ESD-TDR-64-112

W-06871

ENGINEERING SIMULATION OF POWERED FLIGHT

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-112

JULY 1964

R. W. Dix

Prepared for

DEPUTY FOR ADVANCED PLANNING
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 611.1

Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts
CONTRACT AF 19 (628)-2390

AD604865

ESD-TDR-64-112 W-06871

ENGINEERING SIMULATION OF POWERED FLIGHT

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-112

JULY 1964

R. W. Dix

Prepared for

DEPUTY FOR ADVANCED PLANNING
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 611.1

Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts
CONTRACT AF 19 (628)-2390

ENGINEERING SIMULATION OF POWERED FLIGHT

ABSTRACT

This document describes the current effort toward establishing a workable engineering simulation of the space-ground environment applicable to a wide variety of missile and space systems. Two powered flight computer programs have evolved from this effort. This report describes the content, inputs and outputs of each of these programs.

REVIEW AND APPROVAL

This technical documentary report has been reviewed and is approved.

Thomas L Ladd, Maj.
for BORDEAN W. CLINGER

Lt. Colonel, USAF
Acting Director of Special Systems
Deputy for Advanced Planning

TABLE OF CONTENTS

ILLUSTRATIONS

ENGINEERING SIMULATION OF POWERED FLIGHT

SECTION I

INTRODUCTION

This document describes the current effort toward establishing a workable engineering simulation of the space-ground environment (without a large expenditure of manpower) applicable to a wide variety of missile and space systems. This activity breaks into three phases.

Phase 1. Powered-flight and free-flight trajectory simulation.

Phase 2. Simulation of the interactions between the space vehicle, other vehicles, and the ground environment. This may take the form of tracking coverage, telemetry, communications, etc.

Phase 3. Engineering utilization of these two basic tools in the solution of a particular problem. This may take the form of evaluating or optimizing the communication capability of a given system, or evaluating the ground environment in relation to a given set or type of missions.

Phases 1 and 2, although evolutionary processes, are essentially complete. Phase 3, also an evolutionary process, has been under study for some time with respect to various systems planning activities.

The term engineering simulation is used to indicate that this simulation has evolved from a certain balance of three factors:

(a) sophistication of the mathematical model,

(b) computer and programing time, and

(c) projected use of the programs.

This report describes the powered flight simulation capability which has resulted from this effort. The report itself describes the capabilities and limitations of the powered flight programs and the input data required to run these programs.

SECTION II

BASIC APPROACH

Two powered flight computer programs have been generated. One has two and the other three translational degrees of freedom. The two-degree-of-freedom program was generated because it is inherently simple and fast to run on the computer, and, at the same time, gives results which are sufficiently accurate for many large parametric systems studies. The three-degree-of-freedom program is, on the other hand, a better simulation of the physical problem and of value when studying a particular system in detail, as might be done in preparation for a flight test program or in feasibility studies when feasibility is critically dependent on booster performance. Each program is written in modular form, using a number of common subroutines. Each subroutine is complete in itself, performing an independent engineering function, thus creating a high degree of flexibility in program construction. Each subroutine for the two programs has been written for the IBM 7090 in Fortran II language and converted to the IBM 7030 in Fortran IV language.

SECTION III

TWO-DEGREE-OF-FREEDOM PROGRAM

A two-degree-of-freedom trajectory model assumes that the vehicle is a point mass and is constrained to move in a plane. This requires that all forces acting out of the plane are zero. If one considers a nonrotating spherical earth, these out-of-the-plane forces will be zero if the thrust, drag, and lift forces are restricted to the plane of the trajectory. The assumption of a nonrotating earth neglects the centrifugal and coriolis accelerations which are, in fact, present in an earth-fixed coordinate system. This assumption of a nonrotating earth is used only in the integration of the equations of motion during the powered flight calculations. The easterly velocity of the launch point is considered in the computation of inertial quantities (i.e., one must launch west of north to fly over the North Pole).

Comparison with the three-degree-of-freedom program (which includes the accelerations due to earth rotation) shows the following errors at the end of powered flight for a typical ICBM vehicle:

Velocity	2.0 percent
Altitude	1.6 percent
Flight path angle	2.5 percent
Range	2.3 percent

These errors propogate through the free-flight phase of the trajectory following powered flight. At apogee, the errors shown on the following page were observed.

Time	5.3 percent
Altitude	3.5 percent
Range	2.0 percent

One must evaluate the engineering problem to be solved through the use of this program in order to determine whether or not these errors are tolerable. It is clear that for detailed design or flight test simulation prior to launch, it would be desirable to use the three-degree-of-freedom program. On the other hand, it appears that for preliminary system design and feasibility studies these errors will not greatly affect the decision-making process. The great savings in computer time during the early planning stages and parametric analyses of a program may well justify the use of this two-degree-of-freedom model. The combination of programs, i.e., the two- and three-degree-of-freedom programs, provides a flexible tool for the analysis of powered flight.

Restriction of the drag and lift forces to the plane of the trajectory implies that the atmosphere rotates with the earth as a solid body and does not slip (i.e., no consideration is given to winds).

The two equations of motion, along and normal to the flight path, are:

$$\dot{V} = -C_D A/m \frac{1}{2} \rho V^2 - g \sin\gamma + \frac{T}{m} \cos\delta, \text{ and}$$

$$\dot{\gamma} = C_L A/m \frac{1}{2} \rho V - g \cos\gamma/V + V \cos\gamma/\left(R_e + h\right) + \frac{T}{m} \sin\delta/V,$$

where

V = velocity,

γ = flight path angle,

C_D = drag coefficient,

C_L = lift coefficient,

A = reference area,

m = mass,

ρ = atmospheric density,

g = gravity = $g_o \left(\frac{R_e}{R_e + h}\right)^2$,

T = vacuum thrust corrected for atmospheric pressure,

δ = thrust deflection,

R_e = radius of the earth,

h = altitude, and

ϵ = range angle.

These equations, coupled with the two auxiliary equations

$$\dot{h} = V \sin \gamma \text{ , and}$$

$$\dot{\epsilon} = V \cos \gamma/(R_e + h) \text{ ,}$$

are integrated using a Runge-Kutta numerical integration. The question of accuracy of this method of integration will be discussed in Section V. The force coefficients, C_D and C_L , are functions of Mach number; and the atmospheric density, ρ , and gravitational acceleration, g , are functions of altitude. Reference axes are shown in Fig. 1. The vacuum thrust, mass, and thrust deflection must be specified as functions of time. The program also computes the instantaneous vacuum apogee and impact points.

The program is written such that the vehicle flies essentially an uncontrolled trajectory. The vehicle rises vertically (flight path angle = 90 degrees) until a specified velocity is attained (150 ~ 200 ft./sec.). The flight path angle is then changed to some value other than 90 degrees. The powered flight trajectory then follows a gravity turn until burnout or until a specified time (or altitude)

