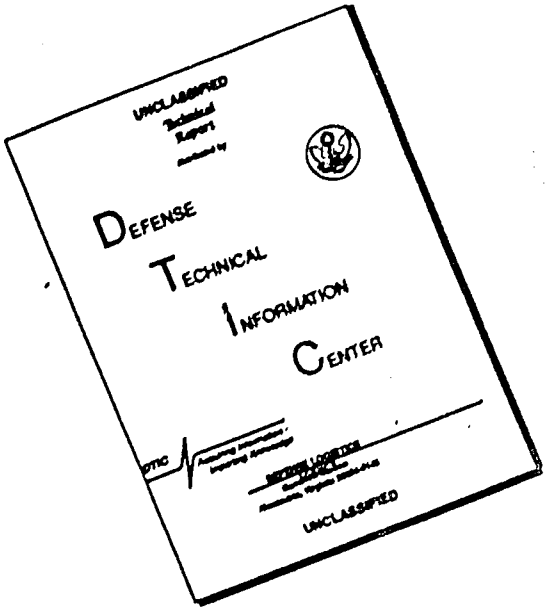
[redacted]

[redacted]

HYDRONAUTICS, incorporated
research in hydrodynamics

Research, consulting, and advanced engineering in the fields of NAVAL and INDUSTRIAL HYDRODYNAMICS. Offices and Laboratory in the Washington, D. C., area: Pindell School Road, Howard County, Laurel, Md.

DISCLAIMER NOTICE



THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

HYDRONAUTICS, Incorporated

TECHNICAL REPORT 117-6

SOME RECENT DEVELOPMENTS IN THEORY OF BULBOUS SHIPS

By

B. Yim

August 1964

Prepared Under

Office of Naval Research
Department of the Navy
Contract No. Nonr-3349(00)
NR 062-266

TABLE OF CONTENTS

LIST OF FIGURES

NOTATION

a	Length of run of wedge strut
a_n	Coefficients of polynomial representing source distribution for a ship
b_n	Coefficients of polynomial representing concentrated singularity distributions for a bulb
B	Beam
$f(x,z)$	Ship hull form
F_H, F_L	Froude numbers with respect to draft and length respectively
g	Acceleration of gravity
H	Draft of ship
H_n	Struve function
k_o	$= gH/V^2$
k_1	$= gL/V^2$
L	Length of ship
m	Nondimensional source strength
R	Nondimensional wave resistance
V	Uniform velocity at $x = -\infty$
x,y,z	Right handed rectangular coordinate system with z positive upward, x in the direction of the uniform velocity V, and the origin on the mean free surface
Y_n	Bessel function of the 2nd kind

α	Half entrance angle
ζ_1	First order wave height
$\zeta_{1\epsilon}$	Second order wave height
ξ, η, ζ	Coordinate system equivalent to O - x,y,z
$-\mu$	Nondimensional doublet strength
λ	Nondimensional quadrupole strength
$\omega(\xi)$	$= (x-\xi) \cos \theta + y \sin \theta$

INTRODUCTION

The history of the bulbous bow on ships may start in the early 19th century with submerged rams on combatant vessels projecting forward along the waterline at the stem, or with the projecting underwater hulls of many old French war ships built about the same time. Later, the British armored cruiser Leviathan had such a projecting ram bow. D. W. Taylor suspected that this ram bow played a definite part in the ships superior performance, and he based the parent model for his famous Standard Series (D. W. Taylor 1911 or 1943) upon the lines of Leviathan. Systematic bulb bow experiments were made by E. F. Eggert in the early 1920's and the general data were reported upon by D. W. Taylor (1923). It had been generally understood that the decrease of resistance due to a bulbous bow is a wave-making phenomenon, such as a decrease in bow wave height due to a bulb wave. This understanding was more strongly supported when Havelock (1928) calculated the surface wave due to a doublet immersed in a uniform stream. A deeply submerged sphere is equivalent to a doublet. Hence according to his calculation, a sphere moving through water at a constant speed causes the surface wave to start with the trough just aft of the sphere. It is natural to imagine that this trough has something to do with the bow wave crest which is seen to start just aft of the bow in ordinary ships. However there was also some other suspicion that the bulb effect is due to a change in the effective ship length owing to the alteration by the bulb of the position of the bow wave. This suspicion was removed by Wigley's mathematical and experimental investigation (1936). He used Havelock's

formula for wave resistance (1934) in terms of the regular wave heights due to the ship hull and a point doublet. He separated the wave resistance into three parts: the hull wave resistance, the bulb wave resistance and the interference resistance of the hull and bulb. The most favorable case occurred when the negative interference resistance was largest. He derived the following six rules for the bulbous bow as the conclusion of his investigation (W.C.S. Wigley, 1936):

"(1) The useful speed range of a bulb is generally from $V/\sqrt{L} = 0.8$ to $V/\sqrt{L} = 1.9$ (or in Froude numbers based on ship length, from 0.238 to 0.563), V being the speed in knots and L the ship's length in feet.

(2) The worse the wave-making of the hull itself is, the more gain may be expected with the bulb and vice versa.

(3) Unless the lines are extremely hollow the best position of the bulb is with its center at the bow, that is, with its nose projecting forward of the hull.

(4) The bulb should extend as low as possible consonant with fairness in the lines of the hull.

(5) The bulb should be as short longitudinally and as wide laterally as possible, again having regard to the fairness of the lines.

(6) The top of the bulb should not approach too nearly to the water surface; as a working rule it is suggested that the immersion of the highest part of the bulb should not be less than its own total thickness."

G. Weinblum (1935) dealt with this same problem by expressing the form of a ship with a bulbous bow in terms of a polynomial according to Michell's thin ship approximation. His theory was also supplemented by model experiments. He expressed a different view from Wigley's, concerning the best vertical position of a bulb, (Wigley's rule (4) and (6)). According to Weinblum's result for an extremely hollow form of ship,a uniformly distributed bulb along the stem line was superior (taking into account the wave resistance only without considering other effects like spray) to the bulb located near the keel, both having the same sectional area. However neither Weinblum or Wigley suggested any optimum variation of bulb size with the speed.

Since then, some experimental investigations on bulbous bows were performed by Lindblad (1944) in calm water and by Dillon and Lewis (1955) in smooth water and in waves. However, after Wigley (1936) and Weinblum (1935), no significant theoretical development on bulbous ships seems to have been made, until Takao Inui and his colleagues made a great contribution on this subject. This will be discussed in a later section in some detail.

In this report, first the necessity of a bulb for minimizing wave resistance will be discussed, followed by a brief review on Inui's explanation of the bulb effect. Inui, using the concept of Havelock's elementary surface waves brought us a clear understanding of the mechanism of bulbs and an easy approach to their design.

Yim (1963) found the ideal bulb or the doublet distribution on a semi-infinite vertical stem line which completely cancels the sine regular waves starting from the stem of a given ship. For the cosine waves from the ship bow, a source line or a quadrupole line are considered. The separation of waves and the wave resistance into the components as in the diagram of Figure 1, simplified the analysis of the bulb effect at the bow or the stern of a ship. The size and the form of the bulb, which are functions of ship shapes and Froude numbers, are supplied extensively. The location of the bulb is of course related to the ship shape and the type of bulb. However, the higher order effect is found to be non-negligible. These are discussed in the next sections.

Throughout this report, inviscid, homogeneous, incompressible, and potential flow around a fixed ship is considered. The origin of the right handed cartesian coordinate system is located on the bow of the ship and on the mean free surface. The intersection of the ship's center plane and the mean free surface is taken as the x axis, with the z axis perpendicular to the free surface, positive upward. The flow at $x = -\infty$ is considered to be uniform with the velocity V parallel to the x axis in the positive x direction (see Figure 2).

SHIPS OF MINIMUM WAVE RESISTANCE AND BULBOUS SHIPS

Since Michell's wave resistance formula (1898) was found, problems of finding the Michell's linearized ship which has the minimum wave resistance have been attacked by many hydrodynamists in various forms and ways. Sretenskii (1935), Pavlenko (1937),

Karp, Votik and Lurye (1958) and Maruo and Bessho (1962) treated symmetric infinite vertical struts. Weinblum (1930, 1957), Krein (1955) and Martin (1961) dealt with three-dimensional symmetric ship with a given vertical distribution of volume. In their solution, they all found either some singularities in the functions representing hull shapes at the ends of ships, or bulb like forms around the bows and the sterns. Wehausen, Webster, and Lin (1962) treated the optimum fore bodies of ships with a given after body as well as three-dimensional symmetric ships without any restriction on the vertical distribution of volume. However they took the ship surface area into account to minimize the wave and friction resistance, and they too found big bulb like forms near the bottom of bows for higher Froude numbers.

Havelock's wave resistance formula (1934) from the regular waves due to the singularity distribution on the center plane of a ship is essentially the same as Michell's , as long as the linear relation of the ship hull form with the singularity distribution

$$m(x,z) = \frac{V}{2\pi} \frac{df}{dx} (x,z) \qquad [1]$$

is used, where $m(x,z)$ is the source strength and $f(x,z)$ is the ship hull form.

Inui (1957) calculated an exact hull form (body streamlines of a double model) from a given source distribution for zero Froude number (flat free surface), and he used this hull form for

his model experiment to test waves and the wave resistance. He compared his experimental results with his calculated wave heights and the wave resistance due to the source distributions. He found, that the calculation agrees better with his experiment on his model than the corresponding Michell's model satisfying (1). The way Karp, Lurye, and Kotik (1958) interpreted their result to a ship form of infinite draft is similar to the idea of Inui's which we have just described. The singular behavior of Michell's ship hull can be easily treated by reinterpreting Michell·s ship hull as the distribution of various singularities like sources or doublets either distributed or concentrated.

Krein (1955) proved in a rigorous manner the existence of a lower bound for the Michell's resistance of ships with a given center plane, a given velocity, a given displacement, and a given vertical distribution of volume. However he concludes that the lower bound of the wave resistance due to a submerged ship is obtained only with generalized functions (i.e. linear combinations of Dirac delta functions) of a ship hull shape: and for floating bodies the wave resistance achieves a lower bound but only for functions of hull shapes having integrable singularities at the ends of the ship.

In the Michell's ship hull representation (1), it is easy to see that the hull shape $f(x,z)$ is proportional to the doublet strength distributed on a given center plane of the ship. Therefore, if we consider the body streamlines due to the doublet distribution in the uniform stream instead of considering $f(x,z)$ as a hull shape, we may be readily convinced that the ship form of

minimum wave resistance has a bulbous bow. In addition, it is worthwhile to note here that, the Dirac delta function of the distributed doublet at the bow is the concentrated line doublet, and the integrable singularity of the doublet distribution at the bow may also be interpreted as a doublet concentrated around the bow.

ELEMENTARY WAVES AND THE WAVE RESISTANCE FORMULA

By Lord Kelvin (1887), it was found that the surface wave due to a point disturbance in a uniform stream consists of two parts: the local disturbance which is limited to the neighborhood of the point disturbance and the regular wave which propagates far aft of the point, mainly restricted to the sector of $|\theta| < 19^{\circ}30'$. This is a mathematical solution of the equation for the potential Φ perturbed by the disturbance,

$$\nabla^2 \Phi = 0 \qquad [2]$$

with linear boundary conditions at the mean free surface $z = 0$, considering the wave height is small compared to the wave length,

$$\frac{\partial^2 \Phi}{\partial x^2} + k_o \frac{\partial \Phi}{\partial z} = 0 \qquad [3]$$

where $k_o = g/V^2$ (g = acceleration of gravity)

and at $x \to -\infty$ and $z \to -\infty$

$$\nabla \Phi = 0. \qquad [4]$$

Now it is well known that a point source of strength m located at a point $(x_1, o, -z_1)$, where $z_1 > 0$, produces a regular wave height ζ at a large x

$$\zeta \sim 4k_o \int_{-\pi/2}^{\pi/2} m \exp(-k_o z_1 \sec^2\theta)\sec^3\theta$$

$$\times \cos\left[k_1 \sec^2\theta \left\{(x-x_1)\cos\theta + y \sin\theta\right\}\right]d\theta \qquad [5]$$

where

$$k_1 = \frac{Lg}{V^2} \qquad k_o = \frac{Hg}{V^2}$$

L is the ship length,
H is the ship draft,
m is nondimensionalized with respect to LHV,
x, x_1, y, ζ is nondimensionalized with respect to L,
$-z_1 \equiv z$ is nondimensionalized with respect to H. } [6]

For a distribution of sources at a ship center plane $S_o(y = 0,\ 0 < z < -1,\ 0 \le x \le 1)$ represented by a series

$$m(x,z) = m_1(x)m_2(z)$$

$$m_1(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$m_2(z) = 1 \qquad [7]$$

the wave height will be, by the integration of [5] with [7] in the domain S_o,

$$\zeta_s = \zeta_{sB} + \zeta_{ss} \qquad [8]$$

$$\zeta_{sB} \sim 4 \int_{-\pi/2}^{\pi/2} \{1 - \exp(-k_o \sec^2\theta)\} \Big[S_1(0) \sin \omega(0) + S_2(0) \cos \omega(0) \Big] d\theta \qquad [9]$$

$$\zeta_{ss} \sim -4 \int_{-\pi/2}^{\pi/2} \{1 - \exp(-k_o \sec^2\theta)\} \Big[S_1(1) \sin \omega(1) + S_2(1) \cos \omega(1) \Big] d\theta \qquad [10]$$

where

$$\left.\begin{aligned} \omega(a) &= k_o \sec^2\theta[(x-a)\cos\theta + y\sin\theta)] \\ S_1(a) &= \sum_{n=0}^{\infty} \frac{(-1)^n\, m^{(2n)}(a)}{k_1(k_1 \sec\theta)^{2n}} \\ S_2(a) &= \sum_{n=0}^{\infty} \frac{(-1)^{n+1}\, m^{(2n+1)}(a)}{k_1(k_1 \sec\theta)^{2n+1}} \\ m^{(n)}(a) &= \left(\frac{\partial^n m(x)}{\partial x}\right)_{x=a} \end{aligned}\right\} \quad [11]$$

According to the theory developed by Havelock, ζ_{sB} and ζ_{ss} are understood as bow waves and stern waves respectively.

The regular wave heights [5], [9] and [10] all have a form

$$\zeta = \int_{-\pi/2}^{\pi/2} S(\theta)\sin[k_1 \sec^2\theta\{(x-a)\cos\theta + y\sin\theta\}]d\theta$$

$$+ \int_{-\pi/2}^{\pi/2} C(\theta)\cos[k_1 \sec^2\theta\{(x-a)\cos\theta + y\sin\theta\}]d\theta \quad [12]$$

Havelock (1934a)showed that the integrands in [12] indicate one-dimensional waves propagating from the point (a,o,O) with the speed V cos θ in the direction θ.

Indeed it can be easily understood if we recognize:

$$(x - a)\cos\theta + y \sin\theta = r \qquad [13]$$

is the equation of the straightline $\ell(r,\theta)$ on the plane z = O with the distance, r, from the point (a,o,O) to the line ℓ, and the angle between the normal to the line ℓ and the x axis, θ; the wave speed C = V cos θ in the deep sea satisfies

$$C^2 = \frac{\lambda g}{2\pi} = V^2 \cos^2\theta \qquad [14]$$

where λ is the wavelength. Hence the one-dimensional wave in the direction angle θ is

$$\zeta_o = A \sin \frac{2\pi}{\lambda}(r - Ct)$$

$$= A \sin\left[\frac{V^2}{g}\sec^2\theta\{(x-a)\cos\theta + y\sin\theta - Vt\cos\theta\}\right]$$

If we replace x - Vt by x and nondimensionalize by L

$$\zeta_o = A \sin [k_1 \sec^2\theta\{(x-a)\cos\theta + y\sin\theta\}] \qquad [15]$$

Therefore these Kelvin regular surface waves are a superposition of the one-dimensional sine and cosine waves with the respective amplitude $S(\theta)$ and $C(\theta)$ in the direction $-\pi/2 \leq \theta \leq \pi/2$. He named these one-dimensional waves "elementary waves" and $S(\theta)$ and $C(\theta)$, amplitude functions. We may omit the word "elementary" in this report except to avoid ambiguities.

