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R. D. Behn

Dynamic-Range Reduction of Video Signals by Amplitude Compression

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

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Abstract

Methods of recording a video signal with large dynamic range on a magnetic tape (or other recording medium) that has a smaller dynamic range than that of the signal are discussed. The useful part of the voltage range of the input signal is mapped onto the linear voltage range of the recorder. Several maps are considered with the thought of minimizing the degradation of the final signal by recorder noise.

Two mapping functions are found to be useful: a logarithmic function, and a part-linear, part-logarithmic one. The use of these maps entails a reduction in signal-to-noise ratio for large input signals. The relationships among the input signal dynamic range, the recorder dynamic range, and the attainable output signal-to-noise ratios are calculated. Results are presented graphically for several particular cases.

Accepted for the Air Force
Stanley J. Wisniewski
Lt Colonel, USAF
Chief, Lincoln Laboratory Office

DYNAMIC-RANGE REDUCTION OF VIDEO SIGNALS BY AMPLITUDE COMPRESSION

INTRODUCTION

This paper is concerned with the recording of a video signal on magnetic tape. The dynamic range of the recorder is less than that of the input signal. The input signal voltage range must therefore be compressed to fit that of the recorder. The input signal is the magnitude of a returned radar signal vector. The noise of the tape recorder is assumed to have a Gaussian distribution and to be white over the bandwidth of the recorder. It reduces the signal-to-noise ratio of all input signals.

The purpose of investigating various methods of compressing the input signal is to ascertain which is characterized by the most desirable output signal-to-noise curve. An optimum output signal-to-noise curve would be as close to that of the input as possible. However, due to the restricted dynamic range available for recording, the output signal-to-noise curve must lie substantially below that of the input in some regions. For an accurate recording to be made of a signal, it is assumed that a particular signal-to-noise ratio, herein termed the critical level, is desirable and sufficient. Signals with an input signal-to-noise ratio that is below this critical level should be corrupted as little as possible by tape noise to allow the maximum amount of information to be obtained from them. Input signals with a signal-to-noise ratio greater than the critical level may be degraded to this level by tape noise. (See Fig. 1.)

The selection of a mapping function is influenced by the relative importance of signals of various amplitudes. We assume here that all signal magnitudes are equally significant over the range of input signal-to-noise ratios from unity to a particular maximum level, and that there is no interest in signals outside this range.

For purposes of illustration, a particular set of signal and recorder characteristics is assumed. The results presented in this paper pertain to them. However, the equations derived and the computer programs written (for the IBM 7094) to calculate output signal-to-noise ratios are general. The particular input signal dynamic range that is considered is 60 db. The linear range of the recorder is assumed to extend from 0.02 (the rms level of the tape noise) to 1.0 (a dynamic range of 34 db).

THEORY

The mathematical procedures employed are shown in Fig. 2. Their basic functions are to (1) map the signal into a voltage range corresponding to that of the recorder, (2) add recorder noise to the signal, and then (3) reverse-map the signal in order that it may be analyzed.

The amount of tape noise in the output signal is dependent upon the shape of the reverse-mapping function (the inverse of the forward-mapping function), since this shape determines the probability distribution of the tape noise in the output. If the mapping curve is approximately linear over a range that includes the most probable values of the tape noise, then the rms value of the tape noise in the output will be the rms value of this noise on the tape multiplied by the slope of the reverse-mapping function. This is true since the shape of the probability distribution function is not affected by a linear map. Its scale, however, is changed by a multiplicative constant, the slope. The probability distribution function of the tape noise broadens with an increase in the slope of the reverse-mapping function. Consequently, the signal-to-noise ratio is increased by decreasing the slope of this function (or increasing the slope of the forward-mapping function).

The Input Signal

The input to the system is a signal vector with components $X = R \cdot \cos \Phi$, $Y = R \cdot \sin \Phi$ * which is corrupted by a Gaussian noise vector with components

* Capital letters indicate pure signal, small letters a signal with noise, or noise alone.

a, b. The resulting signal components, $x = r \cdot \cos \varphi = R \cdot \cos \Phi + a$, $y = r \cdot \sin \varphi = R \cdot \sin \Phi + b$, have a probability distribution:

$$p(x, y) = \frac{1}{2\pi\sigma^2} \exp \left[- \frac{(x - R\cos\Phi)^2 + (y - R\sin\Phi)^2}{2\sigma^2} \right] ,$$

where σ is the rms noise voltage. (This equation and others given in the text are derived in the appendices.)

Upon detection by a peak detector, the probability distribution of the signal is:

$$p(r) = \frac{r}{\sigma^2} \exp \left(- \frac{r^2 + R^2}{2\sigma^2} \right) I_o \left(- \frac{rR}{\sigma^2} \right) ,$$

where I_o is the modified Bessel function of order zero.

Defining D as $R/\sqrt{2}\sigma$, the ratio of the rms input voltage to the rms value of the noise without a signal present, one avoids the necessity of indicating the input rms noise level, and all results can be given as a function of D rather than R. The noise is defined as $n = r - R$, and from this the input signal-to-noise ratio is found to be

$$(S/N)_{IN} = -10 \log \left[2 - \frac{2}{D} \exp(-D^2) \sum_{m=o}^{\infty} \frac{D^{2m}\Gamma(m + \frac{3}{2})}{(m!)^2} + \frac{1}{D^2} \right] .$$

This is plotted in Fig. 4.

