MAGNETOGASDYNAMIC EFFECTS IN THE FLOW BEHIND A SPHERE

A. Goldburg and P. O. Jarvinen

RESEARCH NOTE 415
Contract No. AF 04(694)-414
August 1964

DO NOT PHOTOGRAPH THIS PAGE

prepared for

HEADQUARTERS
BALLISTIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Norton Air Force Base
San Bernardino, California
LIMITATIONS IN REPRODUCTION QUALITY

ACCESSION #4D 606055

☐ 1. WE REGRET THAT LEGIBILITY OF THIS DOCUMENT IS IN PART UNSATISFACTORY. REPRODUCTION HAS BEEN MADE FROM BEST AVAILABLE COPY.

☐ 2. A PORTION OF THE ORIGINAL DOCUMENT CONTAINS FINE DETAIL WHICH MAY MAKE READING OF PHOTOCOPY DIFFICULT.

☐ 3. THE ORIGINAL DOCUMENT CONTAINS COLOR, BUT DISTRIBUTION COPIES ARE AVAILABLE IN BLACK-AND-WHITE REPRODUCTION ONLY.

☐ 4. THE INITIAL DISTRIBUTION COPIES CONTAIN COLOR WHICH WILL BE SHOWN IN BLACK-AND-WHITE WHEN IT IS NECESSARY TO REPRINT.

☐ 5. LIMITED SUPPLY ON HAND: WHEN EXHAUSTED, DOCUMENT WILL BE AVAILABLE IN MICROFICHE ONLY.

☐ 6. LIMITED SUPPLY ON HAND: WHEN EXHAUSTED DOCUMENT WILL NOT BE AVAILABLE.

☐ 7. DOCUMENT IS AVAILABLE IN MICROFICHE ONLY.

☐ 8. DOCUMENT AVAILABLE ON LOAN FROM CFSTI (TT DOCUMENTS ONLY).

☐ 9.

PROCESSOR: SF

TSL-107-10/64
MAGNETOGASDYNAMIC EFFECTS IN THE FLOW BEHIND A SPHERE

by

A. Goldburg and P. O. Jarvinen

AVCO-EVERETT RESEARCH LABORATORY
a division of
AVCO CORPORATION
Everett, Massachusetts

Contract No. AF 04(694)-414

August 1964

prepared for

HEADQUARTERS
BALLISTIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Norton Air Force Base
San Bernardino, California
ABSTRACT

Maxworthy's experiments with the flight of a sphere through an incompressible electrically conducting fluid with an aligned magnetic field showed that transition in the trailing wake was moved to higher fluid mechanical Reynolds numbers. Theoretical and experimental work on suppression of vorticity in uniformly conducting incompressible fluids with imposed uniform transverse magnetic fields is analyzed to find a correlating parameter for $C_D$, $C_f$ and transition to turbulence. The parameter which is found is the square root of the ratio of the fluid mechanical Reynolds number ($R$) to the electromagnetic interaction parameter ($S$), $\sqrt{R/S}$, for the conditions $S \approx 0(1)$ or $RS > 0(1)$. The $\sqrt{R/S}$ is the Reynolds number based on the Hartmann layer thickness, $R_H = \sqrt{\rho V^2 / \sigma B^2 \nu}$. It is shown that for the cases $RS > 0(1)$, the absolute value of $R_H$ for transition is of the same order as the absolute value of $R$ for transition in the zero-field case, $R_X, 0$. Thus, the apparent effect of the transverse magnetic field in the incompressible case is to move $C_D$, $C_f$ and transition to higher values of the fluid mechanical Reynolds number, $R_B$, when the field is on. The apparent correlation leads to $R_B = R_0 (R_0 \cdot S)$.

Extension of the incompressible problem to the compressible one is based on the observed similarity of the transition processes in the hypersonic and incompressible wakes as suggested by Fay and Goldburg. Average values of the fluid properties in the base region were chosen as typical for the calculation of hypersonic wake transition Reynolds numbers, $R$ and $R_H$. The presence of large amounts of easily ionizable ablation products were assumed to give Spitzer conductivity. The calculated values for the relevant nondimensional fluid mechanical and electromagnetic parameters for typical ballistic range conditions indicated they were of requisite magnitude to affect the flow.

Hypersonic experiments were performed covering the range from $R \approx 5R_X, 0$ with zero field to $R_H \approx 1/5R_X, 0$, where $R_X, 0$ is the zero field wake transition Reynolds number. The magneto-hypersonic flow fields were produced by 0.22 caliber nylon pellets traveling at about 14,000 feet per second in argon through the axial magnetic field of a solenoid produced by the discharge of a capacitor bank. Drum camera pictures of the self-luminous wakes and photomultiplier traces were used as the primary diagnostic tools—the self-luminous ablation products color the wake in the same fashion as dye. The camera was oriented such that its field of view encompassed the region before and including part of the coil.

In the first set of runs made at 5 cm Hg pressure, a change in the luminous structure of the flow field occurred possibly suggestive of a change from turbulent to laminar wake flow.

In the second set of experiments made at 5 cm and 2 cm Hg pressure, no MHD effect on the hypersonic turbulent flow field was evident. The paper concludes with a discussion of these results.
I. Correlation Parameter for MHD Flows with Transverse Fields

A. Introduction

The macroscopic magnetohydrodynamic equation of motion for an electrically conducting incompressible medium in which the Hall current effect and the ion slip effect are neglected and in which the current loops close on themselves is

\[ \frac{D\vec{V}}{dt} + \frac{\rho}{\rho} \text{grad} p = \frac{1}{\rho} \cdot j \times B + \nu \nabla^2 \vec{V} \]

in the usual notation. For scalar conductivity the \( j \times B \) force is seen to have a component directed oppositely to \( \vec{V} \), of order \( \sigma \nabla B^2 \).

