Director of Aeronautical Sciences
Air Force Office of Scientific Research
Washington 25, D.C.

Attention: SREP

Contract No.:   AF 49(638)-565
Project No.:   4759
ARPA Order No.: 24-61, Task 2
Contractor: Stanford Research Institute

Subject: Study of Origin and Propagation of Disturbances
            in the Burning of Solid Propellants (SRI Project PRU-2770)
            Narrative Progress Report, No. 13;
            January 1 - March 31, 1962
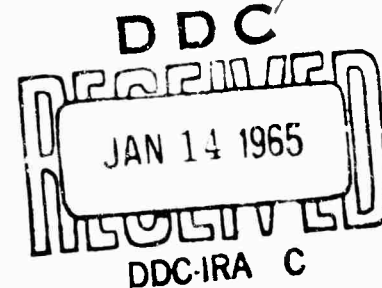

Acoustic Admittance of a Standard Sample

During the previous report period, puls records were obtained from
a standard sample consisting of a piece of grade 50-D garnet sandpaper
cemented to an aluminum block. These records have now been read and the
data processed to obtain the acoustic admittance of the sample. Before
discussing the results, we have to point out that a few recent tests
have indicated that the admittance thus measured was only partly due to
the intrinsic response of the sample surface; the major contribution was
due to leakage around the edges as a result of the particular method of
attaching the sample to the pulse tube. However, all the pulse records
for the sample which are relevant to the present discussion were obtained
under identical mounting conditions; hence we can speak of an _effective_
admittance of this particular sample.

Records were obtained from the sample and from a plain aluminum
surface at peak overpressures of 0.0002 ("low") and 0.002 ("high") atm,
respectively. (These pressures were erroneously reported as 0.002 and
0.02 atm in Progress Report No. 12). The apparent admittances calculated
from these records are shown in Figs. 1 and 2. It is evident that the
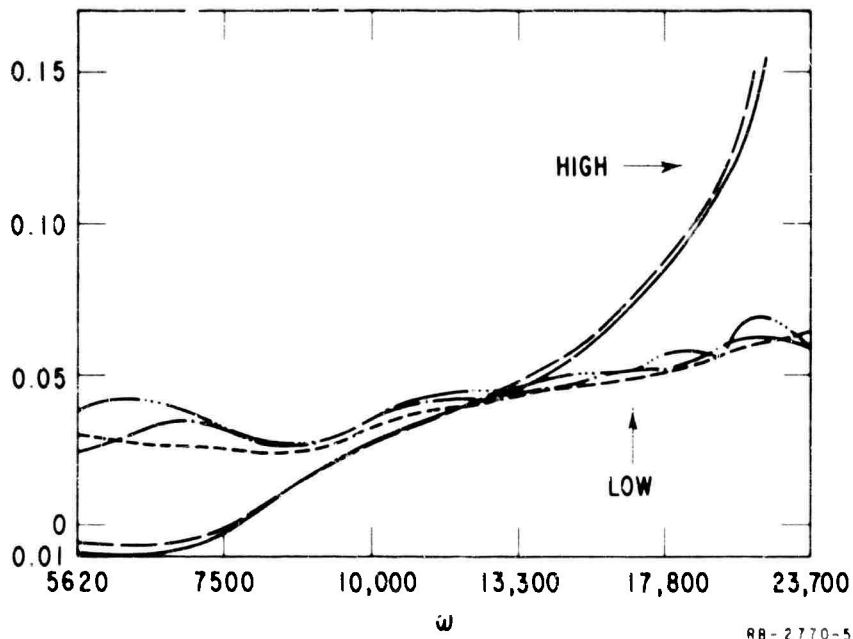
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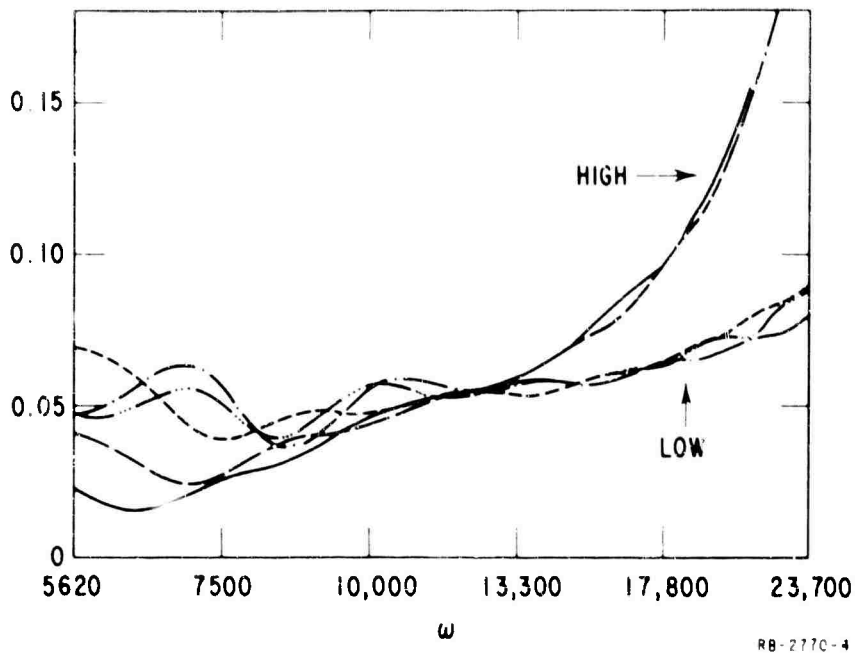
FIG. 1   Re $(Y_{apparent} \, \rho_0 c_0)$ OF 50-D SANDPAPER ON ALUMINUM AT TWO PRESSURE LEVELS

