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ANALYTICAL APPROXIMATIONS

Volume 2

Cecil Hastings, Jr.

P-330

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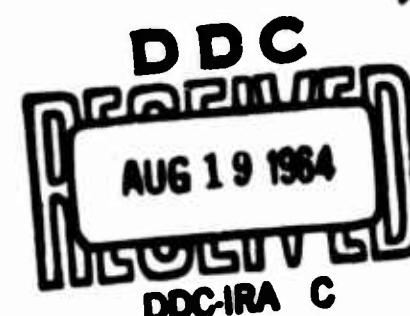
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9-3-52

Analytical Approximation

Common Logarithmic Function: To better than
.000,000,015 over (1, 10)

$$\log_{10} x = \frac{1}{2} + .8685888 \left(\frac{x - \sqrt{10}}{x + \sqrt{10}} \right) + .2395497 \left(\frac{x - \sqrt{10}}{x + \sqrt{10}} \right)^3 \\ + .1731159 \left(\frac{x - \sqrt{10}}{x + \sqrt{10}} \right)^5 + .1314381 \left(\frac{x - \sqrt{10}}{x + \sqrt{10}} \right)^7 \\ + .0547562 \left(\frac{x - \sqrt{10}}{x + \sqrt{10}} \right)^9 + .1832415 \left(\frac{x - \sqrt{10}}{x + \sqrt{10}} \right)^{11}$$

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Analytical Approximation

Descending Exponential Function: To better than
.000,000,11 over $(0, \infty)$,

$$e^{-x} \doteq \left[\frac{1}{1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5} \right]^8,$$

where $a_1 = .125,000,204$, $a_2 = .007,811,604$, $a_3 = .000,326,627$,
 $a_4 = .000,009,652$ and $a_5 = .000,000,351$.

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Analytical Approximation

Segmental Area Function: To better than .0012 over
(-1, 1),

$$A(x) = \int_{-x}^x \sqrt{1-t^2} dt \doteq 2.0083x - .4160x^3 + .1604x^5 - .1808x^7.$$

In terms of elementary functions, $A(x) = \arcsin x + x \sqrt{1-x^2}$.

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Analytical Approximation

Segmental Area Function: To better than .00016
over (-1, 1),

$$A(x) = \int_{-x}^x \sqrt{1-t^2} dt \doteq \frac{1.99916x - 2.39484x^3 + .58673x^5}{1 - 1.03472x^2 + .15634x^4}$$

In terms of elementary functions, $A(x) = \arcsin x + x \sqrt{1-x^2}$.

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Analytical Approximation

Segmental Area Function: To better than .000016
over (-1, 1),

$$A(x) = \int_{-x}^x \sqrt{1-t^2} dt$$

$$\doteq x \left[\frac{1.999872 + 4.143151 \eta - 3.153670 \eta^2 - 1.430807 \eta^3}{1 + 2.901498 \eta - 1.811287 \eta^2 - 1.098016 \eta^3} \right],$$

where

$$\eta = \frac{x^2}{5-4x^2}.$$

In terms of elementary functions, $A(x) = \arcsin x + x \sqrt{1-x^2}$.

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Analytical Approximation

Common Logarithmic Function: To better than .005 over (.1,.1),

$$\log_{10}x \doteq -0.076 + .281x - \frac{.238}{x+15} .$$

This approximation is the result of a request for a very simple formula to use in the reduction of certain data.

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Analytical Approximation

Inverse Tangent: To better than .005 over (-1,1)

$$\arctan x \doteq \frac{x}{1+.28x^2}$$

This approximation is the result of a request for a very simple formula to use in the reduction of certain data.

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