In shear flow, as shown in Fig. 1, a uniform transverse field with a uniform conducting medium will cause eddy currents to circulate such that the fast fluid decelerates more strongly than the slow. The result is to suppress the vorticity in the flow field. In this argument, the assumption is made that the induced field, \( B_i \), caused by the eddy currents is small relative to the imposed field \( B_0 \). Figure 2 shows a schematic of a B field interaction with the vortex ring in the base of a body. It is seen that the component of the \( j \times B \) force oppositely directed to \( \vec{V} \) slows down the speed of rotation of the ring cross section causing a decrease in the strength of the base vortex ring.

The MHD equation of motion (1) is nondimensionalized such that each term of the equation is composed of two factors: a nondimensional parameter indicating the term's physical significance and its magnitude relative to the other terms, and a function of the unknowns and their derivatives of magnitude of order one. This nondimensionalization results in the following:

\[ \frac{D\vec{V}}{dt} + \text{grad} \tilde{p} = \frac{1}{R} \cdot \tilde{\nabla}^2 \vec{V} + S \cdot \tilde{j} \times \tilde{B} \]  

where the tilde represents the usual nondimensional quantities and
\[ R = \frac{V_0 \ell_o}{\nu_o} \] represents the ratio of forces \( \{ \text{inertia} \} \) per unit volume \( \{ \text{viscous} \} \) \hspace{1cm} (3)

\[ S = \frac{\sigma_o B_o \ell_o}{\rho_o \nu_o^2} \] represents the ratio of forces \( \{ \frac{j \times B}{\text{inertia}} \} \) per unit volume \hspace{1cm} (4)

The single subscript (o) represents a characteristic quantity. The characteristic length governing a problem, \( \ell_o \), is the length over which the forces interact to cause the significant change in velocity. In deducing \( S \), the assumption has been made that the velocity perpendicular to \( B_o \) is like \( V_0 \).

If \( 1/R \) and \( S \) are large, then the inertia terms on the left-hand side of Eq. (2) can be neglected and another nondimensional parameter, \( R \cdot S \), appears; the \( \sqrt{R \cdot S} \) is usually called the Hartmann number, \( H_M \).

\[ H_M^2 = \frac{\sigma_o B_o \ell_o}{\rho_o \nu_o} \] represents the ratio of forces \( \{ \frac{j \times B}{\text{viscous}} \} \) \hspace{1cm} (5)

The magnetic Reynolds number, \( R_M \), is the nondimensional parameter which represents the severity of the effect of the electrically conducting fluid flow on the magnetic field.

\[ R_M = \mu_o \sigma_o V_o \ell_o \] represents \( \{ \frac{B_i}{B_o} \} \) \hspace{1cm} (6)

where \( \mu_o \) is the fundamental constant, permeability of vacuum in MKS units.

In a real gas the electrons tend to spiral around magnetic field lines between collisions giving rise to a tensor conductivity rotating and decreasing the current vector and hence the \( j \times B \) force vector. The Hall coefficient, \( \omega_T \), is a measure of the angle between the \( V \times B \) vector and the current vector due to the tensor conductivity.

\[ \omega_T = \frac{\sigma_o B_o}{N_e \cdot e} \] represents ratio of times \( \{ \frac{\text{period for collision}}{\text{period for revolution}} \} \)
where \( \omega \) is the electron spiraling frequency along field lines and \( \tau \) is the mean free time between collisions. \( N_e \) is the electron concentration and \( e \) the electron charge. In liquid metals, this term is effectively zero.

To effect an MHD vorticity interaction the magnitudes of the non-dimensional parameters should be of the following order

\[
S \approx 0(1); \quad R_M < 0(1); \quad \text{and} \quad \omega \tau < 0(1).
\]

It may be that the condition, the ratio of \( |j \times B|/\text{inertia} \) forces be of order one \( [S \approx 0(1)] \), can be replaced by the less severe condition, the ratio of \( |j \times B|/\text{viscous} \) forces be greater than of order one: \( R_S = M^{-2} > 0(1) \).

Available experiments do not encompass a sufficient range of parameter magnitudes to answer this question definitely. However, these experiments do show some evidence that there is an effect when \( S \) is less than order one, so long as \( R_S >> 0(1) \).

B. Theory

Under the conditions of steady flow, with a homogeneous B field in only the y direction, Lundquist\(^4\) following a scheme of Lehner\(^5\) was able to reduce by linearization the complete set of equations governing MHD phenomena to the following "boundary layer" form:

\[
\frac{B_o^2}{\mu_0 \rho_o} \frac{\partial^2 V}{\partial y^2} + \frac{\nu_o}{\mu_0 \sigma_o} \nabla^4 V = 0. \quad (7)
\]

The first term represents the electromagnetic driving force and the second term represents the electromechanical interaction. This "boundary region" equation indicates that changes in \( V \) along the magnetic field direction take place in length of order \( L_M \),

\[
L_M = \sqrt{\frac{\nu_o \rho_o}{\sigma_o B_o^2}} \quad (8)
\]

Lundquist then surmises: The fact that the variation of the velocity is confined to the "boundary region" thickness, \( L_M \), suggests that the length dimension entering the ordinary fluid mechanical Reynolds number be replaced by \( L_M \). Thus, the correlating parameter in the MHD case, \( R_H \),
becomes:

\[ R_H = \frac{V_0}{\nu_0} \cdot L_M = \sqrt{\frac{V_0(\ell_0)}{\nu_0} \cdot \frac{\rho_0 V_0}{\sigma_0 B_0^2(\ell_0)}} = \sqrt{R/S} \]  

where now \( R \) and \( S \) are based on any characteristic length, say a body diameter or channel diameter, \( D \). The nondimensional parameter, \( \sqrt{R/S} \), represented by the symbol \( R_H \), is the fluid mechanical Reynolds number based on the Hartmann layer thickness \( L_M \).

