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THE EFFECTS OF MISMATCH AND DISTORTION ON THE OUTPUT SIGNAL OF A LINEAR FM PULSE-COMPRESSION FILTER

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-149

DECEMBER 1964

M. H. Ueberschaer

Prepared for

DIRECTORATE OF RADAR AND OPTICS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 750

Prepared by

THE MITRE CORPORATION
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FOREWORD

The author wishes to acknowledge the helpful discussions with Mr. R. W. Jacobus.

THE EFFECTS OF MISMATCH AND DISTORTION ON THE OUTPUT SIGNAL OF A LINEAR FM PULSE-COMPRESSION FILTER

ABSTRACT

This report extends the results of two previous TDR's.[1, 2]* The output functions of the pulse-compression filter which correspond to the matched filter case and the mismatched case are compared. Numerical comparison is made for two pulse-compression systems of particular interest. The primary effect of mismatch is found to be a reduction in signal-to-noise ratio which, in the worst case considered in this report, is less than 10 per cent.

Deviations from the idealized rectangular amplitude spectrum and linear group delay of the transmitted signal, which was used in the previous analysis, are considered. These deviations, which are viewed as distortions, are chosen to provide a more realistic representation of the transmitted signal. The output signal thus obtained is expressed as the sum of the original term and a distortion term. An upper bound for the phase difference caused by the distortion term is obtained and is found to be negligibly small, even in the worst case considered. It therefore appears that the equations derived in the previous TDR's (and which were based on the idealized signal) need not be modified, and that the given procedure for relating the phase and delay measurements to the target parameters remains valid.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

Harry M. Byram

HARRY M. BYRAM
Acting Chief
Radar Division

*Numbers in brackets designate references cited at the end of this report.

CONTENTS

SECTION I

INTRODUCTION

In two recent reports[1, 2]* several expressions for the output signal of a pulse-compression filter were derived, compared and interpreted, and a procedure was given for properly relating a sequence of phase measurements (obtained from the compressed pulse) to the range polynomial of a given target. This analysis was made under the assumption that the transmitted signal has a flat band-limited amplitude spectrum and a linear group delay, and that the receiver is matched to the transmitted signal.

It is clear that a mismatch exists in this case between the incoming signal and the receiver for high radial target velocities. It is also clear that the model of the transmitted signal which was chosen may not be a very realistic one. It is therefore desirable to extend the previous results.

This report contains an estimate of the loss due to the mismatch, and a practical scheme of approaching a matched filter more closely. It also discusses the effects of certain deviations from the idealized amplitude spectrum and group delay characteristic; particularly, whether these deviations or distortions affect the output time function enough to require a different procedure for relating the phase measurements to the target parameters.

As indicated by the block diagram in Fig. 1, the same basic notation which was used in Refs. [1] and [2] is used here.

*Numbers in brackets designate References cited at the end of this report.

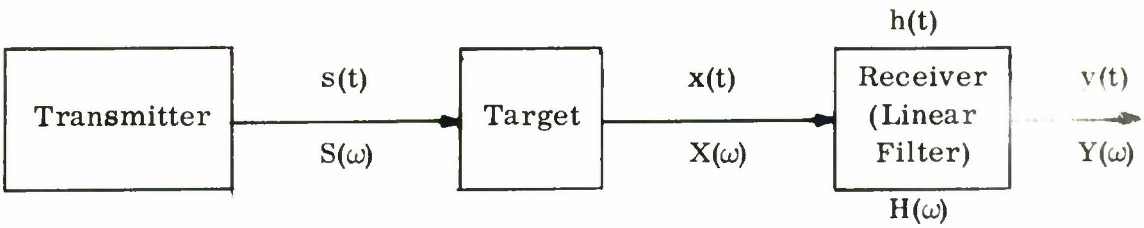

Fig. 1. Pulse-Compression Model

In Ref. [1] it was shown that, under certain realistic assumptions, $X(\omega)$ is related to $S(\omega)$ by the equation

$$S(\omega) = k\, a\, e^{-j\omega \frac{2R}{c}}\, S(a\omega) \qquad (1)$$

where

k = some constant for a given target at a given point in space

$a = \dfrac{c+V}{c-V}$

c = velocity of light in free space

V = radial velocity of the target when the pulse strikes it

R = range of the target when the pulse strikes it.

Figure 1 and Eq. (1) are applicable throughout this report.

SECTION II

MATCHED FILTER CONSIDERATIONS — IDEAL SIGNAL CASE

Assume that the transmitted signal is of the ideal form used in Refs. [1] and [2]. Explicitly, $S(\omega)$ is assumed to have a flat amplitude spectrum of unit height and width W, centered about $\omega = \omega_0$, and a linear group delay with total dispersion T over the band W. Hence, using Eq. (1), $X(\omega)$ will also have a flat amplitude spectrum of height ka and width W/a, centered about $\omega = \omega_0/a$, and a linear group delay with total dispersion aT over the band W/a.

