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NONEQUILIBRIUM SODIUM IONIZATION IN LAMINAR BOUNDARY LAYERS

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ABSTRACT

The injection of sodium as a trace contaminant into a laminar boundary layer is considered. Three ionizing reactions are studied and it is found that the ionization by collision with atomic oxygen is the most significant. The peak electron concentration is determined as a function of the scaling parameter, Px/U_e. Comparisons are made with the predicted clean air variations with the same parameter. Stream tube predictions (neglecting diffusion) are also made and compared with the more exact results. The relative ionization of clean air and sodium is assessed as a function of altitude and amount of injection material. Bortner's rates are used for the nonequilibrium sodium ionization.

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I. INTRODUCTION

The effects of mass transfer in high-speed flow have been a problem of continuing interest in the design of vehicles subjected to intense aerodynamic heating. To gain heat protection, gases or liquids can be actively injected into the boundary layer at controlled rates, a method known as transpiration cooling or mass transfer cooling. Foreign species can also be injected into the boundary layer at a rate not as positively controlled, if the vehicle is provided with an ablating surface.

The injection of species that do not chemically react with the air in the boundary layer has been studied in some detail[1, 2, and 3]. However, because the boundary layer equations become much more complicated and because of a lack of sufficient chemical kinetic data, little work has been done on mass transfer into a chemically reacting non-equilibrium boundary layer. One problem of this type which is quite significant and which lends itself to a ready solution is the problem of trace contaminants.

An ablation heat shield can in manufacturing or handling be contaminated with very small amounts of certain compounds. While insignificant with respect to their total weight, they can be significant, if they are highly ionizable, and when injected into a boundary layer become dominant in the production of electrons. The observable characteristics of a vehicle can be determined by these trace contaminants.

This report evaluates the effect of trace contaminants on the boundary layer with mass addition. It assumes that the trace contaminants do not affect the velocity and enthalpy profiles and these profiles are determined by the gross amount of blowing into the boundary layer. Similar solutions are then derived for the distribution of ionized trace contaminants through the boundary layer. Several types of ionizing reactions are considered. It is assumed that the degree of ionization is much less than equilibrium and is determined by the reactions and rates of Bortner.

II. BASIC EQUATIONS AND SOLUTION

Consider a contaminant being injected into the boundary layer along a cone in small enough amounts so that it does not significantly alter the velocity and enthalpy profiles. Then one need only consider the species equation.

Consider the ions, atoms, and electrons associated with the trace contaminants as one species. The governing equation for the distribution of this species through the boundary layer is then:

$$\frac{d}{d\eta}\left\{\frac{\ell}{Sc}\frac{dC}{d\eta}\right\} + f\,\frac{dC}{d\eta} = 0 \tag{1}$$

where $f(\eta)$ represents a known velocity function determined from the gross blowing into the boundary layer[4].

Boundary conditions:

At $\eta = 0$, $C = C_w$
At $\eta \to \infty$, $C = 0$

$$\text{Let } \frac{\ell}{Sc} \text{ be constant and } S = \frac{C_w - C}{C_w} \tag{2}$$

Then Equation (1) becomes

$$\frac{\ell}{Sc}\,\frac{d^2S}{d\eta^2} + f\,\frac{dS}{d\eta} = 0 \tag{3}$$

Boundary conditions:

At $\eta = 0$, $S = 0$
At $\eta \to \infty$, $S = 1$

The solution to Equation (3) is:

$$S = \frac{\int_0^\eta \exp\left\{-\int_0^\eta \frac{Sc}{\ell} f\,d\eta\right\} d\eta}{\int_0^\infty \exp\left\{-\int_0^\eta \frac{Sc}{\ell} f\,d\eta\right\} d\eta} \tag{4}$$

From the momentum equation for constant ℓ it is readily seen that:

$$\exp\left\{ Sc \int_0^{\eta} \frac{f}{\ell}\, d\eta \right\} = \left\{ \frac{f''(0)}{f''(\eta)} \right\}^{Sc} \tag{5}$$

Substituting into Equation (4), we get a more convenient form of solution

$$S = \frac{\int_0^{\eta} \left[f''(\eta) \right]^{Sc} d\eta}{\int_0^{\infty} \left[f''(\eta) \right]^{Sc} d\eta} \tag{6}$$

This represents the distribution of total injected trace species (atoms, ions, and electrons) across the boundary layer.

Consider now the production of ions through the boundary layer where the source term can be represented as a function of η only. The governing species equation is then (neglecting non-similar terms):

$$\frac{\ell}{Sc} \frac{d^2C}{d\eta^2} + f \frac{dC}{d\eta} + \frac{2\, x\, w}{3 \rho\, U_e} = 0 \tag{7}$$

Boundary conditions:

At $\eta = 0$, $C = C_w$

At $\eta = \infty$, $C = 0$

where $w = w(\eta)$

The solution to this equation is:

$$C = C_w + A \int_0^{\eta} \left[f''(\eta) \right]^{Sc} d\eta - \frac{2x}{3U_e} \frac{Sc}{\ell} \int_0^{\eta} \left[f''(\eta) \right]^{Sc} \int_0^{\eta} \frac{w}{\rho} \left[f''(\eta) \right]^{-Sc} d\eta\, d\eta \tag{8}$$

where

$$A = \frac{\frac{2x}{3U_e} \frac{Sc}{\ell} \int_0^{\infty} \left[f''(\eta) \right]^{Sc} \int_0^{\eta} \frac{w}{\rho} \left[f''(\eta) \right]^{-Sc} d\eta \, d\eta \; - C_w}{\int_0^{\infty} \left[f''(\eta) \right]^{Sc} d\eta} \qquad (9)$$

Thus, if the source term w can be described as a function of η only, then Equation (8) will determine the ion distribution through the boundary layer.

