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a digital computer program

for a generalized inertial

guidance system error analysis

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A DIGITAL COMPUTER PROGRAM FOR A GENERALIZED

INERTIAL GUIDANCE SYSTEM ERROR ANALYSIS

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R. A. Moore

D. F. Meronek

Approved: G. A. Harter, Manager
Inertial Guidance Department

Approved: W. T. Russell, Director
Electromechanical Laboratory

TABLE OF CONTENTS

L I S T O F F I G U R E S

A B S T R A C T

This paper considers the equations relating errors in inertial guidance system measurement instruments to errors in missile burnout position and velocity and consequently to target misses. The relationship is expressed in terms of three linear differential equations with trajectory-dependent coefficients and forcing functions which depend on the measurement instrument errors. The forcing functions are presented in forms sufficiently general to accomodate errors that might arise in widely different types of instruments, both accelerometers and gyroscopes. Furthermore, the forcing functions can be evaluated for arbitrary instrument orientations. Programmed on a digital computer, these equations allow swift computation of target misses for given component tolerances and component orientation and consequently facilitate the optimization of instrument orientation. Such a program has been used extensively by the authors for gyro orientation schemes and general error studies for a number of ballistic missile inertial guidance systems.

A DIGITAL COMPUTER PROGRAM FOR A GENERALIZED

INERTIAL GUIDANCE SYSTEM ERROR ANALYSIS

1. INTRODUCTION

The purpose of this report is to describe and explain the use of a digital computer program which is, at present, on the 1103A. This program relates inertial guidance system component errors to errors in position and velocity at burnout and consequently to target misses. The program has two prime uses, one being to perform complete guidance system error analyses and the other being to study the optimization of instrument orientations. In the past, error analyses and design studies for inertial guidance systems have been performed either by hand or on an analogue computer and for specific instrument (accelerometer and gyros) configurations. Aside from the obvious need for speed and permanency, the need was felt for a program in which the forms of the error functions arising from the instrument errors were sufficiently general to accomodate many different instrument configurations (particularly gyros) where the instruments are arbitrarily oriented with respect to a stable platform. It is felt that this program is sufficiently general to satisfy this need. The authors wish to extend their thanks to Messrs. Frank Meek and Vilas Henderson for their very capable efforts in setting the problem up on the computer.

The inputs to the program are the missile trajectory and the corresponding miss coefficients; and the inertial component errors such as accelerometer errors, gyro drifts, platform misalignments as well as the appropriate angles describing the orientations of the instruments. The outputs are the position and velocity errors at burnout and the corresponding target misses. Section 5 is concerned with a description of the use of the program, particularly the input of instrument errors. Sections 3 and 4 give the derivations of the equations relating burnout position

and velocity errors to component errors and the derivation of the forms of the forcing functions representing the instrument errors. Before deriving the error equations, the coordinate systems used will be discussed in Section 2.

2. COORDINATE SYSTEMS

There are three basic coordinate systems used in this problem. Two of them are represented in the diagram below: the XYZ system which is an earth-centered, inertial coordinate system (ECI) and the xyz system which is also inertial, but centered at the missile launch point at the time of launch. The XYZ system is such that the Z axis is the earth's spin axis and the X axis is such that the X-Z plane contains the launch meridian at the time of launch. The Y axis completes the orthogonal set. In the xyz system, the z axis is the local vertical at the launch point at the time of launch, the x axis is down range in the direction of the target, and the y axis is the lateral axis normal to the x-z plane and completes the right-handed system. The angle A is the firing azimuth measured southward from the launch meridian to the x axis. ϕ_L^* and ϕ_L are, respectively, the geodetic and geocentric latitudes of the launch point. The quantities a and b are, respectively, the semi-major and semi-minor axes of the earth, e the eccentricity, and R_o the distance from the center of the earth to the launch point.

$$\phi_L^* = \tan^{-1} \left(\frac{\tan \phi_L}{1 - e^2} \right)$$

$$\delta = \phi_L^* - \phi_L$$
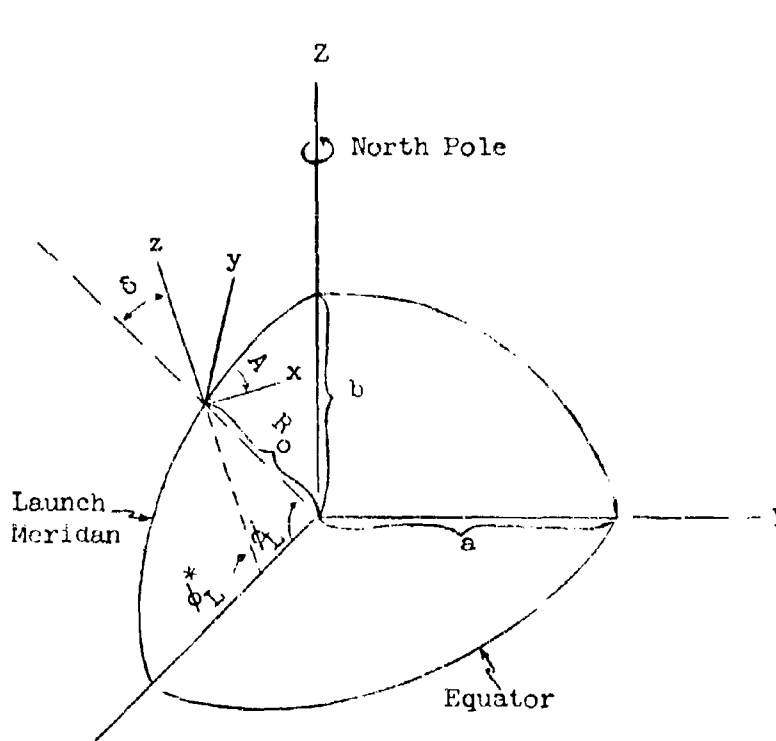
$$R_o = \frac{a (1 - e)}{\sqrt{1 - (2e - e^2) \cos^2 \phi_L}}$$

The coordinate transformation relating the XYZ and the xyz systems is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + C \qquad (2.1)$$

where M is the rotation matrix.

$$M = \begin{bmatrix} -\cos A \sin \phi_L^* & \sin A & \cos A \cos \phi_L^* \\ -\sin A \sin \phi_L^* & -\cos A & \sin A \cos \phi_L^* \\ \cos \phi_L^* & 0 & \sin \phi_L^* \end{bmatrix} \qquad (2.2)$$

The translation vector C is

$$C = R_o \begin{bmatrix} \sin \delta \cos A \\ \sin \delta \sin A \\ -\cos \delta \end{bmatrix} \qquad (2.3)$$

The third coordinate system is the accelerometer coordinate system x_A, y_A, z_A. The axes of this system are the axes along which the range, lateral, and pitch accelerometers' sensitive axes are pointed. The accelerometer coordinate axes are obtained by rotating first about the z axis through an angle ξ_1, which defines an intermediate system $x_1, y_1, z_1 = z$. The axes are then rotated about y_1 through an angle ξ_2 which defines an intermediate system $x_2, y_2 = y_1, z_2$. The final rotation is through the angle ξ_3 about the $x_2 = x_A$ axis.
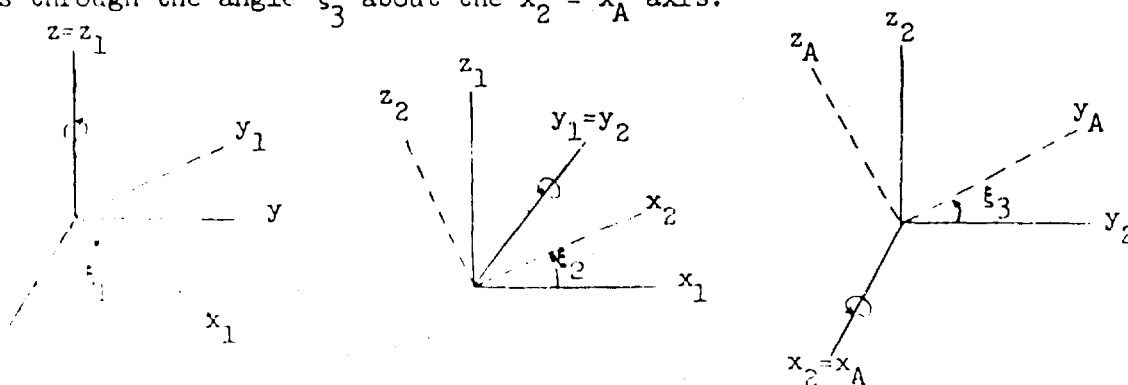
Accelerometer Coordinate System Axes

The coordinate transformation relating the accelerometer coordinate system x_A, y_A, z_A to the xyz system is

$$\begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} = Q \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad (2.4)$$

where the matrix of rotation is

$$Q = \begin{bmatrix} C_1 C_2 & C_2 S_1 & S_2 \\ -C_3 S_1 - C_1 S_2 S_3 & C_1 C_3 - S_1 S_2 S_3 & C_2 S_3 \\ S_1 S_3 - C_1 C_3 S_2 & -C_1 S_3 - C_3 S_1 S_2 & C_2 C_3 \end{bmatrix}$$

where $C_i = \cos \xi_i$ and $S_i = \sin \xi_i$; $i = 1, 2$ or 3. Thus, the accelerometers may be oriented at arbitrary angles to the xyz system. For convenience, the terms range, lateral, and pitch accelerometers are retained.
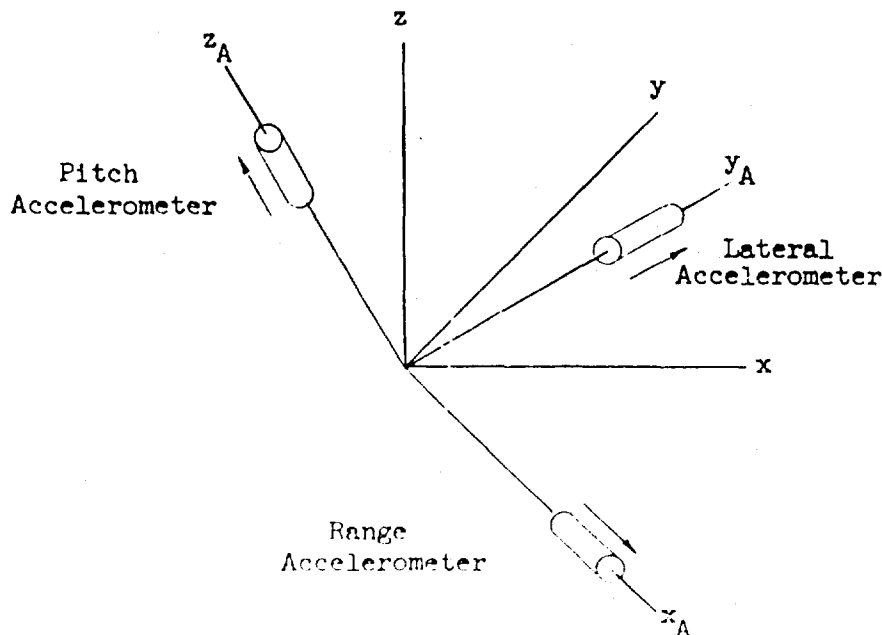
Accelerometer Orientation

x_A, y_A, z_A

An optional means of rotating the accelerometers has been provided in which the rotations are defined as in Figure 2.04.