FIG. 2   Re $(Y_{apparent} \, \rho_0 c_0)$ OF ALUMINUM AT TWO PRESSURE LEVELS

apparent admittance for a given termination is quite different at the two pressure levels; this reflects the significant contribution of nonlinear effects at the higher pressure. Figure 3 shows the true admittance of the sample, calculated as the difference (averaged over all runs) between the apparent admittance of the sample and of the plain aluminum surface at a given pressure level. The agreement between the values of the true admittance obtained at the two levels is remarkably good for angular frequencies from 9000 to 22,000 sec^{-1}. Figure 4 shows the spectral density of the recorded incident pulse (normalized to the same maximum value) for one of the low pressure and one of the high pressure shots. It will be seen that the range of agreement in Fig. 3 corresponds approximately to the range of frequencies for which the spectral density for both the high and the low pressure pulses lies within one-half of the maximum value.

We plan to continue the work on establishing the validity of the pulse method for cold measurements. New standard admittance samples are being prepared by milling the ends of 1-1/2-inch-diameter aluminum cylinders so as to produce patterns of tetrahedal pyramids. These cylinders will be flush-mounted within a cylindrical shell which, in turn, will be attached to the terminal flange of the pulse tube. This will eliminate the problem of leakage at the edge of the pattern.

Instrumentation for Burning Propellant Tests

A reasonably satisfactory microphone enclosure has been built to protect the microphone from the hot combustion products. Essentially it consists of a massive cylindrical brass shell with a channel for cooling water. One end of the shell is shaped so as to provide an unbroken continuation of the interior contour of the pulse tube. At the center of the tube end of the shell there is a 1/16-inch-diameter port which rapidly