GENERALIZED ANALYSIS OF AERIAL CAMPAIGNS AGAINST STRATEGIC TARGETS *

R. Schamberg

P-1017

May 9, 1957

*Presented at the Fifth Annual Meeting of the Operations Research Society of America, May 9-10, 1957.

The RAND Corporation
1700 MAIN ST. • SANTA MONICA • CALIFORNIA

SUMMARY

This paper describes an analytic technique for direct determination of the outcome of optimal aerial bombing or reconnaissance campaigns against strategic targets, without the customary laborious exploration of variations in attack strategy. The scope of this analysis permits wide latitude in the nature of the strategic campaign and its environment, including the following: the criterion defining an optimum campaign may be that of minimum cost, or minimum air crew loss for a fixed level of target destruction, with or without additional constraints such as specified maximum campaign duration; aircraft losses may be due to enemy area and local defenses, non-combat causes, or due to destruction while on their own base. Several examples are used to illustrate how the analytic method presented here leads to a better understanding of the significant features of strategic campaigns, and their sensitivity to variations in assumptions, than can be gleaned from an essentially empirical study of many individual campaigns.

CONTENTS

INTRODUCTION

The analysis of aerial campaigns against strategic targets involves the aggregation of a multitude of detailed factors concerning a variety of subjects. These include the geographic location of bases and targets, design and performance characteristics of aircraft and weapons, the attrition of aircraft by combat and non-combat causes, the resources required to procure and operate the various components of the strategic systems, and a number of others. Usually this process of aggregation, called "campaign analysis," consists of lengthy numerical calculations which consider in sequence each strike of a number of non-optimal campaigns in order to discover by enumeration or interpolation the optimum campaign tactic for the particular weapon system and campaign criterion under consideration. The analyst who has ever performed this laborious process will readily appreciate the need for a tool or mathematical model which is capable of aggregating these diverse campaign ingredients in a systematic manner, designed to achieve two basic aims. These two aims of generalized campaign analysis are: (1) an understanding of the primary physical interactions which characterize strategic campaigns, and (2) a reduction in the amount of computations required to obtain specific results. This paper is intended to describe briefly such a model for generalized campaign analysis and to illustrate its use. This method was developed at The RAND Corporation* in connection with its work for the United States Air Force.

*The author gratefully acknowledges helpful suggestions by his colleagues, especially by G. E. Gompf, R. B. Murrow and L. B. Rumph.

SCOPE

The scope of the analysis we are considering is shown in Table 1. This table shows a number of ways in which strategic campaigns can be classified. Consider the first horizontal row, which defines the purpose of the mission. As indicated, the mission may alternatively have the purpose of bombing or reconnaissance; or, in turn, we may have a combined bombing and reconnaissance campaign, wherein some aircraft are sortied on bombing strikes, and others on reconnaissance strikes. Another way of classifying campaigns is according to the type of primary aerial vehicle being used. The vehicle may be re-usable for more than one strike as is the case for a manned aircraft, or it may be expended on one strike and is hence "non-re-usable;" this type we usually call a missile. The third characteristic used for classifying campaigns, terminal delivery, may be by means of a weapon launched near the target, that is, within the local defense ring; or alternatively, the weapon may be launched outside the local defenses as is usually the case with air-to-surface missiles.

The fourth way of campaign classification called strike progression requires somewhat more explanation. The first type listed in row IV of Table 1, the iso-strike campaign, is one wherein forces of identical magnitude are sortied on each strike; that is, in an expected value model each strike is like every other one, hence the term "iso-strike." This type of strike progression is useful in the analysis of cases wherein a certain link in the system has a fixed capacity such as, for example, the capacity of staging bases used, or the fuel transfer capability of a limited tanker force. The second type of strike progression is that of the "impact campaign," wherein the entire force of available bombers is sent out on the

Table 1

SCOPE OF ANALYSIS

	CLASSIFICATION BY:	CASE IDENTIFICATION NUMBER			
		1	2	3	4
I	MISSION	BOMBING	RECONNAISSANCE	COMBINED BOMBING & RECONNAISSANCE CAMPAIGN	
II	PRIMARY VEHICLE	RE-USABLE	NON-RE-USABLE		
III	TERMINAL DELIVERY	WEAPON LAUNCHED INSIDE LOC'L DEFENSE	WEAPON LAUNCHED OUTSIDE LOCAL DEFENSE		
IV	STRIKE PROGRESSION	ISO-STRIKE CAMPAIGN	IMPACT CAMPAIGN		
V	CRITERION	MINIMUM CREW LOSS	MINIMUM TOTAL COST		
VI	TACTICS RESTRICTIONS	NONE	SPECIFIED NUMBER OF STRIKES	SPECIFIED SURVIVAL PROBABILITY PER STRIKE	SPECIFIED NUMBER OF STRIKES & SPECIFIED SURVIVAL PROBABILITY PER STRIKE

first strike, whatever returns is sent on the next strike, and so on; that is, the greatest possible impact is produced on every strike. Since we are concerned here with the analysis of optimal campaigns, a campaign criterion must first be defined. Row V of Table 1 shows two such criteria: (1) minimum air crew loss for a given level of target destruction, and (2) minimum total cost for specified target destruction. In view of the mathematical relationships involved, it turns out that criteria (1) and (2) are completely equivalent to (1a) maximum target destruction for a given level of air crew loss and (2a) maximum target destruction for a specified total cost.

Finally, the method of analysis under consideration is capable of treating optimal campaigns subject to one or more restraining conditions. The last row of Table 1 thus indicates that campaign tactics may be restricted by specifying the number of strikes in the campaign, and/or the crew survival probability per strike (attrition rate).

In the illustrative examples which follow, we will confine ourselves primarily to bombing missions with a re-usable primary vehicle which launches its weapon inside the local defenses; furthermore, we shall consider iso-strike-type campaigns, that are optimized for minimum total system cost.

CAMPAIGN INGREDIENTS

Whatever the type of campaign, there are a number of important factors or ingredients which must be included. These are summarized in Table 2. The various terms used in Table 2 are defined below. A detailed discussion of these campaign ingredients is beyond the scope of this brief paper which is primarily concerned with the process of their aggregation into a campaign analysis.

"Mission accomplishment" denotes the conditional probability that a bomber or missile successfully accomplishes its mission, provided that the designated target has been reached. Specifically, for bombing missions the conditional probability that a dropped bomb destroy the target is called "target coverage" (P_d). For reconnaissance missions, the analogous "reconnaissance success probability" accounts for such random factors as the effect of weather, malfunction of equipment, etc.

The items listed under attrition in Table 2 account for the several causes by which primary vehicles may be lost in a strategic campaign. Combat attrition is conveniently divided into aircraft shot down by area and local defenses. Since the nature of these defenses has an important bearing on the mathematical structure of the present Campaign Analysis Model, these ingredients are discussed in more detail in the next section. Attrition due to non-combat causes is here defined to include aircraft lost in flying accidents, as well as aircraft destroyed on their bases by possible enemy counter action.