(2.4)

First rotation: Θ_2 about y axis.
Second rotation: Θ_1 about z_A axis.
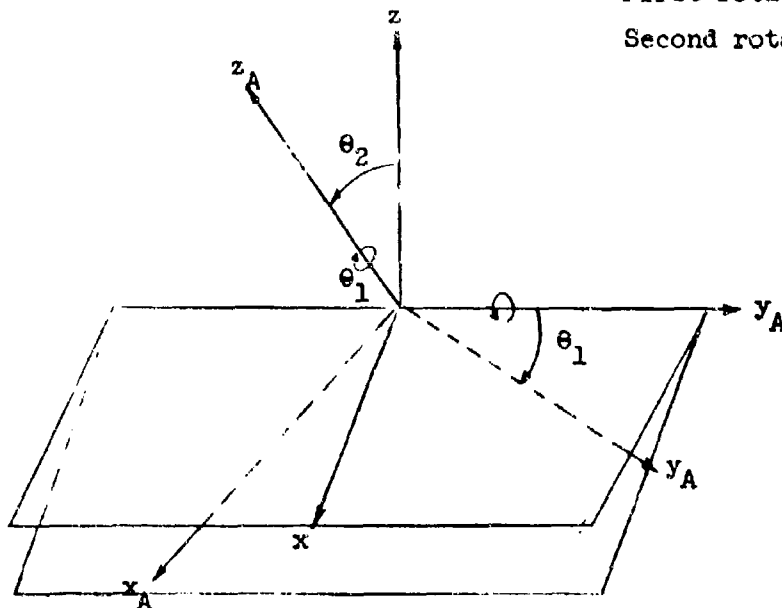
FIGURE 2.04   Optional Accelerometer Orientation

Note that the first rotation has been defined Θ_2. The matrix of rotations is

$$Z = \begin{bmatrix} \cos \Theta_1 \cos \Theta_2 & -\sin \Theta_1 & -\cos \Theta_1 \sin \Theta_2 \\ \sin \Theta_1 \cos \Theta_2 & \cos \Theta_1 & -\sin \Theta_1 \sin \Theta_2 \\ \sin \Theta_2 & 0 & \cos \Theta_2 \end{bmatrix}$$

The first rotation has been defined to be Θ_2 because by setting $\xi_1 = \Theta_1$, $\xi_2 = \Theta_2$, $\xi_3 = 0$ in Q (equation 2.5), and taking the transpose of Q, namely Q', one finds $Z = Q'$. The computer has been programmed to take advantage of this fact.

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The orientation of the gyroscopes with respect to the accelerometer and plat-form coordinate systems will be discussed in Section 4, which is concerned with the forcing functions of the error equations.

3. ERROR EQUATIONS

In the XYZ system let A_I represent the vector of missile inertial accelerations

$$A_I = \begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} \tag{3.1}$$

and let A_{Ia} represent the vector of sensed accelerations

$$A_{Ia} = \begin{bmatrix} a_{Xa} \\ a_{Ya} \\ a_{Za} \end{bmatrix} \tag{3.2}$$

Futher, let P_I denote the missile position vector

$$P_I = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{3.3}$$

and let G denote the gravity vector

$$G = \left(- g_0 \frac{R_o^2}{R^3} \right) P_I = - g_0 \frac{R_o^2}{R^3} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{3.4}$$

where g_o is gravitational acceleration at the launch site, R_o is the distance from the center of the earth to the launch site, and where R is the distance from the center of the earth to the missile. Thus,

$$R^2 = X^2 + Y^2 + Z^2 = P_I' \, P_I \tag{3.5}$$

where P_I' denotes the transpose of the vector P_I.
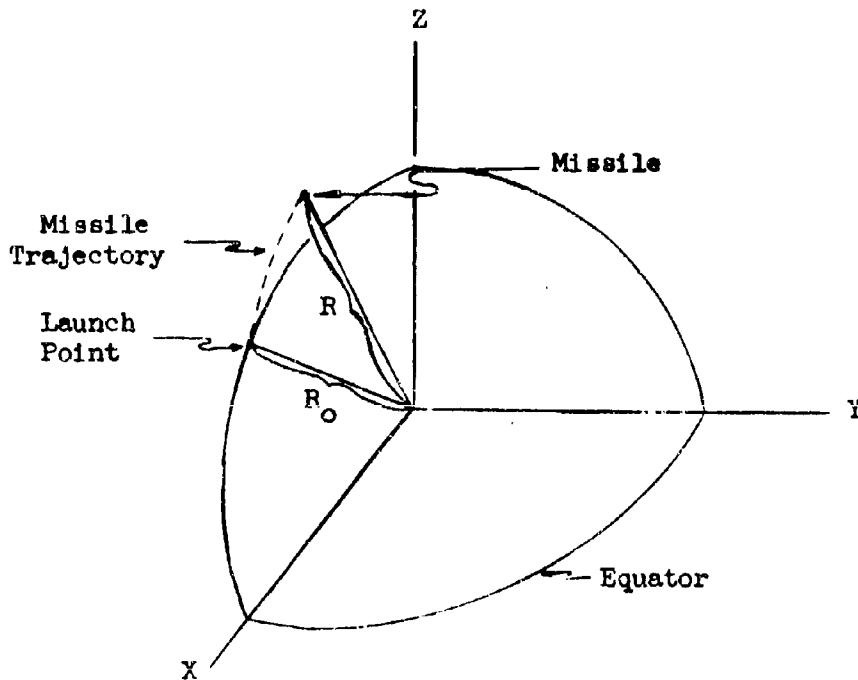
FIGURE 3.01   M ssile Trajectory

Then, from the relationship

$$A_I = A_{Ia} + G \tag{3.6}$$

one obtains

$$\Delta A_I = \Delta A_{Ia} + \Delta G \qquad (3.7)$$

where the errors in inertial accelerations are

$$\Delta A_I = \begin{bmatrix} \Delta \ddot{X} \\ \Delta \ddot{Y} \\ \Delta \ddot{Z} \end{bmatrix} \qquad (3.8)$$

and where the errors in sensed accelerations are

$$\Delta A_{Ia} = \begin{bmatrix} \Delta a_{Xa} \\ \Delta a_{Ya} \\ \Delta a_{Za} \end{bmatrix} \qquad (3.9)$$

The quantity ΔG, the error in the gravitational vector, is

$$\Delta G = (3 g_o \frac{R_o^2}{R^4} \Delta R) P_I - (g_o \frac{R_o^2}{R^3}) \Delta P_I \qquad (3.10)$$

where

$$\Delta P_I = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \qquad (3.11)$$

Equation (3.10) may be written as

$$\Delta G = (3 g_o \frac{R_o^2}{R^5}) (R \Delta R) P_I - (g_o \frac{R_o^2}{R^3}) \Delta P_I \qquad (3.12)$$

the expressions in brackets all being scalars. The scalar $R\Delta R$ may be written as

$$R\Delta R = X\Delta X + Y\Delta Y + Z\Delta Z = P_I{}'\Delta P_I \tag{3.13}$$

Substituting (3.13) into (3.12) yields

$$\Delta G = (3g_0 \ \frac{R_0{}^2}{R^5} \ P_I{}'\Delta P_I) \ P_I - (g_0 \ \frac{R_0{}^2}{R^3}) \ \Delta P_I \tag{3.14}$$

Substituting (3.14) into (3.7) yields the error equations in vector form

$$\Delta A_I + (g_0 \ \frac{R_0{}^2}{R^3}) \ \Delta P_I - (3g_0 \ \frac{R_0{}^2}{R^5} \ P_I{}'\Delta P_I) \ P_I = \Delta A_{Ia} \tag{3.15}$$

This equation written in component form is

$$\Delta\ddot{X} + g_0 \ \frac{R_0{}^2}{R^3} \ \Delta X - 3g_0 \ \frac{R_0{}^2}{R^5} \ X(X\Delta X + Y\Delta Y + Z\Delta Z) = \Delta a_{Xa}$$

$$\Delta\ddot{Y} + g_0 \ \frac{R_0{}^2}{R^3} \ \Delta Y - 3g_0 \ \frac{R_0{}^2}{R^5} \ Y(X\Delta X + Y\Delta Y + Z\Delta Z) = \Delta a_{Ya} \tag{3.16}$$

$$\Delta\ddot{Z} + g_0 \ \frac{R_0{}^2}{R^3} \ \Delta Z - 3g_0 \ \frac{R_0{}^2}{R^5} \ Z(X\Delta X + Y\Delta Y + Z\Delta Z) = \Delta a_{Za}$$

or three simultaneous linear second-order equations which may be solved to yield the relationship between velocity and position errors $(\Delta\dot{X}, \Delta\dot{Y}, \Delta\dot{Z})$ and $(\Delta X, \Delta Y, \Delta Z)$ and the errors in sensed accelerations. The coefficients in the differential equation are trajectory-dependent, i.e., R,X,Y and Z depend on the trajectory.

As will be seen in the next section on forcing functions, the errors Δa_{Xa}, Δa_{Ya} and Δa_{Za} are also trajectory-dependent for the most part.

In the actual computer program the equations are solved in the earth-centered inertial coordinate system XYZ. The position and velocity errors resulting from the solution are printed out in the XYZ system, the xyz or platform system, and the x_A, y_A, z_A or accelerometer system. The target misses are also printed out, but of course these are independent of the coordinate system. In actuality, the misses are computed in the XYZ system, the miss coefficients having been appropriately transformed from the xyz system to the XYZ system.

Since the sensed accelerations are measured in the x_A, y_A, z_A system, it is of interest to point out the relationship between the sensed acceleration errors ΔA_{Ia} in the XYZ system and the sensed acceleration error ΔA_{aA} in the accelerometer system. It is, by virtue of relations (2.1) and (2.4),

$$\Delta A_{aA} = QM\Delta A_{Ia} \tag{3.1}$$

from which it follows that

$$\Delta A_{Ia} = M'Q'\Delta A_{aA} \tag{3.1}$$

since the inverse of QM, viz. $(QM)^{-1} = (QM)' = M'Q'$ where the prime denotes transpose. Thus, equation (3.15) becomes

$$\Delta A_I + \left(g_o \frac{R_o^2}{R^3}\right) \Delta P_I - \left(3g_o \frac{R_o^2}{R^5} P_I'\Delta P_I\right) P_I = M'Q'\Delta A_{aA} \tag{3.1}$$

Equation (3.19) then expresses the relationship between the position and velocity errors in the XYZ system and errors in sensed acceleration in the accelerometer coordinate system. The next section is concerned with the form of ΔA_{aA} and its relation to the trajectory being used.

4. FORCING FUNCTIONS

The term forcing function will be applied to the vector $M'Q'\Delta A_{aA}$ and denoted by the vector F. Thus,

$$F = M'Q'\Delta A_{aA} \tag{4.1}$$

The errors in sensed acceleration in the platform system, ΔA_{aA}, may be expressed as

$$\Delta A_{aA} = E_A + \Phi A_{aA} \tag{4.2}$$

where E_A is the vector of errors arising due to accelerometer inaccuracies such as bias and scale factor. The term ΦA_{aA} represents acceleration errors due to angular errors in the platform. Φ represents the platform angle matrix, and ΦA_{aA} expresses the spurious accelerations sensed by the accelerometers due to the platform misorientation resulting from either initial platform misalignment or from gyro drift. The Φ matrix is

$$\Phi = \begin{bmatrix} 0 & \phi_z & -\phi_y \\ -\phi_z & 0 & \phi_x \\ \phi_y & -\phi_x & 0 \end{bmatrix} \tag{4.3}$$

where the angles ϕ_x, ϕ_y and ϕ_z represent rotations about the x_A, y_A and z_A axes, respectively.

From (2.1) and (2.4) it is seen that A_{aA} may be expressed as

$$A_{aA} = QMA_{Ia} \tag{4.4}$$

but from (3.6) $A_I = A_{Ia} + G$ so that

$$A_{aA} = QM (A_I - G) \tag{4.5}$$

Thus, from (4.1) and (4.2) above, the forcing function F may be expressed as

$$F = M'Q'E_A + M'Q'\bar{\Phi}QM (A_I - G) \tag{4.6}$$

Section 5 is concerned with the general forms of E_A and $\bar{\Phi}$ due to accelerometer errors, platform angles, and gyro drifts. Before proceeding to Section 5, it will be helpful to describe the gyro and accelerometer axes.

