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The Application of P.W. Bridgman's "New emf" to

Exploding Wire Phenomena. *

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ABSTRACT

P.W. Bridgman propounded the possible existence of a "New emf" generated
in a conductor carrying a current due to a time varying temperature. By means
of thermodynamical arguments Bridgman deduced that the emf generated is given
by,

$$V_B = - \frac{\angle I}{H} \frac{dH}{dT} \frac{dT}{dt}$$

This emf was not detected by any researchers due to the fact that the
magnitude of this generated voltage is very small under normal conditions.
However, under the conditions of an exploding wire the magnitude of the emf can
become increasingly important. It is shown in this paper that this effect can
possibly account for the excess energy required to melt and vaporize a wire
under the extreme conditions of a rapid discharge.

INTRODUCTION

The amount of energy a wire receives from a capacitor discharge, before it
undergoes a very rapid expansion, may be much greater than the energy required
to completely vaporize it. This has been a well known experimental fact for some
time, and some aspects of the dependance of the excess energy on various parameters
have been reported by researchers in the field. For example, Kvartskhava and
his co-workers[1] have shown that the magnitude of the excess energy deposited in
a wire prior to the rapid expansion has some dependance upon the inductance of
the discharge circuit.

They have found that in a low inductance discharge circuit, the wire will absorb more energy and have less resistance than a wire undergoing a discharge in a more inductive circuit, if all other parameters are unchanged. The excess energy has also been studied by Kielhacker[2] who has demonstrated that the energy absorbed per unit mass before rupture can be increased by raising the initial voltage or by increasing the wire diameter while keeping the circuit characteristics constant. It is our intention to introduce a little known work of P.W. Bridgman which may lead to a possible description of the superheating in the wire prior to reaching its melting temperature.

THEORETICAL BACKGROUND

In 1932 Bridgman published a paper in the Physical Review entitled, "A New Kind of emf and Other Effects Thermodynamically Connected with the Four Transverse Effects"[3]. In this work Bridgman proposed the theory that there should be an emf, V_B generated in a conductor carrying a current when the conductor experiences a changing temperature. The emf, as proposed by Bridgman, is given by,

$$V_B = -\frac{IL}{H}\ \frac{dH}{dT}\ \frac{dT}{dt}$$

where I is the current in the conductor, L the inductance and H is the Hall coefficient of the conductor. $\frac{dH}{dT}$ and $\frac{dT}{dt}$ are the rates of change of the Hall coefficient with respect to the temperature and temperature with respect to time, respectively. Bridgman arrived at this conclusion through an intuitive thermodynamic argument, which we shall describe briefly, in order to acquaint the reader with this effect.

Consider a Hall effect situation in which a magnetic field B is established perpendicular to a conductor of unit length and width a, carrying a current I. There will exist a Hall potential V_H transverse to both the current, I and field B, whose magnitude is given by,

$$V_H = \frac{HIB}{a} \tag{1}$$

where we have chosen H as the Hall coefficient for clarity. This system can be treated thermodynamically as a battery whose emf is given by eq.(1), since the Hall coefficient is a slowly varying function of the temperature. If we neglect for the moment any joule heating, then we can calculate work done by the system when a charge q flows transversely due to the Hall field. The work done by this transverse flow of charge moving through the Hall potential is,

$$dW = \frac{HIB}{a} dq$$

Now, let us assume that the state of the system can be described by the set of independent variables q and T. This assumption implies that the internal energy, U, of the system is a function of q and T only, i.e., $U = U(Q,T)$. By the application of the first law of thermodynamics we get,

$$d'Q = \left[\frac{\partial U}{\partial q} + \frac{HIB}{a} \right]_T dq + \frac{\partial U}{\partial T} \bigg|_q dt \tag{2}$$

Multiplying eq.(2) by 1/T we can form the entropy dS of the system and by imposing the condition that dS be a perfect differential, i.e., $\partial^2 S / \partial T \partial q = \partial^2 S / \partial q \partial T$, we obtain the relation,

$$\frac{\partial U}{\partial q} \bigg|_T = \frac{TB}{a} \frac{\partial}{\partial T} (IH) \bigg|_q - \frac{HIBT}{a} \tag{3}$$