Among the Operational Factors, "Serviceability" (S) represents the fraction of aircraft on the base which are ready for a sortie, and the reliability (R) denotes the probability that a sortied aircraft will not

Table 2

IMPORTANT FACTORS

MISSION ACCOMPLISHMENT

Target Coverage, Reconnaissance Success Probability

ATTRITION

Combat Causes: Area and Local Defense

Non-Combat Causes

OPERATIONAL FACTORS

Serviceability, Reliability

TACTICAL CONSIDERATION

Reconnaissance Requirement, Minimum Survival Probability

Number of Strikes, etc

COST

"Unit Costs" of Vehicles and Weapons

Total System Cost

abort its mission due to mechanical failures or other causes.

"Tactical Considerations" include several items, primarily related to the imposition of realistic constraints on the variety of possible campaign tactics. The "reconnaissance requirement" (T_R/T_D) denotes the fraction of targets to be destroyed in a bombing campaign which require prior aerial reconnaissance. The other tactical considerations are self explanatory.

The cost ingredients listed in Table 2 are of interest when "minimum total cost" is used as the optimizing campaign criterion (Table 1, row V). The total system cost measures the totality of resources required — primary vehicles, crews, weapons, bases, materiel, etc — to procure and train the strategic force and to maintain it in a combat-ready status for a specified number of (peacetime) years. "Unit costs" are obtained by prorating the applicable portions of the total system cost to each primary vehicle or weapon in the strategic force. Thus the "unit cost" of an aircraft may be several times the procurement cost for one aircraft, since the cost of crew training, aircraft operation and maintenance, and a proration of base costs are also included.

DEFENSES

As indicated above the nature of the defenses which must be penetrated by the strategic force has an important effect on the mathematical structure of this campaign analysis model. Figure 1 illustrates the analytical representation of defenses used in this model. The central problem here is to determine the number of defense units, such as interceptor bases or missile sites that are activated by the defense as a function of the number of targets attacked by the offense on any given strike. The graph in the lower left-hand corner of Fig. 1 shows several such functions. The number of bombers shot down by the defenses is then approximately proportional to the product of the number of defense units activated (DUA) and the bomber kill potential of each defense unit. Consider first the upper left-hand picture which illustrates schematically a "ring defense." The dots in the central area represent individual targets. The shaded circles represent the radius of action of individual defense units, which are deployed peripherally about the entire target area. The optimal offense strategy for penetrating such a ring defense is to pierce it at one point with a single track which then fans out to all the targets. This means that, no matter how many targets are attacked, the number of defense units is essentially constant, so that we can represent this type of defense by the horizontal line on the graph of Fig. 1. The symbol T^* here designates an arbitrary reference number of targets, which is usually taken to be the maximum number of targets in the target system.

Another way of deploying defenses is to have isolated defense units located at each individual target, as illustrated by the sketch in the lower right-hand corner of Fig. 1. This is called "localized defenses"

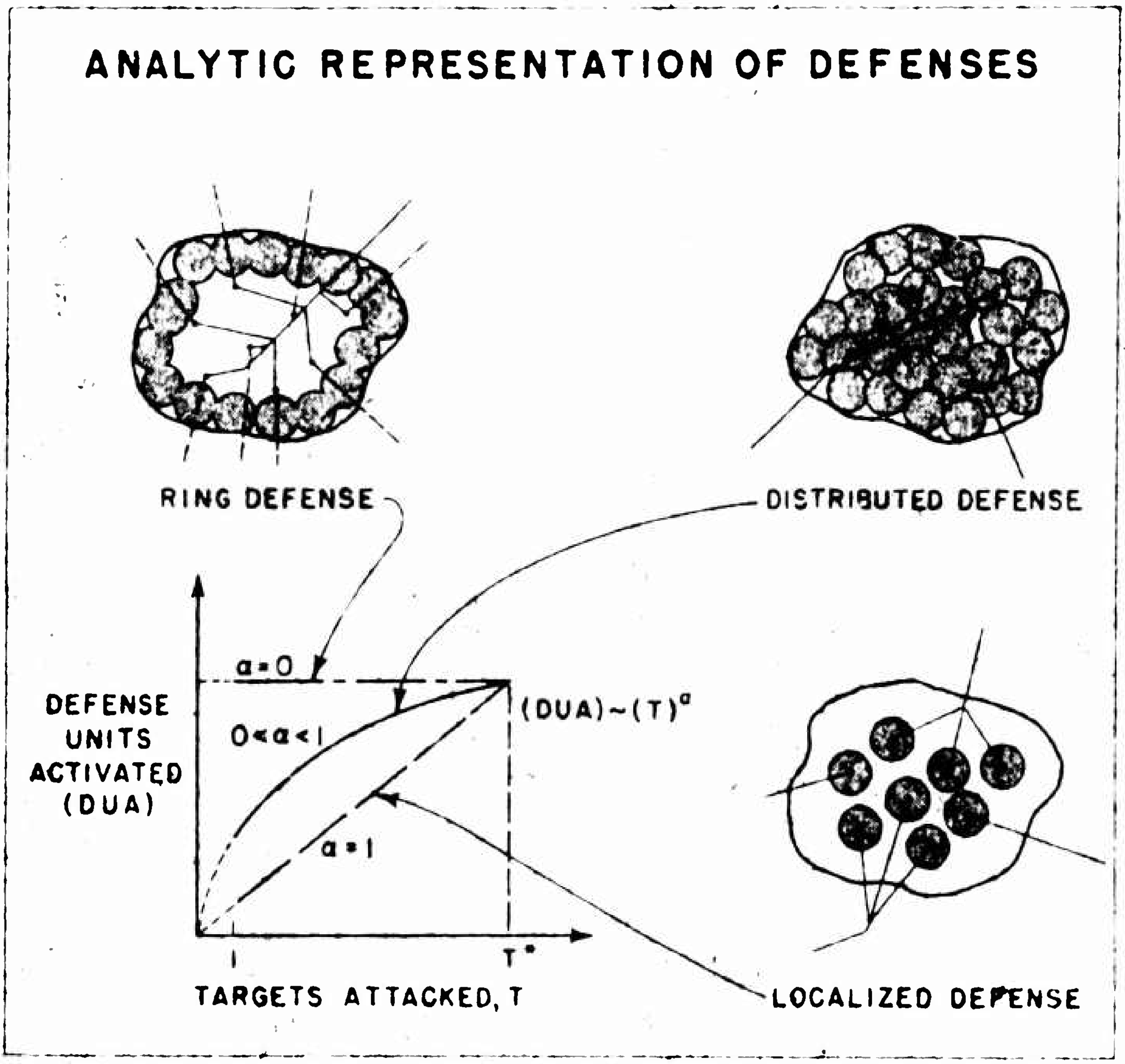

ANALYTIC REPRESENTATION OF DEFENSES
RING DEFENSE
DISTRIBUTED DEFENSE
DEFENSE UNITS ACTIVATED (DUA)
α = 0
0 < α < 1
α = 1
(DUA) ~ (T)^α
1
T*
TARGETS ATTACKED, T
LOCALIZED DEFENSE

Fig. 1

and has the characteristic that the number of defense units activated is directly proportional to the number of targets attacked. This defense behavior is represented by the straight diagonal line of the graph of Fig. 1.

Finally, the defenses may be distributed in a more or less uniform or random manner as illustrated by the sketch at the upper right of Fig. 1. For this type of "distributed defense" the number of defense units activated will increase as the number of targets attacked increases, but not necessarily in direct proportion. The corresponding function looks something like t e middle curve shown in the graph of Fig. 1.

