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FDBennett/GDKahl/EHWedemeyer/jh
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ABSTRACT

The marked increase in electrical resistance occurring at temperatures above melting can be explained by assuming a vaporization, expansion wave proceeding radially inward from the wire surface. This wave reduces the conducting cross section of the wire, thereby increasing the resistance. For wires well matched to the circuit the resistance rise occurs when the condenser voltage is practically zero and can be neglected; and for thin wires that explode rapidly on the linear portion of the current curve, condenser voltage can be taken as constant. Under these assumptions the circuit equation can be integrated and explicit expressions obtained for current, power and deposited energy. The required velocity of the wave, deduced from data on copper wires, is about 250 meters per second, an order of magnitude smaller than sound speed. A theoretical fluid dynamical model of an expansion wave involving a phase change from liquid to wet vapor is investigated; this analysis shows the expected velocity of small amplitude waves to be very close to the experimental value cited above. The theory predicts both the onset of the wave and the wave speed as a function of deposited energy up to the critical temperature. Beyond critical temperature, where the liquid to vapor expansion does not apply, the wave speed should be only a function of deposited specific energy. The experimental data for copper wires under a variety of conditions are found to correlate to a single curve; thus, the supposed resistance anomaly in high temperature copper wires is explained.

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1. INTRODUCTION

For some time it has been understood that the resistance function of an exploding wire is intimately connected with and dependent upon the fluid dynamical phenomena encountered during the expansion of the wire. This view was foreshadowed by W. Müller[1] in his application to the cylindrical wire case of the theoretical calculations made by F. Wecken for spherical explosive charges. Further semi-quantitative development of the basic ideas was given by E. David[2] who estimated the approximate expansion ratio at which electrical conduction in copper wires would cease. A more ambitious attempt at a quantitative theoretical discussion is given by M. Keilhacker[3] who applies to the case of exploding copper wires, the one-dimensional theory of the unsteady, centered, simple-wave expansion of a perfect gas in an effort to calculate the decay of mass density on the wire axis for different experimental conditions. His asymptotic values for density are too high by at least an order of magnitude when compared with the more exact, hydrodynamic, numerical calculations of C. A. Rouse[4], probably because the one dimensional simple wave solutions account for neither the reflection of the expansion wave from the axis nor the cylindrical geometry.

A more complete approximate theory, which provides a clear physical picture of the expansion of a cylinder of high temperature, high density gas into a lower density surrounding medium, is given by M. P. Friedman[5] who accounts for the non-planar geometry by a perturbation treatment based on the planar simple wave. He explains the presence of the internal second shock wave and finds its path using a method developed by G. B. Whitham[6] based on the requirement that there be only a single mapping of the flow region by a given family of characteristic curves.

The shock in the ambient medium and the contact surface behind it are decribed using the approximate theory for the motion of shock waves in channels of varying cross section, which theory is founded on ideas developed by W. Chester,[7] P. Germain and R. M. Gundersen,[8] R. F. Chisnell,[9] G. B. Whitham[10] and others. More recently the magnetohydrodynamic problem of converging or diverging shock waves, with cylindrical or spherical symmetry, in the presence of magnetic fields is solved by Gundersen.[11]

These studies are relevant to various idealizations of the exploding wire problem and enable us to form a fairly definite picture of the fluid motions and shock wave phenomena to be expected, insofar as theory of the inviscid flow of a non-conducting, perfect gas without excitation or nonequilibrium effects can be trusted to provide an accurate representation.

An adequate theory of the exploding wire must regard the energy source, wire and ambient medium as a complete system. The electrical behavior of the circuit can be described either in terms of Maxwell's equations with suitable boundary conditions or, for the low frequency cases ($f \leq 20$ mc) by means of generalized forms of Kirchhoff's laws. Conservation of energy then requires a balance between dissipation of electrical power in the circuit and the thermal heating, conduction, radiation and electromagnetic interactions of the wire material. Given the temperature history of the wire an attempt can be made to solve the fluid dynamical problem posed by the conservation equations for mass, momentum and energy together with suitable initial and boundary conditions which specify the transitions from solid through liquid to vapor within the wire material.

There is a very important missing portion in the development of a complete theory. Even assuming that the electromagnetic and hydrodynamic problems can be solved, there exists no comprehensive, detailed theory relating the resistance of the wire to the phenomena that occur when the metal lattice is heated through its transition points, melts, is further heated, vaporizes and commences to expand. At present there appears to be little prospect that such a theory can be readily developed on the basis of existing knowledge of the solid and liquid states.

We attempt here to shed light on the early heating and expansion phases of exploding wires by a semi-empirical, heuristic approach which bypasses the fundamental theoretical questions raised in the preceding paragraph and replaces them at the outset by an explicit assumption specifying the resistance as a function of time. When the implications of this assumption for the electrical behavior of the circuit and the heating of the wire are understood, refinements aimed at improving the basic hypothesis can be undertaken. Throughout the remainder of the discussion we shall seek the simplest theoretical formulation adequate to represent the selected portions of the wire explosion.

2. THEORY

2.1 Energy Conservation.

Under equilibrium conditions the energy E added to a material can be expressed as a Stieltjes integral. For the exploding wire, which we know from the conditions of the experiment is not heated under equilibrium conditions, this representation may provide a reasonable starting point. Accordingly we write

$$\int_{E_1}^{E_2} dE = \int_{t_1}^{t_2} \frac{dE}{dT}\frac{dT}{dt}\,dt + \sum_j \Delta E_j , \tag{2.1}$$

where $E = E(T)$ and absolute temperature, $T = T(t)$, is a function of time only. ΔE_j refers to the energy absorbed at the j^{th} transition where the $j = 0, 1, 2 \ldots$ correspond to crystal structure, melting, boiling and other transition points of the material.

If we assume that between transition temperatures energy is given by $E = mC_v(T - T_1) + E_1$ where m is mass of wire and C_v its specific heat, and if we neglect radiation, conduction, thermal expansion and other energy losses from the wire, then between transition points

$$\frac{d}{dt}(mC_vT) = Ri^2 , \tag{2.2}$$

where Ri^2 represents the electrical power dissipated in the wire.
During a transition, e.g. melting, we can write

$$\Delta E_j = \int_{t_j}^{t_j + \Delta t_j} Ri^2 \, dt = m H_j , \tag{2.3}$$

where H_j refers to the latent heat at the j^{th} transition, Alternatively,

$$H_j \frac{dm}{dt} = Ri^2 \qquad t_j \leq t \leq t_j + \Delta t_j \tag{2.3a}$$

during a phase change or other transition. For the most part we shall be concerned with exploding wire behavior between transition points even though analysis of the data can shed some light on the rate at which mass is transformed, as in Equation (2.3a), and may show departures from equilibrium conditions.