widens out to the 1/2-inch diameter required to accommodate the microphone. The space between the face of the microphone and the actual opening of the port is filled with crumpled copper thread to eliminate acoustic resonances and to shield the microphone from radiant heating by the combustion gases. The sensitivity of the shielded microphone is
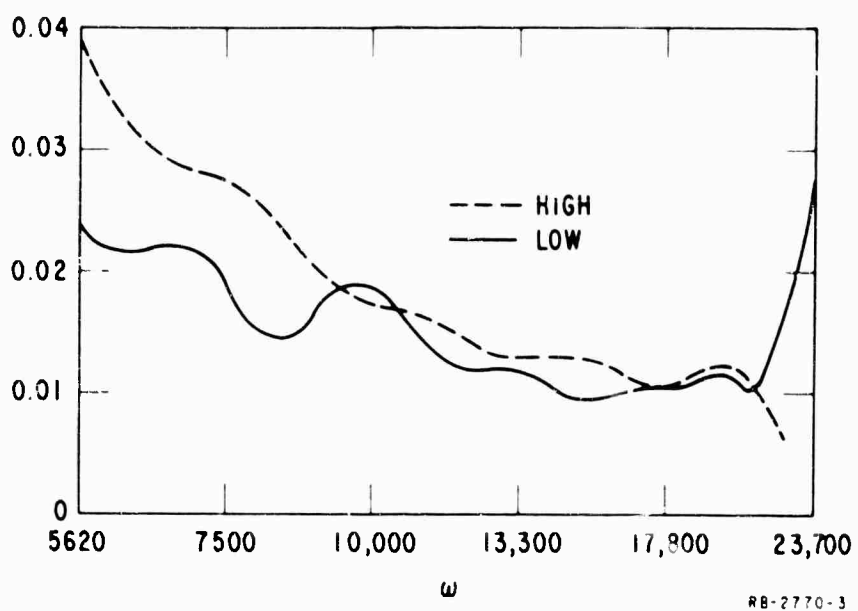
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FIG. 3   AVERAGE VALUE OF Re ($Y_{true}\,\rho_0 c_0$) OF 50-D SANDPAPER ON ALUMINUM
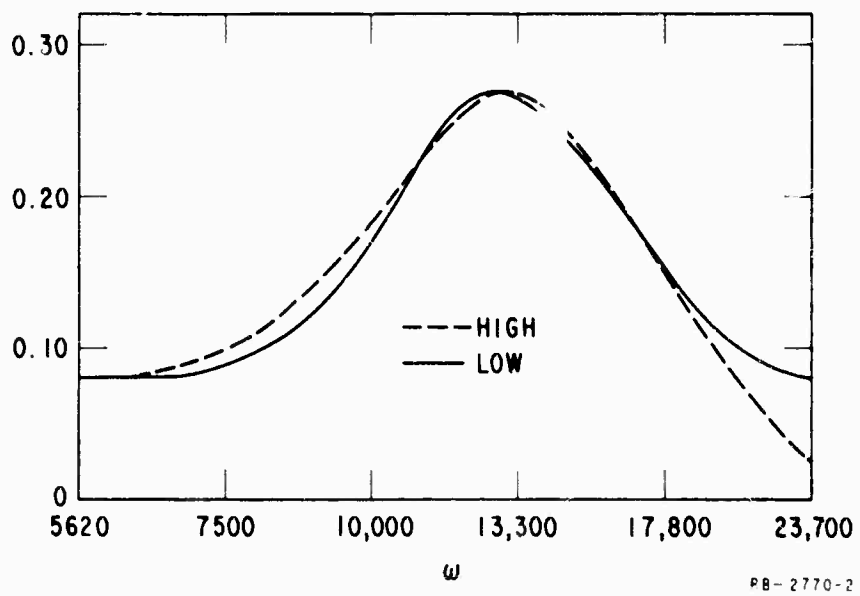AT TWO PRESSURE LEVELS

FIG. 4   SPECTRAL DENSITY OF INCIDENT PULSE (Arbitrary Units)

about the same as that of the unshielded microphone below 1 kc, but falls off very rapidly with increasing frequency. This fact places a serious limitation on the accuracy of measurements obtainable with pulses whose spectrum peaks well above 1 kc, but we hope to improve the performance of the microphone by minor changes in the design of the enclosure, such as enlarging the port.

It will be recalled that the pulse source for the cold tests is a rubber bulb placed over the end of the coaxial cable. For the hot tests, we have replaced the cable with stainless steel tubing which contains two thermocouple wires embedded in magnesium oxide. A steel sleeve capped with a loose plug of crumpled copper thread fits over the end of the tubing to prevent the forming of an ionized path to the pulse tube and the resulting pick-up of electric signals by the condenser microphone. Although no attempt has been made actually to measure it, the peak of the spectrum for the pulses from this source occurs at a considerably higher frequency than the 2 kc for the bulb source (see Fig. 4). This is unfortunate in view of the frequency limitations of the shielded microphone discussed above; however, further modifications in the design of the pulse source, as well as of the microphone enclosure, should eliminate the problem.