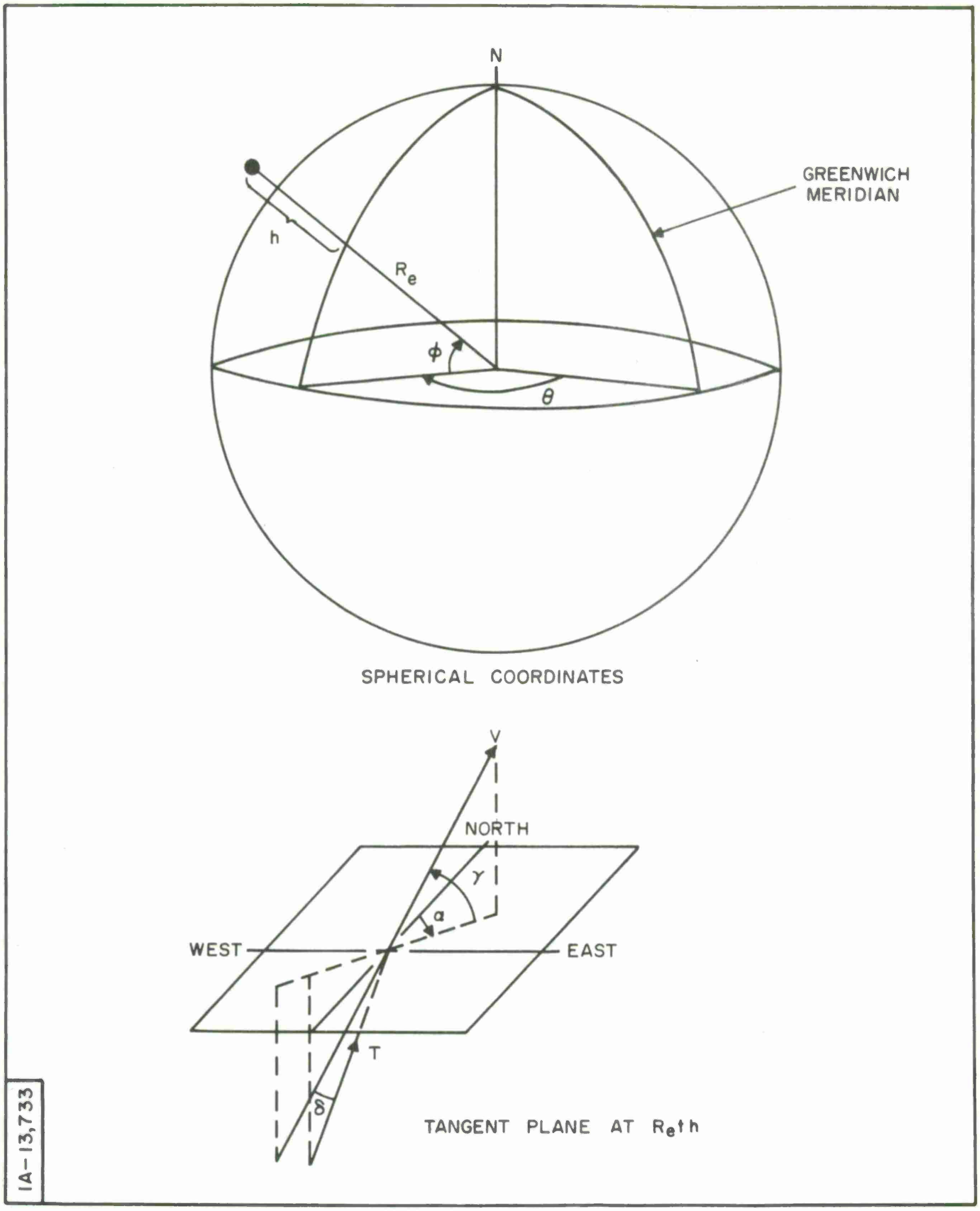

Fig. 1 Reference Axes

when the thrust vector is held at some constant attitude. There is a provision for controlling the trajectory by specifying the deflection of the thrust vector; however, this function must be an input to the program. There is, at present, no closed loop control for this program.

The main program controls the use of the many independent subroutines. This portion of the program receives input data, converts the inputs for internal use, computes time and staging functions, calls various subroutines in proper order, and prepares and delivers output data. In addition, tests are performed at various branch points throughout the program. Figure 2 shows a functional flow diagram for the main program.

Inputs

The inputs required to run this program are described below. The meaning of each input variable is described, and the format for its input is given. Cases may be stacked one after the other without limit. The table of vacuum thrust, mass, and thrust deflection as a function of time may contain up to 500 values. The number of stages is limited to 10. The table of drag and lift coefficients may contain 50 values for each stage. There are artificial limits, since the storage capacity of the computer is not completely used and may be increased by programing changes.

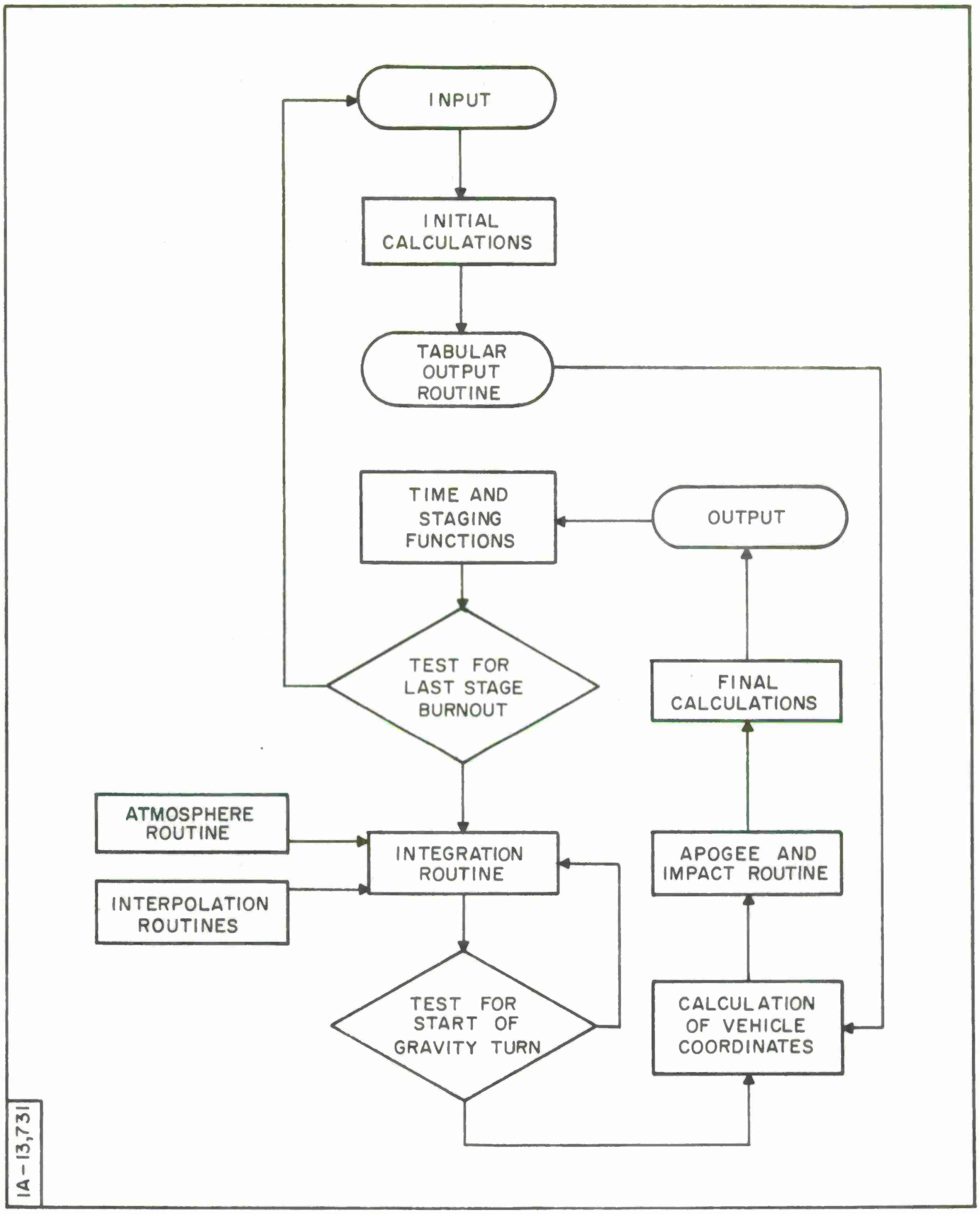

Fig. 2 Functional Flow Diagram

Card No. 1   1 card

Variable*	Column	Format	Definition
NS	1-5	I 5	Number of stages
NC	6-10	I 5	Number of entries in drag and lift tables (NC must be the same for all stages)
NT	11-15	I 5	Number of entries in thrust and mass table
ITAB	16-20	I 5	Index = 1, tables are printed out Index = 0, tables are not printed out
JC	21-25	I 5	Index = 1, drag and lift tables must be an input Index = 0, drag and lift tables are not an input (used to stack cases)
JT	26-30	I 5	Index = 1, thrust and mass table must be an input Index = 0, thrust and mass table is not an input (used to stack cases)
JTEST	31-35	I 5	Control index for thrust alignment (Note No. 1, page 14)
IREAD	36-40	I 5	Control index for single variable change (Note No. 2, page 14)
IBOOST	41-45	I 5	Index = 1, program starts from zero conditions Index = 0, program starts from non-zero conditions

*The variable names are those used in the Fortran program.

