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A GRAPHITE RESISTANCE HEATER
FOR A HYPERSONIC WIND TUNNEL
USING NITROGEN

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SUMMARY

A heater system has been developed for a hypersonic wind tunnel which uses nitrogen as the test gas and can operate continuously at Mach numbers up to 20.

A small pilot hypersonic nitrogen tunnel, which is designed to operate ultimately at a gas stagnation pressure of 10,000 psia, has been used in the development of the heater system. To date the operating stagnation pressures have been limited to 1000 psia. (At such a total pressure, the total temperature required to avoid condensation of the nitrogen in the test section at a Mach number of 20 is about $4500°R$.) The investigations have been concerned mainly with the development of the heater unit and no attempt has been made, so far, to establish high Mach number flow in the nozzle. The heater system uses a small electrically heated graphite element containing a spiral heat transfer gas passage.

The major problems encountered in the development have been caused by chemical reactions in the heater involving substances other than pure nitrogen and pure graphite. The purity of the gas supply, the cleanliness of the equipment (including outgassing before use) and the grade of graphite used are of utmost importance. A thin impermeable coating of pyrolytic graphite over the outside of the heater element has prevented the formation of holes in the outer wall, which occurred earlier using uncoated elements. Such coated elements have regularly been used to

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provide steady gas temperatures up to 5000°R. at 1000 psia pressure. The amount of oxidation evident in the heater passage has been small and can be further reduced by using a new grade of graphite which is now available.

An analysis of the heater performance using an energy balance has yielded an empirical formula, in non-dimensional form, which describes the performance of the heater system over its present range of use. It is suggested that the results of this analysis might be useful in the extension of operation to higher gas pressures (10,000 psia), the problems of which are not considered severe. The present unit has application also in the heating of other gases.

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I. INTRODUCTION

In recent years, the interest in hypersonic flight has stimulated the development of experimental tools which can simulate conditions of ultra high speed flight (Mach numbers of the order of 20) in the laboratory. Several new types of test facilities have evolved (References 1 and 2). Such facilities as the "hot-shot" tunnel, the shock tunnel, and the arc or plasma tunnel have been designed to provide the high enthalpies and high temperatures with many of the real gas effects which are experienced in hypersonic flight. However, the flows produced in these facilities are very complex (there are basic questions of flow composition and steadiness) and measurements are difficult to make because of the extremely short running times in the case of the "hot-shot" and shock tunnels (of the order of milliseconds), or the high heat transfer rates in such equipment as arc tunnels. The short running times preclude many of the types of test which are carried out in conventional wind tunnels, but the more serious restriction is the lack of detailed information on the state of the gas in the test section.

A somewhat different approach to the experimental study of hypersonic flows is being followed by the Gas Dynamics Laboratory of

Princeton University. The method here is to try to isolate the fundamental fluid mechanical effects at high Mach numbers by avoiding the complexities of real gas and very high temperature effects, which, as yet, are not completely understood. This can be done using conventional wind tunnel techniques _if_ the test gas which is used behaves as a thermally perfect gas throughout. The problem then becomes one of designing a high Mach number wind tunnel to operate at conditions where there is neither condensation of the test gas during expansion in the nozzle nor dissociation in the stagnation chamber. For a desired test section Mach number, these considerations determine the lower and upper limits to the stagnation temperature which must prevail in any given gas operating at a desired stagnation pressure. If air is the test gas, some form of heater is necessary to avoid condensation in flows at Mach numbers greater than about 5, and a stagnation temperature of the order of $5000^{\circ}R.$ is required to provide a Mach number of 20. At this temperature some real gas effects would be evident but no dissociation or ionization would occur. If helium, which has a very low condensation temperature, is used as the test gas, flows at Mach numbers greater than 25 are obtainable without using a heater. A helium hypersonic tunnel has been in operation at the Gas Dynamics Laboratory of Princeton University since 1950 (Reference 3). Such a tunnel provides a quite simple apparatus for the experimental study of hypersonic flows. Its operation is directly comparable to that of a conventional supersonic blow-down wind tunnel, running times are long and all of the usual wind tunnel measurements can be made with ease. However, since helium is a monatomic gas with a ratio of specific heats, γ equal to 1.67,

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whereas air is a diatomic gas with γ equal to 1.4, the results from tests using helium are not directly applicable to the simulation of flight in air. Although it has been proposed that the results of tests using helium may be used, in some instances, to predict the results which would be obtained using air (Reference 4), the theoretical understanding of hypersonic flows is not adequate, at the moment, to justify any widespread conversion, particularly for complicated shapes. (For simple shapes, the helium tunnel provides a direct way to check theories in which γ can be explicitly included.) In view of these considerations, there is a need for a hypersonic wind tunnel which uses air, or a similar diatomic gas, and which is capable of continuous operation in the range of Mach numbers from 10 to 20.

Mach numbers up to about 14 have been obtained in wind tunnels using various heating systems such as wire electrical heaters (Reference 5) and pebble bed heaters (Reference 6). For higher Mach numbers, requiring gas stagnation temperatures in excess of 3000°R., there exists the difficulty of finding heater materials which will withstand hot oxidizing atmospheres. The solution proposed in 1957 by Dr. A. G. Hammitt[*], was to avoid the oxidation problem by using a gas which has properties similar to those of air and yet is inert to some particular heater material in the relevant temperature range. Nitrogen and graphite appeared to constitute such a desirable combination of test gas and heater material (References 7 to 9). Since

[*] Dr. A. G. Hammitt, now at Space Technology Laboratories, Inc., initiated the present investigations, carried out the original experiments and continued to be associated with the work until June 1960.

graphite sublimes at a temperature greater than 7000°R., the theoretical limit of this combination is set by the chemical reaction of the graphite and nitrogen, which is predicted to occur only when the nitrogen becomes dissociated (References 7 and 9). Significant dissociation of nitrogen begins to occur at 5400°R. at atmospheric pressure and at correspondingly higher temperatures at higher pressures. Consequently, it was thought possible to use a graphite heater with nitrogen in a hypersonic wind tunnel designed to operate at a Mach number of 20, in which the gas total temperature necessary to avoid condensation at a total pressure of 1000 psi is about 4500°R. Experiments began in 1958 directed towards the development of a simple graphite resistance heater which would heat a continuous flow of nitrogen. The method which was adopted was to house a resistance heating element inside a pressure vessel (which resembled the stagnation chamber of a conventional blow down wind tunnel) in such a way that nitrogen supplied to the surrounding space passed through a heat transfer passage in the element before entering the wind tunnel nozzle. A small pilot hypersonic nitrogen tunnel has been used in the development of the heater system. The tunnel is designed to operate ultimately at a stagnation pressure of 10,000 psi, but for the study reported herein the operating pressure has been limited to 1000 psia. At this stagnation pressure, the mass flow of nitrogen through the tunnel at room temperature is about 0.02 pounds per second. Up to the present time, the emphasis has been placed on heater development and only the most preliminary investigations of the flow in the nozzle have been made.

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The experimental development program has involved the
testing of several different element geometries, and several different
grades of graphite have been used. Early in the investigations,
concentration was put on a simple rod-like element containing a single
spiral heat transfer passage which was made from a very dense grade of
graphite. Steady and repeatable operation of the system at gas temp-
eratures suitable for a test section Mach number of 20 has recently
been achieved following the use of some of the most recently developed
grades of graphite, including pyrolytic graphite.

The purpose of the present report is to outline some of the
experimental work and analysis which has been carried out and to present
the performance achieved with the heater system to date. The details of
the experimental development of the heater element are to be found in
References 10 and 11. The design of a hypersonic nitrogen tunnel is
considered and the present equipment is described in Section II of
this report. In Section III the design of the heater element itself
is discussed and some account is given of the development. Some
particular results from heater performance tests are presented in
Section IV with some considerations of how the experimental test
results may be presented in non-dimensional form. In Section V, a
general equation for the heating system is derived using an energy
balance, which is used in Section VI as a basis for an empirical
correlation of the heater performance. The characteristics, diffi-
culties and future possibilities of this type of heater are discussed
in Section VII.

II. THE PILOT HYPERSONIC NITROGEN TUNNEL

Detailed thermodynamic properties of nitrogen are tabulated
for pressures up to 100 atmospheres and for temperatures up to 5400°R.
(Reference 12). These tables are sufficient for the purposes of the
investigation reported herein, but more information will be required
later for operation at higher pressures (Reference 13).

An examination of the tables confirms that, for the combinations
of pressures and temperatures to be expected in the gas flow, it is
reasonable to consider nitrogen as a perfect gas with variable specific
heat. Moreover, the variation of the ratio of the specific heats of
nitrogen with temperature in the appropriate temperature range is very
nearly the same as that for air (Reference 14), and so it is reasonable
to use the extensive tables already available for air as a thermally-
perfect gas to give the isentropic flow and shock wave properties of
nitrogen. This procedure has been followed wherever permissible in the
calculations performed for this work. The values of total (stagnation)
temperature which correspond to saturation of nitrogen in the test
section of a wind tunnel at various Mach numbers and stagnation pressures
are shown in Figure 1. The corresponding values of Reynolds number
attained in the test section are shown in Figure 2.

Molecular nitrogen should have no reactions with pure graphite
until significant dissociation occurs at about 5400°R. at atmospheric
pressure and at correspondingly higher temperatures at higher pressures
(Reference 7). Atomic nitrogen reacts with graphite to produce cyanogen,
an extremely toxic gas. Since dissociation is to be avoided in the
present work, the generation of cyanogen in the present heater system

should not be expected. However, since there is always the possi-
bility that unwanted impurities will get into the system or that
local hot spots might occur, it is possible that the heated gas might
contain harmful contaminants. During the early work, traces of cyanogen
were found and since that time a hydrocyanic acid gas detector has been
kept on hand to monitor the hot test gas (Reference 15). Safety pre-
cautions for cyanogen are discussed in Reference 16.

A general schematic diagram of the pilot hypersonic nitrogen
tunnel is shown in Figure 3. Nitrogen gas from storage cylinders passes
through a drier and a flowmeter into the high pressure stagnation chamber.
The pressure in the chamber (and, therefore, the flow rate) is manually
controlled by a pressure regulating valve. The gas in the chamber then
passes through the graphite resistance element which is clamped between
an electrical contact at one end and a heavy conical copper nozzle at
the other. The hot gas from the graphite heater flows directly through
the nozzle, then through a cooler, and finally through a two-stage air
ejector at the downstream end of the system. A view of the equipment
is shown in Figure 4.

Details of the stagnation chamber and heater assembly are
shown in Figure 5. A solid copper "O" ring is used to provide a gas-
tight seal between the graphite element and the copper nozzle to which
it is clamped. The other end of the element is he d by graphite
collets in a water-cooled electrical contact. The electrical power
leads are connected to the copper nozzle at one end and to the water-
cooled brass tubes of the electrical contact at the other. A
cylindrical radiation shield of graphite coated on the inside with
0.150 inches of pyrolytic graphite is placed around the element.

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The electrical power for the heater system is supplied by three standard arc welding auto transformers from a 480 volt, 3 phase connection. The primaries are connected in delta and the secondaries suitably in parallel, as shown in Figure 6. This arrangement helps to balance the load on the primary, and hence reduce the peak primary line currents. Mechanically, the transformers are coupled by a chain drive to a small reversible A.C. motor (Figure 7) and the setting of the current output (which is independent of the secondary load resistance) is controlled by remotely activating the motor from the control console. A maximum output of 1200 amps at 40 volts is available.

A single precision Bourdon tube Heise gauge is used to read the pressure in the stagnation chamber and in the short chamber between the end of the heater and the nozzle throat. A Brooks high pressure flowmeter indicates the mass flow of gas through the system. A multi-range ammeter, in conjunction with current transformers, and a multi-range voltmeter are used to monitor the power supplied to the heater element. Tungsten-rhenium thermocouples have been used to measure the temperature of the outer wall of the graphite element, the outputs being recorded graphically on Speedomax pen recording machines.

Following the installation of a new heater element the complete system is first evacuated for several hours in order to eliminate the atmosphere which enters during assembly and to outgas the graphite element and other components of the system. The system is then filled with pure nitrogen. The vacuum gate valve is opened and the flow of cooling water started. The regulator is adjusted to give the required stagnation pressure downstream of the heater element, and

corresponding chamber pressure is also recorded. The cold mass flow
is measured by the flowmeter. The current to the element is then
switched on and set to the desired value, while the stagnation pressure
is held constant at the original value. The particular test program
is then carried through, the air ejectors being used when necessary to
lower the gas densities in the nozzle and test section to reduce the
heat transfer. (No attempt is made here to establish high Mach number
flow in the nozzle.) At the completion of the test, the current is
switched off and the system allowed to cool by continuing a small flow
of cold nitrogen. Finally, the stagnation pressure is adjusted to its
value during the test and the cold mass flow is measured as a check on
the size of the nozzle throat.

