

A DATA REDUCTION PROGRAM FOR HOTSHOT TUNNELS BASED ON THE FAY-RIDDELL HEAT-TRANSFER RATE USING NITROGEN AT STAGNATION TEMPERATURES FROM 1500 TO 5000°K

SEP 6 1967.

By

Martin Grabau, H. K. Smithson, Jr., and Wanda J. Little von Kármán Gas Dynamics Facility ARO, Inc.

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The last line of Equation (I-5), page 28, reads + 2.2566900 $(\log \overline{h_0})(\log s_0/R)^2 + 1.0606850(10)^2(\log s_0/R)^3$

This line should read

+ 2.2566900 $(\log \bar{h}_{0})(\log s_{0}/R)^{2}$ + 1.0606850(10)¹ $(\log s_{0}/R)^{3}$.

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A DATA REDUCTION PROGRAM FOR HOTSHOT TUNNELS BASED ON THE FAY-RIDDELL HEAT-TRANSFER RATE USING NITROGEN AT STAGNATION TEMPERATURES FROM 1500 TO 5000°K

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Martin Grabau, H. K. Smithson, Jr., and Wanda J. Little von Karman Gas Dynamics Facility

ARO, Inc.

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FOREWORD

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ABSTRACT

A method is set forth for the computation of test-section, aftershock, and stagnation conditions in hotshot tunnels of Arnold Engineering Development Center, using nitrogen at stagnation temperatures from 1500 to 5000°K. The basic input data are the stagnation pressure in the reservoir, the pitot pressure behind a normal shock, and the heat-transfer rate at the stagnation point. The value of the overall enthalpy is adjusted by numerical iteration until the measured and computed heat-transfer rates are numerically equal within a prescribed tolerance.

PUBLICATION REVIEW

This report has been reviewed and publication is approved.

Darreld K Calkins

Darreld K. Calkins Major, USAF AF Representative, VKF DCS/Test

ante Jean A. Jack

Jean A. Jack Colonel, USAF DCS/Test

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NOMENCLATURE

A/A*	Effective area ratio
ā	Speed of sound, ft/sec
b _i and ci	Parameters in empirical equations in Appendix I, with numerical subscripts
d*	Throat diameter, in.
h	Enthalpy, atm/amagat units
ħ	Enthalpy, ft^2/sec^2
K	Tunnel constant
k	Dimensionless enthalpy function, $h\rho/p$
log	Logarithm to base 10
Μ	Mach number
m	Symbol for the ratio $\mathrm{k}/\gamma_{\mathrm{E}}$
n	Dimensionless parameter, cf. Eq. (9)
р	Pressure, atm
ġ	Heat-transfer rate, Btu/ft ² sec
R	Gas constant, equal to 3.661 (10) ⁻³ when pressure is given in atm, density in amagats, and temperature in degrees Kelvin
R _a	Radius of model, in.
Re/ft	Reynolds number per foot
r	Density ratio across normal shock, $ ho_1/ ho_2$
s/R	Entropy, dimensionless
Т	Temperature, °K
u	Longitudinal flow velocity in units such that u^2 is given in atm/amagat units
ū	Longitudinal flow velocity, ft/sec
$\overline{\mathbf{v}}$	Average longitudinal signal flow velocity, ft/sec
W	Symbol for the quantity (1/4) ${ m M_2}^2$
$\boldsymbol{\gamma}$	Ratio of specific heats
$\gamma_{\rm E}$	Isentropic expansion coefficient

ix

Viscosity, lb/ft-sec

μ ρ

0

1 2

*

Density in amagat units, a relative scale referred to density at 0°C and one atm pressure. This scale is the same as the ratio ρ/ρ_0 in Refs. 6 and 7, using $\rho_0 = 0.00242$ lb-sec²/ft⁴ for nitrogen.

SUBSCRIPTS

Relates to stagnation conditions. On an unprimed quantity (e. g., ρ_0) it denotes reservoir conditions. On a primed quantity (e. g., ρ_0 ') it denotes stagnation conditions behind a normal shock.

Condition just before a normal shock

Condition just behind a normal shock

Numerical subscripts are also used to identify parameters b; and c; in empirical equations in Appendix I.

SUPERSCRIPT

Throat condition

1.0 INTRODUCTION

Based in part on previous work (Ref. 1), this report presents an improved method of determining the test-section, after-shock, and stagnation conditions in hotshot tunnels at the Arnold Engineering Development Center (Ref. 2), using nitrogen as working gas at stagnation temperatures from 1500 to 5000°K. Besides stipulating thermodynamic equilibrium at all times, as is also being done here, the earlier paper assumed that no temperature or density gradients exist in the arc chamber and that the decay of reservoir conditions as a function of time can be calculated from the rate of mass flow in the throat.

These latter assumptions became suspect in a roundabout way (Ref. 3). In run after run, the results exhibited large and erratic discrepancies between the measured and computed rates of heat-transfer at the stagnation point, the calculated values being derived from Fay-Riddell theory (Ref. 4). At first these discrepancies were attributed solely to contamination of the working gas caused by erosion of metallic surfaces in the arc chamber. As is described in Ref. 3, successive new designs of the arc chamber diminished the contamination and also reduced the differences between the measured and calculated values of the rates of heat transfer at the stagnation point. But as further new designs of the arc chamber reduced the contamination substantially, residual discrepancies remained between the measured and computed rates of heat transfer, which could no longer be ascribed to contamination.

Suspicion now fell on the supposed uniformity of conditions in the arc chamber, and at this juncture it appeared advisable to measure one or another parameter in the free stream. This was successfully done (Ref. 3) by means of timed photographic records of small blast waves in the free stream, which were shown to be moving with the local flow velocity. However, this flow velocity in the free stream could also be calculated in two ways, one in terms of the supposedly uniform reservoir conditions and the other in terms of the measured rate of heat transfer at the stagnation point. The values derived from the heat-transfer rate consistently agreed with the measured velocities to within 5 percent, whereas the values based on reservoir conditions sometimes differed from the measurements by as much as 20 percent.

This led to the conclusion that the measured heat-transfer rate at the stagnation point could be adopted as a basic input datum, which

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would determine the system if, in addition to the measured pitot pressure downstream, the measurement of one reservoir condition could be accepted. By virtue of the high speed of sound in the reservoir, it is reasonable to assume that the pressure gradients in the reservoir disappear in a few milliseconds. The flow in the tunnel is not fully established until about 30 milliseconds after the arc is fired, so the pressure measurements are deemed to be trustworthy. However, heat-transfer theory suggests that the temperature gradients might endure for one second or longer, so the density gradients would be equally slow in disappearing.

The indicated procedure is then as follows: Fay-Riddell theory (Ref. 4) determines a value of the stagnation enthalpy based on the measured heat-transfer rate and pressure at the stagnation point. Corresponding values of density, temperature, and entropy in the reservoir are then given by empirical equations written in terms of pressure and enthalpy as independent variables. Thereafter, a computational program not unlike the one of Ref. 1 determines the system, including a calculated value of the heat-transfer rate at the stagnation point. If this computed value of the heat-transfer rate is unacceptably different from the measured value, the computer is instructed to vary the stagnation enthalpy until the measured and computed values of the heat-transfer rate are in agreement to within, say, 1 percent. Inasmuch as the ultimate value of the overall enthalpy is found by iteration, the value of enthalpy used to start the iteration need be only approximately correct. This procedure is independent of any temperature or density gradients in the reservoir. It also renders it unnecessary to compute rates of decay of reservoir conditions because instantaneous values of these conditions are derived from input data. However, in selecting values of pressure in the reservoir, allowance must be made for the time of passage of a pressure signal from the throat to the stagnation point downstream (see Ref. 1).

Data for the thermodynamic properties of nitrogen are taken from the tables of Smith (Ref. 5), Hilsenrath and Klein (Ref. 6), and Little and Neel (Ref. 7). The tables of Smith are based on the virial coefficients given by Amdur and Mason (Ref. 8), and they do not account for the effects of dissociation and ionization. The referenced tables of Hilsenrath and Klein account for intermolecular forces, as well as dissociation and ionization, but their range of densities extends upward only to about 200 amagats.

The computational program of this report is intended for the following ranges of conditions: The temperature in the reservoir after firing the arc is assumed to be between 1500 and 5000°K, and the corresponding

 $\mathbf{2}$

range of pressures extends from 10 to 2500 atm. Upstream of the shock, the temperature and density are assumed to be between the liquefaction point and 400°K and between 10^{-5} and 10^{-1} amagat, respectively. Behind the shock the pressure is assumed to be between 10^{-3} and one atm.

A detailed outline of the computational program is given in Appendix I. In order not unduly to encumber the ensuing text, some of the equations in the program are given only in the appendix, with appropriate references in the text. Furthermore, lest certain equations in the text be littered with conversion factors, enthalpy (h) and the square of the flow velocity (u^2) are often expressed in atm/amagat units. However, in the working equations in Appendix I, enthalpy appears as h and the square of the flow velocity as \overline{u}^2 , both being expressed in ft²/sec². Some of the empirical equations in Appendix I were written by means of the method of surface-fits developed by Lewis and Burgess (Ref. 9). Appendix II comprises computer printouts from a number of typical runs and for a variety of input conditions.