It has been found that the behavior of the distributed type of defense, which appears to be the most interesting type of area defense, may be adequately represented by the simple function

$$(DUA) \sim T^{\alpha} \quad , \tag{1}$$

where T represents the number of targets attacked and α is an exponent having a value between zero and one. For typical area defenses, $\alpha = 1/2$ has been found to be a reasonable approximation.

Furthermore, equation (1) also represents the behavior of ring and localized defenses, when the exponent α takes on the limiting values of zero and one, respectively.

The present campaign analysis model considers the general case where both area and local defenses must be penetrated to reach the various targets in the target system. Thus B_K the number of bombers (primary vehicles) killed by the defenses wh T targets are attacked or one strike, is given by

$$B_K = B_{K_a} + B_{K_\ell} = aT^{\alpha} + bT \; . \tag{2}$$

The first term, B_{K_a}, represents bombers killed by area defenses, the second term, B_{K_ℓ}, bombers killed by local defenses. The parameters a and b represent the appropriate defense effectiveness per target, or bomber kill potentials, of the type and number of deployed defense weapons against the type of primary vehicle under study. It should be noted that B_K is independent of the number of bombers present, because this model considers only "saturation raids" as required to insure reasonable survival probabilities from the offense point of view.

ANALYSIS

The gist of the generalized analysis will be sketched in this section, by means of a somewhat simplified example. For illustrative purposes consider iso-strike bombing or reconnaissance campaigns with re-usable primary vehicles which penetrate the local defenses; it is desired to optimize these campaigns according to the criterion of minimum total system cost for specified target destruction, subject to the constraint of a specified (expected) crew survival probability per strike.

The two quantities of principal interest are the expected number of targets destroyed in the campaign (T_D), (or the number of targets successfully reconnoitered (T_R)for a reconnaissance campaign) and the total system cost (C) of the associated bombing system. These are given by equations (3) and (4), respectively:

$$T_D = NTP_d \quad , \quad T_R = NTP_r \tag{3}$$

$$C = \gamma B = \gamma \left[(N-1)\ B_K + \frac{1}{S}\ mT \right] \; ; \tag{4}$$

the symbols are defined in Table 3. As shown by equation (3), the number of targets destroyed (T_D) in an iso-strike campaign is simply the product of the number of strikes (N), the number of targets attacked on each strike (T) and the conditional probability P_d that an attacked target is destroyed.

As shown by equation (4), the total system cost is the product of the "unit cost" γ and the number of primary vehicles (bombers) B in the striking force. The number of bombers is given by the square bracket on the right side of equation (4), it being assumed for simplicity that bombers

Table 3

List of Symbols

B	=	Total number (stockpile) of primary vehicles (bombers) in the strike force.
B_K	=	Expected number of bombers killed on a strike.
B_{K_a}	=	Expected number of bombers killed on one strike by area defenses.
B_{K_ℓ}	=	Expected number of bombers killed on one strike by local defenses.
$B_{K_a}^*$	=	Expected number of bombers killed by area defenses when the reference number of T^* targets is attacked on one strike.
$B_{K_\ell}^*$	=	Expected number of bombers killed by local defenses when all T^* targets of the reference target system are attacked on one strike.
C	=	Total system cost of procuring and maintaining for a specified period of time aircraft crews, bases, weapons and support facilities.
DUA	=	Defense units activated.
J	=	Parameter group defined by equation (11).
k	=	Reference value of attrition ratio, defined by equation (8).
L_A	=	Total number of aircraft lost during the campaign.
m	=	Cell size, i.e., average number of aircraft sortied per target scheduled for attack.
N	=	Number of strikes in campaign.
N_{opt}	=	Optimum number of strikes in minimum cost campaign.

Table 3 (Continued)

P_d	=	Target coverage, probability that a delivered bomb destroys the target.
P_K	=	Round-trip kill probability of a bomber that has not aborted.
P_r	=	Probability of obtaining desired reconnaissance information, given that aircraft reached target area.
P_{K_b}	=	Expected fraction of bombers on base destroyed prior to next strike.
R	=	Fraction of sortied bombers which do not abort, reliability factor.
S	=	Fraction of bombers on base which are combat ready, serviceability factor.
T	=	Number of targets scheduled for attack on one strike.
T^*	=	Reference number of targets in target system.
T_D	=	Expected number of targets destroyed in campaign, target destruction potential.
T_R	=	Expected number of targets reconnoitered in the campaign.
x	=	B_{K_ℓ}/B_{K_a}, attrition ratio.
X	=	Optimum value of attrition ratio for minimum cost campaigns.
α	=	Exponent defining area defense behavior.
Γ	=	System cost parameter, defined by equation (12).
γ	=	Unit (system) cost per primary vehicle.
Λ	=	Aircraft loss parameter defined by equation (19).
μ	=	Campaign magnitude, or target destruction, parameter, defined by equation (10).

are lost only by combat causes (B_K), that is shot down by area and local defenses. The quantity $(N-1)B_K$ is the total number of bombers shot down on all strikes but the last one, and mT is the number of bombers which are required to mount the last strike. The expected number of bombers killed on any strike (B_K) are related to the number of targets attacked T by equation (2). In equation (4), "m" is the so-called "cell size" or average number of primary vehicles sortied per target scheduled for attack, and is related to the specified survival probability per strike (P_s) by

$$P_s = 1 - P_K = 1 - \frac{B_K}{RmT} \quad . \tag{5}$$

It can easily be shown that — by solving equations (3) and (5) for N and m, respectively, and substituting these expressions (as well as equation (2) for B_K) into equation (4) — the latter becomes an explicit relationship between the total systems cost C and the number of targets to be destroyed T_D, with the number of targets attacked per strike T as a parameter. In principle then, the optimum number of targets to be attacked for minimum systems cost could then be obtained by the standard method of the differential calculus. However, the algebraic form of the equations involved precludes an explicit solution in closed form. It is, therefore, convenient to resort to a parametric type of solution, making use of reduced or dimensionless parameters to the maximum extent.

It turns out that the most naturally arising independent variable is the attrition ratio x,

$$x = \frac{B_{K_\ell}}{B_{K_a}} \quad , \tag{6}$$

of bombers killed by local defenses to bombers killed by area defenses.

It follows from equation (2) that this attrition ratio is related to the number of targets attacked per strike T by

$$x = \frac{b}{a} \; T^{(1-\alpha)} \tag{7}$$

It is convenient to denote by k the "reference value" of the attrition ratio which occurs when T^* targets are attacked on a strike (see Fig. 1), so that

$$k = x^* = \frac{B_{K_\ell}^{\;*}}{B_{K_a}^{\;*}} = \frac{a}{b} \; T^{*(1-\alpha)} \quad . \tag{8}$$

Equation (7) can then be rewritten in the more convenient form

$$x = k \left(\frac{T}{T^*}\right)^{1-\alpha} \quad . \tag{9}$$

Similarly, the analysis suggests the definition a "campaign magnitude parameter" μ which represents in dimensionless form the magnitude of the strategic job to be accomplished, that is the number of targets to be destroyed or reconnoitered. This parameter μ is defined by

$$\mu = \frac{T_D}{T^* \; P_d \, j} \quad , \quad \mu = \frac{T_R}{T^* \; P_r \, j} \tag{10}$$

for bombing and reconnaissance campaigns, respectively. The factor "j" appearing in equation (10) represents the following combination of previously encountered constants:

$$j = \left[\frac{1}{RSP_K} - 1 \right] \quad . \tag{11}$$

In analogous manner, there is also defined a dimensionless "system cost parameter" Γ by

$$\Gamma = \frac{C}{\Upsilon B^*_{K_a} J} \quad , \tag{12}$$

where $B^*_{K_a}$ represents the area-defense attrition level, that is the number of bombers shot down by the area defense on a strike against the reference number of T^* targets.

