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Final Report

F-49495-8

## TECHNIQUES FOR DETERMINING INFLIGHT THRUST

by

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National Aviation Facilities Experimental Center

and

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36 p  
Ac - 2.00  
Ap - 1.50

January 1963

Prepared for

FEDERAL AVIATION AGENCY  
Systems Research and Development Service  
Experimentation Division  
Atlantic City, New Jersey

Contract FAA/BRD-288 Task 8

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**THE FRANKLIN INSTITUTE**  
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Final Report

F-A2495-8

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ABSTRACT

Methods of measuring the thrust of jet engines were surveyed to determine the state of the art of such measurements and their possible useful application as engine parameters for cockpit presentation. Some other systems for indicating engine performance were also considered but were not studied in detail because of their limitations. An analysis of the theory underlying some of the thrust measuring systems was made to determine if there were any inherent errors. Testing of completely integrated thrust measuring systems was recommended.

PURPOSE

The purpose of this study was to survey the state of the art of thrust measurements and their possible useful application as engine parameters for cockpit presentation. Specifically, the study consisted of a review of known proposed methods of thrust measurement and a determination of limitations due to the underlying theoretical considerations. The program objectives were limited to subsonic, non-afterburning, air breathing jet engines.

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## INTRODUCTION

A reliable and consistent means of measuring inflight output of aircraft engines is required for an overall indication of engine health. Ideally, the measurement should be obtainable through all flight regimes and be continuously and immediately available. Initially a rather simple and highly reliable technique of utilizing engine RPM and exhaust gas temperature and later on the additional monitoring of engine pressure ratio and fuel flow satisfied these requirements. Current operational methods use these four parameters and related manufacturers' charts to determine engine performance and establish efficient operating conditions. These same parameters are employed by ground personnel and flight personnel prior to takeoff as a method of determining engine health and possible malfunctions. The evolution of the modern jet aircraft employing multiple engines with variable inlet and exit geometries and complex control mechanisms results in increased sensitivity of engine output to the effects of engine wear and dimensional changes of critical components.

The useful output of a turbojet engine is the momentum resulting from the discharge of the heated gases through the exhaust nozzle, or exhaust nozzles in the case of a complex engine system. This output is commonly termed thrust. Three significant thrust terms may be distinguished which are usually defined as follows:

Gross Thrust ( $F_g$ ) - The forward force or thrust produced by the momentum efflux of the gases leaving the exhaust nozzle;

Negative Thrust or Ram Drag ( $F_r$ ) - The rearward force or drag produced by the momentum influx of the air entering the diffuser;

Net Thrust ( $F_n$ ) - The net force or thrust (forward) acting on the engine, numerically equal to the difference between the gross thrust and ram drag.

The objectives of this program were (1) to conduct a literature survey, References 1 to 31, (2) to review available test data on existing experimental systems, (3) to compute the theoretical accuracies of several systems, and (4) to propose and recommend, if applicable, the use of, modifications to, and tests of a system as required which would be indicative of the present state of the art for determining engine health.

#### METHODS OF DETERMINING JET ENGINE PERFORMANCE

Methods of measuring and indicating to the pilot the thrust produced by jet engines during take-off, climb, and cruise operations have been the subject of much discussion and great debate. Several schemes have been tried, with more or less success, and many more have been proposed. Basically, these schemes can be grouped in the following categories:

##### 1. Rotor Speed - Turbine Temperature Relationship

This scheme relies primarily on engine rotor speed as a relative indicator of thrust output (Reference 1). Maximum thrust and other ratings were established at fixed rotor speeds for all flight conditions. This system is inadequate for high performance jet aircraft because of the extremely wide variations in thrust versus rotor speed, among engines of a given model, or for a given engine with operating time. Other disadvantages are (1) the low sensitivity of rotor speed as a thrust-indicating parameter (i.e., small change in rotor speed, near 100% RPM, for a large change in thrust at a given flight condition), (2) changes in rotor speed-thrust relationship resulting from an engine improvement program would obsolete the speed ratings and (3) the constant rotor speed rating allows the fuel control characteristics to dictate the thrust ratings at off-design flight conditions.

## 2. Reaction Forces at Engine Mount

Measurement of engine trunnion reaction forces has been proposed as a means for determining net thrust (Reference 32). Consider a jet engine which is mounted on an airplane in such a manner that it is constrained to move only along the thrust axis in relation to the airplane. Along the thrust axis, the jet engine is restrained from motion by an elastic member. Under steady state conditions, if the thrust axis is horizontal then the deflection of the elastic member is a measure of the net thrust. When either acceleration or deceleration of the airplane is taking place or when the thrust axis is not horizontal, the inertia of the engine mass will lead to erroneous results. If  $\ddot{x}$  is the acceleration of an airplane along the thrust axis, then the indicated thrust will be  $F_n + m\ddot{x}$ . Similarly, if the thrust axis makes an angle  $\theta$  with the horizontal while the airplane is in a climb, then indicated thrust will be  $F_n - mg \sin \theta$  and in a dive it will be  $F_n + mg \sin \theta$ . The combinaterial error can be written as  $F_n + m(\ddot{x} \pm g \sin \theta)$ . The quantity in parentheses can be sensed by an accelerometer aligned to read along the thrust axis and this can be used to compensate the net thrust value indicated by the elastic restraint.