One can look at this another way. In the MHD equation of motion (2) there are two independent nondimensional parameters, \( R \) and \( S \). These can be rearranged into \( H_M^2 = RS \) and either \( R \) or \( S \). It can be claimed that the interactions of interest take place over a region in which \( H_M^2 = \{j \times B/viscous\} = 0(1) \). This region is of extent \( L_M \), thus recovering Eqs. (8) and (9).

Lykoudis noticed the similarity of the velocity profiles in the case of the laminar sublayer with mass suction and the laminar channel flow with transverse magnetic field. By taking over the results for the termination of the laminar sublayer, he was able to establish a criterion for transition in the MHD channel flow case. This criterion depended solely on the parameter \( R/H_M = \sqrt{R/S} = R_H \). And, by using the value of the transition Reynolds number for the zero field laminar sublayer, he was able to predict the value of \( R_H \), which agreed with Murgatroyd's experiments to be described next.

C. Incompressible Experiment

The available relevant experiments are incompressible: (1) channel flow with transverse magnetic field and (2) a sphere in free flight with magnetic field aligned with the flight direction. In this discussion \( S \) is based on the assumption that the velocity perpendicular to \( B \) is of the order of the free stream velocity. This is satisfied in the channel flow with transverse field, and also in the sphere base flow with field aligned to the flight direction since the base vortex ring has velocities transverse to the field which are of the order of the free stream velocities, Fig. 2. Further downstream behind the sphere the flow tends to be parallel to the field direction. MHD vorticity interaction with instabilities in the laminar far wake with a flight aligned magnetic field is a separate problem which would have as its channel analogue pipe flow transition with a flow aligned magnetic field. (This last problem has been attacked experimentally by Globe and theoretically by Stuart, and the MHD effect was significantly less than that to be derived below.)
Murgatroyd\textsuperscript{3} performed experiments in which measurements of pressure gradients were taken during the flow of mercury in a rectangular channel of 15:1 aspect ratio in the presence of a transverse magnetic field. The experiment was designed to study the relationship between pressure gradient, mean flow velocity, and field strength. He found that it was possible to suppress turbulence even at the highest obtainable fluid mechanical Reynolds number ($R \approx 25,000$ where the characteristic length was taken as the channel half-width).

Murgatroyd referenced Lundquist's reasoning given in the previous section and plotted his data as $C_f$, the usual skin friction coefficient, against $H_M/R = (\sqrt{R/S})^{-1}$. These experiments show that over a range of $S$ from 1 to 0.4 and for $R$ based on channel half-width of order 25,000, $C_f$ is a function of $RH = \sqrt{R/S}$ alone for $\sqrt{R \cdot S} > 100$. Transition was noted as occurring at a definite value of the parameter, $R_H^X = \sqrt{R/S}^X \approx 225$. $R \times B$ is like 25,000. This agrees with Lundquist's criterion, since for the ordinary fluid mechanical zero field case, $R^X, B = 0$ is like 200. (The value of $R_H^X$ predicted by Lydoudis\textsuperscript{6} was 236.)

Maxworthy\textsuperscript{2} performed a series of experiments in which the drag experienced by metal spheres of several different diameters was measured by determining their terminal velocities as they fall, vertically, through an electrically conducting fluid (liquid sodium) and a solenoid magnetic field. Induction coils were used to detect the moving, perturbed magnetic field associated with the sphere, allowing accurate determination of the latter's position in space and time and the gross nature of the perturbed field.

Now, according to Lundquist's suggestion we expect the onset of wake unsteadiness to occur in the MHD sphere wake at the value of $RH = \sqrt{R/S}$ equal to that value of $R$ at which it occurs in the zero field case. The experiments of Magarvey and Bishop as analyzed by Fay and Goldburg\textsuperscript{9} yield ($R$ based on diameter)

$$R_{\text{crit}} B = 0 \approx 200.$$ \hfill (10)

Thus, the prediction in the MHD case for $S = 0(1)$ or $RS >> 0(1)$ is

$$R_{H}^{\text{crit}} \approx 200.$$ \hfill (11)

Maxworthy's experiments tend to confirm this. Table II is a listing of the test conditions for 11 flights. The $R_{H}^{\text{crit}}$ appears to be between 111 and 143. At $R_{H} \approx 100$ the coil output is smooth and at $R_{H} > 200$ the coil output shows periodicity.\textsuperscript{2} The standard quantitative measurement
of wake periodicity in the zero field case is the behavior of the Strouhal number ($S_t$) defined at the "shedding" frequency ($f$) times the body diameter ($D$) divided by the free stream velocity ($V$) with $R$. The behavior for the zero field case is discussed in Fay and Goldburg\textsuperscript{9} where it is shown that a Strouhal number of 0.1 is observed at an $R$ of about 300. Maxworthy points out that the Strouhal number of the run at an $R_H$ of 210 is also 0.1. It is important to note that the fluid mechanical diameter Reynolds number, $R$, in all of these cases is greater than an order of magnitude above $R_{crit}$ for the $B = 0$ case: 5 to 10 thousand compared with 200.