III. IONIZATION KINETICS

Sodium as a trace contaminant was studied. The particular ionization reactions considered are listed below. Any other trace contaminant could be studied with the existing programmed solution as long as it were ionized by any or all of these mechanisms and the corresponding rate constants were known.

The particular reactions included, which are those considered most significant for sodium ionization[5], are:

1. $Na + M \rightarrow Na^+ + e^- + M$ (10)
2. $Na + e^- \rightarrow Na^+ + 2e^-$ (11)
3. $Na + O + O \rightarrow Na^+ + e^- + O_2$ (12)

where M refers to any third body (except O, and e^- when reactions 2 and 3 are included).

The forward direction is considered only for each reaction, implying that the reactions are not close to equilibrium. As a check on the validity of this assumption the equilibrium concentration can be computed for each particular case studied and compared with the non-equilibrium prediction of this analysis (see Appendix A).

From the described ionization reactions and the law of mass action[6], an expression for the source term follows as:

$$\frac{w}{\rho} = \rho\, C_{Na} \left\{ \frac{k_1}{M_m} (1 - C_O) + k_2 \frac{C_{e-}}{M_{e-}} + k_3\, \rho \frac{(C_O)^2}{M_O{}^2} \right\} \qquad (13)$$

where

ρ = mixture density

M_m = mean molecular weight (Reaction 1)

C_{Na} = mass fraction of sodium atoms

k_j = rate constant associated with reaction j

C_i = mass fraction of species i

M_i = molecular weight of species i

To a first approximation, the distribution of sodium atoms, C_{Na}, is given by Equation (6). This will be a very accurate representation unless the degree of ionization is very high.

The rate constants associated with each reaction are given by:[5]

$$k_1 = 80T^{3/2}\, e^{\frac{-107,500}{T}}\ \text{ft}^3\ (\text{lb-mole})^{-1}\ \text{sec}^{-1} \qquad (14)$$

$$k_2 = 3.1\ (10)^{11}\ T^{1/2}\, e^{\frac{-107,500}{T}}\ \text{ft}^3\ (\text{lb-mole})^{-1}\ \text{sec}^{-1} \qquad (15)$$

$$k_3 = 10.3\ (10)^{12}\ \text{ft}^6\ (\text{lb-mole})^{-2}\ \text{sec}^{-1} \qquad (16)$$

The mass fraction of electrons acting to ionize sodium by reaction 2 are considered to be those electrons produced by reactions 1 and 3 as well as the electrons associated with the ionized NO in the boundary layer. The clean air distributions of atomic oxygen and ionized NO can be determined by the procedure described in Reference 7. Figure 1 describes the distributions used in this report.

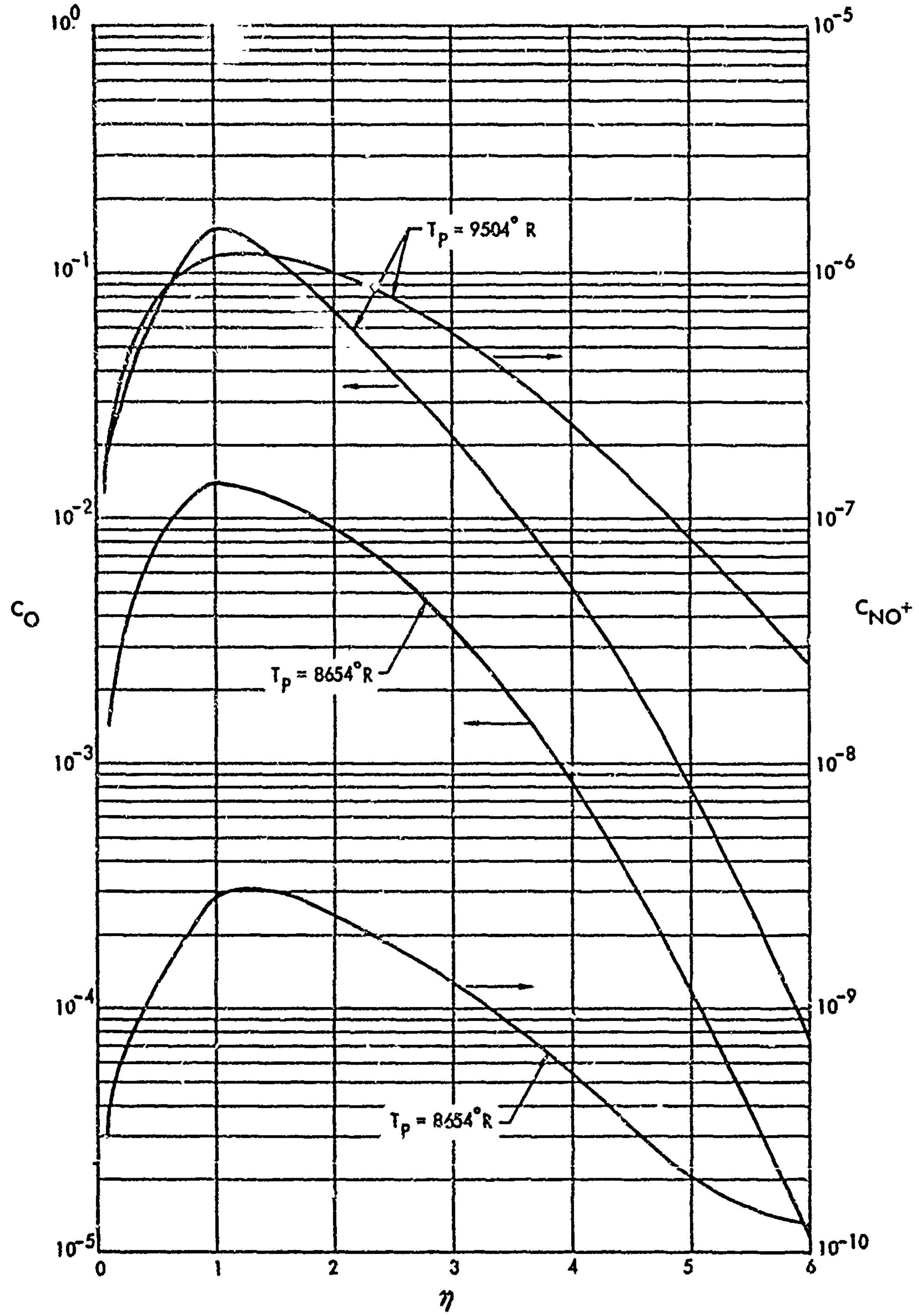

Figure 1. Atomic Oxygen and NO^+ Distributions

The source term is thus completely described as a function of η except for the temperature dependency of the rate constants.