That the magnitude of the error introduced by the engine mass is significant can be seen from the following: The Rolls Royce Spey Engine weighs 2200 lb and provides a thrust of 9850 lb at takeoff. If the airplane assumes a  $20^\circ$  attitude from the horizontal at takeoff, the indicated thrust will be  $(9850 - \sin 20^\circ \times 2200) = 9150$ . Consequently, the error is 7.6%. In addition, forces on the inlet cowling, inlet ducts, and tail pipe nozzle which can be appreciable are not always accounted for in the measurement at the engine trunnions.

## 3. Engine Pressure Ratio

The current EPR method is based on the use of manufacturers' engine performance charts along with inflight measurement of diffuser and nozzle (fixed area) pressure, exhaust gas temperature (EGT), and

engine rotational speed (Reference 1 to 3). Engine performance charts, for the various engine models, are obtained from results of sea level performance on static test stands with extrapolation of performance to other altitudes and ram pressures.

The most common engine setting problems that the pilot must handle is that of setting

- (1) Take-off thrust
- (2) Climb thrust
- (3) Cruise thrust
  - a. Maximum continuous thrust
  - b. Some percentage of maximum continuous thrust.

At present the pilot must enter a chart with values of Mach number, ambient temperature, and pressure to find out what EPR to set up. Also, in many cases, corrections must be made for airbleed, inlet duct loss, and water injection.

#### 4. Gross Thrust

Proposed thrust measuring systems are essentially in the form of analog computers with various inputs from the engine (References 4-14). The inputs may include some or all of the following:

- a. Total pressure at turbine outlet
- b. Static pressure at turbine outlet
- c. Ambient pressure
- d. Nozzle area.

The equations used in the computer design may be either theoretically exact or of a simplified form, the latter being more common. Underlying theory of thrust measurement will be discussed subsequently. The computer output is displayed on an indicator in the cockpit. Some systems provide a digital display.

5. Net Thrust

Net thrust is usually computed as the numerical difference between gross thrust and ram drag (References 8-11, 15). Consequently, in addition to the inputs needed for gross thrust measurement, some or all of the following inputs are required for computing ram drag.

- a. Total pressure at compressor inlet
- b. Static pressure at compressor inlet
- c. Ambient pressure
- d. Diffuser inlet area.

Output of the computer may also be displayed in a digital form as net thrust.

6. Review of Test Data

A review of the available test results (References 16 to 22) indicate that none of the experimental hardware investigated represented an integrated system containing probes, computers, and display units. The operational mode for the hardware tested included non-afterburning and afterburning engines but did not include multiple geometry engines. Further, the references cited above did not contain simultaneous testing of various systems on the same engine to permit comparisons.

THEORY

1. Basic Considerations

From considerations of rate of change of momentum with respect to time, Reference 33 indicates that net thrust developed by a turbojet engine may be expressed as follows:

$$F_n = \frac{W_e V_f}{g} - \frac{W_o V_o}{g} \quad (1)$$

where  $W_o$  and  $W_i$  are the weight flow rates of exhaust gases and intake air respectively and  $g$  is the acceleration due to gravity.  $V$  is the gas velocity with the subscripts denoting the following: "o" refers to free stream conditions and "f" refers to the section where the pressure of the engine exhaust gases is first equal to the ambient pressure. In equation 1, it may be noted that the difference between the weight-flow rates of the inlet and exhaust gases represents the weight-flow rate of the fuel. In most engines, this difference is small and amounts to about 2%.

The two terms contained in the equation for net thrust are commonly referred to by specific names. First, there is negative thrust or ram drag,  $F_r$ . It is defined as

$$F_r = \frac{W_o V_o}{g} \quad (2)$$

Secondly, the remaining term in the net thrust equation is called gross thrust,  $F_g$ . It is defined as

$$F_g = \frac{W_e V_f}{g} \quad (3)$$

Purely from practical considerations, in evaluating gross thrust, measurements would need to be made either in the engine itself or close to it. Consequently, it is desirable to express the gross thrust in terms of conditions at the engine exit section, labelled as station 6 in Fig. 1. The pressure at station 6 can be greater than the ambient pressure, and when this is so, gases at section 6 will be further accelerated. Thus, equation 3 may be written

$$F_g = \frac{W_e V_6}{g} + A_6 (P_6 - P_o) \quad (4)$$

where  $A_6$  is the flow area at station 6. The effective exhaust velocity,  $V_f$ , in equation 3 is

