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PETROLEUM REFINERY OPERATIONS SCHEDULING

Chapter V - A Gasoline Blending Problem¹

Alan S. Manne

1. Basic economic assumptions

The preceding chapter has dealt with one type of intra-refinery decomments balance problem -- the operation of a typical kind of conversion unit. The present one concentrates upon another such intra-refinery study -- the selection of an optimal gasoline blending schedule. Not until the following chapter will any attempt be made to discuss the operations of an integrated refinery.

There were two primary reasons for singling out the blending problems of the Union Oil Company of California: First, this operation is itself a complex large-scale one -- grossing an amount of the order of \$100 millions annually. And second, the data available for this analysis were of tolerable precision. It was agreed that there is little point in making an elaborate economic study when the basic numbers are shaky.

Together with members of the Manufacturin; Economics Division of Union Oil, the problem was formulated in the following terms (see the schematic diagram, Figure 10): First, the quantities and qualities of 19 raw gasoline stocks were assumed given. Together with tetraethyl lead (a purchase; item), these serve as the inputs to the gasoline blending operation. The input materials are combined to form three basic types of products: 7500, premium grade gasoline; 76, regular grade gasoline; and fuel oil cutter stock. As a first approximation, it was assumed that each of these products could be sold at refinery realizations that were independent of the total quantity marketed. The scheduling problem consists of assigning each raw gasoline stock among

^{1.} In connection with this section, a particular lebt of gratitude is owed to Messrs. Reaugh, McCreery, and Norton of the Manufacturin; Economics Division of the Union Oil Company of California. Without their encouragement and active help, it would not have been possible to carry through the work described here.

the three potential products, and also of setting the tetraethyl lead levels for the two motor fuels. This assignment has to take into account not only the absolute quantities of materials that are available, but also certain product specifications on the two finished gasolines.

Each of these assumptions must be reviewed with care. Consider, for example, the matter of bleading stocks. With the exception of just two of the 1-inputs, it is assumed here that the quantities of raw gasolines available and that the quality characteristics of these materials are independent of decisions taken in the bleading department. In fact, from the viewpoint of a refinery superintendent, the quantities and characteristics of all the streams are variables subject to control. By altering the choice of crudes, the reactor temperatures, the recycle ratios, and the assignment of intermediate oil streams, the central management is able to influence the size and composition of the gasoline blending materials. This type of interlock can only be studied in a larger model that cuts across departmental lines.

A second point

additi nol quantities of finished refinery products may be marketed at the prevailing realizations. In the case of a major refiner, this is open to serious doubt. It is not at all obvious that a large company can put additional quantities on the market without disturbing the current price structure.³

is the assumption that

See J. S. Bain, The Economics of the Pacific Coast Petroleum Industry, University of California Press, Part I, 1744; Part II, 1945; and Part III, 147.

During the course of short time intervals, the quantities and qualities of the available blending stocks may change radically. An unforeseen shutdown of a unit can take place. A new unit may be brought on stream. A new source of crude oil may be introduced, etc. For these reasons, in order to keep blending analyses up to date, it is the current practice within the company to recalculate schedules at least once a month. The optimum allocations of raw streams among the various uses could be significantly affected if, say, a fire occurred in a large catalytic cracker.

At the same time, no one -- least of all an economist -- is in a position to make a reasonable quantitative estimate of the relationship between a company's price structure and the demand for its products.

For purposes of computation, an explicit marketing limitation could have been placed upon each finished product. In order to provide a benchmark, and then one however, it was decided to study one basic case in which a sales restriction was introduced. The comparison of these two provides an indication of the extent to which profits could be improved by overcoming the marketing barrier. The comparison also indicates the magnitude of the gap between the refinery realization and the incremental cost in the second instance.

Corresponding to the assumption about the realization upon refinery products, it was agreed that one of the inputs -- tetraethyl lead -- would be considered as available without any quantity restrictions at the Ethyl Corporation's prevailing tank-car delivered price. During wartime, TEL has been rationed among refiners, but at least during 1953, this type of restriction hold true. the If lead were to become short again, A did notA explicit availability limitation would have to be considered as an element in the problem. As shown in the preceding chapter, it is possible for the refiner to substitute reformed and cracked gasolines over a wide range in place of tetraethyl lead in the finished products. Although lead is employed in virtually every gallon of gasoline today, its use in motor fuel could be eliminated -- but only at a distinct cost in dollars and cents to the refiner.

Having decided upon the two major assumptions -- fixed gasoline stock availabilities and fixed refinery product realizations -- the next step was to determine which of the numerous quality characteristics of the finished products

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would be studied explicitly. The Manufacturing Economics Division pointed out that there were many specifications that finished motor fuel must meet -three types of octane numbers; 10%, 30%, 60%, 90%, and end points of the boiling range; sulfur and gum content; wapor pressure, etc. Any of these could conceivably limit the output of either premium or regular grade gasoline. For purposes of this study, the ones that seemed to be worth the most close attention were: Research octane number, the percent sulfur content, and a volatility index. In addition, the end point problem was handled automatically by limiting the end points of the individual raw gasolines. During the course of the computing runs, it turned out that sulfur was not a limiting specification, but that the octane number and volatility index specifications did come into play.

In order to protect proprietary information, the exact house-brand specifications cannot be revealed here. It can, however, be said that the 7600 premium gasoline has a higher minimum octane number and volatility index -and a lower maximum permissible sulfur content and end point than the regular 76 gasoline. On both products, the maximum permissible tetraethyl fluid concentration is 3.0 milliliters per gallon.

Although the blending section must operate within whatever company specifications are currently effective, the reader should not suppose that these limits remain fixed over time. At any moment, "octane wars" between refiners can break out. The effect upon motorists -- and upon refinery profits -- is of much the same nature as an actual price war. For this reason, a separate computation was set up in order to determine the effects of a 2.5 octane number increase in the specifications of the premium gasoline.

Before proceeding to the technology of the problem, two more of the economic assumptions should be brought out -- one having to do with the are manufactured. cutter stocks and the other with transportation costs. The raw gasolines A and the blending operations are performed at two distinct locations -at Wilmington in the Los Angeles area and at Oleum on San Francisco Bay. Raw stocks are continually lifted by tanker for interchange between the two refineries. Union Oil estimates its tanker costs between the two points to be nominal - only a few percent of the refinery realization on previum gasoline. In the analysis of the company's gasoline blending problems, it is evidently a desirable thing to take account of transportation costs and of differences in product realisations at the two refineries. At the same time, it was easy to see that by bringing geography into the problem, the volume of computations would have increased by more than 100%. The storage capacity of the then available computing machine -- the International Business Machine Card Programmed Calculator -- would have been completely inadequate to take care of the enlarged model." As a first effort, therefore, it was agreed that the geography should be left out of account. considered to be

The raw gasoline stocks were Aavailable in one company-wide pool, were and the finished products valued at the company's overall realizations. (Roughly speaking, these revenue figures are of the same order of magnitude as the current spot market refinery tank-car quotations.)

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^{4.} This type of limitation would not have existed if one of the highercapacity electronic computers had been available, e.g., the IBM 701 or the Univac.

Finally, the problem of outter stock realisations. As indicated by Figure 10, eight of the heavy gasolines may be employed, not only for motor fuel components, but also for blanding with heavy residuum into No. 6 fuel oil. The figure also shows that this heavy residuum may be converted into No. 6 oil by the alternative route of blending with No. 2 furnace oil.⁵ Under the assumption that <u>some</u> heavy residuum is always going to be cut down to the viscosity specifications by blending with No. 2 oil, it is possible to establish a value for the heavy residuum, and then to go on to determine a cutter value for any particular heavy gasoline.

P-487 V-6

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To give a purely hypothetical illustration of the procedure, assume the following product values:

No. 2 furnace oil \$4.00/B.
 No. 6 fuel oil \$2.00/B
 f No. 2 oil in No. 6 blend required for cutting beckgresicham to No. 6 oil specifications

\$ of heavy gasoline in No. 6 blend required for outting back heavy residuum to No. 6 oil specifications 20% If No. 2 oil is used for blending, the value of the heavy residuum, pr,

may be established through the following equation:

there is

 Value of No. 6 oil = Value of No. 2
 + Value of residence

 oil component
 component

 \$2.00 = .25(\$\$.00)
 + (1 - .25)(p_r)

 p_r = \$1.333/3

⁵In general, the outter will not be a finished No. 2 cdl, but rather some gas cil component that <u>equid</u> be used for No. 2. When there are several alternative gas cils that may be employed for outter, the problem takes an additional complexity. See Chapter VI below, p. VI - 26



Next, this value for p_r is substituted in a costing equation for the heavy gasoline cutter stock. Out of this relation, there is derived P_g , the cutter value of the particular gasoline.

