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TESTING ORGANIZATION THEORIES

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SUMMARY

✓ A discussion of a few typical theories of decision making in small human groups, and description of a few pilot experiments illustrating the kind of theoretical and experimental problems that are met in testing these theories.

A

1. Introduction

This paper proposes a class of mathematical models to represent the behavior of a group of one or more individuals engaged in a decision-making process. In one sense, this class of models can be thought of as a general theory of organization whose parameters permit description of a wide variety of behavioral situations. There is also some discussion of the experimental problems that are met in the attempt to test the theory for applicability to everyday human situations.

The von Neumann-Morgenstern [1] formulation of the choice situation as an n-person game in normal form with a finite set of choices available to each person at each moment of time, and with a valuation of the expected outcome for each person and each set of choices, is accepted as the underlying description of the behavioral alternatives available to the group. The Bush-Mosteller [2] formulation of the learning situation for a single person, treated as a stochastic process, is accepted as the underlying description of individual behavior in a repetitive choice situation. The Bales-Householder [3] formulation of the group interaction process, treated as a stochastic process, is accepted as the underlying description of group behavior in a repetitive choice situation. The class of models presented in this paper is essentially a synthesis of the essentials from these three broad theories so as to yield an organization theory general enough to relate to the dynamic valuation, learning, interaction, and decision processes

observed in group decision-making.

A few pilot experiments have been conducted to test various special features of the general model. One central problem concerns the methods used by a coalition of persons to establish the division of rewards, derived by forming the coalition, among its several members; some experimental results in connection with this problem have been reported elsewhere [4]. In their most common form, the stochastic learning models describe behavior in terms of a dichotomy of payoffs, termed simply "reward" and "punishment"; the conversion of the game-theoretic model, with its many-valued payoff functions, to this dichotomous form has been discussed elsewhere [5] together with some experimental results pertinent to this aspect of the problem. Several experiments designed to provide parameter estimates, and tests of validity, have shown the stochastic learning models to be promising for description of individual human learning; a sample of these experimental results and references to other authors may be found in an earlier paper [6]. One especially difficult task is the prescription of the rule determining group decision in terms of individual choices, involving such questions as majority rule and the like, and many recent authors have discussed various aspects of this problem; a few experimental results along these lines have been reported in an earlier series of papers [7].

The present paper disposes of the troublesome game-theoretic problem of coalition and division by letting these results be

determined stochastically by other properties of the group decision model. The payoff functions are generally not known to the individuals at the outset of their interaction, and are only partially learned during a sequence of group decisions, so that the model reflects this human feature of learning values from outcomes of similar acts previously experienced. Unfortunately, it has seemed necessary to make an arbitrary selection of the rule for determining group decision in terms of individual choices; this imposition of a rule of conduct on the group by a superior authority may or may not be realistic, and is a feature of the model worth further attention.

A first very crude pilot experiment was conducted to see if the kind of experiment proposed in this paper seems interesting on purely subjective grounds. As one of the two subjects, I found the situation to be surprisingly realistic in the sense that I had real difficulty in making the choices and felt that the two of us should be able to communicate and cooperate so as to increase our returns. More extensive experiments are in progress but it seems likely that they will only serve for some time to come as a means toward improvement of the model and the methods of testing.

Perhaps this paper is a bit premature. It presents no striking results, either of an experimental or theoretical nature. But the key effort is synthesis of various mathematical theories pertaining to components into a first crude general theory of organization. It is hoped that others working in

related fields may find enough of interest here to be of help in furthering such a synthesis.

2. The group decision model

I shall define the decision models only for groups with two members, leaving the rather straightforward extensions to larger groups for another paper. The two members are assumed to be interested in the same set of alternatives, and their choices at each moment of time are determined both by their individual preferences and by the interaction between members in a manner now to be defined precisely.

