600618

AMRL-TDR-63-123

$5.00

3878

HUMAN MECHANICS

FOUR MONOGRAPHS ABRIDGED

W. BRAUNE and O. FISCHER
"Center of Gravity of the Human Body"

•

O. FISCHER
"Theoretical Fundamentals for a Mechanics of Living Bodies"

•

J. AMAR
"The Human Motor"

•

W. T. DEMPSTER
"Space Requirements of the Seated Operator"

DDC
JUN 3 1964
TISIA B

TECHNICAL DOCUMENTARY REPORT No. AMRL-TDR-63-123

DECEMBER 1963

BEHAVIORAL SCIENCES LABORATORY
6570th AEROSPACE MEDICAL RESEARCH LABORATORIES
AEROSPACE MEDICAL DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

Contract Monitor: Kenneth W. Kennedy
Project No. 7184, Task No. 718408

(Prepared under Contract No. AF 33(616)-8091 by
Wilton Marion Krogman, Ph.D., LL.D. and Francis E. Johnston, Ph.D.
Graduate School of Medicine, University of Pennsylvania,
Philadelphia, Pa.)

NOTICES

When US Government drawings, specifications, or other data are used for any purpose other than a definitely related government procurement operation, the government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise, as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Qualified requesters may obtain copies from the Defense Documentation Center (DDC), Cameron Station, Alexandria, Virginia 22314. Orders will be expedited if placed through the librarian or other person designated to request documents from DDC (formerly ASTIA).

Do not return this copy. Retain or destroy.

Change of Address

Organizations receiving reports via the 6570th Aerospace Medical Research Laboratories automatic mailing lists should submit the addressograph plate stamp on the report envelope or refer to the code number when corresponding about change of address.

800 - May 1964 - 162-38-769

FOREWORD

This study was initiated by the Anthropology Branch, Human Engineering Division of the Aerospace Medical Research Laboratories at Wright-Patterson Air Force Base, Ohio. The work was conducted by the Graduate School of Medicine, University of Pennsylvania, under Contract AF 33(616)-8091. Dr. W.M. Krogman, Professor and Chairman of Physical Anthropology, was the principal investigator and Dr. F.E. Johnston, Assistant Professor of Anthropology, Graduate College of Arts and Sciences, was the co-investigator. Mr. H.T.E. Hertzberg, Chief of the Anthropology Branch, recognizing the lasting importance of such monographs to the field of human factors, particularly to applied human mechanics, conceived the plan to have them condensed. Mr. Kenneth W. Kennedy, also of the Anthropology Branch, monitored the contract. Mr. Kennedy and Mr. Hertzberg each critically reviewed the manuscripts. The work was performed in support of Project No. 7184, "Human Performance in Advanced Systems," Task No. 718408, "Anthropology for Design." The work sponsored by this contract was started in June 1961 and was completed in September 1963. The four volumes are the following:

1. Braune, W., and Fischer, O. Über den Schwerpunkt des menschlichen Körpers, mit Rücksicht auf die Aüsrustung des deutschen Infanteristen. (The Center of Gravity of the Human Body as Related to the Equipment of the German Infantry) Abh.d. math.-phys. cl.d. K. Sachs. Gesellsch. der Wiss., Bd. 26, S. 561-672. 1889. Copyright release obtained by permission from S. Herzel Verlag, Leipzig.

2. Fischer, O. Theoretische Grundlagen für eine Mechanik der Lebenden Körper mit Speziellen Anwendungen auf den Menschen, sowie auf einige Bewegungs-vorgänge an Maschinen. (Theoretical Fundamentals for a Mechanics of Living Bodies, with Special Applications to Man, as well as to some Processes of Motion in Machines) B.G. Teubner, Leipzig and Berlin. 1906. Copyright release obtained by permission from B.G. Teubner, Leipzig.

3. Amar, J. The Human Motor: or The Scientific Foundations of Labor and Industry, E.P. Dutton Co., N.Y.; Geo. Routledge and Sons, Ltd., London. 1920. Copyright releases obtained from E.P. Dutton and Company, New York, and Routledge and Kegan Paul Ltd., London.

4. Dempster, W.T. Space Requirements of the Seated Operator. Geometrical, Kinematic, and Mechanical Aspects of the Body with Special Reference to the Limbs. WADC Technical Report 55-159, Wright Air Development Center, Air Research and Development Command, United States Air Force, Wright-Patterson Air Force Base, Ohio. 1955.