2.2 Circuit Equation

For low frequency experiments ($f \leq 20$mc) the usual exploding wire circuit contains variable resistance, inductance and fixed capacitance. In writing a generalized form of Kirchhoff's second law for the circuit, if we assume the magnetic flux to be given by $\phi = Li$, the possibility of an emf arising from motion of the circuit via the $i(dL/dt)$ term must be considered. Since the wire inductance is given by a function whose form is like $\ell \; \ln(\ell/d)$ where the length ℓ remains fixed but diameter d can vary, and since the circuit inductance will usually be an order of magnitude larger than that of the wire it is easily seen that the wire inductance must change by an order of magnitude or more in times like 0.1 μsec before the motional emf term can become important. Accordingly we write

$$L \frac{di}{dt} + Ri + q/C = 0 , \tag{2.4}$$

where the symbols have their usual meaning. The inductance $L = L_c + L_w$ is dominated by the circuit contribution L_c; thus $L_w << L_c$ for almost all times. Resistance in the circuit is, after a brief heating period, quickly dominated by the resistance of the wire; so the symbol R can be thought of as representing the wire resistance without much error except at early times before heating begins.

In this paper we shall be interested only in regimes where the condenser voltage can be considered constant. In particular, around current peak $q/C \approx 0$ and the decay of current is dominated by R and L very much as if the condenser had been crowbarred. This is the situation for the well-matched wires of an earlier study.[12] A second case of interest includes many of the smaller wires which explode during a very short interval, usually on the current rise; and, because the condenser voltage remains nearly constant during this interval, can be treated under this approximation.

2.3 Resistance Function

Because of the great difficulties involved, we make no attempt here to solve either the fluid dynamical problem or that of connecting the internal state of the wire with its electrical resistance. In order to account for both the dynamical and thermal phenomena in the wire we assume that wire resistance, R is a function of absolute temperature T and dimensionless time $\tau s = t - t_o$ where τ is a time constant to be determined later. The form of the function is approximated, to first order in each of the variables, by the simplest possible assumptions in the following way.

When no fluid motions occur, the resistance of a wire is known to increase, over limited ranges, linearly with absolute temperature; thus a product function is suggested and we write

$$R = R(T,s) \tag{2.5}$$

$$= R_o^* (s)[1 + \alpha(T - T_o)]. \tag{2.5a}$$

If no motions occur the resistance depends on time only through the temperature, and

$$R = R_o[1 + \alpha(T - T_o)] \ . \tag{2.5b}$$

To account crudely for fluid motions, we note that the head of an expansion wave propagating into undisturbed medium,[1, 3, 5] travels with constant velocity, certainly the simplest possibility; but, more generally, the front characterized by a critical density below which conduction ceases[2] may travel with a velocity $v = v(s)$ dependent on time.

We now suppose the wire material to exist in <u>two states only</u>, viz., 1) fully conducting material not yet affected by the expansion wave, and 2) expanded, non - conducting material. Thus, for a constant velocity wave the radius of the conducting part diminishes with time as $r = r_o[1 - \frac{v}{r_o}(t - t_o)]$, where t_o is a time chosen to represent the start of the expansion. With this model it is clear that the conduction process must terminate at the finite time when $r = 0$. Since wire resistance varies inversely as the cross-sectional area, we have from Equation (2.5a)

$$R_o^*(s) = R_o/[1 - \frac{\tau}{r_o}\int_o^s v(x)dx]^2 \tag{2.5c}$$

for the case in which the velocity of the expansion front may vary with time. Even to specify the velocity of the front is a formidable task which cannot truly be separated from the complete fluid dynamical problem. To gain insight into the implications of these hypotheses we simplify one step further with the assumption that the front velocity is constant. Then, with $\tau = r_o/v$ and $\tau s = t - t_o$ we can write

$$R_o^* = R_o/(1 - s)^2. \tag{2.5d}$$

2.4 Expansion Wave.

We examine the effects of a constant velocity expansion wave alone by assuming temperature effects are negligible, i.e., $\alpha(T - T_o) \ll 1$, and the condenser voltage sensible constant as explained in §2.2. Then with the definitions $\tau_c = L/R_o$, $a = \tau/\tau_c$, $b = q\tau/LC$ we can combine (2.5d) and (2.5a) with (2.4) to obtain

$$\frac{di}{ds} + \frac{ai}{(1 - s)^2} = - b \ , \tag{2.6}$$

which is a linear, ordinary differential equation of first order. The solution is well known and can be written

$$i = i_o e^{-\frac{as}{1-s}} - b\, e^{-\frac{a}{1-s}} \int_o^s e^{-\frac{a}{1-x}} dx \, , \qquad (2.7)$$

with $0 \leq s \leq 1$ and the understanding that $i = i_o$ and $t = t_o$ refer to the current and time at which the expansion begins.

The case of greatest interest to us is that for which $b = 0$. The resulting current function is plotted in Figure 1 for several values of parameter a. Figure 2 shows the current decay curve for a 5.5 mil Cu wire (see also Fig. 9 of ref. 12 which shows similar data for a 5 mil wire). From about 0.6 μsec on, the experimental curve resembles those for $a \sim 0.5$ in a striking way. It is easily seen that $di/ds = -ai_o(1-s)^{-2} e^{-\frac{as}{1-s}}$, from which $(di/ds)_o = -ai_o$ and $(di/ds)_1 = 0$. By a process of curve fitting we can deduce an average wave velocity from the experimental data. Such fittings yield velocities an order of magnitude smaller than the estimated sound velocities in the wire just prior to expansion and therefore contradict the theoretical expectations of previous authors[1, 3, 5] based on the unconfined expansion of a perfect gas at uniform initial temperature.

Further information can be obtained by examining the voltage and power functions. These are

$$V_R = Ri = R_o i_o (1 - s)^{-2} e^{-\frac{as}{1-s}} \, , \qquad (2.8)$$

and

$$P_R = Ri^2 = R_o i_o^2 (1-s)^{-2} e^{-\frac{2as}{1-s}} \, . \qquad (2.9)$$

One finds easily that these curves have maxima at $s = 1 - \frac{a}{2}$ and $s = 1 - a$ respectively; thus, the maximum in power always precedes the maximum in voltage by $\Delta s = a/2$. Examination of Figure 1 and the curves of reference 12 show that this theory correctly predicts the order of the maxima in every case. For those cases in which restrike is absent, a measurement of the peak separation gives an estimate of the expansion wave velocity, providing r_o, L and R_o are known. The voltage peak may occur too early if there is a restrike; but, on the other hand, the possibility exists that because of the rapidity of the expansion, restrike may not have noticeable effects until after the voltage peak caused by resistance rise is passed. The peak separation is small in any event and the velocity determination cannot be of high accuracy.

We point out again that when $b \neq 0$ we can, in a similar way, discuss the small wires already mentioned in § 2.2; however, for any of the foregoing examples the neglect of thermal effects is serious because these effects are not small. During the time of the hypothetical wire expansion, estimates show that $\alpha(T - T_o) \sim 1 - 10$ and consequently ought not to be neglected.