Individual 1 has an m -component stochastic preference vector $p^1(t)$ at moment t such that he will select alternative x with probability $p_x^1(t)$ if he acts alone. This vector satisfies the stochastic relation:

$$2.1) \quad p^1(t+1) = L^1(i_t, s_t) p^1(t),$$

where $L^1(i_t, s_t)$ is an operator that carries a stochastic vector into a stochastic vector, i_t is the actual choice made at moment t , and s_t is a value attached to the consequences following choice i_t . In this paper, the model will be taken in a special form, as follows:

- 2.2)
- 1) s_t is either 1 or 0 according as the choice is rewarded or non-rewarded,
 - 2) $L^1(i_t, s_t) = s_t R^{i_t} + (1-s_t) P^{i_t}$, where R^{i_t} and P^{i_t} are stochastic matrices.

Member i has a set of j -component stochastic attitude vectors $d^{ijx}(t)$ at moment t , for alternative x , and for source j , such that he will decide on action r with probability $d_r^{ijx}(t)$. These vectors satisfy the stochastic relations:

$$2.3) \quad d^{ijx}(t+1) = D^{ijx}(i_t, j_t, r_t, x_t) d^{ijx}(t),$$

where $D^{ijx}(i_t, j_t, r_t, x_t)$ is an operator that carries a stochastic vector into a stochastic vector. Whenever member i must make a proposal to himself he does this with probability I_y^i for alternative y . In a sentence, $d_r^{ijx}(t)$ is the probability that i will decide on r after j proposes alternative x to i at moment t . Similarly, $D^{ijx}(i_t, j_t, r_t, x_t)$ is applied to $d^{ijx}(t)$ after i_t actually decides on action r_t after j_t proposes alternative x_t to i_t at moment t . In this paper, the model will be taken in a special form, as follows:

- 1) $D^{ijx}(i_t, j_t, r_t, x_t)$ are stochastic matrices,
 - 2) $D^{ijx}(i_t, j_t, r_t, x_t) = 1$ if $x_t \neq x$,
 - 3) $D^{ijx}(i_t, j_t, r_t, x_t) = 1$ if $r_t = 0$,
 - 4) $D^{ijx}(i_t, j_t, r_t, x_t) = 1$ if $i_t = j_t \neq i$ when $r_t \neq 0$.
- 2.4)

These four special restrictions may be interpreted as follows:

- 2.4.1) This is a specialization of mathematical form, so that the operators are matrices with non-negative elements and with columns that sum to unity.

- 2.4.2) No attitudes toward one alternative are changed by reaction to the proposal of some other alternative.
- 2.4.3) No attitudes are changed if the reaction is simply to question a proposal.
- 2.4.4) The attitudes of one member are not changed as a consequence of the reaction of another member to his own proposal.

The ranges of the variables are limited as follows:

$$\begin{aligned}
 i, j, i_t, j_t &= 1, 2, \\
 r, r_t &= 0, 1, 2, \\
 x, x_t &= 1, 2, \dots, m, \\
 t &= 0, 1, 2, \dots, N.
 \end{aligned}$$

The three possible actions may be thought of as question, accept, or reject for $r = 0, 1, 2$, respectively. In further specialization, it is assumed that:

$$\begin{aligned}
 1) \quad D^{ijx}(i_t, j_t, 1, x_t) &= \begin{pmatrix} 1-a_1 & 0 & 0 \\ b_1 & 1 & a_1 \\ 0 & 0 & 1-a_1 \end{pmatrix} \\
 2) \quad D^{ijx}(i_t, j_t, 2, x_t) &= \begin{pmatrix} 1-a_2 & 0 & 0 \\ 0 & 1-a_2 & 0 \\ b_2 & a_2 & 1 \end{pmatrix}
 \end{aligned}$$

It will usually be convenient to omit x and x_t in writing

the D-matrices and to write simply $D^{ij}(i_t, j_t, r_t)$, or even more simply $D^{ij}(t)$.