ABSTRACT

This report condenses four important monographs in the field of applied human mechanics:

The Centers of Gravity in the Human Body, Braune, W., and O. Fischer; is a study of the main center of gravity in the human body and the centers of its several parts. It is based upon the measurement and positional analysis of four frozen adult male cadavers, projected to an x, y, z coordinate system. The basic data are transferred to a living adult male soldier, with and without load, in differing military positions.

Theoretical Fundamentals for a Mechanics of Living Bodies Fischer; O., is the analysis of freely movable joint systems ("n-link systems") in the living human body. The aim is (1) to present the kinetics of joint systems, and (2) the analysis of states of motion and equilibrium. Part I presents a three-link joint system and the n-link plane and solid joint system. Part II is an application to the mechanics of the human body and to motion in machines (latter here omitted).

The Human Motor, Amar; J., is devoted to the application of principles of mechanics to bodily movements, specifically oriented to work-situations. There are discussions of muscle-bone kinetics in structure and function; the physiology of fatigue is stressed. Environmental factors are discussed: external, as climate, temperature, altitude, etc.; internal, as heart, lungs, nutrition, etc. Experimental devices to measure energy exchange are given. All data are finally interpreted in terms of actual work performance in tool use, time, and motion, etc.

Space Requirements of the Seated Operator, Dempster, W.T., is the analysis of the human body, utilizing osteological material, cadavers, and living subjects, in terms of body links and kinetics, differential tissue relationships, physique differences, and the range of normal variation, carried out for the purpose of more precisely defining the work space required by seated individuals in various tasks. The results consist in the presentation of these requirements for a variety of seated functional postures, as well as detailed plans for the construction of kinetically-correct two- and three-dimensional manikins.

TABLE OF CONTENTS

---

* - Page numbers in parentheses refer to the pagination of the original. The page numbers to the right refer to the pagination of this publication.

THEORETICAL FUNDAMENTALS FOR A MECHANICS OF LIVING BODIES

(Condensed from O. Fischer)

THE HUMAN MOTOR

(Condensed from J. Amar)

SPACE REQUIREMENTS OF THE SEATED OPERATOR

(Condensed from W.T. Dempster)

FIGURES

Figure No.		Page No.

Figure No.		Page No.

Figure No.		Page No.

TABLES

Table No.		Page No.

PREFACE

Kenneth W. Kennedy

The works of W. Braune and O. Fischer have long been recognized as basic to the field of Human Mechanics. They are cited as such in much of the literature dealing with this subject. Braune and Fischer's The Center of Gravity of the Human Body, published in Germany in 1889, and Fischer's Theoretical Fundamentals for a Mechanics of Living Bodies, published in 1906, also in Germany are two such basic works. In the former, the investigators discuss their methods regarding the determination of whole body and segment centers of gravity, and report data obtained on a series of four cadavers. Fischer's work presents the analysis of joint systems and its application to the mechanics of human body motion and to motion in machinery.

In 1914, another important book on Human Mechanics was published in France and then translated and published in the United States and England in 1920. This was J. Amar's The Human Motor, which deals with the mechanics of body movements in working situations. It has achieved much the same stature as the foregoing monographs.

In 1955, Professor W.T. Dempster, of the University of Michigan, completed an investigation of the general mechanics of the human body as applied to the seated operator. The results were published as a Wright Air Development Center Technical Report titled Space Requirements of the Seated Operator: Geometrical, Kinematic, and Mechanical Aspects of the Body with Special Reference to the Limbs. These investigations were performed at the request of what was then Anthropology Section, Aeromedical Laboratory. In a relatively short time Dempster's work was recognized as a major contribution.

The works of Braune and Fischer, Fischer, and Amar are now of very limited availability. Although the United States Air Force translation unit at Wright-Patterson Air Force Base, Ohio, translated the books by Braune and Fischer, availability has not been appreciably improved. These two monographs, then, remain in limited supply and primarily in the original German, both factors having greatly reduced their usefulness to English-speaking engineers. Even though Amar's work was translated and published in England and the United States, its availability has also diminshed until it is now quite difficult to find in its original or translated editions. Dempster's report is obtainable through Defense Documentation Center and the Office of Technical Services, U.S. Department of Commerce. However, it is written in the language of a specialist and has been found somewhat difficult to use by some investigators not experienced in the field of Human Mechanics.