2.5 Thermal Effects

The effect of temperature rise on current via the resistance can be examined separately from that of any expansion process by letting $R_o^*(s) = R_o$ in Equation (2.5a) and combining with Equation (2.4). We examine only the case near current maximum, and after wire melting, where $q/C \approx 0$. Furthermore, we restrict the temperature away from any transition points so that Equation (2.2) may apply. The temperature rise may be found by combining Equations (2.2) and (2.4) to yield

$$mC_v(T - T_o) = \frac{L}{2}(i_o^2 - i^2) \tag{2.10}$$

As mentioned before, the energy is stored in the magnetic field as shown by Equation (2.10). If we let $p = 1/\tau_c = R_o/L$ and $g = \alpha R_o/2mC_v$ the equation of the circuit, Equation (2.4), can be written as

$$\frac{di}{dt} + i(p + gi_o^2) - gi^3 = 0, \tag{2.11}$$

which is converted to a linear equation of first order by the substitution $w = 1/i^2$. The solution is

$$i = i_o \left[\frac{p + g\,i_o^2}{p\,e^{2(p+gi_o^2)t} + g\,i_o^2} \right]^{\frac{1}{2}} \tag{2.12}$$

The ratio $gi_o^2/p = \dfrac{Li_o^2/2}{mC_v/\alpha}$ is the quotient of magnetic stored energy by the thermal energy necessary to heat the wire through the interval $1/\alpha \sim 300^oK$. In our typical cases[12] this ratio is of order $10^2 - 10^3$. It is readily shown that the current of Equation (2.12) decays from i_o with the initial slope $(di/dt)_o = - pi_o$ and at large times exponentially as $e^{-gi_o^2 t}$; thus it has a form similar to shape to that already found for the expansion wave hypothesis except that in this case the decay does not terminate at a finite time.

2.6 Expansion and Thermal Effects Combined.

An equation can be obtained for both effects just discussed by using the methods and assumptions of the preceding sections. One has also to consider that the mass being heated as in Equation (2.2) varies like $m = m_o(1 - s)^2$ on account of the expansion wave. The circuit equation becomes

$$\frac{di}{ds} + \frac{ai}{(1 - s)^2} = \frac{\alpha a i}{(1 - s)^4} \left[T_o s(s - 2) - \frac{L}{2m_o C_v} (i_o^2 - i^2)\right], \qquad (2.13)$$

and the term on the right-hand side might be considered a perturbation of Equation (2.6) with $b = 0$ if α were sufficiently small; however, an exact solution can be obtained by use of the variable change $w = 1/i^2$ as before. The result is much less transparent analytically than Equations (2.7) and (2.12) and will not be quoted here. As will appear in the next sections the hypotheses under which Equation (2.13) is derived are probably unrealistic. A combined graphical and analytical approach over limited temperature ranges is found to be more nearly correct.

3. EXPERIMENTAL

3.1 Resistance-Time Data.

The data analyzed here are taken from voltage and current measurements on copper wires described in detail previously.[12] With the assumptions of § 2.4, viz., constant velocity wave and negligible temperature variation of resistivity, plots of $R^{-1/2}$ vs. time should show a linear portion during the interval between the onset of the expansion wave and before any restrike occurs. After restrike, or electrical breakdown in the metal vapor, the resistance drops to low values, and consequently $R^{-1/2}$ will increase.

Representative plots of $R^{-1/2}$ against time are shown in Figure 3, where nearly linear behavior during an appropriate interval is noted. The slope of the linear portion yields directly a value for the wave velocity. Values of the wave speed deduced from plots of data from 3 to 5 mil copper wires at different condenser voltages vary between 200 to 400 meters per second. In one test of a 6.3 mil wire, this deduced velocity was only 80 meters per second.

3.2 Experimental Determination of the Wave Speed

In the preceding sections a model for the expansion is presented which accounts quite well for the rapid resistance increase of the experimental data. On the basis of this model a more refined analysis of the experimental data is undertaken here in order to acquire detailed information about the postulated expansion wave. With the assumption that the electrical conductivity drops to zero behind the expansion wave, and with a simplifying assumption for the temperature dependence of resistivity in the unexpanded molten metal, it should be possible to deduce values of the radius of the conducting core of the wire. Knowledge of these values as a function of time then yields the wave speed after differentiation.

With no vaporization wave the resistance would be simply $\rho \ell/(\pi r_o^2)$ where ρ is specific resistivity and ℓ the length. In what follows, it is assumed that ρ is only a function of temperature; thus, plots of "scaled resistance", $R\pi r_o^2/\ell$, versus temperature should give a universal curve, for different wire sizes and condenser voltages, during the interval prior to onset of the expansion wave. Deviations from this universal curve are taken as an indication that the conducting cross section is decreasing. The wire temperature, which is assumed to be uniform, can be deduced from the known energy input, E, and the mass of the wire from the relation,

$$\int C_v \, dT = \int \frac{dE}{m} . \tag{3.1}$$

So long as the wire does not vaporize, the mass, m, remains constant, equal to m_o, and the right-hand side of Equation (3.1) is just E/m_o. Since the specific heat, C_v, is not very well known at the high temperatures encountered here, it is advantageous to plot the scaled resistance versus E/m_o instead of T. Plots of this kind ($R\pi r_o^2/\ell$ versus E/m_o) have been made previously by Webb et al[13] and by Tucker.[14] The data plot of Webb et al shows nearly a single curve for small values of E/m_o, and deviates for larger values, which deviation is taken as evidence of vaporization of the wire. Data taken at this laboratory[12] on copper wires, agree well with those of Webb et al on copper. A scaled resistance-energy diagram for some of this data is shown in Figure 4 where for small values of E/m_o, all data points cluster about a single curve and the scaled resistance values increase with energy in accord with other experimental results[15] through the heating, melting, and further heating of the liquid copper. Just after melting, the resistivity increases almost linearly with energy up to about 2.5 kilojoules per gram. Beyond this energy, the curves for the different wires deviate from the linear law and from each other. This suggests that wires have begun to vaporize, and that the resistance depends not only on the energy, but also on the radius of the conducting core, according to Equation (2.5c).

In order to find the instantaneous wire radius, and from its derivative the wave speed, it is necessary that the dependence of resistivity on temperature be known. Unfortunately, no reliable values at high temperatures are available; however, from the experimental values up to 2.5 kilojoules/gram and from theoretical expectations, a linear increase of resistivity with specific energy is suggested. This assumed linear law is therefore used to extrapolate to energies higher than the 2.5 kilojoules/gram. Although this extrapolation is somewhat risky, it will certainly not be greatly in error for specific energy values just above 2.5 kilojoules/gram, and for higher values, a given error does not have a large effect on the evaluation of the wave speed. With the notation $e = \int dE/m$, it is thus assumed that the resistivity of the liquid copper is represented by

$$\rho = \rho_0 \, [1 + \beta(e - e_0)] \, , \tag{3.2}$$

where β is the slope of the linear part of the scaled resistance curve just beyond melting. The total resistance for energies beyond vaporization is then

