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ACOUSTIC WAVE-BURNING ZONE INTERACTION IN SOLID PROPELLANTS

Special Progress Report
1 April 1962 - 30 July 1962

AeroChem Research Laboratories, Inc.
Princeton, New Jersey
a subsidiary of Pfaudler Permutit Inc.

Prepared for

OFFICE OF NAVAL RESEARCH
POWER BRANCH
DEPARTMENT OF THE NAVY
WASHINGTON 25, D. C.
ATTN: CODE 429

Contract NOnr 3477(00)
ARPA Order No. 23-61

SUMMARY

It has been determined that the real part of the acoustic admittance of the propellants which have been tested must be less in absolute magnitude than the minimum detectable with the present type of experiment (-2.5×10^{-3} rayls^{-1}), in which the sound wave makes only one pass through the combustion zone.

Additional work toward detecting luminosity waves in the combustion gases from propellants which were being acoustically driven has failed to confirm the presence of these waves. It is postulated that the acoustic amplitude was insufficient to produce luminosity waves of detectable amplitude.

Experiments have indicated that at 1400 cps the acoustic damping constant of propellant combustion product gases is approximately three-fold greater for an aluminized than for a similar non-aluminized propellant.

An experiment which has been used by other investigators to study the acoustic damping constant of gases is discussed relative to its possible use to determine the absolute value of the damping constant for the combustion gases at their combustion temperature.

FORWARD

Work on this project has progressed to the point where results are available which indicate a shift in direction of the work.

The data presently available indicate that the real part of the acoustic admittance of the solid propellant flame is insufficiently large to be measured by the method which has been used on this project. Additional data strongly indicate that a significant increase in the acoustic damping constant of the solid propellant combustion products is caused by the addition of aluminum to the solid propellant formulation. An experiment which can quantitatively measure this constant is available and can be made to suit the requirements of this project.

This report is written to report on the present status of this program and to recommend its future avenue of investigation.

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I. INTRODUCTION

During this reporting period work has progressed on determining the interaction of an acoustic wave with the propellant flame zone. Initial problems of reproducibility were surmounted to the extent that it has been possible to demonstrate that the acoustic amplification caused by the flame zone is less than the minimun detectable value with the present type of experiment.

Initial work was done to estimate the relative damping of acoustic waves by the combustion gases of aluminized and non-aluminized propellants. Because of the apparent delicate balance of acoustic gains and losses in a solid propellant motor (1), continued work directed toward determining the acoustic properties of the combustion gases seems imperative. A standard acoustic experiment has been investigated which appears quite capable of being adapted to measuring the acoustic absorptivity of the combustion gases.

II. PROGRESS DURING THE LAST QUARTER

During this reporting period work was continued upon the problem of measuring the amplification of a sound wave by the propellant flame zone. The detection of "luminosity waves" caused by the passage of sound waves through the burning zone was pursued and, for this particular experiment at least, the problem appears to have been laid to rest. Rather crude experiments were performed to study the relative acoustic absorption of propellant combustion gases, and a more refined experiment to determine the absolute value of the acoustic absorptivity of these gases has been investigated.

III. SOUND WAVE AMPLIFICATION BY THE BURNING ZONE

During this period the work reported in Quarterly Progress Report No. 3 was continued. The installation of propellant guard rings about the propellant sample did not prove to be an adequate cure for the problem of the reproducibility of the experiment. The refractive effect of the jet of hot gases on the sound waves appeared overly subject to the winds of change and it was decided that the microphone pickup would have to be placed in the jet of hot gases itself if this effect was to be avoided. Because of the certain deleterious effect of the combustion gases upon a microphone, it was necessary to provide it with protection. This was done by equipping it with a 21 inch long extension tube which reached from the face of the microphone into the jet of hot gases. The open end of the tube was in the jet of hot gases approximately $1^5/_8$ inches from the face of the propellant sample. The centerline of the extension tube intersected the centerline of the sample of propellant at the sample face.

During the operation of the experiment, the minimum detectable change in the microphone signal which could be recorded was 0.02V while the average signal voltage was approximately 0.6V. Thus the minimum detectable power change was approximately 0.3db. Over some frequency ranges, considerable flutter was superimposed on the data trace and the minimum detectable amplification (or attenuation) was correspondingly increased. However, over most of the frequency range from 500 to 10,000 cps, the 0.3db figure applies. The db change which was produced in the sound signal by the flame zone could not be unambiguously determined in this experiment and it was, therefore, less than the above figure. A further increase in recorder sensitivity will not help, because the signal to noise ratio will preclude further improvement unless this ratio is improved. The noise level, in this apparatus, apparently arises from the poor performance of the propellant and the flame and combustion jet as acoustical transmitter. This appears to be a basic limitation of this type of experiment.

As is shown by the calculation of Appendix I, for the case where the impedance of the burning surface has a zero phase angle, the minimum detectable db change in this experiment corresponds to a real part of the admittance of approximately -2.5×10^{-3} rayls $^{-1}$. Hart (1) quotes a value for this parameter of the order of -10^{-6} rayls $^{-1}$. This would correspond to a power change of 1.13×10^{-4} db in this experiment, and this is 3.91×10^{-4} of the minimum detectable power change.

Based upon this analysis and the reported order of magnitude values of the real part of the acoustic admittance of the burning surface, it has been decided ~~to~~ abandon this phase of the investigation after making the observation that for the propellants tested the order of magnitude of the absolute value of the real part of the acoustic admittance of the burning surface is less than 10^{-3} rayls^{-1}.