He further considered (1934b) the energy carried away by regular waves far aft of a ship in connection with the wave resistance, and he derived the wave resistance formula related to the regular waves [8]. From [9] and [10], [8] can be rearranged as

$$\zeta_s = \int_0^{\pi/2} [A_1(\theta)\sin(k_1 x \sec\theta) + A_2(\theta)\cos(k_1 x \sec\theta)] \times \cos(k_1 y \sin\theta \sec^2\theta)d\theta \qquad [17]$$

where

$$A_1(\theta) = 8\,[1 - \exp(-k_o \sec^2\theta)][S_1(0) - S_1(1)\cos(k_1 \sec\theta) - S_2(1)\sin(k_1 \sec\theta)] \qquad [18]$$

$$A_2(\theta) = 8\,[1 - \exp(-k_o \sec^2\theta)][S_2(0) + S_1(1)\sin(k_1 \sec\theta) - S_2(1)\cos(k_1 \sec\theta)] \qquad [19]$$

Then Havelock's wave resistance formula is

$$R = \frac{1}{2}\int_0^{\pi/2} [A_1^2(\theta) + A_2^2(\theta)] \cos^3\theta \, d\theta \qquad [20]$$

where R is related to the wave resistance R_o by $R = \frac{R_o}{\pi/2 \ \rho L^2 V^2}$.

Since the integrand of [20] is positive definite, R is zero if and only if

$$A_1(\theta) \equiv A_2(\theta) \equiv 0, \quad \text{for } 0 \le \theta \le \pi/2 \qquad [21]$$

The wave resistance [20] can be written as

$$R = R_B + R_s + R_{Bs} \qquad [22]$$

R_B = bow wave resistance

$$= \frac{1}{2}\int_0^{\pi/2} [S_1^2(0) + S_2^2(0)]K^2 \cos^3\theta \, d\theta \qquad [23]$$

R_s = stern wave resistance

$$= \frac{1}{2}\int_0^{\pi/2} [S_1^2(1) + S_2^2(1)]K^2 \cos^3\theta \, d\theta \qquad [24]$$

$$R_{Bs} = \text{stern bow interference resistance}$$

$$= -\int_0^{\pi/2} \Big[S_1(0)[S_1(1)\cos(k_1 \sec\theta) + S_2(1)\sin(k_1 \sec\theta)]$$

$$- S_2(0)[S_1(1)\sin(k_1 \sec\theta) - S_2(1)\cos(k_1 \sec\theta)] \Big]$$

$$\times K^2 \cos^3\theta d\theta \qquad [25]$$

$$K \equiv 8[1 - \exp(-k_o \sec^2\theta)]$$

From [23] we can see that the bow wave resistance consists of the sum of the wave resistance due to sine elementary waves and that due to cosine elementary waves. The same is true of the stern wave resistance in [24]. The expression for the interference resistance [25] shows that there is no interference between the elementary sine waves and the elementary cosine waves starting from the same point either at the bow or the stern. The humps and hollows of the wave resistance are due to the interference resistance, and this is usually very difficult to evaluate. However, if the bow or stern wave resistance is small, the interference resistance is also small. The idea of bulbous bows or bulbous sterns is therefore to reduce the bow or stern wave resistance.

MECHANISM OF BULBOUS BOWS

We consider the bow wave [9], and the bow wave resistance [23] due to a sine ship with its source distribution

$$m_1(x) = \cos(\pi x) \quad \text{in } 0 \le x \le 1,\ 0 \ge z \ge -1 \qquad [26,1]$$

which has no cosine elementary waves but only positive sine waves from the bow in all direction of propagation. Namely $S_2(0) = 0$ in [9] and [23] and $S_1(0) \ge 0$, or we may write

$$\zeta_{sB} = \int_{-\pi/2}^{\pi/2} A(\theta) \sin \omega(0)\, d\theta \qquad [26,2]$$

with $A(\theta) \ge 0$, for $|\theta| \le \pi/2$.

Now we observe the regular wave height due to a point doublet of strength $-\mu$ at $(0,0,z_1)$, which was calculated by Havelock (1928),

$$\zeta_B \sim -4k_o^2 \int_{-\pi/2}^{\pi/2} \mu \exp(-k_o z_1 \sec^2\theta)\sec^4\theta \sin\left[k_1 \sec^2\theta \times (x\cos\theta + y\sin\theta)\right] d\theta \equiv \int_{-\pi/2}^{\pi/2} B(\theta) \sin\omega(0) d\theta \qquad [27]$$

Inui, Takahei, and Kumano (1960) noticed these doublet waves also consist of sine elementary waves and that the amplitude function $B(\theta)$ is purely negative for all θ which is in $|\theta| \leq \pi/2$. Therefore the superposition of two waves [26] and [27] becomes

$$\zeta_{sB} \sim \int_{-\pi/2}^{\pi/2} [A(\theta) - B(\theta)] \sin \omega(0)\, d\theta \qquad [28]$$

and the bow wave resistance is

$$R_B = \int_{-\pi/2}^{\pi/2} [A(\theta) - B(\theta)]^2 \cos^3\theta\, d\theta \qquad [29]$$

By matching $B(\theta)$ to $A(\theta)$ graphically to make $[A(\theta) - B(\theta)]$ as small as possible, especially for small θ, Takahei (1960) found the most favorable doublet strength μ and the position of the doublet z_1 in [27]. They built cosine ship models according to Inui's method, observed the wave patterns by the method of stereo photographs, and tested numerous spherical bulbs fared at the cosine ship. Finally they obtained the models C-201F2 with the so-called waveless bow. Namely, they observed a remarkable reduction in the bow wave heights due to the bulb at the design speed.

If we notice in [7] and [11]

$$m^{(n)}(0) = n!\ a_n \qquad [30]$$

We can readily see in [9] that the bow wave consist purely of sine waves if the source distribution [7] is an even power series and consists purely of cosine waves if [7] is an odd power series. If we consider [7] with only an even power series and in addition,

$$(-1)^n\ a_{2n} \geq 0 \text{ in } [7] \qquad [31]$$

(as in the cosine series) the waves will always be positive sine waves. (However, [31] is a necessary but not a sufficient condition). Yim (1963) showed that these positive sine bow waves due to a source distribution of even power series can be completely eliminated by a doublet distribution along a semi-infinite line $x = 0$, $y = 0$, $-\infty > z \geq 0$, with the doublet strength in the negative x direction,

$$\left.\begin{aligned} \mu(z_1) &= \sum_{n=0}^{\infty} \frac{b_n z_1^{n+1}}{n+1} \quad \text{for } 0 \leq z_1 \equiv -z \leq 1 \\ \mu(z_1) &= \sum_{n=0}^{\infty} \frac{b_n}{n+1} \left[z_1^{n+1} - (z_1 - 1)^{n+1} \right] \quad \text{for } z_1 \geq 1 \end{aligned}\right\} \qquad [32]$$

having the relation

$$b_n = (-1)^n \frac{(2n)!}{n!} \frac{k_o^n}{k_1^{2n+1}} a_{2n} \qquad [33]$$

Namely the amplitude function of the elementary waves from the bow for all angle θ in [28] can be made zero by attaching at the bow a concentrated doublet line which extends to infinite depth. Since the deeply submerged part does not influence too much the surface waves (Yim, 1963) this clarifies the mechanism of the bulb and backs up the approach made by Inui (1962).

SHIPS WITH ZERO BOW WAVE RESISTANCE

Krein (1955) proved that there is no finite ship which has zero wave resistance. Therefore it was essential for the latter to have an infinite doublet line. Nevertheless, ships of zero resistance is not only of academic interest but also gives us a good physical insight and directs us in practical usage.

Although a doublet is good to cancel positive sine waves, it is not applicable to cosine bow waves. Yim (1963) considered one step higher order singularities than a doublet, which is called a quadrupole. The wave height due to a point quadrupole with the strength λ_o (in x direction) at $x = 0$, $y = 0$, $z = -z_1$ in the uniform flow V generates the wave heights

$$\zeta_q \sim -8k_o^3 \int_0^{\pi/2} \lambda \exp(-k_o z_1 \sec^2\theta)\sec^5\theta \cos(k_1 x \sec\theta)$$

$$\times \cos(k_1 y \sin\theta \sec^2\theta)d\theta \qquad [34]$$

where

$$\lambda = \lambda_o/(H^3 LV) \qquad [35]$$

We notice here that [34] consists of cosine elementary waves with the same sign, $-\lambda$ in all direction θ. It was found that the cosine waves due to the source distribution [7] of odd power series can be completely eliminated by a distribution of quadrupoles along the semi-infinite line ($x = 0$, $y = 0$, $-\infty > z \geq 0$ with the strength

$$\left.\begin{aligned} \lambda &= \sum_{n=0}^{\infty} b_{n+1} \frac{z_1^{n+2}}{n+2} \quad \text{in } 0 \leq z_1 \equiv -z \leq 1 \\ \lambda &= \sum_{n=0}^{\infty} b_{n+1} \frac{[z_1^{n+2} - (z_1 - 1)^{n+2}]}{n+2} \quad \text{in } 1 \leq z_1 \leq \infty \end{aligned}\right\} \qquad [36]$$

and

$$b_{n+1} = \frac{(-1)^{n+1} k_o^{n+1} (2n+1)!}{(n+1)! k_1^{2n+3}} a_{2n+1} \qquad [37]$$

A quadrupole itself in a uniform stream does not produce a closed body, but it may, when combined with the doublet line. Therefore these quadrupoles could be used to improve the bulb form used to decrease the cosine wave heights as well as to cancel the sine waves.

Another idea to cancel negative cosine waves is to use a source line. In the same way as we found the infinite doublet or quadrupole line to cancel sine or cosine ship waves, we can find the line source distribution

$$\left.\begin{aligned} m(z_1) &= \sum_{n=0}^{\infty} \frac{b_n z_1^{n+1}}{n+1} \qquad \text{for } 0 \le z_1 \le 1 \\ m(z_1) &= \sum_{n=0}^{\infty} \frac{b_n}{n+1} \left[z_1^{n+1} - (z_1 - 1)^{n+1} \right] \text{ for } z_1 \ge 1 \end{aligned}\right\} \qquad [38]$$

with

$$b_n = (-1)^n \frac{(2n+1)!k_o^{n+1}}{n!k_1^{2(n+1)}} a_{2n+1} \qquad [39]$$

which completely eliminates cosine bow waves due to the source distribution [7], of odd power series. Of course, we have to take care to employ a sink distribution at the ship afterbody in order to have a closed body.

Bulbs at ship sterns can be dealt with exactly in the same manner as for ship bows in an ideal fluid, neglecting the effect of propellers and other attachments. However, the influence of the viscosity and the wake near the stern is so important that the stern problem should really be considered separately. Therefore we deal here only with bulbous bows and bow waves. Henceforth we may omit the word "bow" except to avoid ambiguities.

In all three kinds of bulbs mentioned above, the strength of concentrated singularities along the vertical line increases with the depth, starting with zero strength at the free surface. This suggests the shape of a bulb to be used for a practical ship.

PRACTICAL APPLICATION OF THE THEORY OF WAVE CANCELLATION

In understanding the mechanics of cancelling regular ship waves through the concept of elementary waves and for the practical application we can note here three important characteristics of an elementary wave in each direction of propagation between the the angles $-\pi/2$ and $\pi/2$: (1) the point where the wave starts, (2) the phase of the wave, (3) the amplitude. In general, regular

bow waves consist of elementary waves which have different characteristics in each direction of propagation, despite the fact that point or line singularities by themselves produce negative sine elementary waves (pt. doublet) and cosine elementary waves (pt. source or quadrupole) in all directions of propagation from the point of the singularity's location. Therefore it is impossible to match in all directions the aforementioned three characteristics of elementary waves from bulbs with those from a general ship bow so that all waves are cancelled everywhere. Indeed, we have to choose carefully the ship shapes or the source distributions [7] for ships for which we adopt bulbs: Namely ship shapes for which the bow waves are either positive sine waves ($a_{2n+1} = 0$, $(-1)^n a_{2n} \geq 0$) for the application of a doublet bulb, negative cosine waves ($a_{2n} = 0$, $(-1)^n a_{2n+1} \geq 0$) for a source bulb, or strong positive cosine waves plus weak (positive or negative) sine waves for a doublet bulb combined with either a source (sink) or a quadrupole bulb.

Since no waves from a finite singularity distribution for the bulb can cancel the bow (or stern) waves completely, the best bulb is such a distribution of singularities which produces waves so as to minimize amplitudes in all directions (statistically). This is equivalent to minimizing the bow (stern) wave resistance. In fact, it is not very difficult to obtain the optimum distribution of concentrated singularities in a power series of z along a finite vertical line at the bow such that minimum bow wave resistance is obtained corresponding to a given power series for the ship source distribution.

Indeed, the bow wave resistance [23] can be represented in a quadratic form in a_n of [7] and b_n (coefficients in z for the distribution of singularities as in [32], [36], or [38]) with coefficients represented in terms of Bessel functions. Therefore we have only to solve the simultaneous equations,

$$\frac{\partial R}{\partial b_n}(b_1, b_2, \ldots; a_1, a_2, a_3 \ldots) = 0 \qquad [40]$$

$$n = 1, 2, \ldots$$

for b_n when a_n are given. Since the bow resistance due to sine waves and that due to cosine waves are additive as shown in [23], the concentrated singularities for each case can be dealt with separately.

The optimum distribution of the concentrated singularities at the finite stern line for several given ship source distributions are calculated (Yim 1963) and shown in Figures 3-7. These indicate that the strength of the singularities at the deepest point (the same level as the keel) is the largest. Especially for the higher Froude numbers, the optimum distributions appear to be almost concentrated at the keel. This rather supports Wigley's fourth rule. However the optimum size of the bulb is extremely sensitive to the Froude number. We notice in Figures 3-7 almost a linear distribution of the doublet for the low Froude numbers. If we were given the volume of the bulb, the

optimum distribution would be also sensitive to the Froude number and the displacement of the bulb would gradually move from the keel closer to the surface as the Froude number increases, since the effect of a bulb is stronger at a smaller depth. This would clarify the difference in the opinions of Wigley and Weinblum mentioned before in our introduction. However, in actual ships, the wave resistance is not the only problem.

There are many side problems even with the bulbous bow alone, i.e. spray, slamming, cavitation, form drag due to separation, etc. In this respect, the bulb made of a source line for a hollow ship seems to be more favorable than a doublet bulb, especially for lower Froude numbers, since the source bulb will not produce any marked swan neck shape. It may be worth noting here again that the bulb is not necessarily made of a doublet, but it can be a concentrated source at the bow near the keel, or a doublet plus a source or a quadrupole depending upon the original hull shape.

