Analytical Approximations

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho \, d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .00037 over $(0,\infty)$,

$$q(.5,.5+y) \doteq 1 - \frac{.1045}{[1 + .129y + .079y^2 \pm .056y^3]^4}$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the $q(R, R+y)$ surface for any $R \geqslant 0$ and for y ranging over $(0,\infty)$.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_o(\rho x)\rho \, d\rho$$

in which $I_o(z)$ is the usual Bessel function.

To better than .0007 over $(0,\infty)$,

$$q(1,1+y) \doteq 1 - \frac{.267}{[1 + .203y + .079y^2 + .062y^3]^4}$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the $q(R,R+y)$ surface for any $R \geqslant 0$ and for y ranging over $(0,\infty)$.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^{\infty} e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho \, d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0011 over $(0,\infty)$,

$$q(4,4+y) \doteq 1 - \frac{.45}{\left[1 + .227y + .064y^2 + .065y^3\right]^4}.$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the $q(R, R+y)$ surface for any $R \geq 0$ and for y ranging over $(0,\infty)$.

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Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\,\rho\,d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .0013 over $(0,\infty)$,

$$\lim_{R\to\infty} q(R,R+y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}dt$$

$$\doteq 1 - \frac{.5}{[1 + .209y + .061y^2 + .062y^3]^4}.$$

The parametric form used is convenient for approximating fixed-R semi-cross-sections of the $q(R,R+y)$ surface for any $R \geq 0$ and for y ranging over $(0,\infty)$.

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Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R,x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2+x^2)} I_0(\rho x)\rho \, d\rho$$

in which $I_0(z)$ is the usual Bessel function.

To better than .006 over $(0,\infty)$,

$$\lim_{R\to 0} \frac{1 - q(R,R+y)}{1 - q(R,R)} = e^{-\frac{1}{2}y^2}$$

$$\doteq \frac{1}{\left[1 + .015y + .076y^2 + .040y^3\right]^{.4}}.$$

The above gives information concerning a degenerate limiting case in the approximating of fixed-R semi-cross-sections of the $q(R,R+y)$ surface for y ranging over $(0,\infty)$.

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