IV. LUMINOSITY WAVES IN THE COMBUSTION GASES OF AN ACOUSTICALLY DRIVEN SAMPLE OF PROPELLANT

Additional work was done to detect the existance of luminosity waves in the combustion products from samples of propellant which were being acoustically driven. Considerable care was taken to avoid the uncertainties which were present in the work which was reported in the last quarterly progress report. A photocell and slit device was set to scan a cross-section of the combustion gases in a plane parallel and adjacent to the burning surface, in much the same arrangement as was reported in Quarterly Progress Report No. 3. It was not possible to detect luminosity waves in the propellant gases with this apparatus. Because such waves have been reported in motors which were undergoing unstable combustion, some additional effort will be expended to detect them in an acoustic system. It is felt that the amplitude of the vibrations which were produced in the previous apparatus may have been too small to produce detectable liminosity variations. At present, we have available a system whereby an electromagnetic acoustic driver may be used directly to vibrate a propellant without employing a gas phase couping system. Much higher amplitudes are obtainable with this system than were obtainable with the gaseous acoustic coupler system. This system will be used to vibrate propellant samples in a nitrogen atmosphere and another attempt will be made to detect acoustically induced luminosity waves in the combustion gases.

V. ABSORPTION OF ACOUSTIC ENERGY BY THE PRODUCTS OF COMBUSTION FROM SOLID PROPELLANTS

Because the combustion gases constitute an important acoustic medium in a solid propellant rocket motor, it was decided to initiate some rather crude experiments to determine if additives usch as aluminum materially affect the acoustic absorptivity of these gases. It was felt that in the event such effects could be detected, further and more quantitative experiments should be devised to measure the acoustic absorptivity of solid propellant combustion gases.

In order to accomplish this work, the apparatus of Fig. 1 was constructed. A pipe was coupled at one end through an asbestos diaphragm to an acoustically driven air column. The other end of the pipe was open to an anechoic chamber. At the midpoint of the pipe a tube was connected which was packed with glass wool and which led to a microphone. A tube which contained the solid propellant charge (often a mixture of loose fuel and oxidizer powders) was attached to the pipe near the diaphragm end. The bottom third of the propellant tube was filled with a propellant in which part of the styrene fuel had been replaced by aluminum powder, while the top two-thirds was filled with similar but non-aluminized propellant. The pipe was then driven at the desired frequency and the propellant charge was fired. As the products of combustion filled the pipe, the microphone signal was recorded. The amplitude of the signal when the non-aluminized propellant was burning was compared with that when the aluminized propellant was burning. The results of this work are presented in Table I, and a photograph of the apparatus in operation is shown in Fig. 2. In all cases, save one, the combustion products from the aluminized propellant caused a greater attenuation of the sound level than did those from the non-aluminized propellant.

Although the experiment was crude, and the physical state of the combustion gases was not well defined, an attempt has been made to estimate the magnitude of the effect of aluminum upon the damping constant of the combustion gases. The average of the results of the two runs on batches 26-27 at 1400 cps (see Table I) has been analyzed in Appendix II. In Appendix II, the following assumptions have been made, for the purpose of permitting the calculation:

1) The pipe is sinusoidally driven at $X=0$ by a rigid, flat topped pistol. (This assumption departs from reality in that the pipe was driven by an asbestos diaphragm which was in turn gas-coupled to an electromagnetic acoustic driver through the air column).
2) The products of combustion of the non-aluminized propellant when in the pipe could be adequately represented as carbon monoxide at 100°C and 1 atm. pressure.
3) The acoustic damping of the products of combustion of the non-aluminized propellants arose solely from viscous and thermal damping at the cylindrical walls of the pipe.
4) Only the <u>acoustic damping constant</u> of the combustion gases was significantly changed by the addition of aluminum to the propellant. All other parameters remained constant.

The result of the analysis is that for the propellants of batch 26-27, at 1400 cps, the addition of aluminum increased the damping constant to 2.89 times its value for the combustion products of the non-aluminized propellant.

Although this method was quite crude, it seems to demonstrate that the addition of aluminum to propellants significantly increases the acoustic damping of their combustion products.

The experiment which is described in the next section avoids most of the difficulties associated with this experiment, and it makes possible the absolute determination of the acoustic damping constants of the products of combustion (rather than the relative and calculated values as was the case in the present work).

VI. EXPERIMENTAL DETERMINATION OF THE ACOUSTIC ABSORPTIVITY OF SOLID PROPELLANT COMBUSTION GASES

The work suggested below is adapted from an acoustic experiment of Parker (8). The theory of the experiment is presented in Appendix IV as a somewhat expanded version of that which appeared in (8).

A rigid walled cylindrical tube is fitted with a rigid movable piston at one end and is closed at the other end with a near perfect reflector of acoustic waves (Fig. 2). The tube is filled with the gas for which the acoustic absorptivity is to be determined. Plane, sinusoidal waves are generated by the piston and the fluctuating pressure at the reflector is determined by the microphone. If the frequency of the piston is some multiple of the resonant frequency of the tube, the amplitude of the acoustic pressure at the reflector will be at a maximum. If now the frequency is made either slightly greater than or less than this harmonic frequency, the pressure amplitude at the reflector will decrease. At the frequencies to either side of the harmonic frequency where

$$\frac{|P_{en}|}{|P_{en}|_{max}} = \frac{1}{\sqrt{2}}$$

one may write

$$f_h - f_\ell = \delta = \frac{c\sigma}{\pi} \qquad (1)$$

$|P_{en}|$ = the pressure amplitude at the reflector around the <u>nth</u> harmonic

f_h = the high frequency where the pressure amplitude has fallen to $\frac{1}{\sqrt{2}}|P_{en}|_{max}$

f_ℓ = the low frequency where the pressure amplitude has fallen to $\frac{1}{\sqrt{2}}|P_{en}|_{max}$

C = the velocity of sound in the gas

σ = the acoustic absorptivity of the medium in the tube

Eq. (4) holds under the restriction that $\frac{\delta}{f_a} \ll \frac{2}{\pi}$

where f_a is the fundamental resonant frequency of the tube.