The stagnation temperature of the nitrogen leaving the graphite
heater is controlled by the current passing through the element. The
stagnation temperature at a given current is estimated from the measure-
ment of mass flow at that current in the following way: The mass flow,
m, through the system can be expressed as

$$m = \frac{\Gamma A^* p_t}{(R T_t)^{\frac{1}{2}}} \qquad (1)$$

where A^* is the effective area of the tunnel throat, p_t is the gas
total pressure, R is the gas constant for unit mass and T_t is the gas
total temperature; Γ is a factor which, for a thermally-perfect gas,
depends on T_t only and is constant for a perfect gas with constant
specific heats. It happens that for nitrogen in the range of tempera-
tures and pressure under consideration, Γ is effectively constant.
Consequently, equation (1) gives T_t explicitly in terms of m. Using
subscript $_o$ to denote values in the cold flow, we have

$$m_0 = \frac{\Gamma A_0^* p_{t_0}}{(RT_0)^{1/2}} \tag{2}$$

where T_O is room temperature. During normal operation, control is exercised so that $p_t = p_{t_O}$ and if we assume that the effective throat area does not change during the test, i e. $A^* = A_O^*$, it follows that

$$\frac{m}{m_0} = \left[\frac{T_0}{T_t}\right]^{1/2} \tag{3}$$

from which the formula for the gas total temperature becomes

$$T_t = T_0 \left[\frac{m_0}{m}\right]^2 \tag{4}.$$

Clearly, this method of temperature estimation depends directly on the assumption that A^* is constant throughout the test, but the limitations of the method can be effectively condensed into this single assumption. The measurement of cold mass flow which is taken after a test shows whether A^* changed during the test as a result of deposition or erosion. The change in A^* due to thermal expansion is calculated to be very small, but so far no direct measurement of this effect has been obtained.

III. THE GRAPHITE RESISTANCE HEATER ELEMENT

Graphite is a material which is becoming of increasing importance in advanced projects because of its unique high temperature properties (Reference 9). As a consequence, several new types of graphite have recently been developed, such as pyrolytic graphite and impregnated graphites (References 18 to 20) and these are proving to be useful in the present investigations. Graphite sublimes directly from the solid to the vapor phase at $7200^\circ R$. at atmospheric pressure. Although it has a high creep resistance, at high temperatures it does tend to creep under stress rather than to fracture, and it can withstand severe thermal shock. It is comparatively cheap and can be joined and fabricated fairly readily.

The present design of the heater system is only one of a number of forms which might have been developed to do the same job. Resistance heating was chosen on account of its basic simplicity and to enable the use of a cheap form of electrical power supply. With the method of heating and the heater material fixed, there still remains a wide range of possible element geometries; the overall size and shape of the element and the type of heat transfer passage must be selected. Several factors affect the choice of element geometry: firstly, the power generated within the element by the maximum available current must be sufficient to heat the required mass flow of gas; secondly, the structural properties of graphite influence the choice of wall thickness and overall configuration; thirdly, joints between graphite sections must either be made by threading or by using a graphite cement, and these joints may not be impermeable to gases.

Exploratory tests to evaluate the usefulness of different
types of element geometry and different grades of graphite (Reference 10)
resulted in the adoption of a simple design consisting of a cylindrical
rod of high density graphite containing a simple spiral gas passage.
The single heat transfer passage eliminates many of the stability
problems which are associated with multiple passage designs
(Reference 10) and the spiral configuration offers several advantages:
a long continuous passage is obtained in a small and rugged element;
the gas flow, in circulating around the element, will reduce any
tendency for the current to channel down one side (Reference 21);
and there is an improvement in heat transfer in a spiral passage com-
pared with that in a straight passage (References 22 and 23). Since
the length of heat transfer passage required is much less for turbulent
flow than for laminar flow, care in design has to be taken to select
passage dimensions which ensure that the flow is turbulent (Reference 22).

A guide to the design of the element was found in an extension
of heat transfer results in a straight-through constant-area passage,
(Reference 24) using an empirical correction to take account of the
spiral (Reference 22). The results of calculations for conditions of
constant wall temperature and of constant heat flux to the gas are
shown in Figure 8. It is likely that the condition of heating exist-
ing in the element (away from the ends which have to be cooled) lies
somewhere between these two. From Figure 8, a value of the ratio of
passage length to equivalent diameter can be chosen. Preliminary tests
indicated a range of possible values of overall length and diameter of
element which could be conveniently machined and which gave a suitable
resistance for the available electrical power supply. A gas passage
having a rectangular cross-section was chosen because its equivalent

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diameter was smaller than for a square section with the same area. To avoid large pressure drops, the area of the gas passage must be such that the gas in the spiral has a low Mach number, and for the dimensions chosen the maximum Mach number in the passage was estimated to be about 0.04. From such considerations, the shallow spiral shown in Figure 9 was selected. The passage length is approximately 27 inches, with an equivalent diameter of 0.08 inches. The design operating condition for this element is shown in Figure 8 for a stagnation pressure of 1000 psia. When operating at this stagnation pressure, the pipe Reynolds number based on the equivalent diameter of the passage is 76,500 when the gas is at room temperature and 11,800 at 5000°R. Since the transition pipe Reynolds number for this spiral is about 8,500 it is believed that the flow through the heater element is turbulent (Reference 22).

The element, shown in Figure 10, is constructed in two parts which are machined from blocks of high density graphite. A spiral groove is machined in a cylindrical rod which is slipped into a cylindrical shell, and the two sections, fitted by hand to have good contact throughout their length, are held together by a small amount of graphite glue applied to the joint near the beginning of the spiral passage (Figure 9). The ends of the element are designed specially to reduce conduction losses to the cooling systems while still maintaining good electrical contact. The heated gas leaves the spiral passage through a filter of twelve small holes to eliminate the swirl introduced by the spiral.

Extensive tests using single spiral elements made from different grades of graphite (National Carbon Company) revealed a key problem. Holes developed in the thin outer walls of the elements as a result of chemical reactions, involving impurities within the porous

spaces of the graphite wall. (The graphites used in these tests were not completely impervious and some gas entered the spiral passage through the outer wall due to the pressure difference arising from the pressure loss in the passage.) Time histories of the element wall temperature distributions, obtained using tungsten-rhenium thermocouples recording on self-balancing potentiometers, showed that the appearance of a hole marked the end of the useful life of the element (Reference 10). In an attempt to prevent this early failure from occurring, a series of elements, manufactured from Grade ATJ (National Carbon Company) graphite, were coated over the outside wall with an impermeable layer of pyrolytic graphite, nominally 0.005 inches in thickness. (The coating process was carried out by High Temperature Materials, Inc. of Boston.) Further, a supply of very high purity nitrogen, containing a total or six parts per million of oxygen and water vapor, was procurred. During subsequent tests using coated elements, no holes appeared in the walls of any of these elements during repeated operation, under steady conditions, at gas temperatures greater than $4000^{\circ}R$. The performance obtained using coated elements is typical of that obtained during the most recent tests using elements made from newer grades of very dense graphite. Some results from tests using coated elements, which had the geometry shown in Figure 9, are presented in the following sections as an illustration of the performance of the present heater system.

IV. SOME RESULTS FROM HEATER PERFORMANCE TESTS

Four coated elements, as described in Section III, were
tested in turn. During each test, readings were taken at several
distinct current settings before operating continuously for five
minutes at the highest current setting. Heater 2 was operated for a
total of 25 tests (which implies a total running time at gas tempera-
tures over 3000°R. of over 2 hours and at gas temperatures over 4000°R.
of over 1 hour). For this heater the results from tests 1, 5, 10, 15
and 20 are presented. Heaters 1, 3, and 4, were each used several times
but results are given for only one test of each. The elements were
nominally identical and the stagnation pressure was always 1000 psia.
During the 25 tests using Heater 2, the cold mass flow measurement taken
between tests indicated some slight reduction in throat size resulting
from the deposition of a film of solid material which appeared to contain
graphite. A comparison of the measurements of cold mass flow taken
before and after the series of tests indicated the film was less than
0.0005 inches thick and was thought to arise as a result of the presence
of impurities in the system. Supporting evidence of this was supplied
by the way in which the deposits occurred. There was occasionally a
definite reduction in mass flow after o e test and not after the next,
even though the latter test might have been a repetition of the former
one, or a test carried out at increased values of current and conse-
quently gas temperature. More significant deposits have been observed
in the contraction ahead of the nozzle throat on previous occasions
when appreciable quantities of impurity have been known to be present
in the heater system (Reference 10). Further, after a series of tests,
a light film of graphite dust has generally been found in the corners
of the spiral passage, which is likely to be the result of oxidation

of the graphite by the small amounts of oxygen and water in the
nitrogen supply. The amount and detailed appearance of the dust
depend on the graphite which is used, and it is probable that the
grain size and oxidation resistance of the material are major influences.

The simplest way of expressing the performance of the heating
system is in the form of a graph of gas total temperature, T_t, against
current, i, for a given total pressure. The performance obtained in
tests using Heater 2 at a gas stagnation pressure of 1000 psia is shown
in Figure 11.

In order to express the performance of the system in a more
significant fashion, it is desirable to use non-dimensional quantities
instead of simply temperature and current and it is desirable to plan the
experimental procedure so as to obtain sufficient information to present
the results in non-dimensional form. The non-dimensional version of the
dependent variable T_t is taken to be τ defined by

$$\tau \;=\; \frac{T_t - T_o}{T_o} \tag{5}$$

The non-dimensional version of the independent variable i is taken to be
χ defined by

$$\chi \;=\; \frac{r_o\, i^2}{m_o\, c_{p_o}\, T_o} \tag{6}$$

where r_o, m_o and c_{p_o} are the values of the element resistance, the mass
flow of gas and specific heat of the gas, all taken at room temperature.
From equations (4) and (5), we have also

$$\tau \;=\; \left(\frac{m_o}{m}\right)^2 - 1 \tag{7}.$$

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To be able to obtain experimentally the plot of the total temperature parameter, τ , against the current parameter, κ , for a heater element it is necessary to determine the resistance when cold, r_o. It was found impossible to measure r_o accurately because of the presence of contact resistances which, when the system is cold, are sometimes of the same order as the element resistance. Consequently, an extrapolation procedure is used which is described in detail in Reference 11. The scheme for experimentally determining the performance of an element is the following: in the cold flow, m_o and T_o are measured and hence C_{p_o} obtained; at different current settings, i, m and v are measured, where v is the voltage across the element, and $r = v/i$ is calculated; r is plotted against τ and the curve extrapolated to room temperature to obtain r_o; $\kappa = r_o i^2/m_o c_{p_o} T_o$ is calculated and then τ is plotted against κ .

Results for the four elements are shown in Figure 12. The values of cold resistance, r_o, and the cold mass flow, m_o, for the con- sidered tests are given in Table I. It is seen from the values of r_o that the elements did not initially have the same cold resistances, moreover, that the cold resistance of Heater 2 progressively increased with the number of times it was used. The effect of the film which formed in the orifice on the cold mass flow for Heater 2 is seen in the given values of m_o. Also recorded in Table I is the pressure drop parameter, ω_o , which we define as the ratio of the cold pressure drop down the element to the stagnation pressure (always 1000 psia). The values of ω_o in the first test of each heater were similar but for Heater 2 decreased progressively with the number of tests performed. This effect has been observed with all coated elements which have been

tested, but has not been found in the case of elements made completely

from one type of graphite. The decrease in pressure drop resulted

from a loosening of the spiral section within the outer shell of the

heater element, which is probably an effect of creep. (Creep would

occur as a result of thermal stresses arising from the different coeffi-

cients of expansion of the pyrolytic graphite coating and of the

base graphite.)

A plot of the overall efficiency of the heater system, η

which is defined as the heat received by the gas to the energy put

into the system, against τ is shown in Figure 13. η decreases

from a cold value of about 0.8 to about 0.59 at gas temperature of 5000°R.

V. SOME THEORETICAL CONSIDERATIONS

The present heating system, under steady conditions, can be represented quite generally by a control volume through which flows a mass flow of gas, m, and an electric current, i. The gas enters with specific enthalpy h_0 and leaves through a choked orifice with specific enthalpy h_t. Power is put into the system equal to i^2r, r being the total resistance inside the control volume and heat is lost at a rate, $\dot{Q}$. There is no external work done.

The energy balance for the system is then

$$i^2 r = m \left(h_t - h_0 \right) + \dot{Q} \tag{8}$$

which may be non-dimensionalized by dividing by $m_c c_{p_0} T_0$ to give

$$\lambda = \mu + \nu \tag{9}$$

where

$$\lambda = \frac{r\, i^2}{m_0 c_{p_0} T_0} \tag{10}$$

$$\mu = \frac{m \left(h_t - h_0 \right)}{m_0\, c_{p_0}\, T_0} \tag{11}$$

and

$$\nu = \frac{\dot{Q}}{m_0\, c_{p_0} T_0} \tag{12}.$$

If v: assume the gas is thermally perfect, the quantity $\left(\dfrac{h_t - h_o}{c_{p_o} T_o}\right)$ is a function of τ only, and for the range of temperature of interest it may be assumed that it can be represented analytically by the formula

$$\frac{h_t - h_o}{c_{p_o} T_o} = \tau \left(1 + k\,\tau\right) \tag{13}$$

where k is a small constant which represents the departure of the gas from a calorically-perfect gas; a study of the enthalpy of nitrogen leads to the choice of $k = 0.0185$, for $T_O = 530^\circ R$. Therefore, substituting from equations (7) and (13) in equation (11), it follows that for a given thermally-perfect gas, μ depends on τ only and is explicitly given by

$$\mu = \left(1 + \tau\right)^{-\frac{1}{k}} \tau \left(1 + k\,\tau\right) \tag{14}.$$

Nothing has been said about the way in which the losses occur but we choose to write (Reference 11),

$$\nu = \left(\frac{1}{\alpha} - 1\right)\tau \tag{15}.$$

This is quite permissible since we do not put any restrictions on α. The reason for writing the coefficient of τ in this way is that α is then simply related to the efficiency η of the heating system. $\eta = \mu / \lambda$, and it may easily be shown that, if η_o denotes the limiting value of η in the cold flow, then $\lim\limits_{\tau \to 0} \alpha = \eta_o$, so that α is such that its initial value is the initial efficiency of the system.