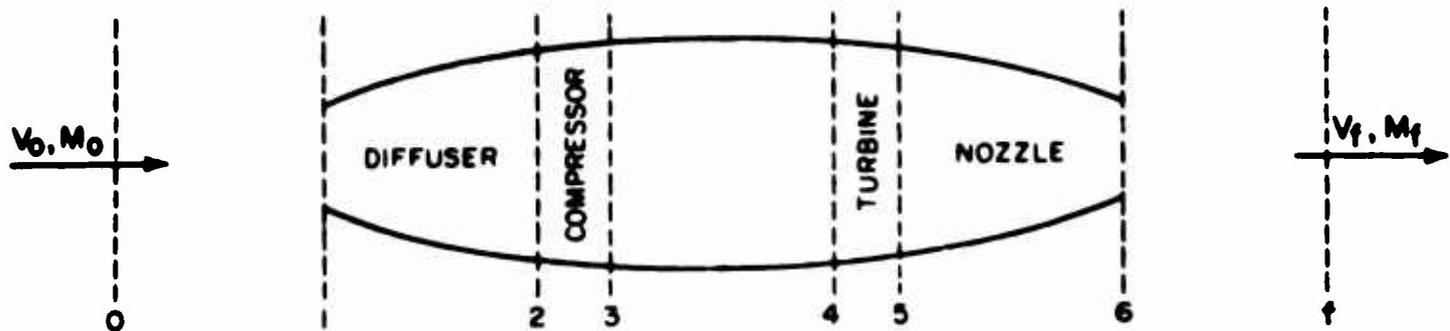


FIG. 1. TURBOJET STATION DESIGNATION

$$V_f = V_6 + \frac{gA_6}{W_e} (P_6 - P_0) \quad (5)$$

Equation 5 shows that the effective exhaust velocity is equal to the exit velocity when there is no pressure unbalance at the rear of the engine. If a pressure unbalance does exist, the effective exhaust velocity is greater than the actual exit velocity.

Either of the equations 3 and 4 may be used to evaluate gross thrust. Experimental evidence indicates that both yield satisfactory results (Reference 34).

It is desirable to express the foregoing thrust equations in terms of gas properties which can be measured. It will be demonstrated that either gross thrust or ram drag can be expressed as a function of pressure ratios, of an area and of some constants. If it were possible to measure the actual values of the pressures, areas and constants, exact values of thrust can be determined. However, it is impractical to do so, and, correction or calibration factors are used to modify the equations. Moreover, in some engines, the throat area of the nozzle and the diffuser inlet area are variable. In such cases, area changes are sometimes measured by a transducer for use in the equations.

The theoretical analysis given here has a twofold objective: First, to minimize the number of measured parameters required and second to eliminate the need for transducing variable areas. The analysis follows:

2. Ram Drag

Let station 1 and 2 identify diffuser and compressor inlet conditions respectively. Objective of the analysis will be to express ram drag in terms of conditions at station 2. Referring to equation 2,

$$W_o = V_2 A_2 \rho_2 \quad (6)$$

where  $V_2$  is the velocity,  $A_2$  is the flow area and  $\rho_2$  is the density at station 2. The velocity at station 2 may be expressed as (Reference 35)

$$V_2 = \left\{ \frac{2\gamma_2 R_2 T_2}{\gamma_2 - 1} \left[ \left( \frac{P_{t2}}{P_2} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} - 1 \right] \right\}^{1/2} \quad (7)$$

where,  $\gamma_2$  is the ratio of specific heats,  $R_2$  is the gas constant and  $T_2$  is the absolute temperature.  $P_{t2}$  is the total pressure and  $P_2$  is the static pressure. Similarly, the density,  $\rho_2$ , is

$$\rho_2 = \frac{P_2}{R_2 T_2} \quad (8)$$

Equation 6 may now be expressed as

$$W_o = \frac{P_2 A_2}{R_2 T_2} \left\{ \frac{2\gamma_2 R_2 T_2}{\gamma_2 - 1} \left[ \left( \frac{P_{t2}}{P_2} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} - 1 \right] \right\}^{1/2} \quad (9)$$

The free stream velocity,  $V_o$ , may be expressed as (Reference 35)

$$V_o = \left\{ \frac{2\gamma_2 R_2 T_o}{\gamma_2 - 1} \left[ \left( \frac{P_{t2}}{P_o} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} - 1 \right] \right\}^{1/2} \quad (10)$$

Equation 2 may now be modified by equation 9 and 10 as follows:

$$F_r = \frac{2\gamma_2 A_2 P_2}{(\gamma_2 - 1)} \left\{ \frac{T_o}{T_2} \left[ \left( \frac{P_{t2}}{P_2} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} - 1 \right] \left[ \left( \frac{P_{t2}}{P_o} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \right\}^{1/2} \quad (11)$$

Moreover,

$$\frac{T_{t2}}{T_2} = \left( \frac{P_{t2}}{P_2} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} \quad (12)$$

$$\frac{T_{t2}}{T_o} = \left( \frac{P_{t2}}{P_o} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} \quad (13)$$