One next converts the basic campaign equations (3) and (4) into dimensionless form, making use of equations (8) to (12). Carrying out the algebraic manipulations described in the paragraph following equation (5), one obtains the following explicit relationship between the cost parameter Γ and the campaign magnitude parameter μ :

$$\left[\Gamma \, k^{\frac{\alpha}{1-\alpha}} \right] = x^{1-\alpha} + x^{\frac{\alpha}{1-\alpha}} + \left(1 + \frac{1}{x}\right) \left[\mu \, k^{\frac{1}{1-\alpha}} \right] \quad . \tag{13}$$

In this relation k and α are constants which define the intrinsic nature and level of the defenses and their interaction with the penetrating offense vehicles (see equations 8 and 1). The variable attrition ratio "x" in equation (13) represents a one-dimensional choice of attack strategy, namely, in view of equation (9), the number of targets T which are attacked on each strike of the iso-strike campaign.

The optimum tactic, $x = x_{opt} \equiv X$, which yields minimum cost (Γ) for fixed value of target destruction (μ) is obtained by differentiation of equation (13) with respect to x.

$$\frac{\partial}{\partial x} \left[\Gamma \, k^{\frac{\alpha}{1-\alpha}} \right] = 0 \tag{14}$$

Although the resulting algebraic equation cannot be solved explicitly for

$x = X$ (for arbitrary values of α), this equation can, nevertheless be used in conjunction with equation (13) to yield the following two simultaneous, parametric equations which define minimum cost iso-strike campaigns:

$$\left\{ \begin{aligned} \left[\mu\, k^{\frac{1}{1-\alpha}}\right] &= f_1(X,\alpha) = \frac{\alpha}{1-\alpha}\, X^{\frac{1}{1-\alpha}} \left(1 + \frac{X}{\alpha}\right) && (15) \\ \left[\Gamma\, k^{\frac{\alpha}{1-\alpha}}\right] &= f_2(X,\alpha) = \frac{1}{1-\alpha}\, X^{\frac{\alpha}{1-\alpha}} (1 + X)^2 \; . && (16) \end{aligned} \right.$$

Here X represents the attrition ratio associated with the optimum tactic for minimum cost campaigns, that is

$$X = k\left(\frac{T_{opt}}{T^*}\right)^{1-\alpha} . \qquad (17)$$

The meaning of the pair of parametric equations (15) and (16) is that, for a specified value of the area-defense behavior exponent α , there is a one to one correspondence between the cost and campaign magnitude parameters. Thus, by assuming arbitrary values of α and X one can compute the functions f_1 and f_2, and plot cost parameter versus target destruction parameter as shown in Fig. 2. This quasi-universal set of curves represents all minimum cost iso-strike campaigns.

It should be pointed out that various other characteristics of these optimal campaigns are uniquely determined for each value of X and α . For example, the number of strikes, N_{opt}, of which the optimal iso-strike campaign is composed, is given by

$$\frac{N_{opt}}{j} = f_3(X,\alpha) = \frac{\alpha}{1-\alpha}\left(1 + \frac{X}{\alpha}\right) . \qquad (18)$$

COST PARAMETER VS TARGET DESTRUCTION PARAMETER
FOR MINIMUM—COST CAMPAIGNS

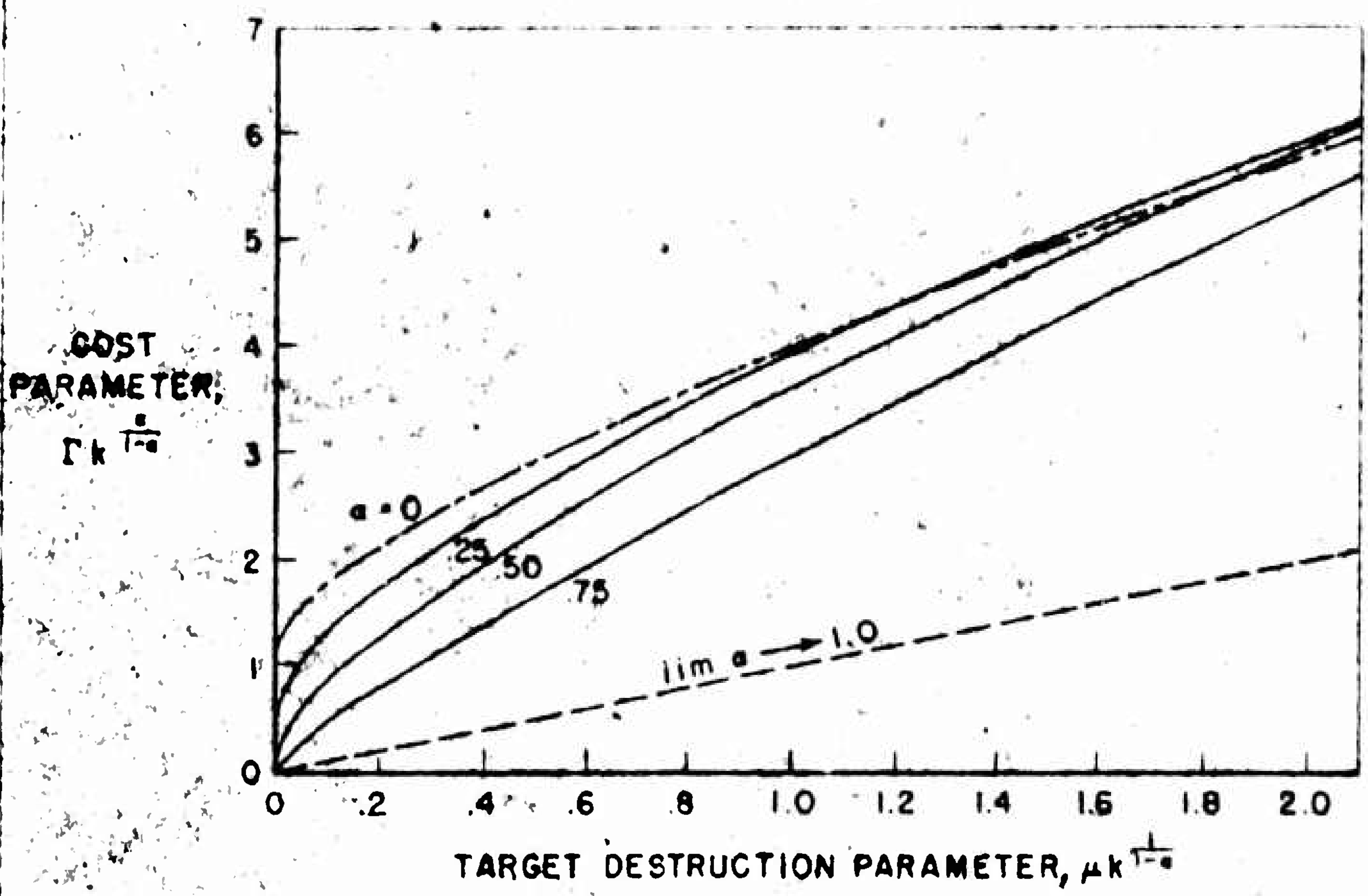


Fig. 2

Similarly, the number of aircraft (or aircrews) lost in this type of campaign, L_A, is determinable with the aid of the "loss parameter" Λ,

$$\Lambda = \frac{L_A}{B_{K_a}^{*} J} , \tag{19}$$

from the equation

$$\left[\Lambda k^{\frac{\alpha}{1-\alpha}} \right] = f_4(X, \alpha) = \frac{\alpha}{1-\alpha} \; X^{\frac{\alpha}{1-\alpha}} \; (1 + X)(1 + \frac{X}{\alpha}) . \tag{20}$$

For repeated use and rapid calculation, a number of other graphs analogous to Fig. 2 may, of course, be computed in order to relate to one another any two of the quantities given by equations (15), (16), (17), (18), and (20).