$$R = R_0 \, [1 + \beta(e - e_0)]/(r/r_0)^2 \, , \tag{3.3}$$

from which

$$(r/r_0)^2 = (R_0/R)(1 + \beta \, [e - e_0]) \, . \tag{3.4}$$

Here e refers to the specific energy of the uniformly heated conducting material within radius r. In Figure 4 the scaled resistance is graphed as a function of E/m_0 rather than e, but in order to evaluate the radius from Equation (3.4), the resistance must be compared with the resistivity from Equation (3.2) at corresponding values of e. Although e and E/m_0 are identical before vaporization, they differ afterwards because the mass m of the conducting core changes. With the assumption that the conductivity drops to zero behind the expansion wave, the mass of conducting core is given by

$$m = m_0(r^2/r_0^2) \; ; \tag{3.5}$$

and since the additional energy is added to the conducting core,

$$dE = m \, de = m_0(r^2/r_0^2)de \, . \tag{3.6}$$

Multiplying Equation (3.6) with (3.3) then gives the differential equation

$$RdE = R_0 m_0 \, [1 + \beta(e - e_0)]de \; ; \tag{3.7}$$

from the graph R is given as a function of $\frac{E}{m_o}$, and hence Equation (3.7) can be integrated to obtain

$$\int_{E_o}^{E} \frac{R}{R_o} \frac{dE}{m_o} = e - e_o + (\beta/2)(e - e_o)^2. \tag{3.8}$$

Equation (3.8) yields the desired function, E(e), via numerical integration. The connection between E and R is given experimentally as in Figure 4; thus (e,R) pairs can readily be obtained for use in Equation (3.4) to obtain values of (r/r_o). Finally, through the experimental R(t) relation, (r/r_o) can be plotted as a function of time and differentiated numerically. Wave speeds so obtained are plotted versus the specific energy, e, in Figure 5. One sees that data for different wire explosions, i.e. different wire sizes and condenser voltages, essentially fall on a single curve. In addition to the specific energy scale on the abscissa, a temperature scale is also shown; this scale is computed assuming constant specific heat, and must therefore be only approximately true, especially for the higher energy. Thus, the wave speed is seen to depend only on the temperture or equivalently on the specific energy, as might be expected on physical grounds.

For temperatures below 4000°K, the wave speed is found to be practically zero; above 4000°K, it increases quite rapidly, and then more slowly reaches values of about 200 meters per second for temperatures around 15000°K. This asymptotic value for wave velocity is surprisingly small being less than one-tenth the sound speed in the liquid. Thus, the supposition of Keilhacker[3] that the expansion wave proceeds with sound speed of liquid copper can be ruled out.

4. EXPANSION WAVES IN A TWO-PHASE SYSTEM

4.1 Thermodynamics of a Vaporization-Expansion Wave.

We give here a thermodynamic analysis of the vaporization-expansion of a liquid. To our knowledge, this approach has not previously been made, but nevertheless appears suitable for interpretation of a variety of phenomena involving superheated liquids. One interesting result is the prediction of the conditions under which relatively slow disturbance waves propagate into the liquid.

A pressurized liquid can expand only slightly without changing its phase from liquid to vapor when the pressure is released. This small expansion of the liquid phase depends on the liquid compressibility, which is ordinarily small.

Greater expansion then exceeds the ability of the fluid to sustain tension, and cavitation occurs which results in some vaporization of the liquid. The fluid then becomes a two-phase thermodynamic system where liquid and vapor coexist. For the case of exploding wires this damp vapor, a mixture of liquid and vapor, may bear a resemblance to the foam suggested by Chace.[16] At early times, the damp vapor is at its lowest temperature and contains a large percentage of liquid. As the vaporization wave proceeds inward, it encounters hotter liquid which has been heated longer by the electrical current. Thus, the inner portion of the damp vapor should contain a relatively smaller fraction of liquid than the outer portion and should exist initially at higher temperatures and pressures, thereby producing a strong pressure gradient causing the mixture to expand energetically into and through the cooler outer parts.

This model does not encounter the structure-insensitive nucleation problem discussed by Chace; for nucleation need only take place at the liquid-wet-vapor separation where surface effects predominate and where the boundary conditions presuppose an interface. Actual conditions at the wire interface prior to and during expansion may be very complicated; for in addition to irregularities due to randomly fluctuating atomic and molecular fields, and favored sites for evaporation due to absorbed or dissolved gases, there will be present thermionically emitted electrons and ions. One therefore would not expect any considerable superheating caused by inability to surmount the energy barrier between the liquid and vapor states.

A situation closely similar to our proposed model of the exploding wire expansion is realized physically in the superheated water shock tube used by Terner.[17] Consider a liquid confined to the driver section of an ordinary shock tube, and heated to high temperature and pressure. Upon release of pressure by breaking the diaphragm, an expansion wave travels from the low pressure side into the liquid. The first part of the expansion wave proceeds with ordinary sound speed in the liquid, lowering the pressure to the vapor pressure of the liquid; this vapor pressure depends only on the liquid temperature. Subsequent expansion will turn part of the liquid into vapor. Thus, the expansion can be thought of as accomplished by a vaporization wave, across which occur changes in the liquid - vapor ratio and the dynamical variables. If the initial pressure of the liquid driver were just the vapor pressure for that temperature, as it would be if a vapor bubble were included, then the first liquid expansion down to the vapor pressure would be avoided; only the vaporization-expansion wave would occur. It is true that a liquid can sometimes be overexpanded in the liquid state below the vapor pressure, just as condensing gases can be somewhat supersaturated, but because the surface is a site of fluctuations that will prevent overexpansion, it is assumed that the effect here is small, and equilibrium thermodynamics can be used. The vaporization wave now proceeds with wave speed different from the sound speed of the liquid because the adiabatic compressibility of the damp vapor is much greater than that of the liquid.

To express these ideas in mathematical form we proceed as follows. Quite generally, the speed of small amplitude disturbances, c, is found from

$$c^2 = \left(\frac{dp}{d\rho}\right)_{ad} , \tag{4.1}$$

where p is pressure, ρ is density, and the subscript "ad" refers to an adiabatic change. Using the specific volume $v = 1/\rho$, Equation (4.1) becomes

$$c^2 = -v^2 \left(\frac{dp}{dv}\right)_{ad} . \tag{4.2}$$

The equation of state is assumed to be

$$p = p(v,T) , \tag{4.3}$$

and

$$\left(\frac{dp}{dv}\right)_{ad} = \frac{\partial p}{\partial v} + \frac{\partial p}{\partial T}\left(\frac{dT}{dv}\right)_{ad} . \tag{4.4}$$

For an adiabatic change, the second law of thermodynamics gives

$$TdS = \frac{\partial e}{\partial T}\, dT + \left(\frac{\partial e}{\partial v} + p\right) dv = 0 , \tag{4.5}$$

where e is specific energy; so

$$\left(\frac{dT}{dv}\right)_{ad} = -\left(\frac{\partial e}{\partial v} + p\right)/\left(\frac{\partial e}{\partial T}\right) . \tag{4.6}$$