By operating the experiment at several harmonic frequencies of the tube and determining δ at each harmonic, it will be possible to determine σC as a function of frequency from Eq. (4). Any harmonic frequency is given by:

$$f_n = \frac{C}{\lambda_n} = n\left(\frac{C}{2\ell}\right) \tag{2}$$

where:

f_n = the <u>nth</u> harmonic frequency

λ_n = the wavelength of the <u>nth</u> harmonic

ℓ = the length of the closed tube

Thus, because the length, ℓ, is known and n can be determined quite easily with the experimental apparatus, C may be determined for each experimental case. Because C is independent of frequency, its value, once determined, may be combined with the values of $\sigma_n C$ (where σ_n refers to the value of σ at the frequency of the <u>nth</u> harmonic), and this will allow the determination of σ as a function of frequency.

If this experiment can be performed upon products of combustion which are typical of solid propellants (with particular emphasis placed upon comparing the effect of the presence of aluminum in the combustion system upon σ), this will shed great light upon the influence of propellant composition upon the gas phase acoustic loss mechanisms in solid propellant rocket motors. Furthermore, the experiment readily adapts to operation over a wide range of pressures.

The theory of the experiment and the method of analysis is both straightforward and available. This in itself is an unusual situation for an experiment in solid propellant combustion instability research. The question remains as to whether or not the experimental technique can be adapted to the requirements which are imposed by a corrosive acoustic medium which is at approximately 2800°K without violating the assumptions which were made in developing the analysis of the experiment.

The two most important assumptions of the analysis which might be violated under these circumstances if one is not cautious are:

1) With the exception of the acoustic fluctuations, the acoustic medium is of uniform temperature and composition throughout its entire volume.
2) There is no net flow of fluid through the acoustic test cavity.

In order to conform to these assumptions, the walls of the acoustic test cavity must be heated to the temperature of the test gases. This may be accomplished by heating the chamber to some initial high temperature and then allowing the combustion gases to flow through the chamber until equilibrium is established. If the test chamber is relatively massive, the gas flow may then be stopped and the temperature will remain constant for a sufficient time to permit the experiment to be run. Thus the test chamber system must incorporate some system for admitting and exhausting combustion gases and for stopping their flow during the testing period. Furthermore, the walls of the chamber will be at the temperature of the combustion products, some 2800°K. The conditions of the experiment will permit the use of graphite for all interior surfaces of the acoustic chamber, and this material will stand temperatures higher than those which are attained by the combustion products of most solid propellants.

From the results of the calculations of Appendix III, we may expect that for the combustion gases from non-aluminized propellants the acoustic absorption constant will be of the order 3×10^{-4} per cm at 300 cps, and the velocity of sound will be of the order (correcting for temperature)

$$C = 3.95 \times 10^{4} \left(\frac{[illegible]}{[illegible]} \right)^{1/2} = 1.0^{5} \times [illegible]$$

$$C \simeq 10^{5} \text{ cm/sec}$$

Thus, from Eq. (4):

$$\delta = \frac{10^{5} \times 3 \times 10^{-4}}{\pi}$$

$$\delta \simeq 10 \text{ cps}$$

Now if we assume a tube 100 cm long Eq. (5) shows that we will have a fundamental frequency

$$f_{a} \simeq \frac{10^{5}}{200}$$

$$f_{a} \simeq 500 \text{ cps}$$

Because we will need to accurately determine frequency changes of magnitude $\frac{\delta}{2}$ we will have to be able, at the least, to determine frequency changes to 0.1 cps (this is 0.02 percent of the fundamental frequency or 0.002 percent of the tenth harmonic), if we wish 2 percent accuracy for our results. This may be done with rather standard electronic apparatus.

It is believed that this experiment represents the logical next step for this project, and the design of the apparatus is about to proceed.

VII. REFERENCES

1 Hart, R. W., Eighth Symposium (International) on Combustion, Williams and Wilkins Company, Baltimore, Maryland, p. 917, 1962.

2 Morse, P. M., Vibration and Sound, McGraw-Hill, New York, pp. 243-244, 1948.

3 Ibid, p. 241.

4 Ibid, p. 247.

5 Rayleigh, The Theory of Sound, Dover Publishing Company, New York, Vol. II, pp. 325-326, 1945.

6 Marks, L. S., Mechanical Engineers' Handbook, McGraw-Hill, New York, pp. 291-292, 1952.

7 Beranek, L. L., Acoustic Measurements, Wiley and Sons, New York, p. 46, 1949.

8 Parker, J. G., "Effect of Several Light Molecules on the Vibrational Relaxation Time of Oxygen", J. Chem. Phys., 34, 1767-1772 (1961).