Two two-degree-of-freedom gyros have been assumed and their orientation restricted to cause the three input axes to form an orthogonal set. The axes h_1, f_1, f_2 are the axes of the number 1 gyro with h_1 the input axis. The axes h_2, h_3, f_3 refer to the number 2 gyro with h_2 and h_3 the input axes. With respect to the x_A, y_A, z_A system and consequently with respect to the xyz system, the gyros may be arbitrarily oriented, subject only to the constraint as mentioned above, so that the three input axes form an orthogonal set. Figures 4.01 and 4.02 illustrate these orientations.

GYRO NUMBER 1
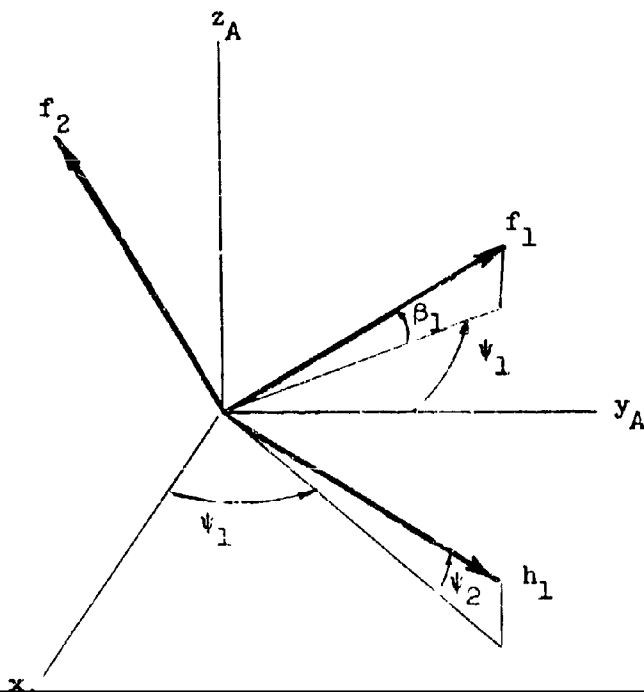
angle	about	when
ψ_1	z_A axis	$\psi_2 = \beta_1 = 0$
ψ_2	f_1 axis	$\beta_1 = 0$
β_1	h_1 axis	

GYRO NUMBER 2
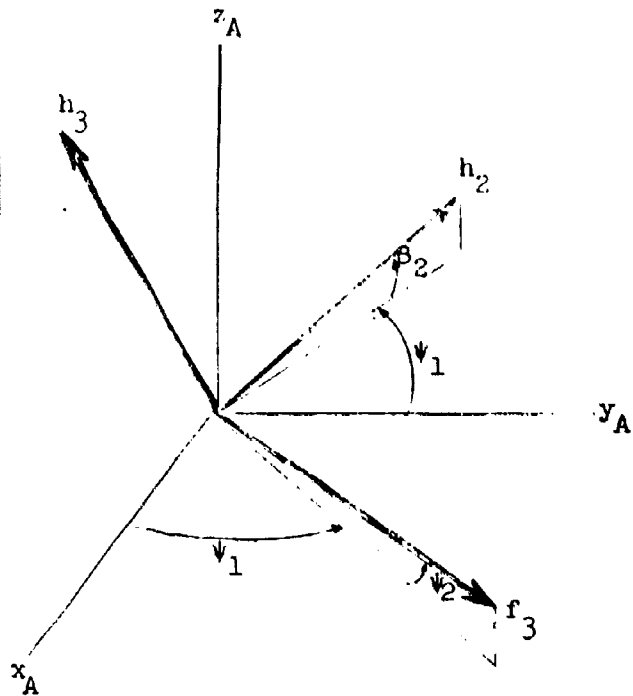
angle	about		when
ψ_1	z_A	axis	$\psi_2 = \beta_2 = 0$
ψ_2	h_2	axis	$\beta_2 = 0$
β_2	f_3	axis	

FIGURE 4.02   Gyro Orientation

Thus, the orthogonal set h_1, h_2, h_3 is related to the x_A, y_A, z_A system by the rotation matrix.

$$R = \begin{bmatrix} \cos\psi_2\cos\psi_1 & \cos\psi_2\sin\psi_1 & \sin\psi_2 \\ \begin{array}{l}(-\cos\beta_2\sin\psi_1 \\ -\sin\beta_2\sin\psi_2\cos\psi_1)\end{array} & \begin{array}{l}(\cos\beta_2\cos\psi_1 \\ -\sin\beta_2\sin\psi_2\sin\psi_1)\end{array} & \sin\beta_2\cos\psi_2 \\ \begin{array}{l}(\sin\beta_2\sin\psi_1 \\ -\cos\beta_2\sin\psi_2\cos\psi_1)\end{array} & \begin{array}{l}(-\sin\beta_2\cos\psi_1 \\ -\cos\beta_2\sin\psi_2\sin\psi_1)\end{array} & \cos\beta_2\cos\psi_2 \end{bmatrix} \qquad (4.7)$$
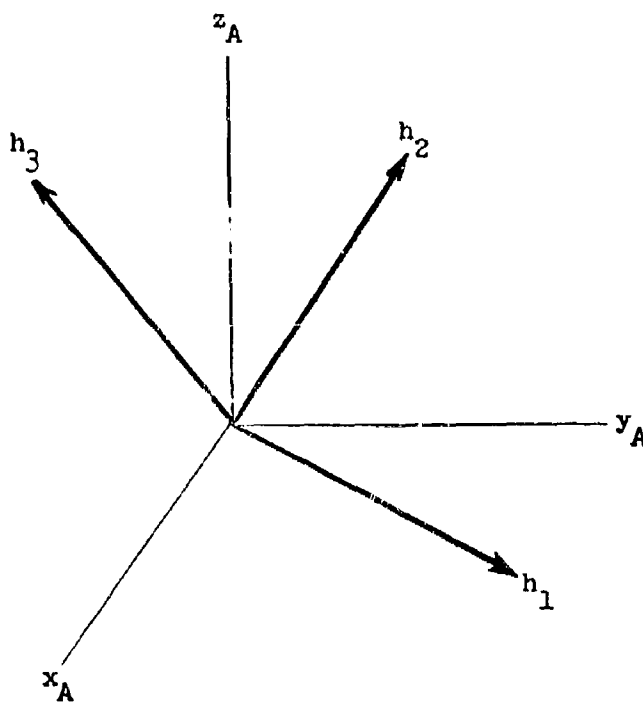
FIGURE 4.03   Gyro Orientation

Thus, rotations about the x_A, y_A, z_A axes, ϕ_x, ϕ_y, ϕ_z, are related to rotations due to gyro drifts about the h_1, h_2, h_3 axes, say ϕ_1, ϕ_2, ϕ_3, by the relationship

$$\begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} = R' \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \tag{4.8}$$

where R' is the transpose of (4.7). It can be shown that as a result of (4.8) the Φ matrix, (4.3), may be written as

$$\Phi = \begin{bmatrix} 0 & \phi_z & -\phi_y \\ -\phi_z & 0 & \phi_x \\ \phi_y & -\phi_x & 0 \end{bmatrix} = R' \begin{bmatrix} 0 & \phi_3 & -\phi_2 \\ -\phi_3 & 0 & \phi_1 \\ \phi_2 & -\phi_1 & 0 \end{bmatrix} R \tag{4.9}$$

Thus, the forcing function (4.6) may be written as

$$F = M'Q'E_A + M'Q'R'\phi_h RQM(A_I - G) \qquad (4.10)$$

where ϕ_h is the matrix in (4.9), viz.

$$\phi_h = \begin{bmatrix} 0 & \phi_3 & -\phi_2 \\ -\phi_3 & 0 & \phi_1 \\ \phi_2 & -\phi_1 & 0 \end{bmatrix} \qquad (4.11)$$

It should be pointed out that the gyro arrangement need not necessarily be two two-degree-of-freedom gyros. Any arrangement wherein there are drifts about three orthogonal gyro axes may be used. As a matter of fact, many cases of non-orthogonal axes may be treated by using one set of values of the angles in the matrix R for the number 1 gyro and in an independent computer run use another set of angles in R for the number 2 gyro.

Certain types of accelerometers and velocity meters are such that the plane in which the pendulous element moves and which contains the sensitive axis may be at any arbitrary angle with respect to the x_A, y_A, z_A system. Denote the axes of the meter itself as 1,2,3, where 1 is the sensitive axis and axes 1 and 2 define the plane of motion of the pendulous element. Axis 3 completes the orthogonal set and is the axis about which the pendulous element rotates

3

1 Sensitive Axis

Pendulous Element

2
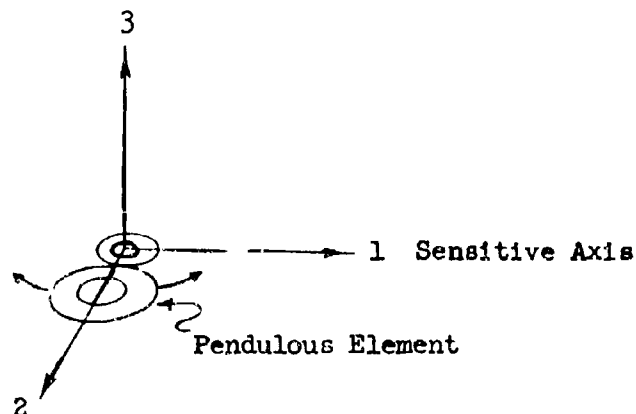
Figure 4.05 illustrates two possible orientations of the 2 and 3 axes for the pitch accelerometer.
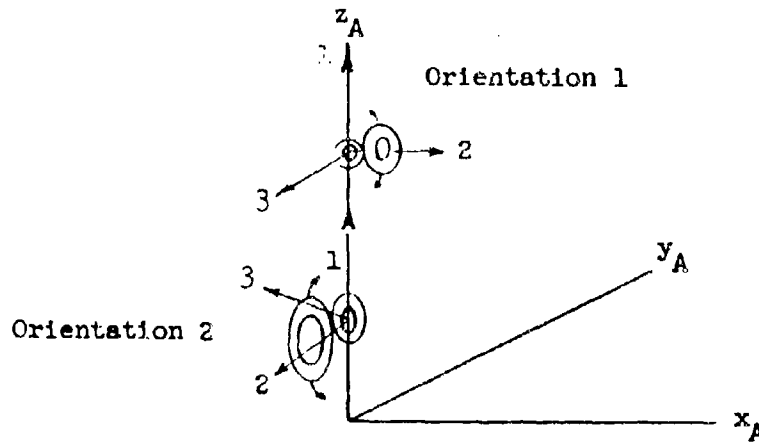
FIGURE 4.05   Accelerometer Alignment