Card No. 2   NC* NS cards

Variable	Column	Format	Definition
TMACH	1-17	E 17.8	Mach number table (zero to highest value)
CDT	18-34	E 17.8	Drag coefficient table vs. Mach number
CLT	35-51	E 17.8	Lift coefficient table vs. Mach number

Card No. 3   NT cards

Variable	Column	Format	Definition
TTIME	1-17	E 17.8	Time table (sec.)
TVAC	18-34	E 17.8	Vacuum thrust table (lb.-force)
ETATI	35-51	E 17.8	Thrust deflection (deg.)
TMASS	52-68	E 17.8	Mass table (slugs)

Card No. 4   NS cards

Variable	Column	Format	Definition
REFA	1-17	E 17.8	Reference area, i^{th} stage (sq. ft.)
ANOZ	18-34	E 17.8	Nozzle exit area, i^{th} stage (sq. ft.)
BTIME	35-51	E 17.8	Stage burn time, i^{th} stage (sec.)
DMASS	52-68	E 17.8	Mass increment, i^{th} stage (lb.-mass)

Card No. 5   1 card

Variable	Column	Format	Definition
ALTI	1-17	E 17.8	Initial altitude (ft.)
VELI	18-34	E 17.8	Initial velocity (ft./sec.)
GAMAI	35-51	E 17.8	Initial flight path angle (deg.)
TIMEI	52-68	E 17.8	Initial time (sec.)

Card No. 6   1 card

Variable	Column	Format	Definition
VLATI	1-17	E 17.8	Initial latitude (deg.)
VLONGI	18-34	E 17.8	Initial longitude (deg.)
ALPHAI	35-51	E 17.8	Initial heading (deg.)
RANGEI	52-68	E 17.8	Initial range (n. m.)

Card No. 7   1 card

Variable	Column	Format	Definition
DELTAT	1-17	E 17.8	Integration increment (sec.)
DGAMA	18-34	E 17.8	Kickover angle for start of gravity turn (deg.)
NPRINT	35-51	I 17	Print interval (sec.)
VPITCH	52-68	E 17.8	Velocity at which gravity turn begins (ft./sec.)

Card No. 8   1 card

Variable	Column	Format	Definition
TESTH	1-17	E 17.8	Test altitude for thrust orientation (ft.) (Note No. 1)
TESTT	18-34	E 17.8	Test time for thrust orientation (sec.) (Note No. 1)
THETAD	35-51	E 17.8	Thrust orientation angle (deg.)

Note No. 1

The index, JTEST, plus the values of TESTH and TESTT control the orientation of the thrust vector according to the following listing.

JTEST	TESTT	TESTH	Description
	0	0	Thrust is deflected from the velocity vector by the specified deflection, ETATI.
0 0	$\neq 0$ 0	0 $\neq 0$	Thrust maintains the orientation that the velocity vector had when h = TESTH or t = TESTT, depending on the test used.
1 1	$\neq 0$ 0	0 $\neq 0$	For polar flights the thrust assumes the orientation THETAD from the polar axis in the plane of the trajectory.

Note No. 2

The index, IREAD, may be used for repeat runs when it is desired to change only one variable. The variables which may be changed in this manner are the initial increment in flight path angle used to initiate the gravity turn, DGAMA; and the initial heading, ALPHAI, and the mass increment for each stage. DMASS.

IREAD	Required Input
0	Data card No. 1 plus all others required by 1.
1	Data card 1-A with X = DGAMA
2	Data card 1-A with X = ALPHAI
3	Data card 1-A with X = DMASS (one for each stage)

Card No. 1-A

Variable	Column	Format	Description
X	1-17	E 17.8	Changing variable
IREAD	18-34	I 17	New value of control index

Outputs

The outputs delivered by this program are described below. The definition of each variable as it appears on the printed output is described.

Variable	Units	Definition
TIME	sec.	Time from first-stage ignition
STAGE		Current stage
ALT	ft.	Vehicle altitude (nautical miles when ALT $>$ 100,000 ft.)
VELOCITY	ft./sec.	Vehicle relative velocity
PATH ANGLE	deg.	Vehicle relative flight path angle measured from local horizontal
HEADING	deg.	Vehicle relative heading measured east from north
RANGE	n. m.	Great circle range measured from launch pad
LATITUDE	deg.	Vehicle latitude (+ for north)
LONGITUDE	deg.	Vehicle longitude (+ for east, - for west)
AP TIME	hr.	Time to apogee
DEFL	deg.	Thrust deflection from velocity vector
APLAT	deg.	Apogee latitude
APLON	deg.	Apogee longitude with respect to a rotating earth
APALT	n. m.	Apogee altitude
ILAT	deg.	Impact latitude
ILON	deg.	Impact longitude with respect to a rotating earth
THRUST	lb.	Instantaneous thrust
MASS	slugs	Instantaneous mass
I VEL	ft./sec.	Inertial velocity
I GAM	deg.	Inertial flight path angle
I HEADING	deg.	Inertial heading

Since the effects of earth rotation are neglected during the integration of the equations of motion, the relative flight parameters at burnout are independent of launch heading. The inertial parameters, apogee, and impact points will be different. When the launch heading is zero, the program will integrate the equations of motion through the boost trajectory once. It will then compute the following parameters for launch headings 0 to 360 degrees every 10 degrees.

Variable	Units	Definition
LAUNCH HEADING	deg.	Relative launch heading
B/O HEADING	deg.	Relative burnout heading
B/O LATITUDE	deg.	Vehicle burnout latitude
B/O LONGITUDE	deg.	Vehicle burnout longitude
IVEL	ft./sec.	Inertial velocity
I PATH ANGLE	deg.	Inertial flight path angle
I HEADING	deg.	Inertial heading
INCLINATION	deg.	Orbital inclination
AP LATITUDE	deg.	Apogee latitude
AP LONGITUDE	deg.	Apogee longitude with respect to a rotating earth
AP ALTITUDE	n. m.	Apogee altitude
AP TIME	hr.	Apogee time
IMP LATITUDE	deg.	Impact latitude
IMP LONGITUDE	deg.	Impact longitude with respect to a rotating earth

SECTION IV

THREE-DEGREE-OF-FREEDOM PROGRAM

A three-degree-of-freedom trajectory model assumes that the vehicle is a point mass with three translational degrees of freedom. Certain characteristics are assigned to the point mass such as drag, lift, and area. This program includes the effects of a rotating earth during the integration of the equations of motion. A spherical earth is used; however, a multicomponent gravity force may easily be included through a small programing change. This program has provision for inclusion of three component wind profiles as a function of altitude.

The three equations of motion, expressed in spherical coordinates, are shown in Fig. 3. These equations are integrated using a Runge-Kutta numerical integration technique. The force coefficients, C_D and C_L , are functions of Mach number; and atmospheric density, ρ , gravitational acceleration, g , and wind components are functions of altitude. The vacuum thrust, mass, and thrust deflection must be specified as functions of time. The program also computes the instantaneous vacuum apogee and impact points and/or orbital parameters.

The program is written such that the vehicle flies an essentially uncontrolled trajectory (gravity turn) similar to the two-degree-of-freedom program. There is, at present, no closed loop control for this program.

$$\ddot{R} = R\,\dot{\phi}^2 + R\,(\cos\phi\,\dot{\theta})^2 + \left[\frac{1}{2}\,\rho\,V_A^2\,A\left(-C_D\sin\gamma_A + C_L\cos\psi\cos\gamma_A\right) - mg + T\sin\left(\gamma_R + \delta\right)\right]/m\ ,$$

$$\ddot{\phi} = -\cos\phi\sin\phi\,\dot{\theta}^2 - 2\,\dot{R}\,\dot{\phi}/R + \Bigg[\frac{1}{2}\,\rho\,V_A^2\,A\Big(-C_D\cos\gamma_A\cos\alpha_A - C_L\cos\psi\sin\gamma_A\cos\alpha_A$$

$$- C_L\sin\psi\sin\alpha_A\Big) + T\cos\left(\gamma_R + \delta\right)\cos\alpha_R\Bigg]/Rm\ , \text{ and}$$

$$\ddot{\theta} = \frac{1}{R\cos\phi}\Bigg[-2\,(\dot{R}\cos\phi - R\sin\phi\,\dot{\phi})\,\dot{\theta} + \Bigg(\frac{1}{2}\,\rho\,V_A^2\,A\Big(-C_D\cos\gamma_A\sin\alpha_A - C_L\sin\gamma_A\sin\alpha_A$$

$$+ C_L\sin\psi\cos\alpha_A\Big) + T\cos\left(\gamma_R + \delta\right)\sin\alpha_R\Bigg)/m\Bigg]\ ,$$

where

R = radius,

ϕ = geocentric latitude,

θ = geocentric longitude,

V = velocity,

γ = flight path angle,

α = heading angle,

δ = thrust deflection angle,

T = thrust,

m = vehicle mass,

ψ = roll angle about relative velocity vector,

ρ = atmospheric density,

A = reference area,

C_D = drag coefficient,

C_L = lift coefficient,

$(\ )_A$ = with respect to air, and

$(\ )_R$ = with respect to earth

Fig. 3 Equations of Motion

The main program is essentially the same as for the two-degree-of-freedom program. The functional flow diagram in Fig. 2 is equally applicable to both the two- and three-degree-of-freedom programs. However, some subroutines and inputs and outputs are different.