2.0 STAGNATION POINT HEAT-TRANSFER RATE AND RESERVOIR CONDITIONS

In the theory of Fay and Riddell (Ref. 4), the boundary-layer equations are developed for the case of high-speed flight, in which the fluid in the external flow is partially dissociated. The analysis includes the effects of diffusion and recombination, and it finally reduces to a set of non-linear equations which are amenable to numerical solution. If the effects of dissociation are neglected, and if the Prandtl number is assumed equal to 0.71, the rate of heat transfer at the stagnation point is given by the relation

$$\dot{q} \sqrt{Ra} = \text{constant } x (\rho_w \mu_w)^{0.1} (\rho'_0 \mu'_0)^{0.4} (\bar{h}_0 - \bar{h}_w)$$

 $x (p'_0 - p_1)^{0.25} (\rho'_0)^{-0.25}$
(1)

in which q is the heat-transfer rate and Ra is the radius of curvature of the model surrounding the heat-transfer gage. The temperature at the wall is assumed to be 300°K, and the pitot pressure is also the wall pressure. Under these conditions, the density at the wall is proportional to the pitot pressure, whereas the viscosity and enthalpy at the wall are constants. Equation (1) then becomes

$$\dot{q} \sqrt{Ra} = \text{constant } x (\mu_0')^{0.4} (\rho_0')^{0.15} (\overline{h}_0 - \overline{h}_w)$$

 $x (p_0')^{0.1} (p_0' - p_1)^{0.25}$
(2)

In this equation, if q is expressed in Btu/ft^2sec , Ra in inches, ρ in amagats, μ in lb/ft-sec, p in atm, and \overline{h} in ft²/sec², the constant is numerically equal to 4.2519(10)⁻⁴ and the enthalpy at the wall is equal to 3.3469(10)⁶. In the case of nitrogen, the conversion from h (given in atm/amagat units) to \overline{h} (ft²/sec²) follows from the relation

$$\bar{h} = 8.722 (10)^5 h$$

In the range of stagnation temperatures from 3000 to 4000°K, numerical evaluations of Eq. (2) in more than 50 runs disclose the empirical fact that the stagnation enthalpy is approximately a linear function of the quantity ($q \sqrt{Ra/p_o'}$). Accordingly, a provisional value of the stagnation enthalpy is given by

$$\bar{h}_{o} = 1.459 (10)^{5} \left(\frac{1}{q} \sqrt{Ra/p_{o}'} \right) + 0.7750 (10)^{7}$$
 (3)

By using this value of \overline{h}_0 and the measured value of p_0 in the reservoir as independent variables, the corresponding values of ρ_0 , T_0 , and s_0/R are found by applying Eqs. (I-2), (I-3), and (I-4) in Appendix I. These equations are shown graphically in Figs. 1, 2, and 3.

In the tables of Ref. 5, entries are clearly indicated by dotted lines for which dissociative effects exceed 1 percent. On this basis, the ranges of reservoir conditions are intentionally limited to exclude such effects almost entirely.

3.0 THROAT CONDITIONS

The present method of determining the throat conditions is a generalization of the one developed in Ref. 1. In writing empirical equations for the product ρu , which has a local maximum in the throat, it is obvious to start with the conditions of constant entropy and to apply the continuity equation for the conservation of energy, which is essentially a relation between enthalpy and flow velocity. The product ρu can be expressed as a function of entropy and density which, at constant entropy, can be differentiated with respect to density.

The argument begins by forming the function

$$\rho^2 u^2 = 2 \rho^2 (h_0 - h)$$

Its derivative is then

$$\left(\frac{\partial (\rho^2 u^2)}{\partial \rho}\right)_{s} = 4\rho (h_{o} - h) - 2\rho^2 (\partial h/\partial \rho)_{s}$$

This derivative vanishes in the throat, so that

$$2(h_o - h^*) - \rho^* \left(\frac{\partial h}{\partial \rho}\right)_s^* = 0$$
⁽⁴⁾

At this point it is advantageous to introduce the identity

$$(\partial h/\partial \rho)_{s} = (h/\rho) \left(\frac{\partial \log h}{\partial \log \rho}\right)_{s}$$
 (5)

On substituting Eq. (5) into Eq. (4), there follows

$$h_{o} = h^{*} \left[1 + \frac{1}{2} \left(\frac{\partial \log h}{\partial \log \rho} \right)_{s}^{*} \right]$$
(6)

Equation (6) expresses the equilibrium throat condition which must hold for real as well as ideal gases and its evaluation is not difficult.

By observing the identity

$$\left(\frac{\partial \log h}{\partial \log \rho}\right)_{\mathbf{s}} = \left(\frac{\partial \log h}{\partial \log p}\right)_{\mathbf{s}} \left(\frac{\partial \log p}{\partial \log \rho}\right)_{\mathbf{s}}$$

and noting that

$$\left(\frac{\partial \log h}{\partial \log p}\right)_{s} = p/h \rho = 1/k$$

and that

$$\left(\frac{\partial \log p}{\partial \log \rho}\right)_{s} = \gamma_{E}$$

where k is the dimensionless enthalpy function $h\rho/p$ and γ_E is the isentropic expansion coefficient, Eq. (6) becomes

 $\overline{h}_{0} = \overline{h}^{*} \left[(2m^{*} + 1)/2m^{*} \right]$ (7)

in which m is the ratio k/γ_E and m* is its value in the throat. Forgetting the asterisks for a moment, the function comprising the entire right-hand side of Eq. (7) can be tabulated (or plotted) isentropically as a function of \overline{h} , and this is the same as a table (or plot) of \overline{h} * as a function of \overline{h}_0 at constant entropy. In like manner, an isentropic table (or plot) of the right-hand side of Eq. (7) as a function of density yields the relation between ρ * and \overline{h}_0 . These relations are shown graphically in Figs. 4 and 5, and the corresponding empirical equations are given as Eqs. (I-5) and (I-6) in Appendix I. The numerical value of the product $\rho^*\overline{u}$ * is then given by

$$\rho^* \overline{\mathbf{u}}^* = \rho^* \sqrt{2(\overline{\mathbf{h}}_0 - \overline{\mathbf{h}}^*)}$$

(8)

Another derivation of Eq. (7) is given in section 4.2. It should also be noted that the present parameter m(= k/γ_E) is akin to the reciprocal of the parameter m used in Ref. 1.

4.0 THE SHOCK CROSSING

4.1 BASIC PARAMETERS AND THEIR INTERRELATION

Although the procedure being set forth here differs somewhat from the one developed in Ref. 1, the shock crossing again involves the use of four dimensionless parameters, namely

$$n = (h_0 - h_1)/h_0 = 1 - h_1/h_0$$
(9)

$$\mathbf{r} = \rho_1 / \rho_2 = \mathbf{u}_2 / \mathbf{u}_1 \tag{10}$$

$$k_{1} = h_{1} \rho_{1} / p_{1} \tag{11}$$

$$k_2 = h_2 \rho_2 / p_2 \qquad (12)$$

In the present instance, the fluid in front of the shock can be taken as a perfect gas, so that the parameter k_1 is substantially constant, its average numerical value being 3.4985 with a maximum error of 0.25 percent. Needless to say, it is also necessary to observe the three continuity equations

$$\rho_1 \mathbf{u}_1 = \rho_2 \mathbf{u}_2 \tag{13}$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$
(14)

$$u_1^2 + 2h_1 = u_2^2 + 2h_2$$
 (15)

These four definitions and the continuity equations give rise to the following relations: From Eqs. (9), (10), (13), and (15)

$$u_1^2 = 2 n h_0$$
 (16)

$$\mathbf{L}_{\mathbf{r}} = \mathbf{r} \mathbf{u}_{\mathbf{r}} \tag{17}$$

$$u_{2}^{2} = 2 nr^{2} h_{0}$$
 (18)

from Eqs. (9), (15), and (18)

$$h_{1} = (1 - n)h_{0}$$
(19)

$$h_2 = (1 - nr^2)h_0$$
 (20).

from Eqs. (11) and (19)

$$p_{i} = h_{i} \rho_{i} / k_{i} = (1 - n) h_{o} \rho_{i} / k_{i}$$
(21)

and from Eqs. (10), (14), and (21)

$$p_2 = p_1 + \rho_1 u_1^2 (1 - r)$$
(22)

$$p_2/p_1 = 1 + 2nk_1(1 - r)/(1 - n)$$
 (23a)

From the definitions of k_1 and k_2 , the pressure ratio across the shock can also be written in the form

$$p_2/p_1 = \left[k_1(1 - nr^2)\right] / \left[rk_2(1 - n)\right]$$
 (23b)

The following four relations are sometimes useful and can be written by inspection

$$p_{1}/\rho_{1} = (1 - n) (h_{o}/k_{1})$$

$$p_{1}/\rho_{2} = r(1 - n) (h_{o}/k_{1})$$

$$p_{2}/\rho_{1} = \left[1 - n + 2nk_{1}(1 - r)\right] (h_{o}/k_{1})$$

$$p_{2}/\rho_{2} = r\left[1 - n + 2nk_{1}(1 - r)\right] (h_{o}/k_{1})$$

Finally, by substituting Eqs. (10), (16), (18), (19), and (20) into Eq. (23), there follows

$$(1 - nr^{2})k_{1} - 2nr(1 - r)k_{1}k_{2} - r(1 - n)k_{2} = 0$$
(24)

which interrelates the four dimensionless parameters across the shock and can easily be shown to be an alternate form of the Hugoniot equation. This form of the Hugoniot equation is especially useful in the present application because k_1 is substantially constant and because k_2 can be determined from observations at the stagnation point downstream. By rearranging terms, Eq. (24) is a quadratic in r, namely:

$$nk_{1}(2k_{2}-1)r^{2} - k_{2}(2nk_{1}+1-n)r + k_{1} = 0$$
(25)

The Hugoniot equation is often written in the more familiar form

$$2h_2 - 2h_1 = (p_2/\rho_1) + (p_2/\rho_2) - (p_1/\rho_1) - (p_1/\rho_2)$$

On substituting from Eqs. (11) and (12) and from the four equations inserted between Eqs. (23b) and (24), this form of Hugoniot's equation becomes a simple, but rather long, expression in which 12 terms annul one another in pairs. Equation (25) then follows by assembling the coefficients of r and r^2 .