Setting $\frac{\partial^2 S}{\partial v \partial T} = \frac{\partial^2 S}{\partial T \partial v}$ in the first of Equations (4.5) gives

$$\frac{\partial e}{\partial v} + p = T\frac{\partial p}{\partial T} , \tag{4.7}$$

and with Equation (4.6) we obtain:

$$\left(\frac{dT}{dv}\right)_{ad} = -\frac{\partial p}{\partial T}\,(T/c_v) . \tag{4.8}$$

With the above relations, Equation (4.2) becomes

$$c^2 = - v^2 [\frac{\partial p}{\partial v} - (\frac{\partial p}{\partial T})^2 \, (T/C_v)] \tag{4.9}$$

or, alternatively,

$$c^2 = \frac{\partial p}{\partial \rho} + (1/\rho^2) \, (\frac{\partial p}{\partial T})^2 \, (T/C_v) \; . \tag{4.10}$$

For a perfect gas where $C_p - C_v = R$ and $C_p/C_v = \gamma$, this yields $c^2 = \gamma RT$, the usual form for the small amplitude wave speed; however, for a two-phase system where the pressure is just the vapor pressure, $p(T)$, independent of v or ρ, Equation (4.10) specializes to

$$c = (1/\rho) \frac{dp}{dT} \, [T/(C_v T)^{\frac{1}{2}} \;] \; . \tag{4.11}$$

In evaluating Equation (4.11) for the head of the vaporization wave traveling into the liquid, values of ρ and C_v are just those for the liquid.

Most vapor pressure curves can be represented fairly well by

$$p = A \, T^{\alpha} \exp[-(T_o/T)] \; , \tag{4.12}$$

where the constants A, α and T_o can be determined from experimental values. For the present tests with copper, the following vapor pressure formula was adapted from reference (15), using logarithms to base 10,

$$\log p = 13.5 - (17{,}700/T) - 1.275 \log T, \tag{4.13}$$

where pressure is in mm Hg and temperature in degrees Kelvin. With $C_v = 0.12$ cal/gm-deg, and $\rho = 5.3$ gm/cm^3, the values for c are found from Equation (4.11), and the results plotted as the theoretical curve in Figure 5. The curve ends at the critical temperature, about 9000°K for copper.

Formula (4.11) is no longer applicable at temperatures higher than critical, and must be replaced by the more general form, Equation (4.10); however, Equation (4.10) cannot be evaluated before more precise thermodynamic data for copper beyond critical temperatures is known. For these reasons the ability of Equation (4.10) to describe the data above critical temperature cannot be determined at present; however, even at these higher temperatures the correlation of the data to a single curve is striking and strongly suggests that the wave speed is uniquely related to specific energy.

In the temperature range of its validity, Equation (4.11) yields results agreeing quite well with the experimental values in Figure 5. In particular, it predicts astonishingly well the onset of significant velocity of the vaporization wave at about $4000^{o}K$. The determining factor here is clearly the large negative exponent in the vapor pressure law, i.e. the term $T_o (\doteq \Delta H_{vap}/\hat{R})$ which depends on the heat of vaporization of the material. A good estimate of the temperature at which the wave velocity becomes appreciable may be obtained as follows.

The exponential contained in dp/dT dominates the expression for c given in Equation (4.11). It is therefore permissible to regard the various powers of T as practically constant in the termperature range of interest. One then finds the inflection point of the curve, essentially the exponential term only, and the slope at that point. Passing a line through the point of inflection with the slope of the tangent there, locates an intercept on the abscissa which may be taken to define the onset of wave velocity. The intercept is $T_w = T_o/4$, or about $10,000^{o}K$ for copper. As seen from Figure 5 this value agrees well with the knee of the data curve and significant velocities occur at temperatures lower by a factor of two. Wire materials can now be ranked in ascending order of heats of vaporization and the temperatures at which the vaporization wave will occur can be predicted with some confidence by use of the inequality $T_o/8 \leq T$. One further observation can be made. For elements with the higher heats of vaporization and correspondingly higher temperatures T_w, the current decay in the exploding wire circuit may be dominated by resistance dependence on temperature as developed to first approximation in § 2.5. Much larger amounts of energy, stored in the magnetic field, would be needed to induce a rapid expansion wave, and in some experimental arrangements may not be available. The result would be a slow, or "weak," expansion at the end of the heating process.

4.2 Discussion.

It is notable that the temperature where significant vaporization begins is considerably higher than the boiling point (for Cu b.p. is $2855^{o}K$). Both David[2] and Keilhacker[3] attribute this occurrence to the elevation of the boiling temperature by the magnetic pinch pressure. In analyzing the present data, no correlation with magnetic fields was made, nor was any needed. Moreover, the present view of the vaporization-expansion wave is independent of the magnetic field. The pressure on a wire carrying uniform current is largest at the center and decreases parabolically to its lowest value at the outermost radius of the conductor. This lowest value is a boundary condition, and for negligible vaporization is just the ambient pressure on the conductor surface. At high temperatures where vaporization is significant, the surface of the conductor is inclosed by a nonconducting damp vapor. The lowest pressure at the outer conducting radius is just the vapor pressure of the material appropriate to its temperature; thus, the vaporization-expansion occurs from a surface whose pressure is simply the vapor pressure, and the magnetic field has no influence in suppressing the onset of significant vaporization. For very rapid exploding wire circuits, where the current distribution is dominated by skin effects, magnetic pressure may be a controlling influence, but this is certainly not the case for most of the experiments cited.

The vaporization-expansion wave appears to be capable of accounting in a satisfactory way for a great deal of the resistance-time data observed in wire explosions; thus, we may consider that the "resistance anomaly," which has been a puzzle for many years, is now satisfactorily explained. There remain, however, several areas for further study with regard to resistance anomalies. For example, some of the highest current density data of Tucker[14] fail to show the expected melting-point rise in the early heating period and thus point to the existence of possible nonequilibrium effects in the metal interior. The effects of composite structure in an exploding wire as in the experiments of Reithel and Blackburn,[18] and Bennett,[19] remain to be explained although here a basic feature will probably be the partial suppression and delay of the vaporization-expansion wave by either a nonconducting coating or one whose onset temperature T_w is higher than that of the core. The effects of adsorbed gases, as in the H-Pd system,[19] will be strongly felt in the resistivity function, where a negative coefficient can result.

We emphasize again that the vaporization-expansion characterized here as a wave motion, differs radically from the gas-dynamic or hydrodynamic expansion waves invoked by other authors.[1, 3, 5, 13] Its velocity is a factor of ten smaller than the sound velocity in a liquid. A phase change occurs across the vaporization wave, in contradistinction to the usual fluid dynamic waves where no change in state occurs*; and the wave velocity depends not only on the specific energy, but also on the heat of vaporization as a parameter.

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* A possible exception would be the condensation shocks encountered in wind tunnel nozzles.

5. SUMMARY

1) A survey of exploding wire research shows that the idea of an expansion wave into a hot, perfect gas has been evident in the idealizations used to interpret exploding wire data.