Hence, from equations (9), (14), and (15), the energy balance gives

$$\lambda = \left(1 + \tau\right)^{-\frac{1}{2}} \tau \left(1 + k\tau\right) + \left(\frac{1}{\alpha} - 1\right)\tau \quad (16).$$

Now, from the definition (6) and (10), it follows that,

$$\lambda = \beta \, \kappa \quad (17)$$

where the quantity β is given by

$$\beta = \frac{\gamma}{\gamma_0} \quad (18)$$

and is regarded, like the efficiency parameter, α , as a quantity which depends on everything.

By combining equations (16) and (17), we have finally

$$\kappa = \frac{1}{\beta}\left[\left(1 + \tau\right)^{-\frac{1}{2}} \tau \left(1 + k\tau\right) + \left(\frac{1}{\alpha} - 1\right)\tau\right] \quad (19)$$

which is a general expression for the performance of the heating system containing two completely unknown quantities, α and β .

VI. AN EMPIRICAL CORRELATION OF HEATER PERFORMANCE

To be able to use the energy balance equation (19) to obtain a useful expression for the performance of the system, empirical expressions for α and β are sought from experimental test results in the following way: β is calculated from equation (18) and λ from equation (17); α is obtained from equation (16) and plots are prepared of α and β against τ . The plots obtained using the results from tests of Heaters 1, 2, 3 and 4 are given in Figures 14 and 15. From Figure 14, it is clear that in the range of τ covered in these tests a surprisingly close representation of the results is obtained if the assumption is made that

$$\underline{\alpha \text{ is independent of } \tau} \tag{20}.$$

It is noted that the constancy of α has been observed in the results of all tests of all heater elements ever tested (more than 40) with one notable exception; on one occasion a heater element was tested without a radiation shield around it, and in this case α decreased by more than ten percent between $\tau = 1$ and $\tau = 6$.

The results for β which are plotted in Figure 15 show more scatter than the corresponding results for α . This is not unexpected since β is sensitive to detailed element geometry and is particularly dependent on the material. The variation of β with τ will largely depend on the form of the variation of the resistivity of the graphite with temperature (Reference 11). The analytical representation which is found to fit closely the results for the coated elements is of the form

- 22 -

$$\beta = \frac{1 + a\left(\tau + \tau^2\right)}{1 + b\left(\tau + \tau^2\right)} \ , \ \text{a and b independent of } \tau \qquad (21).$$

Figure 15 shows this expression to adequately represent the test results over the range of τ for which results were obtained; the inherent errors involved in measurements in the region $\tau = 0$ does not allow a close investigation in this region.

Thus it is possible to obtain an empirical correlation of the present heater performance by incorporating the empirical results (20) and (21) into the theoretical formula (19) to give

$$\chi = \left[\frac{1 + b\left(\tau + \tau^2\right)}{1 + a\left(\tau + \tau^2\right)}\right]\left[\left(1 + \tau\right)^{-\frac{1}{2}}\tau\left(1 + k\tau\right) + \left(\frac{1}{\alpha} - 1\right)\tau\right] \qquad (22)$$

where k depends only on the particular gas, a and b are independent of τ for a given element. A comparison of the curves from equation (22) with the previously given experimental points is shown in Figure 16.

A possible use for the empirical equation (22) appears in extending the use of the heater system to operation at other pressures. First it is necessary to find empirically the form of the β v. τ variation which best fits the particular graphite which is used for the element. It is then necessary only to seek the variation of α , and in the present case a and b, with stagnation pressure (or some non-dimensional representation of it), to obtain an extended correlation formula. In its present form, the empirical representation of the heater performance has been very useful in clarifying the experimental results.

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VII. CONCLUDING REMARKS

Using graphite resistance elements coated externally with pyrolytic graphite, steady operation of the heating system of the pilot hypersonic nitrogen tunnel has been achieved for periods of over five minutes at gas temperatures up to $5000^{\circ}R$. (The operating times were limited by the amount of compressed air available for the ejector system.)

Many difficulties which were encountered in the development of the system are thought to have been caused by chemical reactions in the heater involving substances other than pure nitrogen and pure graphite. The formation of holes in the outer walls of elements used in earlier tests was probably a result of chemical reactions taking place during the passage of impurities through a permeable graphite wall. An impermeable coating of pyrolytic graphite over the outer wall of the element has been entirely successful in preventing the occurrence of these holes, and further significant improvements in heater performance have been obtained as a result of attention being concentrated on methods of eliminating all contamination from the system. The use of very high purity nitrogen, careful procedures for cleaning and handling components, and outgassing of the system before use, are considered necessary to obtain optimum performance. Since there will inevitably be traces of impurities which cannot be entirely eliminated, a dense graphite with high resistance to oxidation should be used so that the effects of any impurities will be minimized. (Oxidation within the element can result in the contamination of the nitrogen stream by solid graphite particles, and in the reduction of the size of the nozzle throat by the deposition of products of chemical reaction. Both these effects are highly undesirable from the point of view of conducting aerodynamic tests.)

A coating of pyrolytic graphite over all surfaces of the element would provide excellent resistance to oxidation, but the technical problems of such a solution have not yet been fully explored. Another promising possibility is offered in the use of a relatively new material, recrystallized graphite Type ZTA (National Carbon Company), which has a very high resistance to oxidation, high density and very low permeability (compared with Grade ATJ). Since the permeability of Grade ZTA is low, and moreover since it can be reduced to near that of pyrolytic graphite by a process of impregnation, it may be possible to use uncoated elements made from this material. Tests are presently being performed using Grade ZTA graphite and the initial results are very promising.

More development work is required to extend the operation of the heater system to higher stagnation pressures (around 10,000 psia) so that the densities and Reynolds numbers in the tunnel test section can be varied over a wide range. Problems concerned with the structural soundness of the element and the current-carrying capacity of the electrical contacts might occur, but these are not likely to be severe. The copper throat of the nozzle will probably melt when the flow there ceases to be laminar, but the ultimate solution to this problem is available in the use of the same material for the throat as for the heater element itself. An attractive possibility is offered in the use of a pyrolytic graphite throat which can be manufactured using a plating process. The present method of estimating gas total temperature, involving the metering of mass flow, might present experimental difficulties at higher pressures and some of the simplifying assumptions and approximations used in the analysis of the present

heater performance may have to be modified. However, the empirical formula with a theoretical basis which describes the performance of the present system at 1000 psia stagnation pressure, may well help greatly in any such extension to higher pressures.

The present heater can be used to heat any inert gas stream (but care must be taken to ensure that properties used in the present work which are peculiar to nitrogen, are not used for other gases). For example, a similar unit has been used with freon in experiments concerned with temperatures of dissociation (Reference 25) and with helium. If helium is sed, the heater can supply gas temperatures ($3000^{O}R.$) which are sufficient to avoid condensation in an expansion to Mach numbers greater than 60, and can also supply gas conditions simulating stagnation point heat flux of an ICBM on re-entry.

It may be said in conclusion that, with the development of a successful heater unit for use with nitrogen, a major problem in the design of a truly continuous wind tunnel to operate with a di-atomic gas at Mach numbers up to 20 has been solved. Also, the experience which has been obtained in the development of this unit will have application in the design of a similar system for heating air as new non-oxidizing materials become available.

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NOMENCLATURE

a   coefficient in the numerator of assumed expression for β

b   coefficient in the denominator of assumed expression for β

c_{p_O} value at room temperature T_O of specific heat of gas at constant pressure; 261.7 joules per pound mass ^{O}R. for nitrogen at 530°R.

h_t specific enthalpy of gas; joules per pound mass

h_O specific enthalpy of gas at temperature T_O; joules per pound mass

i   current; amps

k   constant used in representation of $(h_t-h_O)/c_{p_O}T_O$ as a function of τ; 0.0185 for nitrogen when $T_O = 530^O$R.

m   mass flow of gas; pound mass per second

m_O cold mass flow of gas; pound mass per second

p_t total pressure of gas; pound weight per square inch

p_{t_O} total pressure of gas in cold flow; pound weight per square inch

r   resistance of element; ohms

r_O cold resistance of element; ohms

A*   area of nozzle throat; square inches

A^*_O area of nozzle throat in cold flow; square inches

$\dot{Q}$ rate of heat loss from heater system; joules per second

R   gas constant for unit mass of gas

T_O room temperature, value of T_t in the cold flow; ^{O}R.

T_t total (stagnation) temperature of gas; ^{O}R.

α efficiency parameter, defined such that $\nu = \left(\dfrac{1}{\alpha} - 1 \right) \varkappa$

β resistance parameter, defined as r/r_o

γ ratio of specific heat at constant pressure to specific heat at constant volume for a gas

η efficiency of heater system, equal to μ/λ

η_o limiting value of η in the cold flow

$\varkappa$ current parameter, defined as $i^2 r_o/m_c c r_o T_o$

λ non-dimensional energy input into the heater system, equal to $i^2 r/m_o c_{p_o} T_o$

μ non-dimensional energy input into the gas, equal to $m(h_t - h_o)/m_o c_{p_o} T_o$

ν non-dimensional rate of heat loss from the heater system, equal to $\dot{Q}/m_o c_{p_o} T_o$

τ total temperature parameter, defined as $T_t - T_o/T_o$

ω_o pressure drop parameter, defined as the cold pressure drop in the heater passage divided by p_t

Γ non-dimensional factor which, for a real gas, depends on temperature and pressure

Extra symbols used in Figure 8:

A_p cross sectional area of the gas passage; square inches

d_e equivalent diameter of the gas passage, inches $\left(= \dfrac{A_p}{\text{perimeter of cross section}} \right)$

d_s mean diameter of spiral passage; inches

ℓ total length of the gas passage, inches

T_{t_1} total temperature of gas entering heat transfer passage; ^{o}R.

T_{t_2} total temperature of gas leaving heat transfer passage; ^{o}R.

T_p temperature of passage wall where gas total temperature is T_{t_2}; ^{o}R.

μ_p viscosity of gas at temperature T_p; pounds mass per inch second

	Test No.	r_O (ohms)	m_O (lbs /sec)	ω_O
Heater 1.	1	0.057	0.0186	0.045
Heater 2.	1	0.063	0.0186	0.040
	5	0.064	0.0181	0.035
	10	0.065	0.0178	0.031
	15	0.070	0.0176	0.028
	20	0.072	0.0176	0.026
Heater 3.	1	0.052	0.0187	0.041
Heater 4.	1	0.058	0.0184	0.045

Table I: Data of tested heater elements

BIBLIOGRAPHICAL CONTROL SHEET

1. Originating agency and monitoring agency:

   O.A.: Princeton University, Department of Aeronautical Engineering
   M.A.: Fluid Mechanics, Air Force of Scientific Research

2. Originating agency and monitoring agency report numbers:

   O.A.: Princeton University Report 560
   M.A.: AFOSR 1028

3. Title and classification of title: A GRAPHITE RESISTANCE HEATER FOR A HYPERSONIC WIND TUNNEL USING NITROGEN (unclassified)