For a perfect gas, $k_1 = k_2 = k$, and with this restriction Eq. (24) reduces itself to nr(2k - 1) = 1. Finally, when k is given the numerical value 3.5, Hugoniot's equation for a perfect gas is simply 6nr = 1.

4.2 THE FREE-STREAM MACH NUMBER

The Mach number in the free-stream is a function of the parameters n and $m_1 (= k_1/\gamma_{E1})$. The free-stream velocity is given by

$$u_1^2 = 2 nh_0$$

and the square of the speed of sound is

$$a_1^2 = \gamma_{E1} p_1 / \rho_1 = (1 - n) h_0 \gamma_{E1} / k_1$$

The ratio of these two equations is

$$M_1^2 = 2 n m_1 / (1 - n)$$

or

$$n = M_1^2 / (M_1^2 + 2 m_1)$$

If the medium can be construed to be a perfect gas for which $k_1 = 3.5$ and $\gamma_{E1} = \gamma = 1.4$, so that $m_1 = 2.5$, then

$$M_1^2 = 5n/(1 - n)$$

Such relations are also valid in the throat where the Mach number is equal to unity, so that

$$n^* = 1/(2m^* + 1)$$

1 - $n^* = 2m^*/(2m^* + 1)$

and since

$$h^* = h_0 (1 - n^*)$$

it follows that

 $h_o = h^* (2m^* + 1)/2m^*$

as is set forth in Eq. (7).

4.3 IMPLICATIONS OF ISENTROPIC FLOW BEHIND NORMAL SHOCK

In the hypervelocity regime, the thermodynamic parameters behind the normal shock change very little in going from the back of the shock to the stagnation point. In a typical run at, say, 4000°K and at Mach 12, the enthalpy increases by about 1 percent, the density by about 5 percent, and the pressure by about 6 percent. The change in the dimensionless enthalpy function from k_2 to k_0' is hardly perceptible. Under such conditions the ratio of the specific heats is substantially constant.

This renders it permissible to regard the fluid behind the shock as a perfect gas of some kind, especially since the numerical value of the specific heat ratio is immaterial. Following a procedure commonly adopted in the literature (for example, Ref. 10) and as is carried through in detail in Ref. 1, the argument rests on the isentropic flow equation for a compressible gas

$$p'_{o} = p_{2} \left[1 + \frac{\gamma - 1}{2} M_{2}^{2} \right]^{\gamma/(\gamma - 1)}$$
 (26)

The right-hand side of this equation is expansible in a rapidly convergent series. By discarding negligible terms and applying the relation for a compressible gas, $(\gamma/2)M_2^2 = (1/2)\rho_2u_2^2/p_2$, the specific heat ratio drops out entirely, and Eq. (26) then becomes

$$p'_{0} = p_{2} + (1/2) p_{2} u_{2}^{2} (1 + w)$$
(27)

where w is an abbreviation for the quantity $(1/4)M_2^2$. Then, by applying the mass-flow and momentum continuity equations, Eq. (27) becomes

$$p'_{o} = p_{i} + (1/2) \rho_{1} u_{i}^{2} \left[2 - r(1 - w) \right]$$

The density ratio is always less than 0.16 and the numerical value of (1-w) is slightly less than unity. It follows that the product r(1-w) can be more than 10 percent in error before its effect on the quantity in brackets exceeds 1 percent. This is an important fact, and as will presently be seen, good use can be made of it.

The quantity w is small and it always appears in the expression [2 - r(1-w)] or its equivalent. Since the density ratio is also small, it follows that any reasonably correct approximation for the value of w is adequate. Through the range of conditions for which this program is intended, the Mach number behind the shock will be in the range from 0.28 to 0.38. The corresponding values of w lie in the range from 0.02 to 0.04, so that 0.03 may be taken as an average. On combining Eqs. (16), (21), and (27), there follows

$$\rho_{1} = \frac{k_{1} p_{o}'}{[(1-n) + nk_{1} (2-0.97 r)]h_{o}}$$
(28)

in which the enthalpy is expressed in atm/amagat units. The parameter (1-n) increases from about 0.01 to 0.05 as the free-stream Mach number decreased from 20 to 10. Inasmuch as this program is intended for use at Mach numbers from 10 upwards, Eq. (28) can be viewed from the

point of view that the numerical value of the term (1-n) in the denominator does not exceed 0.05. The remainder of the expression inside the square brackets keeps a numerical value about 6.5. If, in the extreme case at Mach 10, the term (1-n) is neglected, the resulting error is about 0.8 percent, and it decreases as the Mach number increases. If for the moment the term (1-n) is neglected, Eq. (28) becomes

 $\rho_{1} = p_{0}^{\prime} / \left[nh_{0} \left(2 - 0.97 r \right) \right]$ (29)

The omission of the term (1-n) in this equation is adequately taken into account in the ultimate determination of the free-stream density.

4.4 DETERMINATION OF THE PARAMETERS n AND r

It is well known that shock crossing calculations for real gases involve an iteration of one kind or another. The usual procedure is to assume a numerical value of some thermodynamic or flow parameter and with recourse to the continuity equations, to test the implications of the assumption with reference to known stagnation conditions. Various assumptions are tried until a set of results are found which are mutually consistent within prescribed limits.

In principle, the present method also invokes iteration, but the iteration is hidden by the device of numerical estimates in situations which are insensitive to the accuracy of the estimates. Besides, instead of reserving the stagnation conditions for purposes of comparison at the end of an iterative cycle, they are put into the numerical works at the beginning. The method is based on the simultaneous solution of three equations in logarithmic form, as follows: In front of the shock, the perfect gas law can be written in the form

$$RT_{1} = (1 - n) h_{0} / k_{1}$$

 or

$$\log T_{1} = \log (1 - n) + \log h_{0} - \log k_{1} - \log R$$
 (30)

in which h_0 is expressed in atm/amagat units. Likewise, Eq. (29) can be written in the form

$$\log \rho_1 = \log p'_0 - \log n - \log h_0 - \log (2 - 0.97 r)$$
(31)

Next, to use the fact that the upstream entropy is known, it is advantageous to resort to an empirical equation in the form of the theoretical expression for the entropy of a perfect gas, namely,

$$\log T_1 = 0.17364 (s_0/R) + 0.39971 \log \rho_1 - 1.50950$$
(32)

Using the numerical values of R and k_1 and eliminating T₁ and ρ_1 from these three equations, there follows

$$log (1 - n) = 0.17364 (s_o/R) - 1.39971 log h_o$$

+ 0.39971 log p'_o - 0.39971 log n
- 0.39971 log (2 - 0.97 r) - 3.40202 (33)

The purpose of this equation is to yield the value of the parameter n in terms of a somewhat inaccurate value of the term (1-n). If, in the relation

$$n = 1 - (1 - n) \tag{34}$$

the numerical value of (1-n) is approximately 0.05, then a 20 percent error in (1-n) causes only a 1 percent error in n. It should be noted that the right-hand side of Eq. (33) contains a term log n and another involving the density ratio. If 0.97 is taken as an average value of n, it can never be more than 2 percent in error in the Mach range from 10 to 22. However, since this term has the coefficient 0.39971, the use of the average value of n at this point contributes less than a 1 percent error in (1-n).

The density ratio across the shock plays only a minor part in determining the thermodynamic state upstream of the shock, this being the small forward effect of the compressibility of the gas. In the quantity (2 - 0.97 r), the constant 0.97 is valid to 1 percent. On the other hand, the density ratio is relatively small, that is, 0.16 or less. Consequently, the numerical value of the density ratio used at this point needs only to be approximately correct, and such a value is easily found. Equation (24) can be rearranged into the form

$$k_2 r \left[2 n k_1 (1 - r) + (1 - n) \right] = k_1 (1 - n r^2)$$

If, on the left-hand side of this equation, the term (1-n) is assumed to be negligible with respect to the term $2nk_1(1-r)$, the error introduced at Mach 10 is less than 1 percent. With this change, the above equation becomes

$$2k_2 nr(1 - r) = 1 - nr^2$$

By long division, the ratio $(1-nr^2)/(1-r)$ generates the expression

$$1 + r + r^{2}(1 - n)/(1 - r)$$

in which the third term is clearly negligible. The above equation is thus further reduced to

$$2k_2 nr = 1 + r$$
 (35)

By giving the parameter n its average value at this point and using the condition that k_2 is very nearly equal to k_0' , there follows

$$\tilde{\mathbf{r}} = 1/(1.94 \, \mathbf{k}_0' - 1)$$
 (36)

where the tilde in \tilde{r} denotes an approximate value of r. The dimensionless quantity k_0' is determined by an empirical equation in terms of enthalpy and pressure as independent variables. See Eq. (I-8) in Appendix I and Fig. 6. Equation (33) now attains its working form

$$\log N = 0.17354 (s_o/R) - 1.39971 \log h_o + 0.39971 \log p'_o$$

$$-0.39971 \log (2 - 0.97 \tilde{r}) - 3.39673$$
(37)

(38)

where the symbol N is used briefly for the slightly inaccurate value of (1-n). The final value of n is given by

n ==

and the final value of the density ratio is found by solving the quadratic Eq. (25) which is reproduced as Eq. (I-12) in Appendix I.