Table II. MHD SPHERE WAKE\textsuperscript{2}

<table>
<thead>
<tr>
<th>$R_H = \sqrt{R/S}$</th>
<th>R Dia.</th>
<th>Downstream Wake</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.8</td>
<td>5330</td>
<td>Steady</td>
<td>6.95</td>
</tr>
<tr>
<td>35.7</td>
<td>2140</td>
<td>Steady</td>
<td>1.68</td>
</tr>
<tr>
<td>62.5</td>
<td>7270</td>
<td>Steady</td>
<td>1.60</td>
</tr>
<tr>
<td>105.0</td>
<td>5600</td>
<td>Steady</td>
<td>0.48</td>
</tr>
<tr>
<td>111.0</td>
<td></td>
<td>Steady</td>
<td></td>
</tr>
<tr>
<td>143.0</td>
<td>3290</td>
<td>?</td>
<td>0.161</td>
</tr>
<tr>
<td>143.0</td>
<td>10000</td>
<td>Unsteady</td>
<td>0.49</td>
</tr>
<tr>
<td>210.0</td>
<td>11200</td>
<td>(Strouhal=0.10)</td>
<td>0.24</td>
</tr>
<tr>
<td>222.0</td>
<td>11000</td>
<td>Unsteady</td>
<td>0.223</td>
</tr>
<tr>
<td>286.0</td>
<td>21000</td>
<td>Unsteady</td>
<td>0.254</td>
</tr>
<tr>
<td>500.0</td>
<td>3720</td>
<td>Unsteady</td>
<td>0.015</td>
</tr>
</tbody>
</table>

* $R_{crit}$\textsubscript{B = 0} $\approx$ 200 (Ref. 9).

Figure 4 shows $C_D$ for a sphere as a function of $R$ alone when $B = 0$ (the classical result\textsuperscript{10}), and as a function of $R_H$. For the conditions of Maxworthy's experiments, $R_S \geq 2400$, it is seen that $R_H$ correlates the data well (parenthetically, $S < 0(1)$), and that for the same values of $R_H$ and $R$\textsubscript{B = 0} the corresponding values of $C_D$ are always within a factor of two of each other.
**D. The Number }$R_H = \sqrt{R/S}\)**

The above evidence suggests that an indication of how the MHD vorticity interaction problem scales with an $S \approx 0(1)$ or $RS \gg 0(1)$ may be obtained by replacing the usual fluid mechanical Reynolds number of the zero field case with the Hartmann layer Reynolds number, $R_H = \sqrt{R/S}$. This broad role for the number $R_H$ prompts one to search for a general, though perhaps imprecise, deduction of $R_H$ from the equations of motion. Equation (2) might be characterized in the following way:

$$\begin{align*}
\text{(inertia terms)} &= \frac{1}{R} \left( \frac{\text{viscous forces}}{\text{inertia forces}} \right) \cdot \text{(viscous terms)} \\
+ \left\{ \frac{S}{R} \left( \frac{j \times B \text{ forces}}{\text{inertia forces}} \right) \right\} \cdot \text{(j \times B terms)}
\end{align*}$$

Equation (12)

In ordinary incompressible viscous fluid mechanics, most effects scale with the ratio of the first power of driving force to retarding force, i.e., $R$. In the combined MHD vorticity interaction case, one can make a similar number in two ways: an arithmetic combination, or a geometric combination, $R^q/S^{1-q}$, where $q$ is an integer between 0 and 1. The following table lists three geometric combinations.

<table>
<thead>
<tr>
<th>Type of Fluid Dynamics</th>
<th>$RS = \left{ \frac{j \times B \text{ forces}}{\text{viscous forces}} \right}$</th>
<th>$q$</th>
<th>Correlating Parameter $\left( \frac{R^q/S^{1-q}}{\text{driving force}} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ordinary viscous</td>
<td>$0$</td>
<td>$1$</td>
<td>$R$</td>
</tr>
<tr>
<td>combined viscous</td>
<td>$0(1)$</td>
<td>$1/2$</td>
<td>$R_H$</td>
</tr>
<tr>
<td>magneto</td>
<td>$\infty$</td>
<td>$0$</td>
<td>$1/S$</td>
</tr>
</tbody>
</table>

It is curious that although this correlating parameter has appeared in an important way in both theoretical and experimental investigations, it has not been singled out as such. The $q = 1/2$ case has usually been written in terms of the Hartmann number $R_H = H_M/S$. This seems to obscure its fundamental nature as the ratio:

$$R_H = \sqrt{R/S} = \left[ \frac{\text{inertia}}{\text{viscous} \cdot j \times B} \right]^{1/2} = \left[ \frac{\text{driving force}}{\text{retarding force}} \right]^{1/2}$$

Equation (13)
II. Hypersonic Magnetogas Dynamic Wake Experiments

In the above discussion we have suggested that an indication of how the MHD vorticity interaction problem scales with an $S \approx 0(1)$ or $RS > 0(1)$ may be obtained by replacing the usual fluid mechanical Reynolds number of the zero field case with the nondimensional parameter, $R_H = \sqrt{R/S}$. For compressible flow, it is usual to extend the incompressible result to the compressible case with the qualification that the representative local values rather than free stream values are used. Values characteristic of the base region conditions are taken as representative for the present purposes.