Let the output signal y(t) be written in terms of its envelope E and phase θ, viz.,

$$y(t) = E \cos \theta \quad .$$

It can readily be shown that if $X(\omega)$ is passed through a matched filter, E and θ become

$$\left.\begin{aligned} E &= \left(\frac{2k}{\pi}\right) a \left(\frac{W}{2a}\right) \left[\frac{\sin \frac{W}{2a}\left(t - \frac{2R}{c} - aT\right)}{\frac{W}{2a}\left(t - \frac{2R}{c} - aT\right)}\right] \\ \theta &= \frac{\omega_0}{a}\left(t - \frac{2R}{c} - aT\right) \end{aligned}\right\} . \qquad (2)$$

For the mismatched case the output signal y(t) is a rather complicated function involving Fresnel integrals[1]. However, it was shown[1, 2] that for the kind of targets and the radar parameters of interest to us, y(t) can be approximated by a function of the same form as (2). This function differs from (2) in several respects:

(a) The main lobe of E is slightly wider.

(b) The slope of θ is slightly less.

(c) The time when the maximum value of E occurs is different.

(d) The maximum value (i.e., the peak) of E is lower.

The first three points are insignificant and require at most a slightly different interpretation of the data. The fourth point is consistent with the fact that a matched filter yields a higher signal-to-noise ratio (S/N) than any other linear system.

To estimate the amount of increase which would thus be obtained by using a matched filter, it is first necessary to distinguish between the case of an approaching target and a receding target and analyze what happens to both the noise and the signal in either case. Using the following notation, compute the ratio $r = \frac{S_1/N_1}{S_2/N_2}$, assuming white Gaussian noise of noise power N_0 per unit bandwidth:

S_1 = peak signal for the matched filter case

N_1 = rms noise for the matched filter case

S_2 = peak signal for the mismatched case

N_2 = rms noise for the mismatched case

Consider first the case of a receding target. From Eq. (2), $S_1 = kW/\pi$. The bandwidth of the matched filter is W/a, so that

$$S_1/N_1 = ka/\pi N_0 .$$

The bandwidth of the mismatched filter is W regardless of target velocity, so that $N_2 = W N_0$. The peak signal is approximately equal[1, 2] to

$$S_2 = \frac{ka}{\pi} \left[W \frac{c}{c+V} - 2\omega_0 \frac{V}{c+V} \right]$$

so that

$$r = \frac{S_1/N_1}{S_2/N_2} = \frac{W(c+V)}{Wc - 2\omega_0 V} . \tag{3}$$

Similarly, for the case of an approaching target,

$$r = \frac{W(c-V)}{Wc - 2\omega_0 V} . \tag{4}$$

When the radar parameters for both the 1,000:1 system and the 10,000:1 system* are substituted in Eqs. (3) and (4), it is found that the ratio r does not exceed the value 1.10 for $V \leq 10^4$ meters/second.

Thus, at most, a ten percent increase in signal-to-noise ratio could be realized (for cases of particular interest to us) by building a filter which is matched exactly to the incoming signal. It does not seem worthwhile, at least for our applications, to spend much time in modifying the system to approach matched filter performance more closely. However, from Eqs. (3) and (4) it can be seen that the ratio r can become very large for a different choice of radar parameters (or for higher velocities). In particular, it is noted that $r \to \infty$ as $\frac{2V}{c} \omega_0 \to W$, so that in this case the mismatched filter is inadequate.

*c.f. Reference [1] or [2] for a listing of these parameters.

To build a filter which is matched exactly to the incoming signal is impractical because it would be necessary to continuously change the bandwidth and the slope of the group delay characteristic, as well as the center frequency, according to the radial target velocity. A more practical scheme, which would give a fairly good approximation to a matched filter, is to change only the center frequency of the filter to coincide with that of the incoming signal. When the output signal of such a filter is analyzed, it is found to be of exactly the same form as obtained in Ref. [1] since the same amount of residual quadratic phase as before is present. The only difference is that the carrier frequency of the output signal has changed and the bandwidth has increased as a result of shifting the center frequency of the filter. Hence, all the exact and approximate equations in Refs. [1] and [2] are valid for this case, provided the correct values of ω_0'' and W'' are substituted.

SECTION III

DEVIATIONS FROM RECTANGULAR AMPLITUDE SPECTRUM

In this section, assume that the receiver is matched to the transmitted signal and the phase of the transmitted signal is quadratic as before (i.e., linear group delay), but that the amplitude spectrum has a more realistic shape than what had previously[1] been considered. The approach here is to compute an upper bound for the phase difference (of the output function $y(t)$ at the peak of the envelope), resulting from the deviation from the idealized rectangular spectrum.