5.0 BEFORE-SHOCK, AFTER-SHOCK, AND STAGNATION CONDITIONS

The thermodynamic state in front of the shock depends very much on the parameter (1-n) which is easily found after the density and temperature are determined. The density is given by a slight modification of Eq. (28), namely

$$\rho_1 = \frac{k_1 p_0'}{N + nk_1 (2 - 0.97 r) h_0}$$
(39)

in which the substitution of N for (1-n) eliminates practically all the error introduced in going from Eq. (28) to Eq. (29). The temperature may now be found from Eq. (32) (see Fig. 7), and the final value of (1-n) is given by

 $(1 - n) = RT_1 k_1 / h_0$ (40)

The combination of terms in Eq. (33) clearly shows what is involved in attaining a specified degree of accuracy in determining the thermodynamic state in front of the shock. The heart of the problem is seen in Eq. (32) in which the temperature is expressed in terms of the upstream entropy. Let it be supposed that the density is known without error. The quantity log T₁ then depends directly on the product $(0.17364)(s_0/R)$. If the temperature is to be determined to 1 percent, log T₁ must be known to within ±0.004. In the hotshot tunnels for which this program is intended, a value of s_0/R equal to 30.0 is fairly typical, and the product (0.17364)(30.0) is equal to 5.209. It follows that the quantity 5.209 cannot be in error by more than 0.004 and that the entropy must be known to 0.08 percent. For this reason, special care was used in constructing Eq. (I-4) in Appendix I, in which entropy is expressed as a function of pressure and enthalpy. The average deviation of Eq. (I-4) from 125 equitably distributed entries in the tables is 0.06 percent. Only 20 of these entries deviate from the equation by more than 0.1 percent, and all of them lie in the lower-most quartile of the enthalpy range.

The remaining quantities of interest in front of the shock now follow directly. The pressure p_1 is given by the perfect gas law. The enthalpy \overline{h}_1 and the flow velocity \overline{u}_1 follow from Eqs. (I-17) and (I-18) and the speed of sound from Eq. (I-19) (see Fig. 8) in Appendix I. The Mach number is the ratio $\overline{u}_1/\overline{a}_1$, and the effective area ratio is equal to $\rho*\overline{u}*/\rho_1\overline{u}_1$. The viscosity is given by Eq. (I-22) (see Fig. 9) and the Reynolds number by Eq. (I-23) in Appendix I.

Behind the shock, the density, pressure, flow velocity, and enthalpy follow from Eqs. (10), (23), (17), and (20) in the text or from Eqs. (I-24) through (I-27) in Appendix I. The temperature and speed of sound are given by Eqs. (I-28) and (I-29), the Mach number M₂ being thereby determined.

At the stagnation point, the density ρ_0' is determined because k_0' , h_0 , and p_0' are known. Temperature, entropy, and viscosity are computed from Eqs. (I-32) through (I-34); the equation for the entropy is shown graphically in Fig. 10. The Fay-Riddell heat-transfer rate follows by applying Eq. (I-35) in Appendix I. To reconcile the measured and computed rates of heat-transfer, the computing machine is instructed to vary the overall enthalpy until the desired degree of agreement between them is attained. The version of this program in use in the von Karman Gas Dynamics Facility (VKF) specifies agreement within 1 percent, and this result is usually achieved with only two iterations on the computing machine.

6.0 CONCLUDING REMARKS

The pressures in the reservoir and at the stagnation point, as well as the heat-transfer rate at the stagnation point downstream, are recorded electronically as functions of time. The duration of a run is of the order of 50 milliseconds. In selecting corresponding readings, it is necessary to take into account the time required for a pressure

signal to travel from the throat to the test section. This interval of time depends on the average speed \overline{v} with which the signal travels and the distance from the throat to the test section. This average speed can be written in the form

$$\overline{\mathbf{v}} = \mathbf{K} \ \overline{\mathbf{u}}^*$$

in which K is a tunnel constant which depends on the expansion angle of the nozzle, the throat diameter, and the thickness of the boundary layer. The signal is assumed to be progressing through successive elements of volume in the tunnel defined by parallel planes normal to the longitudinal axis. In any such element of volume, the velocity of the signal is given by

$$\overline{u}_i + \overline{a}_i = \overline{u}_i (1 + 1/M_i) = \overline{u}^* (\overline{u}_i / \overline{u}^*) (1 + 1/M_i)$$

where the subscript identifies the successive elements of volume serially. The constant K is then the average value of the quantity $(\overline{u}_i/\overline{u}^*)(1 + 1/M_i)$ which, in fact, is very nearly constant along the length of the tunnel except for a very short distance at the throat. Besides, the numerical value of K is surprisingly insensitive to the presumed thickness of the boundary layer. In the case of Tunnel H (formerly known as Hotshot 2) in the VKF with an assumed boundary-layer thickness of half the radius of the test section, the empirical expression for K is

$$K = 2.55 + 0.20 d^*$$

which, in practice, needs to be evaluated only to two significant figures. The quantity d* is the diameter of the throat in inches.

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Fig. 6 Dimensionless Enthalpy Function k_o'at the Ultimate Stagnation Point as a Function of Overall Enthalpy and Pitot Pressure behind Normal Shock



ч. : с









Fig. 9 Viscosity as Function of Free-Stream Temperature

AEDC-TDR-64-50



Fig. 10 Downstream Entropy as Function of Overall Enthalpy and Pitot Pressure behind Normal Shock

(I-1)

APPENDIX I

DETAILS OF COMPUTATIONAL PROCEDURE

Note the relations

$$\vec{h} = 8.722 (10)^{\circ} h$$

 $\overline{u} = 9.339 (10)^2 u$

Where \overline{h} is expressed in ft^2/sec^2 and \overline{u} in ft/sec.

Calculate \overline{h}_{o} from the measured values of \dot{q} , Ra and Po.

$$\bar{\mathbf{h}}_{o} = 1.459 (10)^{5} (\dot{q} \sqrt{Ra} / \sqrt{P_{o}}) + 0.7750 (10)^{7}$$

Calculate ρ_o from \overline{h}_o and p_o

$$\log \rho_{0} = 2.2415672 (10)^{1} - 7.4788666 (\log \bar{h}_{0})$$

$$- 1.5430265 (10)^{-1} (\log P_{0}) + 8.5032627 (10)^{-1} (\log \bar{h}_{0})^{2}$$

$$+ 4.7065564 (10)^{-1} (\log \bar{h}_{0}) (\log P_{0}) - 3.5687463 (10)^{-1} (\log P_{0})^{2}$$

$$- 3.5600516 (10)^{-2} (\log \bar{h}_{0})^{3} - 4.5287166 (10)^{-2} (\log \bar{h}_{0})^{2} (\log P_{0})$$

$$+ 6.0866340 (10)^{-2} (\log \bar{h}_{0}) (\log P_{0})^{2} - 1.9536922 (10)^{-2} (\log P_{0})^{3}$$
(I-2)

Calculate T_o from \overline{h}_o and p_o

$$T_{o} = (10)^{3} \left[-1.6784483 (10)^{3} + 7.0663450 (10)^{2} (\log |\bar{h}_{o}|) - 1.2484939 (\log p_{o}) - 9.9473139 (10)^{1} (\log \bar{h}_{o})^{2} + 2.3547617 (10)^{-1} (\log \bar{h}_{o}) (\log p_{o}) + 2.4996545 (10)^{-1} (\log p_{o})^{2} + 4.6856478 (\log \bar{h}_{o})^{3} - 1.6175003 (10)^{-2} (\log \bar{h}_{o})^{2} (\log p_{o}) - 2.9673076 (10)^{-3} (\log \bar{h}_{o}) (\log_{4} p_{o})^{2} - 4.3807464 (10)^{-2} (\log p_{o})^{3} \right]$$
(I-3)

Calculate s_o/R from \overline{h}_o and p_o

$$s_{o}/R = (10)^{1} [1.2995109 - 3.8889476 (10)^{-1} (\log \bar{h}_{o}) - 3.9336637 (10)^{-1} (\log P_{o}) + 8.3586816 (10)^{-2} (\log \bar{h}_{o})^{2} + 2.7878653 (10)^{-2} (\log \bar{h}_{o}) (\log P_{o}) - 1.3109092 (10)^{-2} (\log P_{o})^{2}]$$
(I-4)