Substituting this into ea.(2) we can get the heat absorbed per unit charge which has flowed transversely to the applied field, at constant temperature, under the assumption that the wire has not undergone any expansion.

$$\frac{\Delta' q}{\Delta q}\bigg|_T = \frac{TB}{a} \quad \frac{\partial}{\partial T}(IH)\bigg|_q \tag{4}$$

Equation (4) demands the inflow of heat to the system in order to keep the temperature constant when there is a transverse current in the system. Bridgman performed an experiment[3] using bismuth as the conductor at low temperature, in order to see if this effect could be detected, and he found negative results. In his paper, he presents an argument on the basis of symmetry considerations as to why this effect should vanish. We shall not reproduce his reasoning, but shall refer the reader to his paper. However, if we accept this result that $\frac{\Delta' q}{\Delta q}$ must vanish at constant temperature, then we arrive at this result,

$$\frac{1}{I}\frac{\partial I}{\partial T} = -\frac{1}{H}\frac{\partial H}{\partial T} \tag{5}$$

This relation shows that the current in a system will change when the temperature changes, while no work will be delivered by the system during the change in temperature. Therefore, Bridgman concluded that "a New Kind of emf" must be generated in a circuit undergoing a temperature change, while carrying a current, in order to produce this change of current.

In order to arrive at a value for this emf, let us perform a simple mental experiment. We shall prepare a superconductor in the form of a ring and by some means establish a circulating current in the ring. The current will continue to circulate due to the $\frac{1}{2}LI^2$ energy stored in the inductance, assuming the energy dissipated by the ohmic resistance is negligable.

Now, if we increase the temperature uniformly throughout the conductor, without appreciably increasing the ohmic resistance, a counter emf V_H should be generated according to Bridgman, and the primary current will have to do work against this potential. The work done against the potential by the primary current must deplete the energy stored in the inductance, accordingly,

$$IV_H = \frac{d}{dt}(\tfrac{1}{2}LI^2) = LI \frac{dI}{dt} = LI \frac{dI}{dT}\frac{\partial T}{\partial t} \tag{6}$$

But, from eq.(5) substituted into eq.(6) we get the result

$$V_H = -\frac{LI}{H}\frac{dH}{dT}\frac{\partial T}{\partial t} \tag{7}$$

This equation is independent of the externally applied magnetic field B, and hence, this emf should be generated in any circuit undergoing a change in temperature, while carrying a current. The effect is not easily observable, for under normal conditions, temperature changes occur slowly with time, and the current in a circuit is usually not very large in magnitude, and furthermore, the Hall coefficient is a slowly varying function of the temperature. Thus, the magnitude of this emf would not normally be very large. Bridgman was not able to demonstrate the existence of this emf due to these reasons, in addition to the masking of this voltage by ohmic heating effects, which cannot be eliminated in a practical experiment. However, in exploding wire studies the effect becomes increasingly important since we deal with large currents and rapidly varying temperatures.

EXPERIMENTAL

In order to evaluate the applicability of this effect to exploding wire phenomena, a series of energy measurements were taken of wires exploded in a discharge circuit consisting of a 15 microfarad, 5000 joule capacitor. The ringing period of the circuit was 13.55 microseconds, and the values of the resistance and inductance were 19 milliohms and 0.5 microhenries, respectively. The circuit was discharged by means of a flat lovetron type triggered air gap switch. The switch and switching process is the topic of another paper which we shall present to this conference.

The current in the circuit was measured by displaying the integrated output voltage of a small coil on one channel of a dual beam oscilloscope. The coil is placed in close proximity to a current lead of the discharge circuit. This has been found to be a convenient method of obtaining the instantaneous current in the discharge circuit. The voltage waveforms were recorded simultaneously with the current, on the second channel of the dual beam oscilloscope. The voltage across the wire was measured by the method of electronic cancellation[4] which inhibits the introduction of ground loops and spurious inductive voltages into the measuring apparatus. The indicated rise time of the apparatus has been determined to be less than 0.04 microseconds. The determination of the power deposited in the wire was found by plotting the product of the instantaneous values of the current and voltage. Integration of the power curves yields the total energy in the wire, and by subtracting the inductive energy stored in the magnetic field of the current at a given time, we arrive at the energy dissipated in the wire.