For a Lewis number of unity and a constant Prandtl number, the energy equation is (assuming similarity):

$$f\,g' + \frac{\ell}{Pr}\,g'' + \frac{U_e^2\;\ell}{(h_{se} - h_{sw})}\left(1 - \frac{1}{Pr}\right)\frac{d}{d\eta}\left(f'\,f''\right) = 0 \qquad (17)$$

where $g = \dfrac{h_s(\eta) - h_{sw}}{h_{se} - h_{sw}}$

$h_s(\eta)$ = total enthalpy

$= h(\eta) + \dfrac{U^2}{2}$

prime denotes differentiation with respect to η.

Boundary conditions:

At $\eta = 0$: $g = 0$

At $\eta = \infty$: $g = 1$

The solution to Equation (17) is:

$$g(\eta) = C_1\int_0^\eta \left[f''(\eta)\right]^{Pr} d\eta - \frac{Pr}{\ell}\int_0^\eta \left|f''(\eta)\right|^{Pr}\int_0^\eta H(\eta)\left|f''(\eta)\right|^{-Pr} d\eta\, d\eta \qquad (18)$$

where

$$C_1 = \frac{Pr}{\ell}\int_0^\infty H(\eta)\left|f''(\eta)\right|^{-Pr} d\eta + \frac{1 - \dfrac{Pr}{\ell}\displaystyle\int_0^\infty H(\eta)\left|f''(\eta)\right|^{-Pr}\int_0^\eta \left|f''(\eta)\right|^{Pr} d\eta\, d\eta}{\displaystyle\int_0^\infty \left[f''(\eta)\right]^{Pr} d\eta} \qquad (19)$$

and

$$H(\eta) = \frac{\ell \, U_e^2}{h_{se} - h_{sw}} \left(1 - \frac{1}{Pr}\right) \left[f'(\eta) f'''(\eta) + f''(\eta) f''(\eta)\right] \qquad (20)$$

Equation (18) can be used to describe the total enthalpy distribution through the boundary layer and in conjunction with the known velocity profile gives the static enthalpy or temperature profile. Thus, the source term is completely described as a function of η and Equation (8) can be used to give the ion (and electron) distribution through the boundary layer.*

IV. RESULTS

Solutions to Equation (8) were computed for each of the three ionizing reactions. The Schmidt number for the diffusion of the sodium atoms into the boundary layer, as determined from Equation (6), was considered to have the constant value 0.5. The Schmidt number for the diffusion of the ion-electron pairs as determined from Equation (8) was taken as 0.25. The results are described in Figures 2 and 3 which represent the scaling of electron density versus the parameter Px/U_e. The electron density is scaled in the form $N_e/P^m K_w^n$ where:

N_e = electron density (electrons/cc)

P = pressure (atmospheres)

K_w = wall concentration of sodium

m = order of initial reaction

n = 1 except when the sodium is ionized by an electron which itself is a product of the ionization of sodium, in which case n is equal to two.

The non-equilibrium clean air[7] and sodium equilibrium ionization levels are included. As seen in Appendix B, the equilibrium ionization scales as $\frac{(N_e)_{eq}}{P^{1/2} K_w}$ while the clean air non-equilibrium ionization scales as $\frac{N_e}{P}$.

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*This solution and the equilibrium analysis of Appendix A have been programmed in FORTRAN for the IBM 7094.