4. Personal authors: Raymond P. Shreeve, W. Trevor Lord, Simon J. Boersen and Seymour M. Bogdonoff

5. Date of report: June, 1961

6. Pages: 30

7. Illustrative material: 16 figures

8. Prepared for contract number: AF 49(638)-709

9. Prepared for code number: 9781

10. Security classification: Unclassified

11. Distribution limitations: None

12. Abstract: A heater system has been developed for a hypersonic wind tunnel which uses nitrogen as the test gas and can operate continuously at Mach numbers up to 20.
A small pilot hypersonic nitrogen tunnel, which is designed to operate ultimately at a gas stagnation pressure of 10,000 psia, has been used in the development of the heater system. To date the operating stagnation pressures have been limited to 1000 psia. (At such a total pressure, the total temperature required to avoid condensation of the nitrogen in the test section at a Mach number of 20 is about $4500^{\circ}R$.)
The investigations have been concerned mainly with the development of the heater unit and no attempt has been made, so far, to establish high Mach number flow in the nozzle. The heater system uses a small electrically heated graphite element containing a spiral heat transfer gas passage.
The major problems encountered in the development have been caused by chemical reactions in the heater involving substances other than pure nitrogen and pure graphite. The purity of the gas supply, the cleanliness of the equipment (including outgassing before use) and the grade of graphite used are of utmost importance. A thin impermeable coating of pyrolytic graphite over the outside of the heater has prevented the formation of holes in the outer wall, which occurred earlier using uncoated elements.
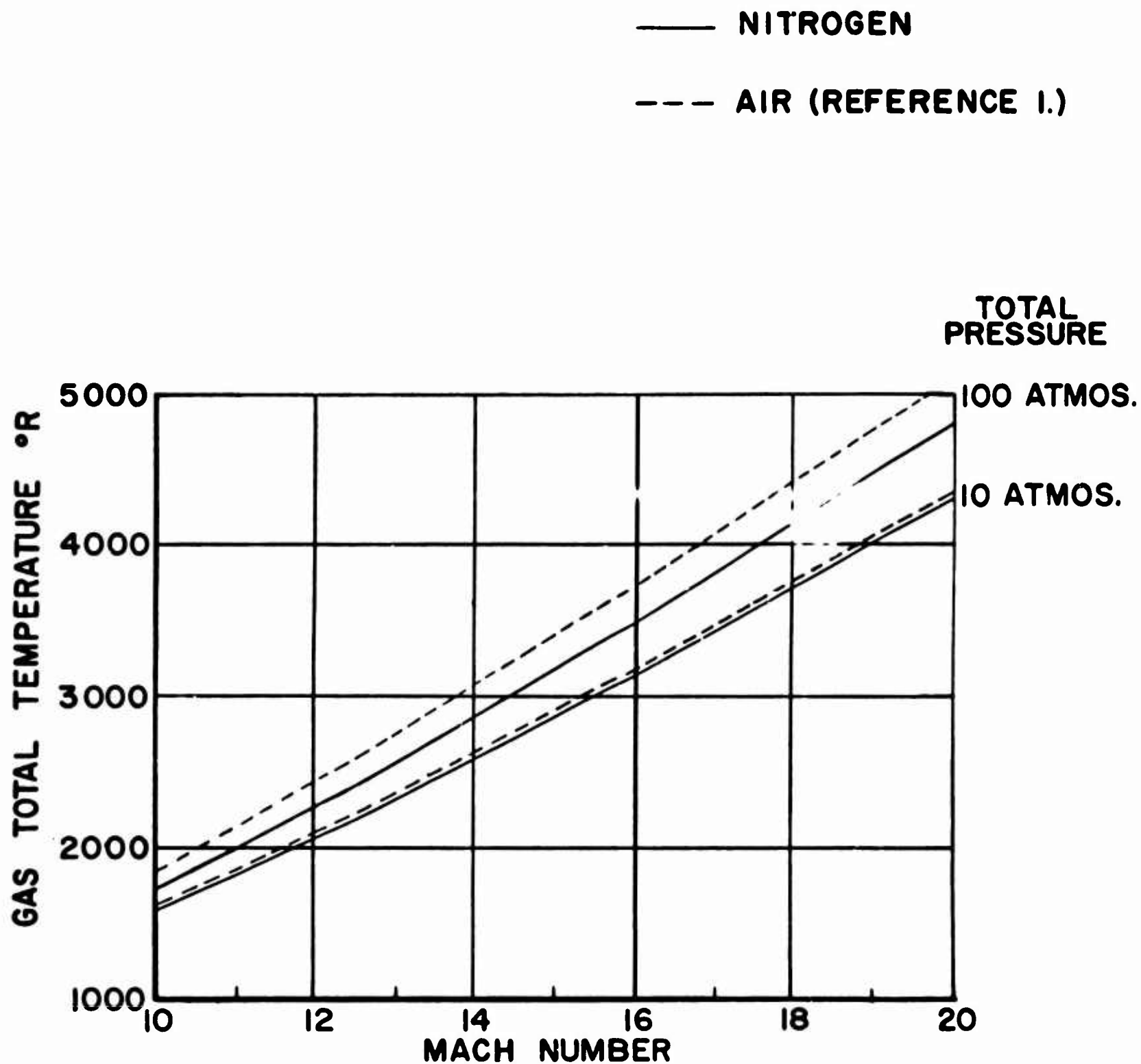
FIG. I TOTAL TEMPERATURES FOR SATURATION
OF NITROGEN IN THE TEST SECTION
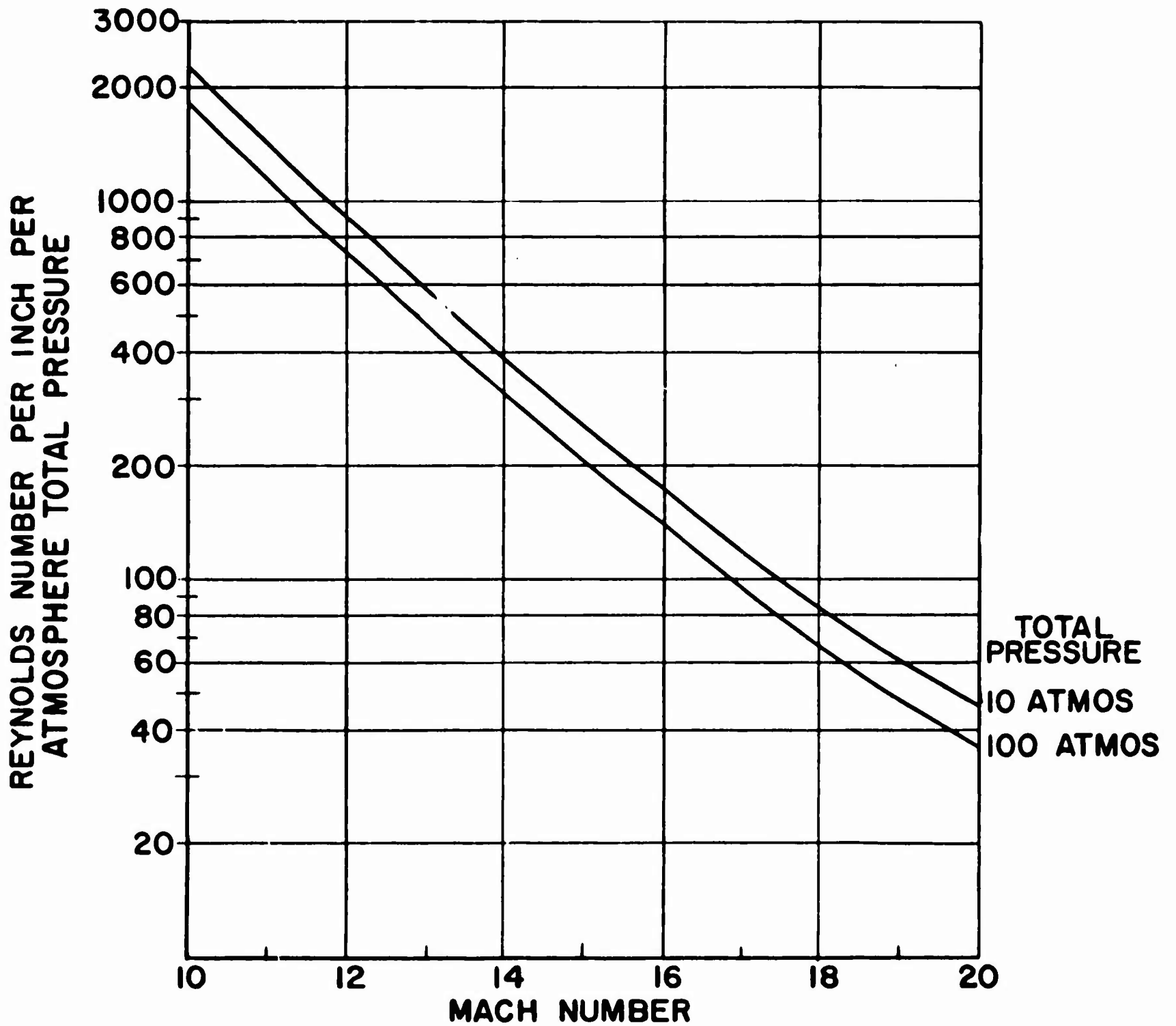AT VARIOUS MACH NUMBERS
AND TOTAL PRESSURES

XIII A-I

FIG. 2 TEST SECTION REYNOLDS NUMBER AT
SATURATION CONDITIONS OF NITROGEN AT
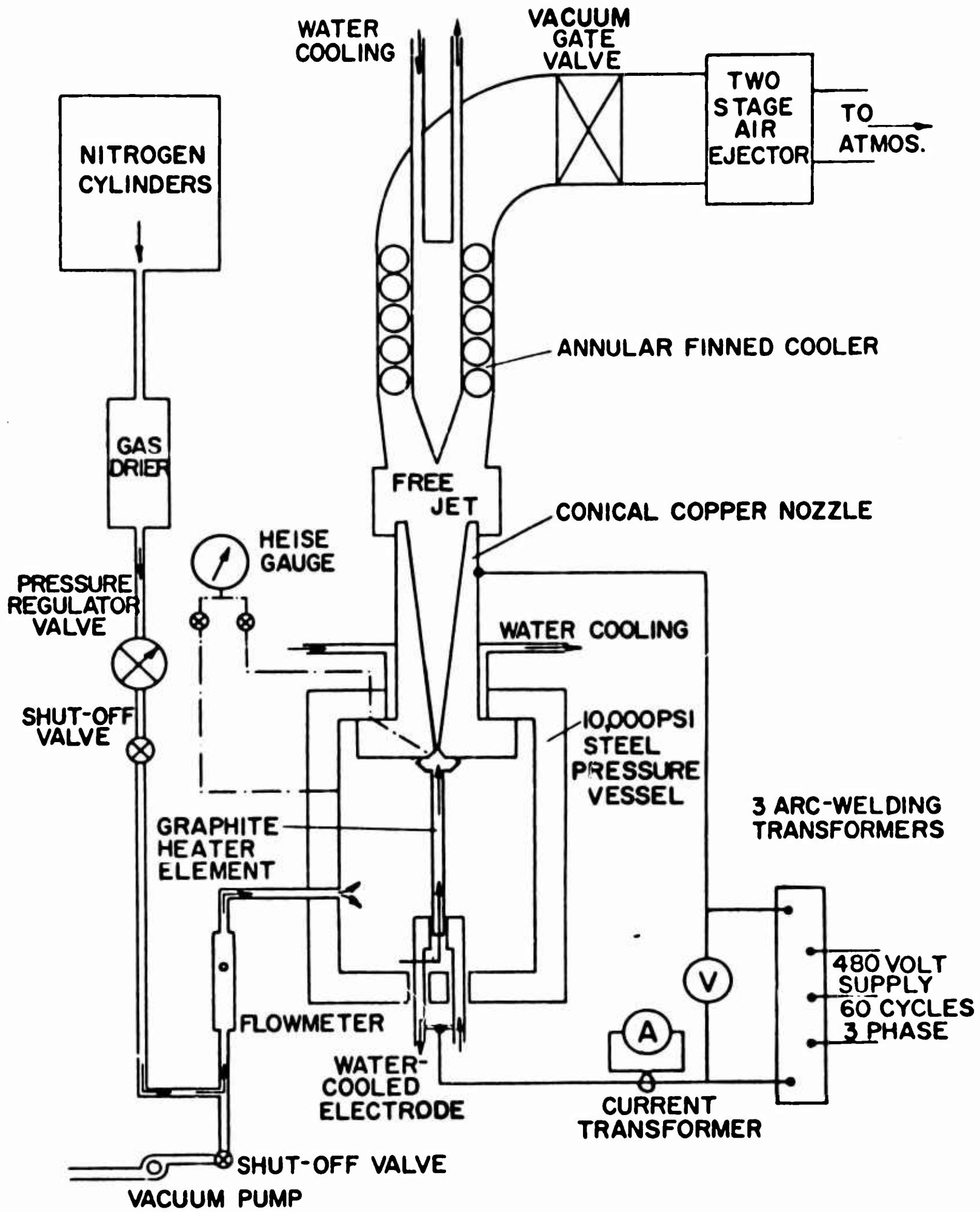VARIOUS MACH NUMBERS
AND TOTAL PRESSURES