During the preparation of WADC Technical Report 56-30, Annotated Bibliography of Applied Physical Anthropology in Human Engineering (1958), by R. Hansen and D. Cornog, the editor, Mr. Hertzberg, decided that these monographs were of too great a magnitude to be represented adequately in the brief treatment required in that publication. Because of their size, limited availability, and the difficulty some engineers experience when interpreting their data and methods, Mr. Hertzberg conceived the idea of having these works condensed and published together in a single volume. His object was twofold: first, such treatment would draw together four basic monographs, with all the essential methods and data found in the originals; and second, the wider distribution would present to many human factors engineers important sources that otherwise might have remained inaccessible to them.

We hope that, through the labors of Drs. Krogman and Johnston, we have accomplished those purposes.

THE CENTER OF GRAVITY OF THE HUMAN BODY

(Condensed from W. Braune and O. Fischer)

DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

OR are Blank pgs that have Been Removed

BEST AVAILABLE COPY

BEST AVAILABLE COPY

THE CENTER OF GRAVITY OF THE HUMAN BODY

(Condensed from W. Braune and O. Fischer)

Introduction and Historical Resumé (pp 561-577)

The Earth's power of attraction exerts a force on every center of gravity in the human body. This force acts along a line which is theoretically connected to the Earth's center and is equal to the weight of the mass at the original point. Because of the great distance from the Earth's center to these points all the lines are virtually parallel. The resultant is a fixed straight line for every position of the body. This line does not change as long as the body is not displaced in space over too great a distance, while still remaining in its original parallel position. The line changes its position in the body as soon as the body rotates around it. The straight line in its changed position intersects the former line at a definite point, and all other and similar lines pass through this point. The point of intersection so established is the common point of attachment for all resultants of the force of the inertia, for every possible body position, i.e., it is the center of gravity of the entire body.

The size of the resultant equals the total body weight of all individual vertical forces. The weights of all separate parts of the body are united at the body's center of gravity: this is the body's gravity line (a line connecting the body's center of gravity with the Earth's center). It does not change its absolute position in space if the center of gravity is unchanged, no matter how the body is rotated. Every rotation of the body on the center of gravity changes the position of the gravity line in relation to the body itself. As long as the line of gravity intersects the body's supporting surface the body remains vertical; if the line is outside the supporting surface of the body it may fall down. The supporting surface of the erect body is its two soles and the space between the double tangent in contact with toe tip and heel.

A supporting center of gravity can have three positions in relation to its supporting point: 1) it can coincide with the point (indifferent equilibrium); 2) it can be above the point, in which case every rotation of the body moves the center of gravity away from the vertical and lowers it (labile equilibrium); 3) it can be below the point, in which case any rotation of the body raises it, and creates a moment of rotation that brings the center of gravity to its original position (stable equilibrium). In labile equilibrium the center of gravity attains its highest position in rotating on the supporting point, whereas in stable equilibrium it attains its lowest position.

The abstract lever arm created by the forces of inertia, with any movement out of labile equilibrium, is shorter the nearer the center of gravity lies to the supporting point; it is equal to zero when the two coincide, which is characteristic of indifferent equilibrium. The closer the body's center of gravity is to the ground the more securely one stands, because the moment of rotation of the force of inertia is smaller the lower the center of gravity lies.

In order to understand the statics and mechanics of the human body one must know: 1) the location of the center of gravity of the entire body; 2) the center of gravity of individual limb sections; and 3) the combined weights in these centers of gravity.* These factors interpret the force to be overcome by each separate muscle, with each body movement. In an average body position the center of gravity is inside the body, but the body can bend over so that the common center of gravity falls outside it, just as in a curved arch. Every calculation of the center of gravity is valid for only one body position; all calculations are individual, because of age, sex, etc.

Historical data concerning the center of gravity are cited from Borellus (1679), the Weber brothers (1836), and H. von Meyer (1853). The data of Harless (1857) are given in some detail. He divided the masses of the limbs at the ends of the lever arms, i.e., at the axes of the joints.

Dimension (cm)		Dimension (cm)	
Height, total	172.685	Foot length	25.369
Foot height	6.0	Entire arm length	86.6
Lower leg length	42.9	Upper arm length	36.4
Upper leg length	44.9	Forearm length	29.889
Line of trochanter to top iliac crest	14.0	Hand length	20.314
Hipbone level to top acromial proc.	39.0		
Head length, chin to crown	21.2		
Front neck length	4.7		

(Braune and Fischer, p. 570)

The bloodless weights of the separate parts are as follows:

Weight of part (gms)		Weight of part (gms)	
Total wt.	63970	Each forearm	1160
Head	4555	Each upper arm	2070
Torso	29608	Each upper leg	7165
Both arms	7540	Each lower leg	2800
Both legs	22270	Each foot	1170
One hand (av.)	540		

(Braune and Fischer, p. 571).