Inputs

The inputs required to run this program are described below. The meaning of each input variable is described, and the format for its input is given. Cases may be stacked one after the other without limit. The table of vacuum thrust, mass, and thrust deflection as a function of time may contain 500 values. The number of stages is limited to 10. The table of drag and lift coefficients may contain 50 values for each stage. These are artificial limits, since the storage capacity of the computer is not completely used and may be increased by programing changes.

The program computes the vehicle position as a function of time and also computes information regarding the trajectory which the vehicle would follow if thrust were terminated at that instant of time. These latter calculations do not include atmospheric effects.

Card No. 1   1 card

Variable	Column	Format	Definition
NS	1-5	I 5	Number of stages
NC	6-10	I 5	Number of entries in drag and lift tables (NC must be the same for all stages)
NT	11-15	I 5	Number of entries in thrust table
NW	16-20	I 5	Number of entries in wind table
JC	21-25	I 5	Index = 1, drag and lift tables must be an input Index = 0, drag and lift tables are not an input (used to stack cases)
JT	26-30	I 5	Index = 1, thrust and mass tables must be an input Index = 0, thrust and mass tables are not an input (used to stack cases)
JW	31-35	I 5	Index = 1, wind table must be an input Index = 0, wind table is not an input (used to stack cases)
JA	36-40	I 5	Index = 1, staging data must be an input Index = 0, staging data is not an input (used to stack cases)

Card No. 1   1 card (con't.)

Variable	Column	Format	Definition
ITAB	41-45	I 5	Index = 1, tables are printed out Index = 0, tables are not printed out
IBOOST	46-50	I 5	Index = 1, program starts from zero conditions Index = 0, program starts from non-zero conditions
NROLL	51-55	I 5	Number of entries in table of roll angle vs. time
JR	56-60	I 5	Index = 1, roll angle table must be an input Index = 0, roll angle table is not an input
IROLL	61-65	I 5	Index = 1, roll angle = f (time) Index = 0, roll angle is an input constant
IDUMP	66-70	I 5	Index = 1, dump is taken between specified times Index = 0, dump is not taken
ICONST	71-75	I 5	Index = 1, nonstandard constants are an input Index = 0, standard constants are used

Card No. 1-A Only if ICONST = 1. 1 card

Variable	Column	Format	Definition
RE	1-16	E 16.8	Earth radius (ft.)
GM	17-32	E 16.8	Gravitational constant (ft.^{3}/sec.^{2})
G0	33-48	E 16.8	Gravity at earth surface (ft./sec.^{2})
CNM	49-64	E 16.8	Conversion (ft./n. m.)
RAD	65-80	E 16.8	Conversion (deg./rad.)

Cards No. 2 Only if JC = 1. NS* NC cards

Variable	Column	Format	Definition
TMACH	1-17	E 17.8	Mach table (zero to highest value)
CDT	18-34	E 17.8	Drag coefficient table vs. Mach number
CLT	35-51	E 17.8	Lift coefficient table vs. Mach number

Cards No. 3 Only if JT = 1. NT cards

Variable	Column	Format	Definition
TTIME	1-17	E 17.8	Time table (sec.)
TVAC	18-34	E 17.8	Vacuum thrust table (lb. -force)
ETATI	35-51	E 17.8	Thrust deflection table (deg.)
TMASS	52-68	E 17.8	Mass table (slugs)

Cards No. 4   Only if JW = 1.   NW cards

Variable	Column	Format	Definition
TALT	1-17	E 17.8	Altitude table (ft.)
TWR	18-34	E 17.8	Radial wind table (+ up) (ft./sec.)
TWP	35-51	E 17.8	North-South wind table (+ North) (ft./sec.)
TWT	52-68	E 17.8	East-West wind table (+ East) (ft./sec.)

Cards No. 5   Only if JA = 1.   NS cards

Variable	Column	Format	Definition
REFA	1-17	E 17.8	Reference area, i^{th} stage (sq. ft.)
ANOZ	18-34	E 17.8	Nozzle exit area, i^{th} stage (sq. ft.)
BTIME	35-51	E 17.8	Stage burn time, i^{th} stage (sec.)
DMASS	52-68	E 17.8	Mass increment, i^{th} stage (lb.-mass)

Cards No. 6   Only if IROLL = 1 and JR = 1.   NROLL cards

Variable	Column	Format	Definition
TTIMER	1-17	E 17.8	Time table (sec.)
TROLL	18-34	E 17.8	Roll angle (deg.)

Card No. 7   1 card

Variable	Column	Format	Definition
ALTI	1-17	E 17.8	Initial altitude (ft.)
VELI	18-34	E 17.8	Initial relative velocity (ft./sec.)
GAMMAI	35-51	E 17.8	Initial relative path angle (deg.)
TIMEI	52-68	E 17.8	Initial time (sec.)

Card No. 8   1 card

Variable	Column	Format	Definition
VLATI	1-17	E 17.8	Initial latitude (deg.)
VLONGI	17-34	E 17.8	Initial longitude (deg.)
ALPHAI	35-51	E 17.8	Initial heading (deg.)
RANGEI	52-68	E 17.8	Initial range (n. m.)

Card No. 9   1 card

Variable	Column	Format	Definition
DELTAT	1-17	E 17.8	Integration increment (sec.)
DGAMA	18-34	E 17.8	Kickover angle (deg.)
VPITCH	35-51	E 17.8	Pitchover velocity (ft./sec.)
ETAXD	52-68	E 17.8	X-axis heading of cartesian coordinate system (deg.)
NPRINT	69-79	I10	Print-out interval

Card No. 10   1 card

Variable	Column	Format	Definition
TESTH	1-14	E 14.6	Test altitude for thrust orientation (ft.): Note No. 1, page 28.
TESTPR	15-28	E 14.6	Test dynamic pressure for thrust orientation (lb./ft.^{2}): Note No. 1
TESTT	29-42	E 14.6	Test time for thrust orientation (sec.): Note No. 1
QLATD	43-56	E 14.6	Latitude of line-of-thrust orientation: Note No. 1
QLOND	57-70	E 14.6	Not used
JTEST	71-80	I 10	Index for thrust orientation: Note No. 1

Card No. 11   Only if IROLL = 0.   1 card

Variable	Column	Format	Definition
ROLLI	1-17	E 17.8	Constant value of roll angle (deg.)

Card No. 12   Only if IDUMP = 1.   1 card

Variable	Column	Format	Definition
DUMPTL	1-17	E 17.8	Lower time for dump (sec.)
DUMPTU	18-34	E 17.8	Upper time for dump (sec.)

Note No. 1

The index, JTEST, plus the values of TESTH, TESTPR, and TESTT control the orientation of the thrust vector according to the following listing.

JTEST	TESTH	TESTPR	TESTT	Description
	0	0	0	Thrust is deflected from the velocity vector by the specified deflections, ETATI.
0	$\neq 0$	0	0	Thrust maintains the orientation that the velocity vector had when Altitude = TESTH, Dynamic Pressure = TESTPR, and/or Time = TESTT.
0	0	$\neq 0$	0	
0	0	0	$\neq 0$	
1	$\neq 0$	0	0	Thrust assumes the orientation (90-QLATD) from the polar axis when Altitude = TESTH, Dynamic Pressure = TESTPR, and/or Time = TESTT.
1	0	$\neq 0$	0	
1	0	0	$\neq 0$	

Outputs

The outputs delivered by this program as a function of time are described on the following pages. The definition of each variable as it appears on the printed output is described.