XIII A-2

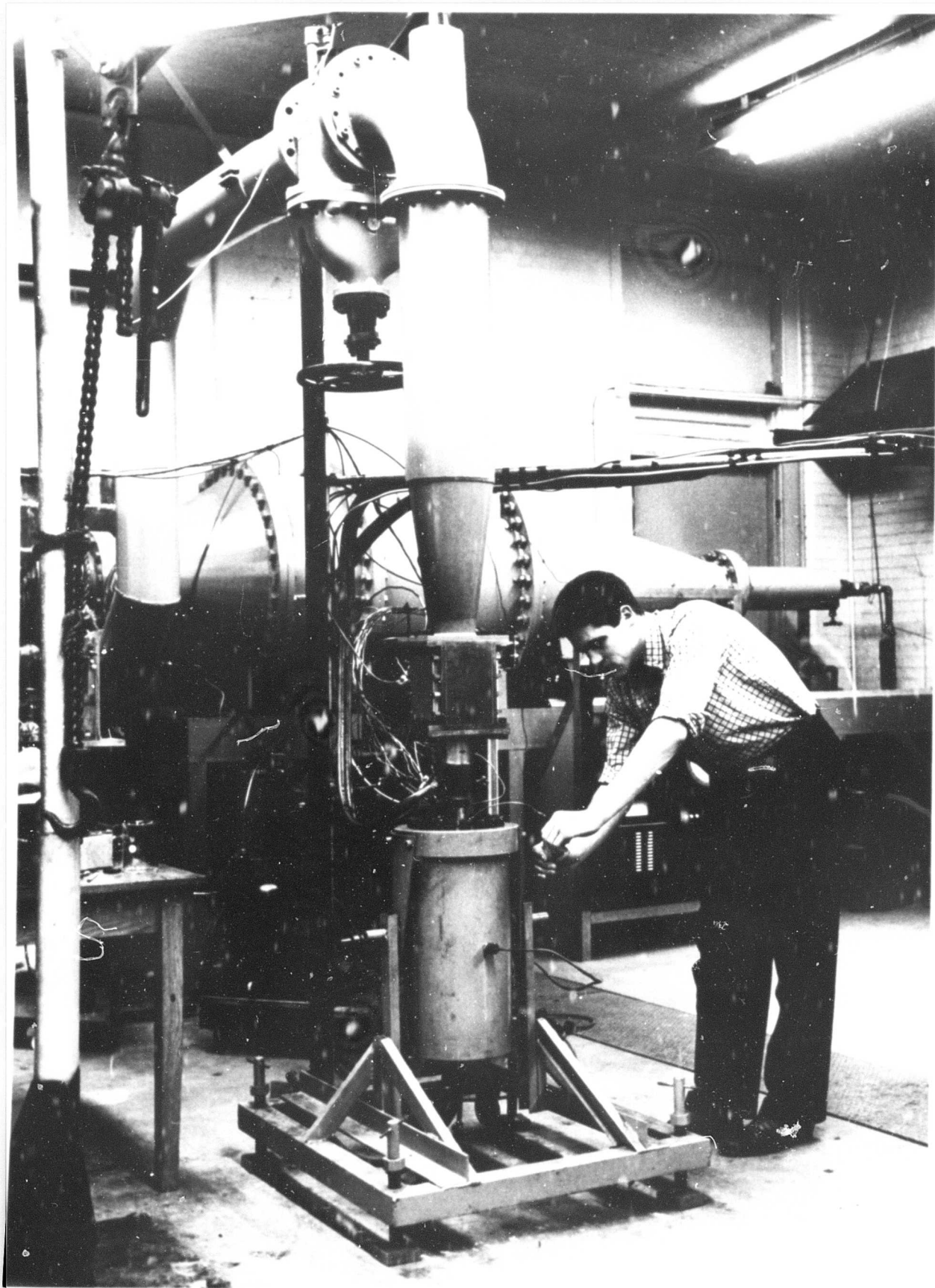FIG. 3 SCHEMATIC DIAGRAM OF THE PILOT HYPERSONIC NITROGEN TUNNEL
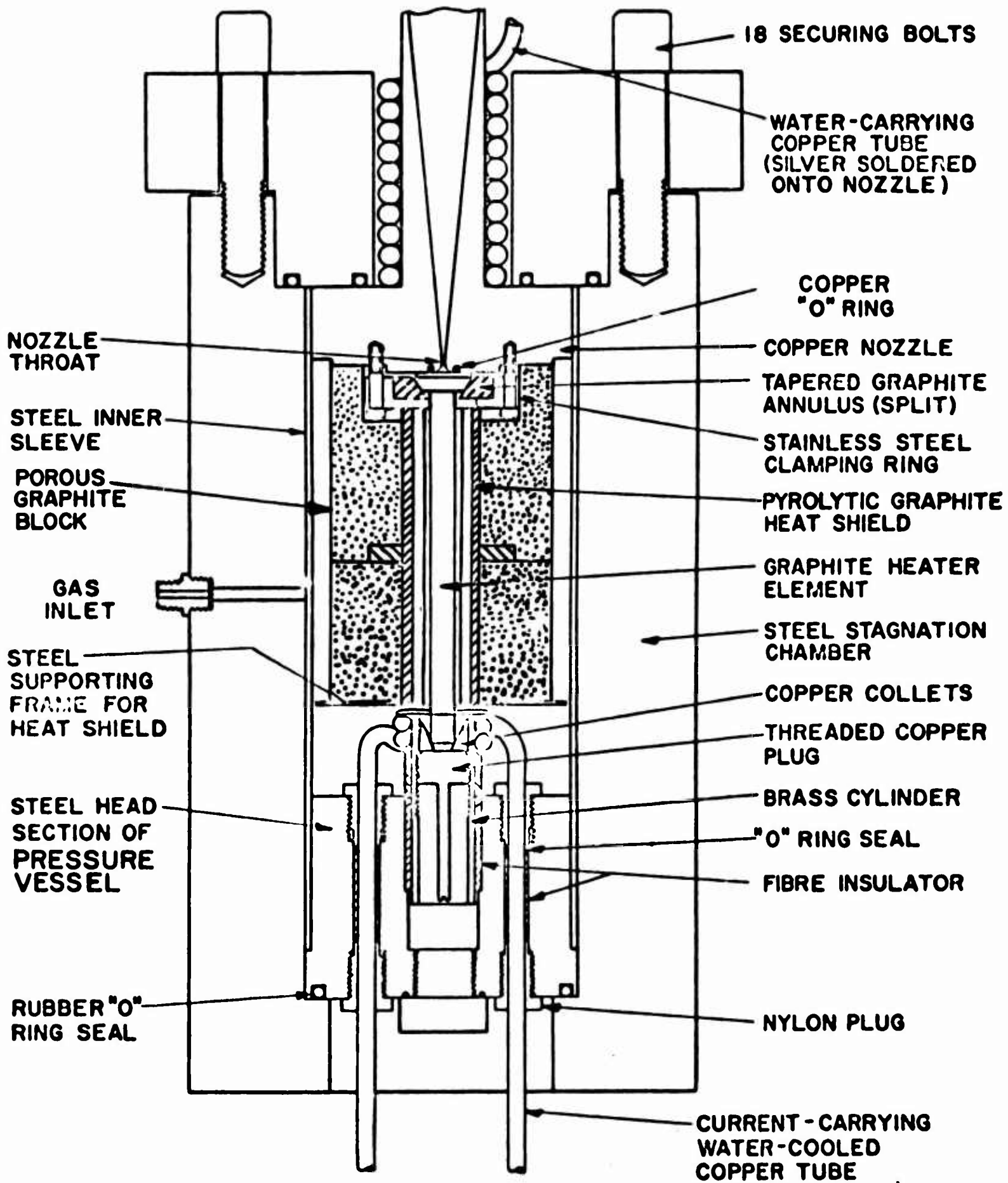
Figure 4. The Pilot Hypersonic Nitrogen Tunnel

18 SECURING BOLTS

WATER-CARRYING COPPER TUBE (SILVER SOLDERED ONTO NOZZLE)

NOZZLE THROAT

STEEL INNER SLEEVE

POROUS GRAPHITE BLOCK

GAS INLET

STEEL SUPPORTING FRAME FOR HEAT SHIELD

STEEL HEAD SECTION OF PRESSURE VESSEL

RUBBER "O" RING SEAL

COPPER "O" RING

COPPER NOZZLE

TAPERED GRAPHITE ANNULUS (SPLIT)

STAINLESS STEEL CLAMPING RING

PYROLYTIC GRAPHITE HEAT SHIELD

GRAPHITE HEATER ELEMENT

STEEL STAGNATION CHAMBER

COPPER COLLETS

THREADED COPPER PLUG

BRASS CYLINDER

"O" RING SEAL

FIBRE INSULATOR

NYLON PLUG

CURRENT-CARRYING WATER-COOLED COPPER TUBE

FIG. 5   PRESSURE VESSEL IN SECTION, SHOWING HEATER ELEMENT ASSEMBLY

XIII A-4

CURRENT DIAGRAM

PHASE DIAGRAM

480 VOLTS 3 PHASE SUPPLY AT 60 CYCLES

T3

T1

T2

i_2

i_1

i_3

i

GRAPHITE HEATER
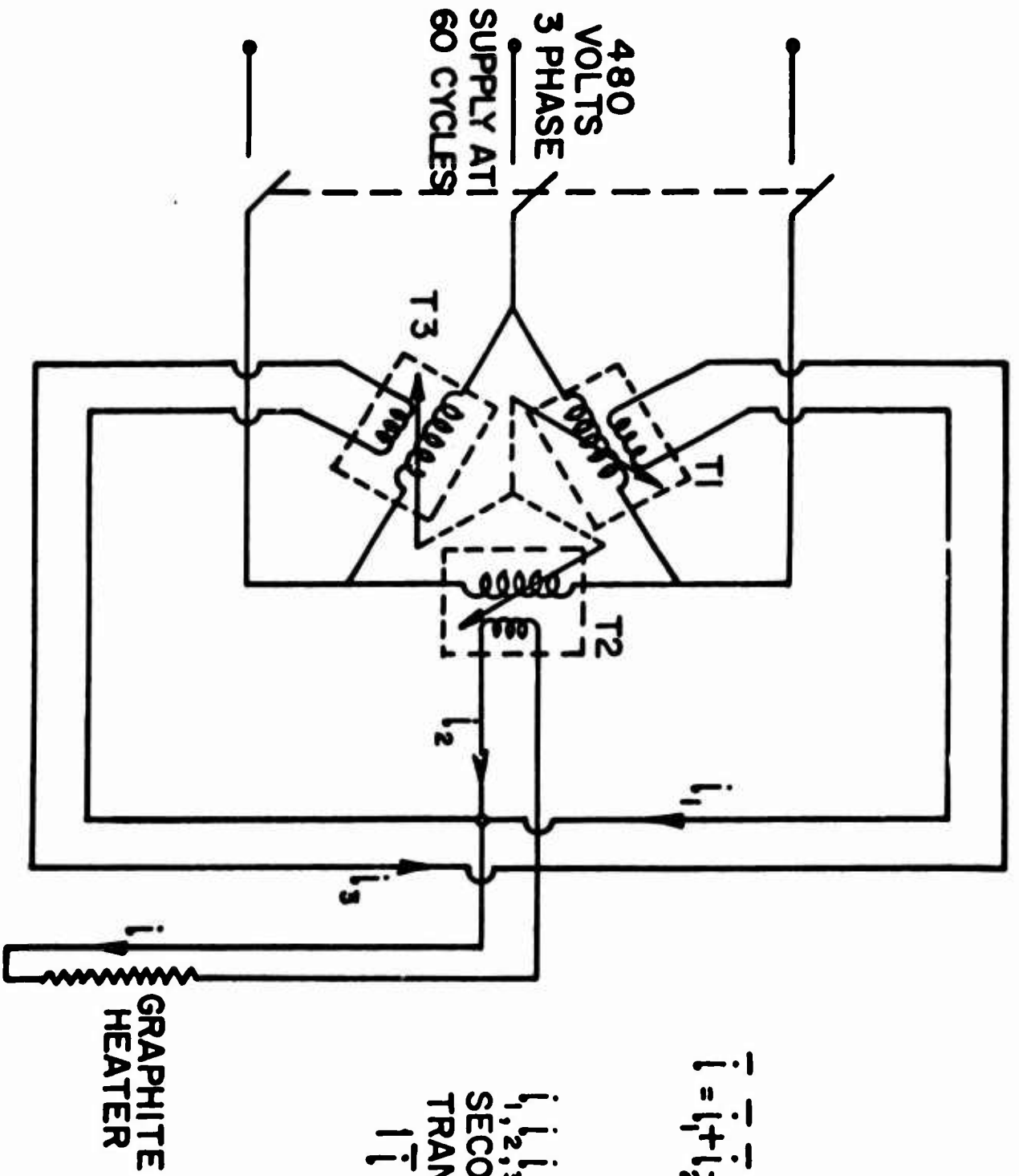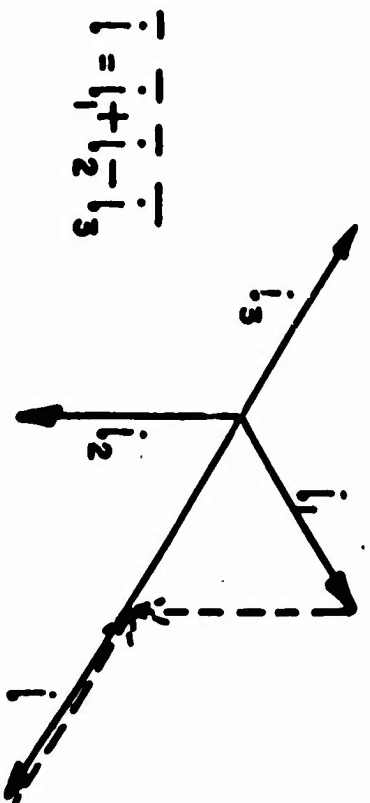
$\bar{i} = \bar{i}_1 + \bar{i}_2 - \bar{i}_3$

i_1, i_2, i_3 ARE THE CURRENTS IN THE SECONDARIES OF ARC WELDING TRANSFORMERS T1, T2, T3

$|\bar{i}| \simeq 2|\bar{i}_1| \simeq 2|\bar{i}_2| \simeq 2|\bar{i}_3|$

i_3

i_2

i_1

i

FIG. 6 SIMPLIFIED CIRCUIT DIAGRAM OF THE POWER SUPPLY FOR THE HEATER SYSTEM

X I I I A–5

Figure 7. Power Supply for the Heater System

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$$\frac{T_{t_2}}{T_p}$$

1.0
0.8
0.6
0.4
0.2
0

$$P_f = 10,000\ \text{psia}$$

$$P_f = 1000\ \text{psia}$$

$$P_f = 100\ \text{psia}$$

$$8 = \frac{T_p}{T_{t_1}}$$

10
8
6
4

0   20   40   60   80   100   120   140   160

$$\left(\frac{A_p\,\mu_p}{\bar{m}\,d_e}\right)^{0.2}\frac{l}{d_e}\left[1+3.5\frac{d_e}{d_s}\right]$$

