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PROPAGATION OF ACOUSTIC-GRAVITY WAVES IN A TEMPERATURE- AND WIND-STRATIFIED ATMOSPHERE

Allan D. Pierce

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AVCO CORPORATION
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OFFICE OF AEROSPACE RESEARCH
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ABSTRACT

A theory is presented which permits the study of the effects of horizontal winds on the dispersion and amplitudes of acoustic-gravity waves in the atmosphere. It is shown that the effective horizontal group velocity for a given frequency in a given normal mode depends on direction of propagation as well as on frequency and that it is not necessarily in the same direction as the horizontal wave number vector. A number of useful integral theorems are derived from a variational principle and one is subsequently applied to the development of a perturbation method for the computation of wind effects on dispersion. Application of the method to a realistic example indicates that winds can appreciably alter the dispersion of the normal modes and that they should be considered in any quantitative interpretation of experimental microbarograms.

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CONTENTS

ILLUSTRATION

NOMENCLATURE

Latin

c	speed of sound, $(\gamma p_o/\rho_o)^{1/2}$
c_m	speed of sound in high altitude layer
$\vec{e}_x$	unit vector in eastward direction
$\vec{e}_z$	unit vector in vertical direction
$f(t)$	function characterizing source time dependence
g	acceleration of gravity
$g(\omega)$	Fourier transform of $f(t)$
$\vec{k}$	horizontal wave number vector
k	magnitude of $\vec{k}$
k_x, k_y	components of $\vec{k}$
$k_n(\omega, \theta_k)$	eigenvalue of residual equations
$k^0(\omega)$	eigenvalue in absence of winds
n	index for normal modes
p	deviation of pressure from ambient
p_o	ambient pressure
$\vec{q}$	vector wind correction to horizontal wave number
q_x, q_y	components of $\vec{q}$
q_x', q_y'	derivatives of $q_x(\omega)$ and $q_y(\omega)$
$\vec{r}$	position of observer
$\vec{r}_o$	position of source

NOMENCLATURE (Concl'd)

Latin

t	time relative to excitation of the source
$\vec{u}$	deviation of particle velocity from ambient
$\vec{v}$	ambient wind velocity
$\vec{v}_m$	ambient wind velocity in high altitude layer
$\langle \vec{v} \rangle_{av}$	weighted average of wind velocity
$\vec{v}_g$	horizontal group velocity
v_g	magnitude of $\vec{v}_g$
v_{gx}, v_{gy}	components of $\vec{v}_g$
v_{gn}	group velocity of nth normal mode
$v_{g\parallel}, v_{g\perp}$	components of $\vec{v}_g$ parallel and perpendicular to wind direction
$v_g^0(\omega)$	group velocity in absence of winds
w	vertical component of $\vec{u}$
x, y	horizontal coordinates
z	altitude
z_o	source altitude
z_m	lower boundary for high altitude layer
A	characteristic atmospheric parameter
A_m	value of A in high altitude layer
A_n	relative amplitude of nth normal mode
A_{12}, A_{21}	coefficients in residual equations
D	constant independent of z

NOMENCLATURE (Concl'd)

Latin

D_t	$\partial/\partial t + \vec{v}.\nabla$, time derivative following wind
$G(\theta, \omega_i)$	time dependent amplitude of normal mode wave
H_m	scale height
I_1, I_2, I_3	characteristic integrals
K	constant
$P(\vec{r}, \omega)$	factor in Fourier transform of p
$P^*(\vec{r}, \omega)$	complex conjugate of $P(\vec{r}, \omega)$
Ph	phase of complex number
Q	factor of A_n
R_T	magnitude of horizontal projection of $\vec{r} - \vec{r}_o$
Re	real part
S, T	functions of k characteristic of high altitude layer
W(z)	Wronskian
Y, Z	solutions of residual equations
Y_l, Z_l	solutions satisfying lower boundary condition
Y_u, Z_u	solutions satisfying upper boundary condition
Y_n, Z_n	eigenfunction pair satisfying both boundary conditions

Greek

α	function characterizing high altitude layer
β_n, β	phase factor of nth normal mode
γ	ratio of specific heats of air

NOMENCLATURE (Concl'd)

Greek

δ	phase of $g(\omega)$
δA, δA_{12}, etc.	small variations in A, A_{12}, etc.
ϵ	0 or $\pi/2$ depending on sign of $\partial^2 \beta/\partial \omega^2$
θ	angle between horizontal projection of $\vec{r}-\vec{r}_o$ and x-axis
$\theta_v(z)$	angle between $\vec{v}$ and x-axis
θ_k	angle between $\vec{k}$ and x-axis
$\theta_{kn}(\omega, \theta)$	saddle point for θ_k integration
$\tilde{\rho}$	deviation of density from ambient
ρ_o	ambient density
σ	focusing factor in normal mode wave amplitude
ω	angular frequency
ω_2, ω_1, etc.	characteristic frequencies of high altitude layer
$\omega_i(\theta, t/R_T)$	frequency arriving at time t
ω_B	Vaisala-Brunt frequency
ω_{Bm}	$(\gamma - 1)^{1/2} \quad g/c_m$
ω_{Am}	$1/2 \ \gamma \ g/c_m$
Λ_1, Λ_2	integrals characterizing variations in c^2 and $\vec{v}$
Φ_n	normalized contribution to waveform
Ω	$\omega - \vec{k} \cdot \vec{v}_m$
Ω_m	$\omega - \vec{k} \cdot \vec{v}_m$

I. INTRODUCTION

Previous attempts to explain the features of the microbarograms of infrasonic waves recorded following nuclear explosions[1-7] and natural atmospheric explosions [8, 9] have for the most part been restricted to atmospheric models with no ambient winds. To what extent this neglect of winds is justified is not entirely clear. Diamond[10] has recently discussed their effect on the apparent sound-speed profile above White Sands and has exhibited data which would seem to indicate that typical winds are of sufficient strength to have an appreciable effect on sound propagation.

A noteworthy beginning in the development of a theory which considers the effects of winds was made by Weston and vanHulsteyn[11] who showed that the linearized equations of hydrodynamics are still separable if the winds are horizontal and vary in direction and magnitude only with altitude. They also indicated how one might calculate the variation of the horizontal phase velocity with frequency for fixed direction of the horizontal propagation vector $\vec{k}$.

Pridmore-Brown[12] also derived the general linearized equations for sound propagation in an atmosphere with arbitrary sound-speed profile and horizontal-wind profile. In some respects, his theory went further than that of Weston and vanHulsteyn, in that it dealt with waves from a point source rather than with free waves (whose wavefronts are vertical planes). However, Pridmore-Brown considered only the steady state case as he was not concerned with dispersion phenomena. Furthermore, since he was interested in sonic frequencies of the order of 100 cps (as opposed to infrasonic frequencies of the order of 10^{-2} cps), he was enabled to make a number of approximations which cannot be justified for lower frequencies.

In the present paper, the theories of Weston and vanHulsteyn and of Pridmore-Brown are extended to the consideration of the propagation of infrasonic waves from an idealized point source characterized by an arbitrary time variation $f(t)$. The expressions derived for the pressure on the ground at a large distance from the source represent an extension of the method of normal modes to the propagation of infrasonic waves from a point source in the presence of horizontal winds. Our derivation of these expressions is similar to that of Pridmore-Brown and is therefore given as briefly as possible. One substantial departure from Pridmore-Brown's method appears in the method of treating the two-fold integration over the components of the horizontal wave number. It is our opinion that the mathematical approximation utilized by Pridmore-Brown as indicated by Eqn. (22) in his paper is not justified. His approximation would indicate that the horizontal wave vector points radially from the source and hence in the same direction as the group velocity. The theory presented in this paper indicates that this is not the case in general and gives a method of computing the angular deviation of the horizontal wave vector from the direction of the horizontal group velocity.