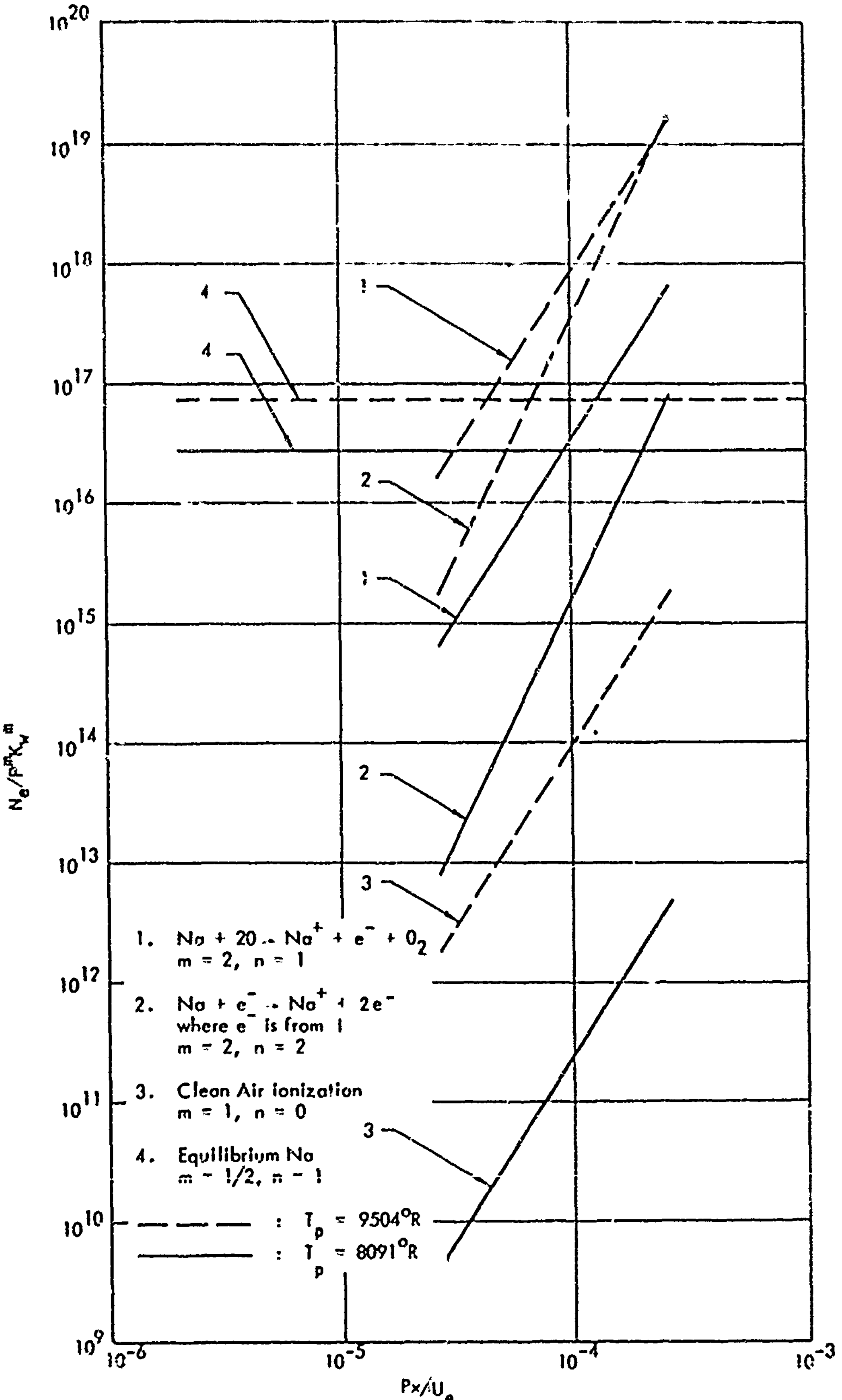

Figure 2. Non-equilibrium Electron Concentration

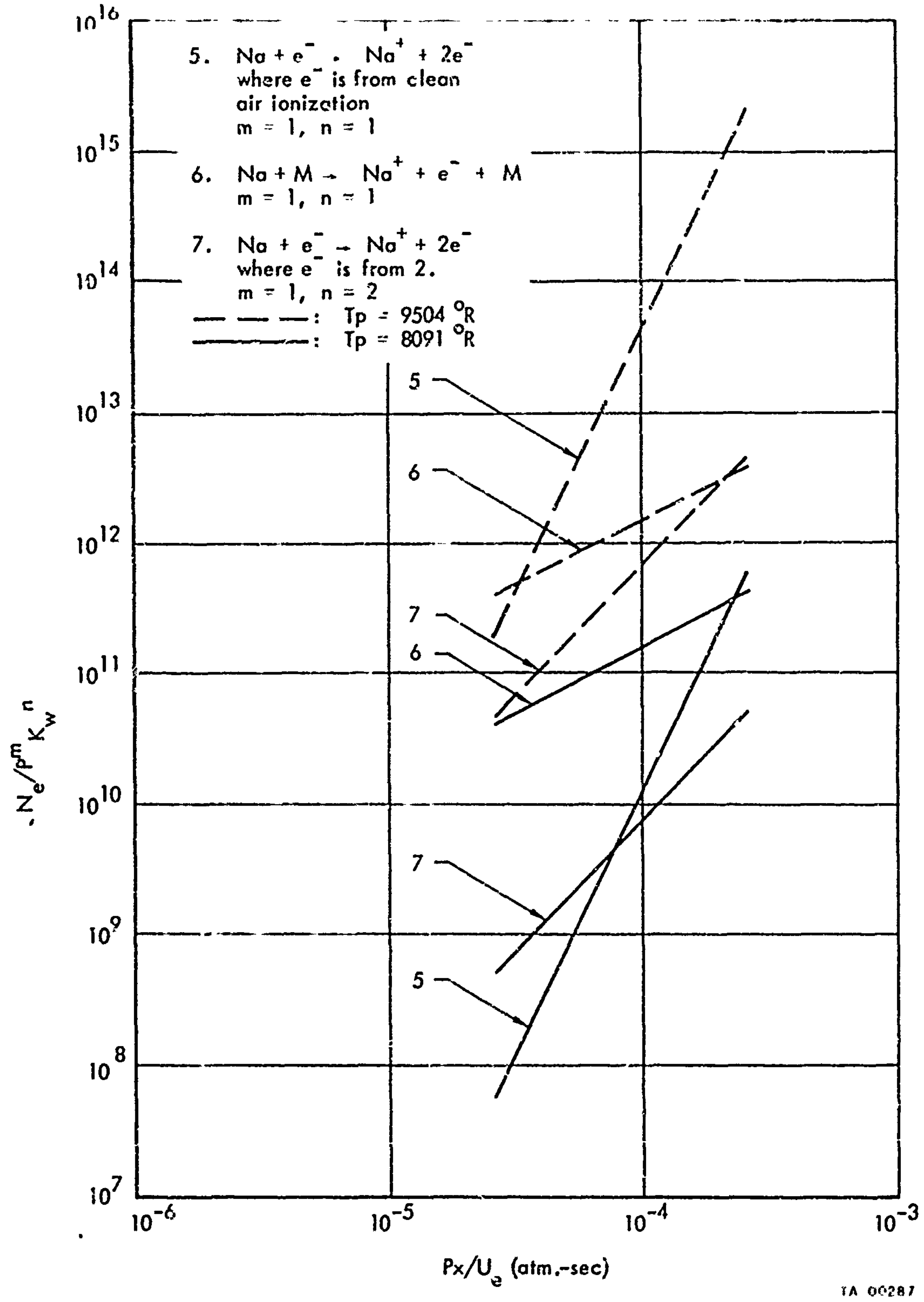

Figure 3. Non-equilibrium Electron Concentration

Each curve represents the peak electron concentration generated by the particular reaction in the boundary layer with a certain peak temperature. For different peak temperatures, the curves would be shifted. To indicate the degree of temperature dependency, two cases of different peak temperatures were studied as indicated in the figures. The peak temperature actually refers to the temperature at the location at which the electron mass fraction is a peak and is not necessarily the peak temperature of the boundary layer. Blasius' type velocity profiles were used although provision was included in the program to use the velocity profile associated with any value of the blowing parameter, f(o). The atomic oxygen and C_{NO^+} distributions corresponding to the given wall and free stream boundary conditions were used. (Figure 1).

The most significant reaction in the non-equilibrium ionization of sodium is the ionization by collision with atomic oxygen. For example, if the contributions from each curve are evaluated for a peak temperature of 8091°R, a wall concentration of 10^{-4}, a pressure of 0.186 atmospheres, and a value of Px/U_e of $0.262(10)^{-4}$, the following is obtained (corresponding to $\theta_c = 12^\circ$, h = 115,000 ft, $U_\infty = 22,000$ fps, $T_w = 1000^\circ K$, $K_w = 10^{-4}$ and a cone length of 3 ft).

Table 1. Electron Ionization Contributions

Curve	Reaction	N_e (electrons/cm^3)
1	Atomic Oxygen	$2.2\ (10)^9$
2	Electron	$2.7\ (10)^3$
3	Clean Air	$8.8\ (10)^8$
4	Equilibrium	$1.2\ (10)^{12}$
5	Electron	$1\ (10)^3$
6	Arbitrary Third Body	$7.7\ (10)^5$
7	Electron	0.9

None of the other sodium-ionizing reactions are within even several orders of magnitude of that by which the sodium is ionized by atomic oxygen. This result applied for all the conditions investigated.