A description of the transmitted amplitude spectrum $A(\omega)$, which is convenient as well as realistic, is given by Fig. 2 and Eq. (5).

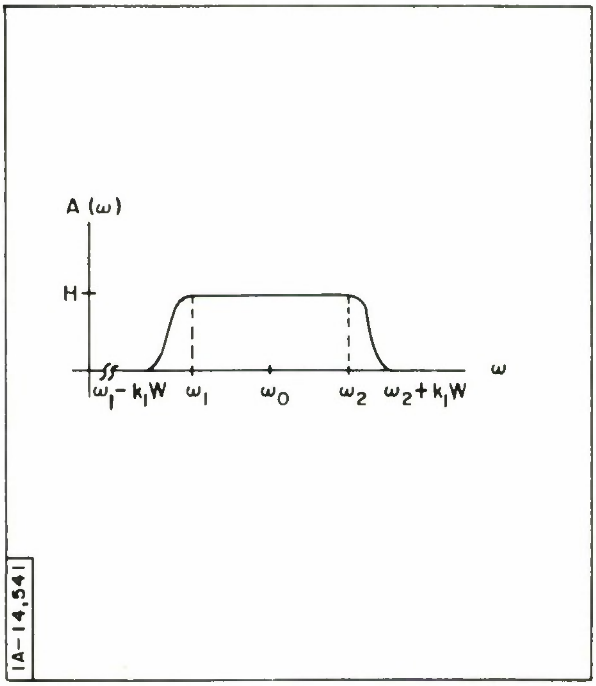

Fig. 2 Transmitted Amplitude Spectrum $A(\omega)$

Only the positive frequency portion is used for convenience.

$$A(\omega) = \begin{cases} \dfrac{H}{2}\left[1 + \cos \dfrac{\pi}{k_1 W} (\omega - \omega_1)\right] & ; \omega_1 - k_1 W \leq \omega \leq \omega_1 \\ H & ; \omega_1 \leq \omega \leq \omega_2 \\ \dfrac{H}{2}\left[1 + \cos \dfrac{\pi}{k_1 W} (\omega - \omega_2)\right] & ; \omega_2 \leq \omega \leq \omega_2 + k_1 W \\ 0 & \text{elsewhere} \quad . \end{cases} \tag{5}$$

It is required that $A(\omega)$ is symmetric about $\omega = \omega_0$ and that the distance between the 6 db points $\left(\text{where } A(\omega) = \frac{H}{4}\right)$ is equal to W. These requirements yield

$$\omega_1 = \omega_0 - \frac{W}{2} + \frac{2}{3} k_1 W$$

$$\omega_2 = \omega_0 + \frac{W}{2} - \frac{2}{3} k_1 W \ . \tag{6}$$

To find H, impose the additional requirement that the energy in this signal is equal to that of the idealized signal whose amplitude spectrum has unit height over the band W; i.e.,

$$2 \int_{\omega_1 - k_1 W}^{\omega_1} \left\{ \frac{H}{2} \left[1 + \cos \frac{\pi}{k_1 W} (\omega - \omega_1) \right] \right\}^2 d\omega + H^2 (\omega_2 - \omega_1) = W \ . \tag{7}$$

Substitution of Eq. (6) into Eq. (7), and integration and solution for H results in

$$H = \left(1 + \frac{1}{6} k_1\right)^{-1/2} . \tag{8}$$

It is instructive to review the relationship between the transmitted and received amplitude spectra, in accordance with Eq. (1). Let these spectra be denoted by $A_s(\omega)$ and $A_x(\omega)$, respectively, and let $A_Y(\omega)$ be the spectrum of the output. The relationship between these quantities is sketched in Fig. 3. The figure applies to a receding target. Again, only the positive frequency portions are shown. $A_x(\omega)$ is seen to be a shifted and compressed version of $A_s(\omega)$. For an approaching target, $A_x(\omega)$ would be stretched (instead of compressed) and shifted to the right (instead of the left).

Figure 3 corresponds to the case where the Doppler shift is larger than $k_1 W$. i.e., $\frac{\omega_1}{a} < \omega_1 - k_1 W$. Thus the tails of $A_x(\omega)$ do not overlap with those of $A_s(\omega)$. For this case,

$$A_Y(\omega) = \begin{cases} \frac{kH^2}{2}\left[1 + \cos \frac{\pi}{k_1 W} (\omega - \omega_1)\right] & ; \ \omega_1 - k_1 W \le \omega \le \omega_1 \\ kH^2 & ; \ \omega_1 \le \omega \le \frac{\omega_2}{a} \\ \frac{kH^2}{2}\left[1 + \cos \frac{\pi}{k_1 W} \left(\frac{\omega - \omega_2}{a}\right)\right] & ; \ \frac{\omega_2}{a} \le \omega \le \frac{\omega_2 + k_1 W}{a} \\ 0 & \text{elsewhere} \end{cases} . \tag{9}$$

