ANALYTICAL APPROXIMATIONS

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The first few items in a cumulative MTAC publication of useful working approximations are listed below. Additional items will be published from time to time. Readers having useful contributions to make of this kind are hereby encouraged to submit their items to the chairman of the editorial committee for publication in MTAC. Needless to say, useful multivariate approximations will be as welcome as useful univariate approximations. In every case, the accuracy of the approximation and the domain of its validity should be clearly stated. ~~It is hoped that this~~ A series of approximations ~~will do much to make values~~ of the higher mathematical functions are presented. ~~readily~~ available to users of modern digital computing machinery.

For convenience in future reference, the approximations listed are numbered consecutively. The first five items have been contributed by Cecil Hastings, Jr. of The RAND Corporation.

(1) Square Root: To better than 1 part in ~~13~~ 12 over (.1,10),

$$\sqrt{x} \doteq \frac{1+4x}{4+x}$$

(2) Pearson Cosine Transformation: To .003 over (0,1),

$$r(x) = \cos\left(\frac{\pi}{1+\sqrt{x}}\right) \doteq \frac{-1-4x+5x^2}{1+8x+6x^2}\,.$$

$r(x^{-1}) = -r(x)$ can be used to obtain function values over $(1,\infty)$.

(3) Common Logarithmic Function: To better than .000,004 over (1,10),

$$\log_{10}x \doteq \tfrac{1}{2} + .86857\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right) + .29059\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right)^3 + .15783\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right)^5 + .20269\left(\frac{x-\sqrt{10}}{x+\sqrt{10}}\right)^7 .$$

(4) Incomplete Gamma Function Type Integral: To better than .000,000,1 over (0,1),

$$F(x) = \int_1^\infty \frac{e^{-t}dt}{t^{1+x}} \doteq \frac{.219384 + x(.024717 + .000803x)}{1 + x(.558651 + .090584x)} .$$

$F(x) + (x+1)F(x+1) = e^{-1}$ can be used to obtain function values outside the range indicated.

(5) Exponential Integral of Negative Argument: To better than .000,000,1 over $(10,\infty)$,

$$xe^x\int_x^\infty \frac{e^{-t}dt}{t} \doteq \frac{1.15198 + x(4.03640+x)}{4.19160 + x(5.03637+x)} .$$