Table 1 also indicates the significance of non-equilibrium sodium ionization in comparison to clean air ionization for the conditions considered. If the wall concentration had been as low as one part per million, the sodium ionization would be negligible. However a wall concentration as high as 10^{-4} can be quite significant.

If diffusion is neglected, a simplified analysis can be accomplished to determine non-equilibrium ionization levels. This has been done in Appendix B and the results are shown in Figure 4 in comparison with the results of the analysis based on the diffusion equation. Included also is a similar comparison for the clean air computation. It is seen that in both cases the streamtube (no diffusion) method over-predicts by roughly two orders of magnitude.

To determine specific regimes in which clean air ionization predominates over sodium ionization, the electron densities at various altitudes were computed for the conditions of the example of Table 1. The results are described in Figure 5. It is seen that clean air ionization predominates at higher altitudes and the sodium ionization at low altitudes. The cross-over point, if any, depends on the wall mass fraction of sodium. This change in predominant ionization is due to the fact that the clean air ionization is a first order reaction while the principal sodium ionization reaction is second order. Thus as altitude decreases, the sodium ionization increases with the square of the pressure as well as with $(Px/U_e)^3$ while the clean air ionization is proportional to the pressure itself as well as to $(Px/U_e)^3$

The above example, and almost all the results of this analysis are highly dependent on the wall mass fraction of the contaminant. If this has a value of 10^{-6} or less, the sodium ionization will be negligible to altitudes below