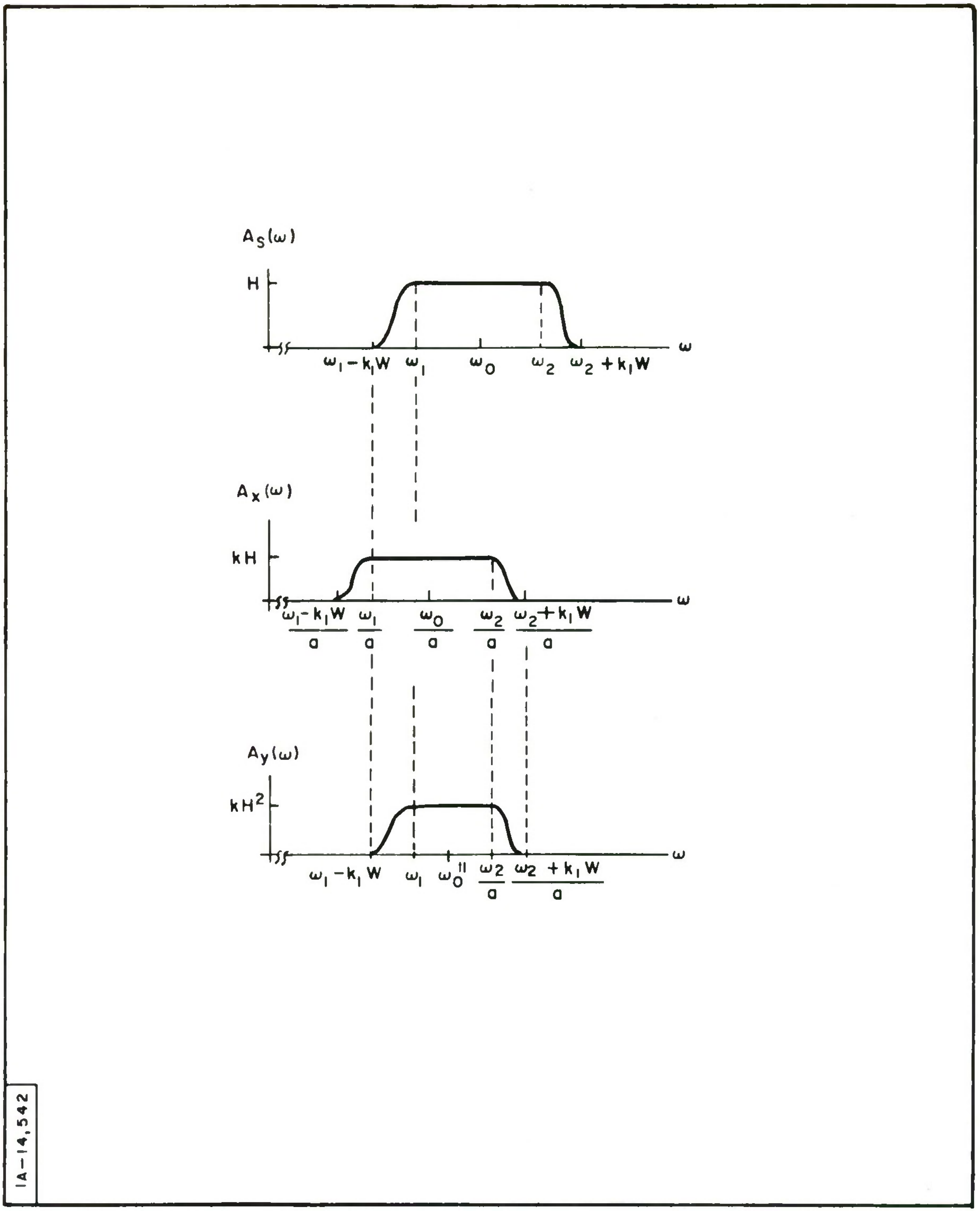

Fig. 3 Relationship between $A_s(\omega)$, $A_x(\omega)$ and $A_Y(\omega)$

Using the subscript 1 for the idealized rectangular case, the corresponding expression is

$$A_{Y_1}(\omega) = \begin{cases} k \quad ; & \omega_0 - \frac{W}{2} \leq \omega \leq \frac{1}{a}\left(\omega_0 + \frac{W}{2}\right) \\ 0 & \text{elsewhere} \quad . \end{cases} \tag{10}$$

Now to write $A_Y(\omega)$ in terms of $A_{Y_1}(\omega)$ and a difference term $\Delta A_Y(\omega)$:

$$A_Y(\omega) = A_{Y_1}(\omega) + \Delta A_Y(\omega) \quad . \tag{11}$$

Since the system is linear, the output time function can also be expressed by the superposition of the corresponding two time functions.