Calculate $\overline{h}\,^*$ from $\overline{h}_{\,0}$ and $s_{\,0}\,/\,R$

$$\log \bar{h}^* = -6.9866080 (10)^i + 1.5500832 (10)^i (\log \bar{h}_0)$$

- + 6.8621580 (10)¹ (log s_o/R) 2.6780798 (log \overline{h}_{o})²
- + 7.8302690 $(\log \bar{h}_{o}) (\log s_{o}/R) 6.7397050 (10)^{i} (\log s_{o}/R)^{2}$
- + $1.6707170 (10)^{-1} (\log \bar{h}_0)^3 8.5424000 (10)^{-1} (\log \bar{h}_0)^2 (\log s_0/R)$
- + 2.2566900 $(\log \bar{h}_{o}) (\log s_{o}/R)^{2}$ + 1.0606850 $(10)^{2} (\log s_{o}/R)^{3}$ (I-5)

Calculate ρ^* from \overline{h}_o and s_o/R

$$\log \rho^* = 9.0011196 (10)^1 - 2.1784020 (10)^1 (\log \tilde{h}_0)$$

$$- 8.9551180 (10)^{1} (\log s_{0}/R) - 1.4084480 (10)^{1} (\log \overline{h}_{0})^{2}$$

- + 1.7752234 (10)² (log \bar{h}_{o}) (log s_{o}/R) 3.8213446 (10)² (log s_{o}/R)²
- + 6.6653368 (10)⁻¹ (log \bar{h}_0)³ + 2.0314374 (10)¹ (log \bar{h}_0)² (log s₀/R)
- $-1.6678346 (10)^{2} (\log \bar{h}_{o}) (\log s_{o}/R)^{2} + 3.6829143 (10)^{2} (\log s_{o}/R)^{3}$ (I-6)

Calculate $\rho^* \overline{u}^*$ from \overline{h}^* and ρ^*

$$\rho^* \,\overline{\mathbf{u}}^* = \rho^* \,\sqrt{2(\overline{\mathbf{h}}_o - \overline{\mathbf{h}}^*)} \tag{I-7}$$

Calculate ko from \overline{h}_{o} and po

$$k_0' = b_1 \log p_0' + b_2$$

Where

$$b_{1} = -0.0557 (\log \overline{h}_{o}/R) + 0.2076$$
$$-\frac{1.1962 (\log \overline{h}_{o}/R - 4.1718)}{1 - \exp \left[-240 (\log \overline{h}_{o}/R - 4.1718)\right]}$$

 $b_2 = 0.8405 (\log \bar{h}_o/R) + 0.6181$

+
$$\frac{3.2083 (\log \overline{h_o}/R - 4.2813)}{1 - \exp \left[-100 (\log \overline{h_o}/R - 4.2813)\right]}$$

+ $\frac{2.4159 (\log \overline{h_o}/R - 4.4750)}{1 - \exp \left[-80.2 (\log \overline{h_o}/R - 4.4750)\right]}$

(I-8)

(I-10)

Calculate the provisional density ratio \tilde{r} from k'_{0}

$$\tilde{\mathbf{r}} = 1/(1.94 \ k_0' - 1)$$
 (I-9)

Calculate the quantity N from so /R, $\overline{h}_{o},\,p_{o}^{\prime}$ and \widetilde{r}

$$\log N = 0.17354 (s_o/R) - 0.23562 \log h_o$$

+ 0.39971 log
$$p'_{0}$$
 - 0.39971 log $(2 - 0.97 \tilde{r})$ - 3.39673

Calculate the quantity n from N

$$\mathbf{n} = \mathbf{1} - \mathbf{N} \tag{I-11}$$

Calculate r by solving the quadratic

$$r^{2} - \frac{k_{o}'(2 n k_{1} + 1 - n)}{n k_{1}(2 k_{o}' - 1)}r + \frac{1}{n (2 k_{o}' - 1)} = 0$$
 (I-12)

NOTE :

The desired value of r is the smaller of the roots. To preserve accuracy, note that the smaller of the roots is equal to $\frac{1}{n(2 k_0' - 1)}$ divided by the larger of the two roots.

Calculate ρ_i from ρ_o , N, n, r, \overline{h}_o and k_i

$$\rho_{1} = \frac{k_{1} p_{0}'}{N + n k_{1} (2 - 0.97 r) [\overline{h}_{0} / 8.722 (10)^{5}]}$$
(I-13)

Calculate T_{1} from $s_{\,\rm O}\,/R$ and ρ_{1}

$$\log T_{1} = 1.7364 (10)^{-1} (s_{0}/R) + 3.9971 (10)^{-1} (\log \rho_{1}) - 1.5095$$
(I-14)

Calculate the quantity (1 - n) from T₁, k₁ and \overline{h}_{0}

$$1 - n = 8.722 (10)^{5} RT_{1} k_{1} / h_{0}$$
 (I-15)

Calculate p_1 from ρ_1 and T_1

$$\mathbf{p}_{\mathbf{i}} = \boldsymbol{\rho}_{\mathbf{i}} \ \mathbf{R} \mathbf{T}_{\mathbf{i}} \tag{1-16}$$

Calculate \overline{h}_1 from (1 - n) and \overline{h}_0

$$\bar{h}_1 = (1 - n) \bar{h}_0$$
 (I-17)

Calculate
$$\tilde{u}_{1}$$
 from n and \bar{h}_{0}
 $\bar{u}_{1} = \sqrt{2 n \bar{h}_{0}}$ (I-18)
Calculate \bar{a}_{1} from T₁
If T₁ < 400°K ·
 $\bar{a}_{1}^{*} = 6.6883 (10)^{4} \sqrt{T_{1}}$
If T₁ > 400°K.
 $\bar{a}_{1} = 1.1055 (10)^{3} \left[-2.3537 (10)^{-2} + 6.4129 (10)^{-2} \sqrt{T_{1}} - 1.2988 (10)^{-4} T_{1} \right]$ (I-19)
Calculate M₁ from \bar{a}_{1} and \bar{u}_{1}
M₁ = \bar{u}_{1}/\bar{a}_{1} (I-20)
Calculate A/A* from $\rho^{*} \bar{u}^{*}$, ρ_{1} and \bar{u}_{1}
A/A* = $\rho^{*} \bar{u}^{*}/\rho_{1}u_{1}$ (I-21)

Calculate μ_1 from T_1

$$\mu_{1} = |(1.1172) (10)^{-5} \left[\frac{373.1}{T_{1} + 100} \left(\frac{T_{1}}{273.1} \right)^{\frac{3}{2}} \right]$$
(I-22)

NOTE :

When
$$T_1 \gtrsim 100^{\circ} K$$
 $\mu_1 = 4.62 \times 10^{-8} T_1$

Calculate ${\rm \mathring{R}e}_1/{\rm ft}$ from ρ_1 , $\overline{\mathfrak{u}}_1$ and μ_1

$$\frac{\text{Re}_{1}}{\text{ft}} = \frac{7.806 \ (10)^{-2} \ \rho_{1} \ \overline{u}_{1}}{\mu_{1}} \tag{I-23}$$

Calculate ρ_2 from ρ_1 and r

 $\rho_2 = \rho_{\rm c} / r \tag{I-24}$

Calculate p_2 from p_1 , n, k_1 and r

$$p_{2} = p_{1} \left[1 + 2 n k_{1} (1 - r) / (1 - n) \right]$$
(I-25)

(I-27)

Calculate \widetilde{u}_2 from r and \widetilde{u}_1

$$\widetilde{\mathbf{u}}_2 = \mathbf{r} \ \widetilde{\mathbf{u}}_1 \tag{I-26}$$

Calculate \overline{h}_2 from n , r and \overline{h}_0

$$\overline{h}_2 = (1 - n r^2) \overline{h}_0$$

Calculate T_2 from p_2 and \overline{h}_2

$$T_2 = (10)^3 [b_3 \log p_2 + b_4]$$
 (I-28)

Where

$$b_{3} = \frac{1.4125 (\log h_2/R - 4.1587)}{1 - \exp \left[-67 (\log \overline{h_2}/R - 4.1587)\right]} + \frac{-1.0759 (\log \overline{h_2}/R - 4.4062)}{1 - \exp \left[-56 (\log \overline{h_2}/R - 4.4062)\right]}$$

 $\mathbf{b_4} = [3.72456 \ (\log \ \overline{h_2} / R) - 12.46890]$

+
$$\frac{4.26469 (\log \overline{h}_2/R - 4.04565)}{1 - \exp [-28.1 (\log \overline{h}_2/R - 4.04565)]}$$

$$+ \frac{-5.41781 (\log h_2/R - 4.37536)}{1 - \exp \left[-33 (\log h_2/R - 4.37536)\right]}$$

Calculate \overline{a}_2 from p_2 and ρ_2

$$\overline{a}_{2} = 1.1055 (10)^{3} \left[3.1491 \ \log p_{2} + b_{5} \right] \\ + \frac{0.4808 \ (\log p_{2} - c_{1})}{1 - \exp \left[-100 \ (\log p_{2} - c_{1}) \right]} \\ + \frac{-1.2419 \ \log p_{2} - c_{2}}{1 + \exp \left[-27.5 \ (\log p_{2} - c_{3}) \right]} \\ + 0.0553 \ \exp \left[-100 \ (\log p_{2} - c_{4}) \right] \right]$$
(I-29)