A hypersonic counterpart of Maxworthy's incompressible experiment is a ballistic range experiment in which pellets fly through an aligned solenoid magnetic field. Evaluation of the relevant MHD parameters in the base region of hypersonic pellets was made for the following assumptions of representative aerodynamics base region conditions (b).

\begin{align*}
P_b &= 5P_\infty \\
V_b &= V_\infty \\
h_b &= \frac{1}{2} h_{\text{stagnation}} \\
\rho_b &= \frac{1}{10} \rho_b \text{ giving Spitzer conductivity}
\end{align*}

where $h$ is the enthalpy and $N_b$ is the total number of particles per unit volume. Argon gas is taken to be the working fluid to maximize gas luminosity and electrical conductivity.

Calculations for the base region values for the parameters $S/B^2 \ell$, $H_M/B \ell$ and $R_H \cdot B$ are shown on a pressure-velocity grid in Fig. 5; $(S/\ell)\omega_T = 1.0$, $(H_M/\ell)\omega_T = 1.0$ and $(R_H)\omega_T = 1.0$ in Fig. 6, and $\omega_T/B$ and $R_M/\ell$ in Fig. 7. (A1) and (A2) are justified on the basis of theory and experiment, the velocity referring to the expansion of flow around the flare and along the base region. (A3) has not been checked definitively, but it seems reasonable and is a commonly used representative value. (A4) is based on a long residence time for fluid in the base region. (A5) is the most questionable assumption. It is based on the assumed presence of large amounts of easily ionizable ablation products.

Fay and Goldburg\textsuperscript{9} indicate that transition from laminar to turbulent wakes behind 0.22-inch diameter spheres in argon at velocities like 14,000 ft/sec occurs at a Reynolds number based on shoulder conditions of 2000. The corresponding base region value is like 600. The corresponding ambient pressure is like 1 cm Hg.
The suggested criterion for suppression of wake transition is that the fluid mechanical Reynolds number based on Hartmann layer thickness, $R_H$, be less than the ordinary fluid mechanical Reynolds number $R_X, 0$. For a range pressure of 5 cm Hg and velocities like 14,000 ft/sec, this condition is met when the magnetic field strength $B$ is chosen to make $\omega_T = 1.0$: $R_H \approx 75 < R_X, 0 \approx 600; (S \approx 0(1), RS \approx 0(10^3))$ fulfilling the requirements for an MHD vorticity interaction.

A set of MHD parameter maps assuming that the gas in the base region is ionized argon alone is contained in Appendix A. These maps indicate that an experiment in the ballistic range is not feasible if the gas properties are assumed to be due solely to ionized argon, Fig. A-2. In the pure argon case, for the same ballistic range conditions, $R_H/\omega_T = 1.0 > 1000$.

The experiment to explore the possibility of suppressing hypersonic wake transition is shown in Fig. 8. Nylon pellets of 0.22-inch diameter are fired at velocities near 14,000 ft/sec through the magnetic field of a solenoid placed about a glass test section in the AERL ballistic range. The magnetic field of the solenoid is produced by passing a current through the wire by the discharge of a capacitor bank. The inductance of the coil and capacitance of the bank were chosen so that the discharge quarter cycle time would be 130 microseconds. This ringing time of the capacitor bank is sufficiently long to produce nearly DC current, while the pellet is in the coil. A maximum magnetic field strength of 2.6 webers/meter$^2$ is obtained on the coil centerline with the present design. The test and coil section are made of transparent material, pyrex and lucite, in order that streak photographs can be taken when the pellet is inside of the solenoid, Fig. 9. The use of streak drum camera photographs to differentiate between laminar and turbulent wakes$^9, 11$ has been reported previously. The streak camera is oriented such that its field of view encompasses a region before and including part of the coil. The appearance of luminous structure in drum camera photographs is interpreted as turbulent wake flow whereas smooth and diffuse luminosity is interpreted as laminar flow.

The base MHD experiment was composed of two sets of runs separated by runs for another experiment on the AERL range. In the first set of runs, a single streak drum camera was oriented such that its field of view encompassed a region before and including approximately one-half of the coil. In the second set, a second drum camera was added so that all of the coil region as well as some region upstream of the coil could be photographed.

The very first runs were made with the coil unenergized. A laminar and turbulent streak photograph is shown in Fig. 10 and 11, respectively. The streak photographs of the pellet within the coil provide sufficient information to reveal the absence or presence of luminous turbulent structure in the wake. The initial vertical break in the streak photograph is the downstream coil support face plate. The subsequent vertical breaks are the wire turns of the solenoid. The luminous streaks which characterize turbulent flow can be recognized within the coil in Fig. 11.
A streak photograph with the coil energized is shown in Fig. 12. Such a photograph was obtained in each of eight runs (Fig. 13). Turbulent luminous streaks are evident in the region before the solenoid where the magnetic field strength is vanishingly small. As the pellet enters the magnetic field of the coil, a definite change in the structure of the luminous flow field occurs. The luminous streaks which characterize the turbulent flow diminish and disappear and a region which is smooth and devoid of streaks appears. The pellet position at which the maximum magnetic field strength is reached is shown with the band of uncertainty. The lower image which runs parallel to the pellet in the figure was usually observed. From the information obtained in this first set of runs, it appears that the originally turbulent wake flow is substantially affected by the presence of the magnetic field, perhaps suggestive of a return to a laminar wake flow.

