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VIBRATIONAL RELAXATION OF NITROGEN IN THE NOL HYPERSONIC TUNNEL #4

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VIBRATIONAL RELAXATION OF NITROGEN IN THE NOL HYPERSONIC TUNNEL NO. 4

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ABSTRACT: The problem of vibrational relaxation of nitrogen in Hypersonic Tunnel No. 4 at the U. S. Naval Ordnance Laboratory has been analyzed for supply conditions of 3600°R, 3960°R, 5400°R, and pressures of 80 and 100 atmospheres. A comparison between equilibrium, relaxing, and frozen flows has been made for flow in the nozzle and around a pitot probe placed in the flow. The results are shown as plots of static pressure, static temperature, free stream velocity, and stagnation pressure ratio versus nozzle area ratio. It has been found that the flow freezes at very short distances downstream of the throat. When plotted versus nozzle area ratio, the ratio of impact to stagnation pressure gives identical results in the relaxing and frozen flow cases. The authors recommend that frozen flow be used as a convenient and reasonably accurate assumption for determining the test section conditions in NOL Hypersonic Tunnel No. 4.

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Vibrational Relaxation of Nitrogen in the NOL Hypersonic Tunnel No. 4

This is a report of a study undertaken to compare the equilibrium, relaxing, and frozen flow in a conical nozzle of Hypersonic Tunnel No. 4 at the U. S. Naval Ordnance Laboratory. Of particular interest was the interpretation of the pressure recorded by a pitot probe placed in the flow.

The authors wish to express their gratitude to Mrs. Carolyn Piper for her many trips to the computer room and her patience in plotting the graphs.

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Captain, USN
Commander

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By direction

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ILLUSTRATIONS

SYMBOLS

A	area of nozzle section, ft^2
a	speed of sound, ft/sec
B	defined in equation (10)
C	thermal speed of molecule, ft/sec
c	average speed of a molecule, ft/sec
C_p	specific heat at constant pressure
E	internal energy of a gas
H	enthalpy
h	Planck's constant
k	Boltzmann's constant
L	mean free path, ft
M	Mach number
N	number of collisions required for equilibrium
P	pressure, atmospheres
R	universal gas constant, 1776 ft/lb/slug^{o}R
S	entropy
T	temperature, ^{o}R
t	time, sec
u	velocity, ft/sec
x	distance from nozzle throat downstream, ft or in as noted
Δ	standoff distance between shock and tip of pitot probe
γ	ratio of specific heats
μ	coefficient of viscosity, slug/ft/sec
ν	vibrational frequency

θ_v	characteristic vibrational temperature, ^oR
ρ	density, lb/ft^3
τ	relaxation time, sec

Subscripts

1	indicates conditions before shock wave
2	indicates conditions after shock wave
F	pertains to frozen flow
o	stagnation or total
r	pertains to rotational mode
t	pertains to translational mode
v	pertains to vibrational mode
*	throat conditions

INTRODUCTION

Hypersonic Tunnel No. 4 at the U. S. Naval Ordnance Laboratory has recently been modified by the addition of a graphite heater. This tunnel operates at a typical supply condition of 4000°R and 100 atmospheres, achieving Mach numbers above 15. It uses nitrogen rather than air as the test gas.

At the above supply condition a considerable amount of the internal energy of the gas is found in the vibrational energy state. Figure 1 shows the percentage of the internal energy found in the vibrational state as a function of temperature. It also shows the vibrational energy as a percentage of the enthalpy. These curves are obtained from equations to be given later in this report.

Unlike the translational and rotational energy states, this vibrational energy requires a considerable number of intermolecular collisions for adjustment to any new state. In this process of adjustment, one can consider two limiting cases: equilibrium flow and frozen flow. In the former the energy in the vibrational degrees of freedom depends only on the translational temperature. In the latter, the energy is locked in the vibrational state and does not change.

The purpose of this report is to consider some effects of vibrational energy on the flow in our hypersonic test facility. We have analyzed the two limiting cases outlined above. We have also considered the gas to be relaxing, that is, adjusting its vibrational energy at a finite rate. In the first part the nozzle flow is considered. The second part discusses the flow around a pitot probe placed in the flow, and finally the results of the first two parts are combined to give useful information on the interpretation of experimental pitot pressure results in Tunnel No. 4.

We have neglected ionization and dissociation since these are not expected for our flow conditions.

NOZZLE FLOW

A. Equilibrium Flow

The internal energy of a gas is composed of a translational, rotational, and vibrational mode:

$$E = E_t + E_r + E_v$$

or, combining translational and rotational:

$$E = \frac{5}{2} RT + E_v \tag{1}$$

The vibrational energy (E_v) associated with the diatomic molecule is usually considered as that of a simple harmonic oscillator. From reference (1), when the flow is in complete thermodynamic equilibrium, E_v may be expressed as a function of temperature in the following way:

$$E_v = R \frac{\theta_v}{e^{\theta_v/T} - 1} \tag{2}$$

where θ_v is a characteristic temperature for a particular gas which depends on Planck's constant (h), Boltzmann's constant (k), and vibrational frequency (ν) in the following way:

$$\theta_v = \frac{h\nu}{k}$$

For diatomic nitrogen, $\theta_v = 6102^{\circ}R$.

The equilibrium flow in the nozzle is treated assuming isentropic flow. This is equivalent to using the usual one-dimensional Euler equations, as seen in the following way:

Consider the energy equation:

$$H + \frac{1}{2} u^2 = \text{constant} \tag{3}$$

where $H = E + P/\rho$.

Differentiation yields:

$$dH = - u \, du \tag{4}$$

On the other hand, the definition of entropy yields:

$$dS = \frac{1}{T}[dE + P \, d \frac{1}{\rho}] = \frac{1}{T}[dH - \frac{1}{\rho} dP] \tag{5}$$

For isentropic flow, $dS = 0$. Therefore:

$$dH = \frac{1}{\rho} dP$$

Using equation (4) one finally obtains:

$$u \, du = - \frac{1}{\rho} dP \tag{6}$$

which is the usual equation for one-dimensional inviscid flow. Thus, the two methods are equivalent.

Let us proceed now to obtain the entropy equation. Consider first the equation of state:

$$P = \rho RT \tag{7}$$

In reality, $P = \rho RTZ$, where Z is the compressibility factor and depends on the virial coefficients. However, from the data in reference (2), a plot was made of P versus T for constant values of Z (fig. 2) showing that for pressures less than 100 atmospheres, the error in assuming $Z = 1$ is very small. We have thus set $Z = 1$ and used equation (7).

By means of equations (1), (3) and (7) one obtains:

$$H = \frac{7}{2} RT + E_v \tag{8}$$

Note that H is a function of T only. From equations (5) and (8):

$$\frac{dS}{R} = \frac{1}{RT}\frac{dH}{dT} - \frac{dP}{P}$$

Integration yields:

$$\frac{S}{R} = \int \frac{1}{RT}\frac{dH}{dT}\,dT - \ln P + (\text{constant of integration})$$

Substituting (8) into this equation and integrating, one obtains:

$$\frac{S}{R} = \frac{7}{2}[\ln T + 1] - \ln P - \ln[1 - e^{-\theta v/T}] + \frac{\theta}{T}\left[\frac{1}{e^{\theta v/T} - 1}\right] + \text{constant}$$

where P is in atmospheres and T in degrees Rankine. The value of the constant was obtained by matching this formula to the NBS data (ref. (3)) at one atmosphere and 540°R. We obtained -2.4767 for pressure in atmospheres and temperature in degrees Rankine. Thus, the entropy equation is:

$$\frac{S}{R} = \frac{7}{2}[\ln T + 1] - \ln P - \ln[1 - e^{-\theta v/T}] + \frac{\theta}{T}\left[\frac{1}{e^{\theta v/T} - 1}\right] - 2.4767 \tag{9}$$

The speed of sound in equilibrium flow is now found in a straightforward but tedious manner:

$$a^2 = \left(\frac{\partial P}{\partial \rho}\right)_S = \frac{\left(\frac{\partial P}{\partial T}\right)_S \left(\frac{\partial T}{\partial \rho}\right)_P}{1 - \left(\frac{\partial P}{\partial T}\right)_S \left(\frac{\partial T}{\partial P}\right)_\rho}$$

Differentiation of (7) and (9) according to the expression above gives for the speed of sound:

$$a^2 = \frac{RT\left[3.5 + \left(\frac{\theta_V}{T}\right)^2 (B + B^2)\right]}{\left[3.5 + \left(\frac{\theta_V}{T}\right)^2 (B + B^2)\right] - 1} \tag{10}$$

where

$$B = \frac{e^{-\theta_V/T}}{1 - e^{-\theta_V/T}}$$

The equilibrium flow in the nozzle is then found as follows: The entropy equation (9) is used to find S in the stagnation chamber. The static pressure in the nozzle as a function of temperature is found from the same equation using this constant value of S. The throat conditions are found by maximizing the mass flow per unit area, ρU. At any station along the nozzle, the area ratio A/A^* is found from the conservation of mass equation:

$$\frac{A}{A^*} = \frac{\rho^* u^*}{\rho u}$$

The results of this section for a typical supply condition are shown in figures 3, 4, and 5 labeled "equilibrium." These figures show pressure, temperature and velocity as a function of the area ratio of the nozzle. The other two curves appearing on these figures will be discussed later.

B. Partial Equilibrium

Whereas in the case of complete equilibrium the vibrational energy depends only on the translational temperature (eq. (2)), in the partial equilibrium case this equation no longer applies. Instead, there exists an additional differential equation (the rate equation) for determining E_V.

In obtaining the rate equation, it is convenient to define two new variables: the relaxation time, τ, and the vibrational temperature, T_v. The concept of relaxation time is generally employed to characterize the time required for the adjustment of the various degrees of freedom to equilibrium. It represents the time required to approach 1 - 1/e of the final equilibrium value. From reference (4) the relaxation time in seconds as a function of temperature and pressure is:

$$\tau = \frac{e^{-0.451\, T^{1/3}}}{13P} \tag{11}$$

where T is in degrees Rankine and P is in atmospheres.

Equation (11) is shown graphically in figure 6. It is noted from this figure that the relaxation time increases as the gas travels through the nozzle, since both pressure and temperature are decreasing. As a result, it is found in hypersonic nozzles that the vibrational energy is unable to follow these rapid changes. Therefore, the vibrational energy can no longer be obtained from equation (2). However, one can always find a temperature such that this equation be true. This temperature is known as the vibrational temperature, T_v. Thus, the vibrational energy in the case of partial equilibrium is:

$$E_v = R \frac{\theta}{e^{\theta_v/T_v} - 1} \tag{12}$$

The rate equation may now be introduced. From reference (5) this is:

$$\frac{dE_v}{dt} = \frac{1}{\tau}[E_v(T) - E_v(T_v)] \tag{13}$$

It will be noted that the right-hand side is proportional to the difference between two vibrational energies--the actual energy and the energy the gas would have if it were in equilibrium with the translational energy.