The implementation of the theory rests on the solution of two coupled first order differential equations, which represent generalizations of the <u>residual equations</u> discussed by Eckart.[13] With appropriate boundary conditions (whose rationale we discuss) these are eigenvalue equations for the magnitude of the horizontal propagation vector $\vec{k}$. The eigenvalues k will depend on the direction of $\vec{k}$ when winds are included as well as on the frequency and the mode index n. The theory presented in sections IV and V shows how the group velocity may be calculated from a knowledge of the partial derivatives of the eigenvalues with respect to frequency and angle of wave normal to a given horizontal direction.

In section VI we introduce a variational technique which leads to a number of integral theorems relating the eigenfunctions of the residual equations and their eigenvalues. In particular, the theorems give methods of computing the partial derivatives of the eigenvalues from a knowledge of the eigenfunctions for a single choice of parameters and therefore eliminates numerical differentiation. An integral expression for the group velocity is then easily obtained.

In section VII we discuss the case of propagation in an isothermal atmosphere with constant winds. This is a particularly simple case and one which should be carefully studied before proceeding on to more complicated model atmospheres. Our analysis shows that both the surfaces of constant phase and constant arrival time are circles whose centers move with the wind velocity and whose radii increase at the sound speed.

In section VIII we use one of the integral theorems derived in section VI to develop a perturbation method for taking winds into consideration. This method makes calculations of dispersion effects of winds highly feasible and requires only that the wind independent eigenfunctions be known. These, however, have been explicitly or implicitly obtained by all previous writers who have computed horizontal phase and group velocities for wind independent model atmospheres. As an example, we make use of the computational results of Pfeffer and Zarichny[5] to find the effects of a wind profile exhibited by Diamond[10] on the dispersion of the fundamental mode.

II. FORMAL SOLUTION OF THE LINEARIZED EQUATIONS OF HYDRODYNAMICS

The atmosphere is assumed to be an ideal gas which obeys the equations of hydrodynamics. The ambient variables p_o, ρ_o, and $\vec{v}$ are assumed to satisfy these equations in the absence of any external perturbation. The winds are assumed horizontal and independent of the horizontal coordinates x and y and of time t. The linearized equations of hydrodynamics as derived by Pridmore-Brown will therefore govern the spatial and temporal variations of the first order quantities p, ρ, and $\vec{u}$. In a somewhat altered notation, these equations are

$$D_t(\rho_o \vec{u}) + \rho_o w d\vec{v}/dz = -\nabla p - g\rho \vec{e}_z \tag{2.1}$$

$$D_t \rho + \nabla \cdot (\rho_o \vec{u}) = 4\pi f(t)\, \delta(\vec{r} - \vec{r}_o) \tag{2.2}$$

$$D_t p - w(g\rho_o + c^2 d\rho_o/dz) = c^2 D_t \rho \tag{2.3}$$

The term on the right-hand side of Eqn. (2.2) has been included to take into account the presence of a source at the point $\vec{r}_o$. The function $f(t)$ is to be chosen such that the linearized equations predict as accurately as possible the known properties of the acoustic field in the near vicinity of the source. A direct intepretation of $f(t)$ may be obtained by integrating both sides of (2.2) over the volume of a small sphere of radius R. Upon applying the divergence theorem, one finds in the limit of sufficiently small R that

$$\int_S \rho_o \vec{u} \cdot \vec{n}\, da = 4\pi f(t) \tag{2.4}$$

where $\vec{n}$ is the unit normal to the surface S of the sphere of radius R. The integral on the left may be interpreted as the mass expelled from the interior of the sphere per unit time. (This assumes that negligible mass has been added to the air by the explosion itself.)

A formal solution of the linearized equations may be obtained by use of Fourier transforms. Using this method, one finds the excess pressure at a point $\vec{r}$ at time t to be given by

$$p(\vec{r}, t) = \frac{\rho_o^{1/2}(z)}{\rho_o^{1/2}(z_o)} \int_{-\infty}^{\infty} e^{-i\omega t} P(\vec{r}, \omega)\, g(\omega)\, d\omega \tag{2.5}$$

where $g(\omega)$ is the Fourier transform of $f(t)$, such that

$$f(t) = \int_{-\infty}^{\infty} e^{-i\omega t} g(\omega)\, d\omega \tag{2.6}$$

and where $P(\vec{r}, \omega)$ may in turn be represented as a two-fold integral over the components k_x, k_y of the horizontal wave number $\vec{k}$,

$$P(\vec{r}, \omega) = \pi^{-1} \iint d^2\vec{k}\, e^{i\vec{k}\cdot(\vec{r}-\vec{r}_o)} Z(z, z_o, \omega, \vec{k}) \quad . \tag{2.7}$$

The expression $Z(z, z_o, \omega, \vec{k})$ must satisfy a differential equation which may be derived in the manner indicated by Weston and vanHulsteyn and by Pridmore-Brown for the quantities $\Pi(z)$ and $\chi(z)$ in their respective papers. The differential equation we find by this method is

$$\left\{ \left(\frac{d}{dz} - A\right) A_{12}^{-1} \left(\frac{d}{dz} + A\right) - A_{21} \right\} Z = -(i/\Omega)\, \delta(z - z_o) \tag{2.8}$$

where

$$A_{12} = \Omega^2 - \omega_B^2, \tag{2.9}$$

$$A_{21} = (k/\Omega)^2 - c^{-2}, \tag{2.10}$$

$$A = (1 - 1/2\gamma)\, g/c^2 - (2c^2)^{-1}\, dc^2/dz \tag{2.11}$$

$$\omega_B^2 = (\gamma - 1)\, g^2/c^2 + (g/c^2)\, dc^2/dz \tag{2.12}$$

and $\Omega = \omega - \vec{k}\cdot\vec{v}$. Both A and ω_B (Vaisala-Brunt frequency) are functions of z which characterize the ambient atmosphere. A somewhat more convenient representation of Eqn. (2.8) is that of two coupled first order differential equations, i.e.,

$$dY/dz - AY - A_{21} Z = -(i/\Omega)\, \delta(z - z_o) \tag{2.13a}$$

$$dZ/dz + AZ - A_{12} Y = 0. \tag{2.13b}$$

The utility of taking the equations in this form has been demonstrated by Eckart[13]. Following Eckart, we shall refer to these two coupled equations as the residual equations. The second of these may be considered as the definition of the auxiliary function $Y(z, z_o, \omega, \vec{k})$.

The delta function on the right-hand side of Eqns. (2.8) and (2.13a) reflects the presence of the source at altitude z_0 and requires there be a discontinuity in Y and dZ/dz at $z = z_0$.

Boundary conditions on the set of coupled equations (2.13) are chosen to insure that w, the vertical component of particle velocity, be zero at the Earth's surface and are chosen to insure that the total solution (2.5) conforms to causality. The former requirement leads to the condition $Y=0$ at $z=0$. (The derivation of this condition is similar to Weston and vanHulsteyn's derivation[11] of $d\chi/dz = 0$ at $z=0$.) The causality requirement has been shown by the author[14] to be satisfied if $Z(z, z_0, \omega, \vec{k})$ for real k_x, k_y is an analytic function of ω for all complex ω in the upper half plane and vanishes as ω approaches $i\infty$.

To relate this requirement to one governing the behavior of Z and Y at large z, it is convenient to assume that the properties of the uppermost region of the atmosphere are such that the z-variation of Z and Y above some height z_m may be written down explicitly. For this reason, we assume that the atmosphere is isothermal above z_m and therefore has a constant sound speed c_m in the uppermost region. The winds at this height will also be assumed to be constant in magnitude and direction.

With this assumption as to the nature of the upper atmosphere, one may show that the causality requirement requires that Z be of the form

$$Z = D\, e^{i\alpha z} \tag{2.14}$$

for $z > z_m$, where D is independent of z and

$$\alpha = \left\{ c_m^{-2} \left(\Omega_m^2 - \omega_{Am}^2\right) - \left(\Omega_m^2 - \omega_{Bm}^2\right)\left(k^2/\Omega_m^2\right) \right\}^{1/2}$$

is a function of ω, k_x, and k_y. Here $\Omega_m = \omega - \vec{k}\cdot\vec{v}_m$, $\omega_{Am} = (\gamma/2)g/c_m$, $\omega_{Bm} = (\gamma - 1)^{1/2} g/c_m$, where c_m and $\vec{v}_m$ represent the ambient sound speed and wind velocity in the isothermal layer.