9 Rayleigh, Loc. Cit., pp. 15-18, 49-51.

10 Ibid., p. 14.

11 Morse, P. M., Loc. Cit., p. 367.

Appendix I

Calculation of Minimum Detectable Value of Flame Zone Impedance

If two electrical signals of voltage E_1 and E_2 are compared when loaded with a given impedance, the db of E_2 versus E_1 is given by:

$$db = 20 \log_{10} \frac{E_2}{E_1}$$

Let $E_2 = E_1 + \Delta E = E_1 (1+\delta)$

$$\delta = \frac{\Delta E}{E_1}$$

$$db = 20 \log_{10} (1+\delta)$$

$$\log_{10}(1+\delta) = \frac{1}{2.3} \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n}\right)(\delta)^n$$

For $\delta << 1$

$$\log_{10}(1+\delta) \simeq \frac{\delta}{2.3}$$

$$db \simeq 8.7\,\delta$$

On the recorder which was used for this experimental work it was possible to detect a least reading of 2mm deflection, which represented 0.02V on the most sensitive range. The average signal was approximately 0.6V.

Thus

$$\delta = \frac{0.02}{0.6} = 0.0333$$

$$db\Big|_{min} = (8.7)(0.0333) = 0.289$$

Now consider a plane acoustic wave which is incident upon a plane wall. Let P_i be the power in the incident wave and P_r be the power in the reflected wave.

Comparing the reflected with the incident power gives:

$$db = 10 \log_{10} \left(\frac{P_r}{P_i} \right) = 5 \log_{10} \left(\frac{P_r}{P_i} \right)^2$$

In the experiment which is being considered, a single interaction between the burning surface and the acoustic wave is reported as a db gain in the intensity of the wave which is sensed by a microphone. Assume that this same db gain would be realized if the acoustic wave were plane and if it were reflected from a plane wall of the burning surface at normal incidence.

Reference (11) gives the following expression for the ratio of reflected to incident power for a plane wave which is reflected from a plane wall.

$$\left| \frac{P_r}{P_i} \right|^2 = \left| \frac{1 - \mathcal{L} \cos \Phi}{1 + \mathcal{L} \cos \Phi} \right|^2$$

$$= \frac{(1 - \theta \cos \Phi)^2 + \chi^2 \cos^2 \Phi}{(1 + \theta \cos \Phi)^2 + \chi^2 \cos^2 \Phi} \qquad (1)$$

Where

$\mathcal{L} = \frac{Z}{\rho c} = \theta + i \chi$ = the specific acoustic impedance of the surface.

ρ = the density of the gaseous medium.

c = the velocity of sound in the gaseous medium.

Z = the acoustic impedance of the surface.

Φ = the angle of incidence.

For normal incidence (any other angle of incidence will increase θ for a given $\left| \frac{P_r}{P_i} \right|^2$) and for zero phase angle of Z (the assumption of zero phase angle also tends to maximize θ for given $\frac{|P_r|}{|P_i|}$), Eq. (1) becomes:

$$\left|\frac{P_r}{P_i}\right|^2 = \left[\frac{1-\theta}{1+\theta}\right]^2 \qquad (2)$$

For the minimum detectable power change in this experiment:

$$5 \log \left|\frac{P_r}{P_i}\right|^2 = 0.289$$

$$\left|\frac{P_r}{P_i}\right|^2 = 1.14$$

Putting this into Eq. (2) and solving the resulting quadratic equation for θ yields:

$$\theta_1 = -30.6$$

$$\theta_2 = -0.0652$$

$\rho c \doteq 42$ rayls for air at 273°K and $\rho c \propto \left(\frac{M}{T}\right)^{1/2}$ where M is the molecular weight of the gas and T is the absolute temperature.

Assuming that the combustion gases may be adequately represented, for this purpose, as air at 3000°K one gets that $\rho c \doteq 13$ rayls.

$\theta = \Re\left(\frac{Z}{\rho c}\right)$ where $\Re$ refers to the real part of a quantity.

$\Re(Z) = \rho c \theta$ and taking θ_1 to be the significant value of θ gives

$$\Re(Z) = (13)(-30.6)$$

$$\Re(Z) = -399 \text{ rayls}$$

If Y = the admittance of the surface = $\frac{1}{Z}$, the assumption of zero phase angle permits the calculation:

$$\Re(Y) = \frac{1}{\Re(Z)} \doteq -2.5 \times 10^{-3} \text{ rayls}^{-1}$$

Thus, to the degree of accuracy of the above assumptions, the experiment can detect a flame zone impedance of absolute value $|2.5 \times 10^{-3}|$ rayls^{-1}. This value is to be compared with that referred to in Ref. (1) where the order of magnitude of the real part of the acoustic admittance of the flame zone is said to be 10^{-6} rayls^{-1}.

If this value of $\mathcal{R}(Y)$ is used and the preceding calculation is reversed one has that:

$$\mathcal{R}(Y) = -10^{-6} \text{ rayls}^{-1}$$

$$\mathcal{R}(z) = -10^{6} \text{ rayls}$$

$$\theta = -\frac{10^6}{13} = -7.69 \times 10^4$$

$$\left|\frac{P_r}{P_i}\right|^2 = \left[\frac{1-\theta}{1+\theta}\right]^2$$

$$= \left[\frac{2}{1+\theta} - 1\right]^2$$

$$= 1 - \frac{4}{1+\theta} + \frac{4}{(1+\theta)^2}$$

Because

$$\theta >> 1$$

$$\left|\frac{P_r}{P_i}\right|^2 \simeq 1 - \frac{4}{1+\theta}$$

$$\left|\frac{P_r}{P_i}\right|^2 = 1 + \frac{4}{7.69 \times 10^4} = 1 + 5.21 \times 10^{-5}$$

$$db = 5 \log \left(\frac{P_r}{P_i}\right)^2 = 5 \log\left[1 + 5.21 \times 10^{-5}\right]$$

$$db \simeq 5\left[\frac{5.21 \times 10^{-5}}{2.3}\right]$$

$$db = 1.13 \times 10^{-4}$$

Therefore:

$$\frac{db\Big|_{R(Y)=10^{-6}}}{db\Big|_{min.detectable}} = \frac{1.13 \times 10^{-4}}{2.89 \times 10^{-1}} = 3.91 \times 10^{-4}$$