The Linear Mapping System

The simplest map is characterized by a linear relationship between the input signal voltage and the voltage recorded, $t = f(r) = \alpha r + \beta$, $r' = (1/\alpha)t' - \beta/\alpha$. (See Fig. 3.) Here α and β are uniquely determined by the voltage ranges in question. Again, it is more convenient to express the input signal in terms of the ratio of the signal voltage to the input rms noise voltage when the signal is not present, $d = r/\sqrt{2}\sigma$. Thus, an equivalent mapping function is $t = g(d) = \gamma d + \beta$, where $\gamma = (\sqrt{2}\sigma) \cdot \alpha$. The expectation value of the output noise is the sum of two terms, one due to the input noise, and the other to that of the tape. The first is identical with that derived when the input noise alone is present. The other is dependent strictly on the rms level of the tape noise, σ_T , and the constant slope of the mapping function. The output signal-to-noise ratio is:

$$(S/N)_{OUT} = -10 \cdot \log \left[2 - \frac{2}{D} \exp(-D^2) \sum_{m=o}^{\infty} \frac{D^{2m} \Gamma(m + \frac{3}{2})}{(m!)^2} + \frac{1}{D^2} + \left(\frac{\sigma_T}{\gamma \cdot D}\right)^2 \right] .$$

This is illustrated in Fig. 4, with the slope of the reverse-mapping function, $1/\gamma$, indicated.

Figure 4 indicates that a single linear map adds so much tape noise as to make low-level signals useless at the output. A second recorder can be added to the system, to record only the lower-level signals (i. e. , those signals whose output signal-to-noise ratios are below a critical level). Since this recorder is handling a smaller dynamic range of input, the slope of the reverse-mapping function is smaller, and consequently, the output signal-to-noise ratio is increased for the low-level signals.

In reference to the illustrated example (Fig. 4), we assume that the critical level is 14 db. Then, since the output signal-to-noise ratio with one recorder is greater than 14 db for D greater than 100, the second recorder handles only the first 40 db ($1 \leq D \leq 100$) of the input dynamic range. The signal-to-noise ratio for the lower 40 db of the dynamic range of the signal is degraded by about 9. 5 db; however, the output signal-to-noise ratio is above the critical level for $D > 10$.

If 20 db is chosen as the critical level, then the lower 46 db of the dynamic range must be mapped onto the second tape. This results in an output signal-to-noise ratio that is above the critical level for only the upper

28 db ($D > 40$) of the dynamic range of the signal. The signal-to-noise ratio of the signals on the second recorder is degraded by about 15 db.

The Logarithmic Mapping Function

As indicated above, an input signal with a signal-to-noise ratio above a particular critical level may be degraded by tape noise down to this level without compromising the utility of the output. However, an input signal with a signal-to-noise ratio below this level should remain as uncorrupted by recorder noise as possible. This suggests a reverse-mapping function with a slope that is small for low input-signal voltages, and increases as the level of the input signal increases. One such map has a reverse characteristic that is exponential, thus making the forward map a logarithmic function. For this function, the degree of compression of the input signal increases as the magnitude of the input signal increases:

$$t = Z \cdot \log(r + \sqrt{2}\sigma G) + \beta = Z \cdot \log(d + G) + Z \log(\sqrt{2}\sigma) + \beta$$

$$r' = \exp\left(\frac{t' - \beta}{p}\right) - \sqrt{2}\sigma G$$

where

$$p = Zk$$

and

$$k = \log e = 0.4343$$

See Fig. 5. Here G, which can be selected arbitrarily, determines the section of the logarithmic curve employed in the map. The constants Z and β are determined by the input signal and the recording voltage ranges.

This mapping curve does not have a slope that is approximately constant over the region of significance for the tape noise probability distribution, and thus, the shape of the mapping curve is significant in determining the output signal-to-noise ratio. Because of the logarithmic characteristic of the forward mapping function, voltages added on the tape will result in the multiplication of their antilogarithms at the output. Thus, we cannot merely add a term for the expectation value of the tape noise to that of the input noise to obtain that of the total noise. The total expectation of the noise will contain terms that are due to both sources of noise. The signal-to-noise ratio of the output is given by:

$$(S/N)_{LOG} = -10 \cdot \log \{\exp(2(\frac{\sigma_T}{p})^2) \cdot [1+(\frac{G}{D})^2 + \frac{1}{D^2} + \frac{2G}{x}\exp(-D^2) \sum_{m=o}^{\infty} \frac{D^{2m}\Gamma(m+\frac{3}{2})}{(m!)^2}]$$

$$- \exp(\frac{1}{2}(\frac{\sigma_T}{p})^2) \cdot (G + \exp(-D^2) \sum_{m=o}^{\infty} \frac{D^{2m}\Gamma(m+\frac{3}{2})}{(m!)^2})(1+\frac{G}{D})\frac{2}{D} + (1+\frac{2G}{D}+(\frac{G}{D})^2)\} \quad .$$

This is plotted in Fig. 6 for a map employing various sections of the logarithmic function (denoted by G) .

The Lin-Log Mapping Function

The signal-to-noise ratio of a logarithmically-mapped signal can be made relatively constant over a portion of the upper part of its range by the proper selection of G . On the other hand, a reverse linear mapping function with a small slope produces an output signal-to-noise-ratio curve that is only slightly below that of the input. These two features are combined in a lin-log mapping function. If a logarithmic map is used for the upper section of the input dynamic range, and a linear one for the lower section (Fig. 7), the output signal-to-noise ratio can be made to rise to a particular level and then remain relatively constant at that level as the magnitude of the input signal increases.

The ranges allotted to the linear map and the logarithmic map of both the input voltage and the tape voltage are first determined. From these regions, all the parameters of the mapping functions, with the exception of G , are determined, while G (the parameter that selects the section of the logarithmic curve employed in the map) is adjusted to provide continuity of the signal-to-noise ratio at the point where the mapping function is changed. Thus, for such a combined map, G is determined uniquely.

The possibility of the tape noise causing a signal that is forward-mapped with the linear function to be reverse-mapped with the logarithmic function has been ignored. The signal-to-noise ratios in the two ranges are assumed to be given by the appropriate formulas previously determined. Two particular examples of lin-log mapping are shown in Fig. 8.

DISCUSSION

This paper has presented only the simplest maps. Other mapping functions may be suggested by a set of signal characteristics, particularly if all signal levels are not considered equally interesting.

The linear mapping function produces an output signal-to-noise-ratio curve that is parallel to that of the input with a separation dependent on the slope of the map (Fig. 4). For the selected signal and recorder parameters, the degradation of the signals is large over a considerable range of input signal magnitudes. The addition of a second recorder to the system doubles the dynamic range of the output signals with signal-to-noise ratio above the critical level. Nevertheless, for the case illustrated, there is considerable degradation of weak signals.