Of course it is possible to consider the adjustment of the location of bulbs instead of considering only the shapes of bulbs with a fixed location. However, in this case, it is not easy to find the best location from the theory of wave cancellation only, since cancellation of elementary waves in one direction of propagation does not mean that cancellation occurs in the other directions. Yim (1962) considered a most simple case of a point source and a doublet in a uniform stream under a free surface as in Figure 8. As mentioned already, a point source produces positive cosine waves while a point doublet produces negative sine waves. By using Lagalley's theorem, he obtained forces at the doublet point and the source point separately as shown in Figure 8

corresponding to the optimum distance "a" between the two points, (shown in Figure 9) which was calculated so as to minimize the total force in the x direction. If we consider only the wave phases along the centerline through the two points, the distance "a" should always be one quarter of the wavelength λ_o for cancellation of phases,

$$\frac{\lambda_o}{4} = \frac{V}{4} \cdot \frac{2\pi V^2}{gr} \equiv \frac{\pi}{2} F_r^2$$

However, it is shown in Figure 9 that the optimum "a" is always less than $\lambda_o/4$. Figure 8 shows the remarkable reduction of the total wave resistance in this case. In addition, the negative force at the doublet is rather an interesting phenomena. The shape of bulbs made of these singularities can be produced by plotting the body streamlines as Inui does for his double model, or we may use an approximate sphere for a point doublet and the head of a Rankine ovoid for a point source.

HIGHER ORDER EFFECT ON THE ELEMENTARY WAVES

In the case of a sine ship [26,1] which has theoretically only positive sine waves starting from the bow and the stern, Inui and his colleagues observed in their experiment with Inui's model of the sine ship a forward shifting of the wave phase. Therefore, they had to stick their bulb quite a bit forward of the bow instead of locating the bulb center at the stern. They

seem to have had a serious concern about this discrepancy between the theory and the experiment. It has been speculated in Japan that the explanation may be in the orbital wave motion on the ship boundary (Takahei, 1960), or in the non-zero Froude number effect (Inui, 1962), since Inui's model is exactly right for his source distribution only in the case of zero Froude number. Inui used two correction factors which are determined by experiments to correct this observed effect together with the influence of viscosity. We will now discuss an explanation for Inui's observations which are based on higher order wave theory.

For a long time since Havelock's representation of a ship by a singularity distribution, people have been very curious about the exact ship form generated by these singularities which satisfy all the conditions including the linearized free surface condition for a non-zero Froude number. Havelock (1936) and Bescho (1957) considered submerged simple bodies including the free surface effect on body representation, and indicated this effect could be large. Sisov (1961) formulated a higher order theory of wave resistance of surface ships. However the calculations involved are so complicated that no one seems to have succeeded yet in producing a significant result from this higher order theory of surface ships.

Recently, in connection with the theory of wave cancellation in bulbous bowed ships, Yim (1964) considered the Froude number effect on the ship representation near the free surface, and its influence on the regular wave far behind the ship.

We consider a uniform source distribution whose strength

$$m = a_o \qquad [41]$$

in $0 \le x \le 1$, $y = 0$, $-\infty < z < 0$, in the uniform flow considered in this report.

The y component of velocity at (x,y,z) is

$$\Phi_y = \int_0^1 \int_0^\infty d\zeta\, d\xi\, a_o \frac{\partial}{\partial y}\left[\frac{1}{r_1} + \frac{1}{r_2}\right.$$

$$\left. - \frac{1}{\pi}\, \mathrm{Re} \int_{-\pi}^{\pi} \int_0^\infty \frac{k\, e^{k(i\omega - |z + \zeta|)}}{k - k_o \sec^2\theta - i\mu \sec\theta}\, dk d\theta\right] \qquad [42]$$

where

$$r_1 = [(x - \xi)^2 + y^2 + (z - \zeta)^2]^{\frac{1}{2}}$$

$$r_2 = [(x - \xi)^2 + y^2 + (z + \zeta)^2]^{\frac{1}{2}} \qquad [43]$$

$$\omega = (x - \xi)\cos\theta + y \sin\theta$$

At a point (x,y,o) which is not on the singularity plane, the last quadruple integral $J(x,y,o;\xi)$, say, can be written

$$J(x,y,o;1) - J(x,y,o;0)$$

$$= \frac{1}{\pi} \mathrm{Re} \int_{-\pi}^{\pi} \int_{o}^{\infty} \frac{\sec\theta \sin\theta \, e^{ik[(x-1)\cos\theta + y \sin\theta]}}{k - k_o \sec^2\theta - i\mu \sec\theta} \, dk d\theta$$

$$- \frac{1}{\pi} \mathrm{Re} \int_{-\pi}^{\pi} \int_{o}^{\infty} \frac{\sec\theta \sin\theta \, e^{ik(x\cos\theta + y \sin\theta)}}{k - k_o \sec^2\theta - i\mu \sec\theta} \, dk d\theta \quad [44]$$

When we consider the limiting case of $y \to 0$ in $J(x,y,o;1)$, this becomes zero for any k_o since the integrand is antisymmetric in θ. Now if we change the variable

$$k \to k_o k$$

$$J(x,y,o;1) = \frac{1}{\pi} \mathrm{Re} \int_{-\pi}^{\pi} \int_{o}^{\infty} \frac{\sec\theta \sin\theta \, e^{ikk_o(x\cos\theta + y \sin\theta)}}{k - \sec^2\theta - i\mu \sec\theta} \, dk d\theta$$

[45]

This is a function of only $k_o x$ and $k_o y$. The case when $x \to 0$, $y \to 0$ for a certain k_o is exactly the same as the case when

$k_o \to 0$ for certain fixed values of x and y. For $k_o \to 0$, or the case of infinite Froude number,

$$\Phi_y = 0 \quad \text{on } z = 0$$

Therefore, for any k_o

$$\Phi_y \underset{\substack{x \to 0 \\ y \to 0}}{} = 0 \quad \text{on } z = 0 \qquad [46]$$

The above argument can also hold for a point (1 + x, y, o) as $x \to 0$, $y \to 0$

Although we considered points only on z = 0, we notice from the potential theory that physical quantities change continuously into the potential flow field from the boundary. This indicates that every surface ship which is represented by a centerplane source distribution has as strong an influence of the free surface on the shape of the ship in a certain neighborhood of the free surface as in the case of infinite Froude number.

The influence of the free surface can be explained much more eloquently by Green's formula for the velocity potential Φ which satisfies the Laplace equation [2] with the boundary conditions [3], [4] and

$$\phi_n \equiv \frac{\partial \phi}{\partial n} = - \bar{n} \cdot \bar{V} \qquad [47]$$

($\bar{n}$ is the normal vector at the ship hull surface into the fluid).

On a given ship hull,

$$\phi = \frac{1}{4\pi} \int_S \int [\phi(\xi,\eta,\zeta) G_n(\xi,\eta,\zeta,x,y,z)$$

$$- \phi_n(\xi,\eta,\zeta) G(\xi,\eta,\zeta,x,y,z)] dS \qquad [48]$$

where S includes the free surface S_F and the ship surface S_s (see Figure 2) G is the well known Green's function (see e.g. Stoker 1957) which is a harmonic function for $\zeta < 0$ except at (x,y,z) where it has the singularity $1/[(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2]^{\frac{1}{2}}$; and G satisfies the boundary conditions [3], [4] and

$$G_\eta = 0 \quad \text{on } \eta = 0$$

The integral on the free surface S_F in [48] can be written by using [3]

$$I \equiv \iint_{S_F} [\Phi G_n - \Phi_n G] dS = - \int\int_{z=0} [\Phi G_\zeta - G\Phi_\zeta] d\xi d\eta$$

$$= \frac{1}{k_o} \int\int_{z=0} \left[\frac{\partial}{\partial \xi}(\Phi G_\xi) - \frac{\partial}{\partial \xi}(\Phi_\xi G) \right] d\xi d\eta$$

$$= - \frac{1}{k_o} \oint_{\ell} (\Phi G_\xi - \Phi_\xi G) d\eta \qquad [50]$$

where ℓ is the intersection of the ship surface and the z = 0 plane. Since the ship beam length ratio B/L = ϵ is considered to be small, in general, [50] is omitted in the first order theory.

Wehausen (1962) considered a systematic, formal, yet thorough estimation of the order of magnitude in the Green's formula with the exact boundary conditions of the potential . For a ship with the draft H as small as the beam B, he estimated I in [50] is $O(\epsilon^3)$ while the main integral around the ship hull in [48] is $O(\epsilon^2)$. In fact it has been known that the effect of the draft behaves like exp(-CH) where C is a function of Froude number and even for the case H/B = 2, the wave heights was comparable to the case $H \rightarrow \infty$ (Wigley, 1931). Therefore the above estimation may be true even for the case of an infinite draft ship, and the

line integral I, in this case will be the most important contribution to the higher order terms which have been previously neglected. Indeed, in [50], I is the influence of the free surface on the potential.

However it is extremely difficult to understand the higher order effect just by the formal estimation of the magnitude and without actual evaluation, since the property of Green's function is very complicated particularly near the free surface. As a simplest case for the evaluation of the line integral Yim (1964) considered a source distribution

$$m = a_o \quad \text{in } 0 \le x \le a,\ y = 0,\ -\infty < z \le 0$$

on the forebody of a semi-infinite wedge shaped strut,

$$y = x \tan \alpha \qquad \text{in } 0 \le x \le a,\ -\infty \le z \le 0$$

$$y = \tan \alpha \qquad \text{in } 0 \le x \le \infty,\ -\infty \le z \le 0.$$

For ϕ or ϕ_ξ inside the line integral [50] he used the first order solution obtained by Havelock (1932),

$$\zeta_1 = -\frac{\pi}{k_o}\frac{a_o}{V}\left[\int_o^{k_o\xi}[H_o(t) + 3Y_o(t)]dt - \int_o^{k_o(a-\xi)}[H_o(t) - Y_o(t)]dt\right]$$

where H is the Struve function and Y is the Bessel function of the 2nd kind. If we use the relation from the pressure condition on the free surface

$$\zeta_1 = \frac{\phi_\xi}{k_o}$$

and the Green's function represented on the free surface

$$G_x(\xi,0,0,x,0,0) = -\frac{k_o}{\pi}\,\mathrm{Re}\int_{-\pi}^{\pi}\int_o^{\infty}\frac{ik\sec\theta\, e^{ki\omega}}{k - k_o\sec^2\theta - i\mu\sec\theta}\,dkd\theta$$

$$= \left[-4\pi k_o^2\frac{d}{dt}Y_1(t) + \pi k_o^2\frac{d^2}{dt^2}[H_o(t) - Y_o(t)]\right]_{t=k_o(x-\xi)}$$

we can evaluate the line integral [50] at large x and y = 0 neglecting higher order terms,

$$\zeta_{1\epsilon} = \frac{I_x}{k_o} \sim -2 \tan\alpha \left[\Phi(k_o x - t) \frac{d}{dt} Y_1(t) \right]_{t=k_o x}^{k_o(x-a)}$$

$$+ 4 \tan\alpha\, k_o \int_0^a \zeta_1 \left[\frac{d}{dt} Y_1(t) \right]_{t=k_o(x-\xi)} d\xi$$

If we take only the lower limit of the above equation, it can be considered from the equation for the surface wave to represent a regular wave starting from the bow due to the influence of the free surface. From here Yim (1964) calculated the amplitude and the phase of the regular bow wave $\zeta_{1\epsilon}$ far behind the ship on y = 0 due to the line integral,

$$\zeta_{1\epsilon} \sim P \sin(k_o x + \frac{\pi}{4} + \beta)$$

It is easy to see from Havelock's result that the regular bow wave ζ_1 from the first order theory is,

$$\zeta_1 \sim Q \sin(k_o x + \frac{\pi}{4})$$

$$Q = \frac{4\pi}{k_o} \frac{a_o}{V} \sqrt{\frac{2}{\pi k_o x}} \quad .$$

In Figure 10 are shown the phase difference β and

$$\frac{P}{Q \tan \alpha} = f(k_o a)$$

which are functions of only $k_o a$. The amplitude of the total wave ζ_t

$$\zeta_t = \zeta_1 + \zeta_{1\epsilon}$$

$$= \sqrt{Q^2 + p^2 + 2Qp \cos \beta} \quad \sin(k_o x + \frac{\pi}{4} + \zeta)$$

and the phase difference ζ between the total wave ζ_t and the first order wave ζ_1 are shown in Figures 11 and 12. β and ζ are shown in radian, considering that one wave length $(2\pi/k_o)$ is just 2π.

These show that the total wave phase is indeed advanced considerably compared with the first order wave, while the amplitude of the total wave height does not differ too much from that of the first order wave. Namely the second order effect is quite large. It is proportional to the slope of the entrance on the free surface, for a given run, a. Therefore, the smaller the entrance slope near the free surface is, the less the second order effect to be expected. As we see in the integrand of the line integral [50], this effect mainly depends on the potential and

the wave on the free surface waterline where the waterline slope is large. Since the local effect is usually big near the bow and the shoulder, the influence of the local effect on the second order wave may be quite important. We notice that when we cancel the regular wave by the bulb, the line integral due to this wave will be also cancelled.

This study of the line integral [50] has just started. However it seems to be quite promising for furtherance of a proper understanding of ship waves and of their reduction.

CONCLUDING REMARKS

Theory and experiment are always stimulating and helping each other. Although this report is on the theoretical side, it does not mean that the influence of experiments are underestimated. This report is merely intended to further appreciation of our great predecessors, Michell, Havelock, Wigley, Weinblum and Inui for the theories related to the bulbous bowed ship, and to add a slight theoretical illumination to them.

The mechanism of the bulb at the ship bow (or stern) is completely clarified. The type of bulb for a given ship hull, and the size and the vertical area distribution of bulb for a given Froude number are derived. The higher order influence is known to be the major reason for the phase shift of the regular waves. Although the stern problem in the non-viscous fluid is exactly the same as the bow problem, it should be studied separately due to the large influence of viscosity, wakes, propellers, etc.

Because of these influences, the bow waves are more important in practice than the stern waves. The humps and hollows of the curve of the wave resistance due to a ship without a bulb may be applied to that for the ship with the bulb without any considerable error. The bulb has an effect of smoothing out the humps and hollows of the resistance curve to a considerable extent (Yim 1962) in the vicinity of the designed speed or for larger speeds. Pien (1962) seems to have obtained this effect using the principle of wave cancellation by distributed singularities rather than concentrated ones. Naturally, a ship with a bulbous bow would have much the better performance if it has a better stern. At the present time, shapes like the transom stern seem to attract the interest of many naval architects for high speed ships.

The higher order effect and the influence of viscosity are extremely difficult to analyze, yet they should and will be gradually exploited in the near future. The theoretical study on the seaworthiness of the bulbous ships remains to be done, although it is known from experiments that a bulbous bows are still effective in waves.

ACKNOWLEDGMENT

This work was kindly sponsored by the Office of Naval Research, Department of the Navy under Contract No. Nonr-3349(00), NR 062-266. Thanks are due to Mr. M. P. Tulin for his helpful discussions.

REFERENCES

1. Bessho, M., "On the Wave Resistance Theory of a Submerged Body," 60th Anniversary Series Vol. 2, SNAJ, 1957.

2. Bessho, M., "On the Minimum Wave-Resistance of Ships with Infinite Drafts," Proceedings of International Seminar on Theoretical Wave-Resistance, University of Michigan, 1962.

3. Dillon, E. S. and Lewis, E. V., "Ships with Bulbous Bows in Smooth Water and in Waves," Trans. SNAME, Vol. 63, 1955.

4. Havelock, T. H., "The Wave Pattern of a Doublet in a Stream," Proc. Roy. Soc. A. 121, pp. 515-23, 1928.

5. Havelock, T. H., "Ship Waves: the Calculation of Wave Profiles," Proc. Roy. Soc. A 135 pp 1-13, 1932.

6. Havelock, T. H., "Wave Patterns and Wave Resistance," TINA, Vol. 76, pp 430-443, 1934a.

7. Havelock, T. H., "The Calculation of Wave Resistance," Proc. Roy. Soc. A 144, pp 514-21, 1934b.

8. Havelock, T. H., "The Forces on a Circular Cylinder Submerged in a Uniform Stream," Proc. Roy. Soc A 157, pp 526-34, 1936.