Figure 2 displays some interesting properties of minimum cost campaigns which are worth noting. With the exception of the limiting case of $\alpha = 1$, wherein the area defense has degenerated to a localized defense (see Fig. 1), these curves are non-linear and have a continuously decreasing slope as the campaign magnitude increases, i.e., as more targets are attacked. This is a direct consequence of the area-defense behavior of Fig. 1 wherein, for $\alpha < 1$, each successive additional target attacked results in the activation of fewer additional area defense units.

For the case of the ring defense, $\alpha = 0$, Fig. 2 shows that there is a threshold cost below which no targets can be destroyed. This threshold represents the cost of the minimum size offense force required to penetrate the ring-defense once and return with the specified crew survival probability.

It is worth noting that the parametric equations (15) and (16) will yield explicit relations between μ and Γ in the two limiting cases of "very small" campaigns ($X \ll 1$) and "very large" campaigns ($X \gg 1$). For these cases it is easily shown that

$$\Gamma \doteq \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \cdot \mu^{\alpha} \quad , \quad \text{for } X \ll 1 \tag{21}$$

$$\Gamma \doteq \mu k \quad , \quad \text{for } X \gg 1 \quad . \tag{22}$$

These expressions verify the several features shown graphically in Fig. 2, namely, the curvature of the curves near the origin (small campaigns); the threshold value of $\Gamma = 1$ for $\alpha = 0$ $X = 0$; and the curves' asymptotic approach to straight lines for large values of μ. Furthermore, these expressions like equations (21) and (22) are most useful to obtain limiting values in sensitivity investigations which aim to determine, for example, the effect on system cost of a change in a parameter such as the area defense strength ($B^*_{K_a}$, see equations 8 and 12), or the crew survival probability (P_s, see equations 5 and 11).

Analogous generalized curves for impact campaigns are quantitatively different from and more laborious to calculate than the iso-strike curves shown in Fig. 2. However, the impact campaign curves have the same qualitative features of non-linearity ($\alpha < 1$) and threshold for $\alpha = 0$ that were pointed out above.

In concluding this section, it should be pointed out that the results obtained for iso-strike minimum cost campaigns (equations 13-22 and Fig. 2) actually are valid for a much broader spectrum of conditions than those assumed here to simplify the exposition. For example, it will be recalled that in applying equation (4) it was assumed that no bombers were destroyed by enemy action against bomber bases. If we waive this restriction and assume instead that, on the average, a fraction P_{K_b} of bombers on the base between strikes is destroyed by enemy counter action, then only two minor changes need be made in the analysis:

(1) The parameter group "j" is redefined in more general form than equation (11) as

$$j = \frac{1}{RSP_K(1-P_{K_b})} - 1$$

(2) The expression $\left[1 + \frac{P_{K_b}}{(1-P_{K_b})\, RSP_K}\right]$ is inserted as another factor into the denomination of the right hand side of equation (19) which defines the aircraft loss parameter Λ.

The generalized campaign equations in their parametric form, such as equations (15) - (18), as well as the curves of Fig. 2, remain applicable without change.

APPLICATIONS

The most straightforward application of the generalized campaign analysis method is, of course, the use of precomputed general campaign curves, such as Fig. 2, for the rapid calculation of specific campaigns. Figure 3 shows schematically the results of such a calculation. The solid curve is a plot of total system cost of a family of bombing systems -- whose members differ only in the number (not the type) of primary vehicles -- versus the number of targets which each system is potentially capable of destroying; it is assumed that the area defense deployment corresponds to $\alpha = 0.5$. Since for this family of campaigns only T_D and C vary, while k, $B^*_{K_a}$, etc remain fixed, and T_D and C are directly proportional to the parameters μ and Γ, respectively, it follows that the solid curve of Fig. 3 has the same shape as the $\alpha = 0.5$ curve of Fig. 2. Figure 3 illustrates more concretely, for a specific case, the non-linear increase total system cost with target destruction potential which was already observed as a general campaign property in Fig. 2.

The dashed curve of Fig. 3 is used to determine the (optimum) number of targets attacked per strike which characterize each of the minimum cost campaigns represented. This curve was computed from equations (17) and (15). As is the case with system cost, the optimum number of targets attacked per strike increases non-linearly with increasing magnitude of the campaign.

As may be seen by combining equations (18) and (17) for the present case of $\alpha = 0.5$, the optimum number of strikes for these iso-strike campaigns is given by

$$N_{opt} = J \left[1 + 2k \sqrt{\frac{T_{opt}}{T^*}} \right] , \; (\alpha = \tfrac{1}{2}) \; ; \qquad (23)$$