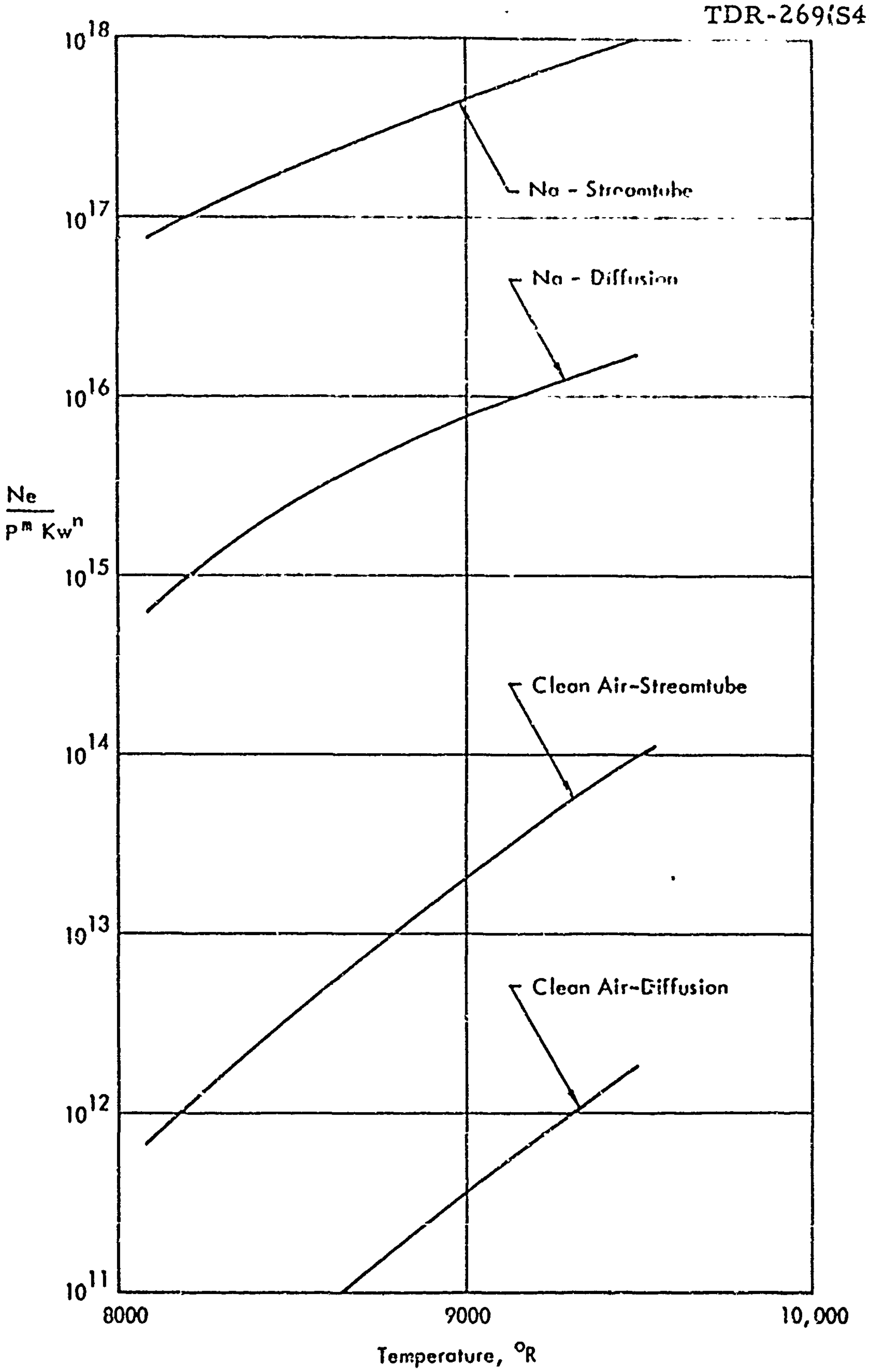

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Figure 4. Comparison of Streamtube and Diffusion Methods

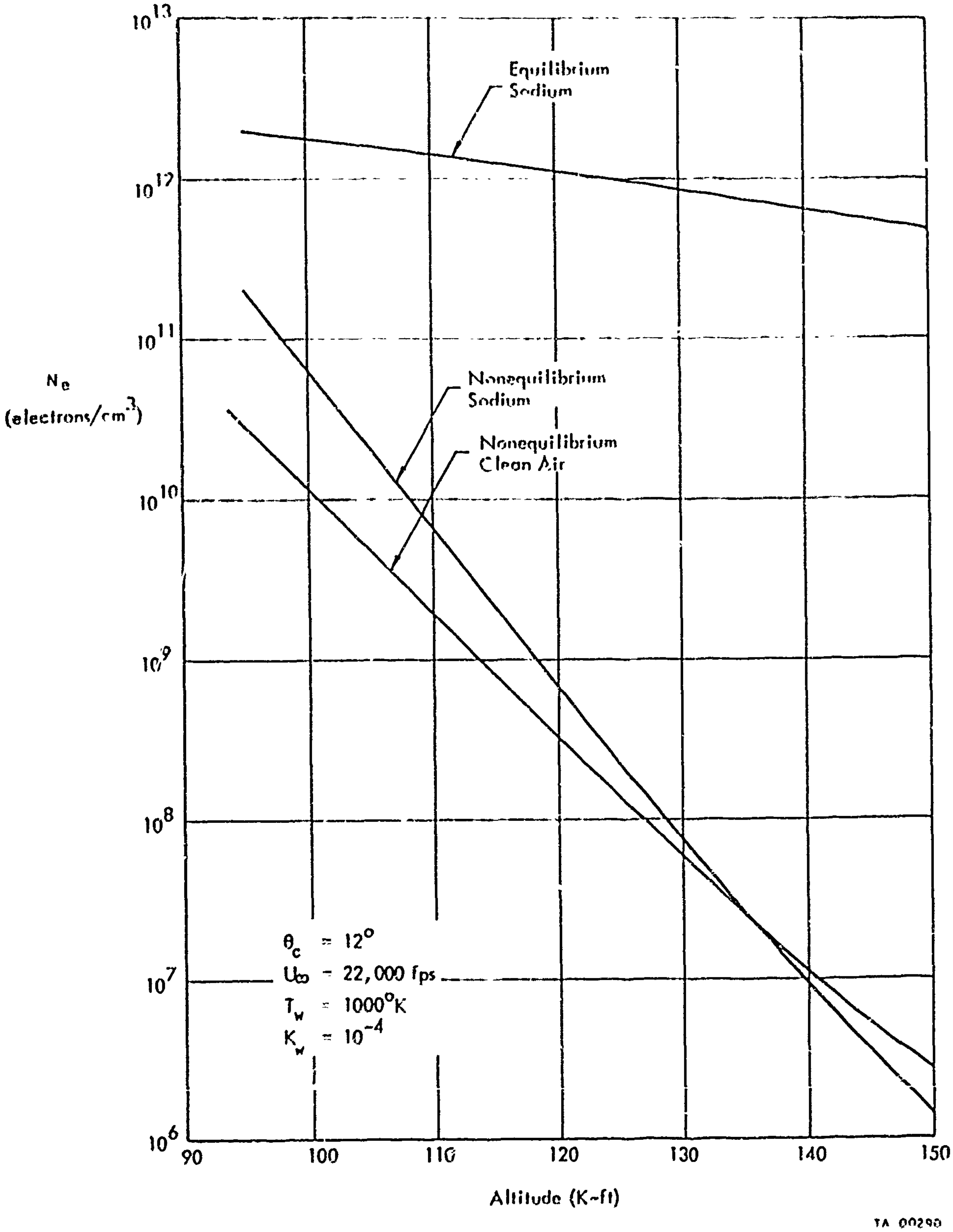

Figure 5. Ionization Levels vs Altitude

90.000 feet. If it has a value of 10^{-3} or greater, the sodium ionization will predominate at altitudes below approximately 200,000 feet. At intermediate values the particular conditions will have to be investigated.