9. Inui, T., "Wave Making Resistance of Ships," Trans. SNAME Vol 70, pp 283-353, 1962.

10. Inui, T, Takahei, T., Kumano, M., "Wave Profile Measurements on the Wave-Making Characteristics of Bulbous Bow," SNAJ (Translation from University of Michigan), 1960.

11. Karp, S., Kotik, J., and Lurye, "On the Problem of Minimum Wave-Resistance for Struts and Strut-Like Dipole Distributions," Third Symposium on Naval Hydrodynamics, ONR, Department of the Navy, 1960.

12. Kostchukov, A. A., "Theory of Ship Waves and the Wave-Resistance," Leningrad, 1959.

13. Krein, M. G., Koklady Akademi Nauk, SSSR, Vol. 100, No. 3, 1955.

14. Kelvin Lord (Sir W. Thomson), "On the Waves Produced by a Single Impulse in Water of any Depth, or in a Dispersive Medium," Proc. Roy. Soc. of London, Set A. 42 pp 80-85, 1887.

15. Lindblad, A., "Experiments with Bulbous Bows," Publication No. 3 of the Swedish State Shipbuilding Experimental Tank, 1944.

16. Lunde, J. K., "On the Linearized Theory of Wave Resistance for Displacement Ships in Steady and Accelerated Motion," Trans. SNAME, 1951.

17. Lunde, J. K., "On the Theory of Wave-Resistance and Wave Profile," Skipsmodelltankens, Meddeleke Nr. 10, 1952.

18. Martin, M., and White, J., "Analysis of Ship Forms to Minimize Wavemaking Resistance," Stevens Inst. of Tech. R-845, May 1961.

19. Maruo, H., "Experiments on Theoretical Ship Forms of Least Wave-Resistance," The International Seminar on Theoretical Wave Resistance, University of Michigan, 1963.

20. Michell, J. H., "The Wave Resistance of a Ship," London, Dublin and Edinburgh, Philosophical Magazine, Ser 5, Vol. 45, 1898.

21. Pavlenko, G. E., "The Ship of Least Wave Resistance," Trudy Vsesoyuz Nanch-Inzhen-Tekhn. Obshch. Sudostroen, 2 No. 3 28-62, 1937.

22. Pien, P. C. and Moore, W. L., "Theoretical and Experimental Study of Wave-Making Resistance of Ships," Proc. of the International Seminar on Theoretical Wave-Resistance, Michigan, 1963.

23. Sisov, V. Isvestiya Akadem Nauk, SSSR, Mechanics and Engineering No. 1, 1961.

24. Stoker, J. J., "Water Waves," Interscience Publishers, Inc., New York, N. Y., 1957.

25. Sretenskii, L. N., "Sub un Probleme de Minimum dans la Theorie du Navire," C. R.(Dokl) Acad. Nauk, USSR 3, 247-248, 1935.

26. Takahei, T., "A Study on the Waveless Bow," SNAJ, (Trans. from University of Michigan), 1960.

27. Taylor D. W., "The Speed and Power of Ships," 3rd U. S. Gov't Printing Office, Washington, D. C., 1943.

28. Taylor, D. W., Marine Engineering and Shipping Age. pp 540-548, Sept. 1923.

29. Wehausen, J. V., "An Approach to Thin-Ship Theory," Proc. of the International Seminar on Theoretical Wave Resistance, The University of Michigan, 1963.

30. Wehausen, J. V., Webster, W. C., and Lin W. C., "Ships of Minimum Total Resistance," Proc. of the International Seminar on Theoretical Wave Resistance, The University of Michigan, 1913.

31. Weinblum, G., "Schiffe geringsten Widerstands," Proc. Third International Congr, Appl. Mech. Stockholm pp 449-458, 1930.

32. Weinblum, G."Die Theorie der Wulstschiffe," Der Gesellschaft fur Angervandte, Mathematik, 1935.

33. Weinblum, G., "Schiffe geringsten Widerstandes," Jahrbuch der Schiffbautechnischen Gesellschaft, V. 51, 1957.

34. Wigley, W.C.S., "Ship Wave Resistance," Trans. N.E.C.I.E.S. Vol. XLVII pp 153-196, 1931.

35. Wigley, W.C.S., "The Theory of the Bulbous Bow and its Practical Application," Trans. N.E.C.I.E.S. Vol. LII pp 65-88, 1936.

36. Yim, B., "Analysis of the Bulbous Bow on Simple Ships," HYDRONAUTICS, Incorporated Technical Report 117-1, 1962.

37. Yim, B., "On Ships with Zero and Small Wave Resistance," Proc. of the International Seminar on Theoretical Wave Resistance, The University of Michigan, 1963.

38. Yim, B., "The Higher Order Effect on the Waves of Bulbous Ships," HYDRONAUTICS, Incorporated Technical Report 117-5, 1964.

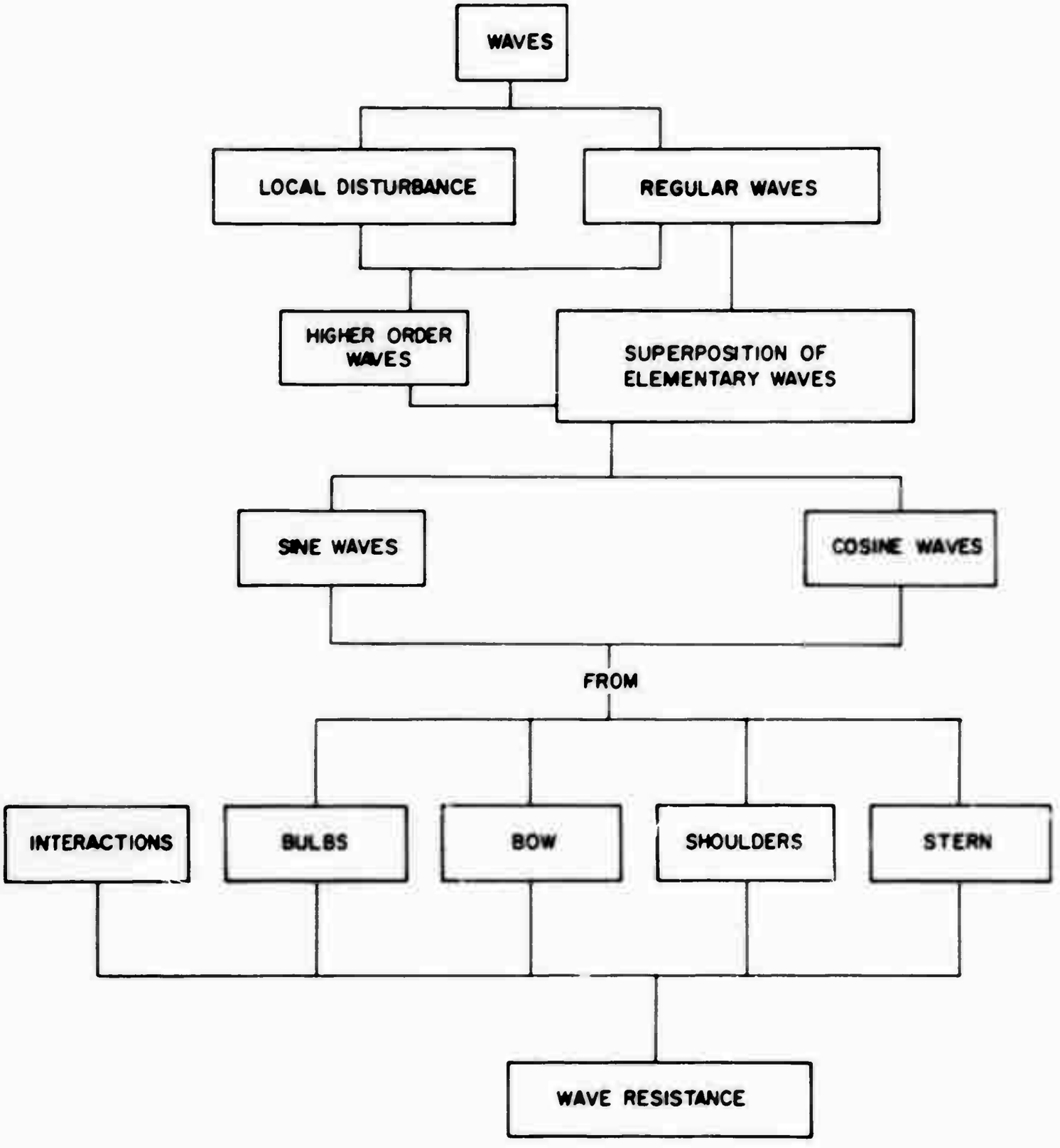


FIGURE 1 - DIAGRAM FOR THE CHARACTERISTICS OF SHIP WAVES

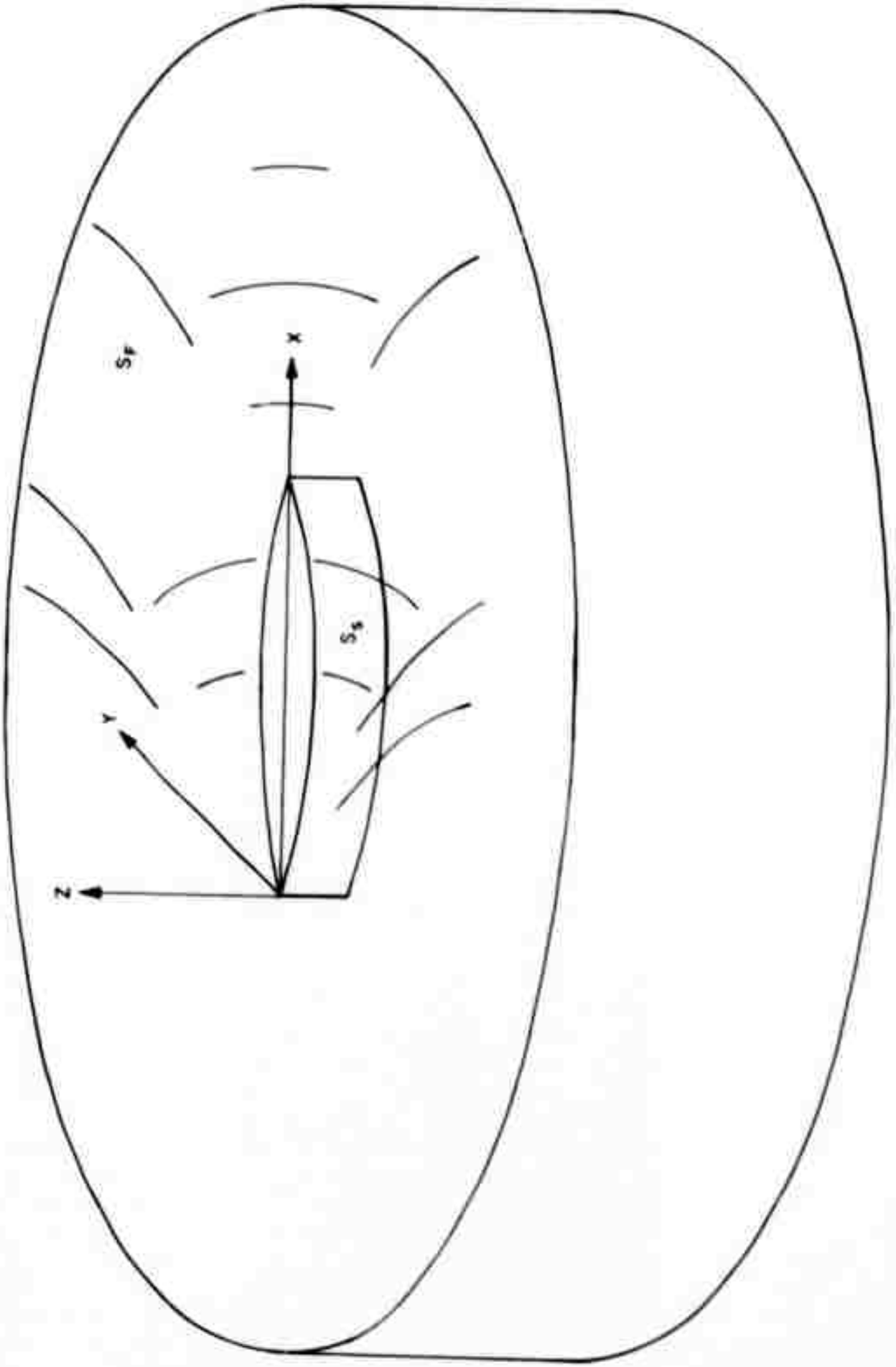


FIGURE 2-SCHEMATIC DIAGRAM FOR A SURFACE SHIP AND THE COORDINATE SYSTEM

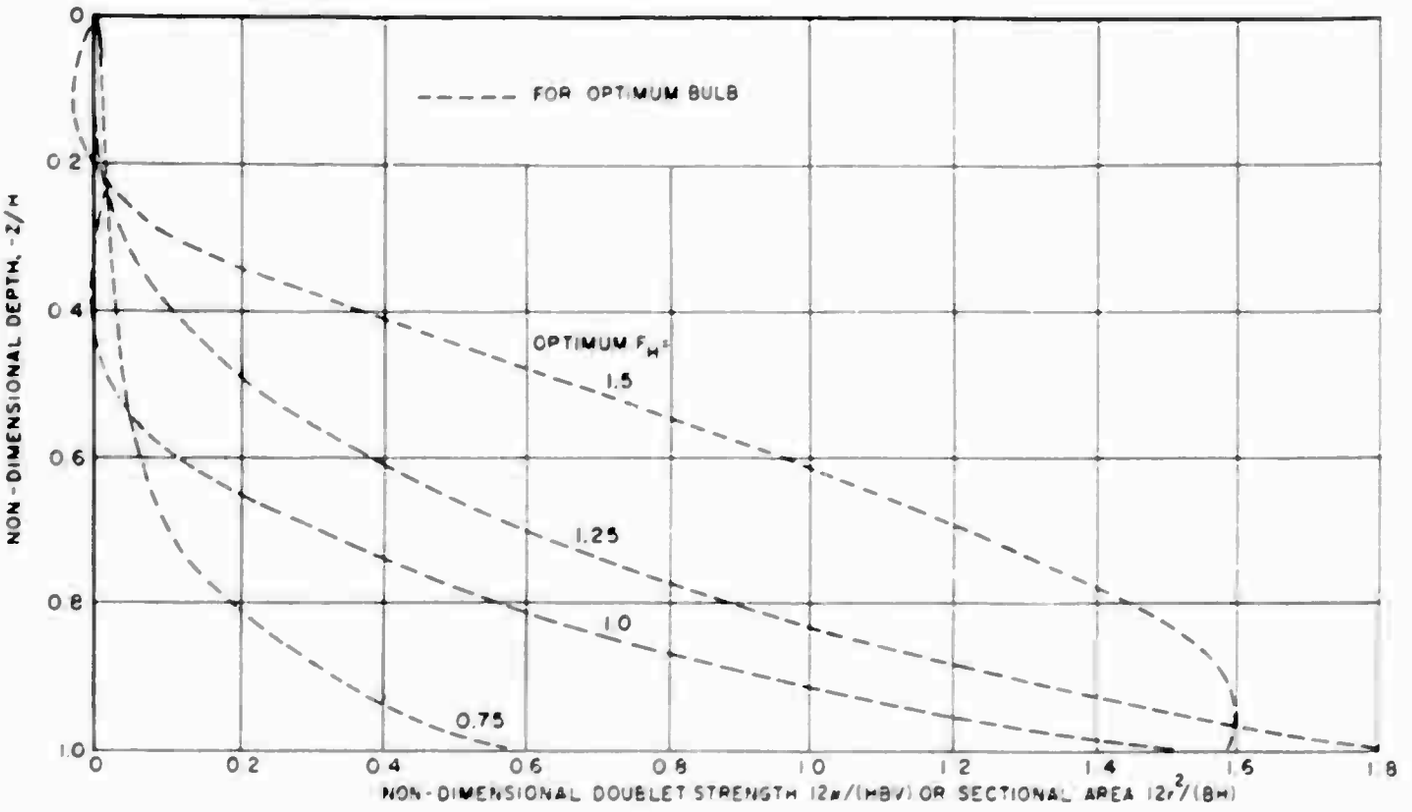


FIGURE 3a DOUBLET DISTRIBUTION FOR SINE SHIP (L/H = 16)