The flow through the nozzle may now be solved by adding the rate equation (13) to the equations used in the equilibrium case and manipulating them to obtain three differential equations in u, Γ, and T_v which are solved simultaneously.

Summarizing, we have the following equations:

Momentum: $$u \frac{du}{dx} = - \frac{1}{\rho} \frac{dP}{dx} \tag{6}$$

Continuity: $\rho uA = \text{constant}$

Energy: $H + \frac{1}{2} u^2 = \text{constant}$

State: $P = \rho RT$ (7)

Enthalpy: $H = \frac{7}{2} RT + E_v$ (8)

Vibrational Energy for Equilibrium: $E_v = R \dfrac{\theta_v}{e^{\theta_v/T} - 1}$ (2)

Actual Vibration Energy: $E_v = R \dfrac{\theta_v}{e^{\theta_v/T_v} - 1}$ (12)

Rate Equation: $\dfrac{dE_v}{dt} = \frac{1}{\tau}[E_v(T) - E_v(T_v)]$ (14)

where τ is given by (11).

The foregoing system of equations which include the vibrational rate equation can be reduced to a set of three differential equations each of which contain only the variables T, T_v, u, and x for a given nozzle with a fixed stagnation condition. These three equations are:

$$\frac{du}{dx} = \frac{1}{\frac{9}{7} u - \frac{1}{u} RT} \left[\frac{RT}{A} \frac{dA}{dx} + \frac{2}{7u\tau}\left(E_v(T) - E_v(T_v)\right)\right] \tag{15}$$

$$\frac{dT_v}{dx} = \frac{1}{uE_v'(T_v)} \; \frac{E_v(T) - E_v(T_v)}{\tau} \tag{16}$$

$$\frac{dT}{dx} = \frac{-2}{7R}\left[u \frac{du}{dx} + E_v'(T_v) \frac{dT_v}{dx}\right] \tag{17}$$

where the prime indicates differentiation with respect to T_v. In this report we shall use an equation for the nozzle area which approximates that used in Hypersonic Tunnel No. 4. It is:

$$\frac{A}{A*} = 1 + 700x^2$$

(A simple calculation from this formula shows the radius of curvature at the throat to be 3.72 in. This is the value used in the actual nozzle.)

We have therefore:

$$\frac{1}{A}\frac{dA}{dx} = \frac{1400x}{1 + 700x^2}$$

where x is any distance downstream of the throat in feet.

These equations were solved simultaneously on the IBM 7090 computer at NOL. It was assumed that the vibrational energy is in equilibrium with the translational and rotational energy modes for the flow process from the nozzle stagnation conditions up to the throat. Downstream of the throat the vibrational energy is allowed to relax.

That this assumption is reasonable is seen in the following way. If t is the average time the gas takes in traversing the upstream region, the ratio t/τ gives us an indication of whether the gas is in equilibrium or not. For example, if $t/\tau \gg 1$, the relaxation time will have elapsed before the gas arrives at the throat, and thus the flow will be in equilibrium. With this in mind, we have plotted a curve of t/τ versus T for values of constant pressures (fig. 7). For t we have used $t = x/u$, where x is the distance from the start of the flow to the throat (.75 in.) and $u = a^*/2$ where a^* is the speed of sound at the throat, a function of the stagnation conditions.

The figure shows that for the range of pressures greater than forty atmospheres, t is greater than 10τ for temperatures above 3400°R. Thus, the assumption of equilibrium flow up to the throat is indeed reasonable for the conditions in Hypersonic Tunnel No. 4.

The results of the partial equilibrium solution are shown in figures 3, 4, and 5 as curves of static pressure, static temperature, and free stream velocity versus area ratio. In figure 4, we have also plotted T_V, and the curve indicates that freezing of the vibrational energy occurs at a short distance from the throat. For the case shown, freezing occurred at $x = 0.05$ ft. The freezing point has been chosen as that point where the vibrational temperature has completed 95 percent of its total change. The total nozzle length used in Hypersonic Tunnel No. 4 is 3 ft. from throat to exit. For other stagnation conditions up to 5400°R and 100 atmospheres, freezing occurred at values of x less than 0.055 ft. In figure 8 we have plotted the final frozen vibrational temperature (T_{V_F}) as a percentage of the supply temperature for various stagnation conditions. It is seen that $64\% < T_{V_F}/T_O < 77\%$ for 40 atm < P < 100 atm for 3600°R ≤ T < 5400°R. This ratio decreases as pressure and temperature increase.

Thus we see that for the most part of this 3 ft. nozzle, the vibrational energy is completely frozen. We shall consider next the completely frozen flow case.

C. Frozen Flow

The other limiting case in high-temperature flow is the case in which the vibrational energy is constant throughout the nozzle. This is known as frozen flow. Since the vibrational energy does not participate in the flow process, this case can be considered identical to perfect gas flow. Mathematically, one can combine the one-dimensional flow equation with the expressions for enthalpy and manipulate them to obtain P/ρ^{γ} constant. This shows frozen flow to be equivalent to perfect gas flow. Reference (6) gives a mathematical proof of this fact.

We have used the same procedure in solving this flow as was used for the equilibrium case, this time making $dE_v = 0$. The expression for entropy is now:

$$\frac{S}{R} = 3.5 \ln T - \ln P - 2.4767$$

The results are shown in figures 3, 4, and 5 as curves of static pressure, static temperature, and free stream velocity versus nozzle area ratio.

PROBE FLOW

A. Shock Wave

Vibrational relaxation in the region near a pitot tube may affect the pressure readings recorded by the probe. The temperature behind the shock wave created by the probe increases as M_1^2 and becomes large enough so that the molecular collisions excite the vibrational degrees of freedom of the diatomic molecules. Immediately behind the shock the vibrational relaxation process will start. In order to determine the effects of this process on the pressure readings, we will consider the flow as it passes through the shock and across the shock layer.

As the gas moves through the shock wave, no vibrational relaxation is expected (the flow is frozen) since the number of molecular collisions within the shock wave is small compared to the number required for equilibrium. This may be shown as follows: An approximate figure for the number of collisions required for equilibrium can be obtained if we know the thermal speed of the molecule (C), the mean free path (L) and the relaxation time. From reference (7):

$$C = \sqrt{2RT}$$

$$L = \frac{16}{5} \frac{\mu}{\rho\sqrt{2\pi RT}}$$

The viscosity of nitrogen is found from Sutherland's relation:

$$\mu = 2.14 \times 10^{-8} \frac{T^{3/2}}{T + 180} \quad \frac{\text{slug}}{\text{ft-sec}}$$

with T in $^{\circ}$R. Since the relaxation time is known (eq. (11)), the number of collisions required for the excited gas to come to equilibrium is:

$$N = \frac{C}{L}\tau = \frac{1.098 \times 10^{11}[T + 180]\ e^{-0.461\ T^{1/3}}}{13\ T^{3/2}} \tag{18}$$

It is noted that N depends only on temperature. This equation has been plotted in figure (9) where it is seen that N is very large, from 30,000 at T = 6000°R to 700,000 at T = 2000°R. On the other hand, reference (8) tells us that the transition across a shock wave from temperature T_1 to T_2 takes place over a distance of the order of one mean free path. That is, a molecule undergoes only a few collisions in its passage through a shock wave, and thus there are not enough of them for any relaxation to occur.

B. Shock Layer

As the gas travels through the shock layer, the relaxation process begins. However, because of the very short standoff distance between the shock and the probe, the gas can be assumed to remain frozen. An indication of this fact may be obtained by calculating t/τ, the ratio of transit time to relaxation time, for this region. If this ratio is very small, the flow can be assumed to be frozen. The transit time for the shock layer is given by

$$t = \frac{\Delta}{\overline{u_2}} \tag{19}$$

where Δ is the standoff distance. From reference (9), for a strong shock wave where $\rho_2/\rho_1 = 6$, the standoff distance is approximately 0.0011 ft. when the tip of the pitot probe is approximated by a sphere of radius equal to 1/8 in. The value of $\overline{u_2}$ is an average value of the velocity behind the shock. The relaxation time is obtained from equation (11) using the temperature and pressure behind the shock, P_2 and T_2. The variables $\underset{\cdot}{u_2}$, P_2, and T_2 are obtained from the supply conditions assuming completely frozen flow from reservoir to shock. This is permissible since we are interested in obtaining only the order of magnitude of the ratio t/τ. Hence, given the supply conditions, we can use perfect gas tables such as reference (10) and obtain u_2, P_2, and T_2 and thus t, τ, and t/τ. This has been done for a typical operating Mach number in Hypersonic Tunnel No. 4 (M=17) and in figure 10 we have plotted the ratio t/τ in the shock layer for various supply conditions. It is seen that for the range $3500^{o}R \le T < 5000^{o}R$ and 40 atm $\le P_O < 100$ atm. The ratio t/τ is in the range of 10^{-4} to 10^{-3}. Thus, we expect frozen flow in the shock layer.

COMPLETE FLOW

The three cases previously discussed separately are now combined and compared using the ratio of the supply stagnation pressure (P_{O1}) to the pressure read by the pitot probe (P_{O_2}). For conditions before the shock we have used the nozzle flow data leading up to figures 3, 4, and 5 and, hence, have assumed the nozzle flow to be in equilibrium, partial equilibrium, and frozen. For the flow through the shock and the shock layer, we have assumed the two limiting cases: equilibrium and frozen. The final result for each case is a curve showing the variation of P_{O2}/P_{O1} with Mach number and area ratio for various typical stagnation conditions. The cases are outlined in TABLE 1.

TABLE 1

CASE	UPSTREAM OF THROAT	DOWNSTREAM OF THROAT	SHOCKWAVE	SHOCK LAYER
1	Frozen	Frozen	Frozen	Frozen
2	Equilibrium	Relaxing	Frozen	Frozen
3	Equilibrium	Equilibrium	Equilibrium	Equilibrium

<u>CASE 1</u>: Since frozen flow is identical to perfect gas flow, C_p is a constant throughout, and the stagnation pressure ratio may be obtained by means of the NACA Tables, reference (10).

<u>CASE 2</u>: C_p is still constant across the shock and shock layer, and reference (10) may also be used. However, the conditions in region 1 (immediately before the shock) must be obtained from the partial equilibrium nozzle flow data.