Value of No. 6 oil = Value of gasoline component + Value of residuum $\$2.00 = .20(p_g)$ + (1 - .20)(\$1.333) $P_g = \$4.668/B$

Values for each of the eight possible gasoline materials were calculated in this manner. Certain treatment cost savings also had to be credited to the cutter operation. The final calculated net cutter values range from 68.2% to 87.4% of the refinery realization on premium gasoline. In effect, this is an opportunity cost calculation of the same nature as the "gasoline replacement" technique described above, pp. II-5-7. Like the gasoline replacement formula, this method applies to the comparison of just two alternative refining operations. (In the one case, the alternatives are fuel cracking versus fuel oil, and in the other, gasoline versus No. 2Aoil were capable of meeting the cutter materials.) In the event that the company's residuumA oil specifications without any additional cutter, the heavy gasoline could not be given the premium value as cutter, but would have to be assigned a lower credit.

fuel

2. The blending technology

Of the three relevant gasoline properties, two -- the volatility index and the percent sulfur content -- give rise to straightforward linear blending relationships. For example, in a 50-50 blend of two raw gasolines with sulfur contents of 0.10% and 0.30%, the resulting mixture will contain 0.20% sulfur. Similarly with the volatility index, a 50-50 blend will have an index that lies midway between those of its two component raw gasolines.

Unfortunately, the octane number of blends cannot be predicited in such a direct manner from the octane numbers of the components. Like most kinds an of engineering work, the forecast of gasoline octane numbers is rather artAthan an exact science. Refiners themselves necessarily make paper predictions

As f the octane number of proposed blends, but before marketing a product they will almost inevitably take the precaution of testing the mixture in an octane rating engine.

One phase of the problem has been investigated extensively by the refining industry -- the relationship between octane numbers and the TEL concentration of a particular gasoline. Just as in the naphtha reforming problem above, it has been observed that, as the lead concentration level is increased, the octane number increases at a decreasing rate. In order to predict this relationship, there is in widespread use today an ethyl blending nomogram put forward by Hebl, Rendel, and Garton in 1939.⁶ fortunately, There is, Na three-parameter analytic expression that gives a close approximation to the results predicted by this blending chart. Where <u>t</u>

^{6.} L. E. Hebl, T. B. Rendel, and F. L. Garton, "Ethyl Fluid Blending Chart for Motor-Method Octane Numbers", <u>Industrial and Engineering Chemistry</u>, Vol. 31, July, 1939, pp. 862-865.

represents the octane number of the leaded gasoline, and \underline{x} the TEL concentration level (in milliliters per gallon):

(V.1)
$$t = a + bx - \frac{c}{1+x}$$

For any unknown gasoline, the three positive parameters, a, b, and c, have to be determined. This may be accomplished readily by taking three the resulting observations of octane number for various lead levels, and solving Asimultaneous linear equations. Using equation (V.1) in this way, sets of calculated constants were_A for all 19 of the Union Oil Company's raw gasolines. Testing seven lead levels in each of these 19 cases, the equation has virtually always given a prediction that lies within $\stackrel{+}{-}$ 0.2 octane number of the one yielded by the blending chart.

No special significance should be attached to the particular form of equation (V.1). It is merely a device for enabling a computing machine to perform the same calculation as a refiner with his ethyl chart and his straight edge.

A second phase of the blending problem is more controversial than the TEL aspect. For a lead level of, say. 3.0 ml., and for a 50-50 mixture of two gasolines A and B, refiners frequently calculate the octane number of the blend to be the 50-50 weighted average of the octane mmber of gasolines A and B -each with 3.0 ml. The weighted average appears satisfactory for many stocks -especially when both components are of a paraffinic nature. There are, however, at least two papers publicly available -- one by Eastman⁷ and the other by Bogen and Nichols -- that call into question the straight-line average method. 7 Du Bois Eastman, "Prediction of Octane Numbers and Lead Susceptibilities of Gasoline Blends", <u>Industrial and Engineering Chemistry</u>, Vol. 33, December, 1.41, pp. 1555-1560.

⁸ J. S. Bogen and R. N. Nichols, "Calculating the Performance of Motor Fuel Blends", <u>Industrial and Engineering Chemistry</u>, Vol. 41, November, 1947, pp. 2629-2635. °-4 °7 ∀-10 Both papers indicate that as the percentage of the high octane component in a binary mixture increases (both components initially at the same TEL concentration level), the octane level of the mixture may increase at a decreasing rate. In other words, the octane number of the blend tends to <u>exceed</u> the weighted average of the octane number of the two components.

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Bogen and Nichols are primarily concerned with three-component mixtures, and they report only five observations at each of three TEL levels for strictly binary blends. The set of 15 points has been plotted, and is reproduced as Figure 11. For each TEL concentration level shown on this chart, straightline interpolation has been used between adjacent observations. In all likelihood, these straight lines also underestimate the octane number that is attainable by blending. The existing data, unfortunately, do not provide a basis for curvilinear interpolation.

At first glance, it would appear to be a trivial matter whether octane numbers were calculated by drawing straight lines between two adjacent points or between two end points of a constant-TEL line. Nevertheless, a sample calculation presented in an earlier paper⁹ indicates that the two methods can lead to TEL input requirements that differ by 20%.

It is evidently worthwhile for a refiner to incur considerable expense in testing gasoline blends, and in attempting to predict the occurrence of curvilinear octane relationships. Nevertheless, at the present stage, the Union Oil Company did not have such blending data available, and it was necessary to revert back to the more conventional straight-line interpolation procedure for calculating the octane numbers of blends.¹⁰ The information (photo-offset A. S. Maane, "Concave Programming for Gasoline Blends", P-383

paper), The RAND Corporation, Santa Monica, California, April 23, 1953.

¹⁰ In going back to the simple straight-line octane interpolation procedure, this paper follows the model proposed by A. Charnes, W. W. Cooper, and B. Mellon, "Blending Aviation Casolines -- A Study in Programming Interdependent Activities in an Integrated Oil Company", Econometrica, Vol. 20, No. 2, April, 1952, pp. 135-159. But unlike the aviation gasoline system, the present one includes ethyl fluid concentration levels as a specific variable with a curvilinear effect.



currently at hand made it possible for the analysis to take account of non-linear TEL relationships, but not of the non-linear gasoline blending features.

The TEL inputs are one of the elements in the Union Oil Company problem that involved a departure from a strictly linear system. In addition, the cutting temperatures on two of the gasoline distillation units introduced non-linear effects. For purposes of blending, just as in the Kellogg Company naphtha reforming problem, the cut point on each of these units is an independent variable that is subject to control. The cutting temperature influences, not only the absolute quantity of material that is available for motor fuel blending, but also the quality of this component. Figures 12a - 12c contain the necessary data for one of these two distillation units -- the one producing gasoline stock #31.

According to Figure 12a, as the cutting temperature moves from 300° to 400° F., the amount of material available for motor fuel blending rises steadily from 1,900 B/CD to 3,050 B/CD. (For each additional barrel of motor fuel obtained, there is approximately a one-barrel decrease in the amount of gasoline cutter stock available.)

Charts 12.b and 12.c contain the original Union Oil Company A also indicate the form of andAthe equations that were fitted to these data. As the make of motor virtually fuel increases, Figure 12.b indicates aAlinear decline in the octane number --both clear and with 3 ml. of TEL/gallon. Also, as the make of motor fuel increases, there is a steady linear decrease in the volatility index, and a quadratically increasing sulfur content.¹¹

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^{11.} Although when distilling this particular stock the sulfur content increases quadratically, on the other material (gasoline #36) the sulfur content only changes linearly.



Fig. 12 a — Cutting temperature vs x_{3i}

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3. The mathematical model

3.1 The basic form

Nuhn and Tucker have proved the following "equivalence theorem": "Let the functions $f_1(x)$, . . . $f_m(x)$, g(x) be concave as well as differentiable for $x \ge 0$. Then x^0 is a solution of the maximum problem if, and only if, x^0 and some u^0 give a solution of the saddle value problem for $\phi(x,u) \equiv g(x) + u'Fx$."