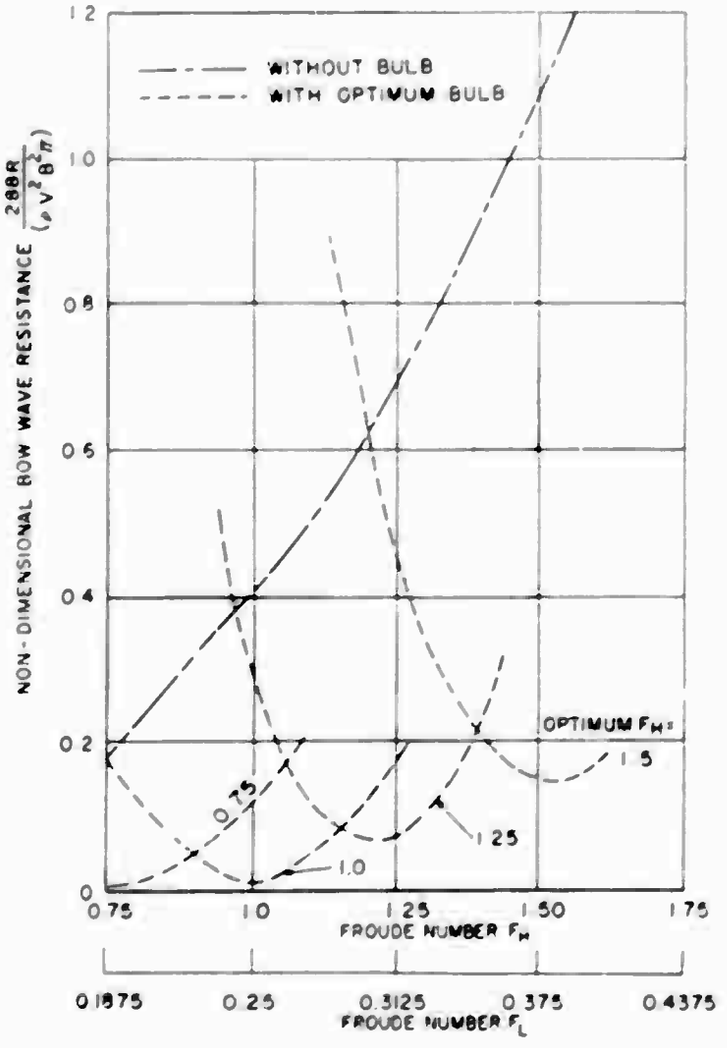


FIGURE 3b - BOW WAVE RESISTANCE OF SINE SHIP (L/H = 16)

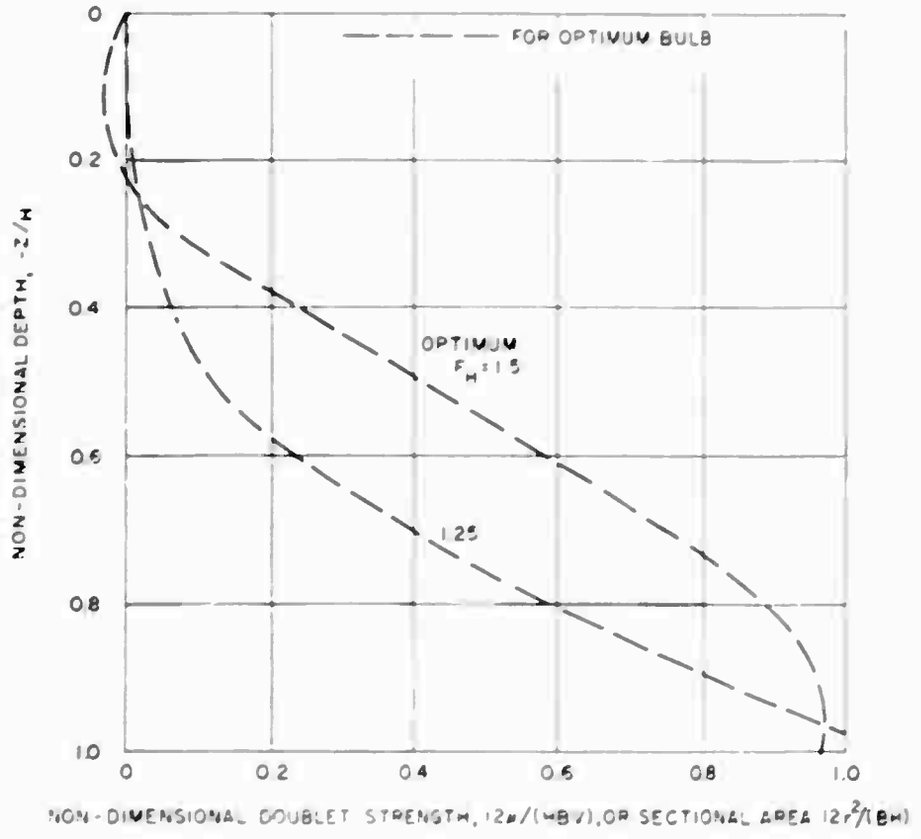


FIGURE 4c - DOUBLET DISTRIBUTION FOR SINE SHIP (L/H = 24)

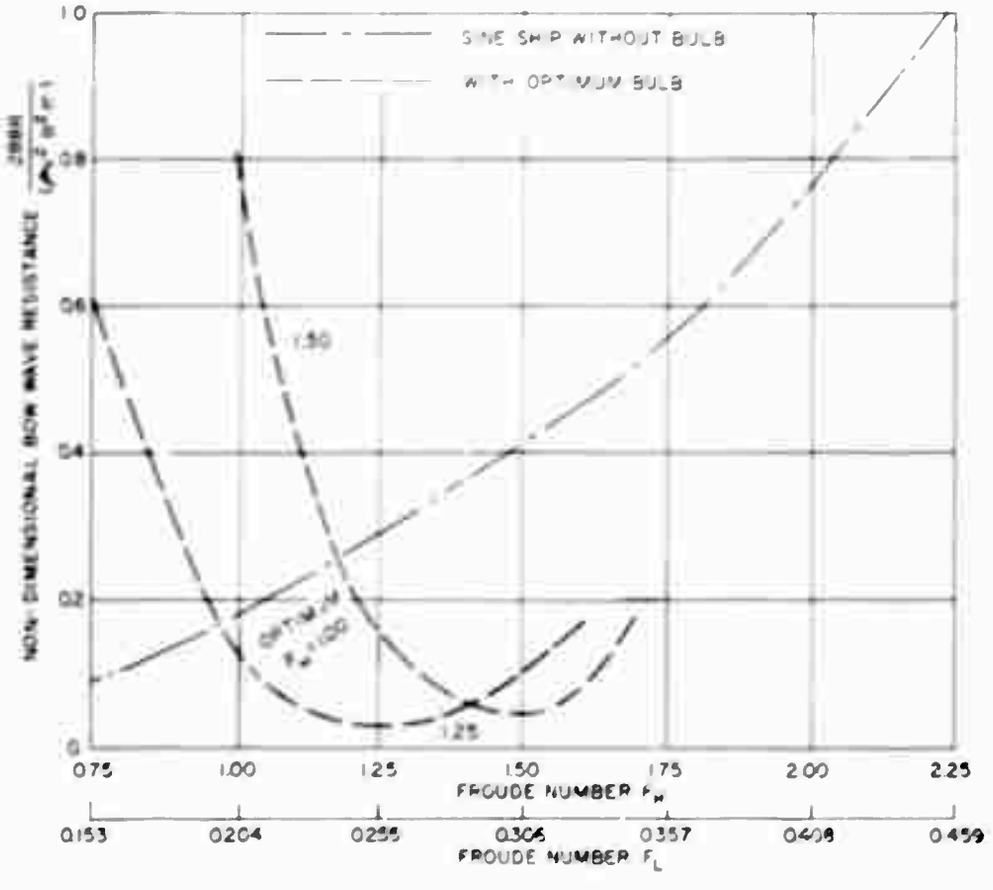


FIGURE 4b - BOW WAVE RESISTANCE OF SINE SHIP ($L/_H$ = 24)

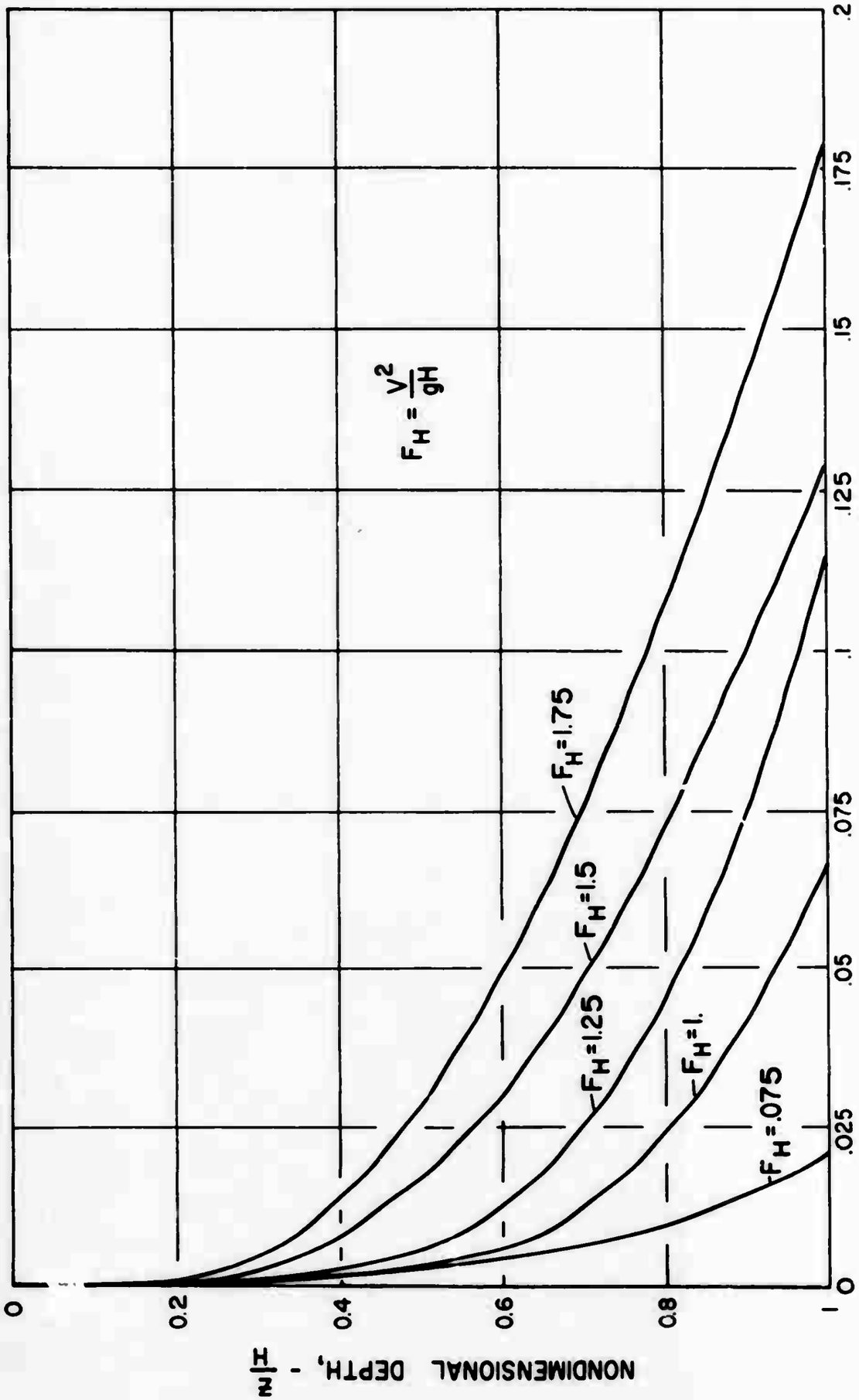


FIGURE 5a — DOUBLET DISTRIBUTION FOR A PARABOLIC SHIP, $Y = \frac{2B}{L}(x-x^2)$, $(\frac{L}{H}=16)$

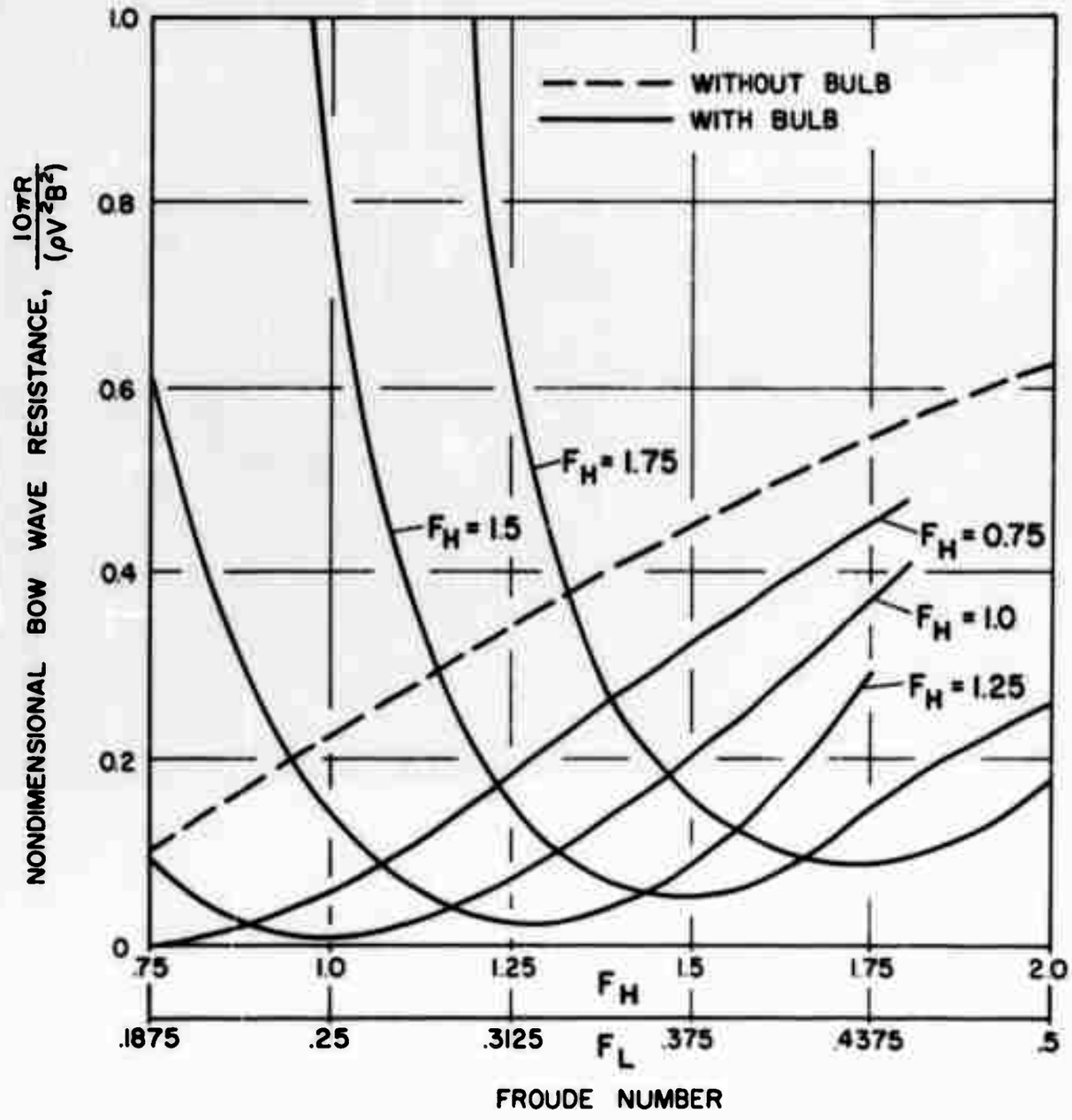


FIGURE 5b - BOW WAVE RESISTANCE DUE TO THE 1st TERM IN THE SOURCE DISTRIBUTION OF A PARABOLIC SHIP, $m = \frac{1}{\pi}\frac{B}{L}(1-2X), (\frac{L}{H} = 16)$