To insure causality and the desired behavior of Z as a function of complex ω, one must require that α be an analytic function of ω in the upper half plane and that it approach $i\infty$ as ω approaches $i\infty$. This requirement may then be used to specify the phase of α for real ω and $\vec{k}$, giving

$$\mathrm{Ph}(\alpha) = 0 \qquad \omega > \omega_2 \tag{2.15a}$$

$$= \pi/2 \qquad \omega_1 < \omega < \omega_2 \tag{2.15b}$$

$$= \pi \qquad \omega_0 < \omega < \omega_1 \tag{2.15c}$$

$$= 0 \qquad \omega_{-1} < \omega < \omega_0 \tag{2.15d}$$

$$\mathrm{Ph}\,(\alpha) = \pi/2 \qquad \omega_{-2} < \omega < \omega_{-1} \tag{2.15e}$$

$$= \pi \qquad \omega < \omega_{-2} \tag{2.15f}$$

where the five characteristic frequencies are given by the following expressions

$$\omega_{\pm 2} = \vec{k} \cdot \vec{v}_m \pm (S + T)^{1/2} \tag{2.16a}$$

$$\omega_{\pm 1} = \vec{k} \cdot \vec{v}_m \pm (S - T)^{1/2} \tag{2.16b}$$

$$\omega_o = \vec{k} \cdot \vec{v}_m \tag{2.16c}$$

with

$$S = (\omega_{Am}^2 + c_m^2 k^2)/2 \tag{2.17a}$$

$$T = (S^2 - c_m^2 \omega_{Bm}^2 k^2)^{1/2} \tag{2.17b}$$

The upper boundary condition on the solution of Eqns. (2.13) is therefore that Z be of the form (2.14) when $z > z_m$, where the appropriate phase of α must be determined from Eqns. (2.15) and (2.16).

Following a method used previously by Haskell[15], we may formally write the solution for $Z(z, z_o, \omega, \vec{k})$ of Eqns. (2.13) in terms of quantities $Z_l(z), Y_l(z)$ and $Z_u(z), Y_u(z)$ which satisfy the homogeneous equations (i.e., Eqns. (2.13) with the omission of the source term). The set Z_l, Y_l is defined as satisfying the boundary condition $Y_l = 0$ at $z = 0$, while the set Z_u, Y_u is defined as satisfying the upper boundary condition. In terms of these quantities, the solution for Z of the inhomogeneous equations which satisfies both upper and lower boundary conditions is given by

$$Z(z, z_o, \omega, \vec{k}) = \frac{-i\, Z_u(z_>)\, Z_l(z_<)}{\Omega(z_o)\, W(z_o)} \tag{2.18}$$

where $z_>$ and $z_<$ refer to the greater or lesser of z and z_o, and the Wronskian $W(z_o)$ is defined to be

$$W(z_o) = Y_u(z_o)\, Z_l(z_o) - Z_u(z_o)\, Y_l(z_o) \tag{2.19}$$

One may show directly from the homogeneous form of Eqns. (2.13) that the Wronskian is independent of altitude. Thus, we may set

$$W(z_o) = W(0) = Y_u(0)\, Z_l(0) \tag{2.20}$$

where we have made use of the fact that $Y_l(0) = 0$.

A more convenient expression for $P(\vec{r}, \omega)$ may now be obtained by inserting the expression above for Z into Eqn. (2.7). Since observations of infrasonic waves are usually made on the ground, we take $z = 0$ in the resulting expression. This gives

$$P(\vec{r}, \omega) = -i/\pi \int_0^{2\pi} d\theta_k \int_0^{\infty} k\, dk\, e^{ikR_T \cos(\theta - \theta_k)} Z_u(z_o)/[Y_u(0)\,\Gamma(z_o)] \qquad (2.21)$$

Here the integration is expressed in cylindrical coordinates; θ_k representing the angle which $\vec{k}$ makes with the x-axis. The magnitude of the horizontal projection of $\vec{r} - \vec{r}_o$ is abbreviated by R_T and the angle between this projection and the x-axis is denoted by θ.

III. THE METHOD OF NORMAL MODES

While, in principle, the order of integration in Eqn. (2.21) is immaterial, great care should be exercised in choosing this order if one or both integrations are to be performed approximately. The technique utilized by Pridmore-Brown was to first integrate over θ_k using the saddle-point method and then to integrate over k using the method of residues. In our opinion, this order of integration leads to incorrect results since the saddle-point method is inapplicable if the saddle-point is close to a pole. This appears to be the case in Pridmore-Brown's method, for, in utlizing the method of residues in the integration over k, it is the behavior of the integrand near its poles which is of principle importance. This objection can be overcome if one first does an integration over k using the method of residues and then does the θ_k integration using the saddle-point method. This is the program we follow here.

The path for the k integration in Eqn. (2.21) is first deformed to one which (in the case of $\omega > 0$) encloses all poles of the integrand in the first quadrant and which includes a contour going from the origin along the positive imaginary axis as well as contours around branch lines. (All branch lines emanating from branch points in the first quadrant are taken as extending vertically upwards.) For large R_T, the predominant contribution to the total integral comes from the residues of those poles which lie on the real axis. The remaining terms may be discarded. The integrand thus obtained for the θ_k integration will consist of a sum of terms, each representing the contribution from one of the poles in the integration over k. Each term is integrated separately, and the saddle-point approximation is assumed to be applicable in each case. The saddle-point is taken as that of

$$\exp\{i k_n R_T \cos(\theta - \theta_k)\} \tag{3.1}$$

where $k_n(\omega, \theta_k)$ is the location of the nth pole in the integrand prior to the performance of the k-integration. (One should note that the saddle-point will generally not be at $\theta_k = \theta$.) In general, the validity of the saddle-point method may be expected to increase with increasing R_T. If no real saddle-point exists for any particular term we may assume the contribution from that term to be negligible for large R_T relative to any other term with a real saddle-point.

The resulting expression for $P(r, \omega)$ appropriate for large R_T appears as a sum over normal modes in the form

$$P(r, \omega) = \sum_n A_n R_T^{-1/2} e^{i\beta_n R_T} \tag{3.2}$$

Both the phase factor β_n and the amplitude factor A_n for each normal mode are functions of ω and θ, but not of R_T.

The method for obtaining the quantities A_n and β_n follows from our preceding remarks concerning the procedure for deriving Eqn. (3.2). For $\omega > 0$, let $k_n(\omega, \theta_k)$ be a real positive root of

$$\frac{\Omega(z_o)\, Y_u(0)}{Z_u(z_o)} = 0 \tag{3.3}$$

where $n = 1, 2, 3, \ldots$ is an index distinguishing the roots. The labelling is chosen such that k_n is piecewise continuous in θ_k and ω. Then, let $\theta_{kn}(\omega, \theta)$ be the saddle-point of the expression (3.1), or, equivalently, a root of

$$\frac{\partial}{\partial \theta_k} \left[k_n(\omega, \theta_k) \cos(\theta_k - \theta) \right] = 0 \tag{3.4}$$

For weak winds (which is the case of physical interest) it should be required that $|\theta_{kn} - \theta|$ is less than $\pi/2$ in the event that one need distinguish among multimple roots of Eqn. (3.4)

In terms of $k = k_n(\omega, \theta_k)$ and $\theta_k = \theta_{kn}(\omega, \theta)$, one may set

$$\beta_n = k \cos(\theta_k - \theta) \tag{3.5}$$

$$A_n = 2(2/\sigma)^{1/2}\, Q\, k\, e^{-i\pi/4} \tag{3.6}$$

where

$$Q^{-1} = \frac{\partial}{\partial k} \left[\Omega(z_o)\, Y_u(0)/Z_u(z_o) \right] \tag{3.7}$$

$$\sigma = -\frac{\partial^2}{\partial \theta_k^2} \left[k_n(\omega, \theta_k) \cos(\theta_k - \theta) \right] \tag{3.8}$$

To compute β_n and A_n for given ω and θ, one first determines the appropriate values of θ_k and k and uses these in the above expressions. The differentiation in Eqn. (3.7) is carried out at constant θ_k, while that in (3.8) is carried out at constant θ. (A more convenient expression for Q is given by Eqn. (6.14).)