This first series of MHD runs raised the following questions:

A. Was the disappearance of the lumionous streaks as the pellet entered the coil in the MHD runs possibly related 1) to an end effect due to the variation of the magnetic field intensity outside of a solenoid; 2) to a stagnation point effect in which the heat transfer to the body was decreased by MHD effects, thus eliminating the luminous "dye" which previously traced the flow pattern; 3) to the presence of objects in the wake of the pellet; or 4) to some other effect, say to one related to the discharge of the LC circuit?

B. What was the origin of the second luminous streak which runs parallel to that of the pellet?

With respect to question A, calculations tend to indicate that the magnetic field end effects and heat transfer effects could not have produced the observed large changes in luminosity.

A second set of experiments was made to help answer the preceding questions. The diagnostic instrumentation was improved with the addition of a photoelectric recorder and shadowgraph. The shadowgraph established the integrity of the model and the location of any other bodies relative to the pellet. Also, a technique of scribing the diaphragm of the light gas gun was developed which eliminated all debris from the wake of the pellet. With respect to question B, additional data taken with streak camera and photoelectric recorders with the coil unenergized indicated that the second image is real and correlates with the focusing of the reflected shock wave from the sides of the pyrex tube test section. The focusing reheats the wake gas producing a region of enhanced luminosity which trails the pellet at a fixed distance.

The effect on wake luminosity in the first set of runs shown in Figs. 12 and 13 did not reappear in the second set. A sufficient number of runs were made at both 5.0 and 2.0 cm Hg pressure to verify that no effect was evidently present. Essentially, all of the runs in the second set with and without
magnetic fields, produced drum camera photographs which appeared as those shown in Fig. 14.

Two significant differences existed between the first and second set of runs. First, was the elimination of the diaphragm debris from the wake of the pellet. It might be conjectured that an interaction of the debris with the magnetic field in some way caused what appeared to be an MHD effect. The diaphragm is 0.015 inches thick and is made of aluminum.

Second, there was an uncontrollable change in the conditions within the range between the first and second set of runs. The first set of runs were made while a sodium vapor oven was attached just upstream of the MHD test section. This was due to the fact that runs for a sodium schlieren experiment were being shot alternately with the base-MHD runs. Although the oven was shut off for the base-MHD experiment, and the oven cleaned, some solid sodium may still have remained in that part of the test section. Between the first and second set of the base-MHD experiment, another ballistic range experiment was carried out which required a clinically clean test section. At that time, the sodium vapor oven was permanently removed from the range, and the range was thoroughly cleaned.

The second set of runs for the base-MHD experiment took place in a clean range in contradistinction to the first set which took place in a range, a part of which may have been coated with solid sodium. As pointed out previously, the assumption of easily ionizable particles leading to Spitzer conductivity (the largest possible) provides $R_H < R_X$, $0$, Fig. 6; while the lack of such particles leads to conductivities which provides $R_H > R_X$, $0$, Fig. A-2. Thus, the controlling factor may have been the respective presence and then absence of sodium dust in the range.

An estimate of the number of 0.1 micron diameter sodium dust particles required to produce the Spitzer concentration of electrons in the base region has been made. Particle sizes of like 0.1 micron diameter remain suspended in the atmosphere for long periods of time due to the effects of Brownian motion. A sodium sphere, 0.1 micron in diameter has a mass of $10^{-16}$ grams and produces $10^7$ electrons when completely ionized. Therefore, $10^8$ sodium particles per cubic centimeter of 0.1 micron diameter are required to produce the $5 \times 10^{15}$ electrons per cubic centimeter for Spitzer conductivity. Engineering handbook values for average dust concentrations in the atmosphere show that $10^8$ particles per cubic centimeter is well within engineering expectations.

Finally, it is possible that (1) one of the other assumptions which went into the calculations is unreasonably optimistic or, (2) that the suggested single criterion $R_H < R_X$, $0$ is too simple for the hypersonic case.
ACKNOWLEDGMENT

The authors wish to thank Professor J. A. Fay for suggesting the experiment and for his encouragement. Also, they wish to thank Mr. P. H. Rose for his interest and aid.
APPENDIX A

MHD Maps for Ballistic Range Experiment. Argon Only.

The MHD parameters in the base region of a pellet, assuming ionized argon only, are calculated using the following assumptions:

\[ P_b = 5P_\infty \]
\[ V_b = V_\infty \]
\[ h_b = \frac{1}{2} h_{stagnation} \]

Equilibrium Chemistry.

The electrical conductivity of the ionized argon gas is calculated using the conductivity equation of Lin, Resler and Kantrowitz. The number of electrons present was determined from the degree of ionization at equilibrium from Saha's equation. The resultant conductivity from Lin, Resler and Kantrowitz differs significantly from that given by Spitzer and Harm only when the temperature is below 6000°K.