A logarithmic mapping function that only slightly degrades signals with low input signal-to-noise ratio can be found by selecting a low value of G. The output signal-to-noise ratio becomes approximately constant for signals of high input level. The smaller the linear range of the recorder, and the smaller the value of G, the lower will be this leveling-off point. If this constant value is not below the critical level, a logarithmic map is adequate. This is true in the particular problem studied if a signal-to-noise ratio of 14 to 17 db is considered sufficient. However, if the critical level is higher (e.g., 20 db), the degradation of the low-level signals becomes excessive. More recorder dynamic range is required.

The shape of the output signal-to-noise-ratio curve for a lin-log map may be made close to the ideal shape discussed in the Introduction. As with the logarithmic map, if the critical level is too high (e.g., 20 db), the portion of the dynamic range of the tape that is allotted to the linear portion of the map is small. Thus, the slope of the forward-mapping function is relatively small, and the low-level signals are badly degraded by the tape noise.

An output signal-to-noise-ratio curve for the logarithmic mapping function can be found that matches a lin-log curve over much of its range. The lin-log curve has a lower signal-to-noise ratio for large signals, but a higher signal-to-noise ratio near the break point where the function changes. The gradual variation of the output signal-to-noise-ratio curve for the logarithmic map may be considered disadvantageous with respect to the lin-log plot, for which the signal-to-noise-ratio curve levels off more rapidly. The lin-log map gives less degradation of signals in the vicinity of the break point.

The characteristics of the logarithmic and lin-log functions make them both useful maps. The lin-log map is superior, but probably not significantly so for most applications. The linear map is relatively inefficient. The selection of an appropriate map for a particular application will depend on the signal and recorder characteristics involved.

Computer programs to calculate output signal-to-noise ratios for other input and tape dynamic ranges are available. Instructions for their operation are found in Appendix IV.

RB:cm

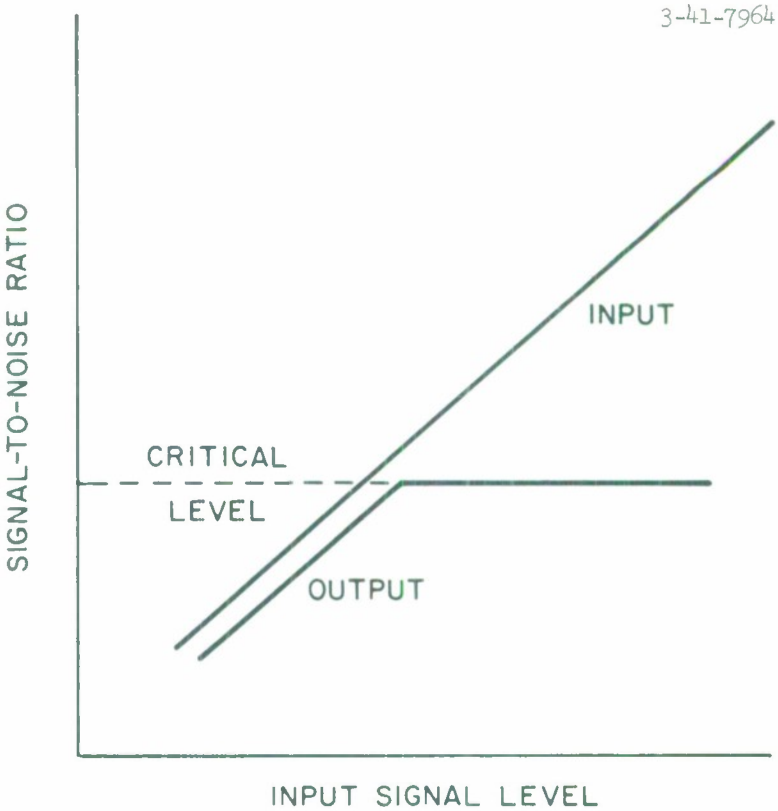

Fig. 1. Ideal Compression Characteristic.

3-41-7965

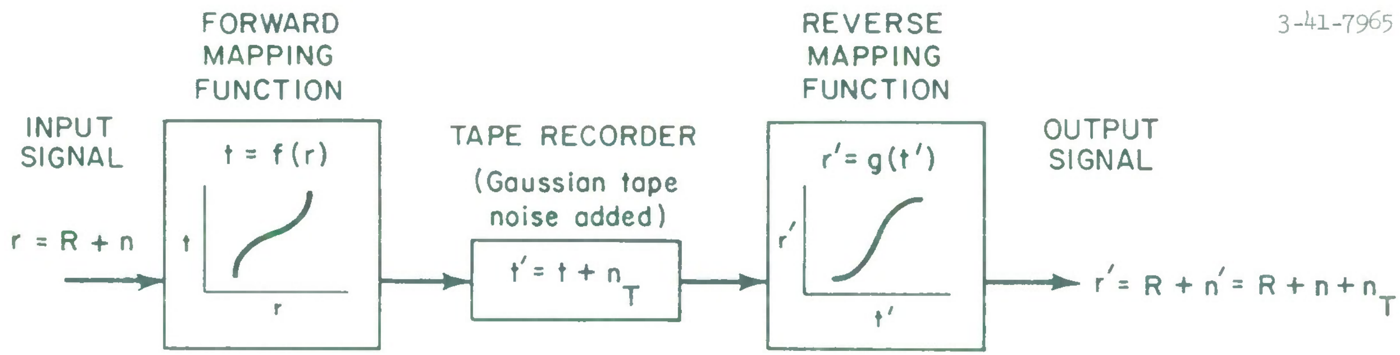

Fig. 2. Mathematical Procedures.