Due to the physical nature of the meters and the unequal accelerations along the x_A, y_A, and z_A axes, there may be some preferred orientation which minimizes the cross-axis acceleration errors. For this reason, provisions in the program have been made for arbitrary orientations of the meters about the sensitive axes. The cross-axis accelerations (accelerations along axes 2 and 3) are

$$
\begin{bmatrix} a_{2x} \\ a_{2y} \\ a_{2z} \end{bmatrix} = \begin{bmatrix} 0 & \cos\theta_x & \sin\theta_x \\ -\sin\theta_y & 0 & -\cos\theta_y \\ -\sin\theta_z & \cos\theta_z & 0 \end{bmatrix} \begin{bmatrix} a_{xA} \\ a_{yA} \\ a_{zA} \end{bmatrix} \tag{4.12}
$$

$$
\begin{bmatrix} a_{3x} \\ a_{3y} \end{bmatrix} = \begin{bmatrix} 0 & \sin\theta_x & -\cos\theta_x \\ \cos\theta_y & 0 & -\sin\theta_y \end{bmatrix} \begin{bmatrix} a_{xA} \\ a_{yA} \end{bmatrix} \tag{4.13}
$$

where a_{2x} is the acceleration along axis 2 of the x or range accelerometer, a_{2y} the acceleration along axis 2 of the y or lateral accelerometer, and where Θ_x is a rotation about the sensitive axis of the range accelerometer, Θ_y is a rotation about the sensitive axis of the lateral accelerometer and Θ_z is a rotation about the sensitive axis of the pitch accelerometer. The angles are described in Figure 4.06. Although the accelerometers used here for illustration have a pendulous element moving in the 1-2 plane, the relationships given in (4.12) and (4.13) may be used for other types of instruments for the study of cross accelerations.
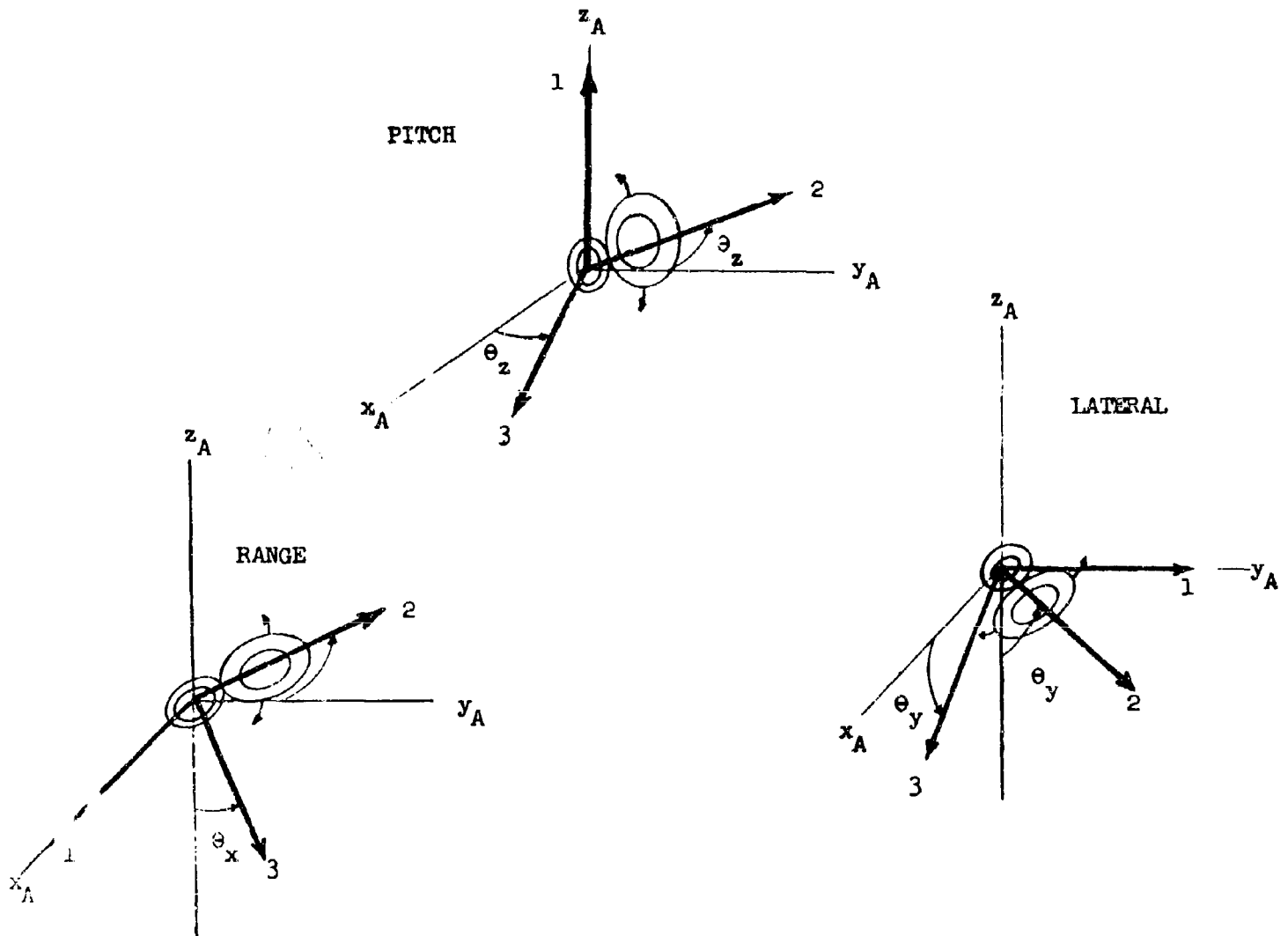
FIGURE 4.06   Accelerometer Alignment

The next section describes the forms of the elements of E_A and ϕ and ϕ_h in terms of missile accelerations and actual component tolerances and the manner in which they are identified and entered into the program.

5. SIMULATION OF ERROR SOURCES

The purpose of an inertial measurement unit is to provide, as its output, a true indication of any acceleration (excepting gravity) the unit is experiencing. To accomplish this, the function relating accelerometer output to input should be well known; and the orientation of each accelerometer, with respect to some coordinate system, should be well defined both in space and time. When these conditions are not satisfied an acceleration error appears, and it is the purpose of the vector

$$F = M'Q'E_A + M'Q'R'\phi_h RQM(A_I - G) \tag{5.1}$$

to evaluate this error.

Broadly speaking, acceleration errors can be catalogued into two classes. The errors arise because:

CLASS 1: The accelerometer output-input relation is not well known (e.g., scale factor uncertainty) or is non-linear, or the accelerometer is misaligned with respect to the platform.

CLASS 2: The stabilized platform on which the accelerometers rest is initially misaligned or has rotated from its correct orientation.

The two classes can be associated with the two parts of the vector F as was discussed in Section 4. Thus, to simulate Class 1 errors E_A must be evaluated; while for Class 2 errors ϕ_h must be formed. The rest of this section will be concerned with the simulation of each class with attention being focused first on Class 1 errors.

Class 1 Errors

Error sources which fall in this category are:

(a) Accelerometer misalignments with respect to the platform.

(b) Uncertainty in zero setting (null shift or bias).

(c) Linear scale factor uncertainty.

(d) Non-linear terms in the input-output relation - proportional to square, cube, cross-products and absolute value of acceleration.

To simulate Class 1 errors the following general vector has been programmed for evaluating E_A.

$$E_A = M_{E1} + \sum_{i=2}^{11} M_{Ei} A_{Ei} \tag{5.2}$$

(5.1)

where the M_{Ei} are 3 x 3 matrices, and the A_{Ei} are vectors whose elements are functions of acceleration.

$$
E_A = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} E_{10} \\ E_{11} \\ E_{12} \end{bmatrix} + \begin{bmatrix} E_{20} & E_{21} & E_{22} \\ E_{23} & E_{24} & E_{25} \\ E_{26} & E_{27} & E_{28} \end{bmatrix} \begin{bmatrix} a_{xA} \\ a_{yA} \\ a_{zA} \end{bmatrix} + \begin{bmatrix} E_{30} & E_{31} & E_{32} \\ E_{33} & E_{34} & E_{35} \\ E_{36} & E_{37} & E_{38} \end{bmatrix} \begin{bmatrix} a_{xA}^2 \\ a_{yA}^2 \\ a_{zA}^2 \end{bmatrix}
$$

$$
+ \begin{bmatrix} E_{40} & E_{41} & E_{42} \\ E_{43} & E_{44} & E_{45} \\ E_{46} & E_{47} & E_{48} \end{bmatrix} \begin{bmatrix} a_{xA}^3 \\ a_{yA}^3 \\ a_{zA}^3 \end{bmatrix} + \begin{bmatrix} E_{50} & E_{51} & E_{52} \\ E_{53} & E_{54} & E_{55} \\ E_{56} & E_{57} & E_{58} \end{bmatrix} \begin{bmatrix} A_{E5} \end{bmatrix} \tag{5.3}
$$

$$
+ \begin{bmatrix} E_{60} & E_{61} & E_{62} \\ E_{63} & E_{64} & E_{65} \\ E_{66} & E_{67} & E_{68} \end{bmatrix} \begin{bmatrix} |a_{xA}| \cdot a_{xA} \\ |a_{yA}| \cdot a_{yA} \\ |a_{zA}| \cdot a_{zA} \end{bmatrix} + \begin{bmatrix} E_{70} & E_{71} & E_{72} \\ E_{73} & E_{74} & E_{75} \\ E_{76} & E_{77} & E_{78} \end{bmatrix} \begin{bmatrix} a_{2x} \cdot a_{xA} \\ a_{2y} \cdot a_{yA} \\ a_{2z} \cdot a_{zA} \end{bmatrix}
$$

Equation (5.3) continued

$$+ \begin{bmatrix} E_{80} & E_{81} & E_{82} \\ E_{83} & E_{84} & E_{85} \\ E_{86} & E_{87} & E_{88} \end{bmatrix} \begin{bmatrix} |a_{2x}| \cdot a_{xA} \\ |a_{2y}| \cdot a_{yA} \\ |a_{2z}| \cdot a_{zA} \end{bmatrix} + \begin{bmatrix} E_{90} & E_{91} & E_{92} \\ E_{93} & E_{94} & E_{95} \\ E_{96} & E_{97} & E_{98} \end{bmatrix} \begin{bmatrix} |a_{3x}| \cdot a_{xA} \\ |a_{3y}| \cdot a_{yA} \\ |a_{3z}| \cdot a_{zA} \end{bmatrix}$$

$$+ \begin{bmatrix} E_{100} & E_{101} & E_{102} \\ E_{103} & E_{104} & E_{105} \\ E_{106} & E_{107} & E_{108} \end{bmatrix} \begin{bmatrix} A_{E10} \\ \\ \end{bmatrix} + \begin{bmatrix} E_{110} & E_{111} & E_{112} \\ E_{113} & E_{114} & E_{115} \\ E_{116} & E_{117} & E_{118} \end{bmatrix} \begin{bmatrix} A_{E11} \\ \\ \end{bmatrix}$$

a_{xA}, a_{yA}, a_{zA} are range, lateral, and pitch accelerometer sensed accelerations; and $a_{2x}, a_{2y}, a_{2z}, a_{3x}, a_{3y}, a_{3z}$ are defined in equations (4.12) and (4.13) with a_{xA}, a_{yA}, a_{zA} again defined to be sensed accelerations. Note that provision has been made for simulating any function of acceleration by leaving A_{E5}, A_{E10}, A_{E11} undefined. Thus, regardless of the error source, if it can be expressed as a function of acceleration it can be simulated in this program.