RESULTS

The wires which we have studied exhibit the phenomena of current dwell. Figure 1 is a typical example of the current and voltage waveforms we record with our apparatus. The verticle deflection sensitivity is 9.75 kV/div and 28.7 kA/div for the voltage and current, respectively, and the sweep speed is 2 microsec./div. The initial voltage on the capacitor was 10 kV and the wire under study in Fig. 1 was a #28 B&S gauge (12.64 mils) copper wire. In order to calculate the Bridgman emf, we took the first discontinuity in the voltage waveform as the indication that the temperature in the wire had reached the melting point of copper, 1356°K. The discontinuity has been interpreted by other researchers[5] to be the point at which the wire commences to change from the solid to the liquid state. Hence, we can evaluate the average change of the temperature with respect to time, by dividing the difference between room temperature and the melting point of copper by the time interval from the initiation of the discharge to the discontinuity, which we obtain from the voltage waveforms. This is truly a rough approximation, since the temperature change is not linear in time, as can be seen by the very rapid rise of voltage at the end of this interval. This point will be discussed further later in this paper. The results of this paper are confined to the interval prior to the first discontinuity. The reason for this is that information of how the Hall coefficient varies with temperature above 600°C is not available. However, it is known to change linearly with temperature below 600°C, hence, we can extrapolate at least to the melting point. The slope of the Hall coefficient with temperature as determined from the data of V. Frank[6] has a value of 12.1 $\times 10^{-5}$ cm^3/coulomb. This slope we assume remains constant during the heating period in which the wire is brought up to its melting temperature.

Therefore, the energy given to the Bridgman effect takes the form of

$$E_B = \frac{L}{H}\frac{dH}{dT}\frac{dT}{dt}\int I^2 dt$$

where the coefficients in front of the integral sign are all constants. This integral is what Anderson and Neilson[7] call the "action integral".

In Fig. 2 the results of the energy calculations are plotted for a number 30 (10.03 mils) and number 32 (7.95 mils) copper wire 5 cm in length against the initial voltage on the capacitor bank. E_T represents the total energy deposited in the wire as calculated from the voltage and current waveforms. E_L is the inductive energy stored in the magnetic field of the wire, and E_{MP} represented by the dashed lines is the required energy to bring the temperature of the wire from room temperature up to the melting point as calculated from handbook values. E_X is the excess energy put into the wire up to the time of the discontinuity and was calculated by taking the difference between the total energy, E_T, and the sum of the inductive energy, E_L, and the energy required to heat the wire to its melting point, E_{MP}, i.e.

$$E_X = E_T - (E_L + E_{MP})$$

A comparison of the excess energy E_X with the energy due to the Bridgman effect E_B shows reasonable agreement under the simplifying approximations made on the temperature changes in the wire. However, the agreement of the excess energy with that calculated for the Bridgman effect is not good with wires having thicker diameters.

DISCUSSION

The results of this study can be taken as an indication that the Bridgman emf may be important in accounting for the excess energy. However, the calculation of the Bridgman energy must be refined to eliminate the errors in the method of calculation. The linear approximations made on the temperature variation is the major source of error. This error can be greatly reduced by the use of a result obtained by Keilhacker[2] for the temperature of the wire as a function of the current, and time. The time rate of change of temperature can be calculated from his results and is given by

$$\frac{dT}{dt} = \frac{i^2}{\pi^2 a^4 \sigma^0 \rho \bar{c}} \exp\left[\frac{\alpha}{\pi^2 a^4 \rho \bar{c}} \int_0^t \frac{i^2}{\sigma} dt\right]$$

where α is the coeficient of linear expansion, σ the conductivity, ρ the density at room temperature for copper. The radius of the conductor is a, and $\bar{c}$ represents the average specific heat over the temperature range in question. This calculation shall be done at some future time.

The divergence of the theory from experimentally determined values of the excess energy calculated for thicker diameter wires is believed to be due to skin effects. The characteristic t, $\sigma \mu \sigma' a^2$ for the thicker wires is of the same order of magnitude as the period of the discharge circuit. Bennett[8] has shown that this is the condition for the transient skin effect to become an important factor in exploding wire phenomena. The skin effect causes a non-uniform distribution of the current density, and temperature gradients are established in the conductor which greatly complicates the situation.