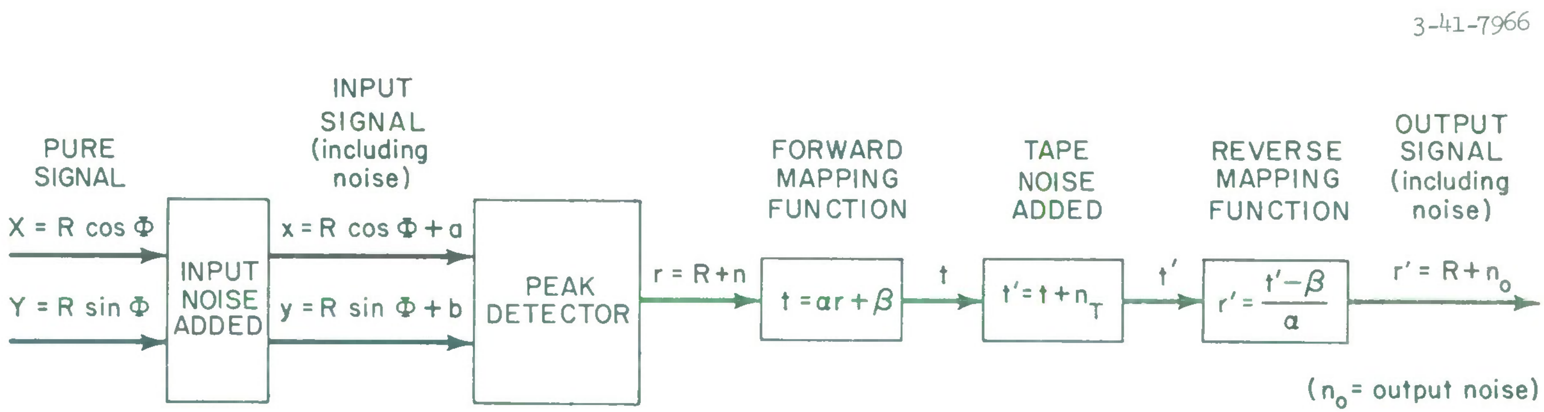

Fig. 3. Linear mapping system.

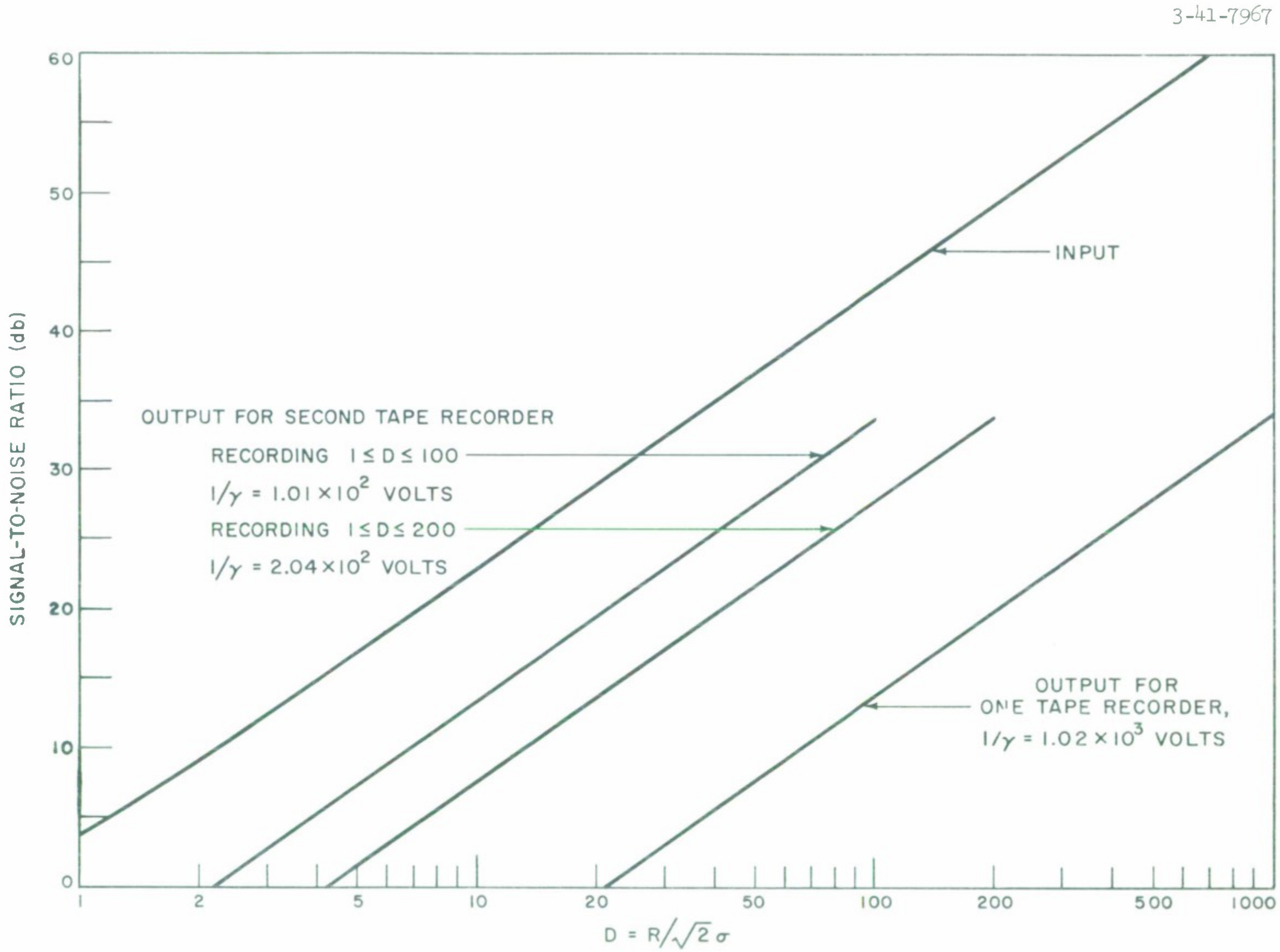

Fig. 4. Signal-to-noise vs input signal strength -- linear mapping.