$$\begin{aligned} y(t) &= 2\,\mathrm{Re}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} A_Y(\omega)\, e^{j\phi_Y(\omega)}\, e^{j\omega t}\, d\omega\right\} \\ &= \frac{1}{\pi}\,\mathrm{Re}\left\{\int_{-\infty}^{\infty}\left[A_{Y_1}(\omega) + \Delta A_Y(\omega)\right] e^{j\phi_Y(\omega)}\, e^{j\omega t}\, d\omega\right\} \\ &= y_1(t) + \frac{1}{\pi}\,\mathrm{Re}\left\{\int_{-\infty}^{\infty} \Delta A_Y(\omega)\, e^{j\phi_Y(\omega)}\, e^{j\omega t}\, d\omega\right\} \end{aligned} \tag{12}$$

where Re { } stand for "the real part of { }", $\phi_Y(\omega)$ is the phase of the output, and $y_1(t)$ is the output time function for the idealized case. Let the second term on the right hand side of Eq. (12) be denoted by $\Delta y(t)$. Then

$$y(t) = y_1(t) + \Delta y(t) \quad .$$

At any given time t_i, $y(t_i)$ may be regarded as the vector sum of $y_1(t_i)$ and $\Delta y(t_i)$. Of particular interest is the time $t = t_p$. This is the time when the peak of the signal occurs and the phase measurement is made. It follows from the principle of stationary phase* that, for the case under consideration, $y(t)$ and $y_1(t)$ peak at approximately the same time.

Now it is desirable to estimate the angle $\Delta\theta(t_p)$ between the vectors $y(t_p)$ and $y_1(t_p)$. This angle is equal to the phase error which would result from transmitting the spectrum of Fig. 2 but interpreting the signal in accordance with Ref. [2] (i.e., as though a rectangular spectrum had been transmitted).

It is clear that $\Delta\theta(t_p)$ is largest when all the contributions which make up $\Delta y(t_p)$ are in phase with each other and are 90 degrees out of phase with $y_1(t)$ at $t = t_p$. In short,

$$\Delta\theta(t_p) \leq \tan^{-1}\left[\frac{\Delta y(t_p)\,\big|_{max}}{y_1(t_p)}\right] \tag{13}$$

where

$$\Delta y(t_p)\,\big|_{max} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\Delta A_Y(\omega)|^2 \, d\omega \, . \tag{14}$$

Now, $y_1(t_p)$ is equal to the energy in $x_1(t)$ which is passed by the filter, i.e.,

$$y_1(t_p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |(A_{Y_1}(\omega)|^2 \, d\omega \, . \tag{15}$$

*c.f. Goldman, Reference [3], pp. 111-117.

Using Eq. (10) and taking the negative frequency portion into account,

$$y_1(t_p) = \frac{1}{\pi} k^2 \left[W \frac{c}{c+V} - \omega_0 \frac{2V}{c+V} \right] . \tag{16}$$

When Eqs. (6), (9) and (11) are substituted into (14) one obtains, after some algebra,

$$\Delta y(t_p)\Big|_{max} = \frac{k^2}{\pi} \left[(H-1)^2 \left(W - \frac{4}{3} k_1 W \right) + 2\left(\frac{H}{2} - 1 \right)^2 k_1 W - \frac{H^2 k_1 W}{4} \right] . \tag{17}$$

Clearly, the largest phase difference will occur when both k_1 and V are largest. It is considered that $V \leq 10^4$ m/sec. The 1,000:1 pulse compression system which is presently in operation was designed so that the tails of the spectrum in Fig. 2 are no wider than 10 percent of the total bandwidth, i.e., $k_1 \leq 0.05$. Using these maximum values of k_1 and V and substituting Eqs. (8), and (16) and (17) into (13), the result for the 1,000:1 system is

$$\Delta\theta(t_p) \leq \tan^{-1} 0.0201 \approx 0.0201 \text{ radians or}$$

$$\Delta\theta(t_p) \leq 1.15 \text{ degrees} .$$

The result for the 10,000:1 system and the same values of k_1 and V is

$$\Delta\theta(t_p) \leq 1.07 \text{ degrees} .$$

Since this is the worst possible case, these errors represent pessimistic upper bounds. Even so, the values are small enough to be considered negligible. It is concluded, therefore, that the phase errors caused by amplitude distortion alone can be ignored.

In the above analysis a particular deviation from the idealized amplitude spectrum has been assumed. This was done because the shape of the spectrum in Fig. 2 is quite realistic and lends itself to mathematical analysis.

From the analysis it is clear that the particular shape of the tails of the spectrum is unimportant. What matters in the error estimation is the energy contained in the difference spectrum, $\Delta A_Y(\omega)$. For example, if a trapezoidal amplitude spectrum is assumed, the resulting $\Delta\theta(t_p)$ is also on the order of magnitude of one degree.