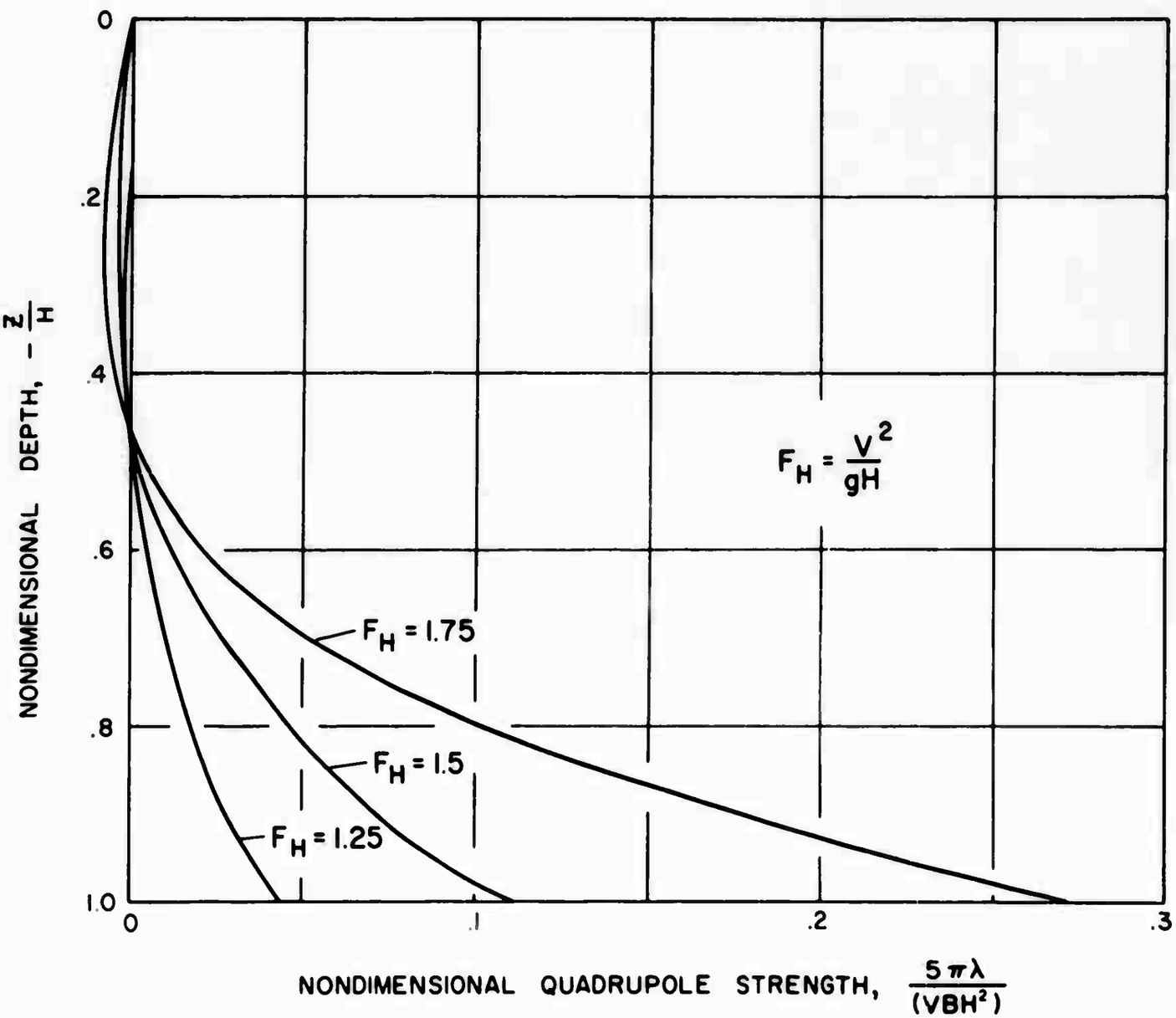


FIGURE 6a- QUADRUPOLE DISTRIBUTION FOR A PARABOLIC SHIP, $Y = \frac{2B}{L}(X - X^2)$, $(\frac{L}{H} = 16)$

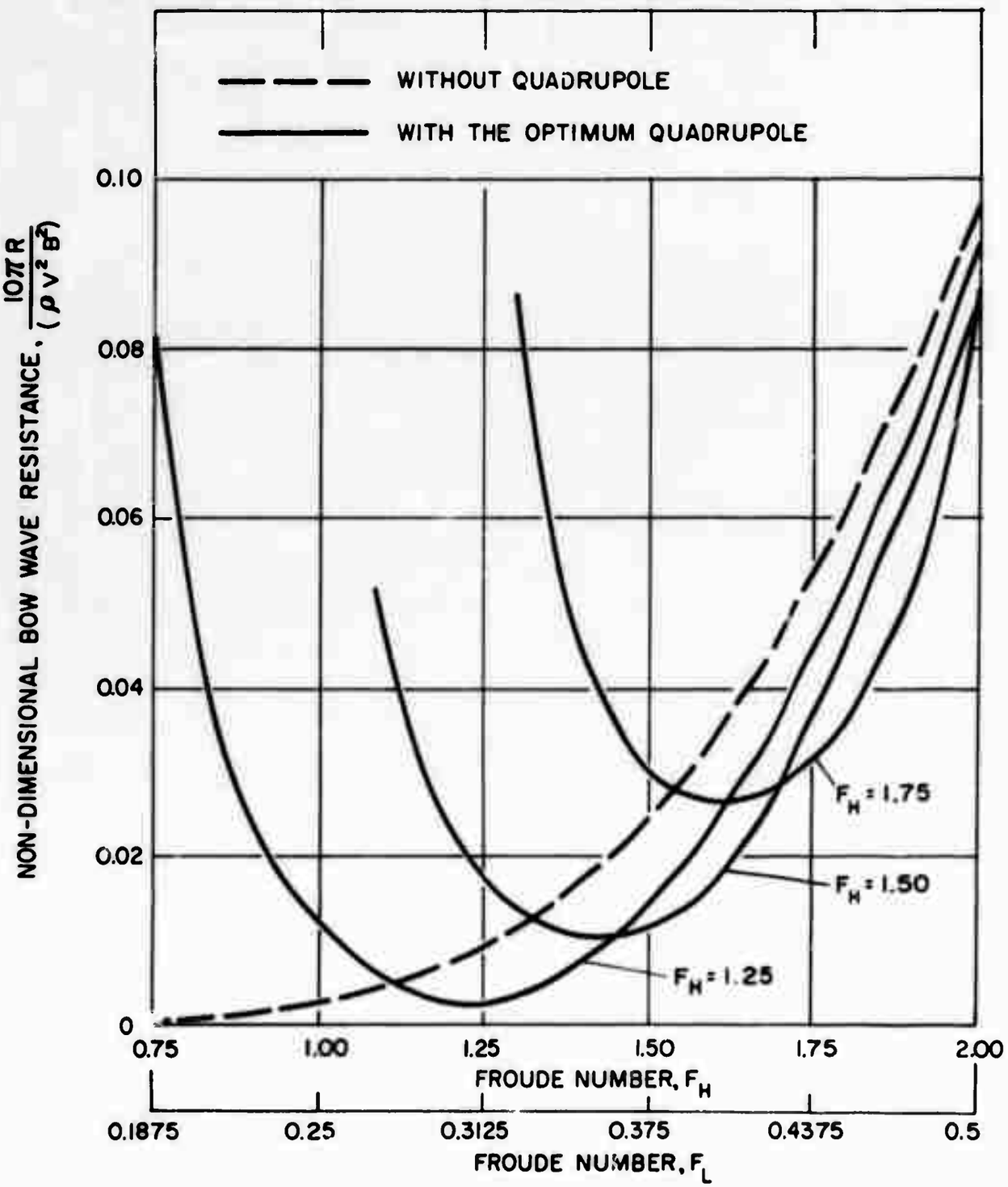


FIGURE 6b– BOW WAVE RESISTANCE DUE TO THE 2nd TERM IN THE SOURCE DISTRIBUTION OF A PARABOLIC SHIP, $m = \frac{1}{\pi}\frac{B}{L}(1-2X), (\frac{L}{H} = 16)$