In other words, they have established that the solution to a concave constrained-maximum problem is equivalent to a certain minimax saddlepoint. At the suggestion of Harry Markowitz, the gasoline blending model was set up in the appropriate minmax form. An iterative digital technique was then devised for solving the problem.¹⁴

From a rigorous mathematical standpoint, the present computing procedure is an unsatisfactory one, but the method does lead to solutions that should be sufficiently accurate from the standpoint of refinery managers. Convergence takes place without an inordinate amount of computing time. All calculations were performed on a machine

¹² The expression u'Fx is shorthand for the following: $u_1 f_1(x) + u_2 f_2(x) + \ldots + u_m f_m(x)$. The u_1 are non-negative Legrangean multipliers, and correspond closely to the economist's notion of "shadow prices".

¹³ H. W. Kuhn and A. W. Tucker, "Nonlinear Programming", <u>Proceedings of</u> the <u>Second Berkeley Composium on Mathematical Statistics</u> and <u>Probability</u>, ed. J. Neyman, University of California Press, 1951, p. 486.

¹⁴ Markowitz has experimented with an electric analogue solution for this class of applications. The present blending model involving 19 gasoline stocks would clearly swamp the capacity of any of the present generation of analogue machines.

that is readily available throughout the United States -- an IBM Card Programmed Calculator, Model 2.

Dr. Martin Beckman has objected to the application of the Kuhn-Tucker theorem in this particular instance. He has made the point that for a function of many variables, f(x), to be truly concave, it is necessary but <u>not</u> sufficient for that function to be concave in <u>each variable taken separately</u>. It has been possible to devise examples that satisfy all conditions of the gasoline blending problem -that is, concave in each variable taken separately -- but which nevertheless violate the strict Kuhn-Tucker conditions.¹⁵ The counter-

15. The following example is not derived from any Union Oil Company data, and serves only for illustrative purposes. Consider two blending stocks, labelled "A" and "B" with the following characteristics:

	Stock A	Stock B
Octane number, clear	74.0	90.0
Octane number, with 3.0 ml. of TEL	90. 0	94.8
Number of ml./gal. required for 90 octane leaded gasoline	3.0	0

Assuming straight line interpolation at constant lead levels for the two stocks, and employing the Hebl-Rendel-Garton blending chart, it will be found that a 50-50 mixture would require 1.6 -- and not 1.5 -ml./gal. in order to reach the 90.0 octane level. This clearly constitutes a violation of the concavity assumption, but a refiner will also recognize that the example requires an unusual configuration of octane numbers and lead susceptibilities. examples give rise to multiple isolated maxima. Athe concavity it is possible property in the various functions, Athe adopt blending schedules that are locally optimal, but which do not, in fact, represent the best of all solutions available.

The problem cannot be dismissed lightly, but I suspect that in most particular applications such local extrema can be detected by the investigator. The difficulties arising out of non-concavity are not unique to the programming method described here. The same argument would apply if the TEL levels were taken as parameters, and a separate linear programming problem then run off for each of many <u>combinations</u> of the two. This difficulty would come up in the case of any trialand-error method that consisted in changing one variable at a time, and observing the effects upon the payoff. And finally, the failure of the proper curvature conditions could frustrate optimizations by vay of a Lange-Lerner shadow price market mechanism.

3.2 Definition of independent variables

It now remains to set up the formal relationships corresponding to the Kuhn-Tucker payoff function, g(x), and the set of restraints, $f_1(x)$. Table V.1 presents schematically the 23 independent non-negative variables, x_j , which must be selected in order to determine the blending schedule. The whole set of 23 will be termed the vector (x).

There are $1 \le raw$ gasoline stocks, labelled $31, 30, 41, \ldots 80$, as indicated by the row headings. In each instance, we are given q_j , the total quantity of this material available for blending. The problem traists of assigning various amounts of the raw gasolines among the

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Stocks			Quantity of stock j assigned to				
Identifi- cation No.	Quan Ava	ntities ilable	three products, B/CD				
	٩J	B/CD	(1) 7600 premium motor fuel	(2) 76 reg motor fue	ular (3) Cutter el Stock		
31	931	3,050	0	x 31	(q ₃₁ -x ₃₁)		
36	^q 36	2,060	0	* 36	(9 ₃₆ -x ₃₆)		
41	q41	1,000	(q41-x41)	x ₄₁	0		
42	942	2,910	×45	×42	(q ₄₂ -x ₄₂ -x ₄₅)		
43	a ₄₃	3,460	×46	x 43	(q43-x43-x46)		
յելե	ATT	1,730	0	×44	$(q_{i_{1}i_{1}}-x_{i_{1}i_{1}})$		
51	٩٦٦	110	(a ₅₁ -x ₅₁)	* 51	0		
5 2	a52	1,240	* 55	×52	(a ₅₂ -x ₅₂ -x ₅₅)		
53	⁹ 53	860	× 56	× 53	(9 ₅₃ -x ₅₃ -x ₅₆)		
54	954	490	0	x 54	(a ₅₄ -x ₅₄)		
60	°-60	6,150	(₉₆₀ -x ₆₀)	x 60	0		
61	°61	3,430	(व् <mark>त-×</mark> त)	×61	0		
62	9 6 2	1,460	(₉₆₂ -x ₆₂)	×62	0		
63	°63	3,200	(4 ₆₃ -x ₆₃)	×63	0		
64	°64	1,680	(q ₆₄ -x ₆₄)	×64	0		
65	°65	11,230	(4 ₆₅ -× ₆₅)	* 65	0		
66	°66	7,440	(9 ₆₆ -x ₆₆)	x 66	0		
70	9 70	1,700	(q ₇₀ -x ₇₀) = q ₇₀	x ₇₀ ■ 0	0		
80	9 9 0	8,710	(q ₈₀ -x ₈₀) ≡ 0	× 80 ^{≢ 9} 60	0		
Totals	_	61,910	۲	z 2			
	TEL conc (ml./ga	entration, 1.)	×21	×22			

Table V.1 Gasoline Blending Problem -- Definition of Various Quantities

three alternative uses -- (1) 7600, premium grade motor fuel, (2) 76, regular grade motor fuel, and (3) cutter stock. In addition, it is necessary to select x_{21} and x_{22} , respectively, the ethyl fluid concentrations in premium and regular grade gasoline.

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The computations were <u>not</u> set up with the full generality of considering 3(19) = 57 individual possible gasoline components. Instead, for mumoses of economizing on machine capacity, it was to agreed make alwance guesses at which of the elements would turn out to have zero values. (Wherever a component was preassigned in this way, there is a zero entry in the corresponding box of Table V.1.) It was also agreed that it would not be economical to throw away any of these particular gasoline stocks. In this manner, it was possible to reduce the number of independent gasoline 17

The Lagrengean multipliers, u_i, taken directly from the basic calculations, enabled us to check up on these particular assumptions at a later stage in the analysis. Fortunately, the initial guesses turned out to be correct ones. Despite the good luck for the particular sets of parameters studied, these assumptions were an unsatisfactory short-cut, and can be justified only on grounds of computational convenience.

¹⁵ Even if a material could not be employed directly as cutter or as regular grade gasoline, under the 1953 price structure, it would always pay to degrade some finished 7000 gasoline for purposes of blending the particular material up to the specifications for regular grade gasoline.

¹⁷ For sympetry in the subsequent work, there are two additional constants that are written as if they were independent variables. The term x_{10} is identically zero, and x_{20} identically equals q_{20} .

Table V.1 always refers to the quantity of stock j in regular grade gasoline as xj. Even in the case of stocks #31 and #36, where the cutting temperature can equally well be regarded as the independent variable, this same convention was employed. (On Figure 12.a. note the one-to-one relationship between the cut point and the quantity of motor fuel available.)