SECTION IV

DEVIATION FROM LINEAR GROUP DELAY

In this section it is assumed that the amplitude spectrum is rectangular again, and that the received is matched to the transmitted signal. As in the last previous section, an upper bound is computed for the phase difference of y (t) at the point of measurement, where this difference is now due to deviations from the idealized linear group delay of the transmitted signal. The group delay $T_S(\omega)$ of the transmitted signal is expressed by

$$T_S(\omega) = T_{S_1}(\omega) + T_R(\omega) \; ,$$

where $T_{S_1}(\omega)$ is the linear term and $T_R(\omega)$ is the ripple in the group delay. $T_R(\omega)$ could be represented by a Fourier series and approximated rather well by a few terms of the form $a_i \cos b_i \omega$. The terms with the largest value of a_i and the smallest value of b_i have the greatest effect on the output. With these values denoted by a_0 and b_0, respectively, the effect on the output of this one term can be derived.

$$T_R(\omega) = a_0 \cos b_0 \omega \; .$$

For the 1000:1 system, pessimistic values of a_0 and b_0 are $a_0 = 1\ \mu\text{sec}$, $b_0 = 5\ \mu\text{sec}$. These numbers shall be inserted later. The phase of S (ω) is found by integrating $T_S(\omega)$, i.e.,

$$\phi_S(\omega) = -\int T_S(\omega)\, d\omega = \phi_{S_1}(\omega) + \frac{a_0}{b_0} \sin b_0 \omega \; . \qquad (18)$$

The phase of the returned signal is then given[1] by

$$\phi_X(\omega) = \phi_S(a\omega) - \frac{2R}{c}$$

$$= \phi_{S_1}(a\omega) + \frac{a_0}{b_0} \sin b_0 a\omega - \frac{2R}{c} , \tag{19}$$

where $a = \frac{c+V}{c-V}$. Hence, the phase at the output of the filter is equal to $\phi_X(\omega) - \phi_S(\omega) - T$, i.e.,

$$\phi_Y(\omega) = \phi_S(a\omega) - \phi_S(\omega) - \frac{2R}{c} - T$$

$$= \phi_{Y_1}(\omega) + \frac{a_0}{b_0} [\sin b_0 a\omega - \sin b_0 \omega] . \tag{20}$$

Here $\phi_{Y_1}(\omega)$ is the output phase under the assumption of linear group delay. Note that $\phi_Y(\omega)$ is equal to $\phi_{Y_1}(\omega)$ when $a = 1$, i.e., the effect of the phase ripple is zero for a stationary target. This is due to matched filter reception.

To obtain the output time function from (20), as in Ref. [1],

$$y(t) = 2 \operatorname{Re} \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} A_Y(\omega) \, e^{j\phi_Y(\omega)} \, e^{j\omega t} \, d\omega \right\} \tag{21}$$

where Re{ } stands for "the real part of { } ", and $A_Y(\omega)$ is the amplitude spectrum of the output. Substitution of Eq. (15) into Eq. (21) produces

$$y(t) = \frac{1}{\pi} \mathrm{Re} \int_{-\infty}^{\infty} A_Y(\omega)\, e^{j\phi_{Y_1}(\omega)}\, e^{j \frac{a_0}{b_0} [\sin b_0 a\omega - \sin b_0 \omega]}\, e^{j\omega t}\, d\omega \; . \tag{22}$$

The following relationship[4] can be used:

$$e^{j x \sin \theta} = \sum_{k=-\infty}^{\infty} J_k(x)\, e^{jk\theta} \tag{23}$$

where J_k represents the Bessel functions of the first kind and order k. For small values of x the infinite series can be approximated by the first few terms.

$$e^{j x \sin \theta} \approx J_0(x) + \left[J_1(x)\, e^{j\theta} + J_{-1}(x)\, e^{-j\theta} \right] \; . \tag{24}$$

Let $x = \frac{a_0}{b_0}$, $\theta_1 = b_0 a\omega$ and $\theta_2 = b_0 \omega$. Using the approximation of Eq. (24),

$$e^{jx[\sin \theta_1 - \sin \theta_2]} = e^{jx \sin \theta_1}\, e^{-jx \sin \theta_2}$$

$$= \left[J_0(x) + J_1(x)\, e^{j\theta_1} + J_{-1}(x)\, e^{-j\theta_1} \right] \left[J_0(-x) + J_1(-x)\, e^{j\theta_2} + J_{-1}(-x)\, e^{-j\theta_2} \right] \; . \tag{25}$$