2) Examination of the conditions in the R-L-C circuits used to explode wires shows that near current maximum, and otherwise only during short time intervals, a simple mathematical treatment is possible for an expansion wave that converts the wire material behind it into a nonconductor. The resulting curves closely resemble the observed current decay in the circuit and from them, or equivalently from the assumed resistance law, curve-fitting to the experimental data provides values of velocity for the expansion wave. These are a factor of ten lower than the sound velocity in the metal wire.

3) With this background, a careful analysis is made of resistance data from exploding wires of copper, and plots of wire resistivity versus specific energy are obtained from which values of the expansion front velocity can be deduced by numerical methods. The velocities derived from several different experimental conditions correlate to a single curve as a function of specific energy.

4) A thermodynamical analysis of the small disturbance velocity in a two-phase, single component system gives an expression dependent mainly on the equilibrium vapor-pressure function for the wire material. Theoretical and observed values agree quite well up to the critical temperature beyond which thermodynamical data are lacking.

5) The quantitative agreement obtained between theoretical and experimental values of wave velocity, and the correlation of data from many separate experiments under different conditions to specific energy as independent variable, support the important conclusion that the so-called "resistance anomaly" of exploding wires can be understood through the vaporization-expansion wave hypothesis.

6) The vaporization-expansion wave characterized here in quantitative terms is a phenomenon new to fluid mechanics and can be expected to play an important role in the understanding of the dynamics of superheated liquids.

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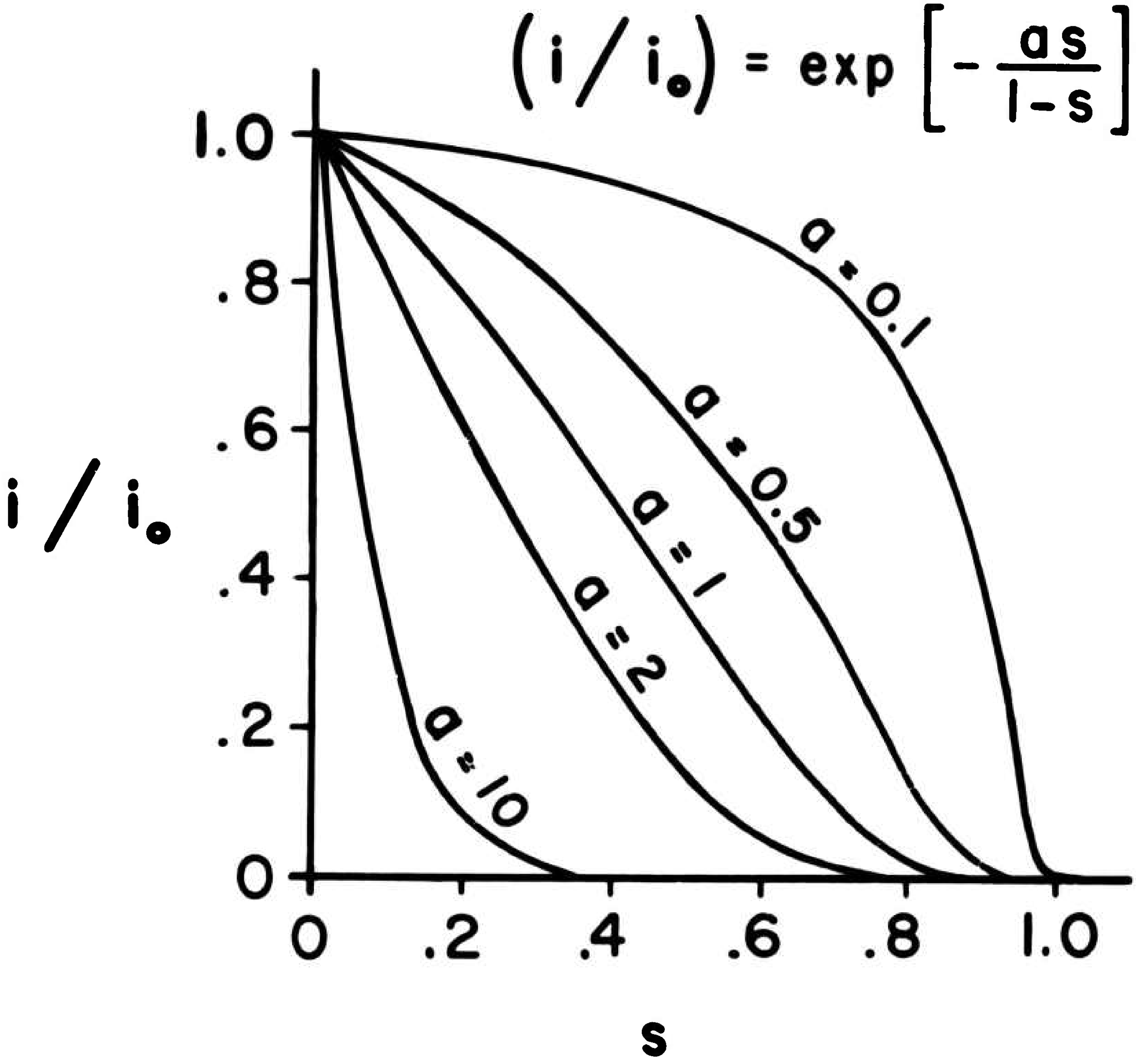