To simplify further reference, two new <u>dependent</u> variables are also defined:

(V.2) $z_1 = \frac{1}{y}(q_j - x_j) + \frac{1}{z} x_k$ = total production rate, B/CD, for premium grade gasoline (j = 41, 51, 60-66, 70) (k = 45, 40, 55, 56) (V.3) $z_2 = \frac{1}{y} x_j$ = total production rate, B/CD, for regular grade gasoline (j = 31, 36, 41-44, 51-54, 50-60, 30)

3.3 Definition of the payoff function and restraints

The payoff, g(x), is determined by crediting the gross refinery realization from the self of premium and regular grade gasoline, and then charging up the ethyl fluid costs and the <u>loss</u> of potential cutter credits associated with the particular blending schedule:

 $(\mathbf{v}.4) \qquad \mathbf{g}(\mathbf{x}) = (\mathbf{p}_1 - \mathbf{p}_{20} \mathbf{x}_{21})\mathbf{z}_1 + (\mathbf{p}_2 - \mathbf{p}_{20} \mathbf{x}_{22})\mathbf{z}_2$ $- \mathbf{p}_{42} \mathbf{x}_{43} - \mathbf{p}_{53} \mathbf{x}_{13} - \mathbf{p}_{53} \mathbf{x}_{13}$ $- \sum_{\mathbf{j}} \mathbf{j} \mathbf{j}^{\mathbf{x}} \mathbf{j}$ $(\mathbf{j} = 31, 2^{j_1}, 42, 4^{j_2}, 4^{j_3}, 4^{j_4}, -2^{j_5}, 5^{j_4}, 4^{j_4})$

In equation (V.4), the various unit costs and prices are defined as follows:

(V.5) $p_1 = refinery realization on 7000, premium$ gasoline, \$/B $<math>p_2 = refinery realization on 76, regular grade$ gasoline, \$/B $<math>p_{20} = cost of ethyl fluid, $/42 ml.$ $p_j = unit cutter credit on stock j, $/B$ (j = 31, 36, 42, 43, 44, 52, 53, 54)

The blending schedule (x) must not call for the production of negative amounts of any motor fuel or cutter component, and so there are certain upper and lower limits imposed upon the various x_j . In addition, the schedule must not require TEL concentration levels exceeding. A 3.00 ml./gallon. And because of the flash point specification on finished No. 6 oil, it must not require cutter components #31 or #36 to fall below q_{31}^0 and $q_{3\zeta}^0$, respectively.

$$(\mathbf{v}.6) \qquad \begin{array}{c} 0 \stackrel{\cdot}{=} & \mathbf{x}_{21} \stackrel{-}{=} & 3.0 \\ 0 \stackrel{\cdot}{=} & \mathbf{x}_{22} \stackrel{-}{=} & 3.0 \\ q_{31} \stackrel{\cdot}{=} & \mathbf{x}_{31} \stackrel{-}{=} & q_{31} \\ q_{36} \stackrel{\cdot}{=} & \mathbf{x}_{36} \stackrel{-}{=} & q_{36} \\ 0 \stackrel{\cdot}{=} & \mathbf{x}_{1} \stackrel{-}{=} & q_{36} \\ 0 \stackrel{\cdot}{=} & \mathbf{x}_{1} \stackrel{-}{=} & q_{36} \\ 0 \stackrel{\cdot}{=} & \mathbf{x}_{1} \stackrel{-}{=} & q_{36} \\ 0 \stackrel{\cdot}{=} & \mathbf{x}_{3} \stackrel{-}{=} & q_{36} \\ (\mathbf{i} = \mathbf{41}, \mathbf{44}, \mathbf{51}, \mathbf{54}, \mathbf{60-66}) \\ (\mathbf{j} = \mathbf{42}, \mathbf{43}, \mathbf{44}, \mathbf{46}, \mathbf{52}, \mathbf{53}, \mathbf{55}, \mathbf{56}) \end{array}$$

quantities of individual

For stocks #:2, 43, 52, and 53, the cutter stocks, $f_1(x)$, must not be negative:

$$(\mathbf{v}.7) \quad \mathbf{f_1}(\mathbf{x}) = \mathbf{q_{42}} - \mathbf{x_{42}} - \mathbf{x_{45}} \ge 0$$
$$\mathbf{f_2}(\mathbf{x}) = \mathbf{q_{43}} - \mathbf{x_{43}} - \mathbf{x_{46}} \ge 0$$
$$\mathbf{f_3}(\mathbf{x}) = \mathbf{q_{52}} - \mathbf{x_{52}} - \mathbf{x_{55}} \ge 0$$
$$\mathbf{f_4}(\mathbf{x}) = \mathbf{q_{53}} - \mathbf{x_{53}} - \mathbf{x_{56}} \ge 0$$

The six remaining equations are connected with the three specifications on each of the two motor fuel products. The functions $f_5(x)$ and $f_6(x)$ relate, respectively, to the octane number of premium and regular grade motor fuel; $f_7(x)$ and $f_8(x)$ to the sulfur content; and $f_9(x)$ and $f_{10}(x)$ to the volatility index. These relationships are all defined in such a way that the particular specification will be satisfied if, and only if, the function $f_1(x) \ge 0$. To simplify notation, the constants in the equations do not refer to the absolute octane number, sulfur content, or volatility index of a particular raw gasoline, but rather to the <u>differences</u> between these values and those of the specifications for a particular motor fuel product.

In the case of the octane ratings, for example, equation (V.1) indicates the relationship between the octane number of a particular blending stock, and the TEL concentration in a finished product. A new function, $t_{jk}(x_{2k})$ is defined in order to indicate the number of octane points by which stock j exceeds the specifications for product \underline{k} , when x_{2k} ml. of lead are added to stock j. The variable t_{jk} is evidently a function of x_{2k} , and is determined through the following relation:

(V.8)
$$t_{jk}(x_{2k}) = a_j + b_j x_{2j} - \frac{c_j}{1 + x_{2k}} - N_k$$
 (k = 1,2)

The terms a_j , b_j , and c_j represent the lead susceptibility constants for stock <u>j</u>, and correspond to the three constants in equation (V.1). The parameter N_k stands for the minimum octane number requirement connected with gasoline blend <u>k</u>.

In order to ensure meeting octane number specifications on the premium grade gasoline, the following condition must then apply:

(V.9) $f_5(x) = \sum_{j \neq j} (q_j - x_j) + \sum_{i \neq j} (1 + i) = 0$ i = 41, 51, 60-66, 70i = 45, 46, 55, 56

Similarly, to make sure that the regular grade gasoline blend will pass its octane number specification, we must have:

(V.10)	$f_{6}(\mathbf{x}) = \sum_{j=1}^{t} j_{2}x_{j} + x_{31} \begin{bmatrix} g_{31} - g_{32}x_{31} \end{bmatrix}$) $j = 31, 36, 41-44$
	$+x_{36}[e_{36} - e_{37}x_{36}] \ge 0$) 51-54,
) 80

The functions in brackets that follow the variables x_{31} and x_{36} are inserted in order to reflect the fact that with these two gasolines, as the cutting temperature changes, both the quantity and the octane number of the motor fuel component will be affected. (See Figure 12.b.) This feature evidently leads to negative square terms involving x_{31} and x_{36} .

The same two stocks give rise to quadratic expressions in connection with the sulfur and volatility specifications for regular grade gasoline, but since these components were assumed to be absent from the premium gasoline blend, those equations remain completely linear. For the sulfur condition reads: specification on the premium grade gasoline, the

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(V.11)
$$f_7(x) = \sum_j d_{j1}(a_j - x_j) + \sum_k d_{k1}x_k \ge 0$$

 $j = 41, 51, 60-66, 70$
 $k = 45, 46, 55, 56$

The constant term d_{j1} represents the algebraic excess of the maximum allowable sulfur content of 7600 gasoline over the amount actually in associated stock j. There is a similar set of constants⁴ with the regular grade gasoline -- d_{j2}, the difference between the allowable sulfur content in the 76 grade product and the content in stock j. The following relationship controls the sulfur level in the finished regular grade product:

$$(v.12) \quad f_{8}(x) = \sum_{j} a_{j2}x_{j} - x_{31} \quad \sqrt{a_{32}} - a_{33}x_{31} + a_{34}x_{31}^{2} - 7$$

$$- x_{36} \quad \sqrt{a_{37}}x_{36} - 7 \geq 0$$

$$j = 31, 36, 41 - 44, 51 - 54, 60 - 66, 80$$

The bracketed terms following x_{31} and x_{36} arise out of the cutting temperature relationship. (See Figure 12. c.) Note that over the entire range between q_{31}° and q_{31} , the terms involving x_{31} bring about a strictly concave function. That is, the following partial second derivative is negative over this particular interval:

$$(\mathbf{v}.\mathbf{13}) \quad \frac{\partial^2 \mathbf{r}_8(\mathbf{x})}{\partial \mathbf{x}_{31}^2} = \mathbf{2d}_{33} - \mathbf{6d}_{34} \mathbf{x}_{31} < 0$$