DESIGN CONDITION FOR GRAPHITE
HEATER ELEMENT USING NOZZLE
THROAT DIAMETER = $\frac{1}{32}$ INCHES

CONSTANT HEAT FLUX
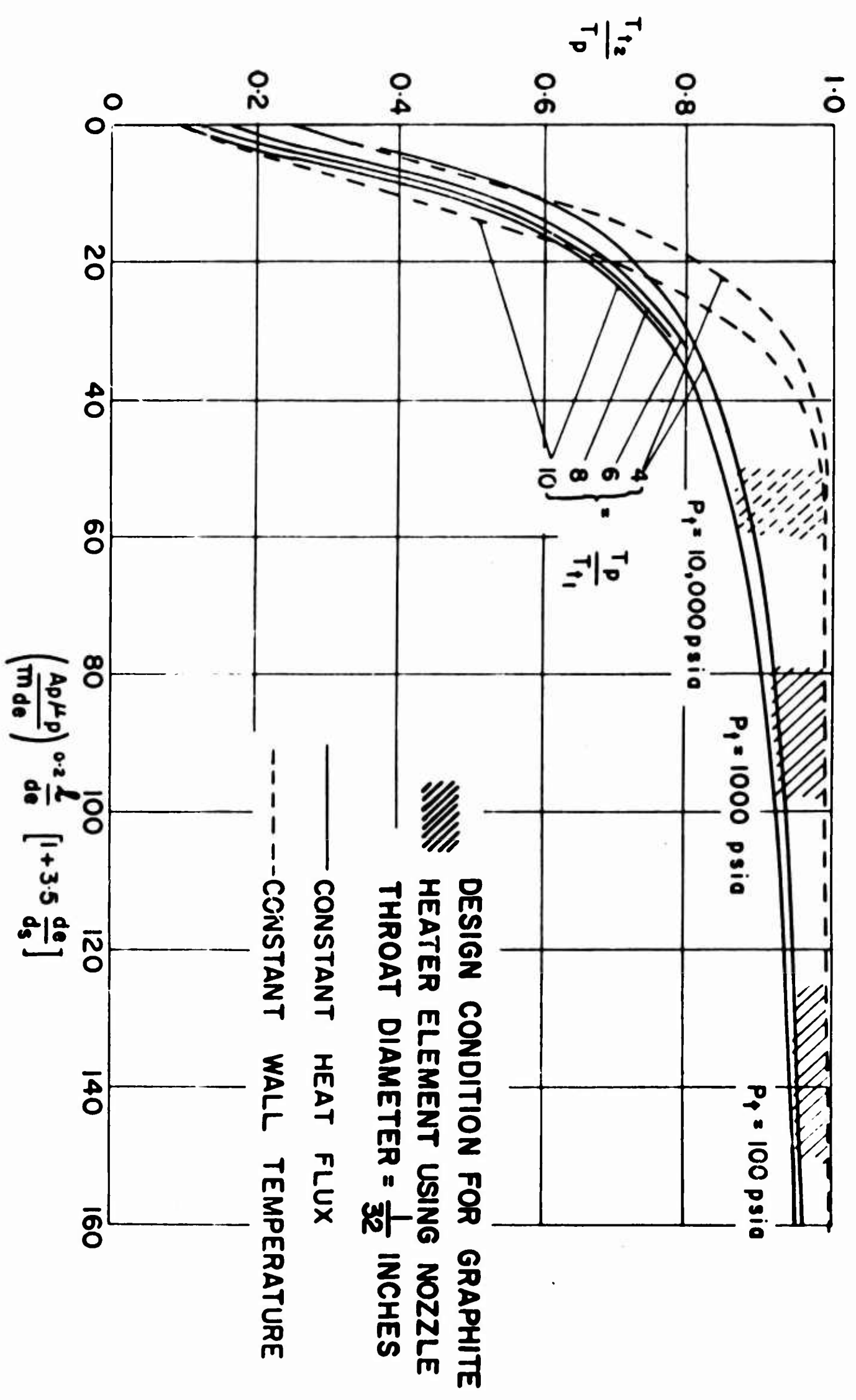
CONSTANT WALL TEMPERATURE

FIG.8   THEORETICAL RELATIONSHIP OF FINAL GAS TEMPERATURE TO FINAL WALL
TEMPERATURE FOR TURBULENT FLOW OF NITROGEN IN A HEATED SPIRAL
PASSAGE

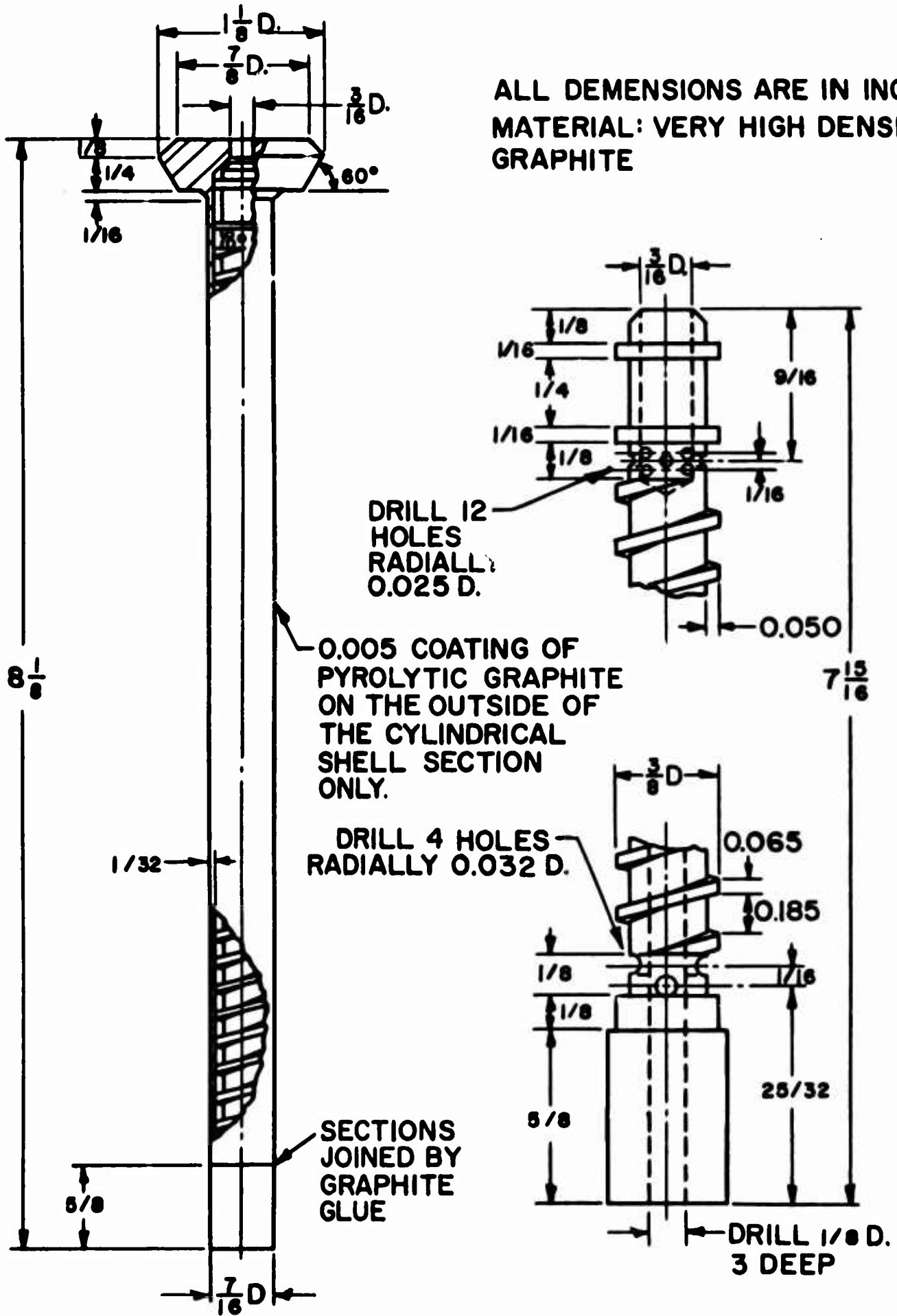HEATER ELEMENT (XI)   CENTER SPIRAL-DETAIL (X2)

ALL DEMENSIONS ARE IN INCHES
MATERIAL: VERY HIGH DENSITY
GRAPHITE

$1\frac{1}{8}$ D.

$\frac{7}{8}$ D.

$\frac{3}{16}$ D.

60°

1/4

1/16

$8\frac{1}{8}$

$\frac{3}{16}$ D.

1/8

1/16

1/4

1/16

1/8

9/16

1/16

DRILL 12
HOLES
RADIALLY
0.025 D.

0.050

$7\frac{15}{16}$

0.005 COATING OF
PYROLYTIC GRAPHITE
ON THE OUTSIDE OF
THE CYLINDRICAL
SHELL SECTION
ONLY.

DRILL 4 HOLES
RADIALLY 0.032 D.