ISO-STRIKE BOMBING CAMPAIGNS FOR MINIMUM COST

$a = 0.5$

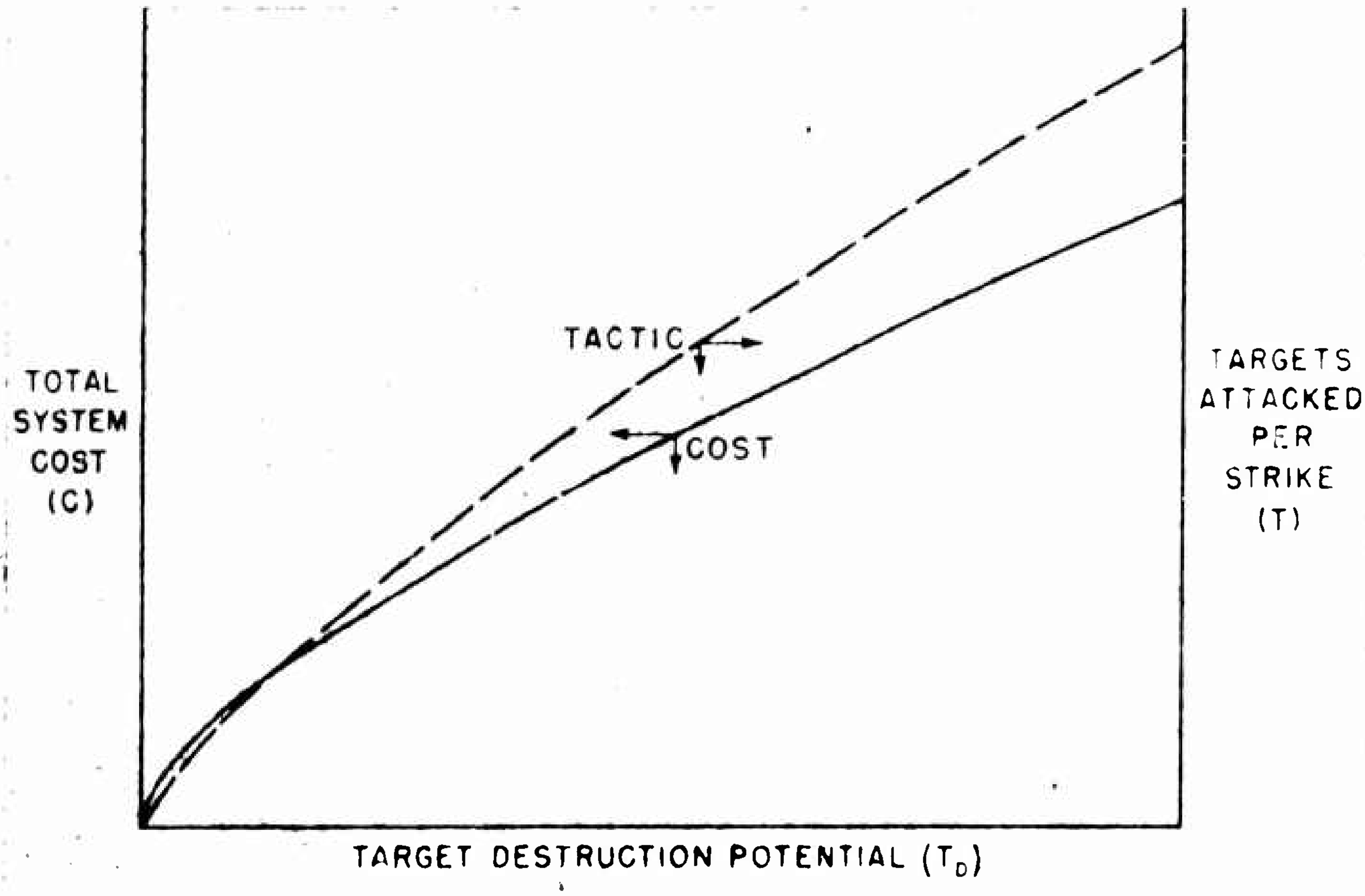


Fig. 3

hence the optimum number of strikes also increases with the magnitude of the campaign, although more slowly than the number of targets (T_{opt}) attacked per strike.

Equation (23) illustrates the relatively simple type of algegraic expression which is generally obtained when the area defense exponent has the value 1/2, that is the value for which $\alpha = (1-\alpha)$. Fortunately, this value also appears to be a satisfactory approximate representation for some realistically deployed area defenses which have been investigated.

Figure 4 is intended to shed some light on the nature of certain campaign efficiencies or ratios which are sometimes used as criteria in systems comparisons. As defined in Fig. 4, the generalized "operational efficiency" $\mu k/\Lambda$ is proportional to the ratio of targets destroyed per aircraft lost, and the economic efficiency $\mu k/\Gamma$ is proportional to the value of targets destroyed per unit system cost. Figure 4 shows that both of these efficiencies increase at an ever decreasing rate with the magnitude of the campaign, and approach unity asymptotically for very large campaigns. This effect is, of course, due to the intrinsic nature of the area defenses, as already pointed out in the discussion of Figs. 1 and 2. In view of this pronounced effect of campaign magnitude on such ratios as targets destroyed per aircraft lost, or targets destroyed per unit systems cost, the use of such ratios as criteria for comparing different strategic systems will not lead to valid results, unless these ratios are evaluated for all competing systems at the <u>same</u> value of target destruction potential.* Only in the latter instance are these ratios equivalent to valid comparison

---

*For a more general discussion on the validity of ratio criteria see Charles J. Hitch, "Sub-Optimization in Operations Problems," Journ. Oper. Res. Soc. of America, Vol. 1, No. 3 (May 1953), p. 94.

CAMPAIGN EFFICIENCIES

ISO-STRIKE BOMBING CAMPAIGNS FOR MINIMUM COST

$\alpha = 0.5$

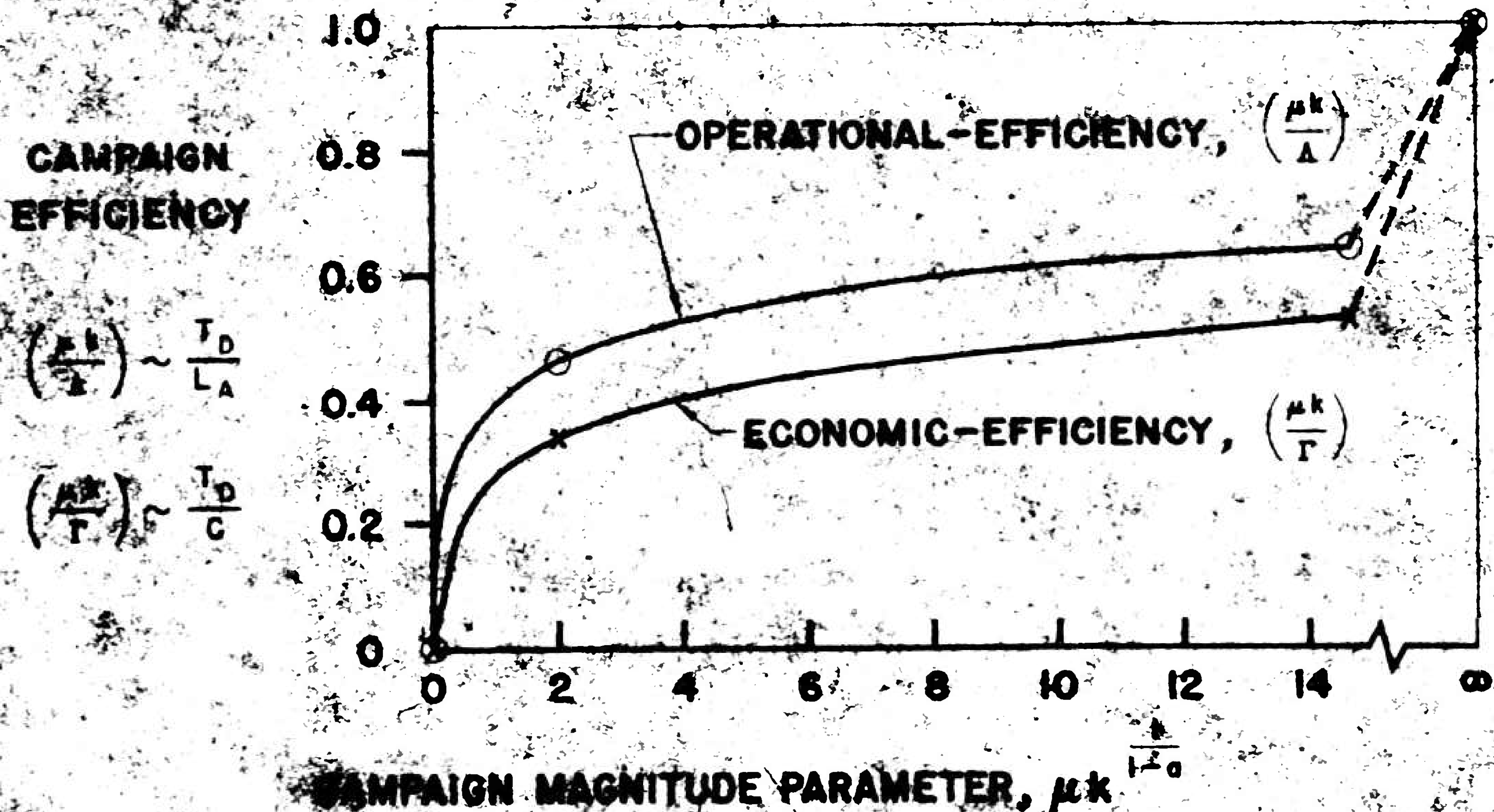


Fig. 4

C9-28

criteria such as aircraft lost for a specified number of targets destroyed, or targets destroyed for a specified total system cost.