The functions $f_{9}(x)$ and $f_{10}(x)$ do the job, respectively, of governing the volatility of the premium and regular grade gasoline:

$$(\mathbf{v}.\mathbf{14}) \quad \mathbf{r}_{9}(\mathbf{x}) = \sum_{\mathbf{j}} \mathbf{e}_{\mathbf{j}\mathbf{1}} (\mathbf{q}_{\mathbf{j}} - \mathbf{x}_{\mathbf{j}}) + \sum_{\mathbf{k}} \mathbf{e}_{\mathbf{k}\mathbf{1}} \mathbf{x}_{\mathbf{k}} \ge 0$$

$$\mathbf{j} = 4\mathbf{1}, 5\mathbf{1}, 6\mathbf{0} - \mathbf{6}\mathbf{6}, 7\mathbf{0}$$

$$\mathbf{k} = 4\mathbf{5}, 4\mathbf{6}, 5\mathbf{5}, 5\mathbf{6}$$

$$(\mathbf{v}.\mathbf{15}) \quad \mathbf{r}_{\mathbf{10}}(\mathbf{x}) = \sum_{\mathbf{j}} \mathbf{e}_{\mathbf{j}\mathbf{2}} \mathbf{x}_{\mathbf{j}} - \mathbf{e}_{\mathbf{32}} \mathbf{x}_{\mathbf{31}}^{2} - \mathbf{e}_{\mathbf{37}} \mathbf{x}_{\mathbf{36}}^{2} \ge 0$$

$$\mathbf{j} = \mathbf{31}, \mathbf{36}, 4\mathbf{1} - 4\mathbf{4}, 5\mathbf{1} - 5\mathbf{4}, 6\mathbf{0} - \mathbf{66}, \mathbf{80}$$

1-27

Here again, because of the cutting temperature-quality relationship, there are square terms connected with x_{31} and x_{36} .

The gasoline blending model has now been reduced to a constrainedmaximum mathematical form. The problem consists of choosing a vector (\mathbf{x}) that will maximize $g(\mathbf{x})$, subject to the side conditions that $\underline{f}_{-1}(\mathbf{x}) \geq 0$, and also subject to certain upper and lower limits on the individual components, \mathbf{x}_{j} . The payoff function $g(\mathbf{x})$ is given by equation $(\mathbf{V}, 4)$. The upper and lower limits on individual variables are stated in $(\mathbf{V}, 6)$, and the lo restraining inequalities, $f_{\frac{1}{2}}(\mathbf{x})$, in $(\mathbf{V}, 1)$, $(\mathbf{V}, 10)$, $(\mathbf{V}, 11)$, $(\mathbf{V}, 12)$, $(\mathbf{V}, 14)$, and $(\mathbf{V}, 15)$.

In this form, the problem <u>almost</u> fits the Kuhn-Tucker conditions. Both the payoff function and the restraints are concave in each variable taken separately. Furthermore, eight of the ten restraints are concave over the whole space -- $f_1(x) \ldots f_{l_1}(x)$, and $f_{r_1}(x) \ldots f_{r_{10}}(x)$.

¹⁸Kuhn and Tucker give the following definition of concavity: "A function is concave if linear interpolation between its values at any two points of definition yields a value not greater than its actual value at the point of interpolation". Op. cit., p. 481. The function is said to be concave in each variable separately if the two points of definition are always chosen so that only one of the components of the vector (x) is altered. A function may be concave in each variable separately without being concave over the whole space of points (x).

The payoff function g(x) and the two octane number restraints -- $f_5(x)$ and $f_6(x)$ -- do not strictly satisfy the full concavity properties. It was decided, nevertheless, to perform the computations as if these also satisfied the rigorous Kuhn-Tucker conditions. Under this approach, will there is always the danger that a local optimum not turn out to be the <u>maximum maximorum</u>. A number of alternative computing schemes were examined, but all of these suffered from precisely the same defect. The primary justification for going ahead in this non-rigorous fashion was that the computational results could always be checked against the experience of the refinery engineers. It was felt that these individuals certainly would be able to detect any errors of a gross nature.

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4. The numerical analysis

4.1 Outline of computational procedure

Following the assumption that this gasoline blending model fits into the format of the Kuhn-Tucker theorem, it is necessary to define the Lagrangean function $\mathscr{G}(x,u)$:

(V.16)
$$\phi(x,u) = g(x) + \sum_{i=1}^{10} u_i f_i(x)$$

The constrained-maximum problem will be solved if, and only if, there is a saddlepoint solution to the function, $\beta(x,u)$. The computational procedure is an iterative one -st each step \underline{t} , converting a vector x(t), u(t) into a new vector x(t + 1) and u(t + 1). The solutions observed have all tended toward a saddlepoint, but I can give no strict proof of the necessity of this convergence. The new vector generated is never exactly "efficient", nor is it "attainable" 19in the sense of Koopmans. The payoff does not increase monotonically as in each successive step of Dantsig's simplex procedure for linear 20programming. Despite these apparent shortcomings, the algorithm has given useful answers in the cases examined to date.

The solution must be started off from some initial point x(0), u(0). In principle, this may be any arbitrary non-negative vector. In practice, though, it is possible to effect a considerable reduction in computing time if a good initial position is selected. At each step, first the (u) vector is determined, and then the (x) vector. For the former, the

19. T.C. Koopmans, "Analysis of Production as an Efficient Combination of Activities", Chapter III, Activity Analysis of Production and Allocation, p. 79.

20. G.B. Dantsig, "Maximisation of a Linear Function of Variables Subject to Linear Inequalities", <u>ibid.</u>, Chapter XXI, pp. 339-347. basic iteration consists of two steps, and for the latter, of four steps. The problem is one of determining $\Delta u_1 = u_1(t+1) - u_1(t)$, and $\Delta x_j = x_j(t+1) - x_j(t)$. The procedure appears cumbersome, but only six minutes of CPC machine time are actually required for generating a whole set of Δu_1 and Δx_j . (The required total number of steps has varied between 50 and 80.) For the Δu_i , the procedure is as follows:

1. If $\frac{\partial \phi(t)}{\partial u_1(t)} \ge 0$, then the "candidate" $\Delta u_1 = -k_1$ If $\frac{\partial \phi(t)}{\partial u_1(t)} < 0$, then the "candidate" $\Delta u_1 = -k_1$

Note: k, is an arbitrary positive constant.

2. If $u_{1}(t) + \Delta u_{1} < 0$, then $\Delta u_{1} = -u_{1}(t)$ If $u_{1}(t) + \Delta u_{1} \ge 0$, then $\Delta u_{1} = \Delta u_{1}(t)$ 1

For the Δx_{j} , the procedure becomes:²¹ 1. If, for any \underline{i} , $f_{1}(t) < 0$, and if $\left[\frac{\partial \phi(t)}{\partial x_{j}(t)} - \frac{\partial f_{1}(t)}{\partial x_{j}(t)}\right] < 0$, then

 $\Delta x_j = 0$; proceed to evaluate Δx_{j+1} . Otherwise, proceed

to step 2.

2. If $\frac{\partial \phi(t)}{\partial x_{j}(t)} \ge 0$, then the "candidate" $\Delta x_{j} = k_{j}$ If $\frac{\partial \phi(t)}{\partial x_{j}(t)} < 0$, then the "candidate" $\Delta x_{j} = -k_{j}$

Note: k, is an arbitrary positive constant.

²¹ The individual $\triangle x_j$ are determined in ascending order of their respective indices, <u>j</u>. To simplify the notation, $\triangle x_{j+1}$ is used to indicate the next $\triangle x_j$ that is to be determined. For example, $\triangle x_{36}$ follows $\triangle x_{31}$ in sequence, and for j = 31, is indicated by $\triangle x_{j+1}$.

3. a. If j = 31 or 36, and if $x_j(t) + \Delta x_j < q_j^0$, then $\Delta x_1 = x_1(t) - q_j^0$; proceed to evaluate Δx_{1+1} . If 1 = 31 or 36, and if $x_1(t) + \Delta x_1 \ge q_1^{\circ}$, then proceed to step 4. b. If j = 31 or 36, and if $x_j(t) + \Delta x_j = 0$, then $\Delta \mathbf{x}_{1} = -\mathbf{x}_{1}(\mathbf{t})$; proceed to evaluate $\Delta \mathbf{x}_{1+1}$. If j = 31 or 36, and if $x_j(t) + \Delta x_j \ge 0$, then proceed to step 4. $\Delta x_{j} = \Delta x_{i}$ 4. **a.** If 1 = 42, 43, 45, 46, 52, 53, 55 or 56, Aproceed to evaluate $\triangle x_{1+1}$. b. If j = 21 or 22, calculate $\sqrt{3} = 00 - x_1(t) - \Delta x_1 / .$ If this expression is non-negative, $\Delta x_1 = \Delta x_1$. Otherwise $\Delta x_1 = 3.00 - x_1(t)$. Proceed next to evaluate Δx_{i+1} . $j \neq 1$ 60, 6., c. 1r j = 41, 44, 51, 54, or A . . . 66, calculate $\int q_1 - x_1(t) - \Delta x_1 / J$. If this expression is nonnegative, $\Delta x_j = \Delta x_j$. Otherwise $\Delta x_j = (j - x_j/t)$.