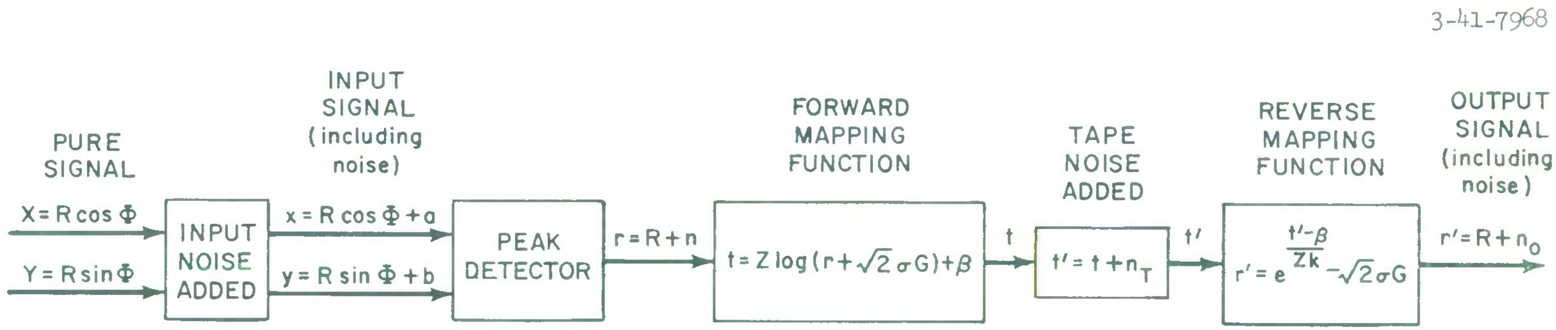

Fig. 5. Logarithmic mapping system.

3-41-7969

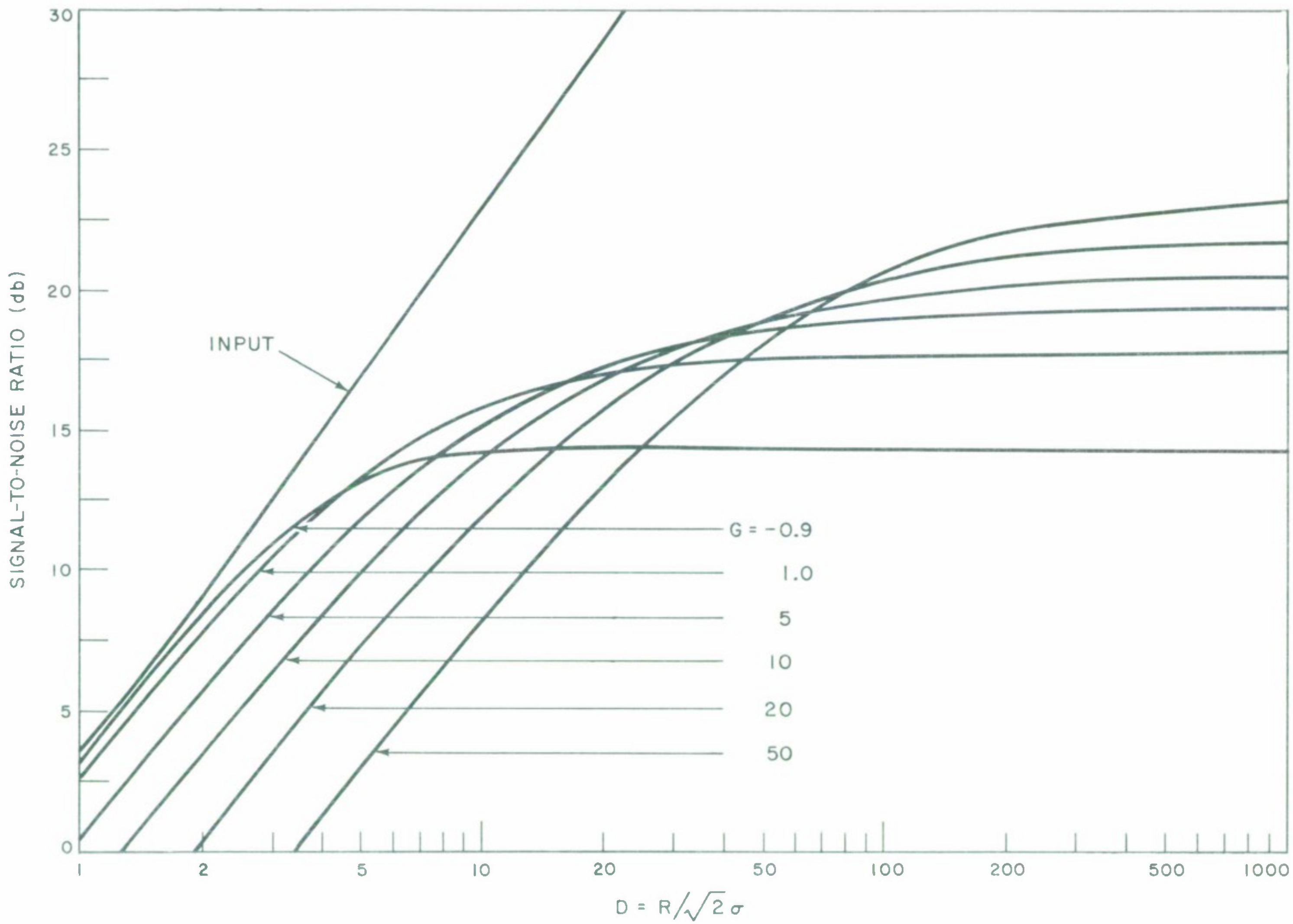

Fig. 6. Signal-to-noise vs input signal strength -- logarithmic mapping.