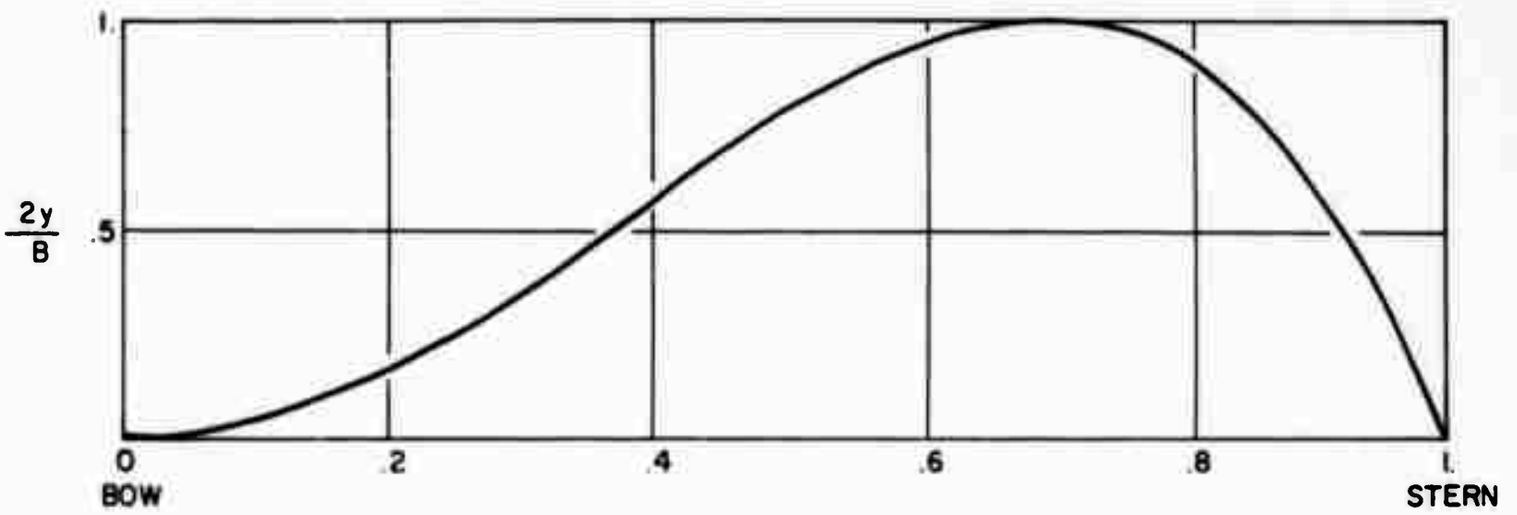


FIGURE 7a -GRAPHICAL REPRESENTATION OF A HOLLOW SHIP

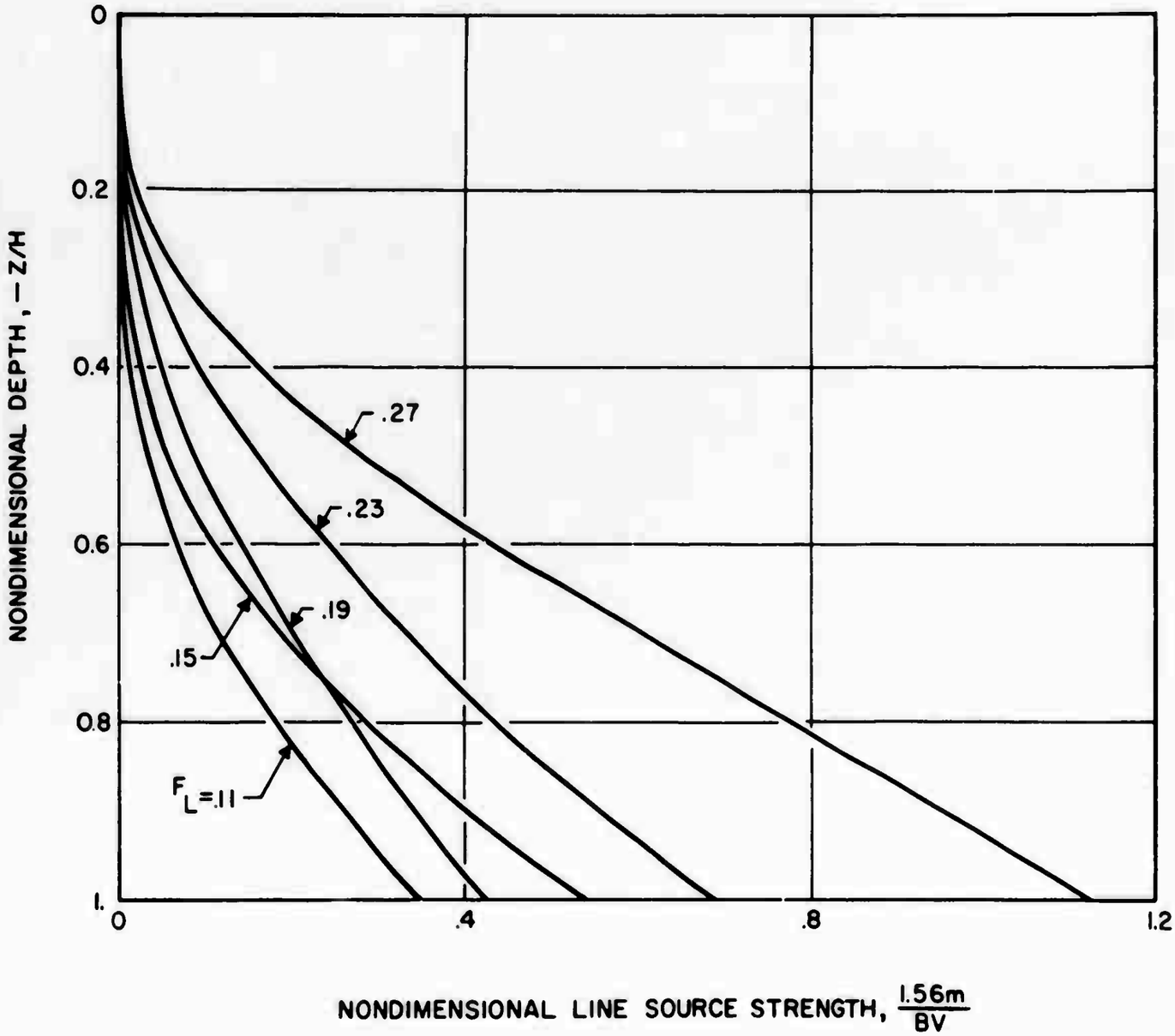


FIGURE 7b-OPTIMUM LINE SOURCE DISTRIBUTION FOR THE HOLLOW SHIP

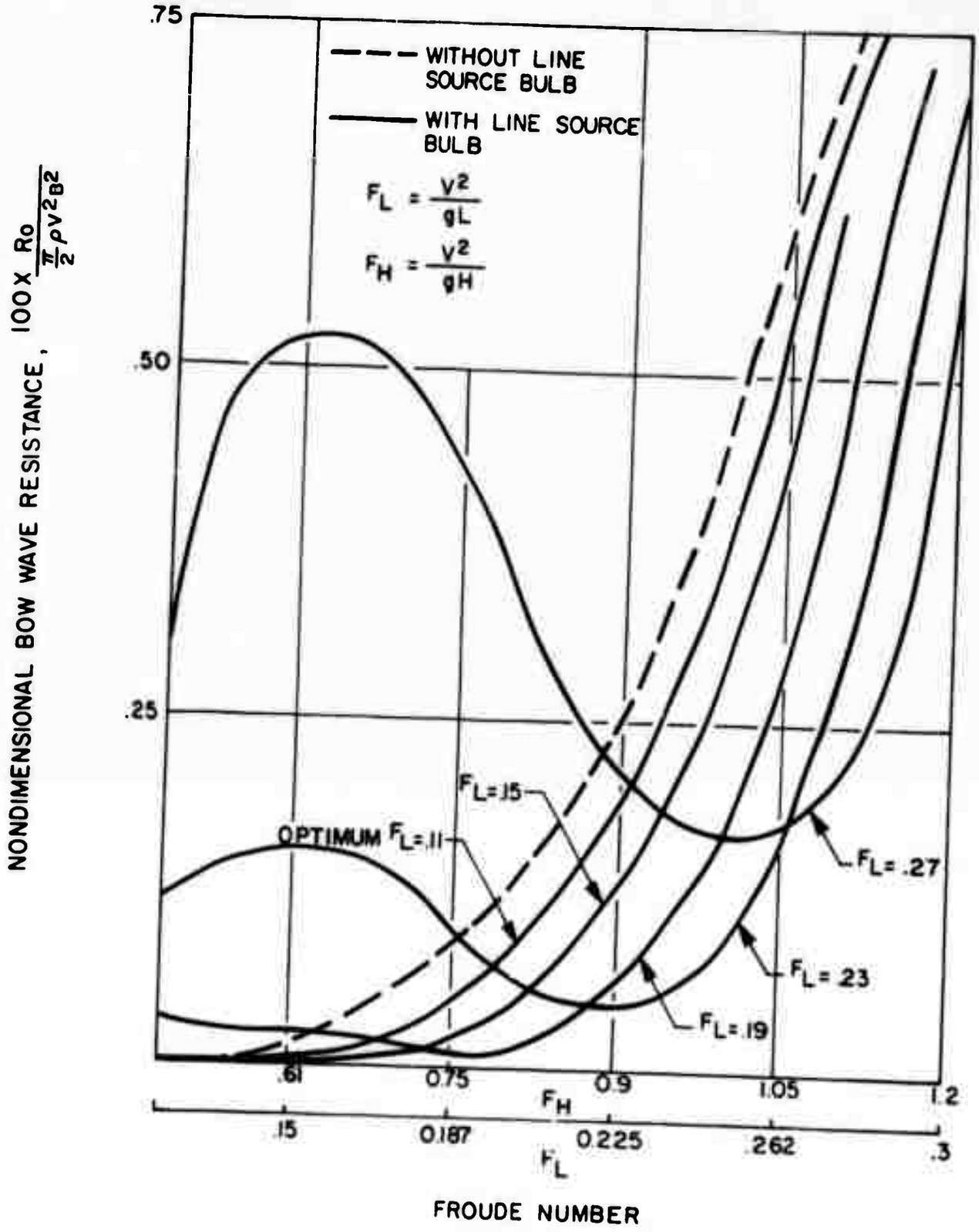


FIGURE 7c–BOW WAVE RESISTANCE OF THE HOLLOW SHIP

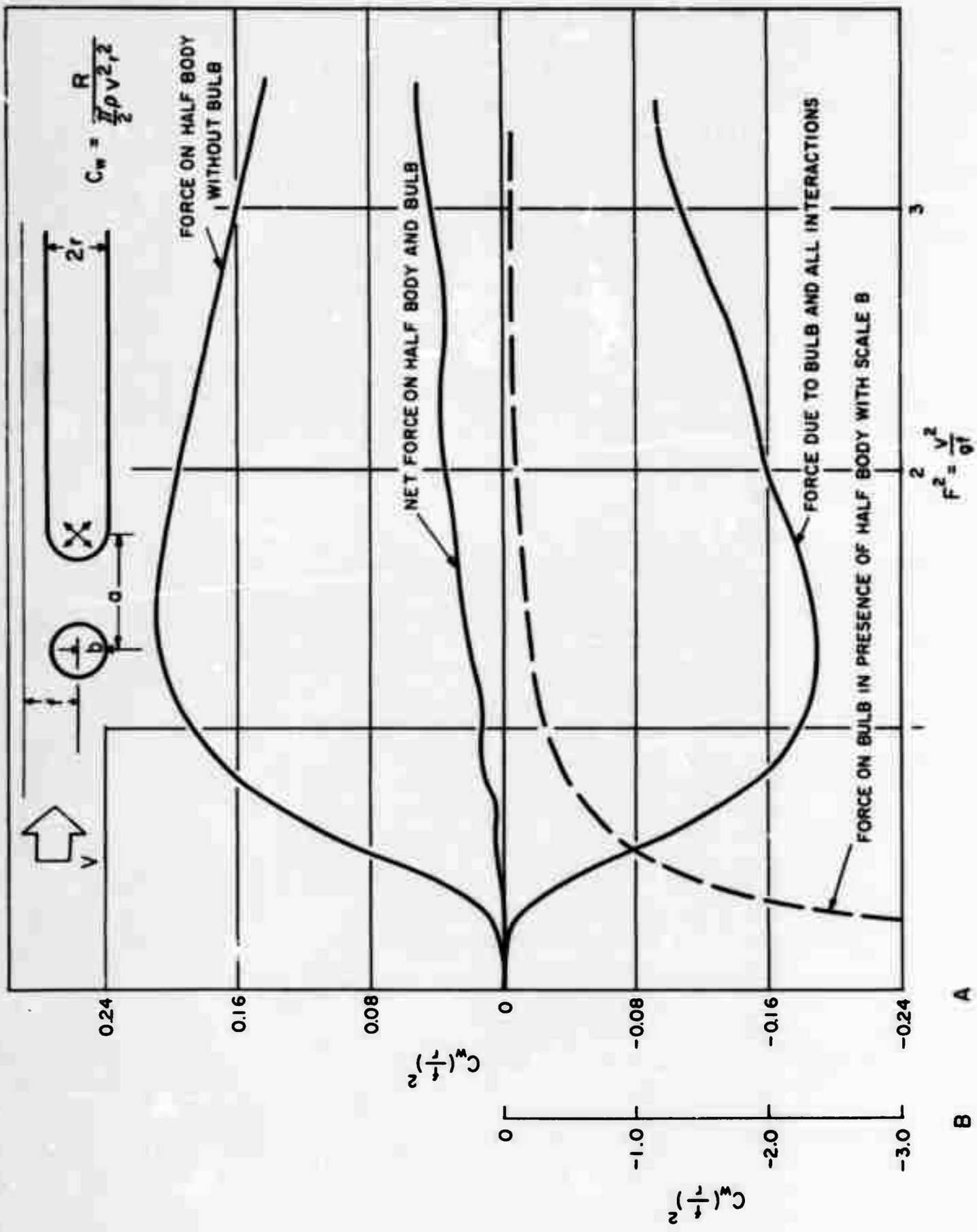


FIGURE 8- WAVE RESISTANCE OF HALF BODY WITH OPTMIZED BULB

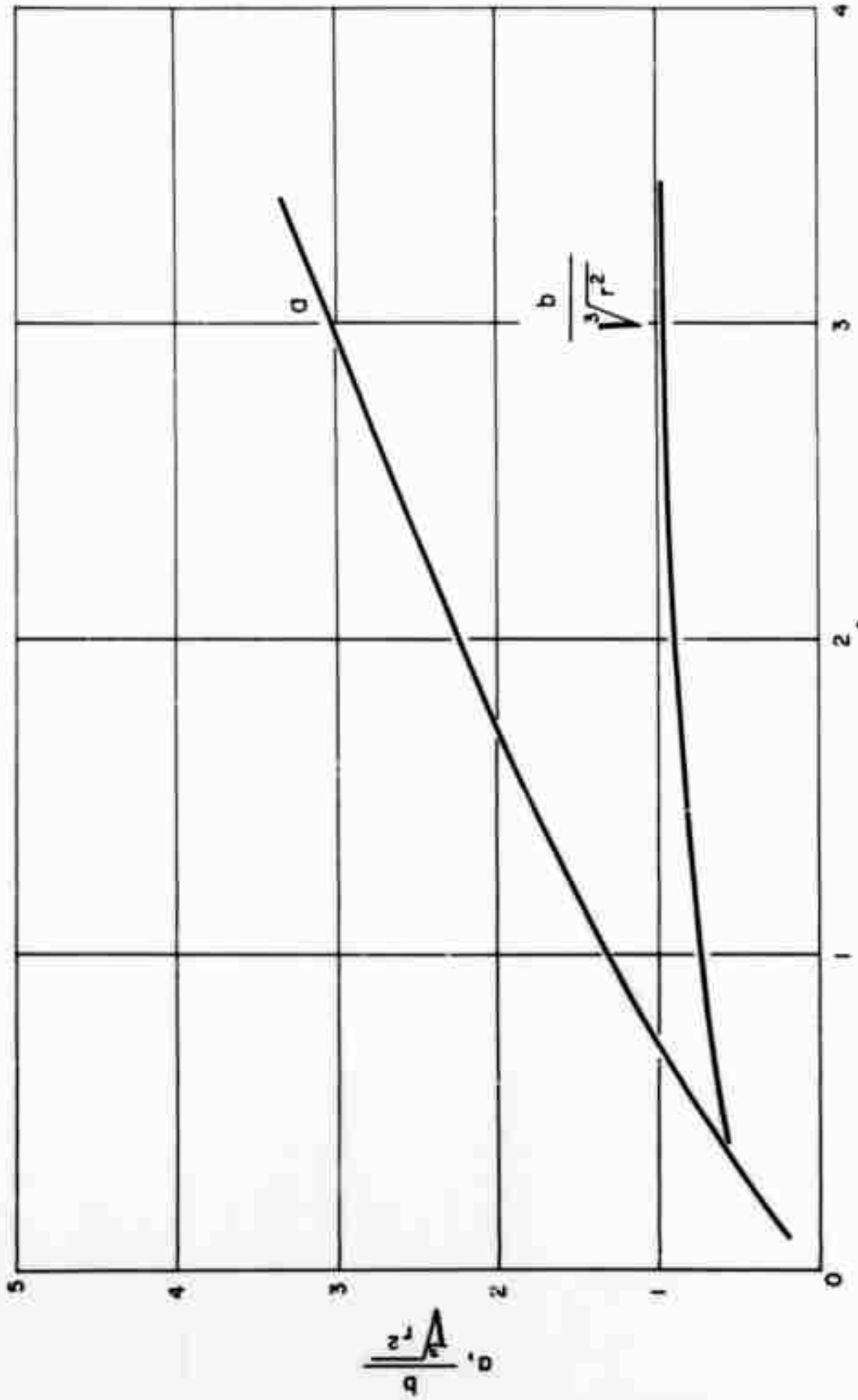


FIGURE 9- OPTIMUM DISTANCE BETWEEN SOURCE AND DOUBLET, OPTIMUM RADIUS OF BULB

a, b, r Are Non-Dimensionalized With Respect To The Depth f

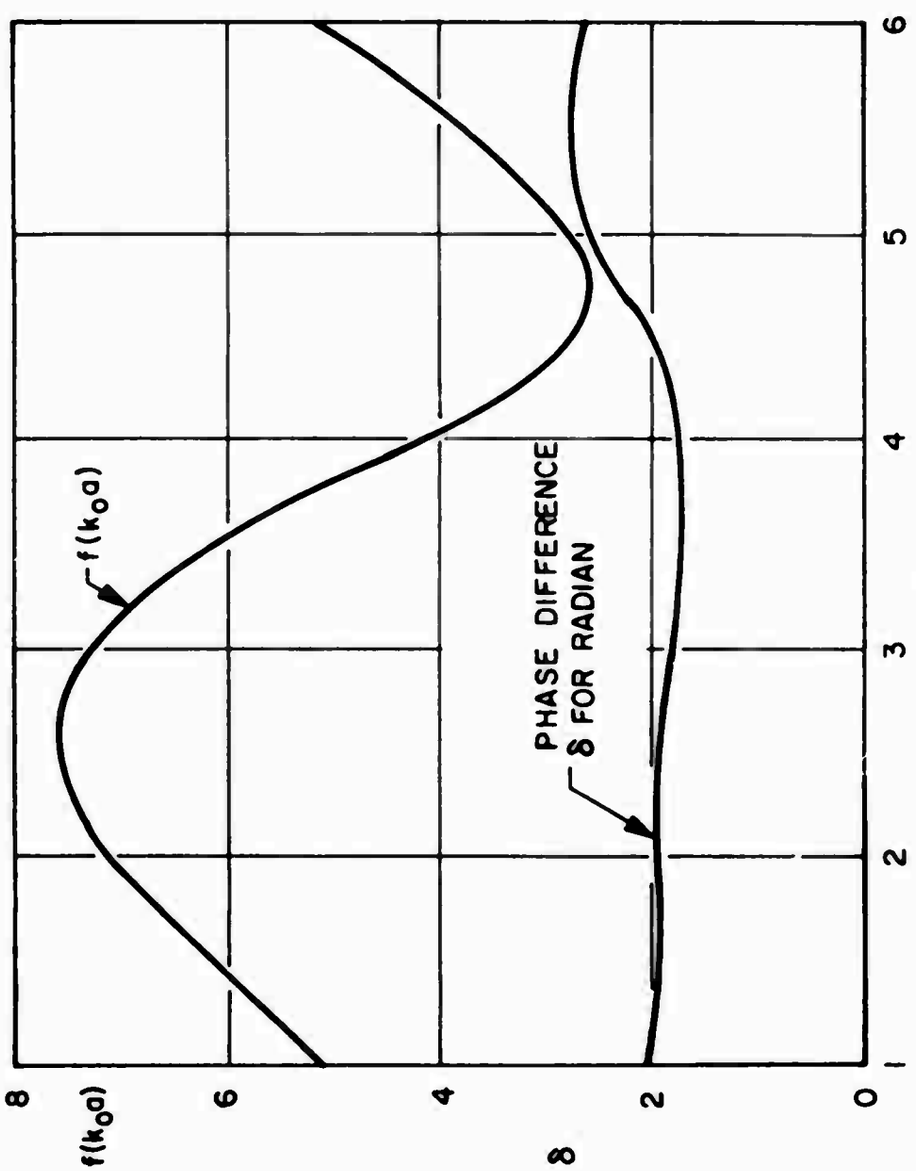


FIGURE 10-PHASE DIFFERENCE BETWEEN THE FIRST AND THE HIGHER ORDER WAVES δ, AND THE AMPLITUDE RATIO/HALF ENTRANCE ANGLE, $f(k_0a)$