Variable	Units	Definition
TIME	sec.	Time from first-stage ignition
STAGE		Current stage
STAGE TIME	sec.	Time from ignition of current stage
STAGE B/O TIME	sec.	Time at which the current stage will burn out.
ALTITUDE	ft. and n. m.	Vehicle altitude above the earth's surface
VELOCITY	ft./sec.	Inertial velocity and velocity relative to a rotating earth
PATH ANGLE	deg.	Inertial flight path angle and flight path angle with respect to a rotating earth
HEADING	deg.	Inertial heading and heading with respect to a rotating earth
LATITUDE	deg.	Vehicle latitude
LONGITUDE	deg.	Vehicle longitude with respect to a rotating earth
G. C. RANGE	n. m.	Great circle range from the launch point to the vehicle position with respect to a rotating earth
THRUST	lb.-force	Instantaneous thrust corrected for atmospheric pressure
MASS	slugs	Instantaneous vehicle mass
C_D		Drag coefficient

(continued from preceding page)

Variable	Units	Definition
C_L		Lift coefficient
REF AREA	sq. ft.	Reference area on which C_D and C_L are based
NOZZLE AREA	sq. ft.	Nozzle exit area
DENSITY	slugs/ft.^{3}	Atmospheric density
PRESSURE	lb./ft.^{2}	Atmospheric pressure
TEMPERATURE	° F.	Atmospheric temperature
SOUND SPEED	ft./sec.	Atmospheric speed of sound
VISCOSITY	lb. sec./ft.^{2}	Atmospheric viscosity
MEAN FREE PATH	ft.	Atmospheric mean free path
THERM COND	BTU/ft. sec.°R	Atmospheric thermal conductivity
MACH NUMBER		Mach number
DYN. PRESSURE	lb./ft.^{2}	Dynamic pressure
STAG ENTHALPY		Stagnation enthalpy
STAG PRESS FS	lb./ft.^{2}	Free steam stagnation pressure
STAG PRESS NS	lb./ft.^{2}	Stagnation pressure after a normal shock at the current mach number
REYNOLDS NUMBER		Reynolds number per foot
ALTITUDE	n. m.	Apogee altitude
VELOCITY	ft./sec.	Apogee and impact velocities

(continued from preceding page)

Variable	Units	Definition
TIME	min.	Apogee and impact time
LATITUDE	deg.	Apogee and impact latitude
LONGITUDE	deg.	Apogee and impact longitude
SAT VEL	ft./sec.	Velocity of a satellite in a circular orbit
DELTA VEL	ft./sec.	Additional velocity required at apogee to inject into a circular orbit
PATH ANGLE	deg.	Flight path angle at impact with respect to a rotating earth
ECCENTRICITY		Vacuum trajectory eccentricity
SEMI-MAJOR	n. m.	Vacuum trajectory semi-major axis
SEMI-MINOR	n. m.	Vacuum trajectory semi-minor axis
PERIGEE ALT	n. m.	Vacuum trajectory perigee altitude
PERIOD	min.	Vacuum trajectory period
INCLINATION	deg.	Orbitual inclination
APOGEE VEL	ft./sec.	Vacuum trajectory apogee velocity
PERIGEE VEL	ft./sec.	Vacuum trajectory perigee velocity
X COORD Y COORD Z COORD	ft. ft. ft.	Vehicle position with respect to cartesian coordinate system located at the launch point

SECTION V

DISCUSSIONS OF ERRORS DUE TO NUMERICAL PROCEDURES

A detailed analytical study of the errors involved in the numerical procedures used in these programs has not been attempted. Instead, the numerical solutions have been compared to known analytical solutions. This approach produces only a limited knowledge of the errors, since analytical solutions are available for only a few restricted cases.

The numerical integration of the equations of motion is accomplished using a Runge-Kutta integration method. More sophisticated integration techniques are available and may be studied at a later time in an effort to reduce the computer time required to produce results with a given accuracy. It is believed that the Runge-Kutta method gives results sufficiently accurate for the present system simulation.

In order to solve the differential equation,

$$\dot{Y} = f(x, y) ,$$

with a given initial value $Y(X_o) = Y_o$, the following method is used to go from Y_K to Y_{K+1}:[1]

$$Y_{K+1} = Y_K + \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right) ,$$

where

$$K_1 = f\left(X_K, Y_K\right) \Delta X \quad ,$$

$$K_2 = f\left(X_K + \frac{\Delta X}{2}, Y_K + \frac{K_1}{2}\right) \Delta X \quad ,$$

$$K_3 = f\left(X_K + \frac{\Delta X}{2}, Y_K + \frac{K_2}{2}\right) \Delta X \quad , \text{ and}$$

$$K_4 = f\left(X_{K+1}, Y_K + K_3\right) \Delta X \quad .$$

The solution of the equations of motion involves the simultaneous integration of the two equations given in Section III for the two-degree-of-freedom program or the three equations given in Section IV for the three-degree-of-freedom program.

The analytical solution presented herein applies to vertical ascent with constant thrust, constant mass flow rate, constant gravity, and no atmosphere.

The equation for vehicle velocity as a function of time for a one-stage vehicle is

$$V = \frac{T}{\dot{m}} \ln \left(\frac{m_o}{m_o - \dot{m}t}\right) - gt \quad ,$$

and for vehicle altitude as a function of time is [2]

$$h = \frac{T}{\dot{m}} t \left[1 - \frac{\ln\left(\frac{m_o}{m_o - \dot{m}t}\right)}{\frac{m_o}{m_o - \dot{m}t} - 1}\right] - \frac{1}{2} gt^2 \quad ,$$

where

V = velocity,

h = altitude,

T = thrust,

$\dot{m}$ = mass flow rate,

t = time,

g = gravity, and

m_o = initial mass.

Figures 4 and 5 show the differences between these solutions and the numerical solutions for the same case after a total powered flight time of 240 seconds.

The errors are a function of integration increment and initial acceleration. There are two primary error sources encountered in numerical integration procedures. One is round-off error which increases as the increment size becomes smaller. The other is discretization error which increases as the increment size becomes larger. These two effects usually produce a minimum error at some increment size[3] (see Fig. 6).

Figures 4 and 5 show the error decreasing as the increment size increases, thus indicating that round-off error is decreasing faster than discretization error is increasing. This would indicate that an increment size of three seconds or more should be used. The level of these errors is quite insignificant for systems planning activities for which these programs were designed. It is not possible to generalize these results to the case where nonlinear effects are included, i.e., gravity, thrust, mass flow rate, atmospheric drag, etc. It is believed discretization error will become more important when these nonlinear effects are included. An integration increment of about one second seems to be a reasonable compromise between accuracy and computation time.