3-41-7970

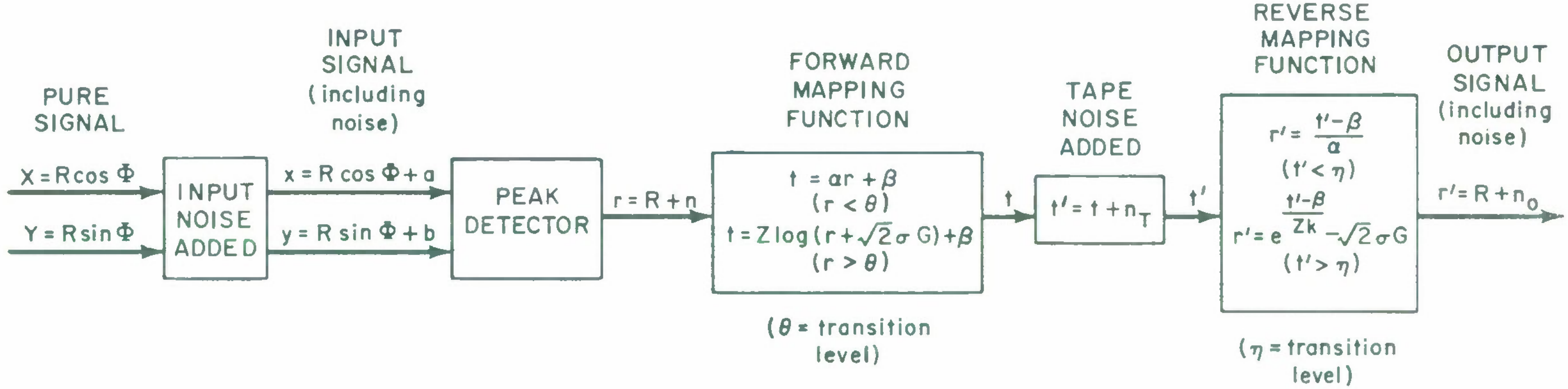

Fig. 7. Lin-log mapping system.

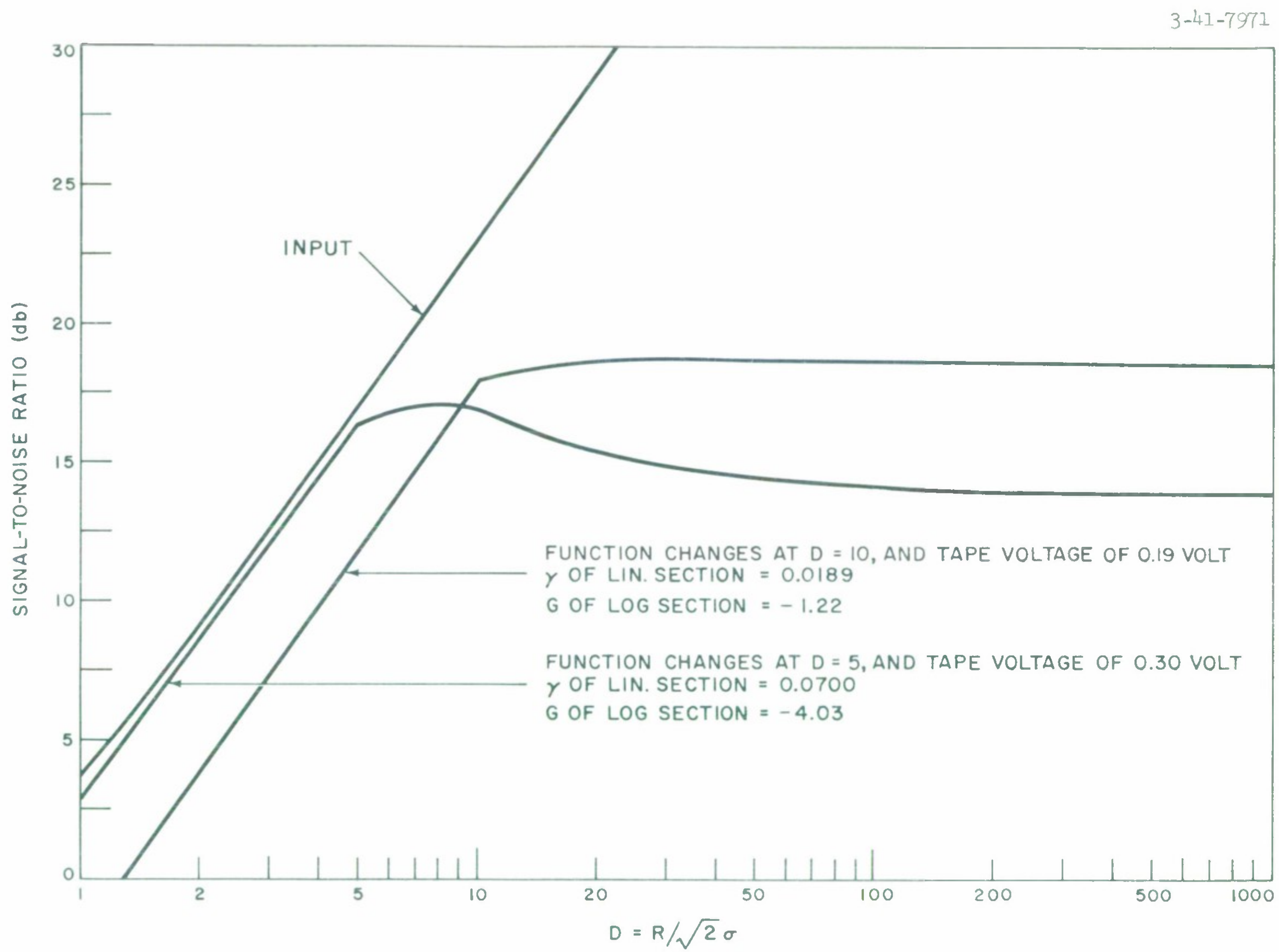

Fig. 8. Signal-to-noise vs input signal strength -- lin-log map.