By use of this general vector the error sources listed above can be simulated in the following ways.

a. Accelerometer Misalignments with Respect to the Platform

Figure 5.01 shows the misorientations of the accelerometers that are to be simulated. β_{xy} is the angular rotation, in radians, of the x accelerometer toward the y_A axis, β_{zx} is the rotation of the z accelerometer toward the x_A axis, and similarly for the others.
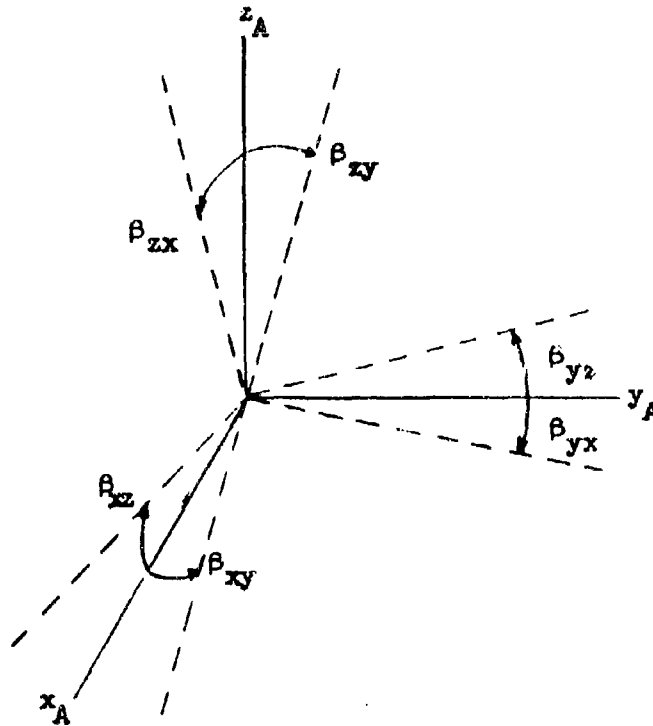
FIGURE 5.01   Accelerometer Misalignments

The acceleration errors resulting from such misorientations are simulated by the following.

$$E_A = M_{E2}A_{E2} = \begin{bmatrix} 0 & \beta_{xy} & \beta_{xz} \\ \beta_{yx} & 0 & \beta_{yz} \\ \beta_{zx} & \beta_{zy} & 0 \end{bmatrix} \begin{bmatrix} a_{xA} \\ a_{yA} \\ a_{zA} \end{bmatrix}$$

(5.4)

b. <u>Uncertainty in Zero Setting (Null Shift or Bias)</u>

If $\alpha_x, \alpha_y, \alpha_z$ represent the shift or bias in the zero settings of the three accelerometers, then the error in the indicated output can be represented by

$$E_A = M_{E1} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \tag{5.5}$$

c. <u>Linear Scale Factor Uncertainty</u>

Uncertainties in scale factor setting give rise to errors which can be simulated by setting the elements of M_{E2} to

$$M_{E2} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \tag{5.6}$$

then

$$E_A = M_{E2} A_{E2}$$

where $\epsilon_x, \epsilon_y, \epsilon_z$ are the uncertainties in the range, lateral, and pitch accelerometers, respectively.

d. <u>Non-linear Terms in the Input-Output Relation</u>

To illustrate the simulation of non-linearities, the hypothetical relation

$$a_{out} = a_A + K_1 a^3_A + K_2 |a_A| \cdot a_A \tag{5.7}$$

will be assumed for each accelerometer. Then

$$\Delta a_A = a_{out} - a_A = K_1 a_A^3 + K_2 |a_A| \cdot a_A \tag{5.8}$$

represents the error in the accelerometer output. Such errors can be simula-ed by setting

$$E_A = M_{E4} A_{E4} + M_{E6} A_{E6} = \begin{bmatrix} K_{1x} & 0 & 0 \\ 0 & K_{1y} & 0 \\ 0 & 0 & K_{1z} \end{bmatrix} \begin{bmatrix} a_{xA}^3 \\ a_{yA}^3 \\ a_{zA}^3 \end{bmatrix}$$

$$+ \begin{bmatrix} K_{2x} & 0 & 0 \\ 0 & K_{2y} & 0 \\ 0 & 0 & K_{2z} \end{bmatrix} \begin{bmatrix} |a_{xA}| \cdot a_{xA} \\ |a_{yA}| \cdot a_{yA} \\ |a_{zA}| \cdot a_{zA} \end{bmatrix}$$

where K_{1i}, K_{2i} are the coefficients for the ith accelerometer. More examples of such simulations will be presented in Section 6.

In all of these illustrations, in fact whenever the vector E_A is being used, it is necessary to specify the angles ξ_1, ξ_2, ξ_3, or Θ_1, Θ_2, depending on the option being used to specify accelerometer orientation (see Section 2).

Class 2 Errors

Error sources considered to fall in this category are:

(a) Initial platform misalignments.

(b) Platform rotations due to platform elasticity.

(c) Platform rotations due to constant or acceleration-dependent gyro drift rates.

When platform rotations are considered to be small, the acceleration error can be found from

$$\Delta \bar{A} = - \bar{\phi} \times \bar{A} \tag{5.10}$$

where $\bar{\phi}$ denotes the angular rotation vector of the stabilized platform with respect to its proper orientation. Expanding the vector cross-product in a coordinate system defined by the axes h_1, h_2, h_3 (see Figure 4.03), and putting the result into matrix form, yields

$$\Delta A_h = \bar{\Phi}_h A_h = \bar{\Phi}_h R A_A = \bar{\Phi}_h R Q M (A_I - G) \tag{5.11}$$

where $\bar{\Phi}_h$ is defined by equation (4.11) and R, defined by equation (4.7), rotates accelerations along accelerometer coordinates into the h_1, h_2, h_3 axes. Multiplying by R' rotates the acceleration errors into accelerometer coordinates, and multiplying by M'Q' completes the rotation into ECI coordinates in which the differential equations are solved. To simulate Class 2 errors the rotation vector, $\bar{\phi}$, must be evaluated. To accomplish this, a general vector G has been programmed from which $\bar{\phi}$ is evaluated by either of two options.

Option 1:

$$\bar{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = G = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \tag{5.12}$$

Option 2:

$$\bar{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \int_0^t G \, dt = \begin{bmatrix} \int_0^t G_1 \, dt \\ \int_0^t G_2 \, dt \\ \int_0^t G_3 \, dt \end{bmatrix} \tag{5.13}$$

$$G = M_{G1} + \sum_{i=2}^{7} M_{Gi} A_{Gi} \qquad (5.14)$$

(5.10)

in which the M_{Gi} are 3 x 3 matrices and the A_{gi} are vectors whose elements are functions of acceleration.

(5.11)

$$G = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} = \begin{bmatrix} G_{10} \\ G_{11} \\ G_{12} \end{bmatrix} + \begin{bmatrix} G_{20} & G_{21} & G_{22} \\ G_{23} & G_{24} & G_{25} \\ G_{26} & G_{27} & G_{28} \end{bmatrix} \begin{bmatrix} a_{f3} \\ a_{h2} \\ a_{h3} \end{bmatrix} + \begin{bmatrix} G_{30} & G_{31} & G_{32} \\ G_{33} & G_{34} & G_{35} \\ G_{36} & G_{37} & G_{38} \end{bmatrix} \begin{bmatrix} a_{h1} \\ a_{f1} \\ a_{f2} \end{bmatrix}$$

1

$$+ \begin{bmatrix} G_{40} & G_{41} & G_{42} \\ G_{43} & G_{44} & G_{45} \\ G_{46} & G_{47} & G_{48} \end{bmatrix} \begin{bmatrix} a_{f3} \cdot a_{h2} \\ a_{h2} \cdot a_{h3} \\ a_{h3} \cdot a_{f3} \end{bmatrix} + \begin{bmatrix} G_{50} & G_{51} & G_{52} \\ G_{53} & G_{54} & G_{55} \\ G_{56} & G_{57} & G_{58} \end{bmatrix} \begin{bmatrix} a_{h1} \cdot a_{f1} \\ a_{f1} \cdot a_{f2} \\ a_{f2} \cdot a_{h1} \end{bmatrix} \qquad (5.15)$$

(5.12)

$$+ \begin{bmatrix} G_{60} & G_{61} & G_{62} \\ G_{63} & G_{64} & G_{65} \\ G_{66} & G_{67} & G_{68} \end{bmatrix} \begin{bmatrix} A_{G6} \end{bmatrix} + \begin{bmatrix} G_{70} & G_{71} & G_{72} \\ G_{73} & G_{74} & G_{75} \\ G_{76} & G_{77} & G_{78} \end{bmatrix} \begin{bmatrix} A_{G7} \end{bmatrix}$$

As in the vector E_A, provision has been made for simulating any function of acceleration desired by leaving A_{G6} and A_{G7} undefined. The acceleration vectors appearing in G are defined by

(5.13)

$$
\begin{bmatrix} a_{f3} \\ a_{h2} \\ a_{h3} \end{bmatrix} = R \begin{bmatrix} a_{xA} \\ a_{yA} \\ a_{zA} \end{bmatrix}
\qquad
\begin{bmatrix} a_{h1} \\ a_{f1} \\ a_{f2} \end{bmatrix} = S \begin{bmatrix} a_{xA} \\ a_{yA} \\ a_{zA} \end{bmatrix}
\qquad (5.16)
$$

The matrix S is identical to R except that β_2 must be replaced by β_1 (see equation 4.7). Reference to Figures 4.01 and 4.02 shows that the effect of the matrices S and R is to rotate accelerometer accelerations into the axes defining gyros 1 and 2, respectively. Note that the matrix R is also used in the vector F to obtain accelerations along the three gyro input axes, h_1, h_2, h_3; or, from another viewpoint it is used, as shown in equation (4.9), to obtain

$$
\bar{\Phi} = R' \bar{\Phi}_h R \qquad (5.17)
$$

where $\bar{\Phi}$ is the matrix containing rotations about platform x, y, z axes. It is important to keep in mind the two roles that the matrix R has when simulating Class 2 errors. This will be demonstrated in the following examples.

a. Initial Platform Misalignments

Normally, initial platform misalignments are given about platform x, y, z axes as $\phi_{xo}, \phi_{yo}, \phi_{zo}$. By setting $\psi_1 = \psi_2 = \beta_2 = 0$ in R, in which case R becomes the identity matrix, the vector $\bar{\phi}$ becomes, from equation (4.8),

$$
\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \end{bmatrix}
\qquad (5.18)
$$

and $\phi = \Phi_h$ from equation (5.17). As can be seen from Figures 4.01 and 4.02, such a choice of angles does indeed cause the h_1 axes to lie along the x, y, z axes. The simulation is completed by choosing Option 1, i.e., $\phi = 0$, and setting

16)

$$
\begin{bmatrix} G_{10} \\ G_{11} \\ G_{12} \end{bmatrix} = \begin{bmatrix} \phi_{xo} \\ \phi_{yo} \\ \phi_{zo} \end{bmatrix}
$$

(5.19)

in equation (5.15).

If, however, the initial platform misalignments were given about axes other than x, y, z, say accelerometer axes, which were rotated through angles ξ_1, ξ_2, ξ_3, then the simulation would be effected by setting

17)

$$
\begin{bmatrix} \psi_1 \\ \psi_2 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} , \quad \phi = G, \text{ and} \quad \begin{bmatrix} G_{10} \\ G_{11} \\ G_{12} \end{bmatrix} = \begin{bmatrix} \phi_{10} \\ \phi_{20} \\ \phi_{30} \end{bmatrix}
$$

(5.20)

In effect, the h_1 axes have been placed parallel to the accelerometer axes, acceleration errors computed, and then rotated back to platform x, y, z axes and finally, as is done in all cases, into ECI coordinates.

b. Platform Rotations Due to Platform Elasticity

If the platform rotates when accelerated, the indicated acceleration errors can be found by choosing the angles ψ_1, ψ_2 and β_2, such that the h_1 axes lie along the axes about which the platform rotates; and using the vector G to simulate the expressions which relate rotations about the h_1 axes to accelerations along these axes. For example, if

$$
\phi_1 = K_1 a_{h1} + K_2 a_{h3} + K_3 a_{h2} a_{h3}
$$

$$
\phi_2 = 0
$$

(5.21)

the simulation would be effected by choosing Option 1, i.e., $\phi = G$, where

$$G = M_{G2}A_{G2} + M_{G4}A_{G4} = \begin{bmatrix} K_1 & 0 & K_2 \\ 0 & 0 & 0 \\ 0 & K_4 & K_5 \end{bmatrix} \begin{bmatrix} a_{f3} \\ a_{h2} \\ a_{h3} \end{bmatrix} + \begin{bmatrix} 0 & K_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{f3} \cdot a_{h2} \\ a_{h2} \cdot a_{h3} \\ a_{h3} \cdot a_{f3} \end{bmatrix} \quad (5.2?)$$

NOTE: $a_{h1} = a_{f3}$ (see Figures 4.01 and 4.02)

In the two examples given thus far, the matrix R has served as a means of obtaining accelerations along the h_1 axes with no mention of gyro axes. The h_1 axes have been used as an intermediate set which in some cases are more convenient to use than platform x, y, z or accelerometer axes. In the case of gyro drift rates, the h_1 axes will be used to represent the three axes stabilized by the gyros. The matrix R, though still used in the vector F to obtain accelerations along the h_1 axes, is looked upon more as a means of obtaining accelerations along the axes of gyro 2. The matrix S is, of course, used to obtain accelerations along the axes of gyro 1. Because the simulation of gyro drift rates is dependent on specific gyro orientations and drift rate equations, examples will not be given until Section 6 in which a specific gyro system will be simulated.