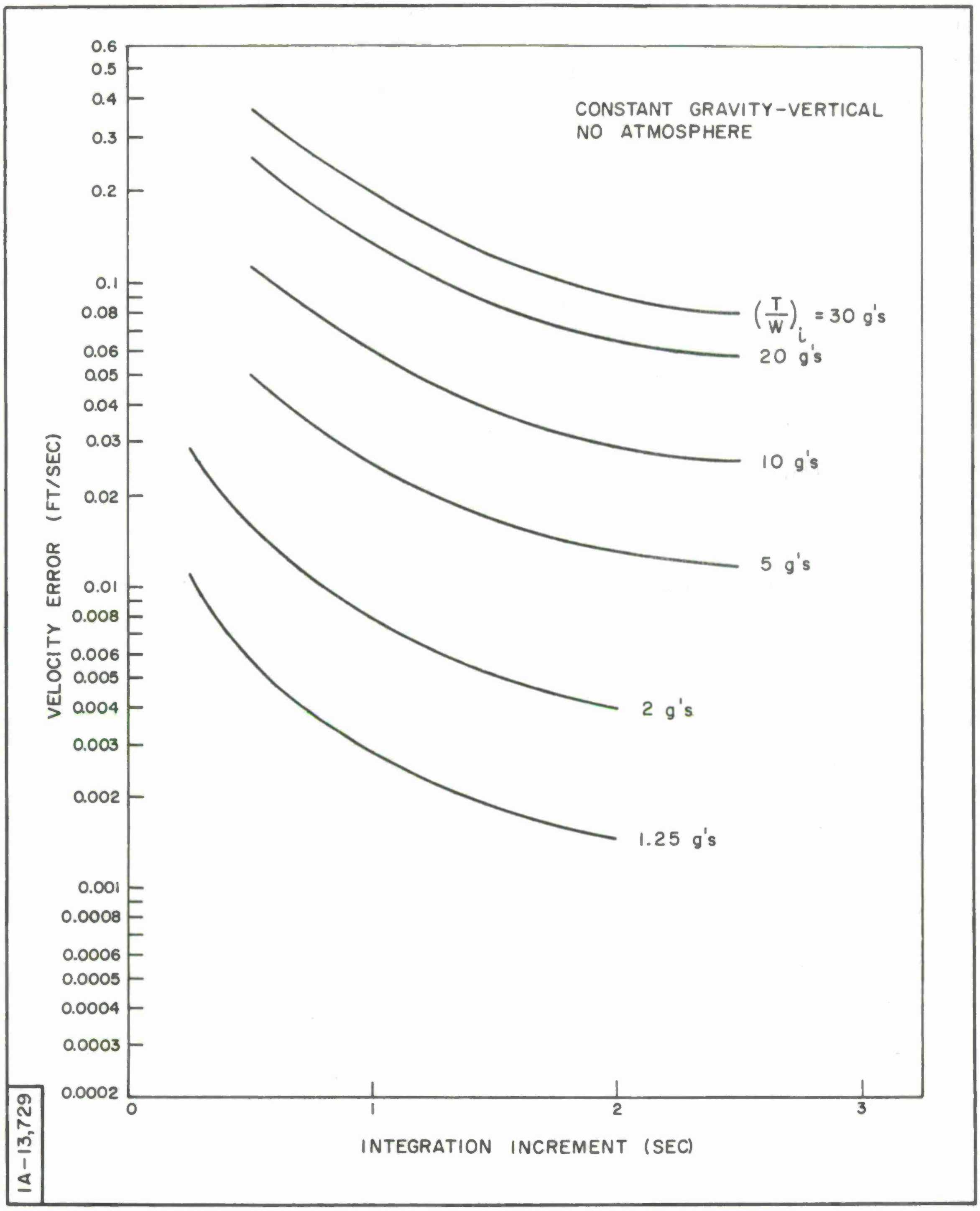

Fig. 4 Total Velocity Error After 240-Second Flight

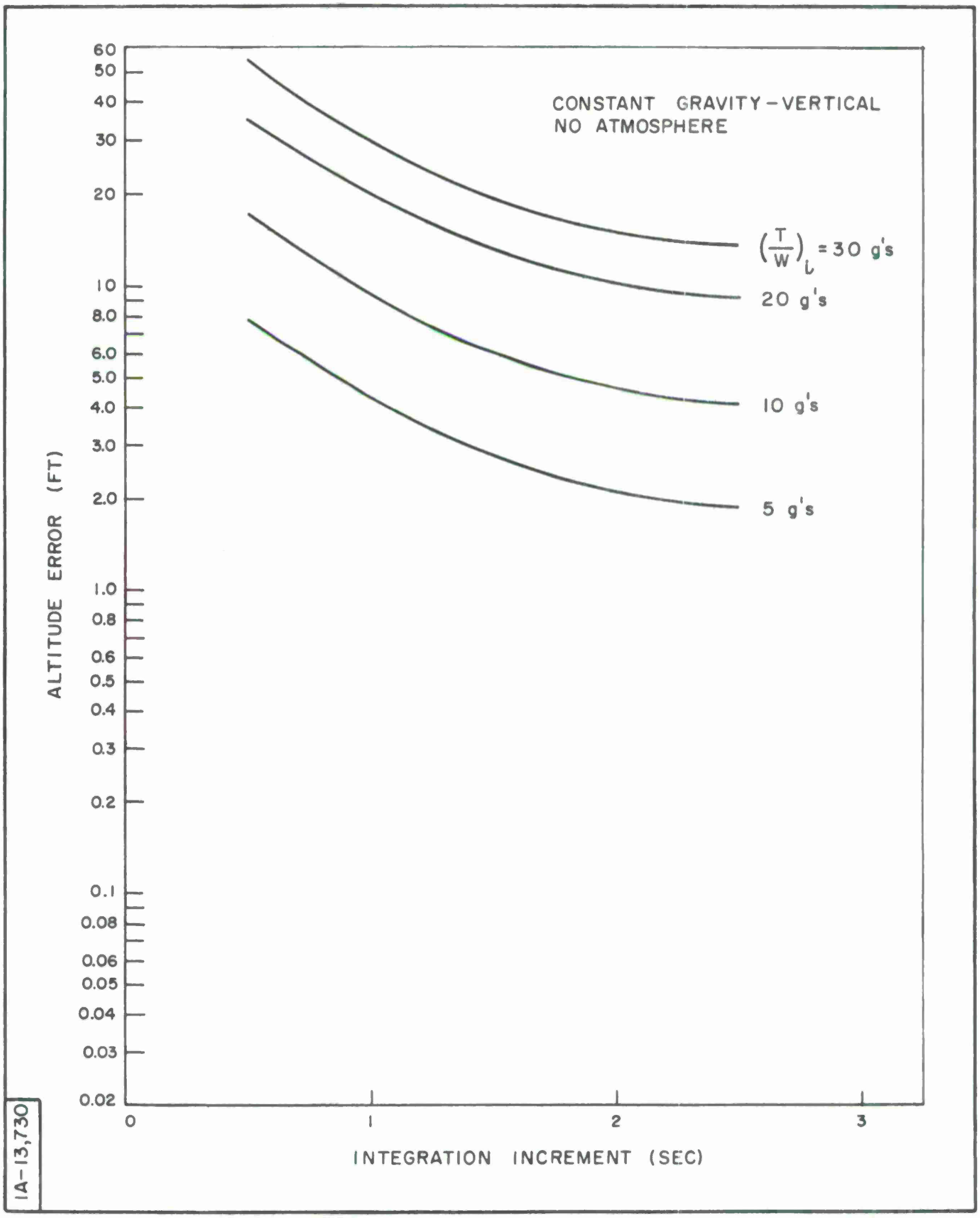

Fig. 5 Total Altitude Error After 240-Second Flight

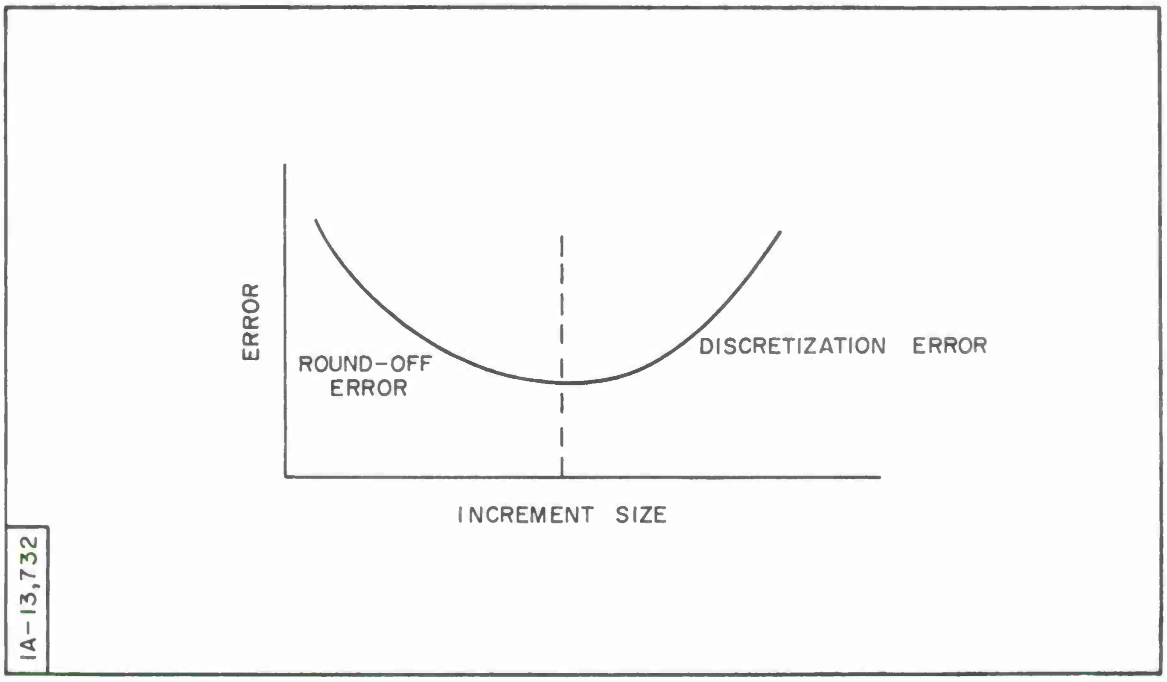

Fig. 6 Primary Error Sources

SECTION VI

SYSTEM SIMULATION

Figures 7 through 10 show the type of data which may be obtained from these programs. These particular results are for a three-stage Scout vehicle with a 200-pound payload launched east from Cape Kennedy. Figure 9 shows the ground track of the powered flight trajectory. Figure 10 shows the ground track of the instantaneous impact point during powered flight with the impact points of the burned-out stages indicated.

The primary use of these booster programs has been in connection with a program which allows evaluation of the tracking or communication coverage during and after the powered flight portion of the trajectory.