Initial Burning Propellant Experiments

Six burning runs have been made with the ammonium perchlorate-polyurethane propellant described in Progress Report No. 12. These runs have been made to test hardware, develop operating procedures, obtain an idea of the noise level during burning, etc. For these tests, a two-foot section has been added to the pulse tube to achieve adequate separation of incident and reflected pulses at the higher sound velocities (the distance between microphone and termination is now 4 feet, and between microphone and source 4-1/2 feet). Figure 5 shows a cold record obtained from a propellant sample, and Fig. 6, the record of the burning run on the same sample. The third pulse is the reflection of the original re-flected pulse (Pulse No. 2) from the open (source) end of the tube, and Pulse No. 4 is the reflection of Pulse No. 3 at the burning surface.
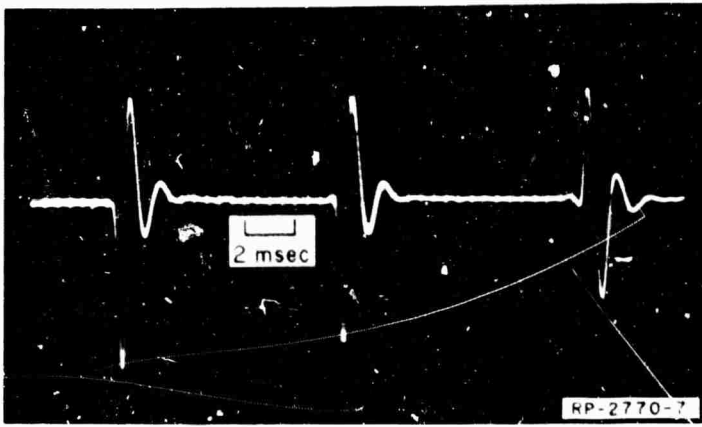
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FIG. 5   PROPELLANT SAMPLE A4 – COLD TEST
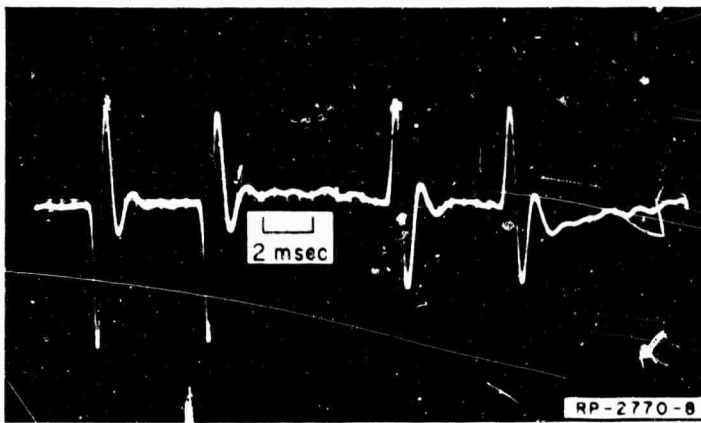

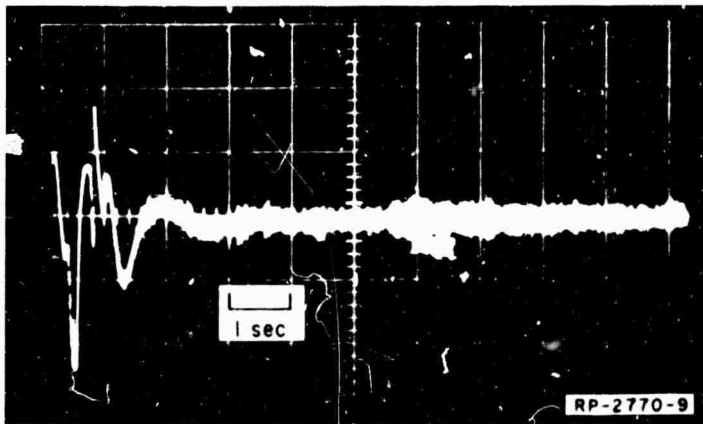FIG. 6   PROPELLANT SAMPLE A4 – BURNING RUN

FIG. 7   PROPELLANT SAMPLE A3 – 10–SECOND BURNING RECORD

Note in particular the pronounced decrease in separation between Pulses
No. 1 and 2 when we compare Figs. 5 and 6, and the much smaller decrease
between Pulses No. 2 and 3. This indicates a strong gradient in sound
velocity (and hence temperature) along the axis of the tube during the
burning run.

It is evident from Figs. 6 and 7 (the latter being a long-scale
record of a burning run on another sample) that something like a 10-fold
increase in recorded pulse height will be required for an acceptable
signal-to-noise ratio. This increase can probably be obtained withou
a corresponding increase in actual peak pressure by modifying the
spectral characteristics of the microphone enclosure and the pulse
source, as discussed in the preceding paragraph.

Obtaining the True Admittance of a Burning Surface

In the case of a burning surface, the contribution to the apparent
admittance which arises from spectral changes in the pulse due to its
passage through the tube cannot be determined by a comparison measurement
with a zero admittance termination. An alternative method is to deter-
mine the apparent admittance at two different microphone stations re-
cording the same pulse. If we call the stations A and B, and if $Y_{A,B}$
and $x_{A,B}$ are, respectively, the apparent admittance at A or B and the
distance from A or B to the termination, the true admittance of the
termination, Y, will be given, to a first approximation, by

$$Y = Y_A - \frac{Y_B - Y_A}{x_B - x_A} \cdot x_A \tag{1}$$

This formula will be valid provided spectral changes of the pulse in the
tube are proportional to the distance traveled. A more detailed dis-
cussion of this point will be given in a future report.