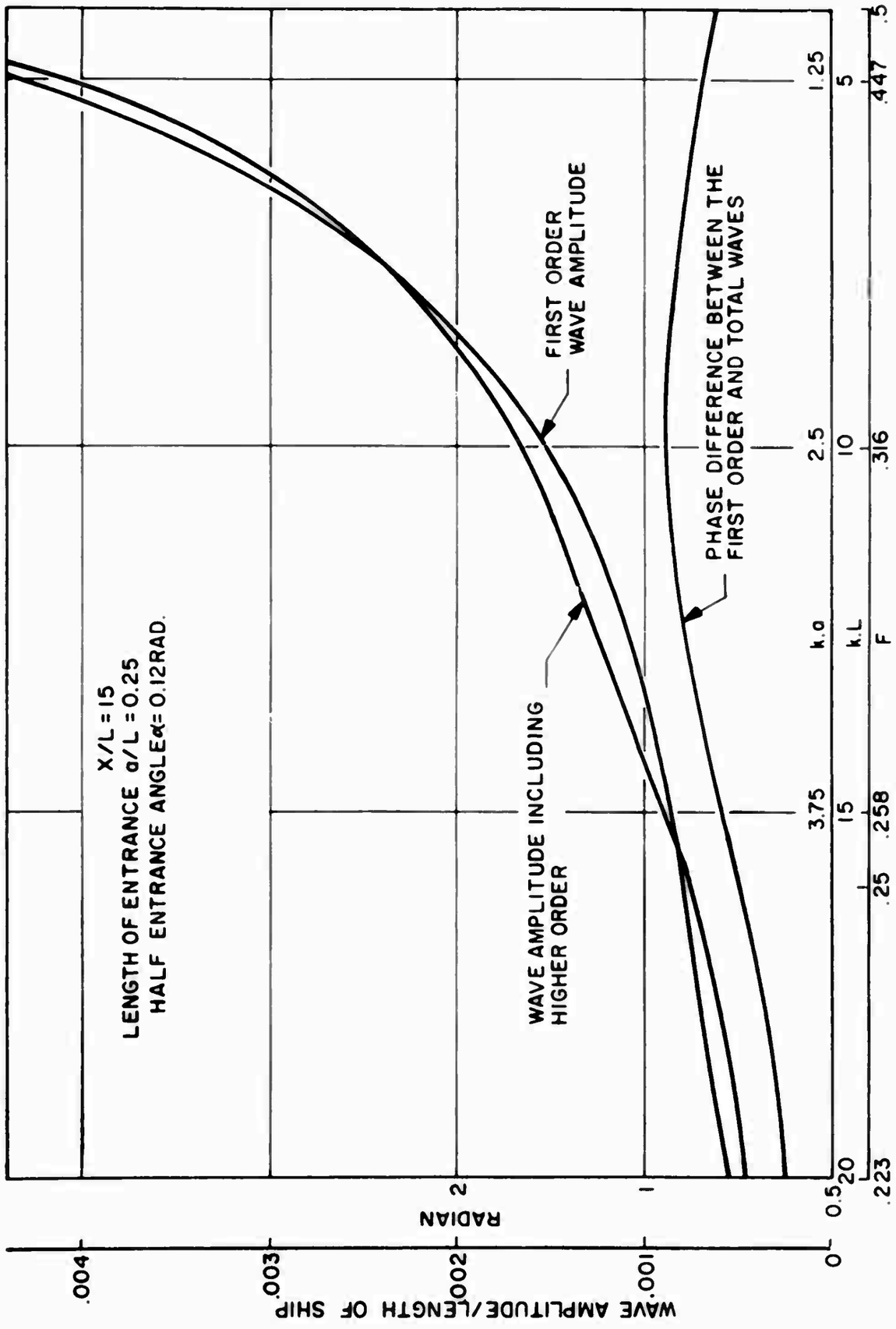


FIGURE 11 - COMPARISON BETWEEN THE FIRST ORDER WAVE AND THE TOTAL WAVE

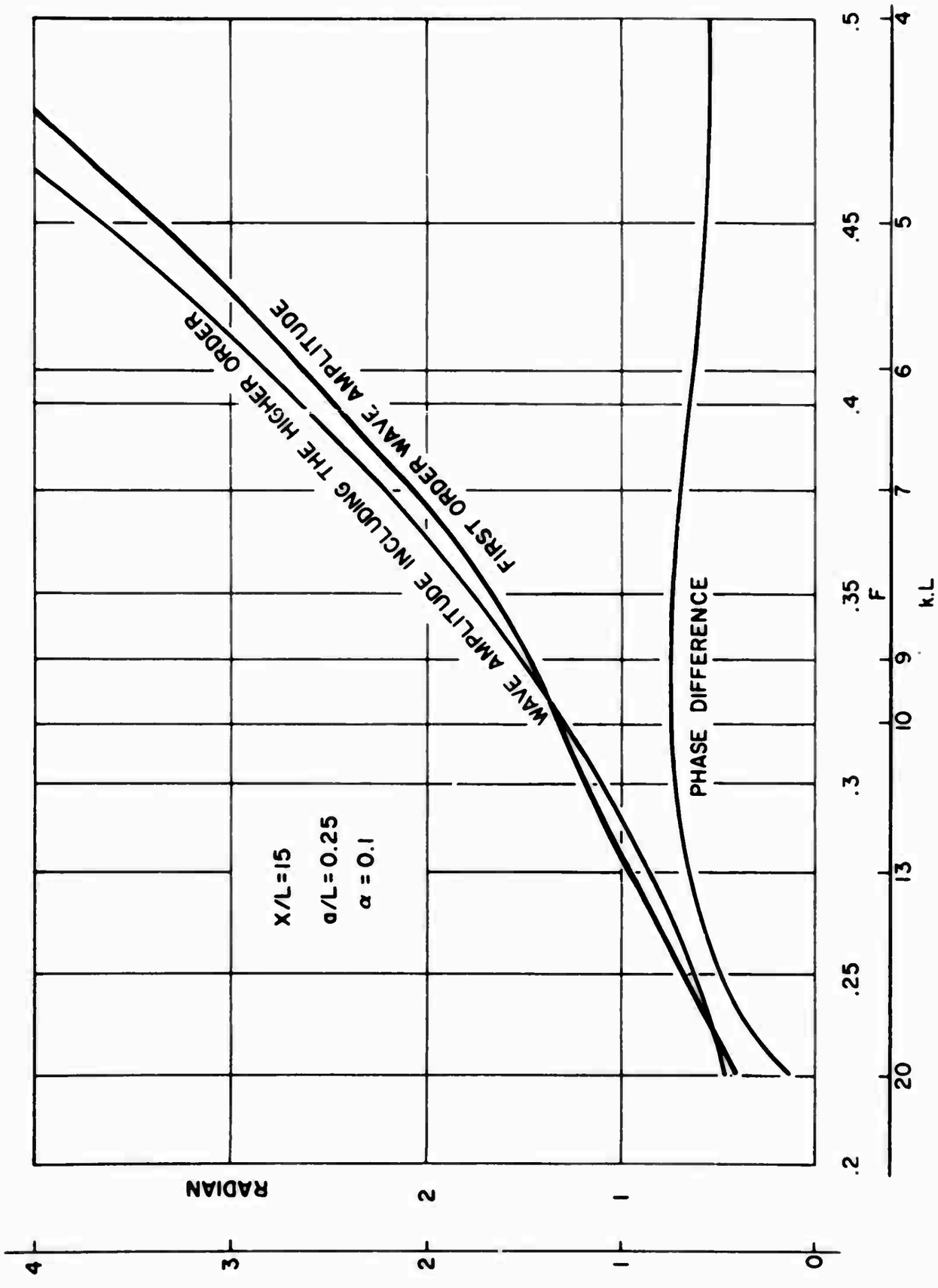


FIGURE 12–COMPARISON OF THE FIRST ORDER WAVE WITH THE TOTAL WAVE

DISTRIBUTION LIST
(Nonr 3349(00))

Chief of Naval Research
Department of the Navy
Washington 25, D. C.
Attn: Codes 438 — 3
463 — 1

Commanding Officer
Office of Naval Research
Branch Office
495 Summer Street
Boston 10, Massachusetts — 1

Commanding Officer
Office of Naval Research
Branch Office
230 N. Michigan Avenue
Chicago 1, Illinois — 1

Commanding Officer
Office of Naval Research
Branch Office
207 West 24th Street
New York 11, New York — 1

Commanding Officer
Office of Naval Research
Branch Office
Navy No. 100, Box 39
Fleet Post Office
New York, New York — 25

Commanding Officer
Office of Naval Research
Branch Office
1030 East Green Street
Pasadena 1, California — 1

Commanding Officer
Office of Naval Research
Branch Office, 1000 Geary St.
San Francisco 9, California — 1

Director
U. S. Naval Research Laboratory
Washington 25, D. C.
Attn: Codes 2021 — 6

Chief, Bureau of Ships
Department of the Navy
Washington 25, D. C.
Attn: Codes 421 — 1
440 — 1
442 — 1
420 — 1

Chief, Bureau of Naval Weapons
Department of the Navy
Washington 25, D. C.
Attn: Codes R — 1
R-12 — 1
RAAD — 1
RR — 1
RRRE — 1
RU — 1
RUTO — 1

Chief, Bureau of Yards and Docks
Department of the Navy
Washington 25, D. C.
Attn: Codes D-400 — 1

Commander
U. S. Naval Ordnance Test Station
China Lake, California
Attn: Code 753 — 1

Commanding Officer
and Director
David Taylor Model Basin
Washington 7, D. C.
Attn: Codes 142 1
500 1
513 1
521 (Dr. Pien) 1
580 1
520 1
530 1
584 (Dr. Newman) 1
584 (Dr. E. Tuck) 1

Superintendent
U. S. Naval Academy
Annapolis, Maryland
Attn: Library 1

Commander , U. S. Naval
Weapons Laboratory
Dahlgren, Virginia
Attn: Comp and Ext. Ball.
Laboratory
(Dr. Hershey) 1

Superintendent
U. S. Naval Postgraduate School
Monterrey, California 1

Commanding Officer
and Director
U. S. Naval Civil Engr. Lab.
Port Hueneme, California
Attn: Code L54 1

Commander
Norfolk Naval Shipyard
Portsmouth, Virginia 1

Commander
New York Naval Shipyard
Naval Base
Brooklyn, New York 1

Commander
Boston Naval Shipyard
Boston 29, Massachusetts 1

Commander
Philadelphia Naval Shipyard
Naval Base
Philadelphia 12, Penn. 1

Commander
Portsmouth Naval Shipyard
Portsmouth, New Hampshire
Attn: Design Division 1

Commander
Charleston Naval Shipyard
Naval Base
Charleston, South Carolina 1

Commander
Long Beach Naval Shipyard
Long Beach 2, California 1

Commander
Pearl Harbor Naval Shipyard
Navy No. 128, Fleet Post Office
San Francisco, California 1

Commander
San Francisco Naval Shipyard
San Francisco 24, California 1

Shipyard Technical Library
Code 303TL, Bldg. 746
Mare Island Naval Shipyard
Vallejo, California 1

Superintendent
U. S. Merchant Marine Academy
Kings Point, Long Island, N. Y.
Attn: Dept. of Engr. 1

Commandant
U. S. Coast Guard
1300 E. Street, N. W.
Washington, D. C. 1

Commander
Hdqs. U. S. Army Trans. Research
and Development Command
Transp. Corps
Fort Eustis, Virginia 1
Attn: Marine Trans. Div. 1

Office of Technical Services
Department of Commerce
Washington 25, D. C. 1

Defense Documentation Center
Cameron Station
Alexandria, Virginia 10

Maritime Administration
441 G. Street, N. W.
Washington 25, D. C.
Attn: Div of Ship Design 1
Research and Dev. 1

Scientific and Technical
Information Facility
Attn: NASA Representative
P. O. Box 5700
Bethesda, Maryland 20014 1

Director
Engineering Science Division
National Science Foundation
Washington, D. C. 1

National Research Council
Montreal Road
Ottawa 2, Canada
Attn: Mr. E.S. Turner 1

Society of Naval Architects
and Marine Engineers
74 Trinity Place
New York 6, New York 1

Webb Institute of Naval
Architecture
Glen Cove, Long Island,
New York
Attn: Prof. E. V. Lewis 1
Dr. Laurence Ward 1
Technical Library 1

Institute of Mathematical
Sciences
New York University
25 Waverly Place
New York 3, New York
Attn: Prof. J.J. Stoker 1
Prof. A. Peters 1
Prof. J. Keller 1

New York University
Dept. of Oceanography
University Heights
New York 53, New York
Attn: Prof. W.J. Pierson,Jr 1

New York State University
Maritime College
Engineering Department
Fort Schuyler, New York
Attn: Prof. J.J. Foody 1

California Inst. of Tech.
Pasadena 4, California
Attn: Prof. T.Y. Wu 1
Prof. C.B. Milliken 1

University of California
Berkeley 4, California
Attn: (Dept. of Engineering)
Prof. H. A. Schade 1
Prof. J.V. Wehausen 1

Director
Scripps Institution of
Oceanography
University of California
La Jolla, California 1

Iowa Institute of Hydraulic
Research
State University of Iowa
Iowa City, Iowa
Attn: Prof. H. Rouse 1
Prof. L. Landweber 1

Harvard University
Cambridge 38, Massachusetts
Attn: Prof. G. Birkhoff 1
Prof. G.F. Carrier 1

University of Michigan
Ann Arbor, Michigan
Attn: Prof. R. B. Couch 1

Director
Ordnance Research Laboratory
Penn. State University
University Park, Penn. 1

Director
St. Anthony Falls Hydraulic
Laboratory
University of Minnesota
Minneapolis 14, Minnesota 1

Massachusetts Inst. of Tech.
Cambridge 39, Massachusetts
Attn: Prof. P. Mandell 1
Prof. M.A.Abkowitz 1

Graduate School of Aeronautical
Engineering
Cornell University
Ithaca, New York
Attn: Prof. W.R. Sears 1

Director
Stevens Institute of Technology
Davidson Laboratory
Hoboken, New Jersey 1

Director
Woods Hole Oceanographic
Institute
Woods Hole, Massachusetts 1

Southwest Research Institute
8500 Culebra Road
San Antonio 6, Texas
Attn: Dr. H.N. Abramson 1

Prof. J. W. Johnson
412 Hesse Hall
Dept. of Civil Engineering
University of California
Berkeley, California 94720 1

Manchester University
Manchester, England
Attn: Prof. F. Ursell 1

Mr. C. Wigley
Flat 103
6-9 Charterhouse Square
London E.C. 1, England 1

Therm, Incorporated
Ithaca, New York 1

HYDRONAUTICS, Incorporated
Pindell School Road
Howard County
Laurel, Maryland
Attn: Mr. P. Eisenberg 1
Mr. M.P. Tulin 1

Oceanics, Incorporated
Plainview, Long Island, N.Y.
Attn: Dr. Paul Kaplan 1

Professor R.C. MacCamy
Department of Mathematics
Carnegie Institute of Tech.
Pittsburgh 10, Pennsylvania 1

Dr. K. Eggers
Institut fur Schiffbau
University of Hamburg
Berliner Tow 21
Hamburg, Germany 1

Dr. R. Timman
Department of Applied Mathematics
Technological University
Julianalaan 132
Delft, Holland 1

Secretary, Ship Structure Comm.
U. S. Coast Guard Hdqtrs.
1300 E. Street, N. W.
Washington 25, D. C. 1

Commander
Puget Sound Naval Shipyard
Bremerton, Washington 1

Commander
Military Sea Transportation Service
Department of the Navy
Washington 25, D. C. 1

Mr. Richard Barakat
Optics Department
Itek Corporation
Lexington, Massachusetts 1

National Academy of Sciences
National Research Council
Committee on Undersea Warfare
2101 Constitution Avenue
Washington 25, D. C. 1

Dean Harvey Brooks
School of Applied Sciences
Harvard University
Cambridge, Massachusetts 1

Netherlands Ship Model Basin
Wageningen, The Netherlands
Attn: Dr. Ir.J.D. Van Manen 1