We must now describe the order of interaction in order to specify the sequence of operations with the D-matrices. The interaction is initiated by member i_0 with probability q_{i_0} , by selecting alternative x_0 with probability $I_{x_0}^{i_0}$. He then decides on action r_0 with probability $d_{r_0}^{i_0, i_0, x_0}(0)$. Both members now apply operators $D^{ijx}(0) = D^{ijx}(i_0, i_0, r_0, x_0)$ to their $d^{ijx}(0)$ to obtain their $d^{ijx}(1)$. If $r_0 \neq 1$ this initial sequence is repeated until at some moment $t_1 (\geq 0)$ it happens for the first time that $r_{t_1} = 1$; then both players apply operators $D^{ijx}(t_1)$ to their $d^{ijx}(t_1)$ to obtain their $d^{ijx}(t_1 + 1)$. Now the other player $i_{t_1+1} \neq i_{t_1} = i_0$ decides on action r_{t_1+1} with probability

$$d_{r_{t_1+1}}^{i_{t_1+1}, i_{t_1}, x_{t_1}}(t_1+1),$$

and both members then apply operators $D^{ijx}(t_1+1)$ to their $d^{ijx}(t_1+1)$ to obtain their $d^{ijx}(t_1+2)$. If $r_{t_1+1} = 2$, then i_{t_1+1} selects alternative x_{t_1+2} with probability

$$I_{x_{t_1+2}}^{i_{t_1+1}},$$

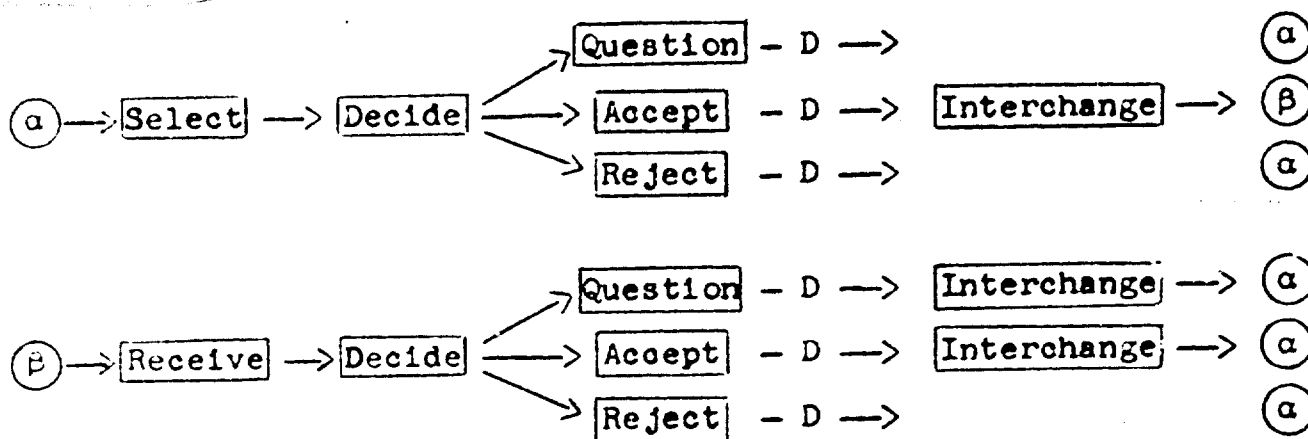
and the sequence of operations continues for i_{t_1+1} in a

manner exactly analogous to that for i_0 at the start, with the roles of the two members interchanged. On the other hand, if $r_{t_1+1} \neq 2$, then the action reverts to member $i_{t_1+2} = i_0$, who repeats the entire sequence of steps starting with an alternative x_{t_1+2} selected with probability

$$I_{x_{t_1+2}}^{i_{t_1+2}}$$

This pattern of actions is shown concisely in the flow diagram following.

Flow Diagram



- D → = Apply appropriate D-matrices to d-vectors for both members, and pass from t to $t + 1$.