Before proceeding to Section 6, it should be pointed out that the vectors E_A and G can be used simultaneously, but that only one option may be used for evaluating ϕ for any given simulation.

6. DETAILS OF COMPUTER PROGRAM

To use the program it is necessary to provide a tape on which are recorded the sensed accelerations, position, and range from the center of the earth of the missile as a function of time, all in ECI coordinates. It is advantageous, but not necessary, to have recorded, at burnout, the missile velocity components in ECI coordinates. Once such a tape is made available, the input required for evaluation of error sources is:

(a) General Input

1. Azimuth angle A

(a)   General Input (continued)

   4.   Gravity components in ECI coordinates if (E_{10}, E_{11}, E_{12}) or (G_{10}, G_{11}, G_{12})
         are to be computed (see examples for use of this feature).

(b)   For Vector E_A

   1.   Angles ξ_1, ξ_2, ξ_3 or θ_1, θ_2

   2. $\theta_x, \theta_y, \theta_z$ if (a_{2x}, a_{2y}, a_{2z}) or (a_{3x}, a_{3y}, a_{3z}) are being used

   3.   Elements of the M_E matrices

(c)   For Vector G

   1.   Angles $\psi_1, \psi_2, \beta_1, \beta_2$

   2.   Whether Option 1 or 2 is being used

   3.   Elements of the M_G matrices

Elements and angles not specified are assumed to be zero. All angles should be
given as positive numbers in degrees.

   Figure 6.01 shows how output information is presented. Blocks 1 and 2 reproduce
whatever input has been provided to the program, while Blocks 3 and 4 present the
errors computed as a result of this input. Position and velocity errors are printed
in accelerometer, platform x, y, z, and ECI coordinates; and if burnout velocity has
been recorded on tape, perturbed position and velocity vectors are printed in
ECI coordinates which can be used in free-flight programs to determine the target
miss. The target miss, in terms of down-range and cross-range miss, is computed
in Block 4 if miss coefficients are provided.

   The term "coupled or uncoupled" is used to denote the form of the differential
equations being solved. That is, one has the option of solving equation (3.16) with
$g_o \neq 0$ or $g_o = 0$, in which cases the equations are said to be coupled or uncoupled.
Uncoupling the equations eliminates whatever effect ΔG (equation 3.12, gravity
computation errors) may have on the final position and velocity errors.

	g_0 (ft/sec^2)		R_{launch} (ft)			ϕ_L (deg)	
	ϕ_L^* (deg)		b (deg)			A (deg)	
ECI Miss Coefficients	(ft/ft)				(ft/ft/sec)		
M_D	ΔX	ΔY	ΔZ		$\Delta \dot{X}$	$\Delta \dot{Y}$	$\Delta \dot{Z}$
M_C	ΔX	ΔY	ΔZ		$\Delta \dot{X}$	$\Delta \dot{Y}$	$\Delta \dot{Z}$

BLOCK 1

	Title (Optional)		Coupled or Uncoupled
	ξ_1	ξ_2	ξ_3
	θ_1	θ_2	all angles in deg.
Trajectory Identification Number	θ_x	θ_y	θ_z
	Ψ_1	Ψ_2	
	β_1	β_2	
	elements of matrices in G		
	elements of matrices in E_A		

BLOCK 2

Errors in Accelerometer Coordinates	t	Δx_A	Δy_A	Δz_A	(ft)
		$\Delta \dot{x}_A$	$\Delta \dot{y}_A$	$\Delta \dot{z}_A$	(ft/sec)
		$\Delta \ddot{x}_A$	$\Delta \ddot{y}_A$	$\Delta \ddot{z}_A$	(ft/sec^2)
Errors in Launch Point Coordinates (x,y,z)	t	Δx	Δy	Δz	
		$\Delta \dot{x}$	$\Delta \dot{y}$	$\Delta \dot{z}$	
		$\Delta \ddot{x}$	$\Delta \ddot{y}$	$\Delta \ddot{z}$	
Errors in ECI Coordinates	t	ΔX	ΔY	ΔZ	
		$\Delta \dot{X}$	$\Delta \dot{Y}$	$\Delta \dot{Z}$	
		$\Delta \ddot{X}$	$\Delta \ddot{Y}$	$\Delta \ddot{Z}$	
Elements of ϕ	t	ϕ_1	ϕ_2	ϕ_3	(rad)
Elements of $\dot{\phi}$		$\dot{\phi}_1$	$\dot{\phi}_2$	$\dot{\phi}_3$	(rad/sec)
ECI Position and Acceleration	t	X	Y	Z	(ft)
	R (ft)	$\ddot{X}$	$\ddot{Y}$	$\ddot{Z}$	(ft/sec^2)

BLOCK 3

ECI Perturbed Position & Velocity	t	X	Y	Z	(ft)
		$\dot{X}$	$\dot{Y}$	$\dot{Z}$	(ft/sec)
Miss		M_D (ft)		M_C (ft)	

BLOCK 4

FIGURE 4.01   Program Output Format
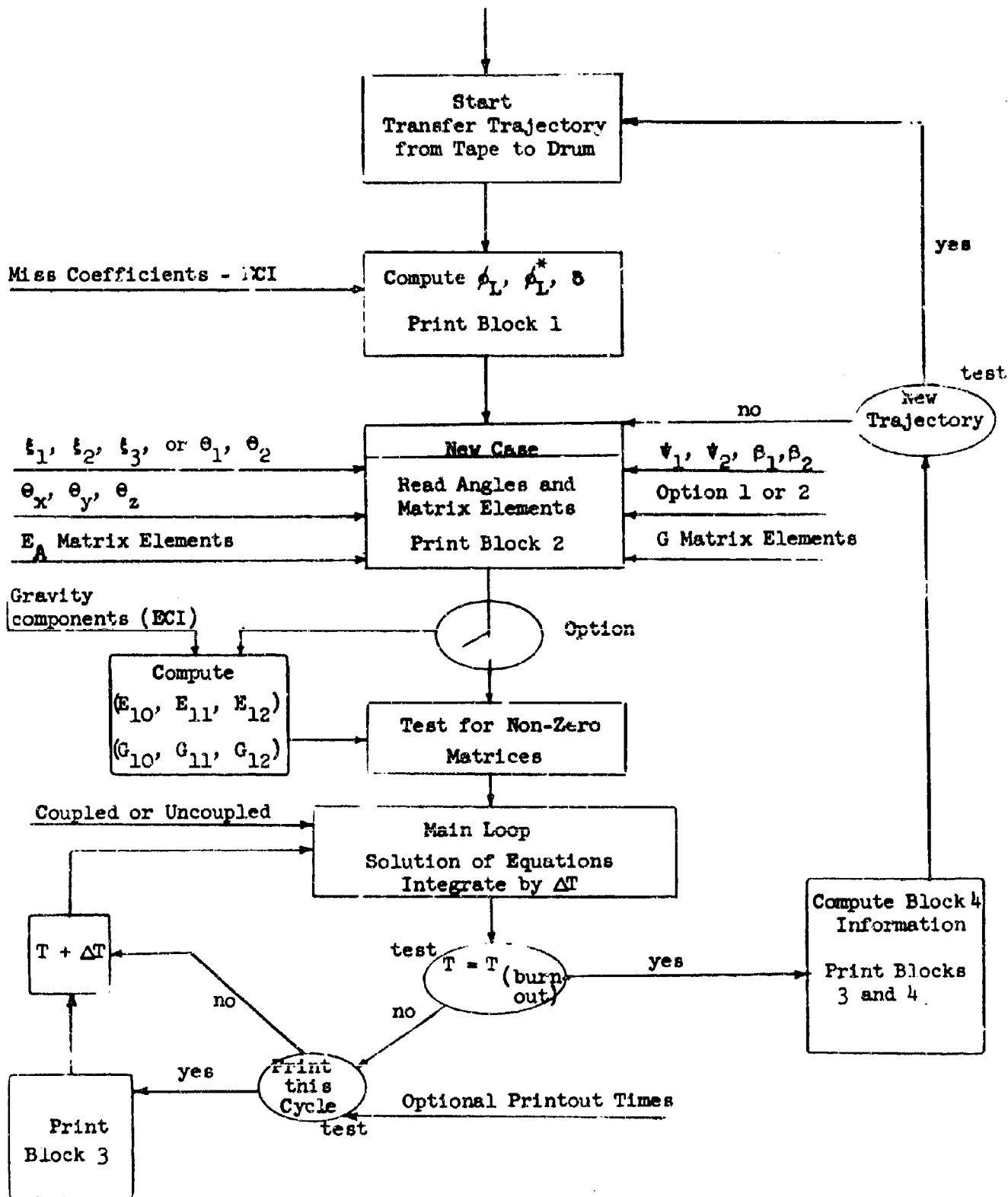
Though not indicated in the output format, provision has been made in the program whereby initial conditions can be placed on $\Delta X, \Delta Y, \Delta Z, \Delta \dot{X}, \Delta \dot{Y}$, and $\Delta \dot{Z}$.

Figure 6.02 indicates the manner in which the computer input is utilized and how the program operates. A test for non-zero matrices is made thereby eliminating time consuming operations with matrices that are identically zero. ₋te the following about the ma᠆er in which output information is printed:

(a) Block 1 is printed only once for any given trajectory.
(b) Block 2 is printed only once for any given case.
(c) Block 3 only is printed at optional printout times.
(d) Block 4 is printed only at burnout.

7. EXAMPLES

To demonstrate the use of this program, several examples of error sources that have been simulated with this program will be presented.

Class 1 Errors

Non-Linear Terms in the Input - Output Relation

The accelerometers used in one guidance system are of the pendulous type shown in Figure 4.04 and are oriented on the platform as shown in Figure 4.06. Three axes, 1, 2, 3 are used to describe the accelerometer, with axis 1 the sensitive axis. The equation

$$a_{out} = C_o + a_1 + C_1 a_1 + C_2 |a_1| a_1 + C_3 |a_2| a_1 + C_3' a_2 a_1 + C_4 |a_3| a_1$$

$$+ C_5 a_1^3 + C_5' a_1^2 - (C_1 g_1 + C_2 |g_1| g_1 + C_3 |g_2| g_1 + C_3' g_2 g_1 + C_4 |g_3| g_1$$

$$+ C_5 g_1^3 + C_5' g_1^2) \qquad (7.1)$$

has been used to relate output to input acceleration for any given accelerometer where $a_1 = a_A$, the acceleration along the sensitive axis, and g_1, g_2, g_3 are gravity components appearing along the three axes at launch. The error in the indicated accelerometer output is then

$$\Delta a = a_{out} - a_1 \tag{7.2}$$

For demonstration purposes, and because the rest of the terms are simulated in a similar manner, only terms involving C_3, C_3' and C_5 will be simulated. For these terms

$$E_A = M_{E1} + M_{E8}A_{E8} + M_{E7}A_{E7} + M_{E4}A_{E4}$$

$$= \begin{bmatrix} E_{10} \\ E_{11} \\ E_{12} \end{bmatrix} + \begin{bmatrix} C_{3x} & 0 & 0 \\ 0 & C_{3y} & 0 \\ 0 & 0 & C_{3z} \end{bmatrix} \begin{bmatrix} |a_{2x}| & a_{xA} \\ |a_{2y}| & a_{yA} \\ |a_{2z}| & a_{zA} \end{bmatrix} + \begin{bmatrix} C_{3x}' & 0 & 0 \\ 0 & C_{3y}' & 0 \\ 0 & 0 & C_{3z}' \end{bmatrix} \begin{bmatrix} a_{2x} & a_{xA} \\ a_{2y} & a_{yA} \\ a_{2z} & a_{zA} \end{bmatrix}$$

$$+ \begin{bmatrix} C_{5x} & 0 & 0 \\ 0 & C_{5y} & 0 \\ 0 & 0 & C_{5z} \end{bmatrix} \begin{bmatrix} a_{xA}^3 \\ a_{yA}^3 \\ a_{zA}^3 \end{bmatrix} \tag{7.3}$$

where

$$\begin{bmatrix} E_{10} \\ E_{11} \\ E_{12} \end{bmatrix} = - \begin{bmatrix} C_{3x}|g_{2x}|\, g_{xA} + C_{3x}'g_{2x}g_{xA} + C_{5x}g_{xA}^3 \\ C_{3y}|g_{2y}|\, g_{yA} + C_{3y}'g_{2y}g_{yA} + C_{5y}g_{yA}^3 \\ C_{3z}|g_{2z}|\, g_{zA} + C_{3z}'g_{2z}g_{zA} + C_{5z}g_{zA}^3 \end{bmatrix} \tag{7.4}$$

In addition, the angles ξ_1, ξ_2, ξ_3 or θ_1, θ_2 must be specified to define the accelerometer sensitive axes (x_A, y_A, z_A) with respect to the platform (x, y, z) axes; and the angles $\theta_x, \theta_y, \theta_z$ must be given to define axis 2 of each accelerometer with

respect to the (x_A, y_A, z_A) system. If, for example, axis 2 were along the z_A axis for the range accelerometer and along the $-y_A$ axis for the pitch accelerometer (see Figure 4.06), one would set $\theta_x = 90°$, $\theta_z = 180°$.