* The authors use the term "combined weights in the centers of gravity", despite the fact that it is the segments, themselves, that have mass and weight. (W.M.K.)

The centers of gravity for the extremities were determined by Harless as follows (all dimensions in cm.)

1.	Upper arm 36.4 cm long	(17.621 from upper end (18.779 " lower "
2.	Forearm 29.889 long	(13.122 from upper end (16.767 " lower "
3.	Hand 20.314 long	(9.623 from upper end (10.691 " lower "
4.	Upper leg 44.9 long	(20.995 from upper end (23.905 " lower "
5.	Lower leg 42.9 long	(15.455 from upper end (27.445 " lower "
6.	Foot 25.369 long	(11.664 from upper end (13.705 " lower "
7.	Head 21.2 long	(7.7 from crown (13.5 " chin
8.	Upper section torso *	(23.465 from lower edge (17.53 " upper "
9.	Lower section torso*	(5.8899 from upper edge (7.6101 " lower "

In an overall calculation of the center of gravity in this man, after reducing height to 1000, it was found to be 413.65 mm from the crown. **

---

* Harless assumed the torso to be two truncated cones; in this B. and F. vigorously disagree. Upper torso extends down to iliac crest, lower torso includes the pelvis.

** Harless did a second man, with data given as in the first.

(Braune and Fischer, pp. 571-572)

The authors worked with frozen bodies, whole and in sections. Their first idea was to swing the body twice by a cord. Each time the center of gravity is under the point of attachment of the cord, hence is a prolongation of the cord. The center of gravity is the point of intersection of the two cords. This method was discarded, for it was too difficult to follow the axis of the cord through the body. Instead they used a body-axis made of a pointed iron rod, strong enough not to bend. These rods were driven through the frozen tissue. Hanging the body or limbs from three different axes made it possible to determine three planes in each case, each at right angles to the other, in each of which the center of gravity must lie; the center of gravity was found at the point of intersection of the three planes. All planes and points of intersection were projected to, and marked on, the surface of the body.

The authors used four bodies, all adult male, all suicides.

Cadaver #1 (18 years, 169 cm , well built) could not be sectioned. The center of gravity was found at the lower level of the second sacral vertebra (S2), almost in the plane of entry into the pelvis. It was 0.5 cm. to the right of the median plane, and 4.5 cm below the promontory and the center points of the two hip joints.

(Braune and Fischer, p. 577)

Cadaver #2 (45 years, 170 cm , 75,100 gms , muscular). Here the center of gravity was near the promontory, but below it. Dimensions of this body are as follows (cm):

1. Total length 170
2. Head length (chin-crown) 21
3. Limb lengths

	Right	Left
upper leg	44	43.3
lower leg	41.4	41.8
foot height	7.8	7.5
foot	28.5	28.3
upper arm	31.7	31.5
lower arm (no hand)	29.5	29.5

4. Distance between centers of hip joints 17

(Braune and Fischer, p. 583)

Table 1. Weights of Cadaver # 2 (Gms.)

	Part		Right	Left
1.	Entire body	75100		
2.	Head	5350		
3.	Torso without limbs	36020		
4.	Entire arm		4950	4790
5.	Upper arm		2580	2560
6.	Forearm + hand		2370	2230
7.	Forearm, less hand		1700	1600
8.	Hand		670	620
9.	Entire leg		12120	11890
10.	Upper leg		7650	7300
11.	Lower leg + foot		4470	4500
12.	Lower leg, less foot		3210	3320
13.	Foot		1100	1160

(Braune and Fischer, p. 583)

Cadaver # 3 (50 years, 166 cm , 60,750 gms; muscular build). Here the center of gravity was on a transverse plane through the lower edge of S1, in front and a bit below the promontory, 0.2 cm to the right of the median plane, 4 cm in front of the upper linea transversa of the sacrum. Dimensions of this body are as follows (in cm) :

	Part	Right	Left
1.	Total length 166		
2.	Head length (chin-crown) 20.2		
3.	Limb lengths		
	upper leg	42.0	41.0
	lower leg	43.0	42.9
	foot height	7.7	7.7
	foot length	26.5	26.9
	upper arm	30.6	30.2
	lower arm (no hand)	26.3	27.1
4.	Distance between centers of hip joints	17.5	

(Braune and Fischer, p. 588)

Table 2. The Centers of Gravity of Cadaver # 2

1. Torso alone, in median plane at lower edge first lumbar vertebrae (LI)
2. Head, in median plane near clivus, near basilar suture, below slope of the sella turcica.