It is not necessary to consider the case $\omega < 0$ separately, since the fact that both $f(t)$ and $p(\ell, t)$ are real implies

$$P(\ell, \omega) = P^*(\ell, -\omega) \tag{3.9}$$

and therefore implies

$$A_n(\omega,\theta) = A_n(-\omega,\theta)^* \tag{3.10a}$$

$$\beta_n(\omega) = -\beta_n(-\omega) \tag{3.10b}$$

(It may be assumed that β_n is real, since we are limiting ourselves to undamped modes.)

The surfaces of constant phase for given ω and mode number n are determined by the condition that $\beta_n R_T$ in the exponent of Eqn. (3.2) be constant, or

$$R_T = \frac{K}{\beta_n(\omega,\theta)} \tag{3.11}$$

where K is a constant. The normal to such a surface at a given value of θ makes an angle

$$\theta - \tan^{-1}(R_T^{-1}\ dR_T/d\theta)$$

with the x-axis. It is readily shown from Eqns. (3.4), (3.5), and (3.11) that this angle is identical with $\theta_k(\omega, \theta)$. Thus the vector $\vec{k}$ is perpendicular to the surfaces of constant phase.

IV. GROUP VELOCITY

The expression (2.5) for the pressure on the ground as a function of time may be rewritten using the approximation (3.2) in the form

$$p(\vec{r}, t) = \sum_{n} \rho_o^{1/2}(0) \quad \rho_o^{-1/2}(z_o) \quad \Phi_n(R_T, \theta, t) \tag{4.1}$$

where

$$\Phi_n(R_T, \theta, t) = R_T^{-1/2}\, 2\, \mathrm{Re} \int_0^\infty e^{-i\omega t}\, g(\omega) A_n(\omega, \theta) e^{i\beta_n R_T}\, d\omega \tag{4.2}$$

represents the normalized contribution to the total waveform from the nth normal mode. Although the integration limits are written as 0 and ∞, it must be borne in mind that, generally, the solution to Eqn. (3.3) corresponding to the nth normal mode will exist only for a limited range of ω. There may be either an upper or a lower cutoff frequency - or possibly both. It is conceivable that the mode may also have a number of frequency gaps, for which the mode does not exist. To allow for such situations, we adopt the convention that the $A_n(\omega, \theta)$ should be considered as zero whenever there is no corresponding root of Eqn. (3.3).

The traditional method of evaluating the integral in Eqn. (4.2) is the method of stationary phase[16]. Although the method in its unmodified form has limited applicability to acoustic-gravity waves, the modifications devised by Scorer[9] and by Weston[2] may generally be incorporated to make the method a valid approximation for the computation of the waveform at distances greater than 5000 km from the source. For many qualitative aspects of the interpretation of the waveforms, the unmodified method appears to be satisfactory. In this paper we restrict ourselves to the traditional method. A direct application gives

$$\Phi_n(R_T, \theta, t) = R_T^{-1} \sum_{i} G(\theta, \omega_i) \cos\left[\beta(\omega_i, \theta) R_T - \omega_i t + \delta(\omega_i) + \epsilon\right] \tag{4.3}$$

where

$$G(\theta, \omega) = \frac{2(2\pi)^{1/2}\, g(\omega) \quad A(\omega, \theta)}{\left|\partial^2 \beta / \partial \omega^2\right|^{1/2}} \tag{4.4}$$

and $\omega_i = \omega_i(\theta, t/R_T)$ is a root of

$$\partial \beta(\omega, \theta)/\partial \omega = t/R_T \tag{4.5}$$

(The subscript n is omitted for brevity.) The sum in (4.3) extends over all such roots if more than one exists. The quantity δ is the phase of $g(\omega)$ and the parameter ϵ is $\pi/2$ or 0, depending on whether $\partial^2 \beta / \partial \omega^2$ is positive or negative, respectively.

The concept of group velocity is derived from the method of stationary phase. The time t obtained from Eqn. (4.5) represents the time relative to the excitation of the source at which a wave of frequency ω in the nth normal mode arrives at a point described by the coordinates R_T, θ. Thus, we may consider the magnitude of the horizontal group velocity as being given by

$$v_g(\omega, \theta) = [\partial \beta / \partial \omega]^{-1} \tag{4.6}$$

Since we are assuming that the medium does not vary in the x or y directions, it must be assumed that the group velocity is in a radial direction away from the source (i. e., in a direction making an angle of θ with the x-axis).

In terms of the parameter θ_k which describes the direction of the horizontal wave vector $\vec{k}$ with respect to the x-axis, the components v_{gx} and v_{gy} of the horizontal group velocity are given by the expressions:

$$v_{gx} = \frac{\cos \theta_k + k_n^{-1} (\sin \theta_k)(\partial k_n / \partial \theta_k)}{(\partial k_n / \partial \omega)} \tag{4.7a}$$

$$v_{gy} = \frac{\sin \theta_k - k_n^{-1} (\cos \theta_k)(\partial k_n / \partial \theta_k)}{(\partial k_n / \partial \omega)} \tag{4.7b}$$

where, in evaluating the partial derivatives, $k_n(\omega, \theta_k)$ is as found from Eqn. (3.3).

The proof of Eqns. (4.7) follows from Eqns. (3.4) and (3.5), which give

$$\tan(\theta_k - \theta) = k_n^{-1} \, \partial k_n / \partial \theta_k$$

$$\partial k_n / \partial \omega = [\cos(\theta_k - \theta)]^{-1} \, \partial \beta_n / \partial \omega$$

(Note that the magnitude of $\partial \beta_n / \partial \omega$ is the same regardless of whether θ or θ_k is kept constant while differentiating.) Insertion of these expressions into Eqns. (4.7) gives a group velocity with the magnitude (4.6) and with a direction making an angle of θ with the x-axis.

It should be noted that surfaces of equal phase are not necessarily surfaces of equal arrival time. If a K independent of θ can be chosen in Eqn. (3.11) such that the resulting value of $R_T(\omega, \theta)$ is equal to $v_g(\omega, \theta)t$ for some time t, this would be the case. However, this would require

$$\frac{\partial}{\partial \theta} \{\beta_n(\omega, \theta) v_g(\omega, \theta)\} = 0$$

which would in turn require that

$$\frac{\partial}{\partial \theta_k} \left\{ \frac{1}{k(\omega, \theta_k)} \frac{k(\omega, \theta_k)}{d\omega} \right\} = 0$$

which is clearly not true in general. Two circumstances where the above would be satisfied are: (a) no winds (k independent of θ_k); and (b), $k(\omega, \theta_k)$ directly proportional to ω. The latter, as we show in section VII, occurs for the case of an isothermal atmosphere with constant winds.

V. EIGENFUNCTIONS AND EIGENVALUES

For all practical purposes, the only modes of interest which may be attained from Eqn. (3.3) are those for which $Y_u(z, \omega, k, \theta_k)$ is zero at $z = 0$. An additional mode apparently exists where $\Omega(z_0) = 0$ which apparently represents a disturbance traveling in the direction of the ambient wind at altitude z_0 with the same velocity as the wind. One may discard this mode if he is interested only in modes which travel with speeds of the order of the sound speed at the ground. (We assume the wind speeds are significantly less than the speed of sound.)

If $k = k_n(\omega, \theta_k)$ is a root of the equation $Y_u(0) = 0$, then the corresponding pair of functions, $Z_u(z)$ and $Y_u(z)$, may be considered as an eigenfunction pair of the coupled differential equations (or residual equations).

$$dY/dz - AY - A_{21} Z = 0 \qquad (5.1a)$$

$$dZ/dz + AZ - A_{12} Y = 0 \qquad (5.1b)$$

and may be denoted by the symbols $Z_n(z, \omega, \theta_k)$ and $Y_n(z, \omega, \theta_k)$. The root $k_n(\omega, \theta_k)$ may be considered as an eigenvalue. The problem of finding the roots of Eqn. (3.3) may therefore be considered as that of finding the eigenvalues of Eqns. (5.1).