Where

$$b_{5} = -3.1491 \log \rho_{2} - 0.1167 \qquad c_{3} = 1.02 \log \rho_{2} + 1.21$$

$$c_{1} = 0.9917 \log \rho_{2} + 1.0003 \qquad c_{4} = \log \rho_{2} + 0.7397$$

$$c_{2} = 1.3697 \log \rho_{2} + 1.5383$$

Calculate M_2 from \overline{u}_2 and \overline{a}_2 $M_2 = \overline{u}_2/\overline{a}_2$ (I-30) Calculate ρ_0' from k_0' , p_0' and \overline{h}_0 $\rho_0' = k_0' [8.722 (10)^5 p_0'/\overline{h}_0]$ (I-31) Calculate T_0' from p_0' and \overline{h}_0 Use Eq. (I-28) Calculate s_0'/R from p_0' and \overline{h}_0 $s_0'/R = b_6 \log p_0' + b_7$ (I-32) Where

$$b_{6} = -0.0231 \log \bar{h}_{0}/R - 2.2089$$

$$+ \frac{-1.2157 (\log \bar{h}_{0}/R - 4.2818)}{1 - \exp [-40.5 (\log \bar{h}_{0}/R - 4.2818)]}$$

$$+ \frac{-4.6987 (\log \bar{h}_{0}/R - 4.5818)}{1 - \exp [-26.4 (\log \bar{h}_{0}/R - 4.5818)]}$$

$$b_{7} = 9.0085 \log \bar{h}_{0} / R - 4.7282 + \frac{1.7300 (\log \bar{h}_{0} / R - 4.2056)}{1 - \exp [-173 (\log \bar{h}_{0} / R - 4.2056)]}$$

Calculate μ_{o}' from T_{o}'

 $\mu_{0}' = 1.1172 (10)^{-5} [1.0256 + 1.4223 (10)^{-3} T_{0}' - 1.8136 (10)^{-8} (T_{0}')^{2}]$ (I-33) Calculate the quantity $\dot{q} = \sqrt{Ra}$ from μ_{0}' , ρ_{0}' , \tilde{h}_{0} , p_{0}' and p_{1}

$$\dot{q}\sqrt{Ra} = 4.2519 (10)^{-4} (\mu_0')^{0.4} (\rho_0')^{0.15} [\bar{h}_0 - 3.3469 (10)^6]$$

 $(p_0' - p_1)^{0.25} (p_0')^{0.10}$ (I-34)

APPENDIX II

COMPUTER PRINTOUTS FOR TYPICAL RUNS

	NITROGEN	Q INPUT	RU	IN NUMBER	1	RADIUS	.50	INCHES	DATA	PROGRAM	006A
1964 NIT	ROGEN Q	DOT PROGRAM	i	•					,		
	REFEREN	CE DENSITY	0.0780641	C LASICUBIC F	00 1						
RESERVO	IR COND	ITIONS					STAGNATI	ON CONDITE	ONS BEHIN	D NORMAL	SHOCK
T SEC	PG PSI	RHO-S ATM	TO DEG K	HO SOFT/SUSEC	SOR	0-00T	P0+P51	RHŨ-J+ATM	TO+DEG K	S0+/R	
•0000	25000.	141.85	2628.	.35716+6d	24.031	200.00	8.000	.0535110	2766.	32.35	
.0000	25000.	129.65	2935.	.40042+08	24.512	200.00	6.000	.0361977	3082.	33.08	
.0000	25000.	88-85	4590.	.63681+08	26.504	200.00	2.000	.0083140	4415.	36.20	

	NITROGEN	Q INPUT	RUN NI	UMBER .	1	RADIUS	.50	INCHES	C)ATA PROGRA	AM 006A
1964 NIT	TROGEN Q DO	T PROGRAM	-								
	REFERENCE	DENSITY C	.07806410 L	BS/CUBIC FOO	T						
FREE S	STREAM CON	DITICNS	MODEL L	ENGTH 12.0	00 INCHE	ES	,			i.	
T SEC	P-1'PSI	RHO-1 ATM	T-1 DEG K	U-1 FT/SEC	M-1	Q-1 PSI	RE/INCH	RE/MODEL	A/A*	PD+/P0	V-1
.0000	.0252838	.0072945	64.42	8366.	15.58	4.298	133377.	1600522.	5318.	.3200-03	.1148-01
.0000	.0173892	.0048666	66.41	8866.	16.27	3.220	91473.	1097675.	7177.	.2400-03	.1447-01
.0000	.0042549	.0010075	78.50	11207.	18.91	1.065	20252.	243026.	22386.	.8000-04	.3576-01

,

1964 NIT	NITROGEN (ROGEN Q DO	Q INPUT F PROGRAM	RUN M	IUMBER 1	RADIUS	• 50	INCHES	Data	PROGRAM	- 006a
	REFERENCE	DENSITY C.	07806410 L	BS/CUBIC FOOT				,		
CONDITI	ONS BEHIND	NORMAL S	НОСК							
T SEC	P-2 PSI	RHO-2 ATM	T-2 DEG K	H-2 SOFT/SOSEC						P.
•0000	7.38404	.050446	2716.	··· 2 30117303EC	U-2 FT/SEC	M-2				
.0000	5.54387	.034141		•401100+02	1209.74	.3668			· ·	
-0000	1 0/170		3027.	•449936+J2	1263.74	• 3565				
	1.003/8 /	•007879	4373.	.718349+02	1433.11	.2314				

	NIRDGEN	Q INPUT.	RU	N NUMBER	2	RADIUS	• >0	INCHES	DATA	PRUGRAM	006A
1964 NIT	RCGEN Q	DOT PROGRAM		1						• •	
	REFEREN	CE DENSITY	0.0780641	0 LBS/CUBIC F	7 0 0						
RESERVO	IR COND	ITIONS	•				STAGNAT	INN CONDIT	IONS BEHIN	NORMAL	SHOCK
T SEC	PO PSI	RHO-0 ATM	TO DEG K	HO SQFT/SQSEC	SOR	Q-DOT	PO+PSI	RHU-0+ATM	TG+DEG K	50+/R	•
•0000	30000.	182.56	2305.	·31521+68	23.297	170.00	8.000	.0599333	2474.	31.86	•
•0000	30000.	167.88	2566.	•35198+ 0 8	23.757	170.00	6.000	.0406906	2729.	32.58	
.0000	30000.	117.66	3984.	•55292+0a	25.682	170.00	2.000	.0091776	4034.	35.51	
.0000	30000.	91.43	5332.	•74984+C8	27.017	170.00	1.000	.0038748	4700.	37.76	

	NITROGEN	Q INPUT	RUN NI	UMBÉR 2	2	RADIUS	.50	INCHES	r.	DATA PROGR	AM 006A
1964 NIT	ROGEN Q DO	T PROGRAM									
	REFERENCE	DENSITY 0	.07806410 LI	BS/CUBIC FOO	r						
FREE S	TREAM CON	DITICNS	MODEL LI	ENGTH 12.00	0 INCHE	≘s					
T SEC	P-1 PSI	RHO-1 ATM	T-1 DEG K	U-1 ⊦T/SEC	M-1	Q-1 PSI	RE/INCH	RE/MODEL	A/A*	P0+/P0	V-1
.0000	.0224080	.0082533	5(.46	7869.	16.56	4.302 /	181199.	2174390.	6173.	.2667-03	.1047-01
.0000	.0153770	•0055293	51.69	8321.	17.30	3.223	125329.	1503951.	8358.	.2000-03	.1315-01
.0000	.0037398	.0011620	59.82	10452.	20.21	1.069	28587.	343047.	26421.	.6667-04	.3215-01
.0000	.0015519	.0004234	68.13	12184.	22.07	•529	10660.	127924.	53979.	.3333-04	.5751-01

1	NITROGEN Q	INPUT (. RUN NU	MBER 2	RADIUS	.50	INCHES	DATA	PROGRAM	006A	
1964 NIT	ROGEN Q DOT	PROGRAM									
	REFERENCE	DENSITY 0.	0780641C LB	S/CUBIC FOOT					·		
CONDITI	ONS BEHIND	NORMAL S	носк							4	
T SEC	P-2 PSI	RHO-2 AIM	T-2 DEG K	H-2 SOFT/SQSEC	U-2 FT/SEC	M-2					
.0000	7.37343	.056445	2432.	.353806+02	1150.54	.3623					
.0000	5.53589	.)38341	2680.	.395301+02	1200-01	.3601					
.0000	1.85612	.008674	3983.	.622694+02	1400.15	.2680					
.0000	•93893	.003688	4671.	.848496+02	1398.69	.1856					
				•							

	NITROGEN	Q INPUT	RUN	NUMBER	3	RADIUS	• 50	INCHES	DATA	PROGRAM	006A	
1964 NIT	1964 NITROGEN Q DOT PROGRAM											
	REFERENC	CE DENSITY	0.07806410	LBS/CUBIC FC	1 O T							
RESERVE	IR COND	ITIONS					STAGNATI	ON CONDITION	S ВЕНІМО	NORMAL	сноск	
T SEC	PO PSI	RHO-9 ATM	TO DEG K	HO SQFT/SUSEC	SOR	Q-DOT	PO+PSI	RHO-0+ATM TO	+DEG K S	0+/R		
.0000	20000.	139.60	2135.	-28427+08	23.344	130.00	6.000	.0493808 2	269. 3	31.74		
.0000	20000.	98.35	3250.	.44105+08	25.177	130.00	2.000	.0110812 3	372.	34.56		
•0000	20000.	77.05	4306.	. 59164+08	26.435	130.00	1.000	.0044017 4	176.	36.54		
.0000	20000.	70.90	4724.	•65233+08	26.860	130.00	-800	.0033385 4	395.	37.26		