Since flow in a jet engine is close to adiabatic and not isentropic,

$$P_{t2} = \eta_r P_{t2} \quad (14)$$

where  $\eta_r$  is the ram recovery factor. But,

$$T_{t2} = T_{t0} \quad (15)$$

Consequently,

$$\frac{T_0}{T_2} = \eta_r \left( \frac{P_0}{P_2} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} \quad (16)$$

Substituting equations 14 and 15 into equation 11 and non-dimensionalizing, the following is obtained for flight velocities at or below sonic velocity:

$$\frac{F_r}{P_0 A_2} = \frac{2\gamma_2}{\gamma_2 - 1} \left\{ \eta_r \left( \frac{P_2}{P_0} \right)^{\frac{1+\gamma_2}{\gamma_2}} \left[ \left( \frac{P_{t2}}{P_2} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} - 1 \right] \left[ \left( \frac{1}{\eta_r} \frac{P_{t2}}{P_0} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} - 1 \right] \right\}^{1/2} \quad (17)$$

### 3. Gross Thrust

Superscripts 5 and 6 identify turbine outlet and nozzle exit conditions respectively. Objective of the following analysis is to express gross thrust in terms of conditions at station 5.

Referring to equation 3,  $W_e$  will be computed on the basis of gas conditions and area of section 5.  $V_f$  will be computed for free stream conditions on the basis of complete expansion.

$$F_g = \frac{W_e V_f}{g} \quad (18)$$

$$V_f = M_f a_f \quad (19)$$

where  $M_f$  is the Mach number and  $a_f$  is the velocity of sound. Moreover,

$$M_f = \left\{ \frac{2}{\gamma_5 - 1} \left[ \left( \frac{P_{tf}}{P_f} \right)^{\frac{\gamma_5 - 1}{\gamma_5}} - 1 \right] \right\}^{1/2} \quad (20)$$

But

$$P_{tf} = \eta_n P_{t5}$$

$$P_f = P_o \quad (21)$$

where  $\eta_n$  is the nozzle adiabatic efficiency.

$$\frac{T_{tf}}{T_f} = \left( \frac{P_{tf}}{P_f} \right)^{\frac{\gamma_5 - 1}{\gamma_5}} \quad (22)$$

$$T_{tf} = T_{t5} \quad (23)$$

Consequently,

$$T_f = T_5 \left( \frac{P_o}{\eta_n P_5} \right)^{\frac{\gamma_5 - 1}{\gamma_5}} \quad (24)$$

Therefore,

$$a_f = \sqrt{g\gamma_5 R_5 T_5} = \left[ g\gamma_5 R_5 T_5 \left( \frac{P_0}{\eta_n P_5} \right)^{\frac{\gamma_5-1}{\gamma_5}} \right]^{1/2} \quad (25)$$

Substituting for  $M_f$  and  $a_f$  from equation 20 and 25 into 19, one obtains

$$V_f = \left\{ \frac{2g\gamma_5 R_5 T_5}{\gamma_5 - 1} \left( \frac{P_0}{\eta_n P_5} \right)^{\frac{\gamma_5-1}{\gamma_5}} \left[ \left( \frac{\eta_n P_{t5}}{P_0} \right)^{\frac{\gamma_5-1}{\gamma_5}} - 1 \right] \right\}^{1/2} \quad (26)$$

Moreover,

$$\frac{W}{g} = A_5 V_5 \rho_5 \quad (27)$$

But,

$$V_5 = M_5 a_5 \quad (28)$$

and

$$\rho_5 = \frac{P_5}{g R_5 T_5} \quad (29)$$

where

$$M_5 = \left\{ \frac{2}{\gamma_5-1} \left[ \left( \frac{P_{t5}}{P_5} \right)^{\frac{\gamma_5-1}{\gamma_5}} - 1 \right] \right\}^{1/2} \quad (30)$$

$$a_5 = \sqrt{g\gamma_5 R_5 T_5} \quad (31)$$

Consequently,

$$\frac{W}{g} = A_5 P_5 \left\{ \frac{1}{gR_5 T_5} \frac{2\gamma_5}{\gamma_5 - 1} \left[ \left( \frac{P_{t5}}{P_5} \right)^{\frac{\gamma_5 - 1}{\gamma_5}} - 1 \right] \right\}^{1/2} \quad (32)$$

Substituting equations 26 and 32 into equation 18 and non-dimensionalizing,

$$\frac{F_G}{A_5 P_0} = \eta_n \frac{1 - \gamma_5}{\gamma_5} \frac{2\gamma_5}{\gamma_5 - 1} \left( \frac{P_5}{P_0} \right)^{\frac{\gamma_5 - 1}{2\gamma_5}} \left\{ \left[ \left( \frac{\eta_n P_{t5}}{P_0} \right)^{\frac{\gamma_5 - 1}{\gamma_5}} - 1 \right] \left[ \left( \frac{P_{t5}}{P_5} \right)^{\frac{\gamma_5 - 1}{\gamma_5}} - 1 \right] \right\}^{1/2} \quad (33)$$