Appendix II

Analysis of the Open Tube Gas Phase Acoustic Absorption Work

If it is assumed that the experiment may be described acoustically as a tube which extends from X = 0 to X = $+\ell$, which is closed at X = 0 by a sinusoidally driven rigid flat topped piston (this is only approximately representative of the true situation in that the driving element was an asbestos diaphragm which was acoustically driven through a long gas filled coupler tube), and which is open at X = ℓ, then the sound pressure at a point X along the tube is given by (2):

$$p(x) = 2 P_+ \exp\left[-\pi\alpha_0 + i\pi\beta_0 - i\omega t\right] \sinh\left[\pi\left(\alpha_0 - \frac{\chi x}{\pi c} - i\beta_0 + i\frac{2x}{\lambda}\right)\right] \tag{1}$$

$p(x)$ = acoustic pressure at position X along the tube.

P_+ = amplitude of the positive traveling pressure wave.

P_- = amplitude of the negative traveling pressure wave.

$-\left(\frac{P_-}{P_+}\right) = \exp\left[-2\psi\right]$ = complex ratio of the amplitudes of the negative and positive traveling waves.

$$\psi = \pi\left(\alpha_o - i\beta_o\right)$$

α_o = measure of the ratio of the absolute amplitude of the negative wave to that of the positive wave at X = 0

β_o = measure of the relative phase of the two waves at X = 0.

$\mathcal{K}$ = the damping constant for the waves.

ω = the circular frequency.

C = the velocity of sound in the medium.

λ = the acoustic wavelength.

Subscripts

0 = conditions at X = 0
ℓ = conditions at X = ℓ

We wish to find an expression for the amplitude of the fluctuating pressure at the position X.

Let
$$A = \pi\left(\alpha_o - \frac{\mathcal{K}X}{\pi C}\right)$$
$$B = \pi\left(\frac{2x}{\lambda} - \beta_o\right)$$
$$C = 2P_+ \exp\left[-\pi\alpha_o + i\pi\beta_o - i\omega t\right]$$

and use the identity
$$\sinh\left[A+iB\right] = \sinh\left[A\right]\cos\left[B\right] + i\cosh\left[A\right]\sin\left[B\right]$$

Eq. (1) becomes
$$p(x) = C\left[\sinh\left[A\right]\cos\left[B\right] + i\cosh\left[A\right]\sin\left[B\right]\right]$$

Differentiating implicitly with respect to y gives:

$$\frac{dx}{dy} = \frac{1}{y} \tag{12}$$

Because $y \geq 1$ for our case, this shows that the value of K^1 which we computed was minimized by the assumption y = 1.

The ratio $\frac{K^1}{K}$ is affected by the calculated value of K. Under the assumption that the term in large brackets in Eq. (10) is unity we have from Eq. (11)

$$K' = 54.6 + K \tag{11'}$$

$$\frac{K'}{K} = \frac{54.6}{K} + 1 \tag{13}$$

Thus, for the case analyzed above the ratio is always greater than unity and it approaches unity as K becomes quite large.

Appendix III

Calculation of the Acoustic Transmission Properties of the Combustion Gases

If for the combustion products of the non-aluminized propellant the acoustic attenuation is considered to be caused wholly by the thermal and viscous interaction of the gases with the walls of the tube, then the acoustic absorption coefficient in units of reciprocal length is given from the theory of Helmholtz and Kirchoff (5):

$$\sigma_w = \left[\frac{(\mu\pi)^{1/2}}{a}\right]\left[\frac{1}{\gamma^{1/2}} + \left(\frac{k}{\mu C_v}\right)^{1/2}\left(\frac{\gamma-1}{\gamma}\right)\right]\left[\frac{f}{P_o}\right]^{1/2} \tag{1}$$

σ_w = the coefficient of acoustic absorption at the walls, cm^{-1}

μ = dynamic viscosity, poise

k = coefficient of thermal conductivity for the gas, cal/sec $cm\,^{o}K$

γ = ratio of specific heats for the gas, C_p/C_v
C_v = specific heat at constant volume, cal/g°K
C_p = specific heat at constant pressure, cal/g°K
a = tube radius, cm
P_o = equilibrium pressure, g/cm sec²
f = frequency, cycles/sec

Because the combustion gases were a complex mixture and because they were cooled to the dew point during their passage down the tube, let us assume, for the purpose of making an order-of-magnitude calculation, that for the non-aluminized propellant the products of combustion may be represented by carbon monoxide at 100°C.

From Ref. (6) after appropriate conversion of units we get the following values of the necessary physical constants for CO at 1 atm. pressure ($1.013 \times 10^6 \frac{g}{cm\,sec^2}$) and 100°C.

$$\mu = 2.1 \times 10^{-4} \text{ poise}$$
$$k = 6.54 \times 10^{-5} \text{ cal/sec cm °K}$$
$$\gamma = 1.41$$
$$C_p = 0.243 \text{ cal/g °K}$$
$$C_v = 0.172 \text{ cal/g °K}$$
$$f = 1400 \text{ cps}$$
$$a = 1.61 \text{ cm}$$

Putting these quantities into Eq. (1) and performing the necessary calculations gives:

$$\sigma_{w} = 7.33 \times 10^{-4} \text{ cm}^{-1}$$

The damping constant K is given by:

$$K = C\,\sigma_{w}$$

where C is the velocity of sound.

