluctuations depend on the mean electron-density distribution and Townsend's intermittency "δ" function.

Application of the above consideration to turbulent scattering by underdense plasma has been made. In contrast to the conventional theory of scattering, two contributions to the scattering cross section are obtained: The first one arises from the intermittency phenomenon and vanishes if the intermittency is omitted; and the second one is due to fluctuations caused by "turbulence". This second contribution also contains the intermittency effect, and reduces to that given by the conventional turbulent scattering theory when the intermittency is not considered.

Numerical results for the scattering cross section have been obtained by using correlation functions and mean electron-density distribution of the Gaussian form. Based on these results, some characteristics of the scattering cross section such as its aspect-angle and frequency dependence are found to be significantly modified by the intermittency. In addition, the magnitude of the cross section will always be larger if the intermittency effects are considered. This increase can be as high as many orders of magnitude in some cases.

KEY WORDS

Turbulent scattering ; Intermittency

BY CUTTING OUT THIS RECTANGLE AND FOLDING ON THE CENTER LINE, THE ABOVE INFORMATION CAN BE FITTED INTO A STANDARD CARD FILE.

AUTHOR K. T. Yen

COUNTERSIGNED H. G. Lew S. M. Scala

H.G. Lew S.M. Scala