For the Δu_j , the explanation of this ritual is straightforward. Step (1) tells us to decrease u_j by an arbitrary amount k_j if, for a "small" change in u_j alone, the effect will be to decrease $\mathcal{J}(x,u)$. Similarly, there is to be an increase in u_j if, for a small increase, \mathcal{J} would decrease. Step (2) prevents u_j from becoming negative.

Proceed next to evaluate Δx_{1+1} .

For the $\exists x_j$, the justification is more_A (Steps (3) and (4) are the obvious ones -- respectively, lower and upper limits on the individual x_j dictated by $(V, \hat{0})$.) As with the $\triangle u_j$ the sign of $\frac{\langle \varphi(t) \rangle}{\langle x_j(t) \rangle}$ is primarily the criterion that determines whether to take $\frac{\partial^2 2}{\partial x_j(t)}$ are a negative step. Preliminary small-scale calculations indicated, however, that the unqualified "direction of ascent (descent) rule" would lead to major oscillations rather than to convergence at equilibrium values. For that reason, rule (1), a relaxation-type principle, was inserted. Translated into English, the statement reads, "If one of the restraints is being violated, and if the direction-ofascent rule would lead to an even greater violation of this restraint, then the particular x_j should not be charged." The relaxation rule is hardly elegant from the views int of pure mathematics, and there are undoubtedly other possibilities for achieving the same effect. T = 3

At the present time, despite their plausibility, there is a serious dilemma concerning the logic of these computational techniques. On the one hand, I have been unable to prove the necessity of convergence for either the relaxation or the unmodified procedure. On the other, I have counterexample been unable to construct a f for a case where the k_i and the k_j may be made arbitrarily small. Readily granting these apparent objections, the fact remains that the relaxation technique has given useful results in a series of computing runs

... Details of the numerical analy to

Three sets of calculations have been performed. The first is to be considered the basic case -- taking as parameters the 1953

The reader should note that \emptyset is being minimized with respect to (u), and maximized with respect to (x).

refinery realizations, product specifications, and raw material availabilities. The second run was concerned with a radical change in the price structure -- an 11.6% drop in the value of premium grade gasoline. In the third, the price structure was taken to be the same as in the basic case, but a 2.5 octane point increase was imposed upon the conditions, premium product. For each of these departures from the basic *A* the

In order to convey some indication of the computational technique employed here, Figures 13.a - 13.e. chart the values of certain of the variables during the last twenty steps of Run #1, i.e., between t = 40 and t = 60. According to Figure 13.a, the payoff and the aggregate make of premium and regular grade gasoline had become stabilized, and "rippled" steadily about certain equilibrium levels. Virtually all of the independent variables exhibit similar rippling around an equilibrium point, but there were exceptions. The worst drifting occurred in the case of x_{42} and x_{45} , respectively, the quantities of stock #42 in regular and premium grade gasoline. Their time paths are shown in Figure 13.b. The more typical pattern was exhibited by the two TEL concentration variables, x_{21} and x_{22} , and is reproduced in Figure 13.c. Two of the Lagrangean multipliers, u_5 and u_5 , connected with the two octane number specifications, are shown in Figure 13.d. The octane number functions themselves, $f_5(x)$ and $f_2(x)$, are given in Figure 13.e.

Before tutting off any individual computation run, it was possible to perform an additional shock upon the stability of the solution. According to the Kuhn-Tucker theorem, in order for the vector $(x)^{\circ}$ to be a saddle value solution, the following condition must hold for all x_{j}° :²³

23. Op. cit., p. 482.

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Fig. 13 e — Run no. 1, relistive octane numbers, $f_{s}(x) \div z_{1}$ and $f_{6}(x) \div z_{2}$

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$$(\mathbf{v}.\mathbf{17}) \quad \frac{\partial \mathbf{g}^{\circ}}{\partial \mathbf{x}_{j}^{\circ}} \leq \mathbf{0}, \quad \mathbf{x}_{j}^{\circ} \quad \frac{\partial \mathbf{g}^{\circ}}{\partial \mathbf{x}_{j}^{\circ}} = \mathbf{0}, \quad \mathbf{x}_{j}^{\circ} \geq \mathbf{0}$$

In the present case, where the x_j are bounded from above as well as below, this condition on the equilibrium x_j^0 becomes:

(V.19) a. If
$$\frac{\partial g^{0}}{\partial x_{j}^{0}} \leq 0$$
, then $x_{j}^{0} \frac{\partial g^{0}}{\partial x_{j}^{0}} = 0$, and $x_{j}^{0} \geq 0$
b. If $\frac{\partial g^{0}}{\partial x_{j}^{0}} > 0$, then $x_{j}^{0} = q$.

Find condition (V.18), it is possible to perform a rough check on the procision of the solution. For those x that are at their lower limits, $\frac{\partial p^0}{\partial x_1^0} = 0$. For those that are at their upper limits, $\frac{\partial x_1^0}{\partial x_1^0}$

 $\frac{\partial p^{\circ}}{\partial x_{j}^{\circ}} \ge 0.$ And for those that are neither at their upper nor at their yx_{j}° lower limits, $\frac{\partial p^{\circ}}{\partial x_{j}^{\circ}} = 0.$ In this latter case, it is not to be expected yx_{j}°

that the partial derivative will be literally at a zero level. Rather, since this is an approximative method of solution the derivative should be in the <u>meighborhood</u> of zero.

Table V.2 contains the actual values for the $x_j(60)$ and for the normalized values of the partial derivatives $\frac{30(60)}{3x_j}$. All of those $\frac{3x_j(60)}{3x_j}$ variables that are at their lower limits exhibit as is to be expected. negative partial derivatives, A On the other hand, the single variable

In the case of (V.18a), where j = 31 or 36, the lower limits q⁰ must be interpreted as equivalent to zero. For (V.18b), where j = 21 or 22, the upper limits of 3.00 ml. are to be regarded as the appropriate q. For those x_j on which there is no specific imposed upper limit -- that is, j = 42, 43, 45, 46, 52, 53, 55, and 56 -- the condition (V.18b) does not apply.

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Table V.2 (w	parison of	Partial	Derivatives	for	t	= (50,	Run	/ 1
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1	Lower Limit on Xj	×j ⁽⁶⁰⁾	Upper Limit on xj	$\frac{2 \phi(60)}{2 x_{j}(60)}$	Adjusted Optimal x
				milliter/pa	allon in premium gasoline
21	0	1.94	3.00	1.57% of cost of 1A	1.92
22	0	2.)8	3.00	-0.61% of cost of ly	2.96
.	1 000			mil.iliter/ga	allon in regular gasoline
٦٤	1,900	1,900	3,050	-22.54% of realization on 1 barre	el or 1,900
٦	1 200	1 220	0 060	o al T	1 32
30	1,320	1,320	2,000	-0.34	1,320
41	0	0	1,000	-5.58% of realization on 1 barre	el of O
			,	premium gasoline	
42	0	540	2,910	-0.88	620
43	0	400	3,460	-0.40	380
44	0	С	1,730	-8.62 •	0
45	0	2,3,0	2,910	2. 35 •	2,290
46	0	850	3,460	1.09 •	845
51	0	0	110	-4.41% of realization on 1 barre	el of O
				premium gasoline	-
52	0	0	1,240	-3.41	0
53	0	280	860	-0.05	200
54	0	4 10	490	-0.08	480
55	0	100	1,240	0.20	120
56	0	000	960	1.27	600
60	0	0	6,150	-6.90% of realization on 1 barre	l of O
61	0	0	3.430	-5.98	0
62	Õ	C	1.460	-9.82 "	0
63	0	0	3.200	-8.55 "	õ
64	Ō	Ō	1.680	-9.10	Ō
65	0	õ	11.230	-7.94 .	õ
6	0	Ō	7.440	-7.35	0

that lies at its upper limit, $\mathbf{x}_{c_{i_{k}}}$, does not have a partial derivative of the proper sign. This derivative is slightly negative, and in fact. during the sequence leading up to t = 60, $\mathbf{x}_{c_{i_{k}}}(t)$ had oscillated between 470 B/CD and 4.00. There appears to be a coincidence between the upper limit on this particular variable, and the point at which $\frac{1}{2} \mathbf{x}_{c_{i_{k}}}$ vanishes.