In accordance with our remarks in section III, we consider only those eigenfunctions which correspond to real eigenvalues. Thus, A_{12} and A_{21} must both be real. This implies that any set of solutions of Eqns. (5.1) which satisfy the lower boundary condition of $Y = 0$ at $z = 0$ must be real functions of z, apart from a multiplicative constant which may be complex. Thus, the upper boundary conditions cannot be satisfied if the phase of α in Eqn. (2.14) is 0 or π. The phase of α must be $\pi/2$. This proves that any real eigenvalue $k(\omega, \theta_k)$ satisfies the condition

$$\omega_1(k, \theta_k) < \omega < \omega_2(k, \theta_k)$$

where ω_1 and ω_2 are given by Eqns. (2.16). The nature of such a constraint is best demonstrated by plotting the functions ω_1 and ω_2. In figure 1, these, as well as ω_0, are plotted versus k for fixed angle between $\vec{k}$ and $\vec{v}_m$. Numerical values used are such that in (a), $\vec{k} \cdot \vec{v}_m = (1/3) c_m k$, and in (b), $\vec{k} \cdot \vec{v}_m = -(1/3) c_m k$. For simplicity, k is plotted in units of $1/H_m$ and ω is plotted in units of c_m/H_m, where H_m is the scale height $c_m^2/\gamma g$. Figure 1 may be considered as representing generalizations of the diagnostic diagram for a quiescent isothermal atmosphere given by Eckart[17].

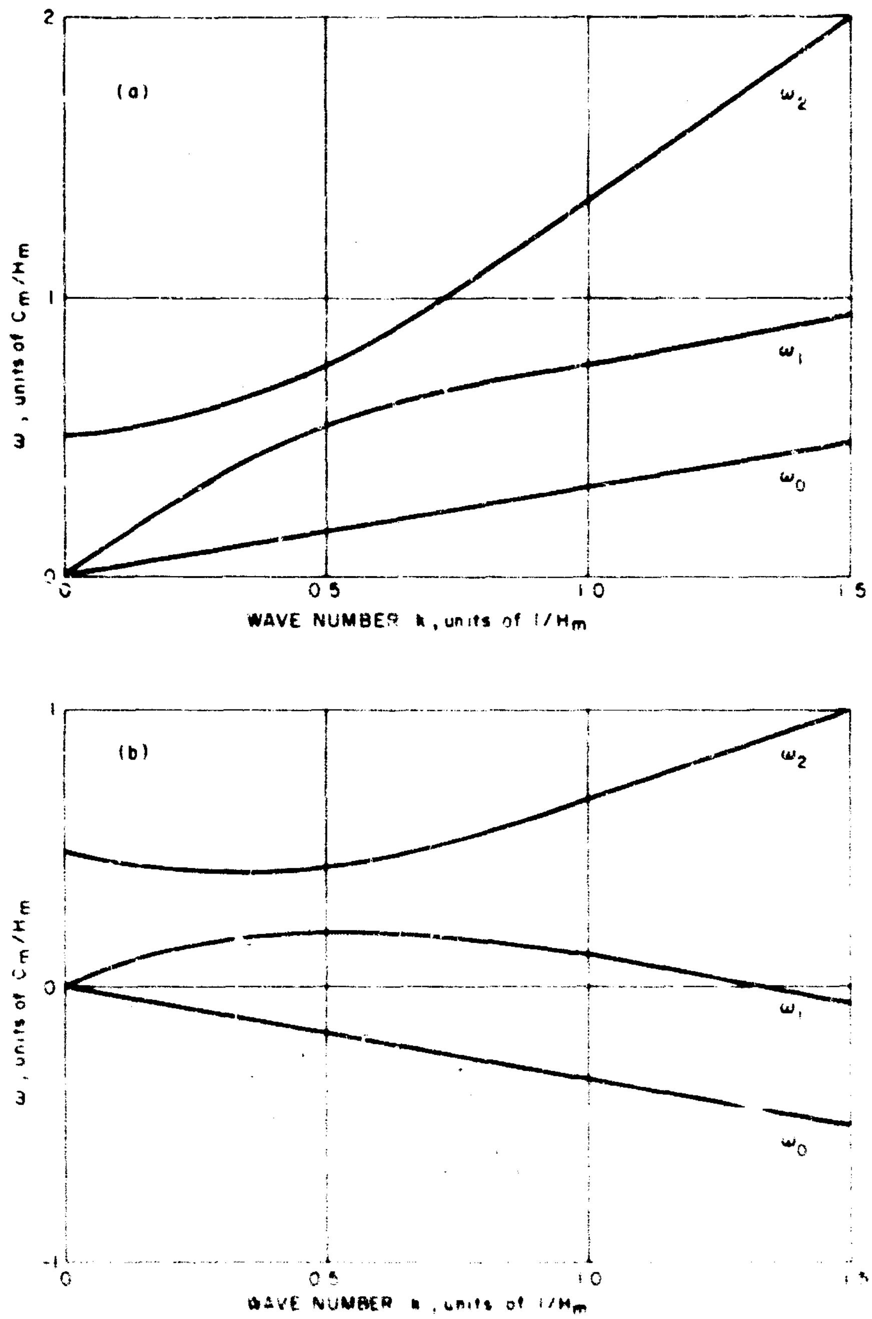

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Figure 1. CHARACTERISTIC FREQUENCIES , , AND VERSUS HORIZONTAL WAVE NUMBER , FOR A LAYER WITH SOUND SPEED , WIND VELOCITY , AND SCALE HEIGHT . IN (a), IT IS ASSUMED THAT (1.3), WHILE IN (b), IT IS ASSUMED THAT (1.3).

VI. INTEGRAL THEOREMS

A number of theorems may be derived relating the variation of the eigenvalues and eigenfunctions of Eqns. (5.1) to small variations in the atmospheric sound speed and wind speed profiles and to small variations in the parameters ω and θ_k. These theorems all follow from a general theorem which we now prove.

We assume that Y and Z are a set of eigenfunctions of Eqns. (5.1) for given $A(z)$, $A_{12}(z)$, and $A_{21}(z)$. Let δA, δA_{12}, and δA_{21} be small variations in the quantities A, A_{12}, and A_{21}. Then let $Z+\delta Z$, $Y+\delta Y$ be solutions of Eqns. (5.1) when A, A_{12}, and A_{21} are replaced by $A+\delta A$, $A_{12}+\delta A_{21}$ and $A_{21}+\delta A_{21}$. (It is not necessarily assumed that $Z+\delta Z$, $Y+\delta Y$ are a set of eignfunctions, but it is assumed that they conform to the upper boundary condition.) To first order, the variations δZ, δY will then satisfy the two coupled inhomogeneous equations

$$d(\delta Y)/dz - A(\delta Y) - A_{21}(\delta Z) = (\delta A)Y + (\delta A_{21})Z \tag{6.1a}$$

$$d(\delta Z)/dz + A(\delta Z) - A_{12}(\delta Y) = -(\delta A)Z + (\delta A_{12})Y \tag{6.1b}$$

It follows directly from Eqns. (5.1) and (6.1) that

$$\frac{d}{dz}\left\{Y(\delta Z) - Z(\delta Y)\right\} = (\delta A_{12})Y^2 - (\delta A_{21})Z^2 - 2(\delta A)YZ \tag{6.2}$$

The desired theorem is now obtained by integrating both sides of the above equation with respect to z from 0 to ∞, giving

$$\left\{Z(\delta Y)\right\}_{z=0} = \int_0^\infty \left\{(\delta A_{12})Y^2 - (\delta A_{21})Z^2 - 2(\delta A)YZ\right\} dz \tag{6.3}$$

where use has been made of the fact that both Y and Z approach zero as z approaches infinity and of the fact that $Y=0$ at $z=0$.