The effect shows qualitative agreement with experiment, for it is very sensitive to the rate of admission of energy into the wire. This is born out by the fact that the excess energy increases with an increase in the initial voltage on the capacitor bank and a decrease in the circuit inductance.

The effect of changing these parameters is to increase the initial slope of the current rise, thus raising the power in the wire. The faster we apply energy to the wire in order to increase the rate of heating, the greater will be the energy expended in doing work against the Bridgman emf generated as a result of the rapidly changing temperature. This energy which is lost to the Bridgman emf cannot go into heating the wire and must be accounted for in a microscopic model of the conduction process.

The changing Hall coefficient with respect to temperature may provide the answer to the excess energy. The Hall coefficient is proportional to the reciprocal of the density of effective charge carriers. Therefore, the Bridgman emf could be the result of increasing the density of the effective charge carriers, and the excess energy goes into the creation of these charge carriers. The approach to the problem must lie in the application of the Boltzman equation to the nonequilibrium process of a rapid discharge of current through a conductor, under the restrictions of Fermi-Dirac statistics.

SUMMARY

The results of this paper indicate that the Bridgman effect is worthy of continued study and with refinement of the calculations may provide a means of calculating the amount of excess energy deposited in the wire. Under a more rigorous analysis, the effect may provide a description showing where the excess energy is utilized in exploding wire phenomena. These investigations are now in progress and the results of these studies will be made available when the work is completed.