The wall mass fraction of sodium can be related to the blowing parameter f(o) and the level of heat shield contamination by (see Appendix C)

$$K_w = \frac{Q\, f(o)}{f(o) - f''(o)} \tag{21}$$

where

Q = fractional sodium contamination of heat shield material (parts per million/10^6).

For a cone,

$$f(o) = -\sqrt{\frac{2}{3}}\, \frac{(\rho\, v)_w \sqrt{x}}{\sqrt{\rho_e\, \mu_e\, U_e}} \tag{22}$$

If a similar solution is to apply, then $(\rho v)_w \sqrt{x}$ = constant (23)

The results presented are based on a Blasius profile $[f(o) = o]$, implying that very little or negligible blowing is taking place. The Blasius profile is an accurate representation to within approximately 10 percent of the profiles for blowing up to f(o) = -0.10. Thus the results obtained using the Blasius profile should be valid for values of K_w/Q up to 0.2, corresponding to a twenty percent mass fraction of total ablation products at the wall.

For a case without negligible blowing, f(o) would be known as a function of altitude. Equation (21) then would describe K_w as a function of altitude and the non-equilibrium analysis could be performed using the proper K_w and velocity profile for each altitude.[4]

V. CONCLUSIONS

The following conclusions apply for slender cones using Bortner's rates for the ionization of sodium.

1. Sodium non-equilibrium ionization can be determined by considering only the ionization of sodium by atomic oxygen.

2. For altitudes down to the vicinity of 100,000 - 90,000 feet, only the forward reactions need be considered Below these altitudes the sodium ionization will be close to equilibrium.

3. Sodium ionization will be significant with respect to clean air ionization depending primarily on the wall mass fraction of sodium atoms. A mass fraction of 10^{-6} will ensure negligible sodium ionization.

4. Diffusion works to reduce the peak ionization in the boundary layer by approximately two orders of magnitude, whether one considers clean air or contaminant ionization.

5. The relative importance of clean air and contaminant ionization is a function of altitude although not to the degree that it is a function of wall mass fraction of contaminant. The lower the altitude the more dominant the sodium ionization. Depending on the wall mass fraction of sodium, there will be an altitude above which the clean air ionization exceeds the sodium ionization.

APPENDIX A

EQUILIBRIUM IONIZATION

The non-equilibrium analysis assumes that the ionization of sodium is far from equilibrium. Before the solution for any particular problem is accepted, the validity of this assumption should be determined.

The equilibrium between an atom and its ions,

$$A \rightleftarrows A^{+} + e^{-} \tag{24}$$

can be shown to be described by:

$$\epsilon = \left[\frac{K_p}{P + K_p} \right]^{1/2} \tag{25}$$

where ϵ = equilibrium degree of ionization
K_p = equilibrium constant
P = pressure (atmospheres)

The equilibrium constant is described by the equation of Saha[8].

$$\log K_p = \frac{-5040E}{T} + \frac{5}{2} \log T - 6.5 + \log \left(\frac{g_e \, g_{a^+}}{g_a} \right) \tag{26}$$

where E = ionization potential (electron volts)
T = temperature (^{o}K)
g = statistical weight

The last term can be considered negligible[9].

The electron concentration is related to the degree of ionization by:

$$N_{e^-} = \rho \, \epsilon \, \frac{C_{Na}}{M_{Na}} \tag{27}$$

where N_{e^-} = electron concentration
C_{Na} = mass fraction of sodium atoms
M_{Na} = atomic weight of sodium
ρ = mixture density

Using $P = \rho R T$ and $\epsilon = \left[\frac{K_p}{P + K_p}\right]^{1/2} \cong \left[\frac{K_p}{P}\right]^{1/2}$ for

$P >> K_p$ (which for typical peak boundary layer temperatures is valid for altitudes below 200,000 feet)

$$\frac{N_{e^-}}{P^{1/2}} = \frac{(K_p)^{1/2}}{RT} \frac{C_{Na}}{M_{Na}} \tag{28}$$

The local mass fraction of sodium atoms, C_{Na}, is directly proportional to the wall concentration. Thus:

$$\frac{N_{e^-}}{P^{1/2} K_w} \sim \frac{(K_p)^{1/2}}{RT\, M_{Na}} \tag{29}$$

where the right side is a function of temperature only. Thus the peak equilibrium concentration for a given peak temperature is constant when considered in the form $\frac{N_{e^-}}{P^{1/2} K_w}$ and is plotted as a horizontal line in Figure 3.

APPENDIX B

EFFECTS OF DIFFUSION

It is of interest to determine how much effect the boundary layer diffusion process has in reducing peak electron concentration levels. The prediction of electron concentrations neglecting diffusion is straightforward.

The conservation of mass for any species i requires that:

$$\operatorname{div} (\rho (\overline{q} + \overline{q}_i) C_i) = w_i \tag{30}$$

where $\overline{q}$ = mass averaged velocity

q_i = diffusional velocity of species i with respect to $\overline{q}$.