In the foregoing, instead of using the nozzle adiabatic efficiency directly in the equations, a polytropic exponent  $n$  may be used in place of the isentropic exponent  $\gamma_5$ . The value of polytropic exponent  $n$ , however, must be determined from the nozzle adiabatic efficiency,  $\eta_n$ . It can be shown that the following relation holds (Reference 33):

$$\frac{1}{n} = 1 - \frac{\ln \left\{ 1 - \eta_n \left[ 1 - \left( \frac{P_0}{P_{t5}} \right)^{\frac{\gamma_5 - 1}{\gamma_5}} \right] \right\}}{\ln \left( \frac{P_0}{P_{t5}} \right)} \quad (34)$$

Under choke flow conditions, the exact expression for gross thrust is as follows:

$$\left[ \frac{F_G}{A_5 P_0} \right]_{\text{Choke}} = \left( \frac{P_5}{P_{t5}} \right)^{\frac{1}{\gamma_5}} \left( \frac{\gamma_5 + 1}{2} \right)^{\frac{1}{\gamma_5 - 1}} \left( \frac{\gamma_5 + 1}{\gamma_5 - 1} \right)^{\frac{1}{2}} \left[ 1 - \left( \frac{P_5}{P_{t5}} \right)^{\frac{\gamma_5 - 1}{\gamma_5}} \right]^{1/2} \times$$

$$\left\{ \left[ (\gamma_5 + 1) \left( \frac{2}{\gamma_5 + 1} \right)^{\frac{\gamma_5}{\gamma_5 - 1}} \left( \frac{P_{t5}}{P_0} \right) - 1 \right] \right\} \quad (35)$$

Use of either equation 34 or 35 in a system depends on a knowledge of the area,  $A_5$ . In general, the geometric area of the section will not represent the actual flow area. In order to compensate for this effect some form of a calibration factor would be necessary in practice.

#### ERROR ANALYSIS

##### 1. General Considerations

Errors in measuring ram drag, or gross thrust can arise from various sources. In the exact equations,  $\gamma$  and  $\eta$  are usually assumed to be constant whereas in practice they are variable. Moreover, the exact equation may be simplified for reducing the complexity of the computing system by assuming choke flow. Some of these errors will be evaluated on the basis of the following equations:

##### 2. Gross Thrust

The polytropic exponent for deviation from isentropic flow is obtained from equation 34.

The exact equation used for gross thrust is,

$$B = \frac{F_G}{A_5 P_0} = \frac{2n}{n-1} \left( \frac{P_5}{P_0} \right)^{\frac{n+1}{2n}} \sqrt{\left[ \left( \frac{P_{t5}}{P_0} \right)^{\frac{n+1}{n}} - 1 \right] \left[ \left( \frac{P_{t5}}{P_5} \right)^{\frac{n-1}{n}} - 1 \right]}$$

The choke flow equation is,

$$H = \left[ \frac{F_G}{A_5 P_0} \right]_{\text{Choke}} = \left( \frac{P_5}{P_{t5}} \right)^{\frac{1}{\gamma}} \left( \frac{\gamma+1}{2} \right)^{\frac{1}{\gamma-1}} \left( \frac{\gamma+1}{\gamma-1} \right)^{\frac{1}{2}} \left[ 1 - \left( \frac{P_5}{P_{t5}} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{2}} \left\{ (\gamma+1) \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \left( \frac{P_{t5}}{P_0} \right) - 1 \right\}$$

The linearized equation is,

$$J = \left[ \frac{F_G}{A_5 P_0} \right]_{\text{Linear}} = \left\{ \left( \frac{P_5}{P_{t5}} \right)^{\frac{1}{n}} \left( \frac{n+1}{2} \right)^{\frac{1}{n-1}} \left( \frac{n+1}{n-1} \right)^{\frac{1}{2}} \left[ 1 - \left( \frac{P_5}{P_{t5}} \right)^{\frac{n-1}{n}} \right]^{\frac{1}{2}} \right\} \left\{ C \left( \frac{P_{t5}}{P_0} - D \right) \right\}$$

where C = 0.9 and D = 0.808.