Another application of the generalized campaign analysis technique is illustrated in Fig. 5. Here is shown the effect on system cost of varying the fraction of targets on which aerial reconnaissance information is required prior to bombing. In Fig. 5 the expected number of targets destroyed T_D is held fixed, while the number of targets to be reconnoitered varies from zero to T_D. Total system cost is referenced to the cost of the pure bombing campaign ($T_R/T_D = 0$). Two cases are shown in Fig. 5: (1) the dash line represents the succession of two separate and independent campaigns; first, one for reconnaissance, followed by a bombing campaign; (2) the solid curve represents the case where bombing and reconnaissance airplanes are used to make up a single strike force, which simultaneously carries out a mixed bombing and reconnaissance campaign. It is evident that, for all fractions of the target system to be reconnoitered, the combined campaign has a lesser total cost than do the two separate campaigns. The reason for this is simply that the combined campaign is larger and hence, in view of Fig. 4, more efficient than each of the two smaller campaigns which characterize the case of separate bombing and reconnaissance campaigns. It may be noted that the slope of the curves of Fig. 5 is arbitrary. For the particular example shown, the cost of the reconnaissance job for the complete target system happens to be about twice that of the bombing job.

Consider now briefly the method used for the calculation of the campaign results shown in Fig. 5. For each point on the dashed curve two separate campaigns are computed and their costs are added; the principal

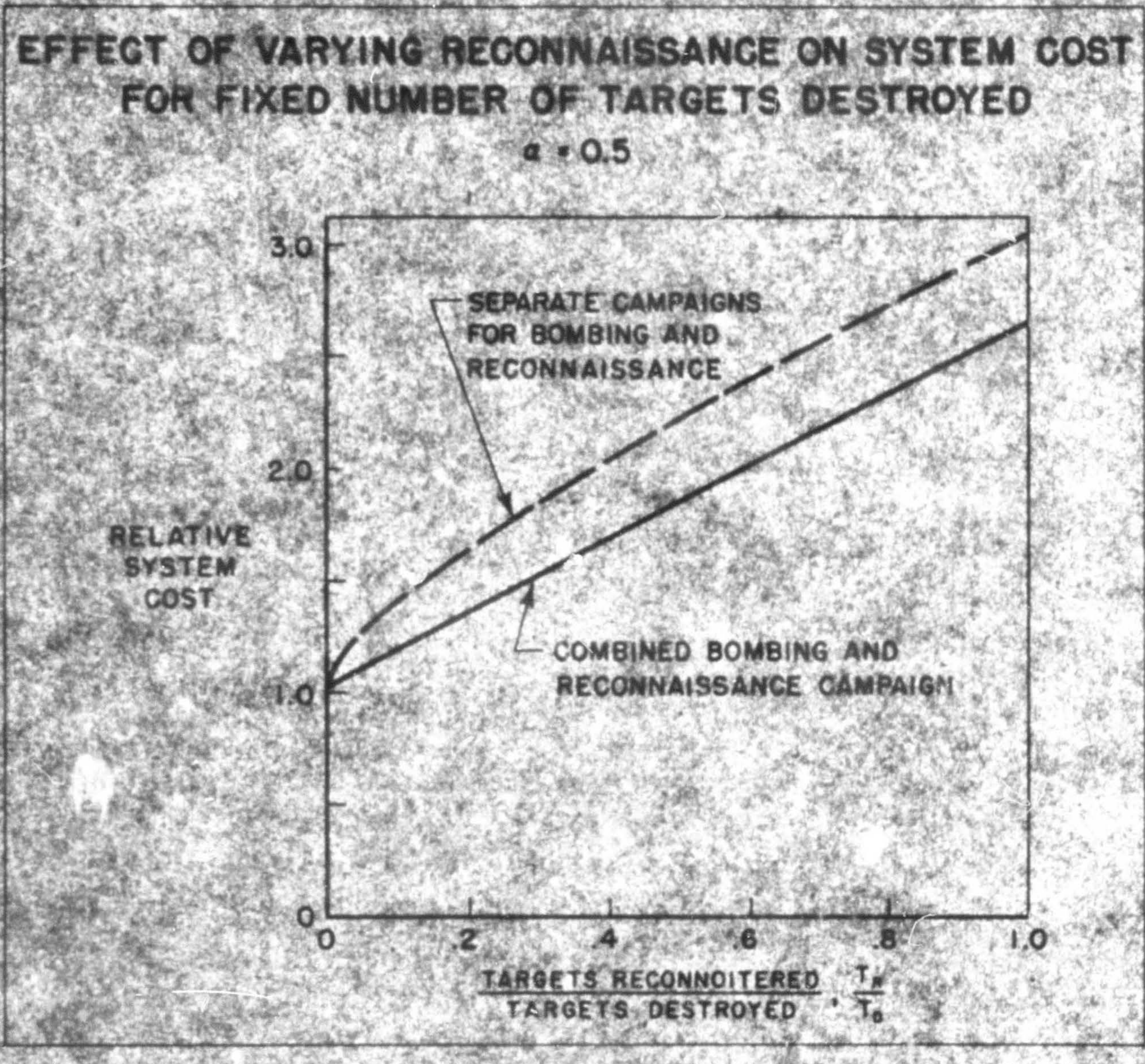

EFFECT OF VARYING RECONNAISSANCE ON SYSTEM COST
FOR FIXED NUMBER OF TARGETS DESTROYED
α = 0.5
3.0
2.0
1.0
0
RELATIVE
SYSTEM
COST
SEPARATE CAMPAIGNS
FOR BOMBING AND
RECONNAISSANCE
COMBINED BOMBING AND
RECONNAISSANCE CAMPAIGN
0
.2
.4
.6
.8
1.0
TARGETS RECONNOITERED / TARGETS DESTROYED, T_R / T_D

Fig. 5

difference between these campaigns is that different definitions for the campaign magnitude parameter μ are used for the reconnaissance and bombing campaigns, in accordance with equations (10) and (3). For the calculation of the combined bombing-reconnaissance campaign the equations and graphs of the preceding section are also applicable when the following modifications are made:

the campaign magnitude parameter μ is redefined as

$$\mu = \frac{T_D}{T_o^* \; P_d \cdot \tau} \quad , \tag{10a}$$

where the factor τ is given by

$$\tau = \frac{1}{1 + \left(\frac{P_d}{P_r}\right) \cdot \left(\frac{T_R}{T_D}\right)} \quad ; \tag{24}$$

the unit cost γ occurring in equations (12) and (4) is replaced by an "effective average" unit cost, γ_e , where

$$\gamma_e = \tau \, \gamma_B + (1-\tau) \, \gamma_R \quad , \tag{25}$$

γ_B and γ_R being the unit costs of bombers and reconnaissance aircraft, respectively. The factor τ defined by equation (24) above has a simple physical meaning; τ is the fraction of the T targets "visited" on a strike, which is attacked by bombers, the remaining $(1-\tau)T$ targets being visited by reconnaissance aircraft. This deviation is based on the implicit assumption that bomber and reconnaissance aircraft, though perhaps differing in unit cost, are similar type aircraft whose attrition characteristics are substantially the same.