Considering one-dimensional flow and neglecting diffusional velocities, this reduces to:

$$\frac{d}{dx} (\rho\ U\ C_i) = w_i \tag{31}$$

which integrates to:

$$C_i = w_i \frac{x}{\rho\ U}, \tag{32}$$

considering w_i to be constant (or for a specific value of η only)

If we consider only what was found to be the most significant reaction, namely the ionization of sodium by collision with atomic oxygen, then:

$$w_{Na^+} = \rho^3\ C_{Na} \frac{(C_O)^2}{M_O^2}\ k_3 \tag{33}$$

and

$$C_{Na^+} = \frac{P\ x}{U}\ \frac{P}{R^2 T^2}\ k_3\ C_{Na} \left(\frac{C_O}{M_O}\right)^2 \tag{34}$$

where C_{Na^+} = mass fraction of sodium ions

Using

$$N_{e^-} = N_{Na^+} = \frac{\rho\ C_{Na^+}}{M_{Na^+}} \tag{35}$$

Equation (34) becomes:

$$\frac{N_e^-}{P^2} = 9.16\ (10)^{31}\ \frac{Px}{U}\ \frac{k_3}{(RT)^3}\ \frac{C_{Na}}{M_{Na^+}}\ \left(\frac{C_O}{M_O}\right)^2 \tag{36}$$

where N_e^- = electron density (electrons/cm^3)

x = distance along surface (ft)

R = gas constant

$= 53.3\ \frac{ft - lbf}{lb_m - {}^oR}$

Subsitituting, and converting x from flat plate to conical surface distances by dividing by three,

$$\frac{N_e^-}{P^2} = 0.35\ (10)^{36}\ \frac{Px}{U}\ \frac{1}{T^3}\ (C_{Na})\ (C_O)^2 \tag{37}$$

It should be noted in using the above equation that C_O varies linearly with $\frac{Px}{U}$. Levinsky and Fernandez [7] show that neglecting diffusion

$$C_O = \frac{Px}{U}\ \frac{(3.58)\ (10)^6}{T^2}\ e^{-\frac{59,400}{T}} \tag{38}$$

where T is in degrees Kelvin.

Substituting for C_O, Equation (37) becomes:

$$\frac{N_e}{P^2} = 4.7\ (10)^{69}\ \left(\frac{Px}{U}\right)^3\ \frac{C_{Na}}{T^7}\ e^{-\frac{212\ 600}{T}} \tag{39}$$

where T is in degrees Rankine.

Levinsky and Fernandez present a similar result for the clean air electron concentration, namely

$$\frac{N_e}{P} = \frac{0.47\ (10)^{48}\ \left(\frac{Px}{U}\right)^3}{(T/1000)^{6.5}}\ e^{-\frac{185,000}{T}} \tag{40}$$

where T is in degrees Kelvin.

APPENDIX C

RELATION BETWEEN WALL BLOWING AND WALL MASS FRACTION

If it is assumed that the velocity of air within and up to the surface is zero[10], then at the wall

$$(\rho v)_w = \frac{-1}{1 - C_w} \left[\rho D \frac{dC}{dy} \right]_w \tag{41}$$

where C here refers to the mass fraction of total material injected into the boundary layer.

Transforming to (η, ξ) where:

$$\eta = \frac{U_e \; r_o \; (x)}{(2 \; \xi)^{1/2}} \int_0^y \rho \; dy \tag{42}$$

$$\xi = \rho_e \, U_e \, \mu_e \int_0^x r_o \; (s) \; ds \tag{43}$$

and introducing the stream function

$$\psi \; (\eta, \xi) = (2 \, \xi)^{1/2} \; f(\eta) \tag{44}$$

where $\rho \; v \; r_o = - \frac{\partial \psi}{\partial x}$ (45)

Then Equation (41) becomes

$$f(o) = \frac{1}{1 - C_w} \; \frac{\ell}{Sc_w} \left[\frac{\partial C}{\partial \eta} \right]_w \tag{46}$$

If C is taken to be non-reacting so that its normalized solution is represented by Equation (6), then

$$\left[\frac{\partial C}{\partial \eta} \right]_w = - C_w \left[\frac{\partial S}{\partial \eta} \right]_w = - \frac{C_w \left[f''(o) \right]^{Sc}}{\int_0^\infty \left[f''(\eta) \right]^{Sc} d \, \eta} \tag{47}$$

Taking $S_c = \ell = 1$, Equation (46) becomes:

$$f(o) = - \frac{C_w \; f''(o)}{1 - C_w} \tag{48}$$

or solving for C_w,

$$C_w = \frac{f(o)}{f(o) - f''(o)} \tag{49}$$

Then

$$K_w = \frac{Q\, f(o)}{f(o) - f''(o)} \tag{50}$$

For a cone,

$$f(o) = -\sqrt{\frac{2}{3}}\ \frac{(\rho v)_w \sqrt{x}}{\sqrt{\rho_e\, \mu_e\, U_e}} \tag{51}$$

where if the solution is to be similar,

$$(\rho v)_w \sqrt{x} = \text{constant} \tag{52}$$

NOMENCLATURE

C_i	mass fraction of species i
D	binary diffusion coefficient
f	velocity function $= \int \frac{U}{U_e} \, d\eta$
g	dimensionless enthalpy defined by equation (17)
H	viscous dissipation and work term defined by equation (20)
h	thermal enthalpy
=	$\int_{T_{ref}}^{T} C_p \, dT$
h_s	total enthalpy
=	$h + U^2/2$
K_w	mass fraction of sodium atoms at the wall
K_p	equilibrium constant
k_i	forward reaction rate constant associated with reaction i
ℓ	density - viscosity ratio
=	$\rho\mu/\rho_e\mu_e$
M	molecular weight
N_i	concentration of species i in moles per unit volume
P	pressure
Pr	Prandtl number
Q	parts per million of sodium contamination of injected material divided by 10^6
R	gas constant

S	normalized atom mass fraction defined by Equation (2)
Sc	Schmidt number
T	temperature
U	velocity parallel to surface
v	velocity normal to surface
w_i	mass rate of formation of species i per unit volume
x	distance along surface
ϵ	equilibrium degree of ionization
ρ	mixture density
η	transformed independent variable (Equation 42)
ξ	transformed independent variable (Equation 43)

SUBSCRIPTS

e	edge of boundary layer conditions
e^-	electron species
eq	equilibrium conditions
m	mean or average
o	atomic oxygen species
w	wall conditions
∞	free stream conditions

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