Robert W. Dix

Robert W. Dix

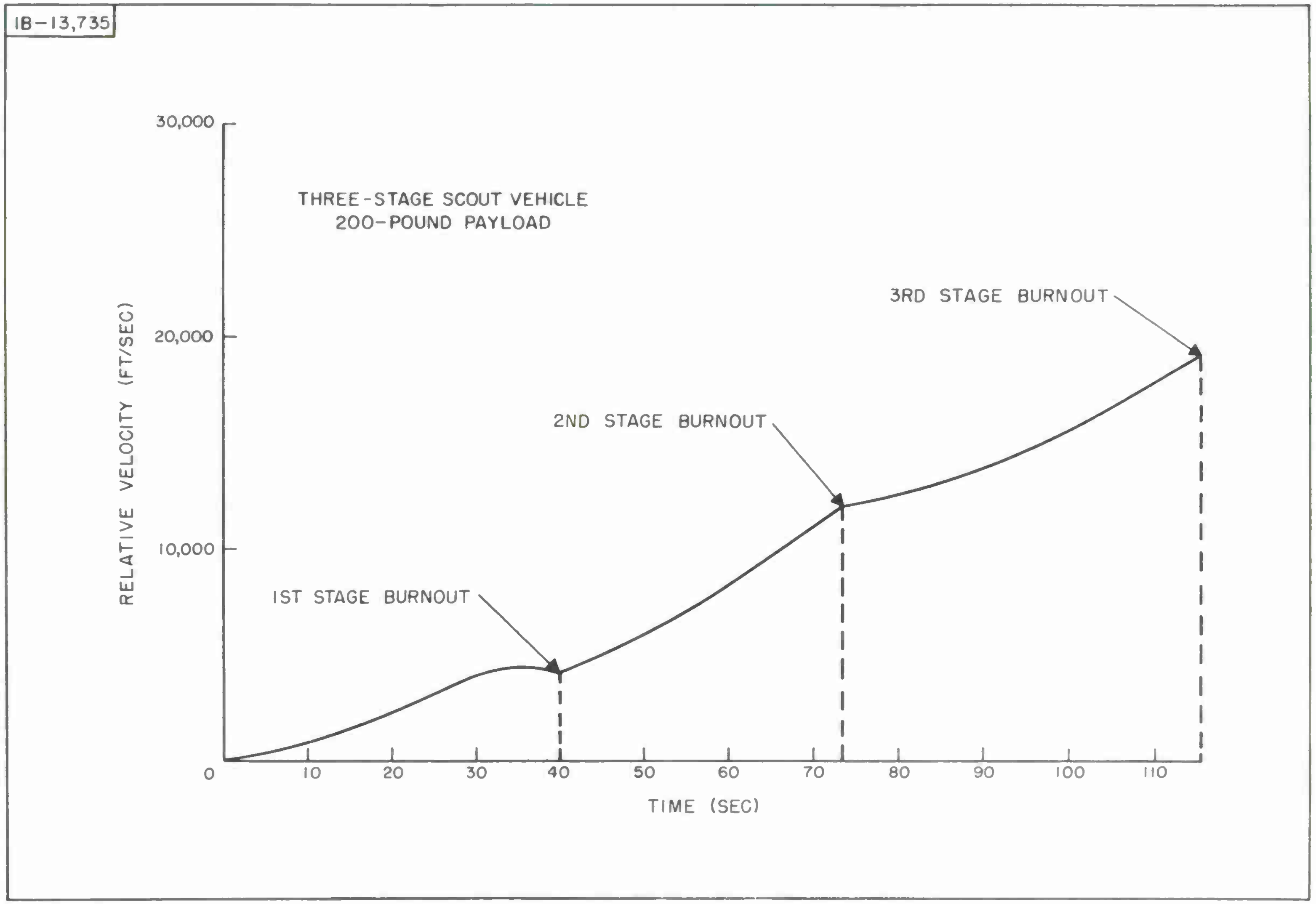

Fig. 7 Relative Velocity Versus Flight Time

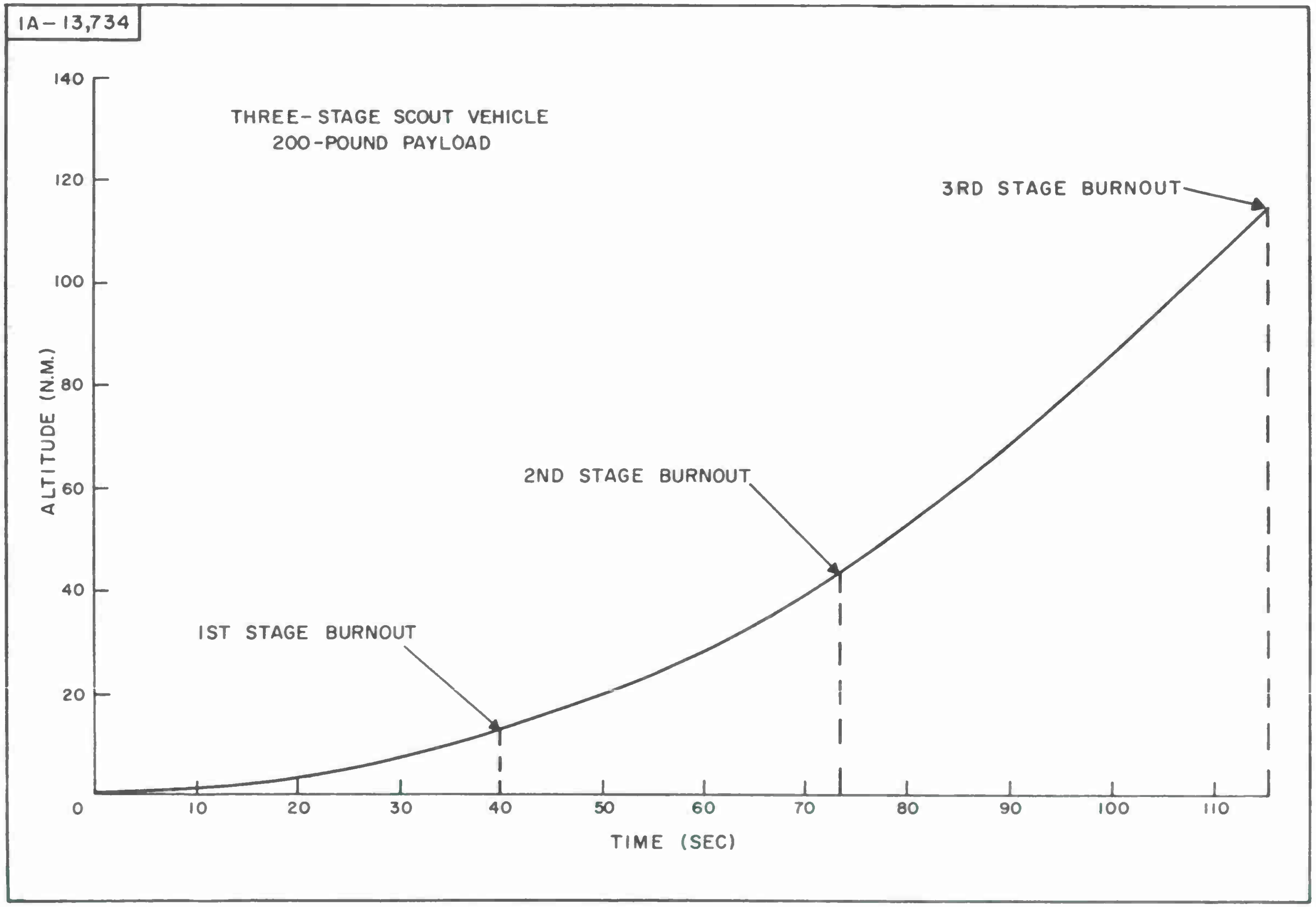

Fig. 8 Altitude Versus Flight Time

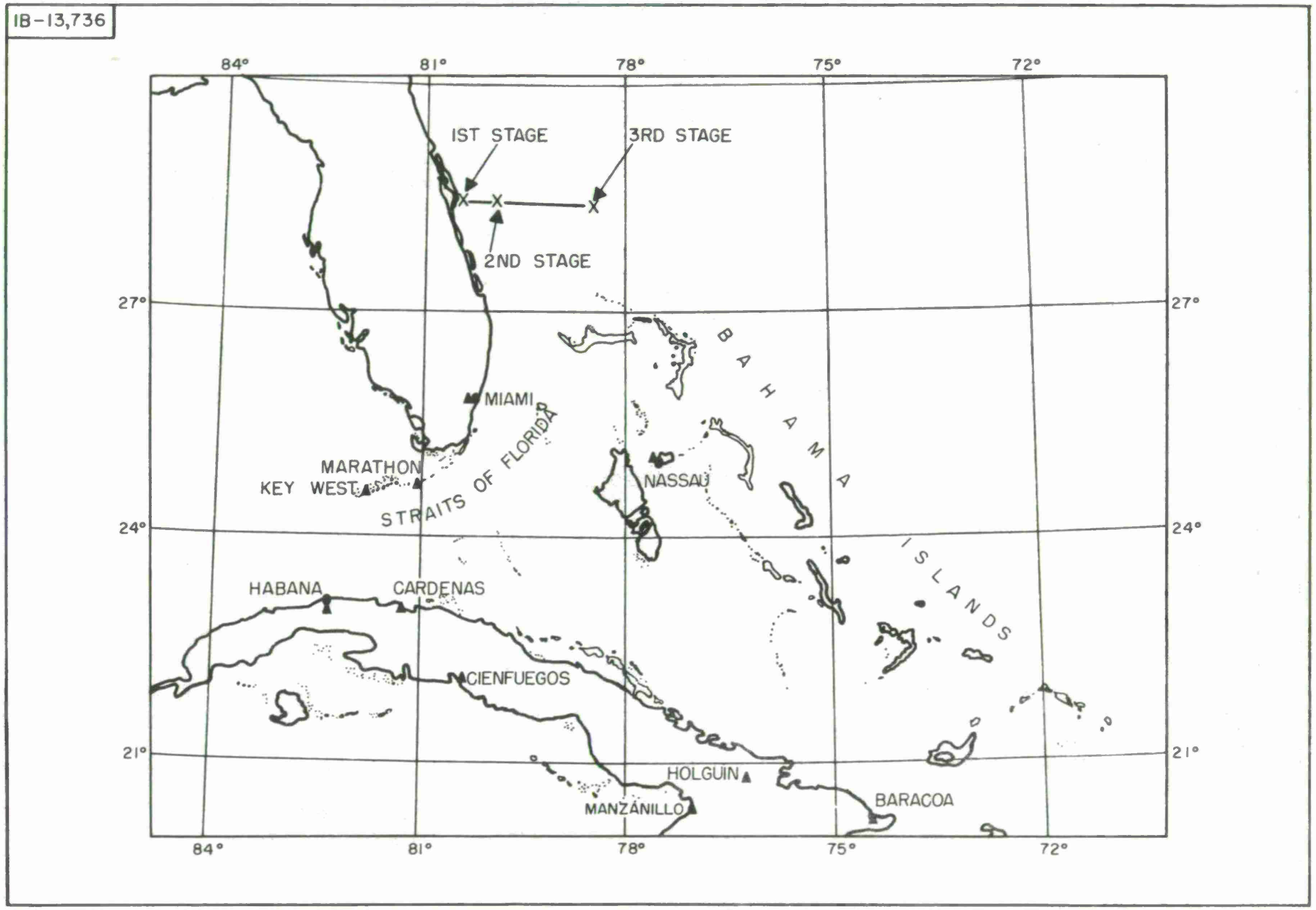

Fig. 9 Powered Flight

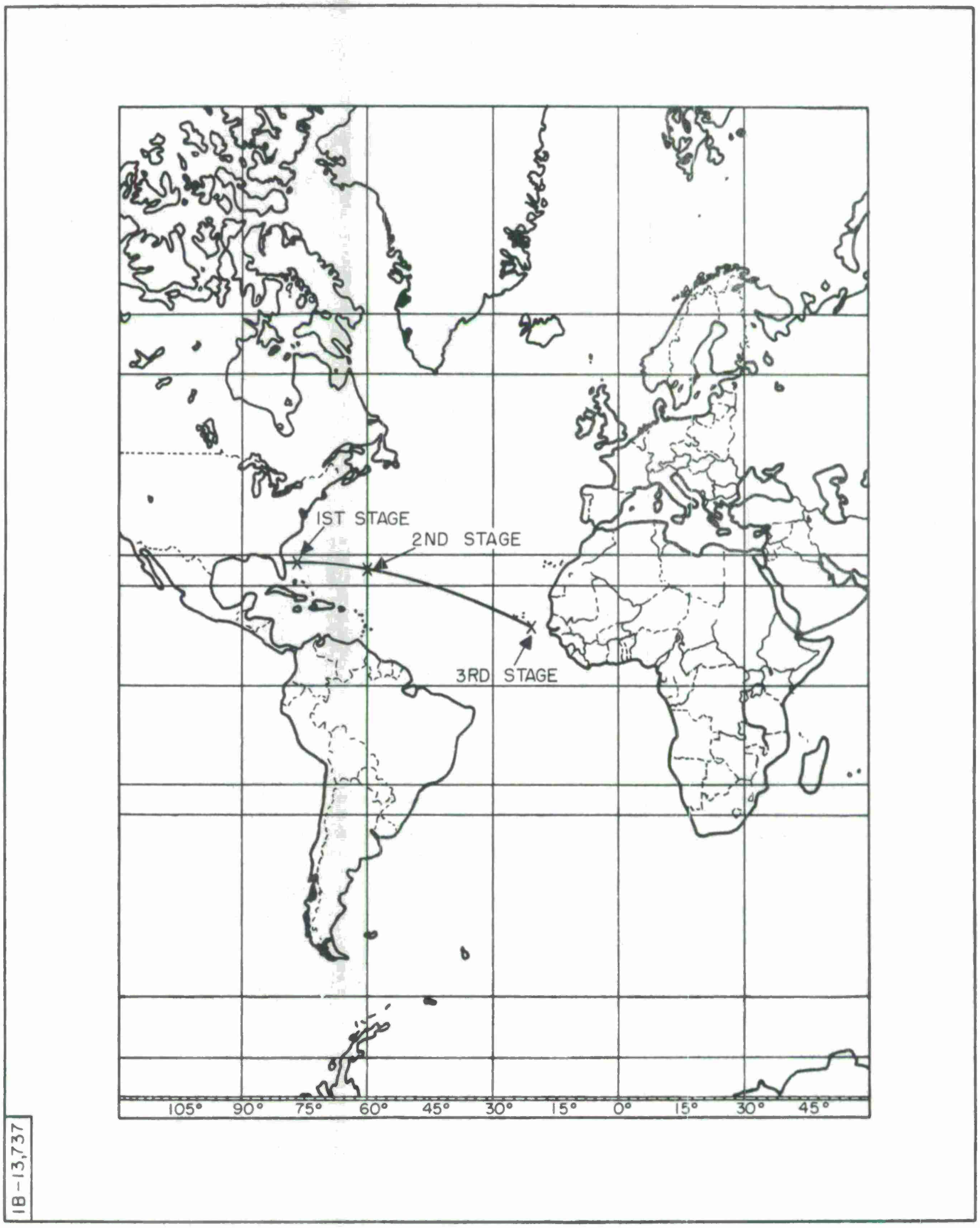

Fig. 10 Predicted Impact Points

REFERENCES

1. Korn and Korn, MATHEMATICAL HANDBOOK FOR SCIENTISTS AND ENGINEERS, McGraw-Hill, New York, 1961.

2. Seifert, SPACE TECHNOLOGY, Wiley, New York, 1959.

3. Crandall, ENGINEERING ANALYSIS, McGraw-Hill, New York, 1956.

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* MITRE CORPORATION BEDFORD, MASS.	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A

3. REPORT TITLE

Engineering Simulation of Powered Flight

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(Last name, first name, initial)*

Dix, R.W.

6. REPORT DATE Jul 64	7a. TOTAL NO. OF PAGES 52	7b. NO. OF REFS 3

8a. CONTRACT OR GRANT NO. AF19(628)2390	9a. ORIGINATOR'S REPORT NUMBER(S) W-06871
b. PROJECT NO. c. 611.1 d.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* ESD-TDR-64-112

10. AVAILABILITY/LIMITATION NOTICES

Qualified Requesters May Obtain Copies from DDC.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Dep for Advanced Planning ESD L.G. Hanscom Field, Bedford, Mass.

13. ABSTRACT

This document describes the current effort toward establishing a workable engineering simulation of the space-ground environment applicable to a wide variety of missile and space systems. Two powered flight computer programs have evolved from this effort. This report describes the content, inputs and outputs of each of these programs.

DD FORM 1 JAN 64 1473

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Flight						
Simulation						
Computers						
Trajectory						
Telemetry Communications						
Mathematical Analysis						
Equations						
Models						
Space Vehicles						
Experimentation						
Data						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
______________________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
______________________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
______________________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, *or (U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.