ANALYTICAL APPROXIMATIONS

Volume 2

Cecil Hastings, Jr.

P-330

24 September 1952

The RAND Corporation
SANTA MONICA · CALIFORNIA

9-3-52

Analytical Approximation

Common Logarithmic Function: To better than .000,000,015 over (1, 10)

$$\log_{10} x \doteq \tfrac{1}{2} + .8685888\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right) + .2395497\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right)^3$$

$$+ .1731159\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right)^5 + .1314381\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right)^7$$

$$+ .0547562\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right)^9 + .1832415\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right)^{11}$$

Cecil Hastings, Jr.
RAND Corporation

Analytical Approximation

Descending Exponential Function: To better than .000,000,11 over $(0, \infty)$,

$$e^{-x} \doteq \left[\frac{1}{1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5}\right]^8,$$

where $a_1 = .125{,}000{,}204$, $a_2 = .007{,}811{,}604$, $a_3 = .000{,}326{,}627$, $a_4 = .000{,}009{,}652$ and $a_5 = .000{,}000{,}351$.

Cecil Hastings, Jr.
RAND Corporation

9-10-52

Analytical Approximation

Segmental Area Function: To better than .0012 over (-1, 1),

$$A(x) = \int_{-x}^{x} \sqrt{1-t^2}\, dt \doteq 2.0083x - .4160x^3 + .1604x^5 - .1808x^7.$$

In terms of elementary functions, $A(x) = \arcsin x + x\sqrt{1-x^2}$.

Cecil Hastings, Jr.
RAND Corporation

Analytical Approximation

Segmental Area Function: To better than .00016 over (-1, 1),

$$A(x) = \int_{-x}^{x} \sqrt{1-t^2}\, dt \doteq \frac{1.99916x - 2.39484x^3 + .58673x^5}{1 - 1.03472x^2 + .15634x^4}$$

In terms of elementary functions, $A(x) = \arcsin x + x\sqrt{1-x^2}$.

Cecil Hastings, Jr.

RAND Corporation

9-15-52

Analytical Approximation

Segmental Area Function: To better than .000016 over (-1, 1),

$$A(x) = \int_{-x}^{x} \sqrt{1-t^2}\, dt$$

$$\doteq x \left[\frac{1.999872 + 4.143151\,\eta - 3.153670\,\eta^2 - 1.430807\,\eta^3}{1 + 2.901498\,\eta - 1.811287\,\eta^2 - 1.098016\,\eta^3} \right],$$

where

$$\eta = \frac{x^2}{5-4x^2}.$$

In terms of elementary functions, $A(x) = \arcsin x + x\sqrt{1-x^2}$.

Cecil Hastings, Jr.

RAND Corporation

9-17-52

Analytical Approximation

Common Logarithmic Function: To better than .005 over (.1, 1),

$$\log_{10} x \doteq -.076 + .281x - \frac{.238}{x+.15} .$$

This approximation is the result of a request for a very simple formula to use in the reduction of certain data.

Cecil Hastings, Jr.

RAND Corporation

9-22-52

Analytical Approximation

Inverse Tangent: To better than .005 over (-1,1)

$$\arctan x \doteq \frac{x}{1+.28x^2}$$

This approximation is the result of a request for a very simple formula to use in the reduction of certain data.

Cecil Hastings, Jr.

RAND Corporation