Except for special orientations, the gravity components g_1, g_2, g_3 are complicated functions of the angles specified above, thus necessitating a lengthy computation to evaluate equation (7.4). To eliminate this computation (especially desirable for optimization studies where the angles are being varied) the computer has been programmed to perform this operation at $t = 0$. The computation is made as follows:

$$
\begin{bmatrix} E_{10} \\ E_{11} \\ E_{12} \end{bmatrix} = - \left\{ M_{E2} \begin{bmatrix} g_{xA} \\ g_{yA} \\ g_{zA} \end{bmatrix} + M_{E3} \begin{bmatrix} g_{xA}^2 \\ g_{yA}^2 \\ g_{zA}^2 \end{bmatrix} + M_{E4} \begin{bmatrix} g_{xA}^3 \\ g_{yA}^3 \\ g_{zA}^3 \end{bmatrix} + \ldots \right\} \tag{7.5}
$$

In other words, the acceleration vectors appearing in E_A are computed at $t = 0$ in terms of gravity, matrix multiplications and summations are carried out, and the negative of this sum put into the cells used for the elements E_{10}, E_{11}, E_{12}. To use this feature, gravity components existing at launch, and expressed in ECI coordinates, must be provided.

Figures 7.01 and 7.02 are examples of the output obtained when C_{3x}, C'_{3y} and C_{5z} are simulated individually. The elements E_{10} and E_{12} have been machine computed by use of the feature described above. The following input was required.

Figure 7.01 - C_{3x} $E_{80} = 0.1 \times 10^{-6}$ $\theta_x = 20°$ $\xi_1 = 315°$ $\xi_2 = 45°$

Figure 7.02 - C'_{3y} $E_{74} = 0.1 \times 10^{-6}$ $\theta_y = 10°$ $\xi_1 = 315°$ $\xi_2 = 45°$

Figure 7.02 - C_{5z} $E_{48} = 0.1 \times 10^{-6}$ $\xi_1 = 315°$ $\xi_2 = 45°$

In addition, gravity components and miss coefficients were provided.

GENERALIZED INERTIAL ERROR ANALYSIS

1935-01

D. MEHONER   V. HENDERSON

	G SUB ZERO	PHI L STAR		M BL OR M O	NELTA		PHI L	AZIMUTH

ECI MISS COEFFICIENTS

3RD STG MAIN

3RD STG CROS

NON-LINEAR SCALE FACTOR ERRORS

	X1 SUB 1	THETA SUB X	E SUB 80	FP SUB 10		A1 SUB 4	THETA SUB Y		COUPLED	A1 SUB 5	THETA SUB 7

ACCELEROMETR ERRORS AND DERIVATIVES

LAUNCH POINT ERRORS AND DERIVATIVES

INERTIAL ERRORS AND DERIVATIVES

PHI SUB M.23
PHI M123 ROT

INERT CMPUTS
SENSD ACCELS

INERT P AND
V CORRECTED

MISSES   DOWN RANGE   CROSS RANGE

FIGURE 7.01   Computer Output for C_{3x}

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NON-LINEAR SCALE FACTOR ERRORS

	XI SUB 1	X1 SUB 2	THETA SUB 3
ACCELEROMETER ERRORS AND DERIVATIVES	279.50105.1	.000220.0062-	.00002049.4-
LAUNCH POINT ERRORS AND DERIVATIVES	279.50105.1		
INERTIAL ERRORS AND DERIVATIVES	279.50105.1		
PHI SUB M123 PHI M123 DOT	279.50105.1		
INERT CROWTS SENSO ACCELS	279.50105.1		
INERT P AND V CORRECTED	279.50105.1		
MISSES	DOWN RANGE		CROSS RANGE

NON-LINEAR SCALE FACTOR ERRORS

	XI SUB 1	X1 SUB 2	THETA SUB 3
ACCELEROMETER ERRORS AND DERIVATIVES	279.50105.1		
LAUNCH POINT ERRORS AND DERIVATIVES	279.50105.1		
INERTIAL ERRORS AND DERIVATIVES	279.50105.1		
PHI SUB M123 PHI M123 DOT	279.50105.1		
INERT CROWTS SENSO ACCELS	279.50105.1		
INERT P AND V CORRECTED	279.50105.1		
MISSES	DOWN RANGE		CROSS RANGE

FIGURE 7.02

In another guidance system the accelerometers are oriented such that their sensitive axes lie along the platform (x, y, z) axes, i.e., $\ell_1 = \ell_2 = \ell_3 = 0$. The equations used to describe the error in their outputs are

$$\Delta a_{xA} = K_{ox} + K_{1x}a_x + K_{2x}a_x^2 + K_{3x}a_x^3 + K_{4x}|a_x a_z|$$

$$\Delta a_{yA} = K_{oy} + K_{1y}a_y + K_{2y}a_y^2 + K_{3y}a_y^3 \tag{7.6}$$

$$\Delta a_{zA} = K_{oz} + K_{1z}a_z + K_{2z}a_z^2 + K_{3z}a_z^3 + K_{4z}|a_x a_z|$$

Since $|a_x a_z| = a_x a_z$ for this system, the equations can be simulated as follows:

$$E_A = M_{E1} + M_{E2}A_{E2} + M_{E3}A_{E3} + M_{E4}A_{E4} + M_{E7}A_{E7}$$

$$= \begin{bmatrix} K_{ox} \\ K_{oy} \\ K_{oz} \end{bmatrix} + \begin{bmatrix} K_{1x} & 0 & 0 \\ 0 & K_{1y} & 0 \\ 0 & 0 & K_{1z} \end{bmatrix} A_{E2} + \begin{bmatrix} K_{2x} & 0 & 0 \\ 0 & K_{2y} & 0 \\ 0 & 0 & K_{2z} \end{bmatrix} A_{E3} + \begin{bmatrix} K_{3x} & 0 & 0 \\ 0 & K_{3y} & 0 \\ 0 & 0 & K_{3z} \end{bmatrix} A_{E4} + \begin{bmatrix} K_{4x} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_{4z} \end{bmatrix} A_{E7}$$

$$\tag{7.7}$$

where $\theta_x = 90^\circ$, $\theta_z = 270^\circ$.

Class 2 Errors

Platform Rotations Due to Platform Elasticity

The platform in one guidance system is such that when subjected to accelerations it tends to rotate, with the rotation being directly proportional to acceleration. The rotations are about platform (x, y, z) axes where

$$\phi_x = -\alpha_x a_z$$

$$\phi_y = 0 \tag{7.8}$$

PLATFORM COMPLIANCES

	G SUB 22	G SUB 20	G SUB 20	COMPLED
	279.5010531	-.2000000A-06	-.2000000D-06	.2000000D-06
ACCELEROMETR ERRORS AND DERIVATIVES	279.5010531	.0014025412- .0006830166 .050507620-06	78.45547214- .0913593560- .0001044035	.0001851241- .000516A352 .5597A005-05
LAUNCH POINT ERRORS AND DERIVATIVES	279.5010531	.0014025412- .0006830166 .050507620-06	78.45547214 .0913593560- .0001044035	.0001851241- .0005146352 .559760005-05
INERTIAL ERRORS AND DERIVATIVES	279.5010531	.0004249542 .0001745636 .4757728-05	78.45547214 .0913593560 .0001044035-	.0013493437- .0006177361A .2760u984-05
PHI SUB M123 PHI M123 ROT		-.2302A955-06 -.2302A955-06		.6220u734-06 .6220u734-06
INERT CHANTS SENSO ACCFLS	279.5010531 217963635	18089342"7 .2710u405700-	382208.604 .0249716682-	4.1003204.0 5.305u42386
INERT P AND V CORRECTED	279.5010531	18589382"7 2628.17792-	382347.062 1319.2356M7	1190.3204.0 22050.0270
MISSES	DOWN RANGE	210.1240234-	CROSS RANGE	658.1240605-

FIGURE 7.03   Computer Output for α_x, α_z

From equation (4.8), when $\psi_1 = \psi_2 = \beta_2 = 0$

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} \tag{7.9}$$

With such a choice of angles, the error source is simulated by choosing Option 1, i.e., $\phi = G$; and

$$G = M_{G2}A_{G2} = \begin{bmatrix} 0 & 0 & -\alpha_x \\ 0 & 0 & 0 \\ \alpha_z & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{f3} \\ a_{h2} \\ a_{h3} \end{bmatrix} \quad (a_{f3} = a_{h1} = a_x) \tag{7.10}$$
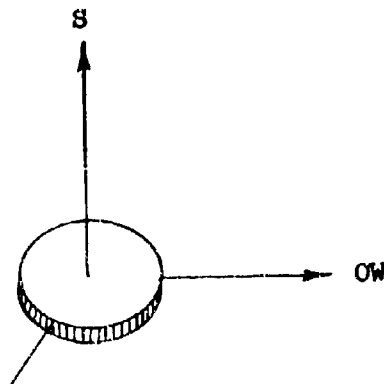
The only input required to simulate this error source is

$$\phi = G, \qquad G_{22} = -\alpha_{x'} \qquad G_{26} = \alpha_z$$

Figure 7.03 is an example of such a simulation where $\alpha_x = \alpha_z = 0.2 \times 10^{-6}$. Note that since all angles are zero, none have been printed.

Gyro Drift Rates

A typical gyro can be defined by three orthogonal axes, S, IW, OW, as shown in Figure 7.04.

The acceleration-dependent drift rates about axes IW, OW can be represented for a certain system as

$$\dot{\phi}_{IW} = B_S a_{IW} + G_S a_{IW} - BG_{IW} a_S - K_{IW} a_S a_{IW}$$

(7.11

$$\dot{\phi}_{OW} = B_S a_{OW} + B_{OW} a_S - K_{OW} a_S a_{OW}$$

B_i, G_i, BG_i are ball, gimbal, and ball-gimbal unbalances along the ith axis, and $K_i = (K_S - K_i)$ is the difference in compliance between the spin and i^{th} axis.