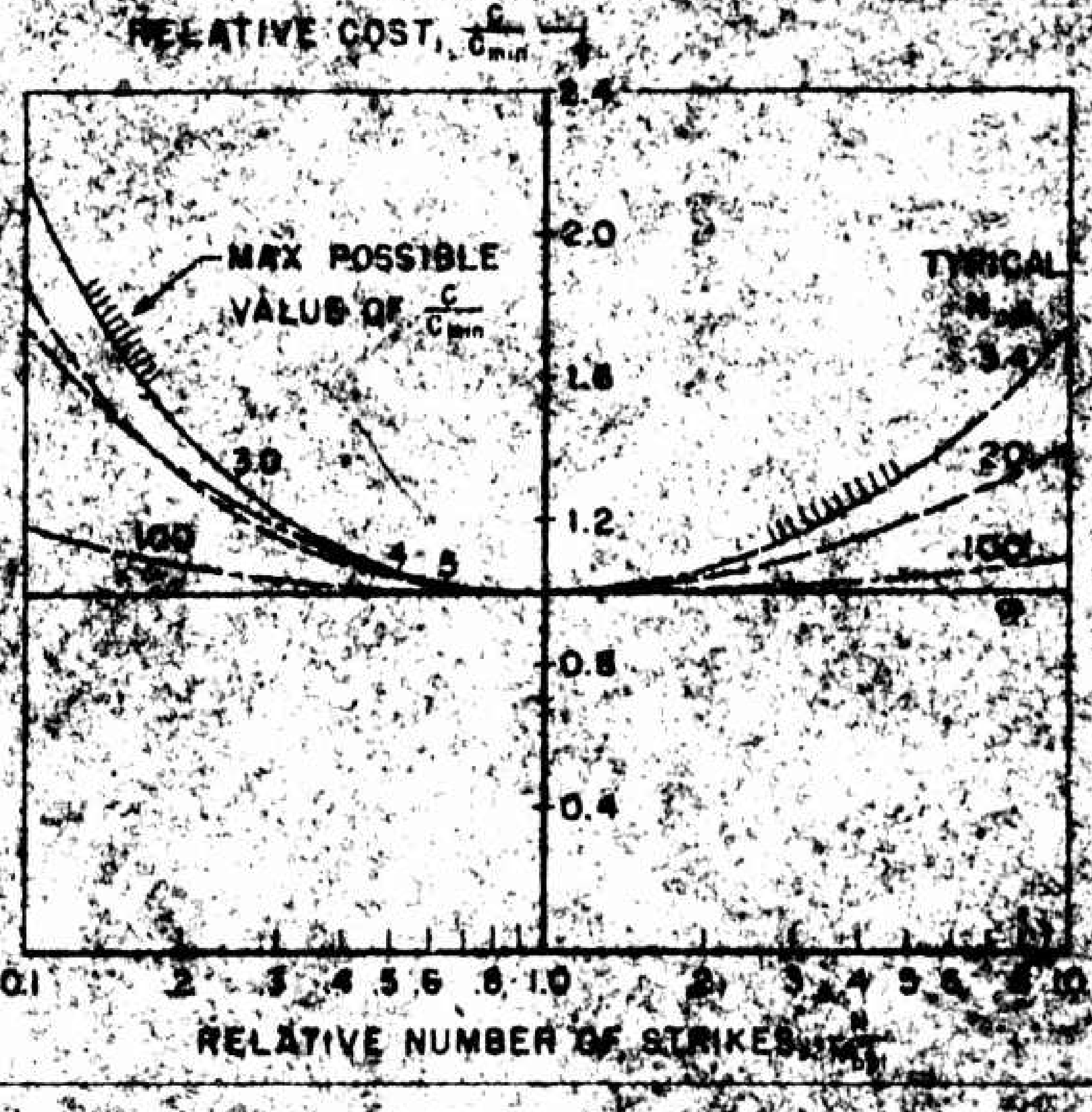
Another important group of applications of the generalized campaign analysis method are sensitivity investigations such as are generally required to determine the effects on campaign results of a specified departure from one or more arbitrary assumptions of the analysis. As an example of this, the generalized relations of the preceding section have been applied to an investigation of the sensitivity of campaign cost to the number of strikes used for a campaign to destroy a specified number of strategic targets. The results are summarized in Fig. 6. The abscissa represents the arbitrary number of strikes (N) to which a campaign is restrained divided by the "optimum" number of strikes associated with the minimum cost campaign. The ordinate is the campaign cost C for an arbitrary number of strikes, divided by minimum campaign cost. The uppermost curve of Fig. 6 represents for any value of N/N_{opt} the greatest possible campaign cost, relative to C_{min}, for the particular area defense behavior considered, $\alpha = 0.5$. Figure 6 shows that campaign cost is relatively insensitive to deviations in the number of strikes from the optimum value. For example, if the number of strikes is increased or decreased from N_{opt} by a factor of 3, the campaign cost is increased above the minimum value by at most 20 per cent. Figure 6 shows that the precise magnitude of this sensitivity depends on the value of the parameter N_{opt} characterizing a particular campaign.* In particular, the greater the number of strikes

*The mathematically significant parameter defining the several curves of Fig. 6 is N_{opt}/j rather than N_{opt} itself, as may be inferred from equation (18). In Fig. 6 a typical value of $j = 3.4$ was chosen to indicate the magnitudes of N involved in typical campaigns. Figure 6 can thus be used for other values of j by relabeling the curves.

EFFECT OF NUMBER OF STRIKES ON SYSTEM COST

150 — STRIKE BOMBING CAMPAIGNS

d = 0.5



which is optimum, the less the percentage increase in cost when the number of strikes is changed from optimum by a specified factor.

One conclusion to be drawn from Fig. 6 is that minimum cost iso-strike campaigns may in many cases be used as adequate approximation for campaigns in which the number of strikes is constrained for operational or tactical reasons to some specified number, which is non-optimal from the economic point of view. However as indicated by Table 1, the generalized method can also be used to derive campaign equations which apply precisely to such campaigns with the number of strikes constrained to a specified value.

CONCLUDING REMARKS

This brief paper does not permit the discussion of other interesting applications of this generalized campaign analysis method. Nevertheless, it is hoped that the preceding illustrations have served to demonstrate these principal features of generalized campaign analysis:

1. The generalized equations and graphs lead to a better understanding of the principal characteristics of strategic campaigns than can be gleaned from an essentially empirical study of many individual campaigns.
2. Rapid campaign calculation is achieved by means of computation of a small number of quasi-universal curves representing an infinity of individual campaigns.
3. The methods described may be used effectively in sensitivity investigations to determine the effect of variations in certain assumptions on the campaign results. Thus, with a given expenditure of effort it is possible to obtain a broader coverage of such sensitivity investigations when the generalized method is used, than when assumptions are varied in a few specific sample campaigns.