There are many ways in which we could combine the learning and interaction models to get a group decision model. We shall identify I^1 with $p^1(T)$, and think of the intervals between

consecutive values of T as interaction times involving moments $t = 0$ to $t = N(T)$; hence, the sequence of events is an interaction stage from $T = t = 0$ to $t = N(0)$ resulting in a joint decision by both members to choose an alternative $x(1)$; as a consequence of this choice, the preference vectors $p^1(0)$ are modified according to 2.1) and the next interaction stage occurs starting with $I^1(1) = p^1(1)$. The rule for a joint decision is here taken simply to be acceptance by one member of a proposal by the other, although stronger requirements such as two (or more) consecutive acceptances of the same alternative could be imposed. It is not assumed that one member is always rewarded if and only if the other member is rewarded; the two schedules of reward and non-reward may be completely independent, or they may be correlated. After each choice it is assumed that the interaction process starts over again with the same initial values $d^{1jx}(0)$, and that the probability is always q_1 that member 1 will be the first to make a proposal; of course $q_1 + q_2 = 1$. All these quite arbitrary assumptions have been made in order to end up with a single group decision model whose properties can be studied mathematically and whose applicability can be tested experimentally.

3. A group experiment

I shall propose a type of group experiment that could provide the data necessary for estimating values of the parameters entering into the two-member group decision model defined in § 2, hereafter called Model G. The extension to more than two

members is involved but straightforward, and will be postponed until later.

Two subjects play a game together with the following rules for moves and payoffs:

- Move 1. A chance move determining that Subject i_1 has Move 2 with probability q_{i_1} , where $q_1 + q_2 = 1$.
- Move 2. Subject i_1 sends to the other subject a positive integer $x_1 \leq m$.
- Move 3. Subject $i_2 \neq i_1$ sends one of the following three messages to Subject i_1 :
0. Pass,
 1. Accept x_1 ,
 2. Reject x_1 and propose $x_2 \neq x_1$, where $0 < x_2 \leq m$. He also pays the referee an amount $2V$ if he rejects, and V if he passes.
- Move 4.
- a. If Move 3.1, the referee pays one unit to Subject i with probability $\frac{V}{x_1}$, or receives one unit from Subject i otherwise, and this terminates a play of the game,
 - b. If Move 3.2, then Subject i_1 sends one of the following three messages to Subject i_2 :
0. Pass,
 1. Accept x_2 ,
 2. Reject x_2 and propose $x_3 \neq x_2$.

He also pays the referee $2V$ if he rejects, and V if he passes.

- c. If Move 3.0, then Subject 1_1 sends an x_3 to Subject 1_2 , as for Move 2.

Move 5.

- a. If Move 4b.1, the referee pays one unit to Subject 1 with probability $V_{x_2}^1$, or receives one unit from Subject 1 otherwise, and this terminates a play of the game,
- b. If Move 4b.2, then Subject 1_2 has the options of Subject 1_1 listed in Move 4a.2.
- c. If Move 4b.0, then Subject 1_2 has the options of Subject 1_1 listed in Move 2.

Moves 6-l.

A play of the game is either terminated by a sequence of moves like Moves 1-5, by one player accepting a proposal of the other within l_0 moves, or a play is terminated at the l_0^{th} move.

The quantities V_x^1 are actually chosen so as to represent the payoffs in a 2-person game, where x provides an enumeration of the strategy-pairs available to the two players. For example, if the payoff matrices for the game in normal form are $P_{\alpha\beta}^1$, for $(\alpha = 1, 2, \dots, a; \beta = 1, 2, \dots, b)$, then x is some enumeration of the ab pairs (α, β) . The payoff units are chosen so that $|P_{\alpha\beta}^1| \leq 1$ and, in the x -enumeration, $V_x^1 = \frac{1+P_x^1}{2}$. For some purposes, it may be easier to use the $\alpha\beta$ -enumeration

and we shall denote it $V_{\alpha\beta}^1$.

