COMMENTS ON "SOLUTION OF THE QUOTA PROBLEM BY A SUCCESSIVE-REDUCTION METHOD"

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SUMMARY

It is pointed out that the "Quota Problem" as described by D. F. Votaw, Jr., in JORSA, Vol. 6, No. 1, can be looked upon as a special kind of maximum flow problem. As such it can be solved more efficiently by the methods of Ford and Fulkerson than by the reduction method proposed by Votaw.

In "Solution of the quota problem by a successive-reduction method," JORSA, Vol. 6, No. 1, D. F. Votaw, Jr., outlines an algorithm for solving the following problem:

> "Consider a set of persons and a set of job categories, and suppose that for each job category there is a quota. Suppose further that for each person it is known with regard to each job category whether he is 'qualified' or 'not qualified'. The quota problem can be stated as follows: (a) Does there exist an assignment of persons to jobs such that each person is qualified on the job to which he is assigned? (b) If the answer to (a) is yes, find such an assignment."

In Votaw's method of solution, one first writes down feasibility conditions which answer (a), and then uses these conditions (with a vengeance) to solve (b). (If there are n job categories, the feasibility conditions are a set of $2^n - 1$ inequalities, one for each non-empty subset of job categories).

It is our contention that this approach to the problem is computationally inefficient — that the way to solve such a problem is <u>not</u> to answer (a) before (b), but rather to answer (a) by attempting to solve (b). An iterative approach, based on an algorithm for constructing maximal network flows (problem (b) is a maximal flow problem), that rapidly constructs a solution to (b), if one exists, or, if the problem is infeasible, singles out a violation of the feasibility conditions, can be found on pp. 215-216 of [1]. (In case of infeasibility, the labelling process of [1] terminates with some quotas unfilled. At this point, the sum of the personnel availabilities over the set of unlabelled rows is strictly less than the sum of quota requirements over the unlabelled columns, violating one of the $2^n - 1$ feasibility inequalities.)

For a comparison of the efficiency of the two methods, we note that the 32 by 5 quota example, which Votaw states took three hours of hand computation by the successive-reduction method, required approximately twenty minutes of hand computation (including copying time for the original data) by the labelling process of [1].

We also point out that the feasibility conditions stated (without proof) by Votaw for the quota problem are a special case of the network flow supply-demand feasibility theorem proved in [2].

REFERENCES

1. Ford, L. R., Jr., and D. R. Fulkerson, "A Simple Algorithm for Finding Maximal Network Flows and an Application to the Hitchcock Problem," Can. J. Math., Vol. 9, 1957, pp. 210–218.

2. Gale, D., "A Theorem on Flows in Networks," Pacific J. Math., Vol. 7, No. 2, 1957, pp. 1073–1082.