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	NITROGEN	Q INPUT	RUN NU	MBER	3	RADIUS	. 50	INCHES		DATA PROC	RAM 006A
1964 NI	TROGEN Q DO	T PROGRAM									
	REFERENCE	DENSITY 0	.07806410 L8	S/CUBIC FOO	г		•				
FREE S	STREAM CON	DITIONS	MODEL LE	NGTH 12.C	00 INCHE	E S					
T SEC	P-1 PSI	RHO-1 ATM	T-1 DEG K	U-1 FT/SEC	M-1.	Q-1 PSI	RE/INCH	RE/MODEL	A/A*	P0+/P0	V-1
.0000	.0176898	.0068760	47.82	7469.	16.15	3.229	151223.	1814672.	5664.	-3000-03	.1117-01
.0000	.0042203	.0014635	53.60	9328.	19.05	1.072	35862.	430344.	17621.	.1000-03	.2706-01
- 0000	.0017358	.0005414	59,59	10816.	23.95	.533	13837.	166038.	35942.	.5000-04	.4792-01
•0000	.0013056	.0003913	62.02	11361.	21.57	.425	10092.	121104.	45116.	.4000-04	.5777-01

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٨	ITROGEN Q	INPUT	RUN NU	18ER 3	RADIUS	.50	INCHES	, DATA	PROGRAM	006A-
1964 NITE	ROGEN Q DOT	PROGRAM		- ,						
	REFERENCE	DENSITY 0.0	7806410 LB	S/CUBIC FOOT						
CONDITIO	INS BEHIND	NORMAL SH	IDCK							1
T SEC	P-2 PS1	RHO-2 ATM	T-2 DEG K	H-2 SQFT/SQSEC	U-2 FT/SEC	M-2				
.0000	5.52479	.046484	2231.	•318925+02	1104.83	•3644				
.0000	1.84942	.010453	3316.	.495901+02	1306.04	.3379			-	
.0000	.93028	.004166	4131.	-667006+02	1405.66	•2229				
.0000	. 74714	.003167	4357.	.736617+92	1403.62	•1958				
20000										

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	NITROGEN	Q INPUT	RUI	N NUMBER	4	RADIUS	.50	INCHES	DATA	PROGRAM	006A
1964 NIT	RCGEN Q	DOT PROGRAM									
	REFEREN	CE DENSITY	0.07806410	D LBS/CUBIC F	υστ						
RESERVO	IR COND	ITIONS					- STAGNAT	ION CONDIT	IONS BEHIN	D NORMAL	SHOCK
T SEC	PO PSI	RHO-C ATM	TO DEG K	HO SQFT/SGSEC	SOR	Q-DOT	PO+PSI	RHO-C+ATM	TO+DEG K	S0+/R	
.0000	4000.	33.16	2139.	.27050+68	24.953	50.00	1.005	.0086272	2178.	33.34	•
.0000	4000.	30-42	2340.	•29859+0d	25.350	50.00	.800	.0063165	2363.	33.95	
•0000	4000.	27.65	2585.	•33279+ūd	25.789	50.00	.600	.0042986	2594.	34.66	
.0000	4000.	24.02	2994.	•39017+0d	26.440	50.00	.400	.0024850	3005.	35.69	
.0000	4000.	18.56	3912.	•51968+C8	27.633	50.00	.200	.0009811	3781.	37.54	

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	NITROGEN	Q INPUT	RUN NUM	BER	4	RADIUS	.50	INCHES		DATA PROGR	AM 006A
1964 NIT	ROGEN Q DO	T PROGRAM									
	REFERENCE	DENSITY O	.07806410 L8S	/CUBIC FOO	L						•
FREE S	TREAM CON	DITIONS	MODEL LEN	GTH 12.0	00 INCH	ES					
T SEC	P-1 PSI	RHO-1 ATM	T-1 DEG K	U-1 FT/SEC	M-1	Q-1 PSI	RE/INCH	RE/MODEL	A/A*	P0+/P0	V-1
.0000	.0029380	.0012045	45.34	7286.	16.18	•538	27256.	327067.	6890.	.2500-03	•2637-01
.0000	.0021873	.000.8710	46.68	7660.	16.76	•430	20126.	241514.	8631.	.2000-03	.3179-01
.0000	.0014920	.0005846	47.44	8093.	17.57	• 322	14042.	168502.	11556.	.1500-03	.3989-01
.0000	.0008740	.0003313	49.04	8771.	18.73	.215	8342.	100108.	17440.	.1000-03	.5516-01
•0000	.0003539	.0001235	53.27	10136.	20.76	.107	3309.	39708.	35148.	.5000-04	.9712-01

	NITROGEN (Q INPUT	RUN NU	MBER 4	RÁDIUS	• 50	INCHES	ΔΑΤΑ	PROGRAM	0064
1964 NIT	ROGEN Q DO	T PROGRAM	!				•			
	REFERENCE	DENSITY 0.	07806410 LB	S/CUBIC FOOT						
CONDITI	ONS BEHING	D NORMAL SI	носк						•	
, T SEC	P-2 PSI	RHO-2 ATM	T-2 DEG K	H-2 SQFT/SQSEC	U-2 FT/SEC	M-2				
.0000	•92053	.008120	2141.	•303442+02	1080.83	•3650			·	
.0000	.73718	•005948	2324.	.335126+02	1121.70	.3619	r			•
.0000	•55350	■004050	2549.	-373730+02	1168.19	.3582				
.0000	.36959	.002343	2953.	.438519+02	1240.11	.3444				
.0000	.18569	.000927	3741.	.585378+02	1349.73	•2225				

	NITROGEN	Q INPUT	RU	N NUMBER	5	RADIUS	.50	INCHES	DATA	PROGRAM	006A
1964 NIT	ROGEN Q	DOT PROGRAM						•		· .	
	REFEREN	ICE DENSITY	0.0780641	O LBS/CUBIC F	00 r						
RESERVO	IR COND	DITIONS					STAGNAT	ION CONDIT	IONS BEHIN	D NORMAL	SHOCK
T SEC	PO PSI	RHO-0 ATM	TO DEG K	HO SQFT/SQSEC	SOR	Q-DOT	PO+PSI	RHO-0+ATM	TO+DEG K	S0+/R	
.0000	6000.	76.16	1302.	.15907+08	22.417	60.00	6.000	.0833787	1300.	29.47	
.0000	6000.	54.68	1880.	-23630+08	23.967	60.00	2.000	+0194727	1946.	32.11	
.0000	6000.	42.81	2442.	.31480+08	25.122	60.00	1.000	.0075258	2471.	33.93	
.0000	6000.	39.77	2642.	.34281+08	25.470	60.00	.800	.0055774	2664.	34.49	
.0000	6000.	36.05	2935.	.38385+08	25.936	60.00	.600	.0037796	2959.	35.22	
.0000	6000.	31.18	3424.	-45270+08	26.622	60.00	•400	.0021733	3440.	36.28	

NITROGEN Q INPUT RUN NUMBER 5 RADIUS -50 INCHES DATA PROGRAM 006A 1964 NITROGEN Q DOT PROGRAM

		REFERENCE	E DENSITY	0.07806410	LAS/CUBIC FOOT	ſ						
I	FREE	STREAM CCM	NDITICNS	MODEL	LENGTH 12.00	CO INCH	ES					•
	T SEC	P-1 PSI	RHO-1 ATM	б Т-1 DEG К	U-1 FT/SEC	M-1	Q-1 PSI	REVINCH	REZMODEL	A/A*	P0+/PJ	V-1
	.0000	.0231673	.0124949	41.90	5357.	12.84	3.245	233337.	2800045.	1760.	.1000-02	.7149-02
	.0000	.0063419	.0027654	42.62	6805.	15.58	1.078	62164.	745970.	5211.	.3333-03	.1682-01
	•0000	.0025282	.0010308	45.59	7870.	17.43	.537	25057.	300688.	10485.	.1667-03	.2962-01
	.0000	.0018821	.0007558	46.28	8217.	13.06	.430	16894.	226733.	13143.	.1333-03	.3535-01
	.0000	.0012894	.0005051	47.45	8701.	18.39	.322	13039.	156473.	17596.	.1000-03	.4450-01
	.0000	.0007602	.0002846	49.64	9457.	20.07	•214	7634.	91607.	26530.	.6657-04	.5179-01

t	NITROGEN (INPUT	RUN NU	MBER 5	RADIUS	.50	INCHES	DATA	PROGRAM	006 A
1964 NITH	ROGEN Q DOI	PROGRAM			•		. •	,		
	REFERENCE	DENSITY 0.0	7806410 LB	S/CUBIC FOOT						
CONDITIO	ONS BEHINE	NORMAL SH	HDCK							
T SEC	P-2 PSI	RHO-2 ATM	T-2 DEG K	H-2 SQFT/SQSEC	U-2 FT/SEC	M-2				
.0000	5.49216	.078279	1262.	.177872+02	887.04	•0306				
.0000	1.83830	.018314	1910.	•264875+02	1027.59	•3701				
.0000	.92188	-00708E	2429.	.353412+02	1144.62	.3605				
.0000	.73814	.005255	' 2618.	-385027+02	1181.91	.3574				
.0000	•55423	.003563	2907.	.431372+02	1233.37	.3488				
.0000	.37009	•00205C	3388.	.509153+02	1312.62	.3036				
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	NITROGEN	Q INPUT	RU	N NUMBER	6	RADIUS	• 50	INCHES	DAT	A PROGRAM	006A
1964 NIT	RCGEN Q	DOT PROGRAM									
	REFEREN	CE DENSITY	0.0780641	C LBS/CUBIC F	00 T						
RESERVO	IR COND	ITIONS					STAGNAT	ION CONDIT	IONS BEHIN	ND NORMAL	SHOCK
T SEC	PO PSI	RHO-0 ATM	TO DEG K	HO SOFT/SUSEC	SOR	Q-DOT	PO+PSI	RHO-O+ATM	TO+DEG K	\$0+/R	
.0000	7060 -	83.93	1363.	•16758+CE	22.443	100.00	14.000	.1856524	1385.	28.83	
.0000	7000.	75.73	1532.	·18983+C8	22.929	100.00	10.000	. 1184994	1587.	29.65	
.0000	7000.	64.15	1843.	•23176+0e	23.718	100.00	6.000	•0593945	1913.	30.94	
.0000	7000.	44.44	2737.	.35716+08	25.470	100.00	2.000	.0134135	2766.	33.73	
.0000	7000.	34.79	3560.	•47299+08	26.639	100.00	1.000	.0052155	3574.	35.53	5

