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SUMMARY

A simplified oil refinery example is used to illustrate the principles of building a linear programming model.

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FORMULATING A LINEAR PROGRAMMING MODEL

G. B. Dantzig

I. Linear Programming Defined

One of the reasons why the programming tool has assumed importance, both in industry and in the military establishment, is that it is a method for studying the behavior of systems. In philosophy it is close to what some describe as the distinguishing feature of management science or operations research, to wit: "Operations are considered as an entity. The subject matter studied is not the equipment used, nor the morale of the participants, nor the physical properties of the output, it is the combination of these in total as an economic process."*

To many the term "linear programming" refers to mathematical methods for solving linear inequality systems. While this may be the <u>central</u> mathematical problem it is not its definition. Linear programming is a technique for building a model for describing the interrelations of the components of a system. As such it is probably the simplest mathematical model that can be constructed of any value for broad programming problems of industry and government. Thus the importance of the linear programming model is that it has wide applicability.

Suppose that the system under study (which may be one actually in existence or one which we wish to design) is a complex of machines, people, facilities, and supplies. It has certain

Operations Research for Management, C. C. Hermann and (J. F. Magee, Harvard Bus. Rev., July, 1953.



over-all reasons for its existence. For the military it may be to provide a striking force or for industry it may be to produce certain types of products.

The linear programming approach is to consider the entire system as decomposable into a number of elementary functions called "activities"; each type of activity is abstracted to be a kind of "black box" into which flow tangible things such as supply and money and out of which may flow the products of manufacture or trained crews for the military. What goes on inside the "box" is the concern of the engineer or the educator, but to the programmer only the rates of flow in and out are of interest.

The next step in building a model is to select some unit for measuring the quantity of each activity. For a production type activity it is natural to <u>measure</u> the quantity of the activity by the amount of some product produced by it. This quantity is called the <u>activity level</u>. To increase the activity level it will be necessary, of course, to increase the flows into and out of the activity. <u>In the linear programming model the</u> <u>quantities of flow of various items into and out of the activity</u> <u>are always proportional to the activity level</u>. Thus it is only necessary to know the flows for the <u>unit activity level</u>. If we wish to <u>double the activity level</u>, we simply <u>double</u> all the corresponding flows for the unit activity level.

While any <u>positive</u> multiple of an activity is possible, <u>negative quantities of activities are not possible</u>. The Mad Hatter, you may recall in "Alice of Wonderland," was urging



Alice to have some more tea, and Alice was objecting that she couldn't see how she could take <u>more</u> when she hadn't had any. "You mean, you don't see how you can take <u>less</u> tea," said the Hatter, "it is very easy to take more than nothing." Lewis Carrol's point, of course, is that the activity of "taking tea" cannot be done in negative quantity.

One of the items in our system is regarded as <u>precious</u> in the sense that the total quantity of it produced by the system measures the payoff. The contribution of each activity to the total payoff is the amount of the precious item that flows into or out of each activity. Thus if the objective is to maximize profits, activities that <u>require money</u> contribute negatively and those that <u>produce money</u> contribute positively to total profits.

Next, it is required that the system of activities be <u>complete</u> in the sense that a complete accounting by activity can be made of each item. To be precise, for each item it is required that the total amount on hand equals the amount flowing into the various activities minus the amount flowing out. Thus, each item, in our abstract system, is characterized by a <u>material</u> <u>balance equation</u> — the various terms of which represent the flows into or out of the various activities.

The programming problem is to determine values for the levels which are positive or zero such that flows of each item (for these activity levels) satisfy the material balance equations and such that the value of the payoff is maximum. It is clear that what we have done is to reduce the programming problem to

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a well-defined mathematical problem which is called the LINEAR PROGRAMMING MODEL.

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II. The L. P. Model Illustrated

To illustrate these principles of the linear programming approach to model building, let us turn to an application in the petroleum industry where linear programming methods have been very successful. The complicated piece of plumbing of figure 1 is a simplified flow diagram of an oil refinery. The problem facing management is this. By turning valves, setting temperatures, pressures, and starting pumps, crude oil will be drawn from one or several oil fields under the control of the refinery (shown on the left). Like the old song about the music, it "will go around and around" and come out as several streams of pure oils (shown on the right). The latter can be marketed at varying prices. By changing the controls, the quantities in various streams of pure oils can be altered. This will change the costs of operating the equipment and the revenues from the sales of the final products. The various components are interrelated, however, in such a complicated manner, that it is not obvious what is the best way to operate the equipment to maximize profits. In spite of these complex interrelations, when this system is decomposed into elementary functions as the first step in building a model, it turns out that there are essentially only three main kinds of activities taking place: Distillation, Cracking, Blending.