CASE 3: C_p is now not constant, and reference (10) may not be used. Instead, we shall write:

$$\frac{P_{O2}}{P_{O1}} = \frac{P_1}{P_{O1}} \frac{P_2}{P_1} \frac{P_{O2}}{P_2} \tag{20}$$

a. The ratio $\frac{P_1}{P_{O1}}$ is obtained from the equilibrium nozzle flow data leading to figure 3.

b. The ratio $\frac{P_2}{P_1}$ is found using an iteration procedure outlined below:

According to reference (11), the velocity ratio $\frac{u_2}{u_1}$ is a good indicator of the equilibrium state of the gas. The value of this ratio, it is shown, decreases as additional degrees of freedom are excited. The shock equations may be expressed in terms of this ratio, and once its value is found (by iteration on these equations) they may be solved to obtain the values of the variables behind the shock. The iteration procedure is as follows:

The conservation equations across the shock are:

$$\rho_1 u_1 = \rho_2 u_2$$

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

$$H_1 + \frac{1}{2} u_1^2 = H_2 + \frac{1}{2} u_2^2$$

In terms of the velocity ratio one obtains:

$$\frac{P_2}{P_1} = 1 + \gamma_1 M_1^2 \left[1 - \frac{u_2}{u_1}\right]$$

$$\frac{H_2}{H_1} = 1 + \frac{\gamma_1 - 1}{2} M_1^2 \left[1 - \left(\frac{u_2}{u_1}\right)^2\right]$$

where, from reference (12):

$$\gamma_1 = \frac{7R + 2E_v'(T_1)}{5R + 2E_v'(T_1)} \tag{23}$$

where

$$E_v' = R \frac{\theta_v^2}{T_1} \frac{e^{\theta_v/T_1}}{(e^{\theta_v/T_1} - 1)^2}$$

It may be seen in equation (23) that for our range of Mach number ($10 < M < 20$) giving values of temperature greater than $50^{o}R$ and less than $250^{o}R$ the value of E_v' is zero and $\gamma_1 = 1.4$.

One also needs the equation of state and the enthalpy equation:

$$P = \rho RT \tag{7}$$

$$H = \frac{7}{2} RT + E_v \tag{8}$$

The iteration procedure is carried out in the following way:

One first assumes $u_2/u_1 = 0$ and calculates P_2 and H_2 from equations (21) and (22). From a plot of equation (8) (fig. 11), T_2 is obtained and ρ_2 calculated from (5). Because of the continuity equation, a new value of u_2/u_1 is given by ρ_1/ρ_2. The procedure is then repeated using the new value of u_2/u_1 until two successive values of this ratio differ by less than the desired accuracy. Usually three or four iterations are sufficient. This final value of u_2/u_1 is used in equation (21) to obtain P_2/P_1.

c. In equation (20), the ratio P_{O_2}/P_2 is obtained in a similar manner to the way in which P_{O_1}/P_1 was obtained in the equilibrium nozzle flow calculations. That is, once P_2 and T_2 are known from the iteration procedure, the value of S is calculated from equation (9). This value is kept constant (isentropic flow) and the pressure at the stagnation point P_{O_2} is found from equation (9) using this constant value of S.

Equation (20) is then finally used to obtain the stagnation pressure ratio P_{O_2}/P_{O_1}. This ratio has been plotted for each case and various stagnation conditions versus Mach number (figs. 12, 13, and 14) and versus area ratio (fig. 15). In figure 14, all the results for the range $3960 \le T_o \le 5400$ and $80 \text{ atm} \le P_o \le 100$ atm fell on the two curves shown.

DISCUSSION

The purpose of these calculations as stated in the introduction was to provide some information on the effect of vibrational energy on the nozzle flow in NOL Hypersonic Tunnel No. 4. It was hoped that this information would assist in the interpretation of experimental results. Basically, the problem is this: once the stagnation pressure ratio P_{O_2}/P_{O_1} is experimentally determined, how can one obtain the static pressure, static temperature, free stream velocity, and density in the hypersonic nozzle with vibrational relaxation present?

In attempting to interpret the results found from our calculations, one encounters the complication that even in an inviscid, one-dimensional flow with relaxation present the usual quantities, P, ρ, T, and u are not uniquely related to the Mach number. See, for example, figures 12, 13, and 14. However, in figure 15 one sees that the parameter A/A^* has succeeded in collapsing any differences between the frozen flow and the partial equilibrium cases to a single curve. Hence, it seems appropriate to discuss the results in terms of area ratio. But it must be noted that in a physical flow there is always a boundary layer and one never really knows what the area ratio is. Furthermore, a physical flow in a conical nozzle is not one-dimensional. Thus, we shall think of the area ratio as an "effective" area ratio, with no real physical significance, but which serves as a convenient quantity to plot quantities against. Its real utility is as an intermediate parameter.

With this in mind, we may give an answer to the question posed in the first paragraph. We know that case 2 is closest to reality, but it requires solution of the rate equation for the vibrational energy. On the other hand, figure 14 tells us that when plotted versus area ratio, the ratio of pitot to supply pressure is the same in the completely frozen flow case as in relaxing flow. It is of interest, therefore, to estimate how much error is introduced when the pressure, temperature, and velocity are calculated using the "frozen flow area ratio." This area ratio is obtained from figures like 15, or equivalently, from frozen flow tables, such as reference (10). As far as the pressure is concerned, figure 3 shows that the error is negligible. For temperature, from figure 4, it can be as much as 9 percent, whereas figure 5 shows that the error can be as much as 5 percent for velocity. It should be noted that these figures are given on the basis of only a limited number of calculations. It is noted, furthermore, from figures 12, 13, and 14, that for Mach numbers around 15, the error introduced in Mach number calculations using the frozen flow assumption is between 1 and 2 percent.

CONCLUSIONS

In order to determine the effects of vibrational energy in the modified NOL Hypersonic Tunnel No. 4, we have considered the flow to be first in equilibrium, then relaxing, and finally completely frozen. Plots of pressure, temperature, and velocity versus area ratio for each of these three cases show that in going from frozen to equilibrium flow, these three variables increase at any particular station in the nozzle. If one considers the relaxing case, it is found that the flow remains in equilibrium up to the throat, then freezes shortly downstream, remaining frozen throughout.

The flow around a pitot probe has been examined for these three cases, and it is found that when the ratio of pitot to supply pressure is plotted versus area ratio, the relaxing flow case gives identical results to the frozen flow case. Therefore, the authors recommend that the frozen flow area ratio be used to determine the flow variables in the nozzle once the stagnation pressure ratio is known. If this is done, for the cases considered, the percentage error in assuming frozen rather than relaxing flow is estimated to be negligible for pressure, a maximum of 9 percent for temperature, and 5 percent for velocity.

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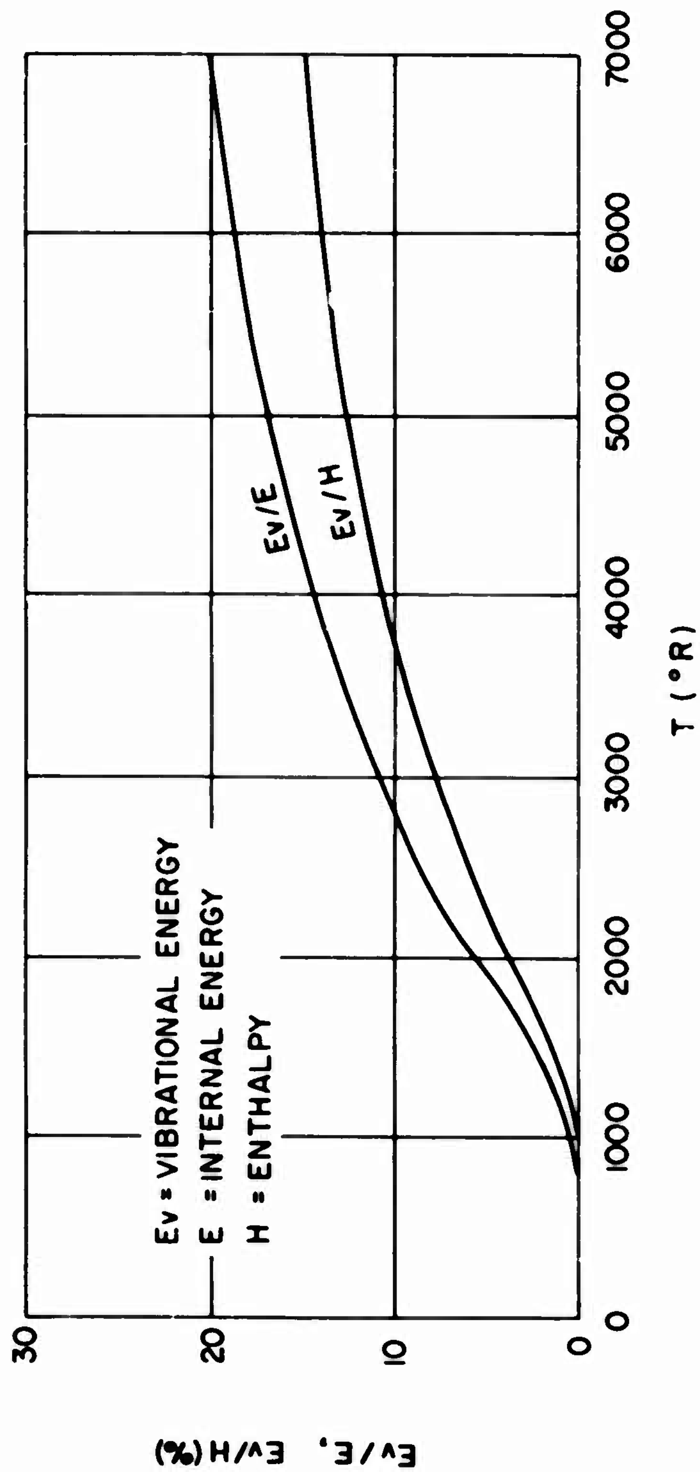