Under the most general variation we will consider, c^2, $\vec{v}$, k, ω, and θ_k go to $c^2+\delta c^2$, etc. The corresponding variations δA, δA_{12}, δA_{21} to first order may be found from Eqns. (2.9-2.12) to be

$$\delta A = -(A/c^2)\,\delta c^2 - (2c^2)^{-1}\,(d/dz)\,(\delta c^2) \tag{6.4}$$

$$\begin{aligned}\delta A_{12} = {} & (\omega_B^2/c^2)\,\delta c^2 - (g/c^2)\,(d/dz)\,(\delta c^2) - 2\,\Omega\,\vec{k}\cdot\delta\vec{v} \\ & - 2\,\Omega\,(\vec{k}\cdot\vec{v}/k)\,\delta k + 2\,\Omega\,\delta\omega + 2\,\Omega\,kv\,\sin(\theta_k - \theta_v)\,\delta\theta_k \end{aligned} \tag{6.5}$$

$$\delta A_{21} = c^{-4}\,\delta c^2 + (2k^2/\Omega^3)\,\vec{k}.\,\delta\vec{v} + 2k^3\,\Omega^{-3}\,\delta k$$

$$- (2k^2/\Omega^3)\,\delta\omega - (2k^3 v/\Omega^3)\,\sin(\theta_k - \theta_v)\,\delta\theta_k \qquad (6.6)$$

where $\theta_v\,(z)$ is the angle between $\vec{v}$ and the x-axis.

Insertion of expressions (6.4-6.6) into (6.3) with an additional integration by parts to eliminate terms with $(d/dz)\,\delta c^2$ gives

$$\Big\{Z\,(\delta Y)\Big\}_{z=0} = -I_1\,\delta k + I_2\,\delta\omega + I_3\,\delta\theta_k + \Delta_1 - \Delta_2 \qquad (6.7)$$

where

$$I_1 = 2\int_0^\infty \Big\{\Omega\,(\vec{k}.\vec{v}/k)\,Y^2 + k\,\omega\,\Omega^{-3} Z^2\Big\}\,dz \qquad (6.8)$$

$$I_2 = 2\int_0^\infty \Big\{\Omega\,Y^2 + (k^2/\Omega^3)\,Z^2\Big\}\,dz \qquad (6.9)$$

$$I_3 = 2k\int_0^\infty v\,\sin(\theta_k - \theta_v)\,\Big\{\Omega\,Y^2 + (k^2/\Omega^3)\,Z^2\Big\}\,dz \qquad (6.10)$$

$$\Delta_1 = \int_0^\infty \Big\{\Big[\gamma\,g^2/c^4 - \Omega^2\,c^2\Big]\,Y^2 - c^{-2}(k/\Omega)^2 Z^2 \qquad (6.11)$$

$$+ \Big[(2g\,c^2)\,(k/\Omega)^2 - g\,c^4\Big]\,YZ\Big\}\,(\delta c^2)\,dz$$

$$\Delta_2 = 2\int_0^\infty \Big\{\Omega\,Y^2 + (k^2/\Omega^3)\,Z^2\Big\}\,\vec{k}\cdot\delta\vec{v}\,dz$$

In obtaining the expression for Δ_1, use has been made of the differential equations (5.1) to eliminate terms in dZ/dz and dY/dz.

We may now apply Eqn. (6.7) to a number of special cases:

1. Expression for dY/dk at $z = 0$.

We consider $\delta\omega, \delta\theta_k, \delta c^2, \delta \vec{v}$ as being zero. Then (6.7) gives

$$(\partial Y/\partial k)_{z=0} = - I_1/Z(0) \tag{6.13}$$

The factor Q which appears in Eqn. (3.7) is therefore

$$Q = -Z(z_o)\, Z(0)/[\Omega(z_o)\, I_1] \tag{6.14}$$

2. Expressions for $\partial k/\partial\omega$ and $\partial k/\partial\theta_k$.

The variation of k with ω for a given normal mode and for fixed θ_k may be obtained from (6.7) by letting $(\delta Y)_{z=0} = 0$. Thus

$$\partial k/\partial\omega = I_2/I_1 \tag{6.15}$$

In a similar manner, one finds

$$\partial k/\partial\theta_k = I_3/I_1 \tag{6.16}$$

3. Expression for group velocity.

Insertion of Eqns. (6.15) and (6.16) into Eqns. (4.7) gives expressions for the components of the group velocity. After some manipulation, one may write the resulting two equations as a single vector equation in the form

$$\vec{v}_g = (I_1/I_2)\,\vec{k}/k + \langle \vec{v} \rangle_{av} \tag{6.17}$$

where

$$\langle \vec{v} \rangle_{av} = (2/I_2) \int_0^\infty \vec{v}\,[\Omega Y^2 + (k^2/\Omega^3)\, Z^2]\, dz \tag{6.18}$$

One should note that, in the event $\vec{v}$ is constant, $\langle \vec{v} \rangle_{av}$ is $\vec{v}$.

4. Effect of Atmospheric Perturbations on k.

If the eigenfunctions Z and Y and corresponding eigenvalue k, for given θ_k and ω and for a given model atmosphere, are known, then the effect on the eigenvalue k of varying the atmospheric sound and wind profiles is given to first order by

$$\delta k = (\Lambda_1 - \Lambda_2)/I_1 \qquad (6.19)$$

The consequences of this equation are discussed in section VIII.

VII. PROPAGATION IN AN ISOTHERMAL ATMOSPHERE WITH CONSTANT WINDS

The simplest model atmosphere including winds is one with a constant temperature and a constant wind velocity. For such an atmosphere, the upper layer with constant sound speed c_m and wind velocity $\vec{v}_m$ coincides with the entire atmosphere. The solutions of Eqns. (5.1) which satisfy the upper boundary condition are of the form (2.14). The lower boundary condition requires

$$i\alpha = -A_m = -(1 - \gamma/2)g/c_m^2 \tag{7.1}$$

It follows that there is only one eigenvalue; it being given by

$$k = \omega / [c_m + v_m \cos(\theta_k - \theta_{vm})] \tag{7.2}$$

The corresponding functions Z and Y may be taken as

$$Z = e^{-A_m z} \tag{7.3a}$$

$$Y = 0 \tag{7.3b}$$

The relationship (3.4) between θ and θ_k becomes

$$\tan(\theta_k - \theta) = \frac{v_m \sin(\theta_k - \theta_{vm})}{c_m + v_m \cos(\theta_k - \theta_{vm})} \tag{7.4}$$

or (omitting the subscript m)

$$c \sin(\theta_k - \theta) = v \sin(\theta - \theta_v) \tag{7.5}$$

The corresponding expression for $\beta(\omega, \theta)$ is readily found to be

$$\beta = \frac{(\omega/c)}{\{1 - (v/c)^2 \sin^2(\theta - \theta_v)\}^{1/2} + (v/c)\cos(\theta - \theta_v)} \tag{7.6}$$

and the group velocity is therefore

$$v_g = (c^2 - v^2 \sin^2(\theta - \theta_v))^{1/2} + v\cos(\theta - \theta_v) \tag{7.7}$$

which is independent of frequency. The significance of the above equation is much clearer if it is expressed in terms of $v_{g\parallel} = v_g \cos(\theta - \theta_v)$ and $v_{g\perp} = v_g \sin(\theta - \theta_v)$. With a little manipulation, Eqn. (7.7) then assumes the form

$$(v_{g\parallel} - v)^2 + v_{g\perp}^2 = c^2 \tag{7.8}$$

which shows, as might well be expected, that the propagation is equivalent to that from a source moving with velocity v in a medium with sound speed c. The surfaces of equal arrival time are circles whose centers are displaced from the origin in the direction of the ambient wind. The ratio of the radius of any circle to the distance of its center from the origin is c/v. Since $\beta(\omega, \theta)$ is directly proportional to ω, the surfaces of constant phase will coincide with surfaces of equal arrival time.