For CO at 100°C, Ref. (7) gives (upon appropriate calculation):

$$c = 3.95 \times 10^{4}\ cm/sec$$

$$K = [7.33 \times 10^{-4}][3.95 \times 10^{4}]$$

$$K = 28.9\ sec^{-1}$$

Table III gives σ_{w} as a function of frequency for CO at 100°C and 1 atm.

Appendix IV

Theory of the Acoustic Resonance Tube as a Device for Determining the Acoustic Absorption Constant of a Gas

This is a somewhat more detailed presentation of the theory of this device than that which appeared in Ref. (8) although a similar analysis was implicit in that work.

Consider a rigid walled cylindrical tube such as the one shown in Fig. (1) which extends along the X-axis from X = 0 to X = ℓ. The tube is closed at X = 0 by a rigid flat topped piston which is driven in the X direction with the velocity

$$u = u_0 = U \cos \omega t$$

The following notation will be used.

- c = acoustic propagation speed, cm/sec
- f = frequency, cycles/sec
- f_a = fundamental frequency, cycles/sec
- f_n = frequency of the nth harmonic, cycles/sec
- n = a positive integer
- λ = the acoustic wavelength, cm
- ℓ = the inside length of the tube, cm
- u = the acoustic velocity = $\nabla \Phi$, cm/sec
- Φ = the acoustic velocity potential, cm^2/sec

ω = the circular frequency, radians/sec
U = the driver velocity amplitude, cm/sec
ρ_o = the equilibrium density of the gas, g/cm^3
P_o = the equilibrium pressure, dynes/cm^2
δP = the acoustic pressure, dynes/cm^2
t = time, sec.
P = acoustic pressure amplitude, dynes/cm^2
P_ℓ = acoustic pressure amplitude at x = ℓ, dynes/cm^2

For this system, which may be solved in terms of plane waves we have (9):

$$\frac{d^2\Phi}{dt^2} = c^2\frac{d^2\Phi}{dx^2} \tag{1}$$

Now if we assume a sinusoidal solution:

$$\Phi = \phi(x)\exp[i\omega t] \tag{2}$$

Equation (1) becomes

$$\frac{d^2\phi}{dx^2} + \left(\frac{\omega}{c}\right)^2\phi = 0$$

$$\frac{d^2\phi}{dx^2} + k^2\phi = 0 \tag{3}$$

where $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

The solution to Eq. (3) may be given as:

$$\phi(x) = A\cos(kx) + B\sin(kx) \tag{4}$$

And Φ is therefore:

$$\Phi = \left[A\cos(kx) + B\sin(kx)\right]\exp\left[i\omega t\right] \qquad (5)$$

To this one applies the boundary conditions:

$$x = 0\,, \quad u = u_0 = \left(\frac{\partial\Phi}{\partial x}\right)_{x=0} = U\cos(\omega t)$$

$$x = \ell\,, \quad u = u_\ell = \left(\frac{\partial\Phi}{\partial x}\right)_{x=\ell} = 0 \qquad (6)$$

Under these conditions the expression for Φ becomes:

$$\Phi = \frac{U}{k}\left[\frac{\cos\left[k(\ell - x)\right]}{\sin\left[k\ell\right]}\right]\exp\left[i\omega t\right] \qquad (7)$$

Now from Ref. (10) we have that:

$$\frac{\delta P}{\rho_0} = -\frac{d\Phi}{dt} \qquad (8)$$

Thus $$\delta P = -iU\rho_0 c\left[\frac{\cos\left[k(\ell - x)\right]}{\sin\left[k\ell\right]}\right]\exp\left[i\omega t\right]$$

$$\delta P = -iP\exp\left[i\omega t\right]$$

$$P = U\rho_0 c\left[\frac{\cos\left[k(\ell - x)\right]}{\sin\left[k\ell\right]}\right] \qquad (9)$$

At $x = \ell$ this becomes:

$$P_e = \frac{U\rho_0 c}{\sin\left[k\ell\right]} \qquad (10)$$

Account is now taken of dissipation by taking K to be complex in Eq. (10). Because only steady state solutions are being considered (the time rate of acoustic energy dissipation is equal to the rate of energy addition at X = 0), the decay term which appears in Eq. (9) is discarded. This defines our absorption coefficient, σ .

$$K = \beta - i\sigma \tag{11}$$

$$P_e = \frac{\rho_o c U}{\sin[(\beta - i\alpha)\ell]} \tag{12}$$

From Eq. (12) we may get the expression for the absolute value of P_e, $|P_e|$.

$$|P_e| = \frac{\rho_o c U}{\left[\sin^2[\beta\ell] + \sinh^2[\alpha\ell]\right]^{1/2}} \tag{13}$$