Figure 1

Current function for constant velocity expansion wave

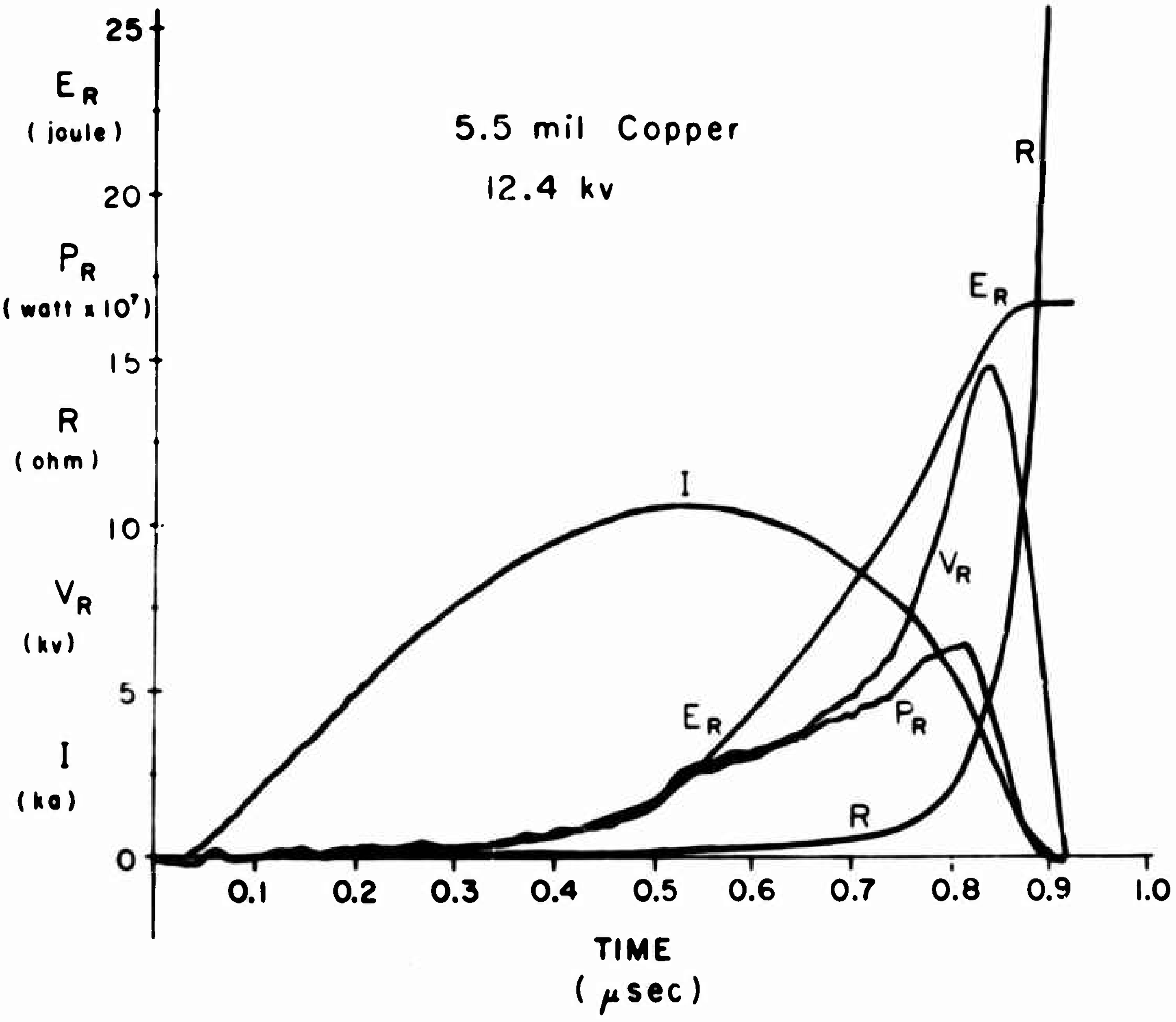

Figure 2

5.5 mil Cu wire exploded at 10.8 kv. I - current, V_r - voltage across the wire, R - wire resistance, P_r - power, E_I - energy deposited.

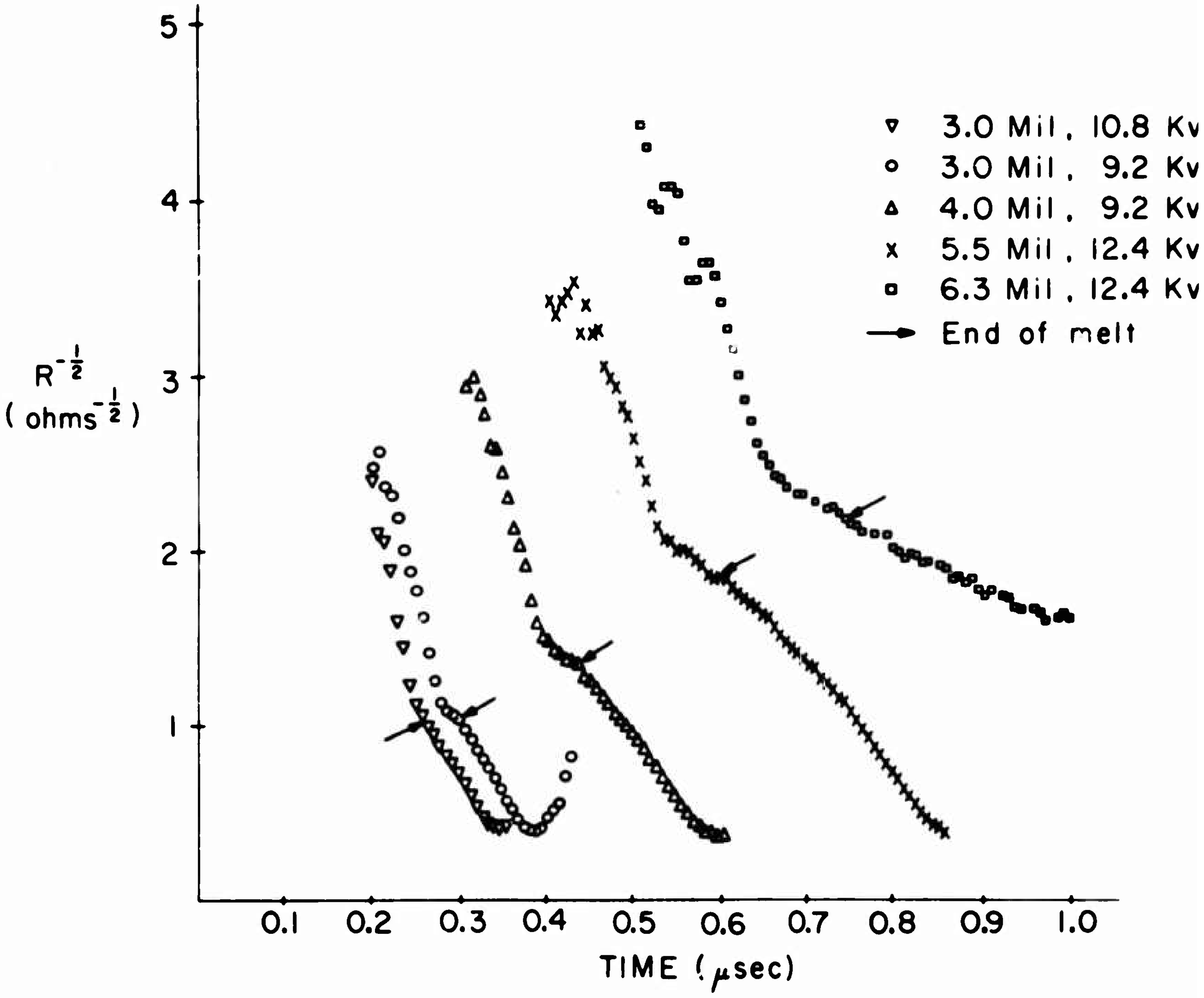

Figure 3

Plots of $R^{-1/2}$ versus time.