### 3. Ram Drag

Ram drag is calculated from the following expression:

$$R = \left[ \frac{F_r}{P_0 A_2} \right] = \frac{2\gamma}{\gamma-1} \sqrt{\eta_r^{\frac{\gamma-1}{\gamma}} \left( \frac{P_2}{P_0} \right)^{\frac{1+\gamma}{\gamma}} \left[ \left( \frac{P_{t2}}{P_2} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \left[ \left( \frac{1}{\eta_r} \frac{P_{t2}}{P_2} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}$$

Ram drag based on choke flow at the diffuser is calculated from the following expression:

$$S = \left[ \frac{F}{P_o A_2} \right]_{\text{choke}} = \frac{2\gamma}{(\gamma^2 - 1)^{1/2}} \left( \frac{P_2}{P_o} \right) \frac{\left[ 1 - \left( \frac{P_2}{P_{t2}} \right)^{\frac{\gamma-1}{\gamma}} \right]^{1/2}}{\left( \frac{P_2}{P_{t2}} \right)^{\frac{\gamma-1}{\gamma}}}$$

4. Results

The foregoing expressions were programmed on a Univac I computer. The results listed in the following tables represent the range of variables used in the computations. Discussion of these results is presented in the next section.

Table 1

GROSS THRUST VARIATION WITH  $\gamma$

$\gamma$	$\eta$	$\frac{P_{t5}}{P_o}$	$\frac{P_{t5}}{P_5}$	$\underline{B}$	$\underline{H}$	$\underline{J}$	
1.30	1	1.666	1.25	0.9089	0.90908	0.6434	Sub Critical
1.34	1	1.666	1.25	0.9089	0.910045	0.639031	
1.36	1	1.666	1.25	0.9102	0.910515	0.63691	
1.30	1	2.000	1.42857	1.42377	1.42183	1.01356	Super Critical
1.34	1	2.000	1.42857	1.42706	1.42698	1.00671	
1.36	1	2.000	1.42857	1.42864	1.42858	1.00448	

Table 2

GROSS THRUST VARIATION WITH DEVIATION FROM ISENTROPICITY

$\eta$	$\gamma$	$\frac{P_{t5}}{P_o}$	$\frac{P_{t5}}{P_5}$	$\underline{B}$	$\underline{H}$	$\underline{J}$	
1	1.34	1.666	1.25	0.909803	0.910095	0.639031	
.95	1.34	1.666	1.25	.909277	0.90967	0.6416	
.9	1.34	1.666	1.25	.908757	0.908917	0.644109	
.85	1.34	1.666	1.25	.90824	0.908371	0.64656	Sub Critical
1	1.34	2.000	1.42857	1.42706	1.42698	1.00671	
.95	1.34	2.000	1.42857	1.42508	1.42497	1.00944	
.9	1.34	2.000	1.42857	1.42313	1.42298	1.01207	
.85	1.34	2.000	1.42857	1.42121	1.42101	1.01459	Super Critical

Table 3

GROSS THRUST VARIATION WITH PRESSURE RATIO

$\frac{P_{t5}}{P_o}$	$\frac{P_{t5}}{P_5}$	$\gamma$	$\eta$	$\underline{B}$	$\underline{H}$	$\underline{J}$
1.42857	1.25	1.34	1	0.657914	0.6619	0.461837
1.666	1.25	1.34	1	0.909803	0.910045	0.639031
2.00	1.25	1.34	1	1.25750	1.25743	0.887102
2.5	1.25	1.34	1	1.78273	1.77852	1.25920
3.33	1.25	1.34	1	2.67760	2.64699	1.87938

Table 4

VARIATION OF RAM DRAG WITH  $\gamma$

$\gamma$	$\eta_n$	$\frac{P_o}{P_{t2}}$	$\frac{P_2}{P_{t2}}$	$\underline{R}$	$\underline{S}$
1.382	1	0.625065	0.7	0.956293	1.11222
1.406	1	0.621818	0.7	0.968086	1.13744
1.418	1	0.620215	0.7	0.973939	1.14502
1.382	1	0.531332	0.7	1.29086	1.29086
1.406	1	0.527273	0.7	1.30983	1.30983
1.418	1	0.525268	0.7	1.31928	1.31928

Table 5

VARIATION OF RAM DRAG WITH  $\eta_r$

$\eta_r$	$\gamma$	$\frac{P_o}{P_{t2}}$	$\frac{P_2}{P_{t2}}$	$\underline{R}$	$\underline{S}$
1	1.406	0.6218	0.7	0.968086	1.13744
0.96	1.406	0.639396	0.7	0.953348	1.11063
0.84	1.406	0.6277	0.7	1.10026	1.112831

Table 6

VARIATION OF RAM DRAG EXPRESSIONS IN THE FLIGHT MACH NUMBER

$\underline{M_o}$	$\underline{M_2}$	$\underline{A_o/A_2}$	$\underline{R}$	$\underline{S}$
1	0.453	0.8	1.11	1.11
0.3	0.450	0.8	0.91	0.997
0.8	0.440	0.8	0.71	0.854
0.7	0.405	0.8	0.55	0.749
0.6	0.370	0.8	0.40	1.595
0.5	0.325	0.8	0.28	0.523
0.4	0.27	0.8	0.18	0.611
0.3	0.207	0.8	0.102	0.312
0.2	1.137	0.8	0.044	0.203
0.1	0.073	0.8	0.011	0.104
0	0		0	0

DISCUSSION

1. EPR

The engine pressure ratio system is based on the use of manufacturer's engine performance charts along with inflight measurements of ram and nozzle pressures, tailpipe temperature and engine rotational speed. Use of charts for determining inflight thrust, in general, involves errors because of the extrapolation of static performance, which is usually obtained at sea level pressures, to other altitudes and ram pressures. Moreover, engine installation may differ from that used in calibration in regard to tailpipe length, exit area, kinking, etc.