FIG. 1 VIBRATIONAL ENERGY AS A PERCENTAGE OF INTERNAL ENERGY AND ENTHALPY

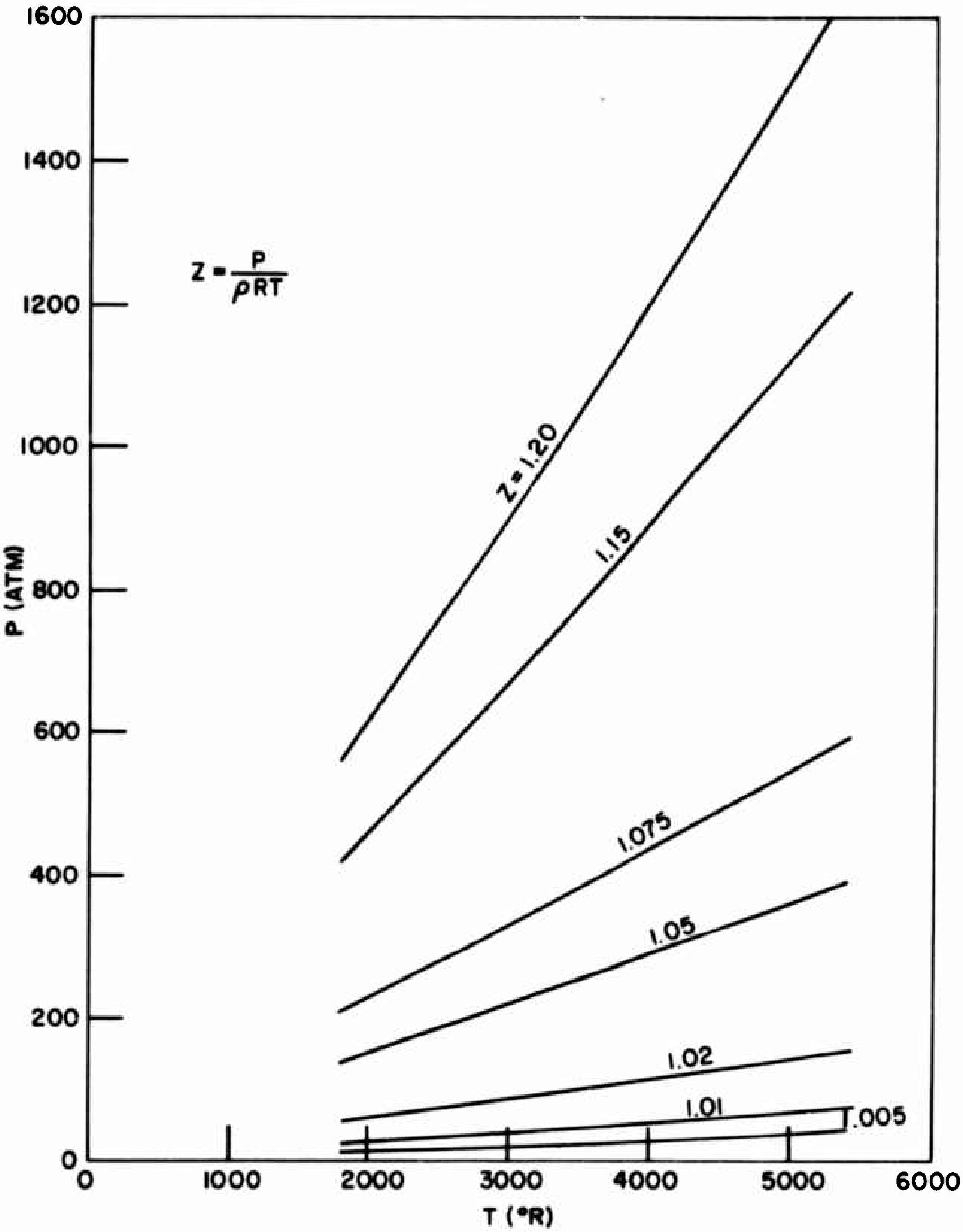

FIG. 2 COMPRESSIBILITY FACTOR FOR NITROGEN

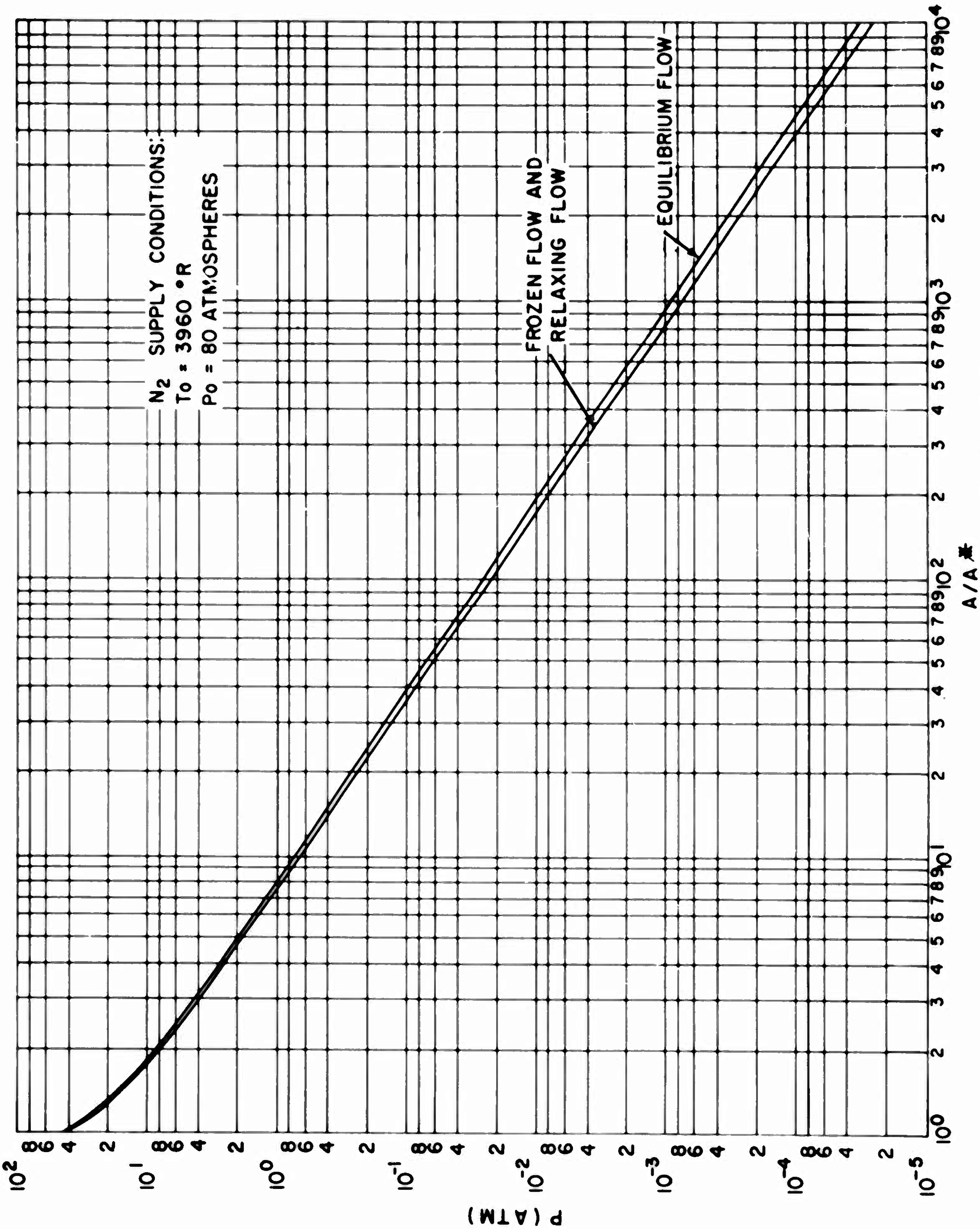

FIG. 3 STATIC PRESSURE VERSUS NOZZLE AREA RATIO

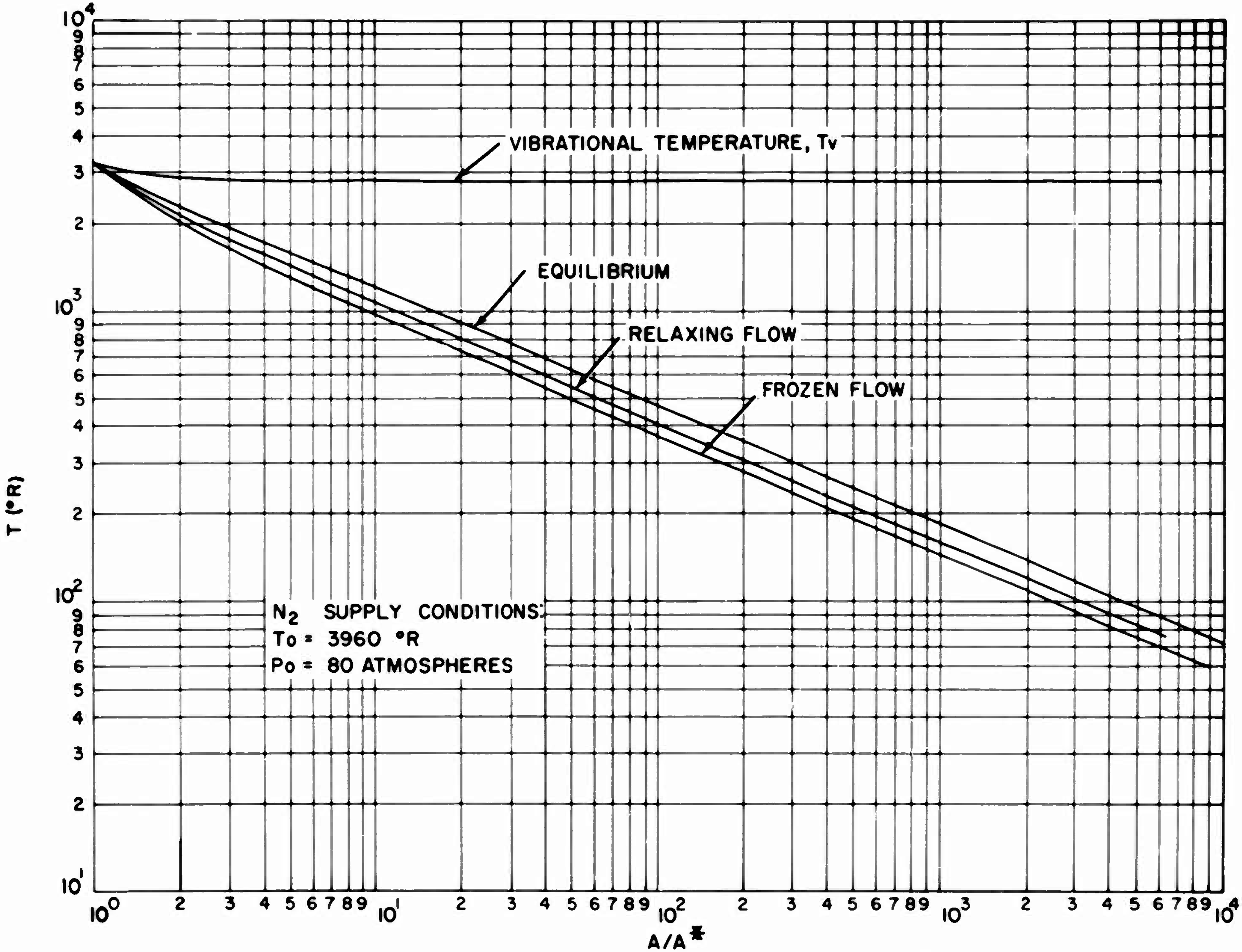