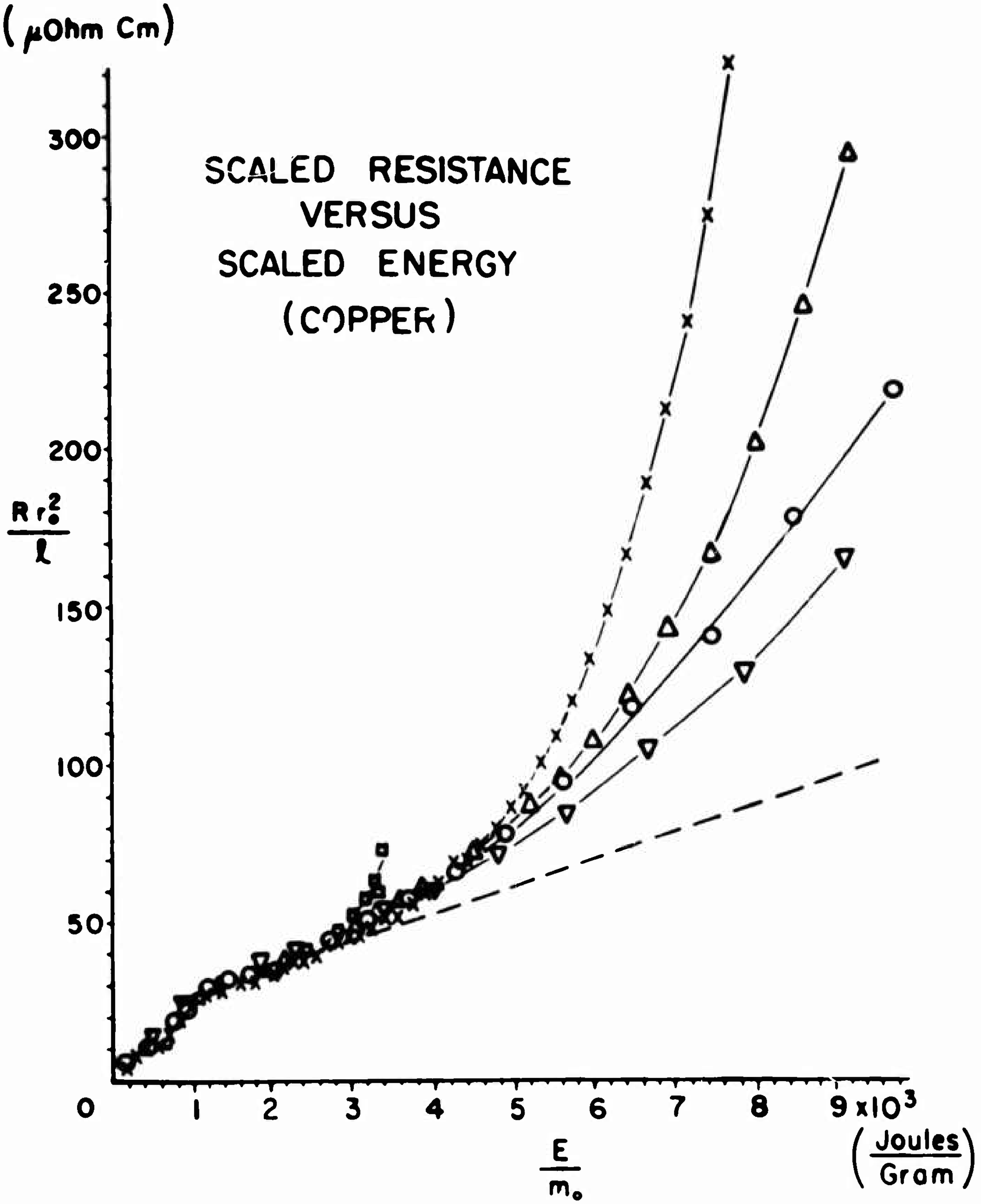

Figure 4

Scaled resistance versus energy. Symbols correspond to the examples of Fig. 3

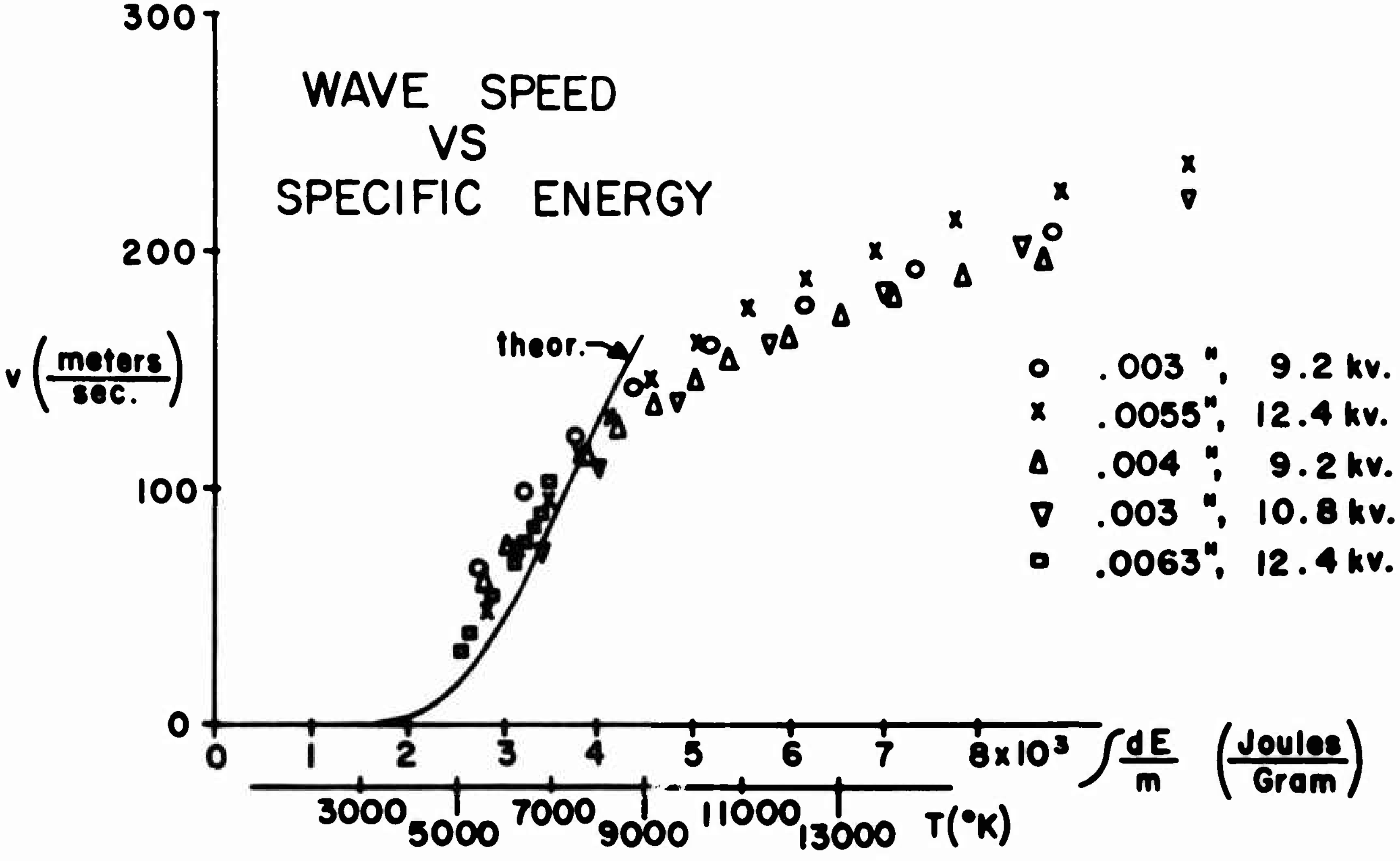

Figure 5

Vaporization-expansion wave velocity versus specific energy