VIII. PERTURBATION METHOD

An approximate method of incorporating winds into a theory of acoustic-gravity wave propagation is a perturbation method based on Eqn. (6.19). The unperturbed atmosphere is taken as one in which $\vec{v} = 0$. The eigenvalues are then approximately given (to first order in $\vec{v}$) by

$$k(\omega, \theta_k) = k^0(\omega) - q_x(\omega)\cos\theta_k - q_y(\omega)\sin\theta_k \tag{8.1}$$

where q_x and q_y are components of the vector

$$\vec{q}(\omega) = \frac{\int_0^\infty \{[(k^0)^2/\omega]\, Z^2 + \omega^3 Y^2\}\, \vec{v}\, dz}{\int_0^\infty Z^2\, dz} \tag{8.2}$$

The first term $k^0(\omega)$ in Eqn. (8.1) is the unperturbed eigenvalue. The second and third terms are the first order correction to $k(\omega, \theta_k)$ and are derived from Eqn. (6.19) with $\delta\vec{v}$ replaced by $\vec{v}$. The quantities Z and Y in Eqn. (8.2) are the zeroth order eigenfunctions and are computed assuming there are no winds. The quantities $k^0(\omega)$, $q_x(\omega)$, and $q_y(\omega)$ will be independent of θ_k but will depend on the mode index n as well as ω.

The apparent uncoupling between the θ_k and ω dependences in Eqn. (8.1) makes the resulting formulas for $\beta(\omega, \theta)$ and the surfaces of constant phase relatively simple. To first order in q_x/k^0 and q_y/k^0, one has

$$\beta(\omega, \theta) = k^0(\omega) - q_x(\omega)\cos\theta - q_y(\omega)\sin\theta \tag{8.3}$$

The surfaces of constant phase (see Eqn. (3.11)) to the same order of approximation are given by the equation

$$\left\{x - (K q_x/(k^0)^2)\right\}^2 + \left\{y - (K q_y/(k^0)^2)\right\}^2 = K^2/(k^0)^2 \tag{8.4}$$

where K is a constant and $x = R_T \cos\theta$, $y = R_T \sin\theta$. The surfaces are therefore approximately circles whose centers are displaced from the origin in the direction of $\vec{q}$ and which are characterized by the number $|\vec{q}|/k^0$ which gives the ratio of the distance of each circle's center from the origin to its radius.

The group velocity may be readily computed from Eqns. (4.6) and (8.3). To first order one finds

$$v_g = v_g^0(\omega) + \left\{v_g^0(\omega)\right\}^2 \left[q_x'(\omega)\cos\theta + q_y'(\omega)\sin\theta\right] \tag{8.5}$$

where v_g^0 is the zero order group velocity, or $(\partial k^0/\partial\omega)^{-1}$, while q_x' and q_y' are the derivatives with respect to ω of q_x and q_y.

The surfaces of constant arrival time are circles to the same degree of approximation

$$\left[x - (v_g^0)^2 q_x' t\right]^2 + \left[y - (v_g^0)^2 q_y' t\right]^2 = (v_g^0)^2 t^2 \tag{8.6}$$

The center of the circle moves with a velocity $(v_g^0)^2\, \partial\vec{q}/\partial\omega$ while the waves move out from the center with a velocity v_g^0.

To demonstrate the utility of the method outlined above, we apply it to the computation of the effects of winds in a realistic case. The wind profile is taken as measured by Diamond[10] above White Sands and as exhibited in figure 2 of his paper. The cited figure indicates that north-south components may be neglected. Taking x to be in the eastward direction, we therefore have $q_y = 0$.

In the computation of q_x we rely on the numerical computations of Pfeffer and Zarichny[5]. In particular, we use the plots of kinetic energy versus altitude for the 52 km model as given by figure 5a in their paper for periods of 48, 87, 225, and 3i5 seconds. The kinetic energy they tabulate should be proportional (as regards variation with z) to the quantity

$$(k^0/\omega)^2 Z^2 + \omega^2 Y^2$$

in the notation used here. For low frequencies and for the 52 km model atmosphere used by Pfeffer and Zarichny, it appears that one may safely neglect the second term in the above. Thus the quantity $\dot{q}(\omega)$ is approximately

$$\dot{q}(\omega) = \left[(k^0)^2/\omega\right] \frac{\int_0^\infty (KE)\, \dot{v}\, dz}{\int_0^\infty (KE)\, dz} \tag{8.7}$$

Curves of v_g^0 and ω/k^0 versus period are given in figure 4 of Pfeffer and Zarichny's paper.

Using Eqn. (8.7), we have computed $\vec{q}(\omega) = q_x \vec{e}_1$ for the four frequencies for which KE is plotted by Pfeffer and Zarichny. A curve fitted to these values is given in figure 2a. A good fit to this curve over the frequency range of interest is

$$q_x(\omega) = -2.7 \times 10^{-4} \omega^{1.23}$$

where ω is in radians/sec and q_x is in meter^{-1}. Thus the magnitude of $dq_x/d\omega$ increases slowly with increasing frequency.

In figure 2b we plot the factor $(v_g^0)^2 q_x'$ versus period. Curves obtained using Eqn. (8.5) of group velocity versus period are given for various values of θ in figure 3. It should be noted that the principal effect of winds in this particular example is to increase the group velicity for downwind propagation and decrease it for upwind propagation by an increment of roughly 15 meter/sec. However, the winds also strongly affect the dispersion of the wave. Since the group velocity curve for downwind propagation is flatter than that for upwind propagation, the signal observed to the east (upwind) will be more dispersed than that observed to the west (downwind).