Spectral Change in a Plane Acoustic Pulse of Finite Amplitude

In Progress Report No. 10 we presented a very rough estimate of the
spectral change which occurs during the propagation of a finite amplitude

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pulse headed by a weak shock. During the current report period we have developed a rather complete theory of the spectral change which a smooth pulse of small but finite amplitude undergoes in moving through a non-viscous fluid prior to the formation of a shock. Although a few details remain to be worked out, we hope during the coming report period to prepare a paper as a Technical Note under this Contract and for submission to a technical journal. A few of the highlights may be given here. Let $p(t)$ and $\bar{p}(t)$ be the pressure-time profiles of a pulse at x_1 and x_2, respectively; time is to be counted from the moment of arrival of the pulse at x_1 or x_2, and pressure measured in units of the ambient pressure. Let c_0 be the ambient sound velocity, and γ the adiabatic index of the fluid. Then

$$\bar{p}(t) \;=\; p(t) + \mu \frac{d}{dt}\, p^2(t) \tag{2}$$

where $\mu = (x_2 - x_1)(\gamma + 1)/(4\gamma c_0)$. Roughly speaking, this formula is valid provided p and $\mu dp/dt$ are both small.

For sufficiently small changes in the spectral density $\left|S(\omega)\right|$, the real part of the apparent admittance is given by

$$\eta \;=\; \mathrm{Re}(Y\rho_0 c_0) \;=\; -\frac{1}{4}\frac{\triangle \left|S(\omega)\right|^2}{\left|S(\omega)\right|^2} \quad . \tag{3}$$

We can calculate

$$\left|S(\omega)\right|^2 \quad \text{and} \quad \triangle \left|S(\omega)\right|^2 \;=\; \left|S(\omega)\right|^2_{\bar{p}} - \left|S(\omega)\right|^2_{p}$$

from Eq. (2). If $p(t)$ is a slightly damped sinusoidal wave of fundamental angular frequency ω_0 and maximum amplitude k, then except for the immediate neighborhood of $2\omega_0$,

$$\eta(\omega) \;\simeq\; - k\mu\omega_0 \frac{(\omega/\omega_0)^2 - 1}{(\omega/\omega_0)^2 - 4} \quad . \tag{4}$$

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This formula predicts large positive values of η just below and large negative values just above $2\omega_0$, provided the amplitude is sufficiently large for the nonlinear effects to outweigh the viscous damping. The pulses used in our measurements made with the bulb source are, approximately, damped sinusoidal waves of fundamental angular frequency about 13,000 sec^{-1}. Qualitatively at least, the prediction of Eq. (4) near $\omega = 2\omega_0$ is seen to be strikingly confirmed in the high-level curve of Fig. 8.
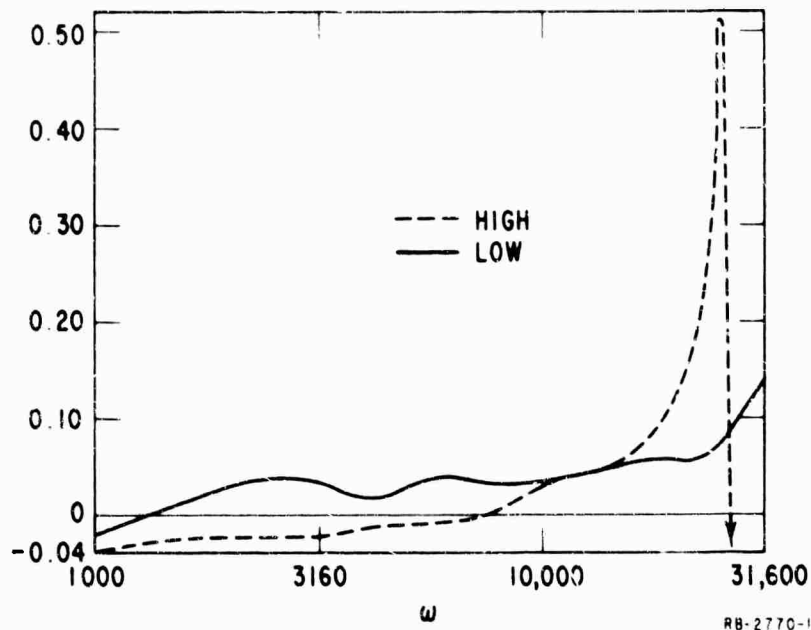

FIG. 8   Re $(Y_{apparent}\ \rho_0 c_0)$ OF ALUMINUM OVER AN EXTENDED FREQUENCY RANGE

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PRU-2770

Approved:

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