1/32

$\frac{3}{8}$ D

0.065

0.185

1/8

1/16

1/8

5/8

25/32

SECTIONS
JOINED BY
GRAPHITE
GLUE

5/8

$\frac{7}{16}$ D

DRILL 1/8 D.
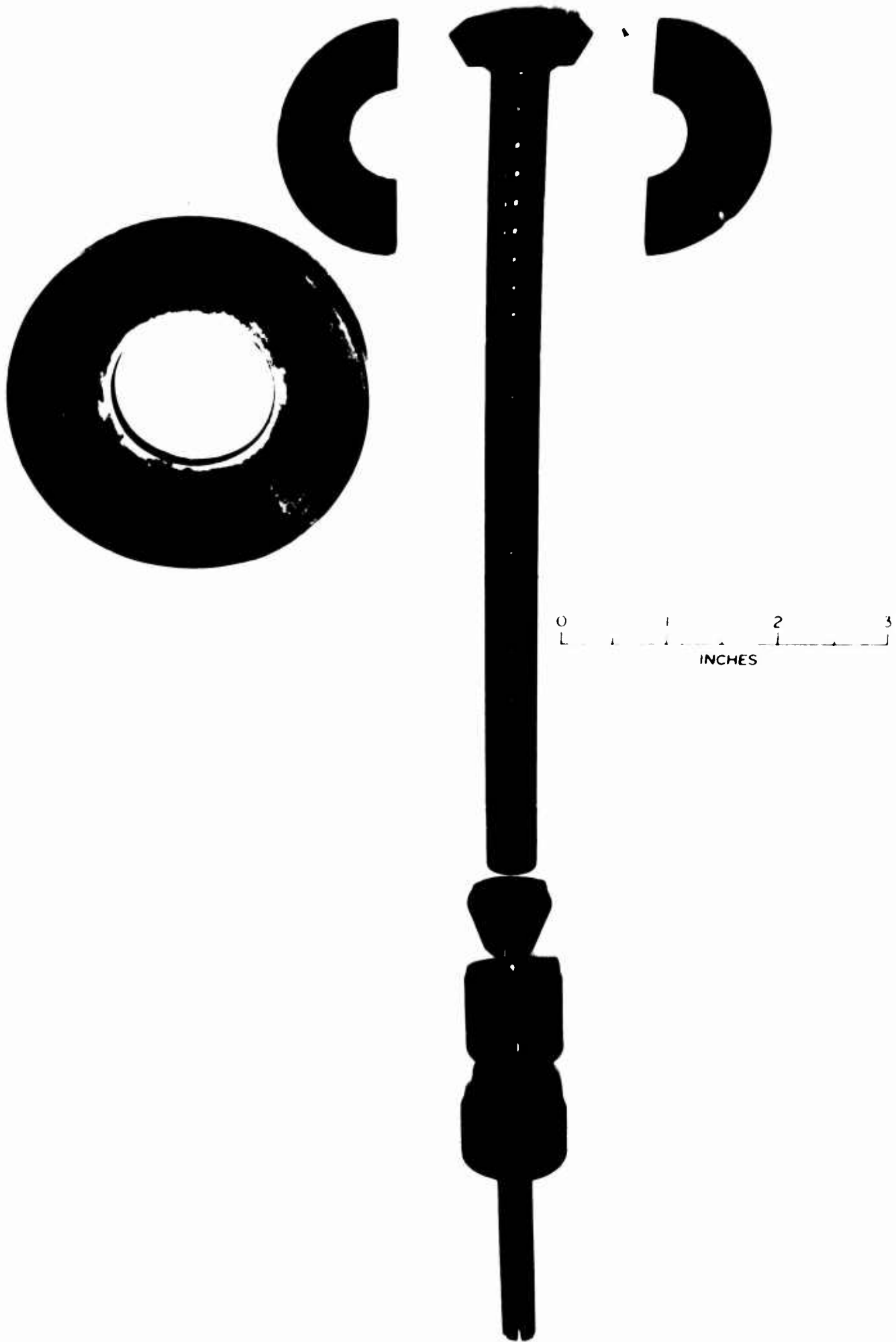3 DEEP

FIG. 9 GRAPHITE RESISTANCE HEATER ELEMENT

Figure 18. Graphite Heater Element (Sectioned)
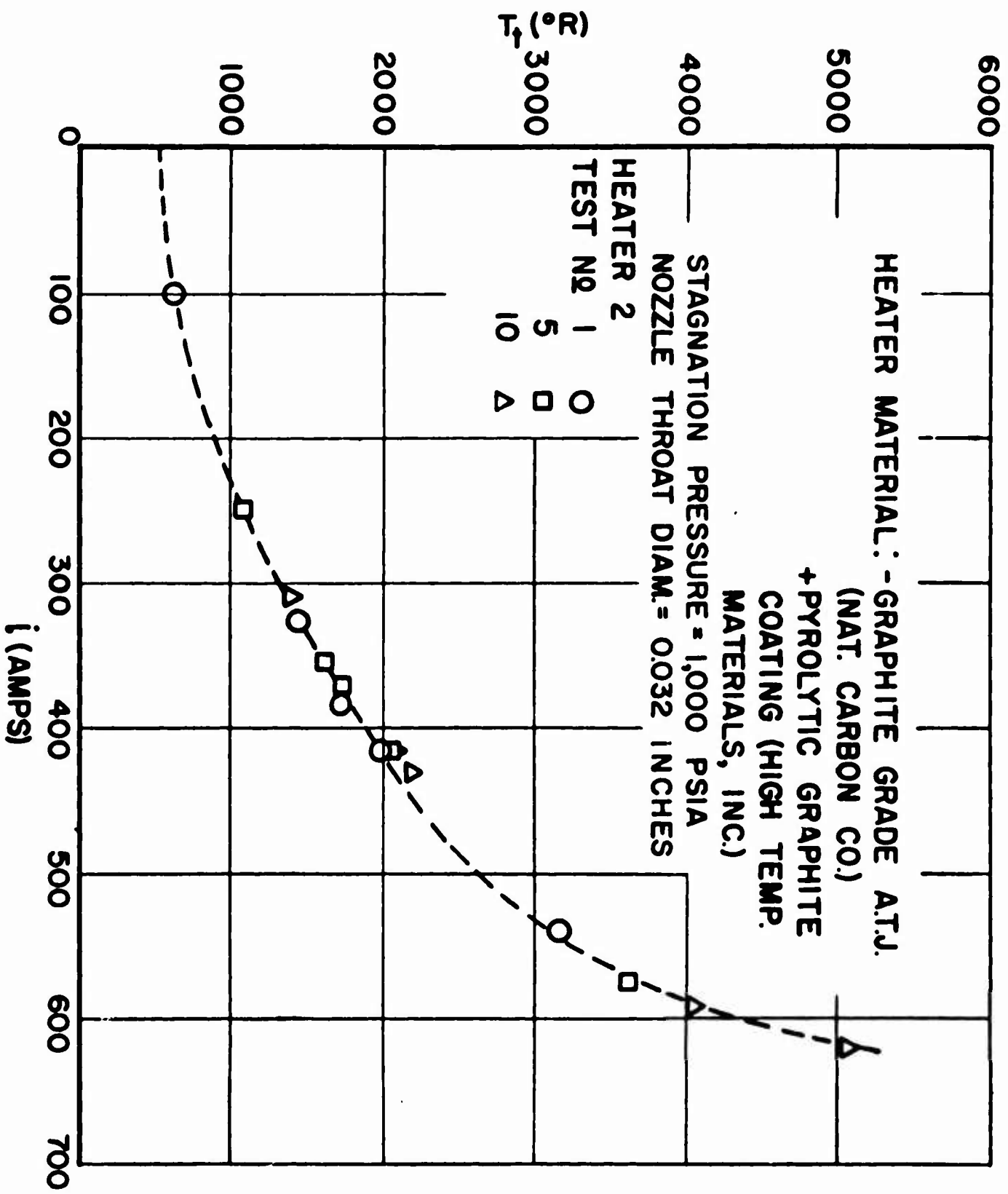with Mounting Flange and Collet

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FIG. II TYPICAL PERFORMANCE OF THE HEATER SYSTEM OF
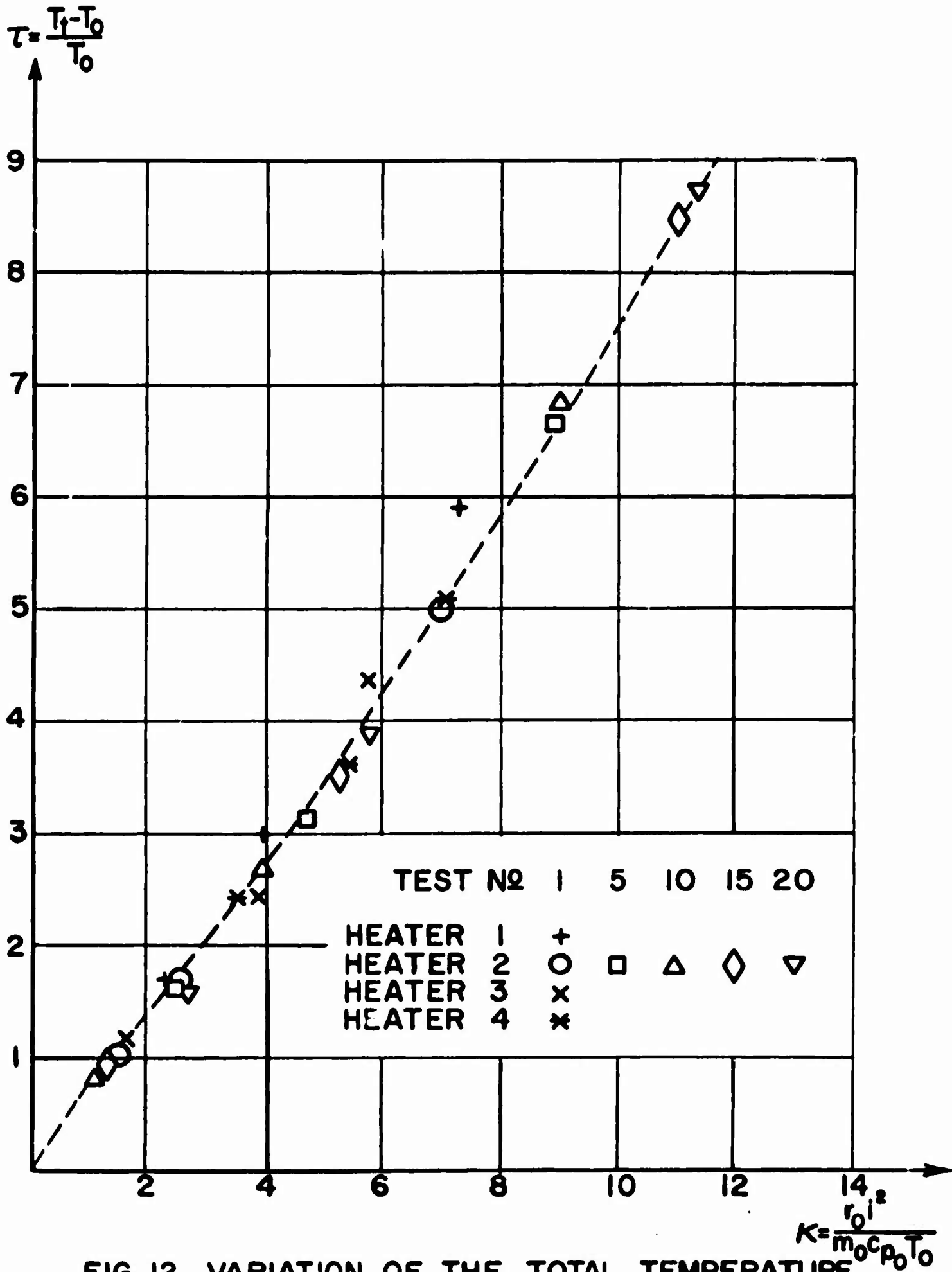THE PILOT HYPERSONIC NITROGEN TUNNEL

FIG. 12 VARIATION OF THE TOTAL TEMPERATURE
PARAMETER τ WITH THE CURRENT
PARAMETER K, FROM
TEST RESULTS

$$\tau = \frac{T_1 - T_0}{T_0}$$

$$K = \frac{r_0 i^2}{m_0 c_{p_0} T_0}$$

TEST Nº   1   5   10   15   20

HEATER	1	+				
HEATER	2	O	□	△	◇	▽
HEATER	3	×				
HEATER	4	✳				

EFFICIENCY

$\eta = \dfrac{\mu}{\lambda}$

1.0

0.8

0.6

0.4

0.2

0

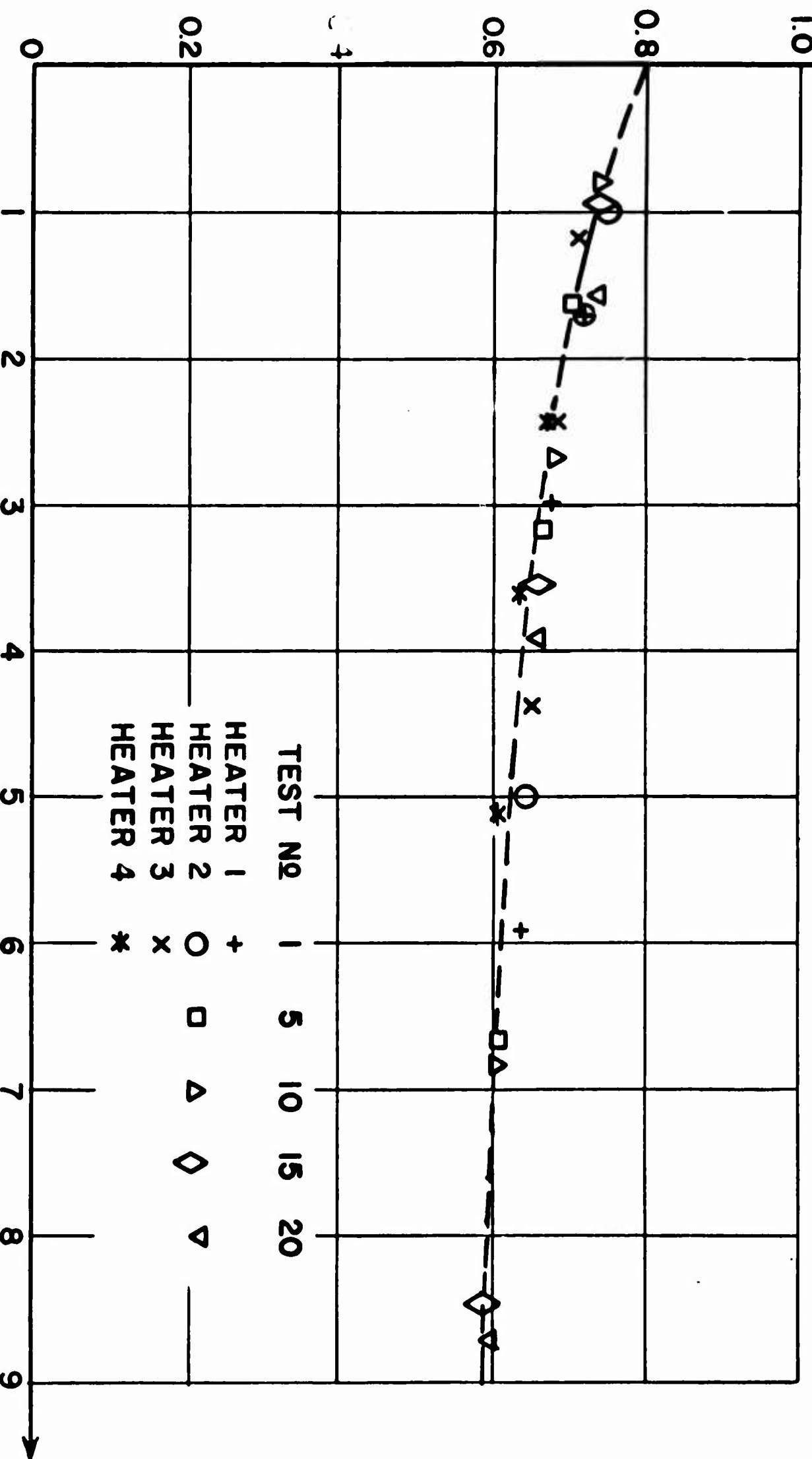| TEST NO	1	5	10	15	20
HEATER 1	+				
HEATER 2	O	□	△	◇	▽
HEATER 3	×				
HEATER 4	✳				

