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# TRANSLATION

FORMULA FOR REPRESENTING THE PRODUCT OF ORIGINALS

By

V. Ya. Natanson

## FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE

OHIO

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# FORMULA FOR REPRESENTING THE PRODUCT OF ORIGINALS

V. Ya. Natanson

(Moscow)

The derivation of a formula of the product of originals, as it is known, leans on the inversion formula. We shall show how one can obtain another formula of the product.

Let $f(x) = \varphi(x)h(x)$, $F(p) \div f(x)$ and

$$F(p) = p\int_0^\infty \varphi(t)\, h(t)\, e^{-pt}\, dt \qquad (1)$$

We shall introduce the original $\varphi(p)$, which we shall designate by $\Phi(t)$ ([1] page 283), i. e., let

$$\varphi(p) = p\int_0^\infty \Phi(\xi)\, e^{-p\xi}\, d\xi \qquad (2)$$

Replacing p by t in this equality we substitute expression $\varphi(t)$ in formula (1):

$$F(p) = p\int_0^\infty h(t)\, t e^{-pt}\, dt \int_0^\infty \Phi(\xi)\, e^{-t\xi}\, d\xi$$

In the second integral we shall replace variable $u = \xi + p$ and change the order of integration; then we shall obtain

$$F(p) = p\int_p^\infty \Phi(u-p)\, du \int_0^\infty h(t)\, t e^{-tu}\, dt$$

Let $H(p) \dot{\to} h(t)$, then

$$\left[\frac{H(u)}{u}\right]' = -\int_0^\infty h(t)\, t e^{-ut}\, dt \qquad (3)$$

consequently, finally we shall obtain such a formula:

$$F(p) = -p \int_{p}^{\infty} \Phi(u-p) \left[\frac{H(u)}{u}\right]' du \qquad (4)$$

From formula (4) we can obtain an expression of the integral from the product of two functions. Indeed, we have, obviously, the equality

$$\frac{F(p)}{p}\bigg|_{p=0} = \int_{0}^{\infty} \varphi(t) h(t)\, dt = -\int_{0}^{\infty} \Phi(u) \left[\frac{H(u)}{u}\right]' du \qquad (5)$$

Formula (4) is easily generalized in the case of the product of three and more numbers of functions. Let, for example

$$f(t) = \varphi(t)\varphi_1(t)h(t)$$

$$F(p) \div f(t),\ \varphi(p) \div \Phi(t),\ \varphi_1(t) = \Phi_1(t)$$

Then, analogous to the preceding, we shall write

$$F(p) = p\int_{0}^{\infty} h(t)\varphi_1(t)\varphi(t)e^{-pt}\,dt =$$

$$= p\int_{0}^{\infty} t^2 h(t)\,dt \int_{0}^{\infty} \Phi_1(\xi)e^{-t\xi}\,d\xi \int_{0}^{\infty} \Phi(\eta)e^{-t(\eta+p)}\,d\eta$$

Replace the variable in the third integral u= $\eta$ +p, then change the order of integration, after which it is possible to write

$$F(p) = p\int_{p}^{\infty} \Phi(u-p)\,du \int_{0}^{\infty} t^2 h(t)\,dt \int_{0}^{\infty} \Phi_1(\xi)e^{-t(u+\xi)}\,d\xi$$

In the last integral we make the following replacement of the variable integration: (v= u + $\xi$,) then we shall change the order of integration. After that

$$F(p) = p\int_{p}^{\infty} \Phi(u-p)\,du \int_{u}^{\infty} \Phi_1(v-u)\,dv \int_{0}^{\infty} t^2 h(t)e^{-tv}\,dt$$

Finally, let us note that

$$\int_{0}^{\infty} t^2 h(t)e^{-tv}\,dt = \left[\frac{H(v)}{v}\right]'' \qquad (H(p) \div h(t))$$

Thus, we obtain a formula representing the product of three originals:

$$F(p) = p\int_{p}^{\infty} \Phi(u-p)\,du \int_{u}^{\infty} \Phi_1(v-u)\left[\frac{H(v)}{v}\right]'' dv \qquad (7)$$

Submitted
19 May 1956

Literature

1. B. Van der Pol' and H. Bremmer. Operation calculus on the basis of a bilateral Laplace transform. Foreign Literature Publishing House, 1952.

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