FIG. 4 STATIC TEMPERATURE VERSUS NOZZLE AREA RATIO

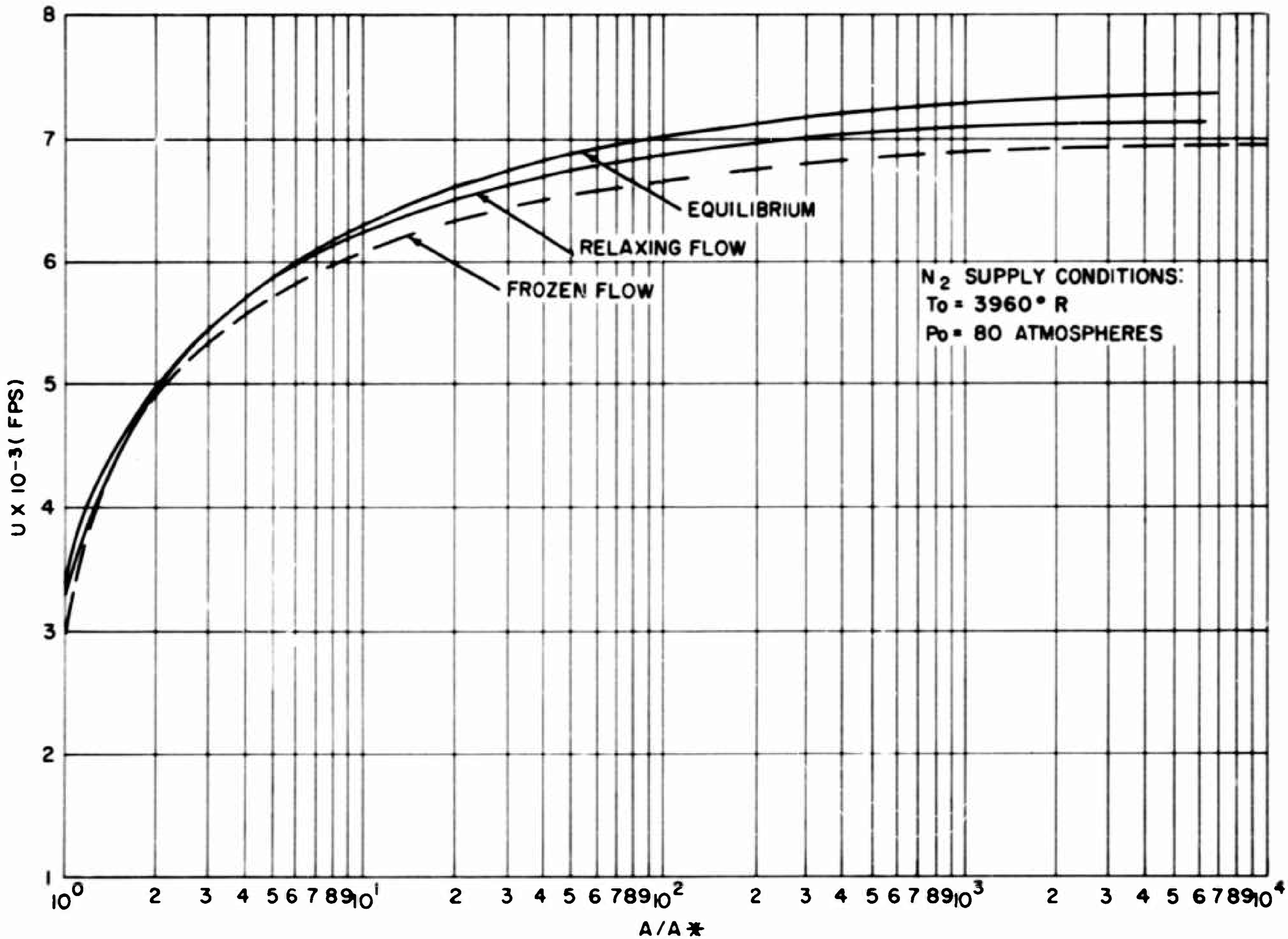

FIG. 5 FREE STREAM VELOCITY VERSUS NOZZLE AREA RATIO

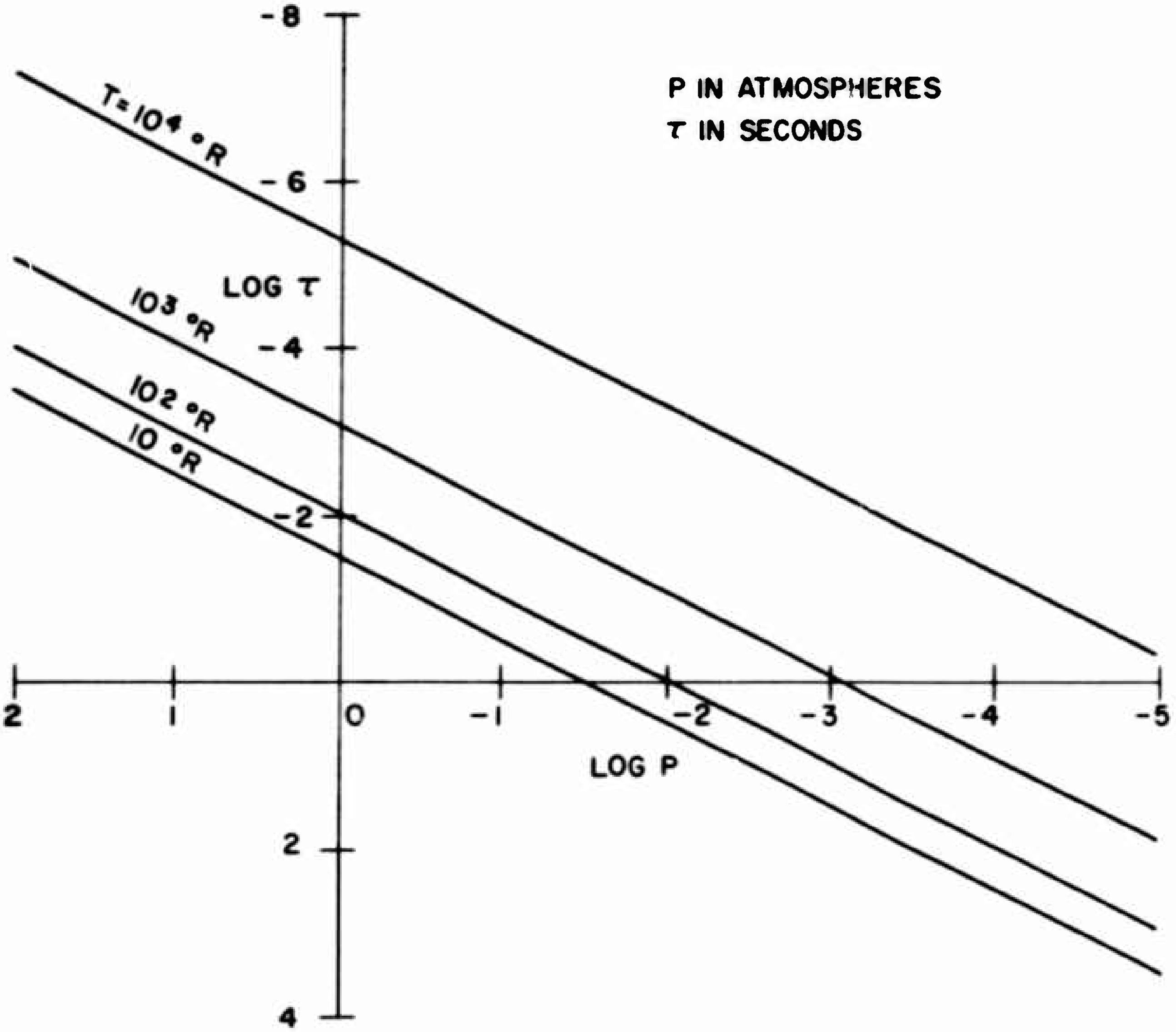

FIG. 6 VIBRATIONAL RELAXATION TIME FOR NITROGEN

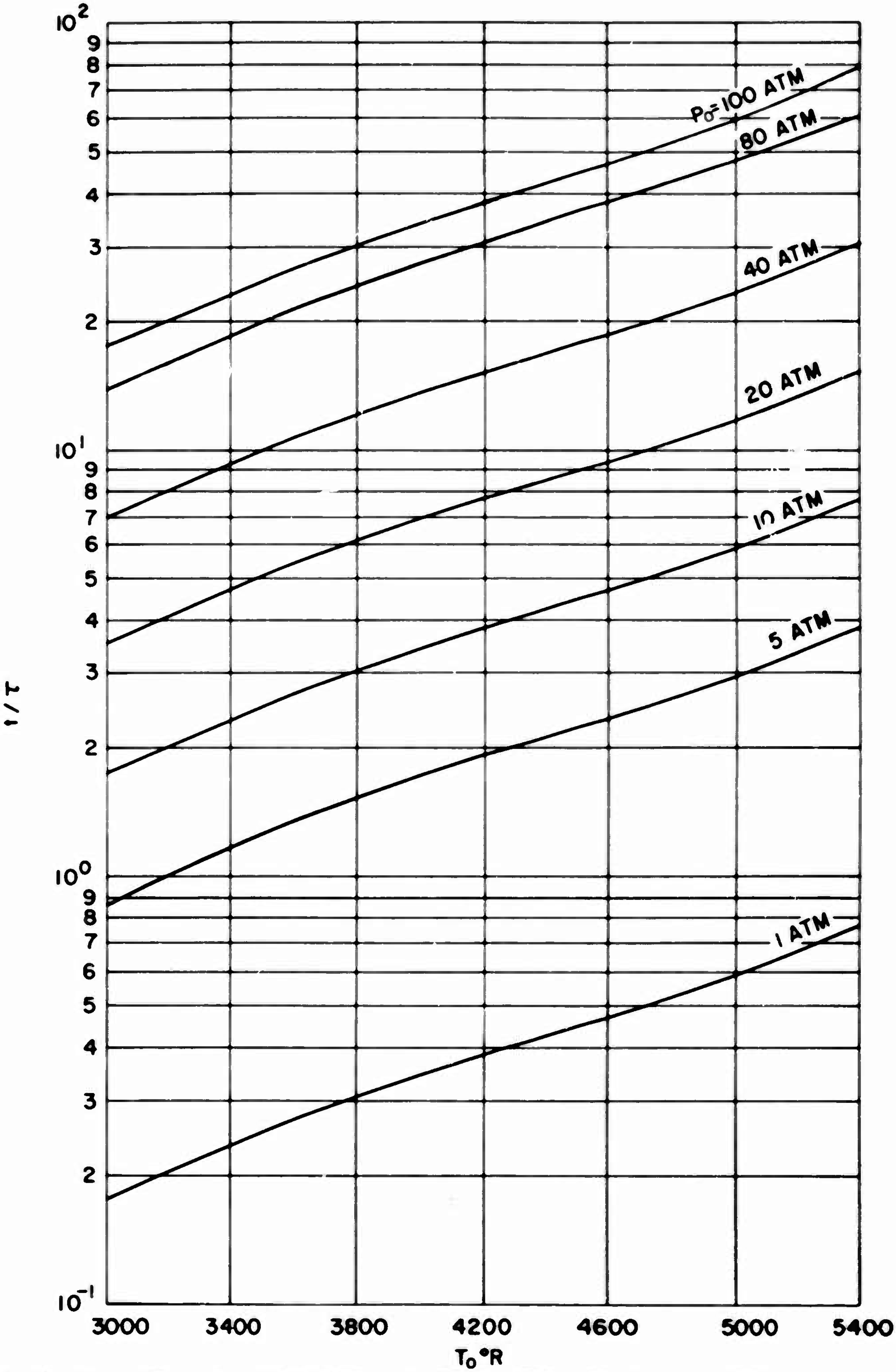