[&]quot;Refinery example taken from a term paper of R. J. Ullman.



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- Distillation Activity: The net effect of the flash tower, heater, fractionating towers, strippers, etc., is to sepparate the crude into varying amounts of pure oils of which it is composed. Crudes drawn from different oil fields will nave different decompositions. Hence there must be separate distillation activity developed for each type crude. The maximum amount of crude that can be distilled depends on which, of the varying pieces of equipment it passes through, will be the bottleneck. In our case we will suppose it is the heater and that it has a fixed capacity of 14,000 bbl's per day independent of type crude processed. From this description it is evident if the level of distillation activity is measured in number of barrels of crude input, then a unit level of activity can be pictures as in Figure 2a. It is seen that I bbl of Crude No. 1 will use 1 bbl of distillation capacity, and will cost \$1.80 (to purchase and to distill); the outputs will be a stream of pure oils in the amount shown. These outputs are princippally the heavier oils; fuel, diesel and stove and smaller amounts of the lighter types used to make gasoline. If instead of 1 bbl, it is desired to distill 10 or X bbls of crude, all input and output quantities of Figure 2a would have to be multiplied by 10 or X.
- <u>Cracking</u>: The net effect of the cracking equipment is to take one of the heavier type oils and to cause it to be broken down into lighter type oils. In the case of fuel oil it will produce a small amount of the lighter types and a larger amount of stove oil which, if desired, can in turn be

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recycled back into the cracker and made into lighter oils. It is seen from Figure 2b that 1 unit of fuel oil requires 1 unit of cracking capacity, will cost \$.16 and will produce the pure oils in the amounts shown on the right. A separate type activity must be set up for cracking of fuel, diesel and stove oils.

Blending: Gasoline is not a pure oil but is a blend of several of the lighter types of pure oil (see Figure 2c). It will be noted the only output shown is the <u>net revenue</u> from marketing 1 bbl of gasoline. The latter is assumed to be the sales price at the refinery less the cost of the blending operation.

Once the flows for these major activities have been determined on a per-barrel basis, it is a simple matter to set up the linear programming model by means of which the managers can détermine the best manner to operate the refinery to maximize profits. In Figure 3 each column represents an activity. The input and output quantities per unit level of activity are shown in the column; to distinguish outputs from inputs, outputs are shown with a minus sign. For example, the data of Figure 2a is shown in column "Distillation --- Crude 1"; the data of Figure 2b is shown in column "Cracking --- Fuel Oil"; the data of Figure 2c is shown in column "Product Marketing --- Gasoline." The other activity columns are self-explanatory. The amounts available of various items to the system are shown on the right.

The unknown activity levels to be determined are denoted by x_1, x_2, \ldots, x_{20} . By multiplying these unknowns by the





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corresponding numbers found in any row and summing the terms across, the total obtained should equal the availability shown on the right.

For example, the first material balance equation reads

 $1.x_1 + 1.x_4 = 9500$,

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which means the amount of crude No. 1 available, 9500 bbls, is completely accounted for by the amount left in the ground, x_1 , plus the amount distilled, x_h .

The fourth material balance equation, referring to the, item distillation capacity, reads simply

$$1.x_{h} + 1.x_{5} + 1.x_{5} + 1.x_{7} = 14,000$$

which means that the distillation capacity of 14,000 barrels is completely accounted for by the amount used in distilling the various types of crude plus any excess capacity not used.

Finally the profit equation states the revenue obtained from marketing various products, $(1.8x_{12} + 4.0x_{13} + 4.2x_{14} + 5.5x_{15} + 4.0x_{16} + 4.1x_{17} + 4.2x_{18} + 4.3x_{19} + 3.3x_{20})$, less the cost of distilling and crude purchases, $(1.8x_4 + 1.9x_5 + 2.0x_6)$, less the cost of cracking, $(.16x_8 + .21x_9 + .21x_{10})$, is the amount of profit. The problem, of course, is to choose the program of activity levels in such a way that the material balance equations are satisfied and the profits maximized.