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In the case of those variables that lie meither at their upper nor their lower limits, the maximum absolute deviation from zero amounts to 2.35% of the value of one barrel of premium grade gascline. This deviation occurred in the case of x_{45} , the variable shown in Figure V.45 that had been slowly drifting upward. It did not seem worthwhile to continue the iterations in order to refine upon this one estimate.

The blending schedule selected at step 60 is neither literally "attainable". The "level recorded for step 45, A this reason, the final blending schedule was taken over from that earlier step. Certain minor corrections were made by hand in order to bring about a feasible and efficient solution. The resulting vector is listed in the final column under the heading "Adjusted optimal x_1 ".

5. Results of the numerical analysis

5.1 Interpretation of results -- blending programs

The operations schedules derived from the three computation runs presented area in Table V.3. For reference purposes, the first three columns indicate the program that was effective at the beginning of the year 1953. The other sets give the results for run #1, the basic case; run #2, a decrease of 11.5% in the realization on premium grade gasoline; and run #3, an increase of 2.5 octane numbers in the specification for the premium product.

The outstanding features of these different schedules are summarized in bar charts, Figures 14.a. - 14.d. Of all four situations, the output of 7600 gasoline is largest in the case of run #1. and smallest under "1953 operations". Just the reverse is true for the regular grade **Asoline.** Its production is largest under the 1953 operations than, and smallest in the basic case. Going along with this considerable shift in the general production strategy, there is an apparently small shift in payoff, for the basic case would yield a return to the company of only 3.99 percentage points in excess of that shown in actual practice. At first glance this figure of 3.49 does not seem impressive, but nevertheless, during the course of a single year's operation a differential of this magnitude amounts to several millions of dollars! From the company's standpoint, it would look eminently worthwhile to shift over to the blending schedule derived from run #1.

^{25.} In order to avoid revealing proprietary information, no absolute dollar figures are presented here. Both the unit and the total refinery realizations are stated in terms of index numbers.

Btocks							
Identification Mumber		1 ··· Operation:					
		2500	ίų.	Cutter Stock	-++		
3 1 . 32.	3,0%0 2,0~1		2,260 1,340	7.0 170			
42. 43. 44.	1,000 2,910 3,440 1,730	\$00 1 \$95 0	00 1,45 3,460 1,730	0 C O			
52. 53. 54.	110 1,2% 800 4.0	55 0 20 0	-√20 -√0 4_+)	0			
60. F1. 62.	6,1%C 3,430 1,4 6	3,0 0 0 0	3,1 0 3,430 1,40				
े 3 . तम . रू.	3,200 1,685 11,230 7,440	0 8,7 2 0 4,80	3, 2 % 1, 430 2, 10 680				
.0. 8c.	1,700 8,710	1,500	8, 1 0				
Totals	101, 10	C2,910	040,81	9642			
TEL Concentration (ml./gal.)		1.13	1.0				
Total Consumption of TEL (liters/ day)	,	i i	2,705				
Payoff, g(x), percent of total for Run #1, Basic Case							
Refinery realization on motor gas percent of 105: realization 7600 product	oline, on	10CO	5 2. 🙀				
Octane rating of motor gasoline p number of points in excess of housebrand specifications	9 7 9ເຊ ງູບເຈ	C.	h				

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Results of Computeting

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Quantity of	. *B 10	ed. 1881	t Ihree	Pr	Hunts,	B/CD
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	Basic Case Run		•)	Frier Dec Rui	።የዜያር ጋ.	•)• ()rtane II Run	. Increate
7300	16	Catter Stock	12		Cutter State	- 11. 	•,	Cutter Stuck
	1,000 1,320	1,1		1 hon 2, v	1,1		1, 2 1, 2	1,1,2
1,000 2,200 84	ତ ି 20 380 ତ	2,2	1, J 1, 1, 1	1, .2 · 1. · ·	,	1,2% 2, * 1.	-220 1, 2	2,73
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Fig:14b — Total output of 7600 gasoline, four cases (thousands of B/CD)









Unhappily, the life of the operations analyst does not run a simple course. From the very beginning of the study, it had been acknowledged that the company could not realistically expect to sell unl'mited quantities of premium gasoline at the prevailing refinery realization. At the same time, it had not been fully appreciated how large an increase in the production of this item might be in order. (The cutput in the one case is almost double that of the other.) For this reason, it seems desirable to incorporate an additional formal restriction into the problem, and to require that the total production of premium product remain below some upper limit, WO'L DA L. In this way, it possible to determine how large an increase in 5 profits attainable even under the limitation of current sales of the one product. By varying this upper limit parameter, the carst could

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trace out the effects of alternative sales volumes.

Rather than revise the computing setup in order to impose a direct marketing restriction of this nature, it seemed a more economical alternative to calculate a second point along the company's "supply curve" for premium gasoline. That is, for run #2 all parameters were maintained at the same level as in run #1, except for the unit realization upon '+00 gasoline. This was dropped to 88.42% of its previous level. The change had the effect of reducing the differential between promission and regular grade gasoline to less than half of its previous amount. This alteration in the price relations had a result analogous to that of a direct quantity limitation upon the make of premium gasoline. The optimal output of this product was reduced from \$1,25° B'CD

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down to 26,385, while the total of regular grade 76 gasoline went up from 13,670 B/CD to 33,590. In this second case, not only premium gasoline but also cutter stock production declines. It now becomes profitable to incorporate most of the heavy gasolines into motor fuel rather than to blend them into residual fuel oil.

The results of run #2 are equivalent to those that would be derived by employing an outright sales limitation of 26.385 B/CD. The unit mice imputed to 7600 -- 00.420 -- indicates the marginal opportunity cost of this product. In other words, if the marketing restriction were raised by one barrel from 26,385 B/CD, and if materials were available for making this extra barrel, the company would be able to obtain an additional payoff of 88.425 of the current average realization on 7600 gasoline. In the jargon of economists this value is termed the "shadow price" of the premium product. Evidently, there is a marked gap here between the shadow price and the average realization upon the product -- a matter of considerable interest to A. P. Lerner and his followers. 26 This group has stressed the fact that such discrepancies are symptomatic of market restraints. At the same time, in themselves these discrepancies constitute no proof of either the absence or the presence of workable competition.²⁷ From the viewpoint of internal management, they do provide the company with an indication

See especially A. P. Lerner, "The Concept of Monopoly and the Measurement of Monopoly Power", <u>Review of Economic Studies</u>, June 1934, I, pp. 157-175.

For a discussion of this concept, see J. M. Clark, "Toward a Concept of Workable Competition", <u>American Economic Review</u>, June, 1940, Vol. XXX, No. 2, pp. 241-256. of the gross gains that might be achieved by reorienting sales policy.

One of the favorite avenues for sales rivalry in the gazoline industry consists of alteration in the specifications for the housebrand product. These modifications occasionally take the form of changes in the volatility index, but more typically center about the octane ratings. For this reason, it was of particular interest to explore the effects of an octane number increase upon the optimal the blending schedule and upon_Apayoff. In run #3, it was assumed that the specification of the premium gasoline had been increased by 2.5 octane points -- a plausible jump in this requirement. Referring back to the bar charts, Figures 14.a. - 14.d., the reader can compare the results with the basic case: a 2.54% decrease in total payoff, a moderate decrease in the output of premium 7600 fuel, and an increase in the output of 76 gasoline. The make of cutter stock remains the same magnitude in both cases.

The outstanding difference between this situation and the others consists of the high ethyl fluid consumption. It amounts to 6,755 liters/day here -- about a one-third increase as against 5,026 liters/ day in the basic case. The TEL concentration level is directly up at the prescribed limit of 3.00 ml./gallon for the premium grade product, but despite this fact, it is only at 2.60 ml. for regular gasoline. Like the results of the reformer analysis in chapter IV, these computations do not yield much comfort to those who would put faith in an often-quoted rule of thumb -- that the optimum blending policy is to assign the maximum allowable TEL concentration to each gallon of product.

P-487 V-50 Only in making premium grade gasoline in run #3, and in making the regular product in run #1 was the 3.00 ml. limitation approached. In all other cases, it paid to employ less than this concentration.

Not only is the ethyl fluid consumption closely connected with the octane specification of the two motor fuels. It is also sensitive to a variation in the price structure for premium grade gasoline. Both in the 1953 operations and in run #2, the 7600 product commands a low shadow price, and by comparison with the basic case, there is accordingly a low consumption of ethyl fluid. Runs #1 and #3

indicate that even during the ordinary peacetime course of market conditions, there is a high degree of flexibility possible in the use of this item. During wartime emergencies, it follows that there are substantial opportunities for cutting down on the use of the material in motor fuel in order to release TEL for aviation blends.