FIG. 7 TRANSIT TIME COMPARED TO RELAXATION TIME IN UPSTREAM REGION

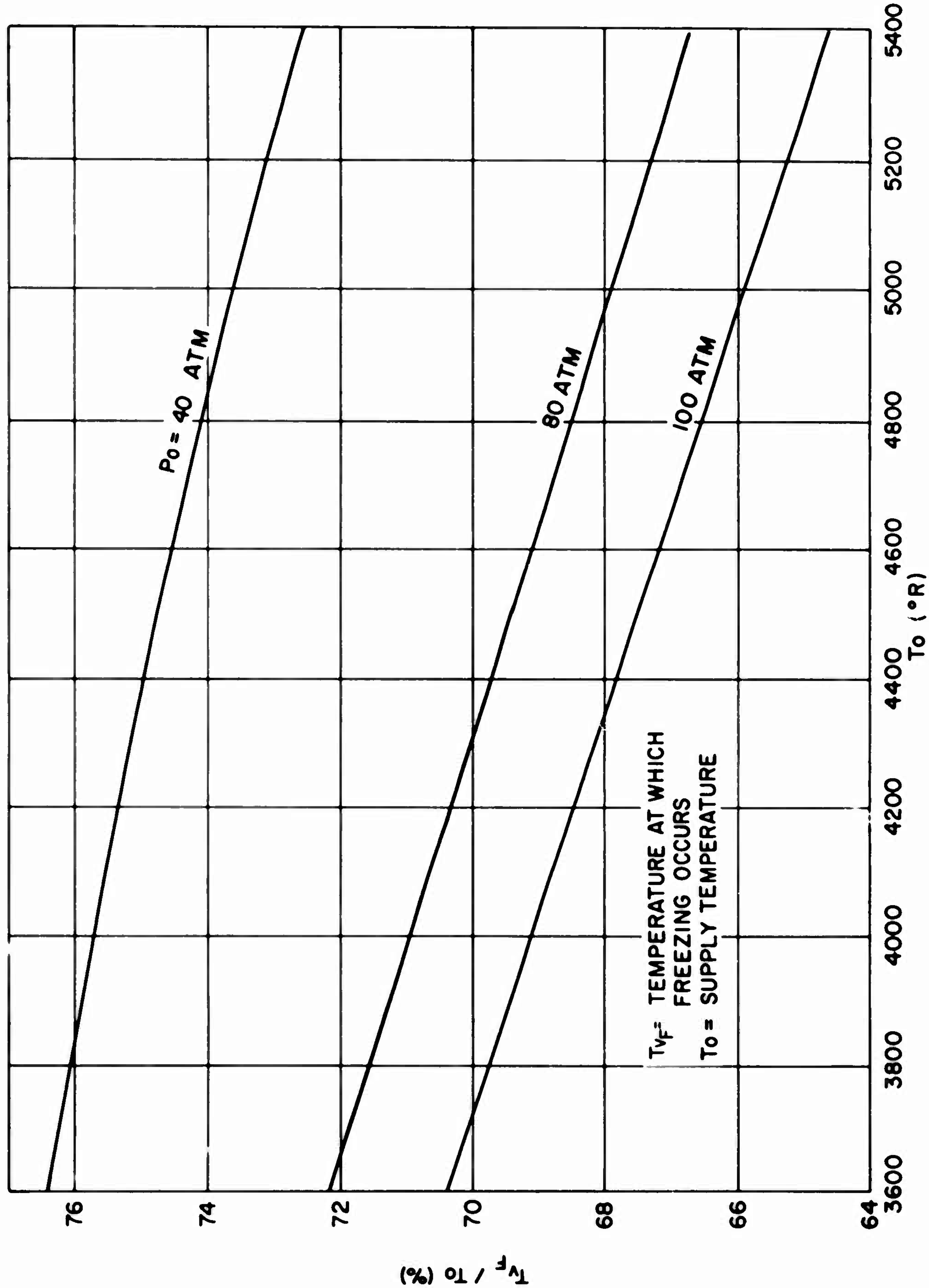

FIG. 8 FREEZING TEMPERATURE AS PERCENTAGE OF SUPPLY TEMPERATURE

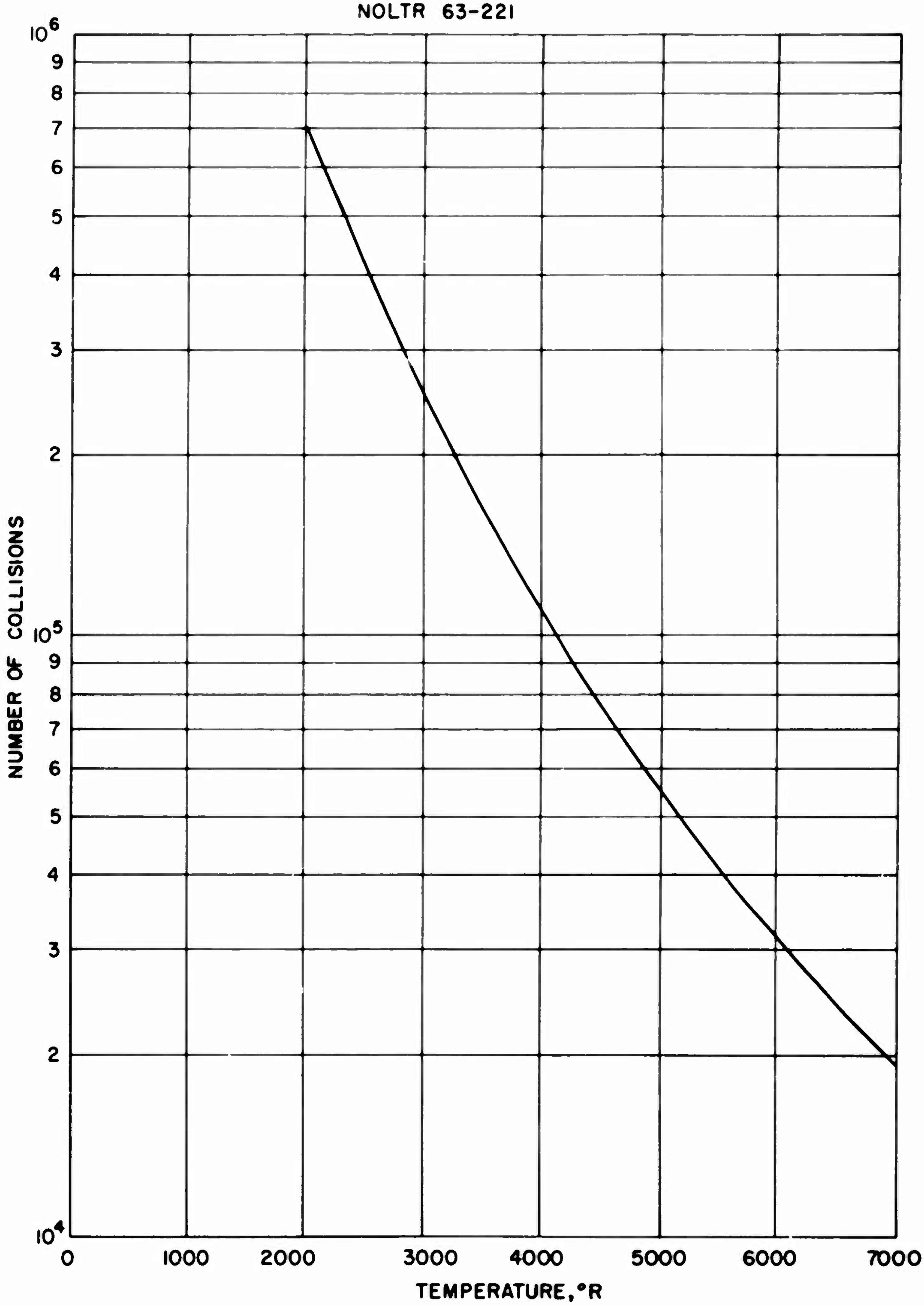

FIG. 9 NUMBER OF COLLISIONS REQUIRED FOR NITROGEN VIBRATIONAL EQUILIBRIUM