A set of specific values will now be assigned for the constant quantities in the experiment, as an illustrative example, as follows:

- a. $q_1 = 1/4, q_2 = 3/4,$
- b. $l_0 = 10,$
- c. 1 unit = 1 cent
- d. $V = 0.1,$
- e.

α, β	1,1	1,2	2,2	2,1
x	1	2	3	4
$8V_x^1$	2	5	6	4
$8V_x^2$	8	6	2	5

The best that the two players can do jointly, as partners, is always to choose alternative 2 with an expected net gain per play for Subject 1 of 1/4 cent and for Subject 2 of 1/2 cent; what they will or should actually do, with sidepayments prohibited, is an open question within the game-theoretic framework. [4].

The experiment should be arranged so that neither subject ever knows the identity of the other, nor the actual payoffs made to the other. They communicate with each other only through the strictly limited language represented by the words: accept, reject, pass, propose x, etc. They are both told that what they do affects the outcome for both, but that a choice that is profitable for one subject may also be profitable for the other.

They are urged to behave so as to maximize their total individual net receipts in a series of 100 plays and are told that they should both come out ahead of the game if they get together properly, although one subject may profit more than the other.

4. Parameter estimation

The group decision Model G may be thought of as a function $G(t, \theta)$ that will yield a sequence $x^N(t)$ of choices for $t = 1, 2, \dots, N$ among $m + 1$ alternatives X_1 , where X_0 is the case in which no acceptance is obtained within ℓ_0 moves. The quantity θ represents a set of parameters $\theta_1, \theta_2, \dots, \theta_p$ and $x^N(t)$ has a probability distribution $F(x^N, \theta)$ that determines the relative frequency with which each possible sequence x^N may be expected in repeated applications of G producing sequences of N choices each.

The group experiment proposed in §3 also yields a set of sequences x^N , say with observed frequencies $f(x^N)$, so the estimation problem is that of determining the particular set $\hat{\theta}$ of parameter values that makes for the best agreement between $F(x^N, \hat{\theta})$ and $f(x^N)$. Standard statistical methods, such as maximum likelihood or moments, may be used to obtain the required estimates $\hat{\theta}$.

The parameters θ that are involved are as follows:

$$p_1^j(0) \quad \text{for } j = 1, 2; \quad i = 1, 2, 3, 4.$$

$$d_k^{1jx}(0) \quad \text{for } i, j = 1, 2; \quad k = 0, 1, 2; \quad x = 1, 2, 3, 4.$$

The assumptions of 4.1) may be justified somewhat, as follows:

4.1.1) At the outset neither subject would seem to have any reason for choosing one alternative instead of another, and so they are assumed equally likely at the start. Indeed, by prior conditioning, the subjects can surely be brought to this initial state.

4.1.2) There seems to be no reason for the subject who moves first to pass any alternative, nor to reject any alternative proposed to himself. We are left with a parameter d^1 that measures the initial "agreeability" of player 1, and this may well differ from subject to subject.

4.1.3) The parameters a^a and b^a determine the rates at which the attitude vectors are modified after each move, and making them independent of a amounts to assuming that an acceptance makes the next acceptance just as much more likely as a rejection makes the next rejection more likely. Of course, we might have done better to allow the parameters in R^{ix} and P^{ix} to depend upon i so as to provide for variation between subjects.

4.1.4) This is the "mixed" learning model, and it has been chosen because it has seemed to fit the data from various choice experiments better than other single-parameter models when the subject is convinced that the system of rewards may be varied from time to time so that his past

experience is not a too reliable indicator of the future. The parameter c^1 measures the rate at which subject 1 changes his preferences as a consequence of reward or non-reward; the use of the same c^1 in both R^{1x} and P^{1x} means that the rate of change with reward is taken to be the same as the rate of change with non-reward, the so-called "equal alpha" case in the Bush-Mosteller theory.

