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ANALYTICAL APPROXIMATIONS
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Analytical Approximation

Unnamed Definite Integral: To better than .00055 over $(0,\infty)$,

$$N(x) = \frac{30}{\pi^4} \int_0^\infty \frac{e^{-\left(\frac{x}{t}\right)^7} t^7 \, dt}{e^{t^2}-1}$$

$$\doteq \frac{1}{1+.38382x^6-.55605x^7+.34791x^8-.10369x^9+.01245x^{10}} .$$

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Analytical Approximation

Pearson Cosine Transformation: To better than .00017 over (0,1),

$$r(x) = \cos\left(\frac{\pi}{1+\sqrt{x}}\right)$$

$$\doteq \frac{-1-4.828\eta+7.866\eta^2-2.038\eta^3}{1+5.560\eta-4.985\eta^2+.385\eta^3}, \quad \eta = \frac{x}{.16+.84x}.$$

$r(x^{-1}) = -r(x)$ can be used to obtain function values over $(1,\infty)$.

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Analytical Approximation

Bessel Function: To better than .00008 over $(0,\infty)$,

$$e^{-x}I_1(x) \doteq \frac{x}{\sqrt[4]{15.4+74.8x+67.2x^2+235.8x^3+43.5x^4+59.4x^5+39.6x^6}}.$$

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Analytical Approximation

Mach Number in Terms of Pressure Ratio: To better than .0021 over $.3 \leq M \leq 3$, the inverse of the function defined by

$$x = \frac{P_S}{P_R} = \left[1 + \left(\frac{\gamma-1}{2}\right) M^2\right]^{-\frac{\gamma}{\gamma-1}}$$

over $.3 \leq M \leq 1$ and

$$x = \frac{P_S}{P_R} = \frac{\left[\left(\frac{2\gamma}{\gamma+1}\right) M^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\frac{1}{\gamma-1}}}{\left[\left(\frac{\gamma+1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

over $1 \leq M \leq 3$ where $\gamma = 1.4$, is given by

$$M = \frac{8.11 + 23.60x - 39.66x^2 + 8.98x^3}{1 + 28.70x - 15.99x^2 - 5.74x^3}.$$

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10-5-53

Analytical Approximation

Natural Addition Logarithm: To better than .00026 over $0 \leq x \leq \infty$,

$$\ln(1+e^{-x}) \doteq \frac{\ln 2}{(1+.3581x+.1151x^2+.0094x^3+.0052x^4)^2}.$$

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10-6-53

Analytical Approximation

Natural Addition Logarithm: To better than .000,045 over $0 \leq x \leq \infty$,

$$\ln(1+e^{-x}) \doteq \frac{\ln 2}{(1+.36123x+.10204x^2+.02411x^3-.00055x^4+.00069x^5)^2}.$$

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10-7-53

Analytical Approximation

Natural Addition Logarithm: To better than .000,008 over $0 \leq x \leq \infty$,

$$\ln(1+e^{-x}) \doteq \frac{\ln 2}{(1+.360571x+.105546x^2+.018760x^3+.002654x^4-.000100x^5+.000066x^6)^2}.$$

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