An interesting sidelight to the motorist: Under the schedule set up in the basic case, there is actually a lower TEL concentration in the 7600 "ethyl" gasoline than in the regular grade product. This does <u>not</u> mean that the consumer fails to get his money's worth on the premium product. The ethyl fluid concentration by itself is meaningless as an indicator of gasoline quality. In order to determine the knock rating, it is also necessary to specify the composition of the product -- the relative amount of components derived from straight-run gasoline, from catalytic cracking, from thermal reforming, and so on.

5.2. Interpretation of results -- Lagrangean multipliers

Because of the need for withholding cost and revenue figures, it is not possible to indicate the precise numerical values for the ten Lagrangean multipliers, u_i . Nevertheless, the general nature of several results may be mentioned. Six of the Lagrangean multipliers turned out to be positive in all three runs: u_i , the material balance for stock #2; u_{ij} , the material balance for stock #53; u_5 , the octane number of 7600 gasoline; u_6 , the octane number of 76 gasoline; u_9 , the volatility index of 7600; and u_{10} , the volatility index of 76. Since the Lagrangean multipliers were always positive in these six cases, the restraint conditions were always satisfied exactly. It never paid to give away octane numbers or volatility index points, and it never paid to employ stock #42 or #53 as cutter.²⁰

A rather different general result held true for u_7 and u_8 -- the "shadow prices" associated with the sulfur content of the two finished gasolines. These two u₁ consistently turned out to be zero₃ with the blending stocks currently available, the company as a whole maintained a confortable position with respect to meeting its sulfur specifications. This result had not been foreseen in advance, and it turned out to be fortunate that the requirement was phrased in terms of a maximum sulfur content rather than an exact one. Had it been specified as an exact requirement, a distinctly non-optimal program might have been generated.

Concerning the two material balance equations associated with stocks #43 and #52, no general rule can be laid down. Both u₂ and u₃ turned out to have zero values in runs #1 and #3, but achieved 28

Table V.3 shows certain quantities of stock #53 used as mutter in the case of runs #2 and #3. This is non-optimal, but is a natural consequence of the approximative type of solution.

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positive levels in the case of run #2. That is, for an optimal solution under the parameters of the second run, neither of these stocks should be employed as cutter material.

The u₁ are of interest, not only in indicating whether or not the restraint conditions were satisfied as exact equalities, but also as a method for evaluating the use of additional blending material in either of the two motor fuel end products. These variables establish a scale for determining the dollar bonus or the penalty that should be attached to <u>small incremental</u> quantities of a material that either exceeds or falls short of the finished product specifications. The first use of these Lagrangean multipliers was to test the earlier assumptions as to which blending stocks could be excluded from consideration as components for 7600 and which others could be ruled out as components in the 76 gasoline product. Under the existing configuration of blending stocks and market conditions, these assumptions did check out.

It is expected that the Lagrangeans will also be of use during the interval before the company is in a position to schedule the blending problem simultaneously with the intra-refinery cracking and conversion operations. The company's Los Angeles refinery, for example, has a polyformer unit that has economic features similar to the thermal reformer discussed in Chapter IV. As with the Kellogg Company's reformer, there is a continuing problem of selecting the optimum operating conditions. The higher the reactor temperature, the better will be the octane rating of the polyformer gasoline, lower quantity yielded. but the will be the A. The outstanding difference between the two problems is the fact that the one reformer was considered as an isolated piece of conversion equipment, and that the other forms just a single component within an integrated company. In this second case, because of the possibilities for blending, there is no simple linkage between the market realization on the end products and the operations of the one unit. Ideally, the company-wide optimization doubtful whether this should be treated as a single calculation, but it is will be achieved within the foreseeable future, Falling

short of this ideal, it would nevertheless be a useful step to attack the polyformer unit problem through the shadow prices derived from the blending calculation. Theory assures us that this is a legitimate assumption if the company, as a whole, is at an optimal point in a concave programming problem. Theory does not assure us of this result if the company fails to be in equilibrium, but it strongly suggests that the same principle holds true even when the company's current 29position is not an equilibrium one.

5.3 Company policy implications

This much is already evident from the computations: that it have paid would Afor Union Oil to forego several millions per annum if a sales have been for disposing strategy could Adevised Aof an

increased volume of premium gasoline. For this reason, the problem was

^{29.} For a discussion of the potentialities for decentralized decisionmaking through the use of shadow prices, see Oskar Lange, <u>On the</u> <u>Economic Theory of Socialism</u>, Minneapolis, 1938, and also A. P. Lerner, <u>Economics of Control</u>, New York, 1944. Also see C. J. Hitch, "Suboptimisation in Operations Problems", <u>Journal of the Operations</u> <u>Research Society of America</u>, Vol. I, No. 3, May, 1953, pp. 87-99; and A. W. Marshall, "A Mathematical Note on Sub-optimization", <u>ibid</u>., pp. 100-102.

taken up with the company's sales service manager. A number of alternatives were discussed with him: price reductions, quality improvements, and an advertising campaign. He agreed that all of these steps could conceivably increase the sales of premium grade gasoline, and that the potential increase in revenues might exceed the extra costs that were entailed. He was particularly concerned of forecasting with the difficulties

the response of consumers to such changes in strategy, but did not bring up the parallel question of how the company's immediate rivals might react.

Rather than risk a major departure in price policy, he indicated Union Oil that it would be useful for *A* to explore the possibility of altering its retailer gasoline discount structure. On any sales in excess of certain goals, the service station operators might be allowed an additional discount from the tank wagon price. By readjusting the product quotas and the sales bonuses, he believed that it would be feasible for the retail operators to shift sales in the direction of 7600 and away from 76 gasoline. This policy is one that need not immediately provoke retaliation from the company's major rivals, and it does not depend upon a large-scale publicity campaign. There is the added advantage that the strategy can be altered at short notice without making consumers aware of a deliberate attempt to influence their choices.

Even aside from major departures in company-wide sales policy, was the gasoline blending model yielded one result that, expected to produce an immediate reduction in cost. In all three computing runs, it turned out to be optimal to cut motor gasoline component #31 at i.e., the lowest possible end point,⁴ at 300° rather than the current 332° . This change in operating practices could be effected without making any radical alteration in the end-item product mix. At the same time, the improvement in payoff that would result from this single step amounts to a sum which, in less than two days, could justify the entire computing machine expense involved in the study.

6. Summary and conclusions

Employing the assumption of fixed availabilities of raw blending materials and of fixed realizations upon finished products, the Union Oil Company's gasoline blending operations were formulated in terms of a constrained-maximum problem.

The outstanding result of the numerical analysis was the fact that it would be worth several millions of dollars per annum if the company were able to shift sales so as to sell additional quantities of premium grade gasoline. As a corollary of this, it follows that the company has considerable latitude in its ability to shift production between the two grades of motor fuel. Marketing problems aside, the ideal pattern of production is quite sensitive to the price structure that prevails.

A second result of the calculations was the finding that the company could increase the octane specification of its premium product, and that even after such a change, the optimal production rate of this one item would remain in excess of current sales.

Neither of these results leads to the implication that the company "ought" to change its price policy or its product specifications. Major changes of this nature cannot be evaluated unless the company's executives also take some account of the response of consumers and of other petroleum refiners. Readily granting the uncertainty that must attend any such conjectures, the formal mathematical treatment seems like a promising approach to the blending problem. It did not turn up with solutions that are absolutely unobtainable by the customary trial-anderror techniques. Rather, it brought to light numerous assumptions that had previously lain hidden, and holds out the possibility for closer pordination between the manufacturing operations and the activities of other branches of the corporation. It is particularly important for the sales group to have quantitative information available on the possibilities for shifting between various classes of refined products, and for changing the specifications of those products. It is equally significant for them to know the cost implications of such changes.

Experience within the Union Oil Company suggests that mathematical models have considerable utility in dealing with complex production problems. This study also indicates that economists and businessmen alike are a long distance from having any satisfactory theory of how a market will respond to a change in the company's sales policies. In the absence of such knowledge, it is dangerous to place too much reliance on formal techniques. This same danger exists, though, with back-of-the-envelope calculations. In none of these matters can a mathematical model make important business decisions by itself. The most that can be claimed is that it will summarize the volume of information that executives have to act upon.

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