In general, errors in EPR indication may arise from the following sources: Probes, display units, and differences in engine installation. Besides these, the pilot may introduce additional errors in reading the display units and the charts. Magnitude of these errors are difficult to estimate at the present time.

## 2. Thrust

Besides errors arising from variation in such assumed constants as  $\gamma$  and  $\eta$ , inaccuracies may arise from the probes and also the display unit. Moreover, the exact theoretical equations require complex computing systems; consequently, the equations are simplified for facilitating computer design and thus introduce other errors.

## 3. Ram Drag

In the ram drag error analysis, for a  $\gamma$  range of 1.38 to 1.42, at a Mach number of 0.85, the error was less than .5%. For a  $\eta$  range of 1 to 0.9, at the same Mach number, the error was again of the same magnitude. Ram drag was computed on the basis of choke flow at the inlet for a flight Mach number range of 0 to 1. At low flight velocities, choke flow assumption resulted in extremely large errors (Table 6). Since flight velocity is low during the critical period of take-off, net thrust computed from a choke-flow ram drag value, though conservative, will be extremely erroneous. Consequently, it cannot be considered to be a desirable approach.

## 4. Gross Thrust

Gross thrust was computed for a  $P_0/P_{t5}$  range of 0.3 to 0.7 from exact equations, from expressions based on choke flow, and also from linearization. In the subcritical range, for a  $\gamma$  variation of 1.30 to 1.36, the exact equation as well as the choke-flow expression showed a variation of only 0.1% whereas the linearized equation showed a variation of 0.7%. Similarly, for a  $\eta$  range of 1 to 0.85, the variation in the exact and choke-flow expressions was only 0.4% and, for the linearized

expression the variation was 1%. Under supercritical operation, the variation was less than 0.4% for a  $\gamma$  range of 1.30 to 1.36 and a  $\eta$  range of 1 to 0.85. In the same ranges, the linearized expression showed less than 1% variation. Consequently, changes in  $\gamma$  and  $\eta$  are not a major source of inaccuracy in computing thrust from measured values of total and static pressures.

In the investigated ranges of  $\gamma$  from 1.30 to 1.36,  $\eta$  from 1 to 0.85 and  $P_0/P_{t5}$  from 0.7 to 0.3, the isentropic choke-flow equation differed from the exact equation by less than 1.8%. The linearized expression, however, showed a variation in error of 2%. Selection of proper constants for the linearized expression will reduce this error considerably. Consequently, error from this source also is not a major consideration with the present state-of-the-art of thrust measurement. Thus, it may be concluded that limitations in the theory alone are not a stumbling block in the development of adequate thrust measurement systems.

#### 5. Net Thrust

Since net thrust is to be evaluated from equation 1, which shows it as the numerical difference between gross thrust and ram drag, it is subject to errors of a cumulative nature. A high degree of accuracy would be needed in the computed values of gross thrust and ram drag to produce adequate accuracy in net thrust indication.

In a completely integrated thrust measurement system, present state-of-the-art permits an accuracy of  $\pm 2\%$  or better in the sensors. Computer and display unit errors are yet unestablished. Moreover, some manner of accounting for the aerodynamic areas will be needed. Consequently, in order to obtain an overall accuracy of  $\pm 2\%$  in indicated thrust, some form of calibration is definitely needed. This would, in all likelihood, have to be conducted on an actual airplane installation rather than on a test cell installation because differences can be expected between the two in regard to tailpipe length, exit area, etc.

6. Other Parameters

Some form of thrust indication in the cockpit would result in a number of benefits.

It would be a good aid in checking engine performance before and during take-off, and inflight; facilitate trimming multiengine aircraft for zero asymmetric thrust and in setting schedules for maximum performance flights.

At the present time, a pilot monitors four parameters, viz., EPR, EGT, fuel flow and RPM. Instead of four parameters, if a pilot monitors only one instrument per engine, so that in a four engined plane, he is monitoring four rather than sixteen parameters, his work load will be considerably reduced. Moreover, he will function much more effectively, especially during the critical period of take-off. However, neither gross thrust nor net thrust alone can accomplish this. Gross thrust increases with flight velocity and a pilot would notice an increase during take-off and he needs to watch the air speed indicator also. Net thrust, like EPR, decreases with velocity, so he has to once again monitor air speed for establishing engine performance. Human factors requirements, such as these, have not been sufficiently emphasized in past studies. Instead of net thrust or gross thrust, it is possible to choose among other parameters, namely, percent maximum net thrust, percent available net thrust, percent maximum gross thrust, or percent available gross thrust. A detailed study on parametric evaluation seems to be still desirable. This study may include a detailed analysis of the extent to which each of the parameters would fulfill the requirements of the engine and airplane manufacturers, the airline operators, the pilot and the FAA. A start has been made along these lines in reference 1. Further effort would be highly useful.