	NITROGEN	Q INPUT	RUN NUM	BER (5	RADIUS	- 50	INCHES		DATA PROGR	AM 006A
1964 NIT	ROGEN Q DO	T PROGRAM								,	•
	REFERENCE	DENSITY 0	.07806410 L8S	CUBIC FOO	r.						•••
FREE S	TREAM CON	DITICNS	MODEL LEN	GTH 12.0	DO INCH	ES					
T SEC	P-1 PSI	RHO-1 ATM	T-1 DEG K	U-1 FT/SEC	M-1	Q-1 PSI	RE/INCH	RE/MODEL	A/A#	P0+/P0	V-1
.0000	.0878415	.0279491	58.42	5676.	11.10	7.581	382398.	4588777.	873.	-2000-02	.4831-02
•0000	.0554959	.0175208	58.87	6055.	11.80	5.407	253718.	3044612.	1216.	.1429-02	.6302-02
.0000	.0278375	.0085444	60.56	6709.	12.89	3.237	133280.	1599363.	2021.	.8571-03	.9499-02
.0000	.0064582	.0018240	65.81	8364.	15.42	1.074	32644.	391724.	6135.	-2857-03	.2296-01
.0000	-0026125	.0006841	70.98	9644.	17.12	• 536	13089.	157071.	12397.	.1429-03	.4025-01

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	NITROGEN C	INPUT	RUN NU	1BER 6	RADIUS	.50	INCHES	DATA	PROGRAM	006A
1964 NIT	ROGEN Q DOT	PROGRAM								
	REFERENCE	DENSITY 0.0	7806410 LB	S/CUBIC FOOT						
CONDITI	ONS BEHIND	NORMAL SH	IDCK						•	
T SEC	P-2 PSI	RHO-2 ATM	T-2 DEG K	H-2 SQFT/SQSEC	U-2 FT/SEC	M-2				
.0000	12.83189	.174531	1346.	. 187395+02	909.01	•1565			· .	
.0000	9.17673	.111449	1550.	.212454+02	951.87	-3804				
•0000	5.51680	.055904	1877.	•259689+0 ²	1025.34	•3728				
.0000	1.84660	.012648	2717.	.401148+02	1206.27	•3586				
.0000	.92585	.004924	3521.	.532008+02	1339.87	•3068				

!	NITROGEN	Q INPUT	RUN	NUMBER	7	RADIUS	50	INCHES	DATA	PROGRAM	006A
1964 NIT	RCGEN Q I	DOT PROGRAM									
	REFERENC	CE DENSITY	3.07806410	LES/CUBIC F	του				•		
RESERVO	IR COND	IFIONS					STAGNAT	ION CONDITIC	INS BEHIN	D NORMAL	SHOCK
T SEC	PO PSI	RHO-C: ATM	TO DEG K- H	O SOFT/SUSEC	SOR	Q-DOT	PO+PSI	RHO-O+ATM 1	0+DEG K	S0+/R	
.0000	12500.	114.43	1695.	·21690+0s	22.794	140.00	14.000	.1470502	1804.	29.84	
.0000	12500.	103.86	1897.	.24486+08	23.280	140.00	10.000	•0941474	2005.	30.65	
.0000	12500.	87.27	2317.	.30354+08	24.154	140.00	6.000	.0465334	2396.	32.00	
.0000	12500.	60.70	3493.	.46902+05	25.967	140.00	2.000	.0104859	3558.	34.80	

	NITROGEN	Q INPUT	RUN	NUMBER	7	RADIUS	. 50	INCHES		DATA PROGRA	M 006A
1964 NI	TROGEN Q DO	T PROGRAM									
	REFERENCE	DENSITY	0.07806410 1	LBS/CUBIC FO	τα						
FREE	STREAM CON	DITICNS	MODEL I	LENGTH 12.	000 INCH	ES					
T SEC	P-1 PSI	RHO-1 ATM	T-1 DEG K	U-1_FT/SE	C M-1	Q-1 PSI	RE/INCH	RE/MODEL	A/A#	PD+/P0	V-1
.0000	.0693751	.0213607	60.37	6484.	12.48	7.559	323037.	3876443.	1534.	.1120-02	.5906-02
•0000	.0441154	.0134529	60.95	6900.	13.22	5.392	214444.	2573331.	2155.	.8000-03	.7678-02
.0000	•0224525	.0064704	64.50	7699.	14.33	3.228	108747.	1304964.	3625.	. 4800-03	.1169-01
.0000	.0053350	.0013808	71.82	9602.	16.94	1.072	25994.	311927.	11125.	.1600-03	.2827-01

	NITROGEN Q	INPUT	RUN NU	JMBER 7	RADIUS	• 50	INCHES	DATA	PROGRAM	006A	1
1964 NIT	ROGEN Q DOT	PROGRAM	a								
	REFERENCE	DENSITY 0.0	7806410 LE	SICUBIC FOOT							1
CONDITI	ONS BEHIND	NORMAL SH	DCK	·							
T SEC	P+2 PSI	RHO-2 ATM	T-2 DEG K	H-2 SOFT/SQSEC	U-2 FT/SEC	M-2					
•0000	12.86374	.138371	1768.	.242935+02	1000.90	.3763			· .		
.0000	9.19886	.088630	1969.	.274445+02	1047.39	.3706					
.0000	5.53072	.243846	.2355.	.340621+02	1136.11	.3639	•				
.0000	1.85115	.009899	3503.	-527459+02	1339.32	.3243					

NITROGEN Q INPUT RUN NUMBER 8 RADIUS .50 INCHES DATA PROGRAM CO6A 1964 NITROGEN Q DOT PROGRAM REFERENCE DENSITY 0.07806410 LBS/CUBIC FODT

RESERVOIR CONDITIONS STAGNATION CONDITIONS BEHIND NORMAL SHOCK . T SEC PO PSI RHO-O ATM TO DEG K HO SQFT/SUSEC SOR Q-DOT PO+PSI RHO-O+ATM TO+DEG K SO+/R .0000 16000. 122.36 2001. .26225+08 23.274 180.00 14.000 .1238183 2123. 30.58 .0000 16000. 109.19 2289. .30262+68 23.859 180.00 10.000 .0777099 2390. 31.48 .0000 16000. 93.20 2756. .36813+08 24.671 180.00 6.000 .0390684 2845. 32.75

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	NITROGEN	Q INPUT	RUN N	IUMBER 8	3	RADIUS	.50	INCHES	(DATA PROGR	AM 006A
1964 NI	TRCGEN Q DC	T PROGRAM									
	REFERENCE	DENSITY	0.07806410 L	BS/CUBIC FOOT	ſ			а. С			
FREE	STREAM CCN	DITICNS	MODEL L	ENGTH 12.00	DO INCHE	ΞS	· ·				
T SEC	P-1 PSI	RHO-1 ATM	T-1 DEG K	U-1 FT/SEC	M-1	Q-1 PSI	RE/INCH	RE/MODEL	A/A*	P0+/P0	V-1
.0000	.0640644	•0175946	67.68	7138.	12.97	7.545	261273.	3135282.	1951.	.8750-03	.6828-02
.0000	.0411212	•0108446	70.48	7678.	13.67	5.382	166351.	1996213.	2755.	.6250-03	.9020-02
.0000	.0209816	.0053183	73.33	8485.	14.81	3.223	86647.	1039760.	4658.	.3750-03	.1354-01

NITROGEN Q INPUT RUN NUMBER 8 RADIUS .50 INCHES DATA PROGRAM DO6A 1964 NITROGEN Q DOT PROGRAM

REFERENCE DENSITY 0.07806410 LBS/CUBIC FOOT

CONDÍTI	ONS BEHIND	NORMAL SH	IOCK	• .		
T SEC	P-2 PSI	RHO-2 ATM	T-2 DEG K	H-2 SQFT/SQSEC	U-2 FT/SEC	M-2
•0000	12.88849	.116623	2086.	.294030+02	1076.83	.3686
.0000	9.21844	.073237	2349.	.339550+02	1136.96	.3646
.0000	5.54097	.036848	2794.	.413471+02	1224.61	.3601

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