A platform can be stabilized by two such gyros oriented with respect to the platform as shown in Figure 7.05.
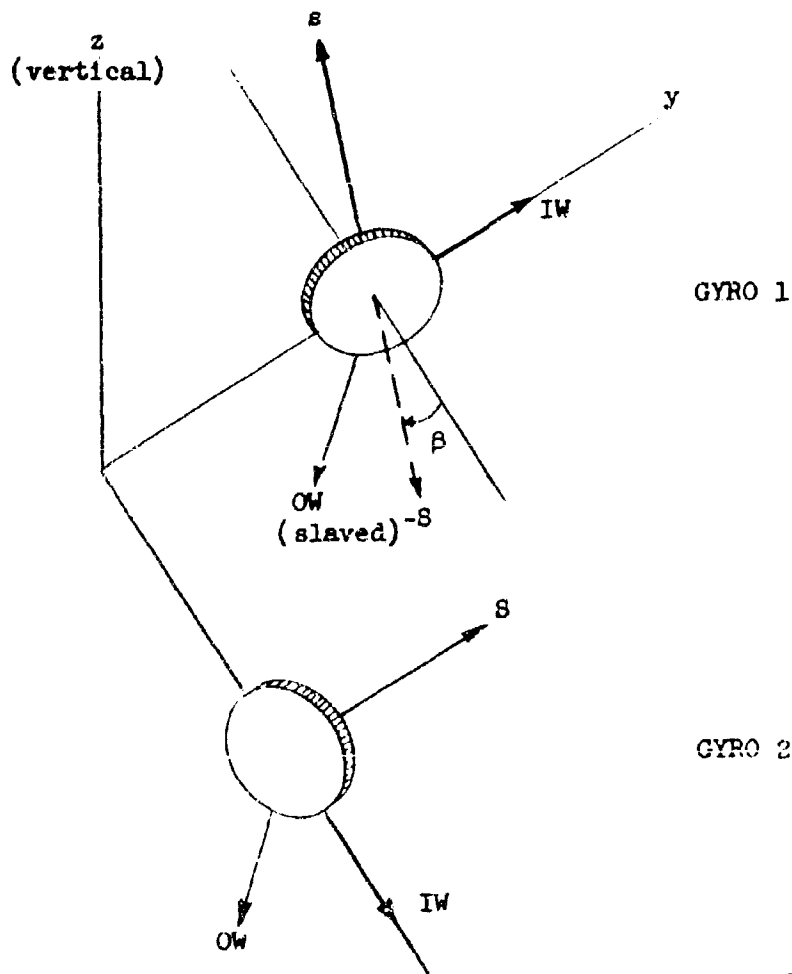
The angle β is commonly used to define the (-S) axis of gyro 1 (pitch gyro). The OW axis of gyro 1 is slaved; thus, three drift rate equations suffice to describe gyro drifts.

$$\dot{\phi}_{IW1} = B_{S1}a_{IW1} + G_{S1}a_{IW1} - BG_{IW1}a_{S1} - K_{IW1}a_{S1}a_{IW1}$$

$$\dot{\phi}_{IW2} = B_{S2}a_{IW2} + G_{S2}a_{IW2} - BG_{IW2}a_{S2} - K_{IW2}a_{S2}a_{IW2} \qquad (7.12$$

$$\dot{\phi}_{OW2} = B_{S2}a_{OW2} + B_{OW2}a_{S2} - K_{OW2}a_{S2}a_{OW2}$$

This orientation can be simulated in the error analysis program by choosing ψ_1, ψ_2, and β_2, and defining the gyro axes as shown in Figure 7.06 (refer to Figures 4.01 and 4.02).

GYRO 1

GYRO 2

$$\psi_1 = \frac{\pi}{2}$$

$$\psi_2 = 0$$

$$\beta_2 = \pi$$

$$\beta_1 = \frac{3\pi}{2} + \gamma$$
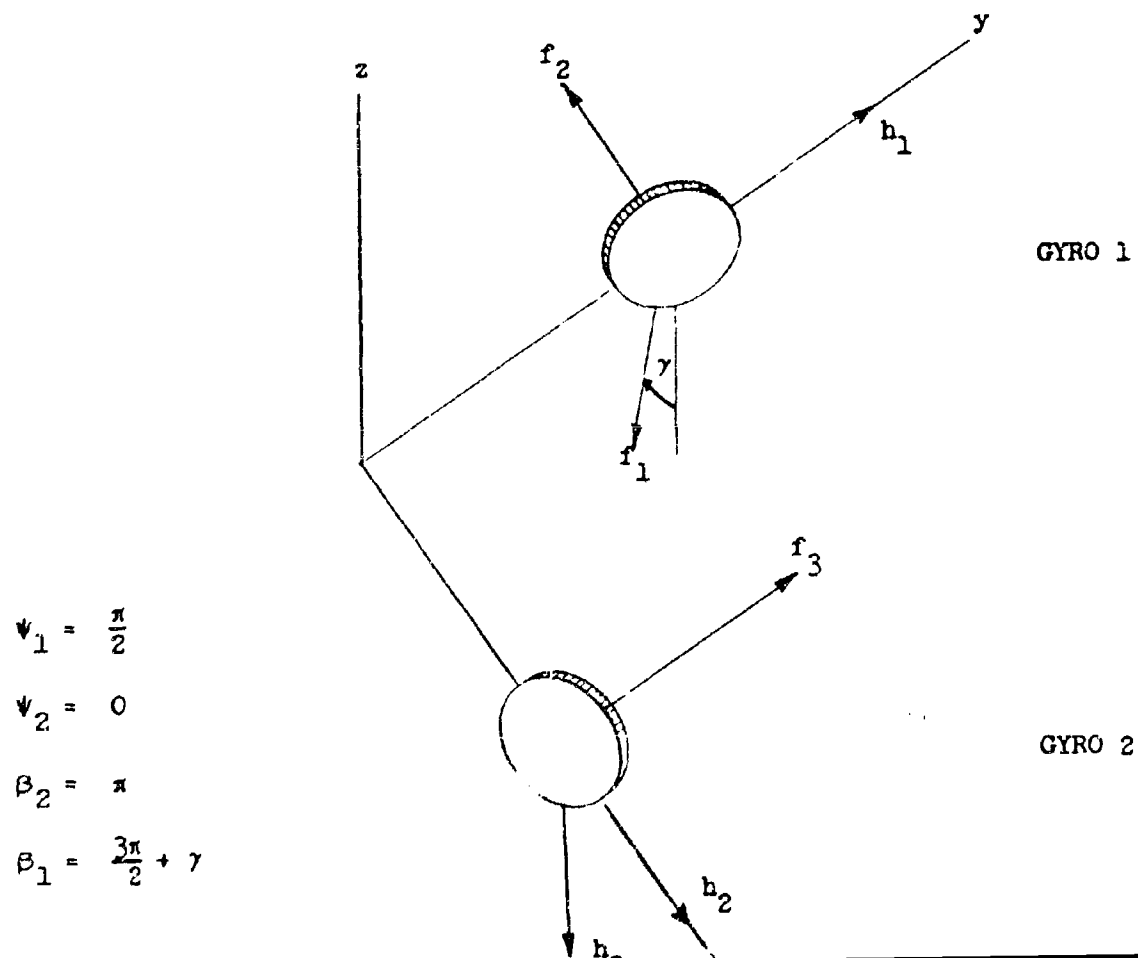
It is necessary to add the 270° offset in defining β_1 because in the simulation β_1 is used to define axis f_1 with respect to the $(-x)$ axis, where β_1 is measured counterclockwise about h_1. When defined in this manner, the angle β, defined in Figure 7.05, is equal to 7. Equations (7.12) now become

$$\ddot{\phi}_1 = B_{S1}a_{h1} + G_{S1}a_{h1} - BG_{IW1}a_{f2} - K_{IW1}a_{f2}a_{h1}$$

$$\ddot{\phi}_2 = B_{S2}a_{h2} + G_{S2}a_{h2} - BG_{IW2}a_{f3} - K_{IW2}a_{f3}a_{h2} \tag{(}$$

$$\ddot{\phi}_3 = B_{S2}a_{h3} + B_{OW2}a_{f3} - K_{OW2}a_{f3}a_{h3}$$

Under the assumption that $a_y \approx 0$, then $a_{f3} = a_{h1} = 0$, and equations (7.13) reduce to

$$\ddot{\phi}_1 = BG_{IW1}a_{f2} \qquad \text{(dropping the minus sign)}$$

$$\ddot{\phi}_2 = B_{S2}a_{h2} + G_{S2}a_{h2} \tag{(}$$

$$\ddot{\phi}_3 = B_{S2}a_{h3}$$

Equations (7.13) can be simulated by choosing Option 2, i.e., $\phi = \int_0^t G dt$ and

$$G = M_{G2}A_{G2} + M_{G3}A_{G3} + M_{G4}A_{G4} + M_{G5}A_{G5}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -BG_{IW2} & (G_{S2} + G_{S2}) & 0 \\ B_{OW2} & 0 & B_{S2} \end{bmatrix} A_{G2} + \begin{bmatrix} (B_{S1} + G_{S1}) & 0 & -BG_{IW1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} A_{G3}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ -K_{IW2} & 0 & 0 \\ 0 & 0 & -K_{OW2} \end{bmatrix} A_{G4} + \begin{bmatrix} 0 & 0 & -K_{IW1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} A_{G5} \tag{(7}$$

Figure 7.07 shows the errors that arise due to BG_{IW1}, where $G_{32} = 0.3014 \times 10^{-6}$, $\beta_1 = 322^O$ corresponding to $\beta = 52^O$.

Constant Gyro Drift Rates

Constant gyro drifts are simulated by choosing the same angles and option and setting.

(7.13)

$$G = M_{G1} = \begin{bmatrix} \dot{\phi}_{10} \\ \dot{\phi}_{20} \\ \dot{\phi}_{30} \end{bmatrix}$$

Though drifts due to gravity are not subtracted out at launch in this system, provision has been made to evaluate the elements G_{10}, G_{11}, G_{12} in terms of gravity components just as the elements E_{10}, E_{11}, E_{12} can be evaluated in the E_A vector.

(7.14)

(7.15)

GYRO DRIFT RATE ERRORS

	PSI SUB 1	BETA SUB 1	PSI SUB 2	BETA SUB 2
711				COUPLED
PSI SUB 1	90.0000000	322.0000000	180.0000000	
G SUB 32	.3014000D-06			
ACCELEROMETER ERRORS AND DERIVATIVES	279.5019531	636.284523- 1.233766517- .002402273	.233212417- .0039508836- .0000386751-	619.440895 1.4411044457- .0021145224-
LAUNCH POINT ERRORS AND DERIVATIVES	279.5019531	636.284523- 1.233766317- .00240222173	.233212417- .0039508836- .0000386751-	619.440895 1.4411044457- .0021145224-
INERTIAL ERRORS AND DERIVATIVES	279.5019531	830.306435 .7846891954- .0029316205-	.233212417 .0039508836 .0000386751	314.8908424- 1.72720283- .0012835507
PHI SUB H.23 / PHI H123 DOT	279.5019531	.001117831- -.30573079-06	383268.609 .02987716682-	113832204.0 3.305642485
INERT CRDNTS SENSD ACCELS	279.5019531 21798303.5	18583424.7 .27104057000-	383268.844	11382889.1
INERT P AND V CORRECTED	279.5019531	18594655.0 5628.96277-	1313.548279	22058.1990
MISSES	DOWN RANGE 7254.01532-		CROSS RANGE 168.7909889	

FIGURE 7.07   Computer Output for BG IWl

D I S T R I B U T I O N

Anderson, S. C.
Bachar, J. M.
Ba kin, F.
Bers, N. S.
Brady, H.
Brown, R.
Brotemarkle, D. W.
Burnett, J. R.
Christensen, N. G. (4)
Christensen, R.
Clegg, J. C.
Culp, R. J.
Drucker, A. N.
Epple, R. G.
Graybill, A. B. (4)
Halliday, R. G.
Harter, G. A.
Heilfron, J.
Henderson, V.
Hendrickson, R. M.
Hoy, E. C.
Jensen, L. K. (4)
Joseph, J. A.
Lanzer, L. F.
Layton, R. W.
McCarthy, J. W.
McLaughlin, W. J.
Meek, F.
Meronek, D. F.
Moore, R. A.
Robb, E. J. (4)
Russell, W. T.
Schultz, P. R.
Sorrels, J. D.
Southworth, R. N.
Spanier, B.
Westlake, R.
Whitcombe, D. W.
Zacharias, R.

STL Library (3)
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File (23)