Now since $J_0(-x) = J_0(x)$, $J_1(-x) = -J_1(x)$, and $J_{-1}(x) = -J_1(x)$, Eq. (25) can be rewritten as

$$
\begin{aligned}
e^{jx[\sin\theta_1 - \sin\theta_2]} &= \left[J_0(x) + J_1(x)\, e^{j\theta_1} - J_1(x)\, e^{-j\theta_1}\right] \\
&\quad \left[J_0(x) - J_1(x)\, e^{j\theta_2} + J_1(x)\, e^{-j\theta_2}\right] \\
&= J_0^2(x) + J_1^2(x)\left[-e^{j(\theta_1 - \theta_2)} + e^{j(\theta_1 - \theta_2)} + e^{-j(\theta_1 - \theta_2)} - e^{-j(\theta_1 + \theta_2)}\right] \\
&\quad + J_0(x)\, J_1(x)\left[e^{j\theta_1} - e^{-j\theta_1} + e^{-j\theta_2} - e^{j\theta_2}\right]. \qquad (26)
\end{aligned}
$$

Substituting for θ_1 and θ_2, this becomes

$$
\begin{aligned}
e^{jx[\sin\theta_1 - \sin\theta_2]} &= J_0^2(x) + J_1^2(x)\left[-e^{jb_0(a+1)\omega} + e^{jb_0(a-1)\omega}\right. \\
&\qquad \left. + e^{-jb_0(a+1)\omega} - e^{-jb_0(a-1)\omega}\right] \\
&\quad + J_0(x)\, J_1(x)\left[e^{jb_0 a\omega} - e^{-jb_0 a\omega} + e^{-jb_0\omega} - e^{jb_0\omega}\right]. \qquad (27)
\end{aligned}
$$

Going back to Eq. (22), let

$$
y_1(t) = \frac{1}{\pi}\,\mathrm{Re}\int_{-\infty}^{\infty} A_Y(\omega)\, e^{j\phi_{Y_1}(\omega)}\, e^{j\omega t}\, d\omega
$$

be the output time function for the case of zero distortion in the phase characteristic (i.e., for linear group delay).

Using Eq. (27), y(t) in Eq. (22) can then be written as

$$y(t) = J_0^2(x)\, y_1(t) + J_1^2(x) \{y_1(t - [a+1]\, b_0) - y_1(t + [a+1]\, b_0)$$
$$+ y_1(t + [a-1]\, b_0) - y_1(t - [a-1]\, b_0)\}$$
$$+ J_0(x)\, J_1(x) \{y_1(t + b_0 a) - y_1(t - b_0 a) + y_1(t - b_0) - y_1(t + b_0)\}. \quad (28)$$

It can be seen that the output consists of the original function $y_1(t)$, modified in amplitude by $J_0^2(x)$, plus several "paired echoes." Eq. (28) can be expressed in somewhat more meaningful quantities. Since $a = \frac{c+V}{c-V}$, $a + 1 = \frac{2c}{c-V} \approx 2$ and $a - 1 = \frac{2V}{c-V} \approx \frac{2V}{c}$. Thus, Eq. (12) is approximately equal to

$$y(t) = J_0^2(x)\, y_1(t) + J_1^2(x) \Big\{ y_1(t - 2\, b_0) - y_1(t + 2\, b_0)$$
$$+ y_1\left(t + \frac{2V}{c}\, b_0\right) - y_1\left(t - \frac{2V}{c}\, b_0\right)\Big\} + J_0(x)\, J_1(x) \Big\{ y_1\left(t + \frac{c+V}{c-V}\, b_0\right)$$
$$- y_1\left(t - \frac{c+V}{c-V}\, b_0\right) + y_1(t - b_0) - y_1(t + b_0)\Big\}. \quad (29)$$

The primary interest here is in estimating the effect of this distortion on the phase measurement which is made at the peak of the compressed pulse. It can be argued, as in the case of amplitude distortion, that the greatest possible effect on the measured phase occurs when all the contributions due to the distortion are in phase with each other and are 90 degrees out of phase with the main signal at the time of the peak, t_p, when the phase measurement is made. In Eq. (29) $y(t)$ can be written as

$$y(t) = J_0^2(x)\, y_1(t) + \Delta y(t) \ .$$

The difference in phase, $\Delta\theta$, between $y(t)$ and $y_1(t)$ at the time $t = t_p$ can be bounded as follows:

$$|\Delta\theta| \leq \tan^{-1} \frac{|\Delta y(t_p)|}{J_0^2(x)\, |y_1(t_p)|} \ .$$

Since $y_1(t)$ gets smaller as $|t|$ increases, $|\Delta y(t_p)|$ can in turn be bounded, assuming that all its contributions add in phase at $t = t_p$, by

$$|\Delta y(t_p)| \leq J_1^2(x) \left\{ 2\, |y_1(t - 2b_0)| + 2 \left| y_1\left(t - \frac{2V}{c}\, b_0\right) \right| \right\}$$

$$+ |J_0(x)|\, |J_1(x)|\, \{4\, |y_1(t - b_0)|\} \ . \qquad (30)$$

Now to estimate $J_0(x)$ and $J_1(x)$. For integral values of n, the Bessel function $J_n(x)$ can be expressed[4] by