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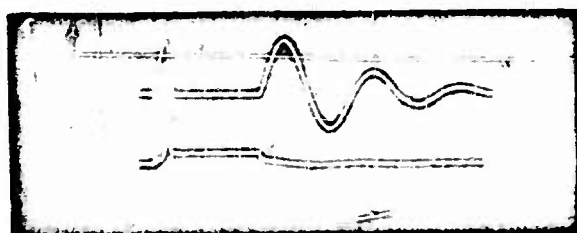
Figure 1. Current and Voltage for #28 B & S gauge Copper Wire
A - Vertical Deflection 28.7 KA/Div.
B - Vertical Deflection 9750 V/Div.
   Horizontal Deflection 5 μ Sec/Div.
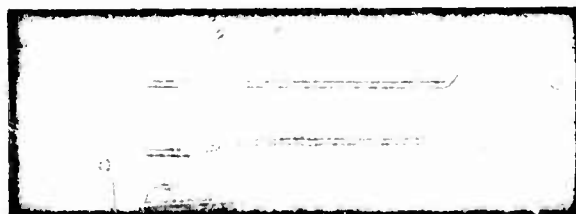
Figure 2. Current and Voltage for #28 B & S gauge Copper Wire
A - Vertical Deflection 28.7 KA/Div.
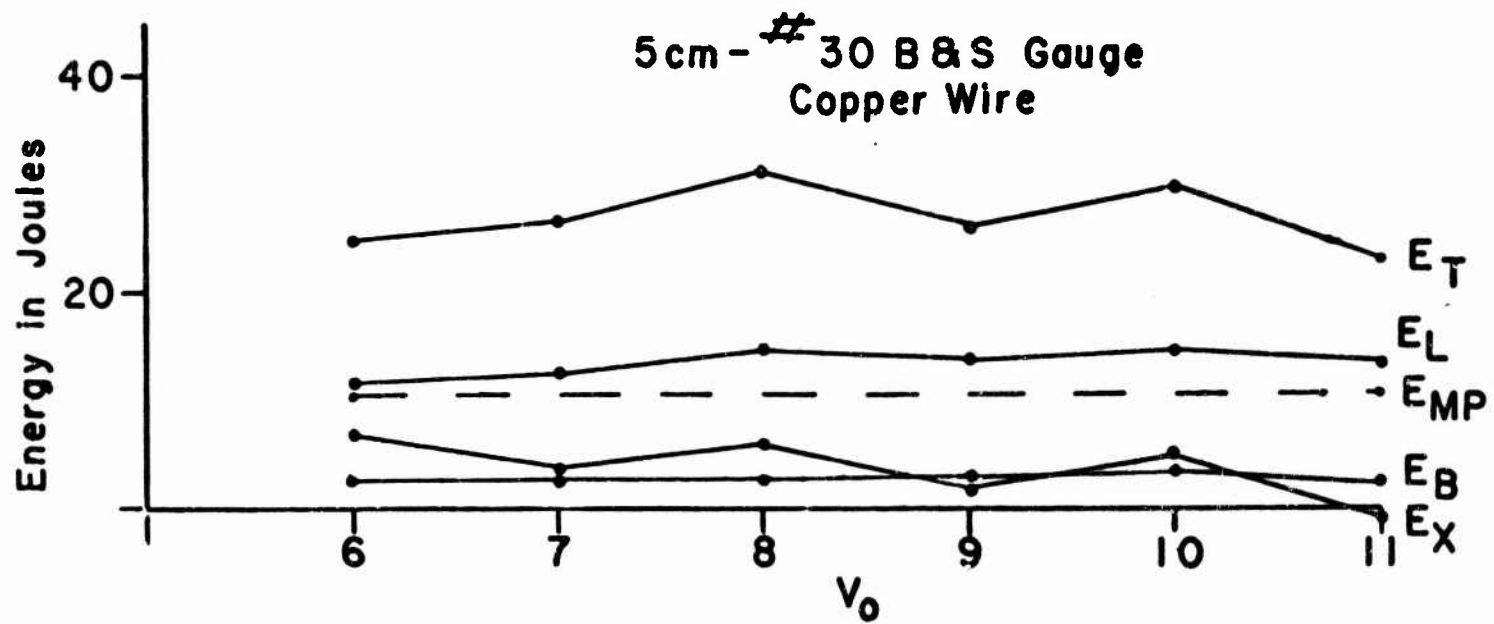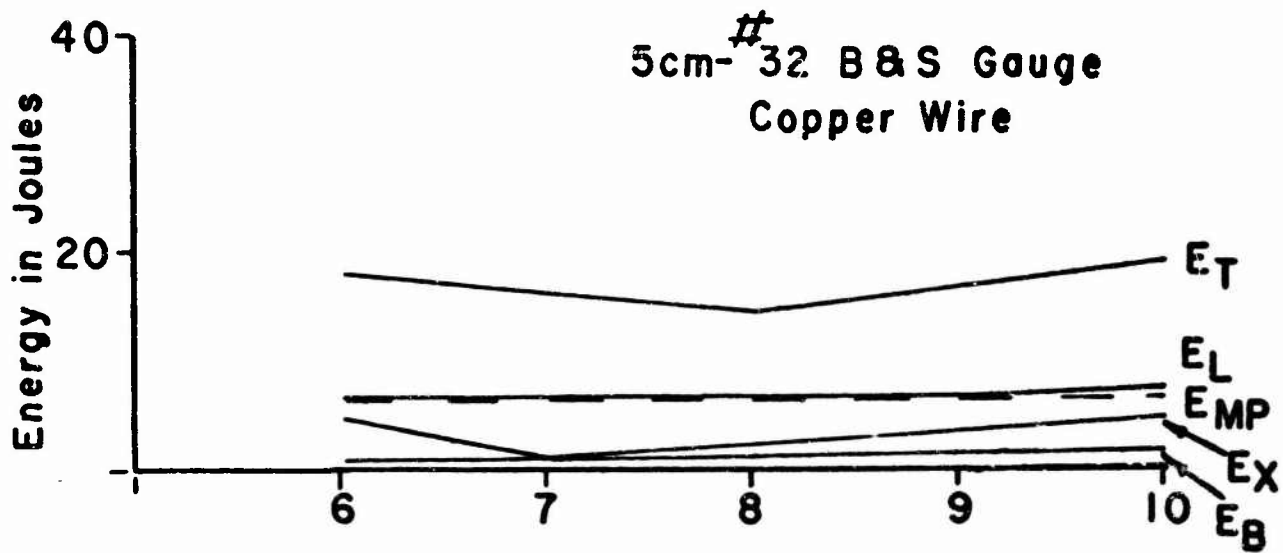B - Vertical Deflection 9750 V/Div.
   Horizontal Deflection 2 μ Sec/Div.

Figure 3