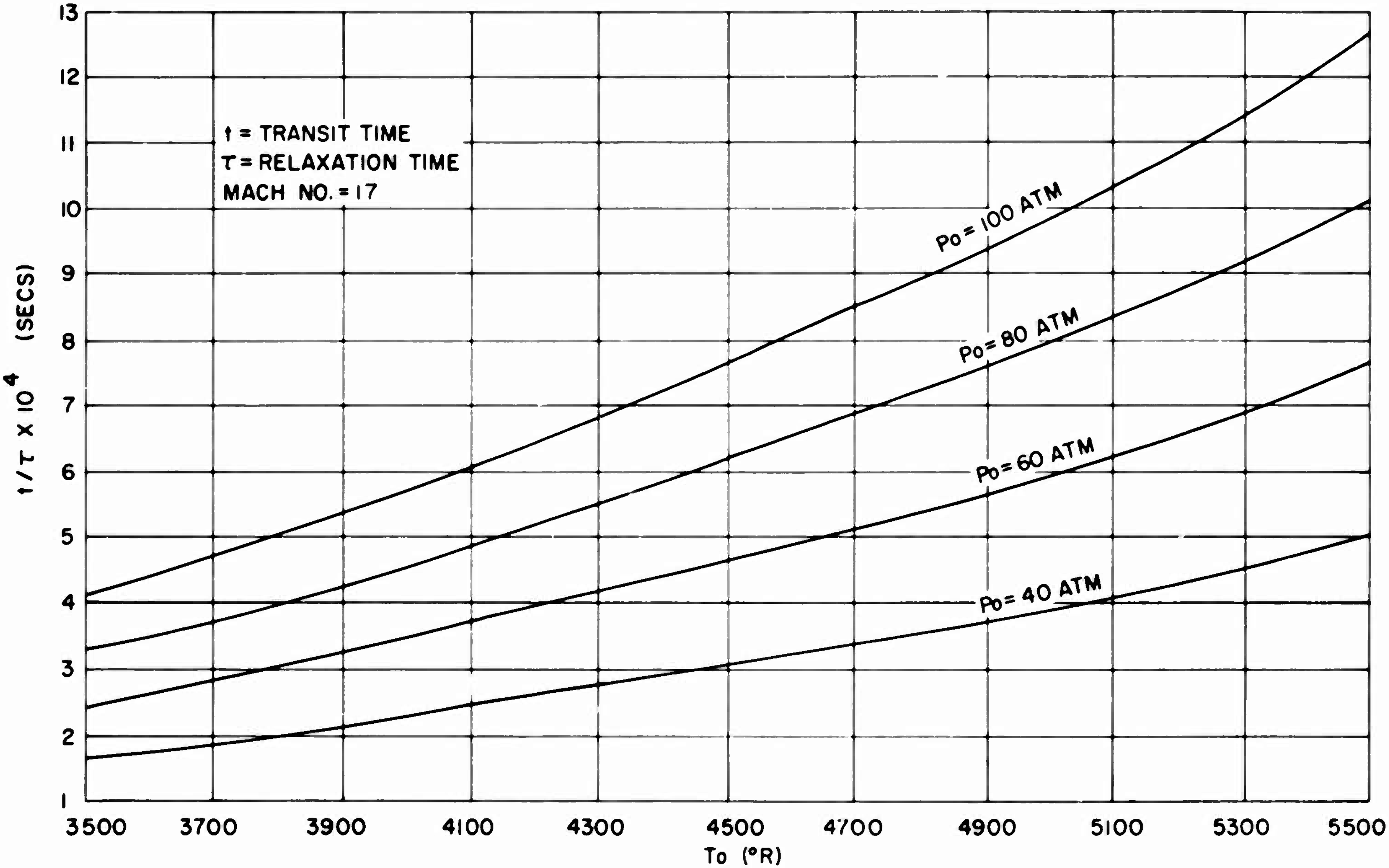

FIG. 10 TRANSIT TIME COMPARED TO RELAXATION TIME IN SHOCK LAYER

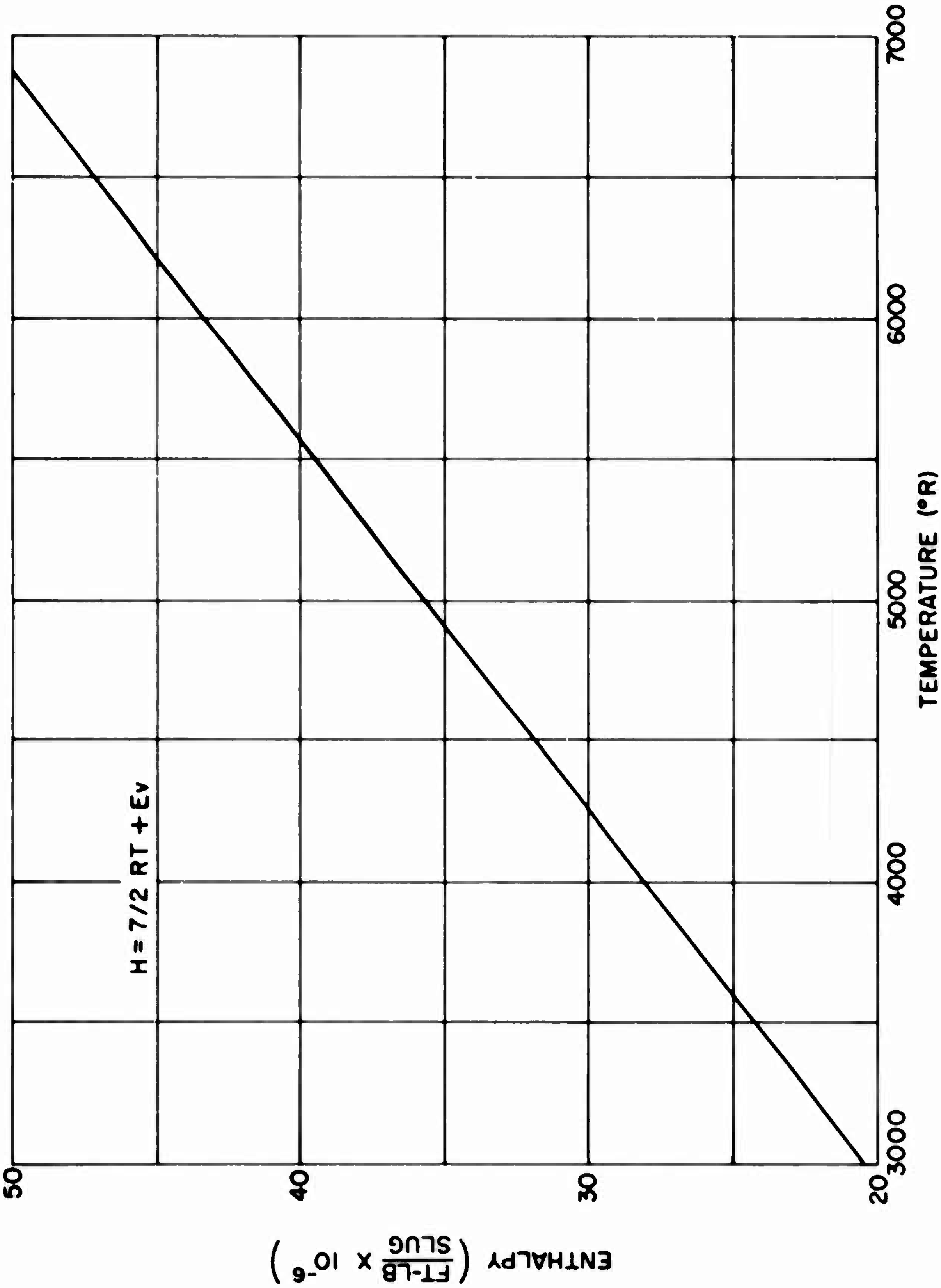

FIG. 11 ENTHALPY OF NITROGEN

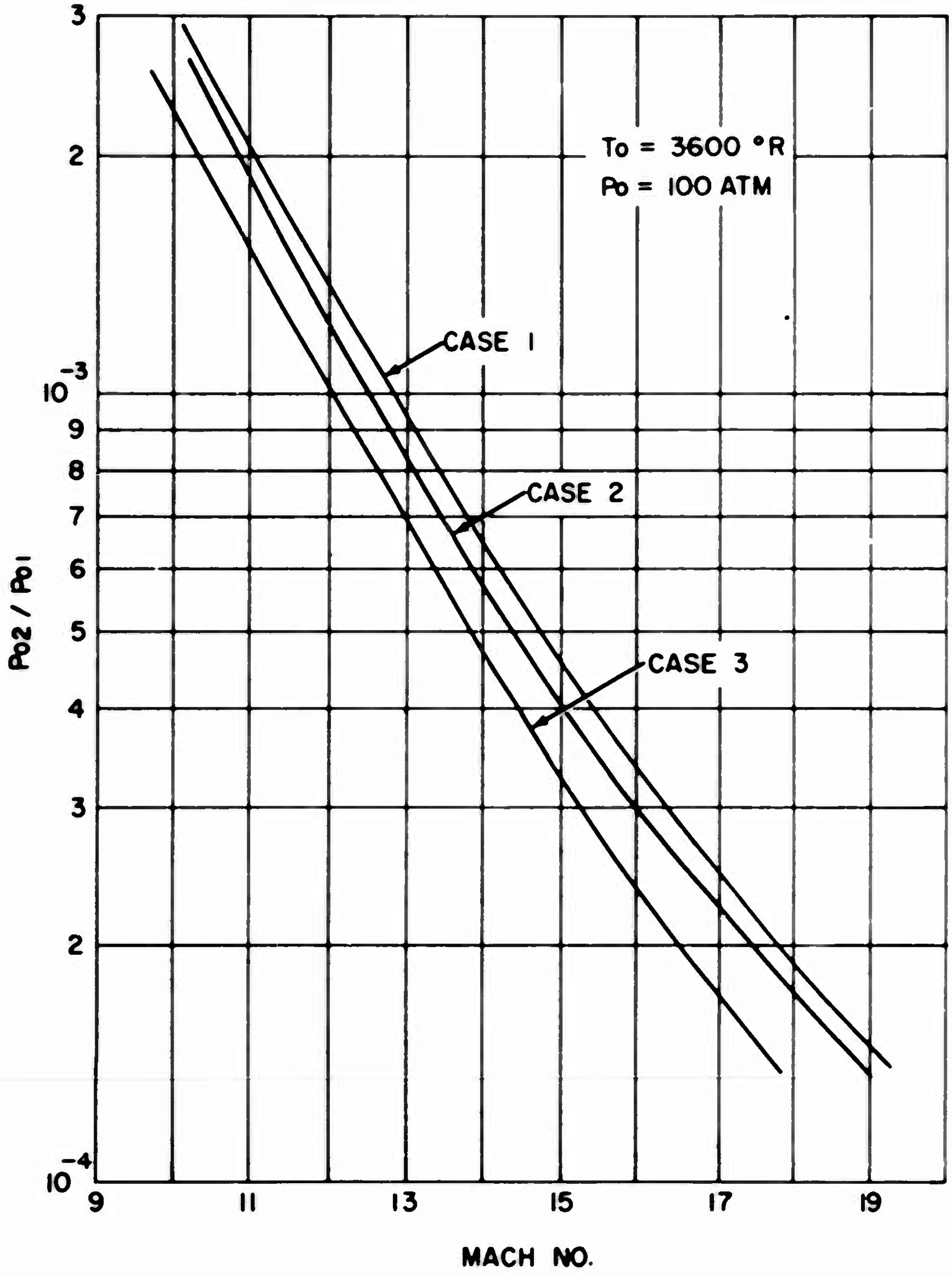

FIG. 12 STAGNATION PRESSURE RATIO VERSUS FREE STREAM MACH NO.