The standing waves in the tube are given by:

$$\lambda_n = \frac{2\ell}{n}, \quad n = 1, 2, 3, 4, \ldots \tag{14}$$

and

$$f_n = \frac{c}{\lambda_n} = n\left(\frac{c}{2\ell}\right) \tag{15}$$

From the definition of K and β

$$\beta\ell = \left(\frac{2\pi f}{c}\right)\ell$$

$$(\beta\ell)_{f_n} = n\pi$$

Now consider frequencies not far removed from f_n

$$\beta \ell = (\beta \ell)_{f_n} + \frac{d(\beta \ell)}{df}(f - f_n) \qquad (16)$$

$$\beta \ell = n\pi + \frac{2\pi \ell}{c}(f - f_n) \qquad (17)$$

If one works under the assumption that $\sigma \ell << 1$ and $\frac{2\pi \ell}{c_2} << 1$ Eq. (13) may, after inserting the proper expansions for $\sin^2$ and $\sinh^2$ and discarding higher order terms, be given as:

$$|P_{en}| = \frac{\rho_o c^2 U}{2\pi \ell}\left[(f - f_n)^2 + \left(\frac{\alpha c}{2\pi}\right)^2\right]^{-\frac{1}{2}} \qquad (18)$$

and

$$|P_{en}|_{max} = \frac{\rho_o c U}{\sigma \ell} \qquad (19)$$

Equations (18) and (19) give:

$$\frac{|P_{en}|}{|P_{en}|_{max}} = \left\{\left[\left(\frac{2\pi}{c\sigma}\right)(f - f_n)\right]^2 + 1\right\}^{-\frac{1}{2}} \qquad (20)$$

If now we define the "half power points" as the frequencies about f_n where $\frac{|P_{en}|}{|P_{en}|_{max}} = \frac{1}{\sqrt{2}}$, and if the frequency interval between these points is $\delta = f_{high} - f_{low}$, then at the half power points Eq. (20) gives:

$$\delta = \frac{c\sigma}{\pi} \tag{21}$$

Returning to the requirement that $\frac{2\pi \ell}{c}(f - f_r) \ll 1$, in terms of Eq. (21) this becomes: $\frac{2\ell}{c}\delta \ll \frac{2}{\pi}$

Inserting Eq. (15) into this expression gives the requirement as:

$$\frac{\delta}{f_a} \ll \frac{2}{\pi} \tag{22}$$

TABLE I

db CHANGE IN MICROPHONE SIGNAL FOR WAVE PROPAGATED IN PRODUCTS OF COMBUSTION OF AN ALUMINIZED PROPELLANT REFERRED TO THE PRODUCTS OF A NON-ALUMINIZED PROPELLANT

Acoustic Frequency cps	Non-Al Prop. No.	Al Prop. No.	db Signal In Al Products Ref. Non-Al Products Ref. Non-Al.
290	26	27	-1.56
290	26	27	-1.36
1400	26	27	-1.43
1400	26	27	-0.78
250	31	30	-2.50
475	31	30	-5.00
1425	31	30	+0.527
1425	31	30	-0.834

TABLE II

PROPELLANT COMPOSITIONS USED FOR WORK OF TABLE I

Propellant Batch No. 1	Percent Composition					
	Ammunium Perchlorate unground	Ammunium Perchlorate ground	P-13 and wetting agent	Al	Styrene Powder	Comment
26	58.2	22.8	19.0	0.0	--	cured prop.
27	58.2	22.8	17.1	1.9	--	cured prop.
30	81.1	--	--	3.8	15.1	powder mixture
31	81.0	--	--	0.0	19.0	powder mixture

TABLE III

COEFFICIENT OF ACOUSTIC ABSORPTION, σ_{ac}, AND ACOUSTIC DAMPING CONSTANT K VERSUS FREQUENCY FOR CO AT 100°C AND 1 ATM. PRESSURE

Frequency, cps	$(10^4)(\sigma_{ac})$ cm^{-1}	K sec^{-1}
290	3.33	13.2
1000	6.22	24.6
1400	7.33	28.9
2000	8.81	34.8
3000	10.8	42.7
4000	12.4	49.0
5000	13.9	54.9
6000	15.2	60.1
7000	16.4	64.9
8000	17.6	69.6
9000	18.6	73.6
10000	19.6	77.5