FIG. 13 VARIATION OF THE EFFICIENCY OF THE HEATER SYSTEM
WITH THE TOTAL TEMPERATURE PARAMETER
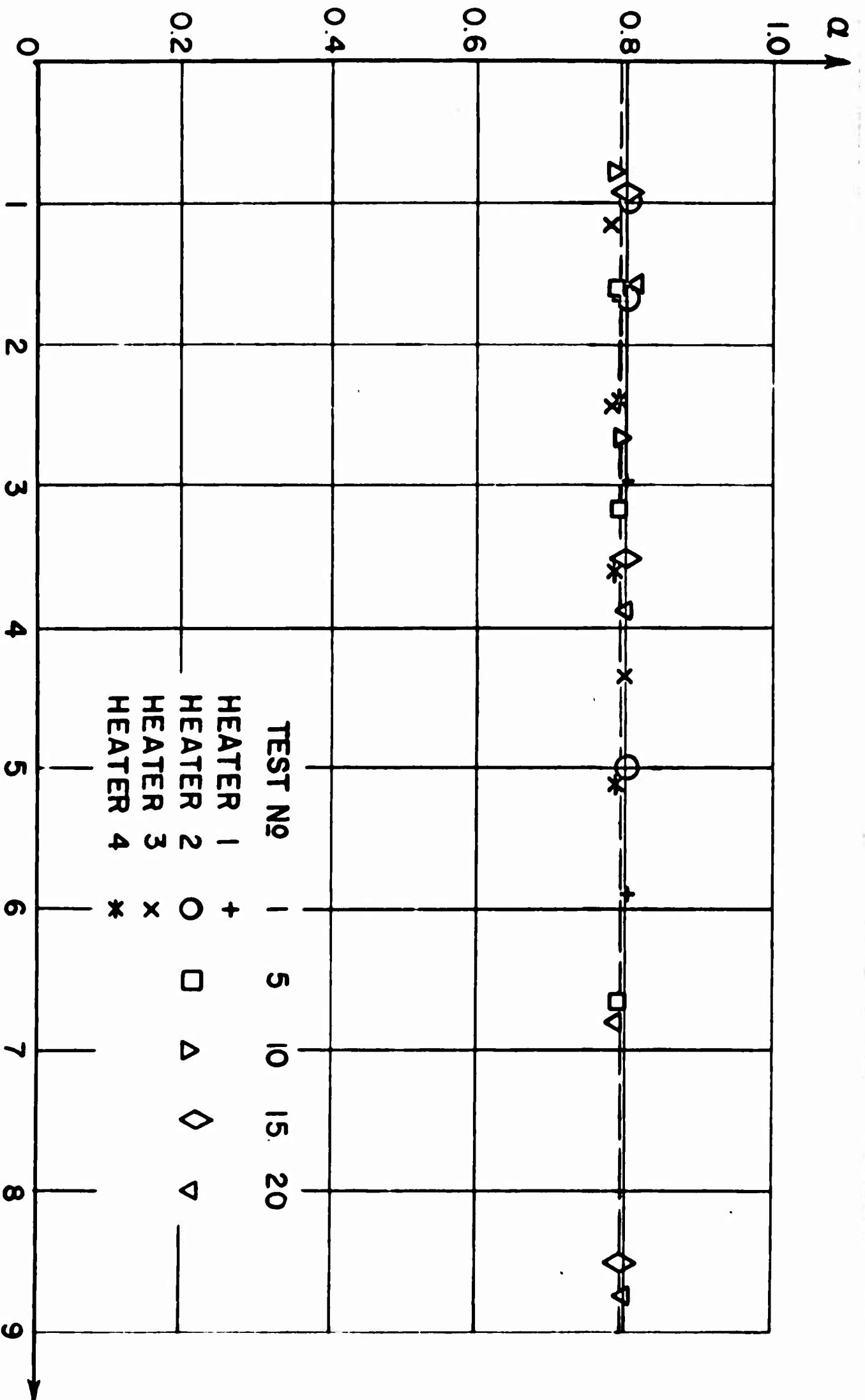FROM TEST RESULTS

$\tau = \dfrac{T_f - T_0}{T_0}$

FIG. 14 VARIATION OF THE EFFICIENCY PARAMETER α (DEFINED
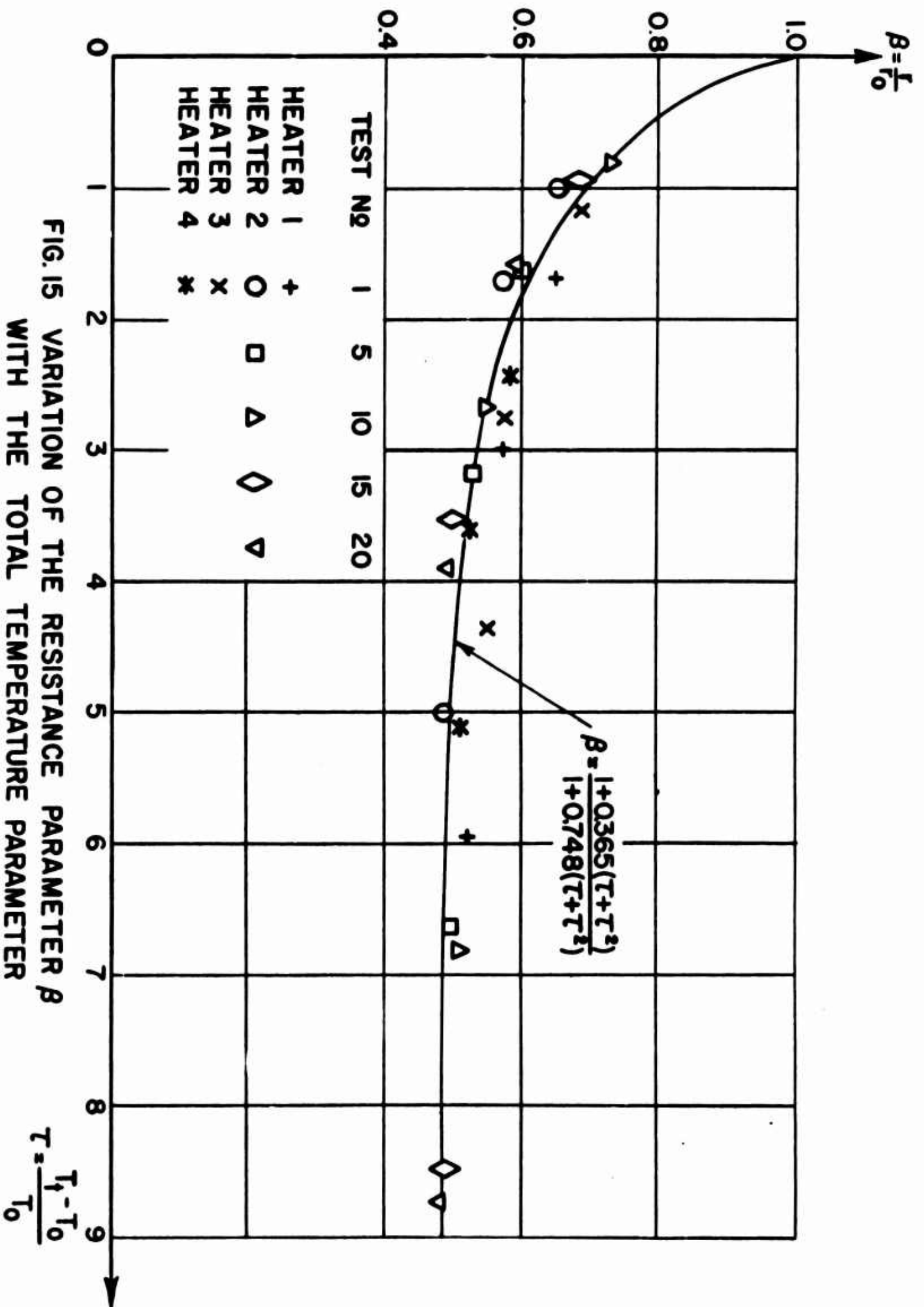BY EQUATION (5) WITH THE TOTAL TEMPERATURE
PARAMETER T , FROM TEST RESULTS

FIG. 15 VARIATION OF THE RESISTANCE PARAMETER β WITH THE TOTAL TEMPERATURE PARAMETER τ FROM TEST RESULTS

$$\beta = \frac{1+0.365(\tau+\tau^2)}{1+0.748(\tau+\tau^2)}$$

$$\beta = \frac{r}{r_0}$$

$$\tau = \frac{T_f - T_0}{T_0}$$

TEST No.

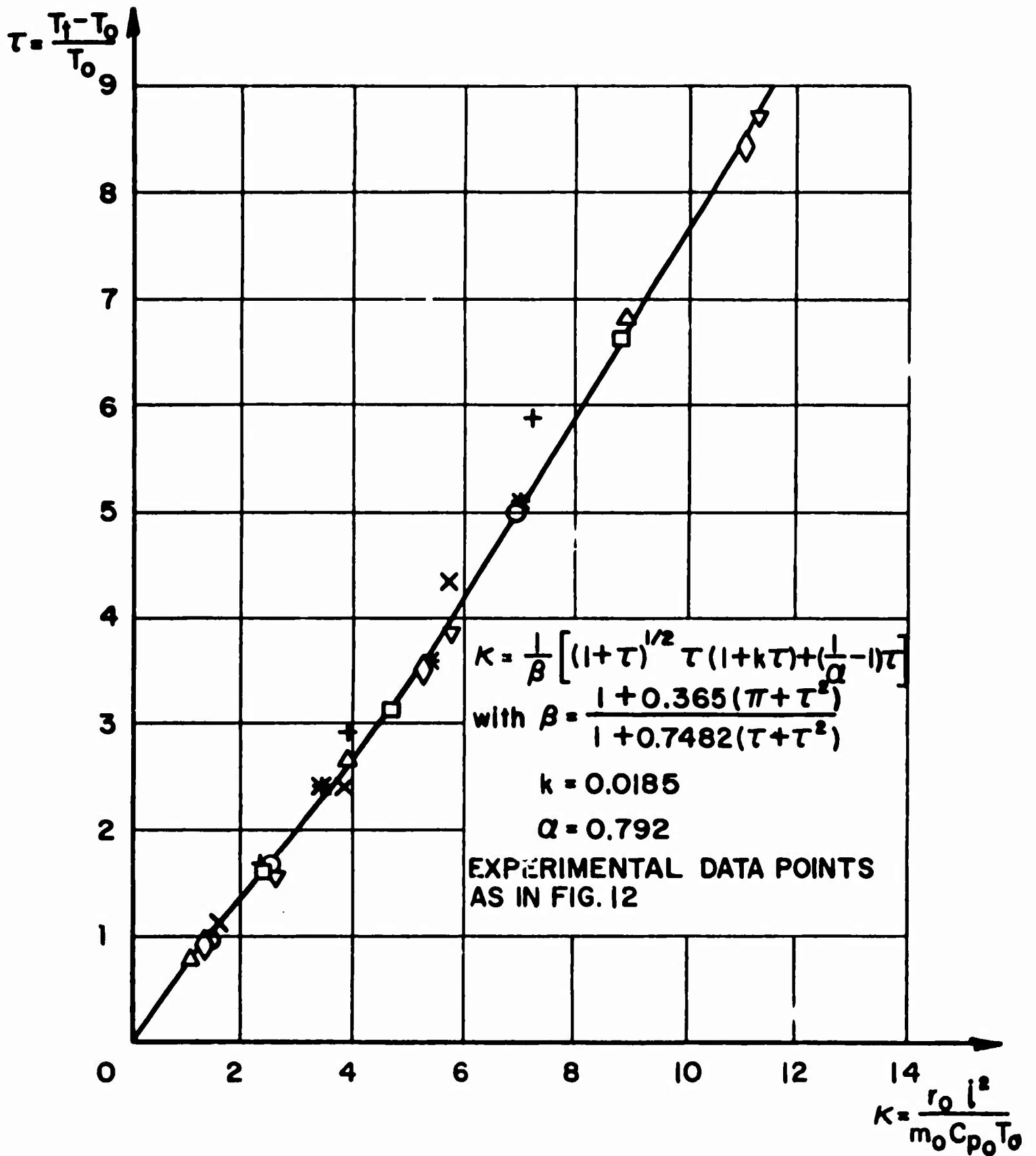|	HEATER 1	HEATER 2	HEATER 3	HEATER 4
	+	○	×	✳
1				
5		□		
10		△		
15		◇		
20		▽		

FIG. 16 COMPARISON OF EMPIRICAL CORRELATION FORMULA WITH TEST RESULTS

XIII A-13