### CONCLUSIONS

Based on the preceding analysis and discussion, it is concluded that:

1. The use of rotor speed and exhaust gas temperature as the sole performance indicator is inadequate.
2. Practical considerations preclude trunnion reaction measurements for indicating engine performance.
3. Engine pressure ratio, while a measurement of gas generator performance, is not necessarily indicative of the useful power output of a turbojet engine.
4. Thrust is a useful indicator of engine power plant performance. However, it is yet to be determined, whether measurements, computation and display of net thrust, gross thrust or some other thrust parameter would be most useful.
5. Deviation from isentropic flow and variations in the ratio of specific heats do not place a serious limitation on the theory of thrust measurement.
6. The literature search indicates that a comprehensive experimental program of evaluation of a complete system including all components is yet to be accomplished.

### RECOMMENDATIONS

Based on the preceding conclusions, it is recommended that one or more thrust measurement systems be experimentally evaluated on both fixed and variable geometry engines. These tests should include all system components, such as probes, computers, and display units. Furthermore, the systems should be tested simultaneously and on the same engine where possible.

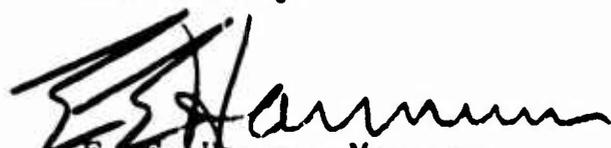
Following the laboratory tests, reliability of the resulting prototype hardware should be established by flight testing.

ACKNOWLEDGEMENTS

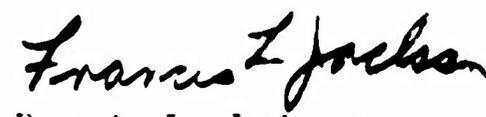
The author wishes to acknowledge the valuable assistance of Howard R. Breuer who reviewed the literature and of Dr. Mullapudi Reddi who conducted the error analysis and contributed the chapter on theory. The comments and suggestions by Mr. Jack J. Shrager, our technical supervisor, were most helpful in the performance of this project and the writing of this report.

  
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SYMBOLS

- $A_x$  = Area at station x ( $\text{ft}^2$ )  
 $F_g$  = Gross Thrust (lb)  
 $F_n$  = Net Thrust (lb)  
 $F_r$  = Ram Drag (lb)  
 $g$  = Acceleration due to gravity ( $\text{ft sec}^{-2}$ )  
 $M_x$  = Mach number at station x  
 $n$  = Polytropic exponent  
 $P_{tx}$  = Total pressure at station x ( $\text{lb ft}^{-2}$ )  
 $P_x$  = Static pressure at station x ( $\text{lb ft}^{-2}$ )  
 $R_x$  = Gas Constant at Station x ( $\text{ft lb/lb}^\circ\text{R}$ )  
 $T_{tx}$  = Total Temperature at Station x ( $^\circ\text{R}$ )  
 $T_x$  = Static Temperature at Station x ( $^\circ\text{R}$ )  
 $V_f$  = Free stream velocity of exhaust jet ( $\text{ft sec}^{-1}$ )  
 $V_o$  = Airplane velocity ( $\text{ft sec}^{-1}$ )  
 $W_e$  = Weight flow rate of exhaust gases ( $\text{lb sec}^{-1}$ )  
 $W_o$  = Weight flow rate of intake air ( $\text{lb sec}^{-1}$ )  
 $\gamma_x$  = Ratio of specific heats at station x  
 $\eta_n$  = Nozzle adiabatic efficiency  
 $\eta_r$  = Ram recovery factor  
 $\rho_x$  = Density

APPENDIX

PROPOSED TEST PLAN

The purpose of the test program outlined here is to evaluate the performance of thrust measuring systems in the laboratory. To accomplish this objective, various thrust measuring systems should be evaluated on a test stand to simulate conditions encountered throughout the flight regime. Tests should be performed as follows:

Engines Both fixed and variable geometry non-afterburning engines should be used.

Systems All available thrust measuring systems should be tested including all of their components such as probes, computers, and display units.

Controls To achieve results which can be compared meaningfully, all of the units should be tested simultaneously and on the same engine if at all possible. In addition, temperatures and pressures at each station should be measured and recorded independently. In case of the variable geometry engine, the changeable areas should also be measured.

Simulated flight conditions The following conditions should be simulated  
take off (sea level to 10,000 feet, up to 120°F)  
climb out  
cruise (sea level to 50,000 feet, changing velocity ranges up to  
.92 Mach Number  
descent