APPENDIX I

DERIVATION OF INPUT SIGNAL PROBABILITY DENSITIES AND SIGNAL-TO-NOISE RATIO

The Cartesian pure-signal components are $(X, Y) = (R \cos \Phi, R \sin \Phi)$. The Gaussian noise components (a, b) added to these have a probability density

$$p(a, b) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(a^2 + b^2)}{2\sigma^2}\right] .$$

Thus, the input signal has components (x, y), where $x = R \cos \Phi + a = r \cos \varphi$, and $y = R \sin \Phi + b = r \sin \varphi$, with a probability distribution

$$p(x, y) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(x - R\cos\Phi)^2 + (y - R\sin\Phi)^2}{2\sigma^2}\right] .$$

(The procedures followed here are similar to those outlined in Davenport and Root.)[1] This can be represented in polar coordinates (r, φ) with the appropriate distribution:

$$p(r, \varphi) = \frac{r}{2\pi\sigma^2} \exp\left[-\left(\frac{r^2 + R^2}{2\sigma^2}\right)\right] \exp\left[\frac{rR\cos(\Phi - \varphi)}{\sigma^2}\right] .$$

$$r \geq 0 , \quad 0 \leq \varphi \leq 2\pi$$

Integration of $p(r, \varphi)$ over φ from 0 to 2π, gives the probability density of the signal after it has been passed through a peak detector:

$$p(r) = \frac{r}{\sigma^2} \exp\left[-\frac{r^2 + R^2}{2\sigma^2}\right] I_o\left(-\frac{rR}{\sigma^2}\right) , \quad r \geq 0 .$$

Now, defining the noise voltage as the difference between r and R, i.e., $n = r - R$, we find that the distribution function for the noise is:

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1. W. B. Davenport, Jr., and W. L. Root, An Introduction to the Theory of Random Signals and Noise, McGraw-Hill Book Co., Inc. (New York) (1958).

$$p(n) = \frac{n + R}{\sigma^2} \exp\left[-\frac{n^2 + 2nR + 2R^2}{2\sigma^2}\right] I_o\left(\frac{nR + R^2}{\sigma^2}\right) \quad , n \geq -R \quad .$$

For $R = 0$,

$$p(n) = \frac{n}{\sigma^2} \exp\left(\frac{-n^2}{2\sigma^2}\right) \quad ,$$

which is the Rayleigh distribution.

The mean square value of the noise, $E(n^2)$, is given by

$$E(n^2) = E\left[(r - R)^2\right] = E(r^2) - 2RE(r) + R^2 \quad .$$

Evaluating the expectation value of r and r^2 , we get

$$E(r^2) = 2\sigma^2 + R^2$$

and

$$E(r) = \sqrt{2\sigma^2} \exp\left(\frac{-R^2}{2\sigma^2}\right) \sum_{m=o}^{\infty} \left(\frac{R^2}{2\sigma^2}\right)^m \frac{\Gamma(m + \frac{3}{2})}{(m!)^2} \quad ,$$

resulting in

$$E(n^2) = 2R^2 + 2\sigma^2 - 2R\sqrt{2\sigma^2} \exp\left(\frac{-R^2}{2\sigma^2}\right) \sum_{m=o}^{\infty} \left(\frac{R^2}{2\sigma^2}\right)^m \frac{\Gamma(m + \frac{3}{2})}{(m!)^2} \quad .$$

Thus, since the signal-to-noise ratio is defined by

$$S/N = 20 \log \frac{R}{\sqrt{E(n^2)}} \quad ,$$

we have

$$S/N = -10 \cdot \log\left[2 + \frac{2\sigma^2}{R^2} - 2\frac{\sqrt{2\sigma^2}}{R} \exp\left(\frac{-R^2}{2\sigma^2}\right) \cdot \sum_{m=o}^{\infty} \left(\frac{R^2}{2\sigma^2}\right)^m \frac{\Gamma(m + \frac{3}{2})}{(m!)^2}\right] \quad .$$

However, it is more convenient to express the signal-to-noise ratio as a function of the ratio of the signal to the rms noise voltage when the signal is not present, $D = R/\sqrt{2}\,\sigma$.

$$S/N = -10 \log \left[2 + \frac{1}{D^2} - \frac{2}{D} \exp(-D^2) \sum_{m=o}^{\infty} D^{2m} \frac{\Gamma(m + \frac{3}{2})}{(m\,!)^2} \right] \quad .$$

APPENDIX II

DERIVATION OF LINEAR MAPPED OUTPUT SIGNAL-TO-NOISE RATIO

At the output of the linear mapping system (see Fig. 3), the signal r' can be defined as the sum of the uncorrupted input signal R and the total noise at the output n_o, i.e., $r' = R + n_o$. Thus, the total output noise is

$$n_o = r' - R = \frac{t + n_T - \beta}{\alpha} - R = n + \frac{n_T}{\alpha} \quad ,$$

where n_T is the tape noise voltage, and its mean square value is

$$E(n_o^2) = E(n^2) + \frac{2}{\alpha} E(n)\, E(n_T) + \frac{1}{\alpha^2} E(n_T^2) \quad .$$

Since n_T is assumed Gaussian,

$$E(n_T) = 0 \quad , \quad E(n_T^2) = \sigma_T^2 \quad .$$

Thus, $E(n_o^2)$ is

$$2R^2 + 2\sigma^2 - 2R\sqrt{2}\sigma \exp\left(\frac{-R^2}{2\sigma^2}\right) \sum_{m=o}^{\infty} \frac{\left(\frac{R^2}{2\sigma^2}\right)^m \Gamma(m + \frac{3}{2})}{(m!)^2} + \frac{\sigma_T^2}{\alpha^2} \quad ,$$

and the signal-to-noise ratio, expressed as a function of $D = R/\sqrt{2}\sigma$ is

$$S/N = -10 \cdot \log\left[2 + \frac{1}{D^2} - \frac{2}{D} \exp(-D^2) \sum_{m=o}^{\infty} \frac{D^{2m}\Gamma(m + \frac{3}{2})}{(m!)^2} + \left(\frac{\sigma_T}{\gamma D}\right)^2\right] \quad ,$$

where

$$\gamma = (\sqrt{2}\sigma)\, \alpha \; .$$

APPENDIX III

DERIVATION OF THE LOGARITHMIC MAPPED OUTPUT SIGNAL-TO-NOISE RATIO

Again, at the output of the system (see Fig. 5), we define the total noise, n_o, as the difference between the signal present, r , and the uncorrupted input signal R . Thus,

$$n_o = r - R = \exp\frac{t + n_T - b}{Zk} - \sqrt{2}\,\sigma G - R = (R + \sqrt{2}\,\sigma G)(e^{n_T/Zk} - 1) + n e^{n_T/Zk},$$

where

$$k = 0.4343 \quad .$$

Taking the expectation value of the output noise squared gives:

$$E(n_o^2) = [R^2 + 2\sqrt{2}\,\sigma RG + 2(\sigma G)^2]\,[E(e^{2n_T/Zk}) - 2E(e^{n_T/Zk}) + 1]$$

$$+ E(n^2)\,E(e^{2n_T/Zk}) + 2(R + \sqrt{2}\,\sigma G)\,[E(e^{2n_T/Zk}) - E(e^{n_T/Zk})]\,E(n)$$