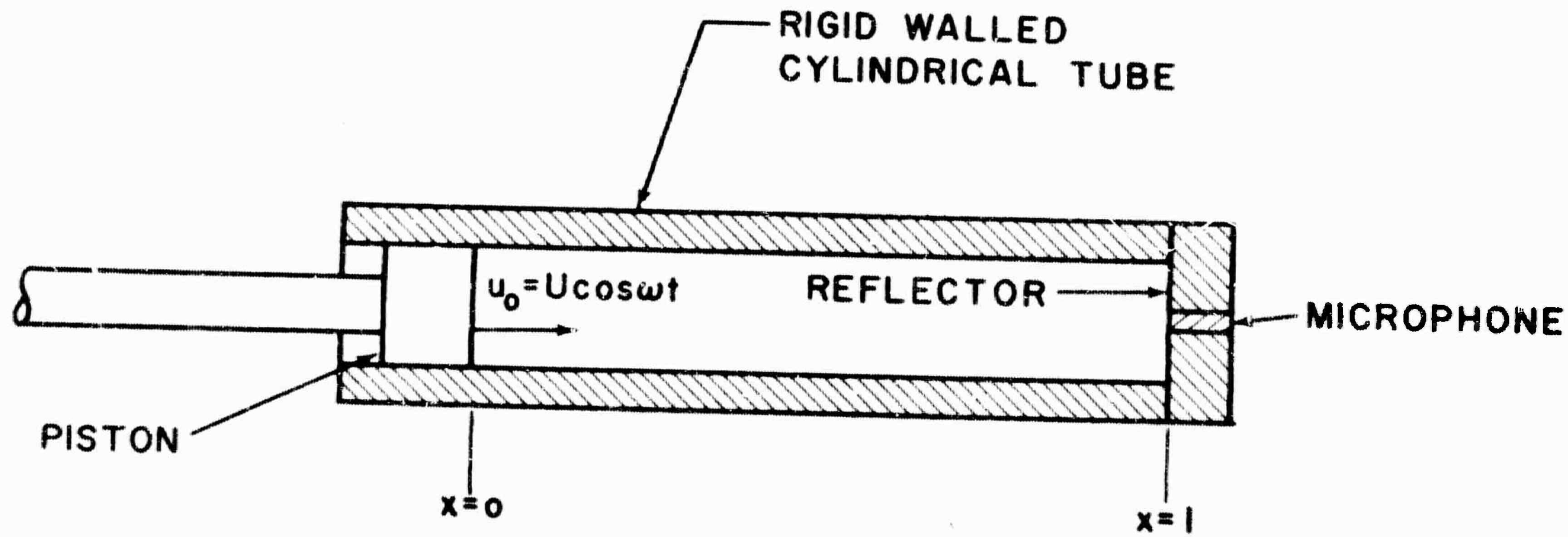

FIG. 1 TUBE FOR DETERMINING RELATIVE ACOUSTIC ABSORPTION OF PROPELLANT COMBUSTION GASES.

FIG. 2 GAS PHASE ATTENUATION APPARATUS IN OPERATION.

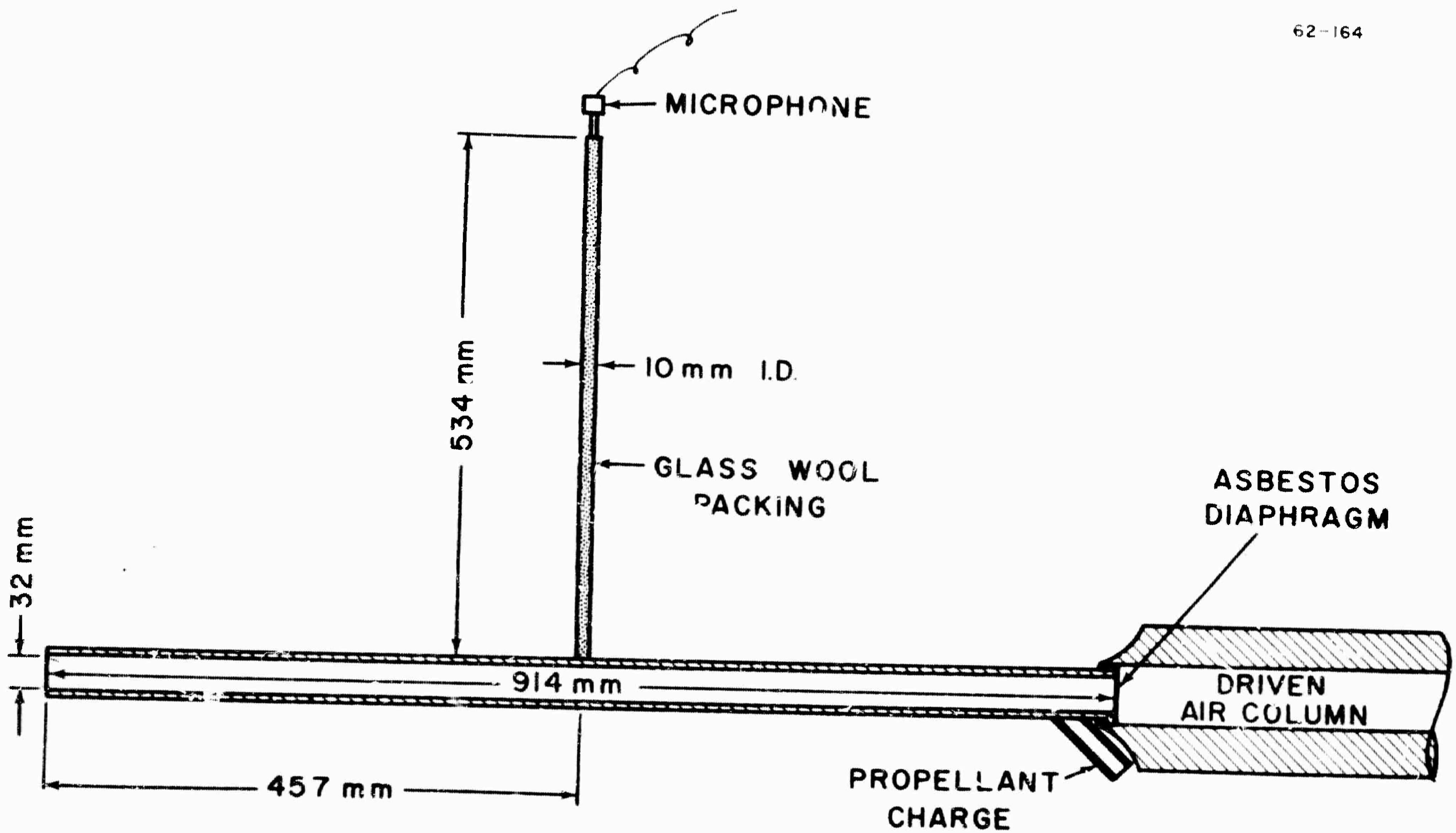

FIG. 3 EXPERIMENTAL APPARATUS FOR DETERMINING ACOUSTIC ABSORPTIVITY OF PROPELLANT COMBUSTION GASES.