We must have a measure of reward in order that r_t be defined. The net amount actually received by member 1 for play T is either +1 or -1, less the total payments $P^1(T)$ he makes for his rejections and passes, so the limits on his net receipts $R^1(T)$ are such that:

$$1 \geq R^1(T) \geq - (1 + 10V).$$

The simplest way is to ignore the costs $P^1(T)$, as I shall do in this paper, and suppose that their effect is merely to alter the values observed for a^a and b^a . One alternative would be to take account of the costs $P^1(T)$ by a separate application of the operator P^{1x} with probability $\frac{P^1(T)}{10V}$ if x was chosen on play T. In the experiment described in 63 the charges $P^1(T)$ were included to simulate the "hurt" experienced by a member of a group when he rejects or questions a proposal made by another, and it is consistent with this simulation to let any effects of the $P^1(T)$ be reflected in the a^a and

b^a as I have done.

5. Experimental design

There are many different experiments, of course, that would yield data adequate for parameter estimation and test of goodness of fit for each group decision model. As experience is gained with the mathematical characteristics of the model, and with the experimental difficulties inevitably met, the experimenter will be better and better able to choose among the many available designs. I can here only discuss certain general considerations that will help to determine the selection of early designs.

Even a model as simple as G, with its very severe restrictions on the parameters, involves estimation of six parameters in each type of experimental situation. It will be desirable, if indeed it is not necessary, to select a sequence of experiments that will provide estimates of just one, or at most a very few, of the parameters in each experiment. For example, each individual might be first investigated in a simple learning experiment so as to estimate the parameters entering into his learning operator. Next the parameters entering into his attitude operators might be estimated one or two at a time from experiments involving only group interaction and no learning. Finally, the validity of these stepwise estimates for individuals singly and in pairs might be checked experimentally by testing them in different combinations or in different experi-

mental situations. The hope would be that eventually the experimenter would find a parametric structure and a way to estimate that would yield values that were essentially invariant from one experimental situation to another quite different one.

Even when a single parameter has been isolated for estimation there are many experiments that might serve for the purpose. For example, in some of the learning experiments already done to provide estimates of parameters in the learning operator, it was found simpler to test whether or not one entire range of values was admissible than to estimate the actual value; because the asymptotic behavior was qualitatively different inside the range and outside of it. In fact, qualitative tests of this sort, that depend on mathematical properties derived for particular parameter sets, will probably provide a quicker and surer way toward acceptance and rejection of classes of models than will detailed parameter estimation until we have gained a far better general understanding of the properties and applicability of the general type of models under consideration.

Specifically, for the learning operator of Model G, I would expect that the size of c^1 would depend upon the strength of reward and non-reward; perhaps the larger the reward the larger the value of c^1 , as is suggested by the results of some of the experiments reported by Mosteller [6]. In our model, we have made no direct provision for a functional dependence of c^1 on strength of reward, although we could easily do so in a variety of ways. At present, since theory does not suggest a functional

form, it will probably be necessary to explore the dependence of c^1 on strength of reward by conducting experiments designed to yield data relevant to this question. With success in this, it might then be possible to replace Model G by a new one in which strength of reward s is taken into account directly in such a way that $c^1(s)$ will be invariant from situation to situation.

It will be especially important to test tentative parameter estimates for predictive validity in rather different situations. For example, if the learning and attitude operators are estimated for each of several individuals in pairwise group experiments, the ideal case is that in which the parameter estimates for a particular individual are essentially the same when derived from several pairwise experiments in which he is a participant. One way to study this kind of predictive validity is to conduct experiments in which one member of the pair is a human subject and the other is a known mathematical model; in this way, the parameters for a subject can be estimated over a sequence of situations under the control of the experimenter and for extreme or critical values of parameters. One especially useful device, that can sometimes be used, is to have the subject assume both roles in the group without his knowledge so as to ensure symmetry in the model and inter-comparability of utilities and rewards.

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