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**ANALYTICAL APPROXIMATIONS**

Volume 26

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✓ P-1217  
*Bar*

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### Analytical Approximation

Incomplete Elliptic Integral: To better than .0000,0003 over  $45^\circ \leq \phi \leq 90^\circ$  and  $45^\circ \leq \alpha \leq 90^\circ$ ,

$$F(\phi, \alpha) = \int_0^\phi \frac{d\phi}{\sqrt{1 - \sin^2 \alpha \sin^2 \phi}}$$

$$= A(\alpha) + B(\phi, \alpha) \sqrt{1 - \sin^2 \alpha \sin^2 \phi}$$

$$+ C(\alpha) \ln \left[ \frac{1}{\cos \phi \sin \alpha + \sqrt{1 - \sin^2 \alpha \sin^2 \phi}} \right].$$

$$A(\alpha) = 1.3862,9436$$

$$C(\alpha) = 1.0$$

$$+.0965,7350(\frac{\pi}{2} - \alpha)^2$$

$$+.2499,9980(\frac{\pi}{2} - \alpha)^2$$

$$-.0013,0437(\frac{\pi}{2} - \alpha)^4$$

$$+.0572,9764(\frac{\pi}{2} - \alpha)^4$$

$$-.0013,7074(\frac{\pi}{2} - \alpha)^6$$

$$+.0149,5866(\frac{\pi}{2} - \alpha)^6$$

$$-.0002,7227(\frac{\pi}{2} - \alpha)^8$$

$$+.0047,0849(\frac{\pi}{2} - \alpha)^8$$

$$-.0000,9495(\frac{\pi}{2} - \alpha)^{10}$$

$$+.0008,2724(\frac{\pi}{2} - \alpha)^{10}$$

$$+.0010,8108(\frac{\pi}{2} - \alpha)^{12}$$

Analytical Approximation

$$B(\rho, \alpha) = f_1(\rho)g_1(\alpha) + f_2(\rho)g_2(\alpha) + f_3(\rho)g_3(\alpha)$$

$$f_1(\rho) = -.3277,5088 \left(\frac{\rho}{4}\right)^3$$

$$f_1(\rho) = -.0638,1682 \left(\frac{\rho}{4}\right)^5$$

$$f_1(\rho) = -.0080,8686 \left(\frac{\rho}{4}\right)^7$$

$$f_1(\rho) = -.0008,5024 \left(\frac{\rho}{4}\right)^9$$

$$f_1(\rho) = -.0000,9551 \left(\frac{\rho}{4}\right)^9$$

$$g_1(\alpha) = .7674,7222$$

$$g_1(\alpha) = +.3184,7443 \left(\frac{\alpha}{4}\right)^2$$

$$g_1(\alpha) = +.0820,7991 \left(\frac{\alpha}{4}\right)^4$$

$$g_1(\alpha) = +.0179,8074 \left(\frac{\alpha}{4}\right)^6$$

$$g_1(\alpha) = +.0034,7561 \left(\frac{\alpha}{4}\right)^8$$

$$g_1(\alpha) = +.0013,5976 \left(\frac{\alpha}{4}\right)^{10}$$

$$f_2(\rho) = .0298,811 \left(\frac{\rho}{4}\right)^3$$

$$f_2(\rho) = -.0676,009 \left(\frac{\rho}{4}\right)^5$$

$$f_2(\rho) = -.0206,017 \left(\frac{\rho}{4}\right)^7$$

$$f_2(\rho) = -.0040,129 \left(\frac{\rho}{4}\right)^9$$

$$f_2(\rho) = -.0007,920 \left(\frac{\rho}{4}\right)^9$$

$$g_2(\alpha) = .0516,668$$

$$g_2(\alpha) = -.1689,896 \left(\frac{\alpha}{4}\right)^2$$

$$g_2(\alpha) = -.0998,444 \left(\frac{\alpha}{4}\right)^4$$

$$g_2(\alpha) = -.0344,697 \left(\frac{\alpha}{4}\right)^6$$

$$g_2(\alpha) = -.0065,684 \left(\frac{\alpha}{4}\right)^8$$

$$g_2(\alpha) = -.0043,974 \left(\frac{\alpha}{4}\right)^{10}$$

$$f_3(\rho) = -.000727 \left(\frac{\rho}{4}\right)^3$$

$$f_3(\rho) = +.004444 \left(\frac{\rho}{4}\right)^5$$

$$f_3(\rho) = -.003986 \left(\frac{\rho}{4}\right)^7$$

$$f_3(\rho) = -.002520 \left(\frac{\rho}{4}\right)^9$$

$$f_3(\rho) = -.000621 \left(\frac{\rho}{4}\right)^9$$

$$g_3(\alpha) = .005752$$

$$g_3(\alpha) = -.075208 \left(\frac{\alpha}{4}\right)^2$$

$$g_3(\alpha) = +.073873 \left(\frac{\alpha}{4}\right)^4$$

$$g_3(\alpha) = +.088197 \left(\frac{\alpha}{4}\right)^6$$

$$g_3(\alpha) = -.014601 \left(\frac{\alpha}{4}\right)^8$$

$$g_3(\alpha) = +.044161 \left(\frac{\alpha}{4}\right)^{10}$$

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