Evaluating the individual expectation values, we find

$$E(n) = E(r) - R = \sqrt{2}\sigma e^{-R^2/2\sigma^2} \sum_{m=0}^{\infty} \frac{(R^2/2\sigma^2)^m\,\Gamma(m + \frac{3}{2})}{(m\,!)^2} - R$$

$$E(e^{2n_T/Zk}) = \frac{1}{\sqrt{2\pi}\sigma_T} \int_{-\infty}^{+\infty} e^{2n_T/Zk} \cdot e^{-n_T^2/2\sigma_T^2}\, dn_T = \exp 2\left(\frac{\sigma_T}{Zk}\right)^2$$

$$E(e^{n_T/Zk}) = \frac{1}{\sqrt{2\pi}\sigma_T} \int_{-\infty}^{+\infty} e^{n_T/Zk} \cdot e^{-n_T^2/2\sigma_T^2}\, dn_T = \exp \frac{1}{2}\left(\frac{\sigma_T}{Zk}\right)^2 \quad .$$

Therefore, $E(n_o^2)$ is

$$[R^2 + 2\sqrt{2}\,\sigma RG + 2\sigma^2 G^2][e^{2\sigma_T^{\,2}/(Zk)^2} - 2e^{\frac{1}{2}(\sigma_T/Zk)^2} + 1]$$

$$+ [2R^2 + 2\sigma^2 - 2R\sqrt{2}\,\sigma e^{-R^2/2\sigma^2} \sum_{m=o}^{\infty} \frac{(R^2/2\sigma^2)^m\,\Gamma(m+\frac{3}{2})}{(m\,!)^2}]\, e^{2(\sigma_T/Zk)^2}$$

$$+ 2(R+\sqrt{2}\,\sigma G)\,[e^{2(\sigma_T/Zk)^2} - e^{\frac{1}{2}(\sigma_T/Zk)^2}][\sqrt{2}\,\sigma e^{-R^2/2\sigma^2} \sum_{m=o}^{\infty} \frac{(R^2/2\sigma^2)^m\,\Gamma(m+\frac{3}{2})}{(m\,!)^2} - R]\,.$$

Finally, expressing the signal-to-noise ratio as a function of D , we have

$$S/N = -10 \cdot \log\, [(1 + \frac{2G}{D} + (\frac{G}{D})^2)(e^{2(\sigma_T/Zk)^2} - 2e^{\frac{1}{2}(\sigma_T/Zk)^2} + 1)$$

$$+ (2 + \frac{1}{D^2} - \frac{2}{D}\, e^{-D^2} \sum_{m=o}^{\infty} \frac{D^{2m}\,\Gamma(m+\frac{3}{2})}{(m\,!)^2})\; e^{2(\sigma_T/Zk)^2}$$

$$+ 2(1+\frac{G}{D})(e^{2(\sigma_T/Zk)^2} - e^{\frac{1}{2}(\sigma_T/Zk)^2})(\frac{e^{-D^2}}{D} \sum_{m=o}^{\infty} \frac{D^{2m}\,\Gamma(m+\frac{3}{2})}{(m\,!)^2} - 1)]\,.$$

APPENDIX IV

THE ARRANGEMENT OF DATA CARDS FOR THE 7094

To use the Input and Linear Mapped Output Signal-to-Noise Ratio Program on the 7094, the data cards must be arranged as follows: the initial card contains (1) the lowest usable tape voltage (which is the rms tape noise voltage) in columns 1 to 10, (2) the highest usable tape voltage in columns 11 to 20, (3) the highest input D in columns 20 to 30, and (4) the lowest input D in columns 31 to 40. These four numbers are punched in fixed-point notation. The remaining cards contain in a fixed-point notation in the first six columns the various values of D for which the signal-to-noise ratio is to be evaluated. These cards also contain another term in spaces 7 through 23, which is approximately equal to D. It is the product $\exp(-D^2) \sum_{m=o}^{\infty} \frac{D^{2m}\Gamma(m+3/2)}{(m!)^2}$ and its format is E 16.9. This is specifically given as data to avoid the necessity of computing the series.

The initial data card for the Logarithmic Mapping System Program is identical through the fortieth column with the initial card for the Input and Linear-Mapped Output Program. However, columns 41 to 50 contain G in fixed-point notation, and columns 51 to 60 contain N, the number of D cards which are to follow, in integer notation. The D cards are identical with those for the Input and Linear-Mapped Output Program. To obtain the output signal-to-noise ratio curve for other values of G, the first set of data cards is merely followed by other sets with the various G's punched in the respective initial cards.

The initial card for the Lin-Log Mapping Function Program is again identical to that of the Input and Linear-Mapped Output Program, through the fortieth column. In addition, columns 41 to 50 contain the tape voltage where the function changes, and 51 to 60 that input D for which the function changes.

Both are in fixed-point notation. Columns 61 to 70 indicate the numbers of D cards to follow in integer notation. The D cards are the same as in the other programs, but there should be two cards for that value of D for which the function changes. The first of these two cards must have a 1 (one) punched in column 30, and the second a 2 (two). Again, many ranges for the two maps may be investigated by stacking the sets of cards behind each other, each set being headed by an appropriate initial card.

RB:cm

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Methods of recording a video signal with large dynamic range on a magnetic tape (or other recording medium) that has a smaller dynamic range than of the signal are discussed. The useful part of the voltage range of the input signal is mapped onto the linear voltage range of the recorder. Several maps are considered with the thought of minimizing the degradation of the final signal by recorder noise.

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