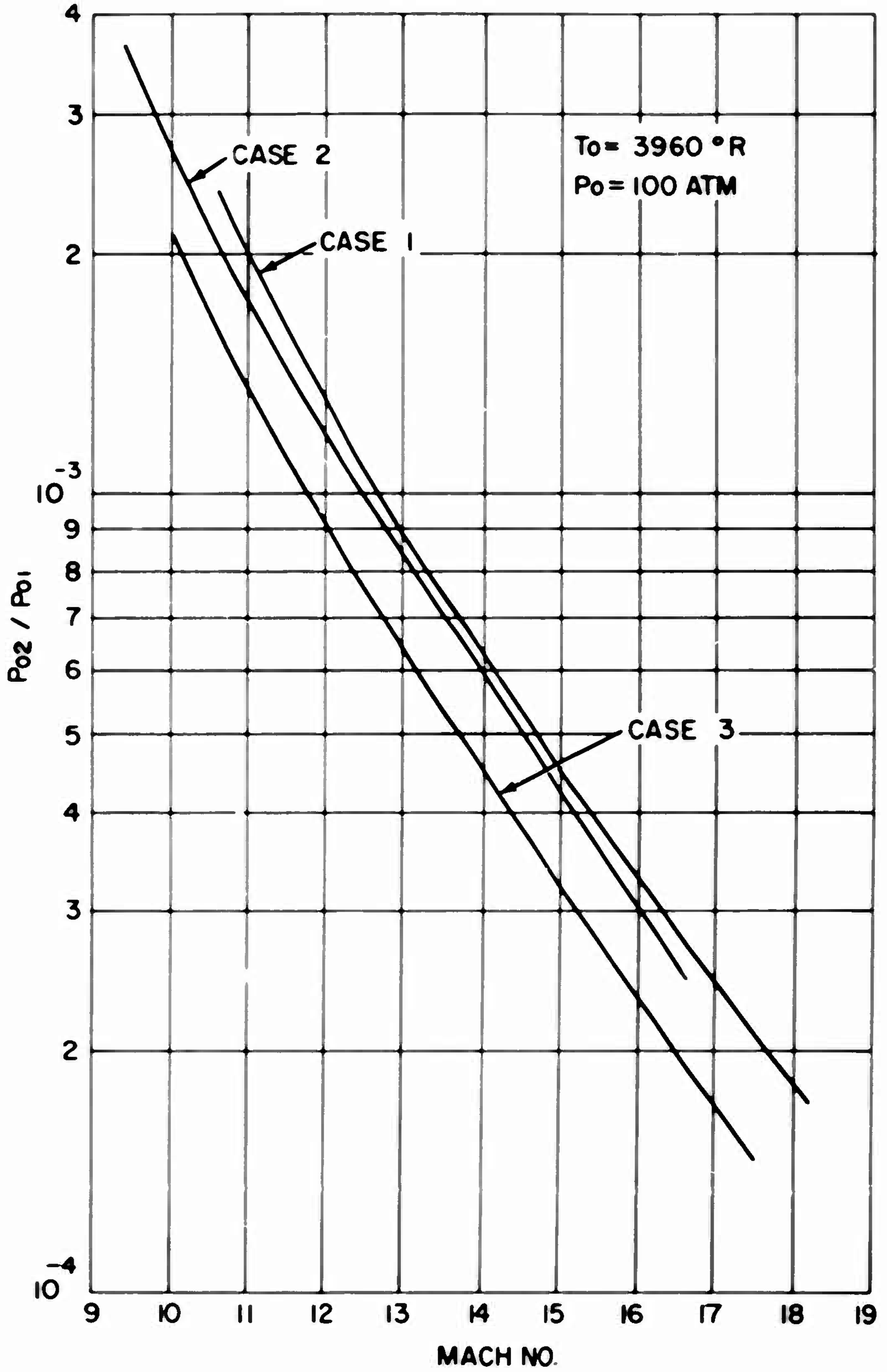

FIG. 13 STAGNATION PRESSURE RATIO VERSUS FREE STREAM MACH NO.

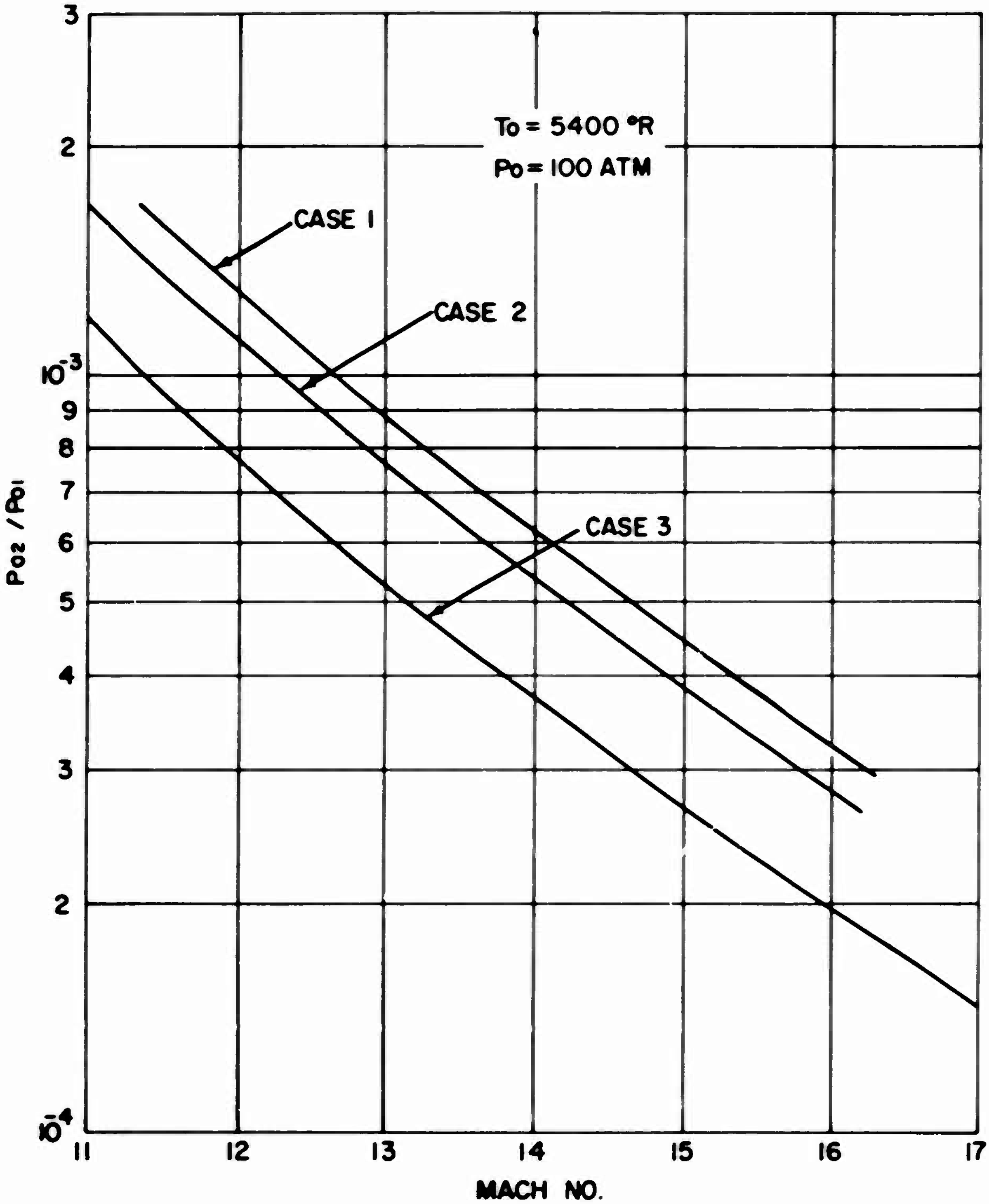

FIG. 14 STAGNATION PRESSURE RATIO VERSUS FREE STREAM MACH NO.

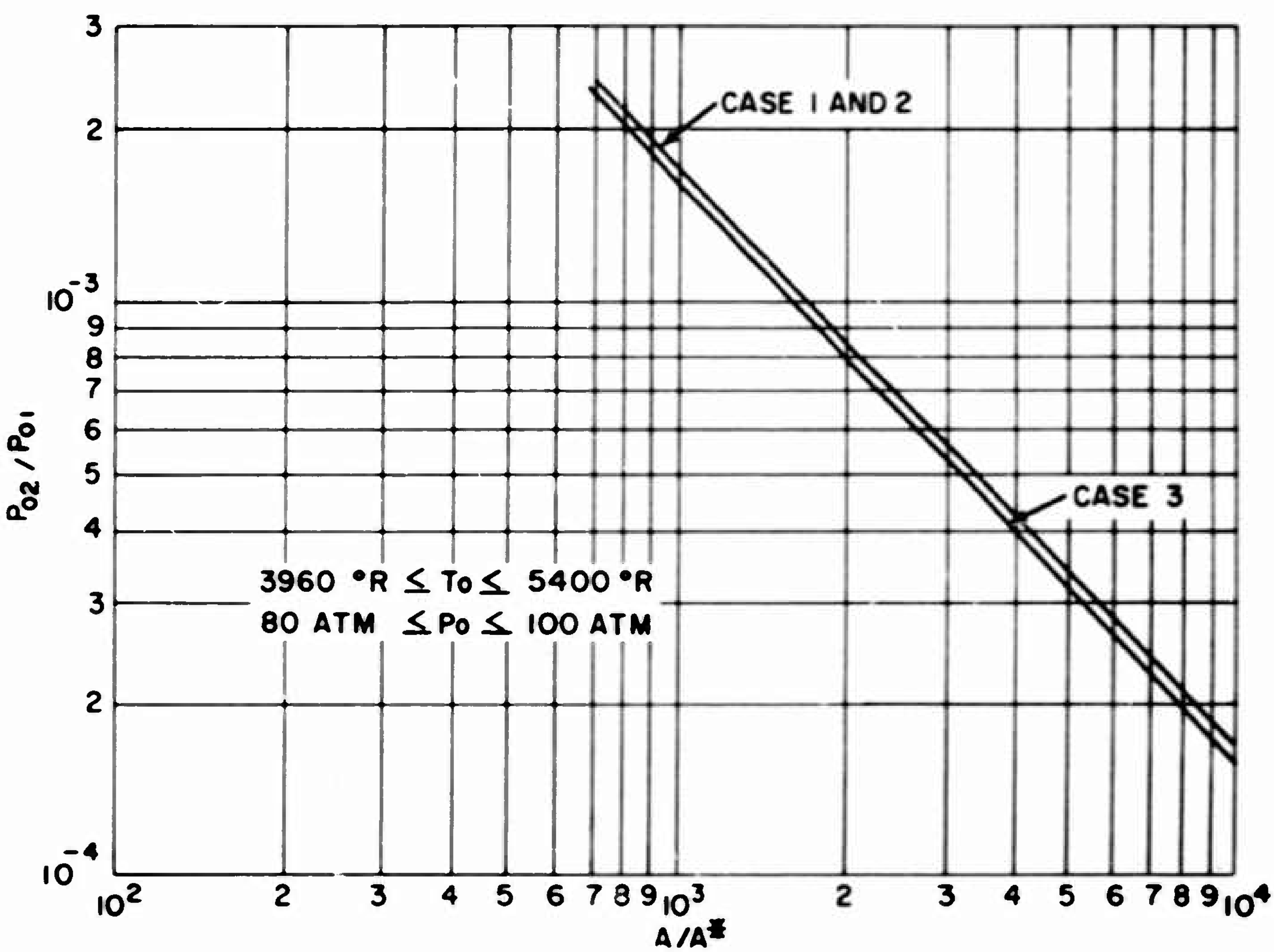

FIG. 15 STAGNATION PRESSURE RATIO VERSUS NOZZLE AREA RATIO