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k\, x^{n+2k}}{2^{n+2k}\, k!\, (k+n)!} \ .$$

In particular

$$J_0(x) = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \ldots + \ldots$$

$$J_1(x) = \frac{x}{2} - \frac{x^3}{16} + \ldots - \ldots$$

For $0 < x < 1$.

$$1 - \frac{x^2}{4} < J_0(x) < 1$$

$$\frac{x}{2} - \frac{x^2}{16} < J_1(x) < \frac{x}{2} .$$

Substituting for x,

$$1 - 0.1 < J_0(x) < 1$$

$$0.1 - 0.0005 < J_1(x) < 0.1$$

so that $J_0(x) = 1$ and $J_1(x) = 0.1$ for all practical purposes. (Note that the largest paired echoes are "down" by 20 log 0.1 = 20 db. This indicates that the values of a_0 and b_0 which were chosen are extremely pessimistic, as the largest observed echoes are approximately 30 db down.) Substituting the above values into Eq. (30) and using the fact that $y_1(t)$ has a $(\sin x)/x$ envelope with a 1 μsec pulse width, $|\Delta y(t_p)|$ can be bounded as follows:

$$|\Delta y(t_p)| \leq 0.01 \left\{ 2 \left(\frac{4}{19\pi} \right) + 2 \right\} + 0.1 \left\{ 4 \left(\frac{4}{9\pi} \right) \right\} = 0.036$$

so that

$$|\Delta\theta| \leq \tan^{-1} .036 \approx .036 \text{ radians} \approx 2.06 \text{ degrees} \quad .$$

This phase error, which is extremely pessimistic, is still small enough to be ignored. As mentioned before, the largest observed sidelobes are approximately 30 db down. These sidelobes are due to the combined effects of amplitude and phase distortion as well as all other imperfections in the system. It is expected that the sidelobes can be reduced to 40 db in the near future. The sidelobes we have computed here as a result of the dominant phase ripple term are only 20 db down. This more than compensates for the omission of any higher order terms in the Fourier expansion of the ripple in the group delay as well as truncating the series in Eq. (23) by Eq. (24). Furthermore, the calculations were made on the basis that all the echo contributions add in phase at the time of measurement, and that all the echo delays are such that the peak of the nearest sidelobe of the $(\sin x)/x$ envelope occurs at the time of measurement. It is fairly safe to say that the value of $\Delta\theta$ which would be obtained by a more detailed analysis would be several orders of magnitude less than 0.2 degrees.

SECTION V

SUMMARY

For the two pulse-compression systems and the radial target velocities under consideration, the following conclusions can be drawn:

(a) At most a ten percent increase in signal-to-noise ratio would be obtained by using a filter which is matched to the incoming signal instead of the transmitted signal. No other advantage would be gained since a satsifactory procedure of interpreting the radar data for the mismatched case is available.

(b) The upper bound on the phase error (under the worst cases which are expected to be encountered) due to both amplitude distortion and phase distortion, taken separately, is negligibly small. The sum of the two phase errors is then also negligible.

(c) Interpretation of the delay and phase measurements in accordance with the procedure given in Ref. [2] should yield highly accurate estimates of target parameters even though the actual transmitted signal differs from the idealized model used in the analysis.

A word of caution is in order. In the foregoing analysis it has been assumed that the receiver (which includes everything from the antenna on back) can be described by a linear, time-invariant filter. This is only an approximation made for convenience and for lack of knowledge. It is hoped to be a good approximation insofar as the measured quantities (which are related to the target parameters) are concerned. This will have to be tested experimentally.

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13. ABSTRACT

This report extends the results of two previous TDR's. [1, 2]* The output functions of the pulse-compression filter which correspond to the matched filter case and the mismatched case are compared. Numerical comparison is made for two pulse-compression systems of particular interest. The primary effect of mismatch is found to be a reduction in signal-to-noise ratio which, in the worst case considered in this report, is less than 10 per cent.

Deviations from the idealized rectangular amplitude spectrum and linear group delay of the transmitted, signal, which was used in the previous analysis, are considered. These deviations, which are viewed as distortions, are chosen to provide a more realistic representation of the transmitted signal. The output signal thus obtained is expressed as the sum of the original term and a distortion term. An upper bound for the phase difference caused by the distortion term is obtained and is found to be negligibly small, even in the worst case considered. It therefore appears that the equations derived in the previous TDR's (and which were based on the idealized signal) need not be modified, and that the given procedure for relating the phase and delay measurements to the target parameters remains valid.

*Numbers in brackets designate references cited at the end of this report.

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