The base region values for the parameters \( S/B^2 \), \( H_M/B \ell \), and \( R_H \cdot B \) are shown on a pressure velocity grid in Fig. A1; \( (S/\ell) \omega_T = 1.0 \), \( (H_M/\ell) \omega_T = 1.0 \) and \( (R_H) \omega_T = 1.0 \) in Fig. A2; and \( \omega_T / B \) and \( R_M / \ell \) in Fig. A3.
REFERENCES


Fig. 1  Schematic of a vortical velocity distribution with a uniform transverse magnetic field.
Fig. 2  Schematic of the transverse MHD vorticity interaction in the base region of a sphere with flight velocity aligned with a uniform magnetic field.
Fig. 3  Results of Murgatroyd's experiments: Channel flow of mercury with transverse magnetic field. $C_f$, the usual skin friction coefficient, against $H_M/R = (\sqrt{R/S})^{-1}$. $L$ is the channel half width.
Fig. 4  $C_D$ vs $H_M/R$ and $R_H$ after Maxworthy\(^2\) and $R|_B = 0$ after Goldstein.\(^{10}\)
Fig. 5  MHD pressure-velocity Maps for Ballistic Range Experiment: 
Fig. 6 MHD pressure-velocity Maps for Ballistic Range Experiment: 
$$(R_H)_{\omega T} = 1, \ (H_M/\ell)_{\omega T} = 1, \ (S/\ell)_{\omega T} = 1.$$ Calculations Based on Assumptions A1 - A5.
Fig. 7  MHD pressure-velocity Maps for Ballistic Range Experiment: $\frac{R_M}{l}, \frac{\omega \tau}{B}$. Calculations Based on Assumptions A1-A5.
Fig. 8  Schematic of MHD Base Experiment.
Fig. 9  Magnetic Coil Test Section.
Fig. 10  Laminar Drum Camera Photograph. Argon Gas. No MHD.

Fig. 11  Turbulent Drum Camera Photograph. Argon Gas. No MHD.
Fig. 12 Turbulent Drum Camera Photograph. Argon Gas. With and Without MHD. FIRST SET.
FREE STREAM

\( P_{x,0} \approx 1 \text{ CM Hg} \)

\[
\begin{align*}
B &= 1.25 \text{ WEBERS/M}^2 \\
B &= 1.25 \text{ WEBERS/M}^2 \\
B &= 1.25 \text{ WEBERS/M}^2 \\
B &= 2.5 \text{ WEBERS/M}^2 \\
B &= 2.5 \text{ WEBERS/M}^2 \\
B &= 2.5 \text{ WEBERS/M}^2 \\
B &= 2.5 \text{ WEBERS/M}^2 \\
B &= 2.5 \text{ WEBERS/M}^2 \\
B &= 2.5 \text{ WEBERS/M}^2
\end{align*}
\]

\[
\begin{align*}
P_{\infty} &= 5.0 \text{ CM Hg} \\
P_{\infty} &= 5.0 \text{ CM Hg} \\
P_{\infty} &= 5.0 \text{ CM Hg} \\
P_{\infty} &= 5.0 \text{ CM Hg} \\
P_{\infty} &= 5.0 \text{ CM Hg} \\
P_{\infty} &= 5.0 \text{ CM Hg} \\
P_{\infty} &= 5.0 \text{ CM Hg} \\
P_{\infty} &= 5.0 \text{ CM Hg}
\end{align*}
\]

\[
\begin{align*}
V_{\infty} &= 14,265 \text{ FT/SEC} \\
V_{\infty} &= 13,650 \text{ FT/SEC} \\
V_{\infty} &= 13,880 \text{ FT/SEC} \\
V_{\infty} &= 12,400 \text{ FT/SEC} \\
V_{\infty} &= 12,900 \text{ FT/SEC} \\
V_{\infty} &= 13,550 \text{ FT/SEC} \\
V_{\infty} &= 12,000 \text{ FT/SEC}
\end{align*}
\]

BASE REGION

\[
\begin{align*}
R_h &= 100 \\
H_M &= 30 \\
S &= 0.3 \\
R_M &= 0.08 \\
\omega T &= 0.8
\end{align*}
\]

\[
\begin{align*}
R_h &= 50 \\
H_M &= 60 \\
S &= 1.2 \\
R_M &= 0.08 \\
\omega T &= 1.6
\end{align*}
\]

Fig. 13 Summary of Streak Photographs with MHD. FIRST SET. The arrow represents the calculated position of maximum B. The uncertainty of the location is four wire segments to each side of the arrow.
**Fig. 14** Laminar and Turbulent Drum Camera Photographs with MHD. Argon Gas. SECOND SET.
Fig. A2  MHD Base Maps - Argon Only.
Fig. A3  MHD Base Maps - Argon Only.
DISTRIBUTION LIST for Contract No. AF 04(64)-414

   C. E. McCall (1 copy)
   Mr. Hertfie11 (1 copy)

Defense Research Laboratories, General Motors Corporation, Santa Barbara, California - Attn: Cam Scharr (1 copy)

   BSTA (1 copy)

Air Force Cambridge Research Labs., Laurence G. Hanscom Field, Bedford, Massachusetts - Attn: Lew Block (1 copy)

Aerospace Corporation, Post Office Box 1108, San Bernardino, California - Attn: Paul Doherty (2 copies)

Systems Engineering Group (SEP), Wright-Patterson Air Force Base, Ohio 45435

Arnold Engineering Development Center, Arnold Air Force Station, Tennessee - Attn: AEVD (1 copy)
   ARS (1 copy)

MIT Lincoln Laboratories, Post Office Box 4188, San Bernardino, California - Attn: J. Vernor (1 copy)

U. S. Air Force Weapons Laboratory, Kirtland Air Force Base, Albuquerque, New Mexico - Attn: SWOC (1 copy)
   WLAAX (1 copy)

Defense Documentation Center, Cameron Station, Alexandria, Virginia - (20 copies)