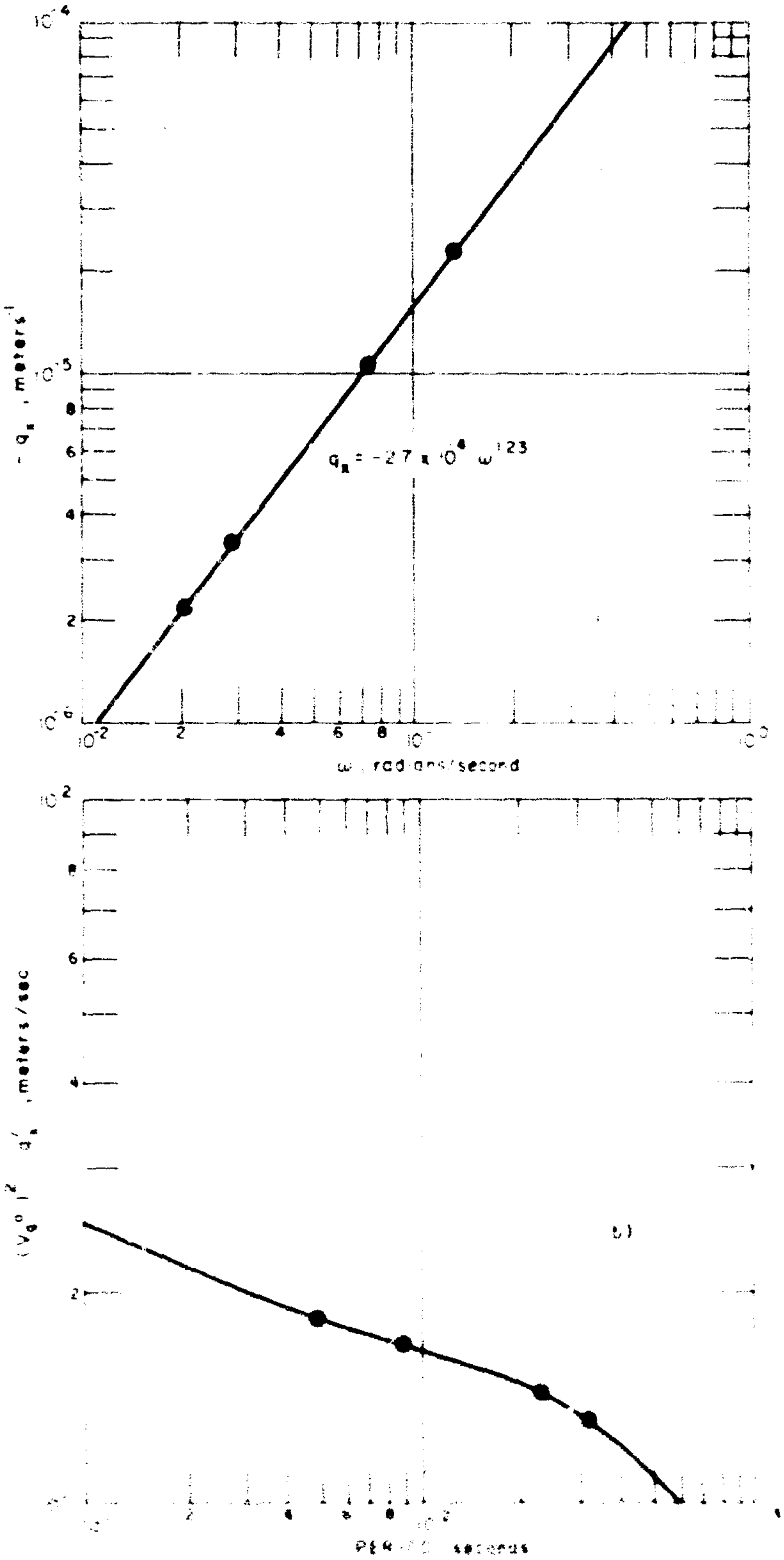

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Figure 2. (a) APPROXIMATE VARIATION OF PARAMETER [illegible] WITH ANGULAR FREQUENCY FOR THE FUNDAMENTAL MODE. THE DOTS REPRESENT VALUES COMPUTED USING PROFILES OF KINETIC ENERGY COMPUTED BY PFEFFER AND ZARICHNY. (b) PLOT OF [illegible] VERSUS PERIOD [illegible] DERIVED FROM THE GRAPH IN (a) AND THE GROUP VELOCITY CURVES COMPUTED BY PFEFFER AND ZARICHNY. HERE [illegible] REPRESENTS THE DERIVATIVE OF [illegible] WITH RESPECT TO [illegible]. NOTE THAT BOTH [illegible] AND [illegible] ARE NEGATIVE

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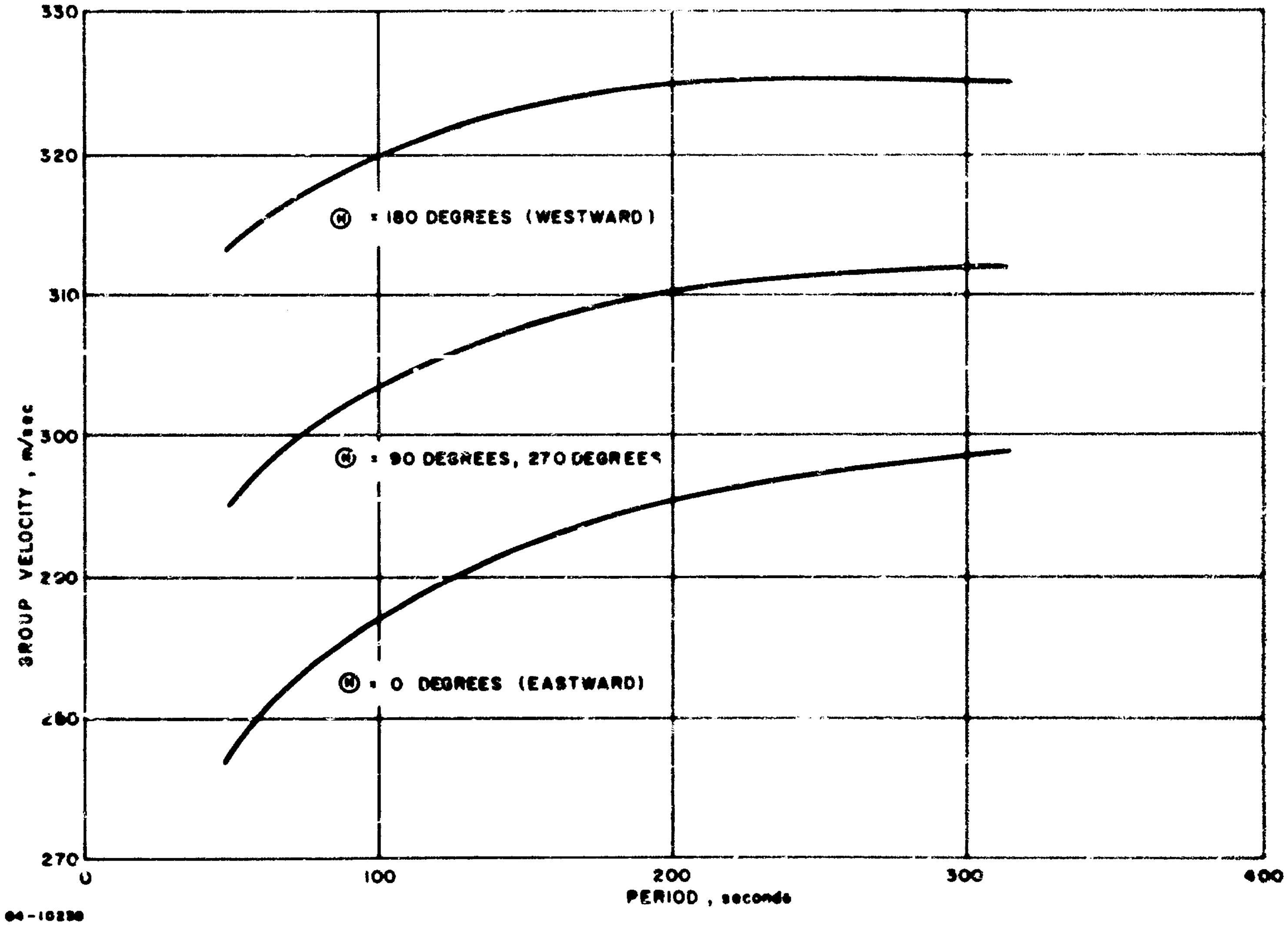

Figure 3 THE NET PERTURBED GROUP VELOCITY AS OBSERVED ON A LINE MAKING ANGLE $\theta = 0°$ 90°, 180°, and 270° WITH THE EASTWARD DIRECTION VERSUS PERIOD IN SECONDS

IX. CONCLUDING REMARKS

The theory outlined in this paper gives a method whereby the effects of winds may be readily incoporated into the study of the propagation of acoustic-gravity waves. Furthermore, the example treated in the previous section indicates that the consideration of winds may be necessary in a quantitative interpretation of actual microbarograms.

The principal complication introduced by winds is that they transform the atmosphere into an anisotropic medium. The magnitude of the horizontal wave number vector (which acts as an index of refraction) depends on the vector's direction as well as on frequency. This would appear to make the computation of the phase and group velocities more difficult by an order of magnitude. However, the perturbation theory developed here (which takes advantage of the small ratio of wind velocity to sound speed) requires only the computation of two functions $q_x(\omega)$ and $q_y(\omega)$ in addition to the wind-independent wave number $k_n^0(\omega)$. With this simplification the consideration of winds becomes feasible.

The perturbation method is but one application of the integral theorems developed in this paper. These show promise of being useful in the numerical calculation of phase and group velocities as well as in the development of approximate methods of solving the residual equations.

The question now remains as to whether or not a model atmosphere with winds independent of horizontal coordinates is a satisfactory model for the actual atmosphere. Certainly, it should be more satisfactory than a model atmosphere without winds. However, a glance at the flow patterns of the atmospheric winds on a global scale given in the Handbook of Geophysics[18] indicates that some modification of the theory may be required to take into account the curvature of the streamlines of the ambient winds. Such a modification should be necessary for propagation over horizontal paths of 7,000 km or greater. The present theory may be readily extended to cover such situations by using the mathematical techniques discussed by the author[19] in a previous theory of wave propagation in an almost-stratified medium. This extension will be given in a later article.

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Unclassified report

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