General Electric Company, Missile and Space Division, 3198 Chestnut Street, Philadelphia, Pennsylvania - Attn: J. Pezah (1 copy)

General Electric Company, Valley Forge Space Technology Center, Space Sciences Laboratory, Post Office Box 8595, Philadelphia, Pa.
   Lawrence L. Chace, Manager
   MSTD Library (2 copies)

Headquarters U.S. Army Missile Command, Redstone Arsenal, Alabama - Attn: A. Jenkins (ASMSM-RRX) (1 copy)

Massachusetts Institute of Technology, Lincoln Laboratory, Post Office Box 73, Lexington, Massachusetts - Attn: G. B. Pippert (1 copy)
   Mary A. Granese, Document Librarian (1 copy)
   Dr. Ellen Bressel (1 copy)
   Dr. Frank McNamara (1 copy)

University of Michigan, Institute of Sciences and Technology, Post Office Box 618, Ann Arbor, Michigan - Attn: Infrared Information and Analysis Group (1 copy)
   BAMIRAC Library (1 copy)
   Richard Jamron (1 copy)


Polytechnic Institute of Brooklyn Aero. Lab., 527 Atlantic Ave., New York, N.Y., 11201 (1 copy)
   New York - Attn: Prof. Martin Bloom (1 copy)

Helioide Corporation, 2365 Westwood Boulevard, Los Angeles 64, California - Attn: Dr. Saul Feldman (1 copy)

Hughes Aircraft Company, Group Systems Group, Fullerton, California - Attn: Library, Bldg. 604 (1 copy)

Hughes Aircraft Company, Florence and Teale, Culver City, California - Attn: Mr. Nicholas F. Devereaux, Technical Document Center (1 copy)

The Mitre Corporation, Post Office Box 208, Bedford, Massachusetts - Attn: Library (1 copy)

Plasmadyne Corporation, 3429 South Main Street, Santa Ana, California - Attn: Document Control (1 copy)

Aeronautical Research Associates of Princeton, Inc., 50 Washington Road, Princeton, New Jersey - Attn: Dr. Coleman deP. Donaldson (1 copy)

Sandia Corporation, Livermore Laboratory, Post Office Box 969, Livermore, California - Attn: Technical Library (1 copy)

Stanford Research Institute, Menlo Park, California - Attn: Acquisitions (1 copy)

Bendix Corporation, Bendix Products Division, 3300 West Sample Street, South Bend, Indiana - Attn: M. Katz (1 copy)

Bendix Corporation, Bendix Systems Division, 3100 Plymouth Road, Ann Arbor, Michigan - Attn: Library (1 copy)

The Boeing Company, Aerospace Division, P.O. Box 3707, Seattle 24, Washington - Attn: Library Unit Chief (1 copy)

Chrysler Corporation, Missile Division, Post Office Box 2628, Detroit 31, Michigan - Attn: Technical Information Center (1 copy)

General Dynamics/Astronautics, Post Office Box 1128, San Diego 12, California - Attn: Library and Information Services (128-001) (1 copy)

Cornell Aeronautical Laboratory, 444 Geneva Street, Buffalo, New York - Attn: Library (1 copy)

Defense Research Corporation, 4040 State Street, Santa Barbara, California - Attn: Technical Information Office (1 copy)

Grumman Aircraft Company, 1001 Corporate Park Boulevard, Santa Monica, California - Attn: Library (1 copy)

Electro-Optical Systems, Inc., 125 North Via De Aqua Avenue, Pasadena, California - Attn: Mr. M. Richard Dentum, Head, Aerospace Physics Dept., Fluid Physics Division (1 copy)

Avco Corporation, Research and Advanced Development Division, 201 Lowell Street, Wilmington, Massachusetts - Attn: D. Walker (1 copy)
   R. Dohre (1 copy)
   J. Luceri (1 copy)

Best Available Copy
Maxworthy's experiments with the flight of a sphere through an incompressible, electrically conducting fluid with an aligned magnetic field showed that transition in the trailing wake was more to higher fluid mechanical Reynolds numbers. Theoretical and experimental work on suppression of vorticity in uniformly conducting incompressible fluids with imposed uniform transverse magnetic fields is analyzed to find a correlation parameter for Cg. Cg transition and turbulence. The parameter which is found is the square root of the fluid mechanical Reynolds number (Rf) to the electromagnetic interaction parameter (Rs). Current work on the foundations of this theory and the close relationship of the problems to the creative one is based on the observed similarity of the transition processes in the hypersonic and incompressible wakes as suggested by Fay and Goldburg. Average values of the fluid properties in the base region were chosen for typical values of hypersonic wake transition Reynolds numbers, Rf and Rs. The presence of large amounts of easily ionizable particle products were assumed to give a unique solution to the characteristic problem. The calculated values for the relevant nondimensional fluid mechanical and electromagnetic parameters for typical baseline range conditions indicated that there were of sufficient magnitude to affect the flow. Hypersonic experiments were performed covering the range from Rs = 30 Rr, with zero field to Rs = 5 Rr, where Rs is the zero field transition Reynolds number. The hypersonic flow fields were produced by 0.22 g steel sphere at 70,000 feet per second. In the first set of runs at 5 and 7.5 cm pressure, a change in luminous structure of the flow field was observed suggestive of a change from turbulent to laminar. In the second set of experiments made at 10 cm and 2.1 atm pressure, no BHI effect on the hypersonic turbulent flow field was evident. The paper concludes with a discussion of these results.