		Right	Left
3.	Total arm	just below elbow axis, 1.5 cm in front.	at elbow axis, 2 cm in front.
4.	Upper arm	14.5 cm below center humeral head, 17.2 cm above articular sur'acc elbow axis, in medullary cavity	13.3 below and 18.2 above (as in right)
5.	Forearm	12.5 cm below elbow axis, 17 cm above capitulum head, 1 cm in front interosseous lig.	12.4 cm below, 17.1 cm above, 1 cm in front (as in right)
6.	Forearm + hand	19 cm below elbow axis, 10.5 cm above capitulum head, 0.5 cm from attachment interosseous lig. to radius	19 cm below, 10.5 cm above, 1 cm from attachment (as in right)
7.	Hand, fingers half flexed	5.5 cm below center head of metacarpal III, 1 cm in front of center of bone	(as in right)
8.	Total leg	39 cm below center of femoral head, 5 cm above knee axis at center of rear edge of femur	38.5 below, 4.8 above (as in right)
9.	Upper leg	19 cm below center of femoral head, 25 cm. above knee axis, 1.5 cm behind linea aspera	19.3 below, 24 cm above (as in right)
10.	Lower leg	17 cm below knee axis, 24 cm. above ankle axis, 1 cm behind center of interosseous lig.	17.4 below, 24.4 above (as in right)

Table #2 (continued)

		Right	Left
11.	Lower leg + foot	24.6 cm below knee axis, 16.8 cm above ankle axis, just behind attachment of interosseous lig. to tibia	25.5 cm below, 16.3 cm above (as in right)
12.	Foot	11.5 cm from rear edge foot, 17 cm from tip, 6.5 cm in front ankle joint, cuneiform II - III, at forward surface navicular	12 cm from rear edge, 18 cm from tip (as in right)

(Braune and Fischer, pp. 578-583)

Table 3 Weights of Cadaver # 3 (Gms)

	Part		Right	Left
1.	Entire body	60750		
2.	Head	4040		
3.	Torso without limbs	28850		
4.	Entire arm		3550	3480
5.	Upper arm		1990	1880
6.	Forearm + hand		1550	1600
7.	Forearm, less hand		1050	1120
8.	Hand		500	470
9.	Entire leg		10650	10250
10.	Upper leg		6690	6220
11.	Lower leg + foot		3950	3980
12.	Lower leg, less foot		2870	2880
13.	Foot		1060	1090

(Braune and Fischer, p. 588)

Table 4 The Centers of Gravity of Cadaver # 3

1. Torso alone, at center L1, 2 cm from the front surface, 1.4 cm. from the rear surface, 0.3 cm to the right of the median plane.
2. Head, in the Fossa Tarini behind the slope of the sella turcica, in the median plane.

		Right	Left
3.	Total arm	28 cm below center of humeral head, 2.6 cm below elbow axis, 1.8 cm in front of humerus	29.1 cm below, 1.1 cm above, 0.4 cm in front center of humerus (as in right)

Table 4 (continued)

		Right	Left
4.	Upper arm	13.4 cm below center of humeral head, 17.2 cm above elbow axis at rear edge humerus	13.7 cm below, 16.5 cm above, and just behind humerus (as in right)
5.	Forearm	10.9 cm below elbow axis, 15.4 cm above head of capitulum, 1 cm from center of interosseous lig.	11.0 cm below, 16.1 cm above (as in right)
6.	Forearm + hand	17.8 cm below elbow axis, 8.5 cm above head of capitulum, 0.7 cm in front of attachment of interos-lig. to radius	17.6 cm below, 9.5 cm above, 1.0 cm in front of interosseous lig. nearer radius (as in right)
7.	Hand, fingers half flexed	5.9 cm below center capitulum, 1 cm above head of metacarpal II, between metacarpal II-III, near skin of palm.	5.9 cm below, 0.8 cm from center head metacarpal III 1 cm in front of palmar edge metacarpal III
8.	Total leg	37.7 cm below center of femur head, 4.3 cm above knee axis, just behind rear edge of femur	38.5 cm below, 2.5 cm above, 0.7 cm in front of rear edge of femur, in the bone
9.	Upper leg	19 cm below center of femur head, 22.3 cm above knee axis, 2.2 cm behind rear edge of femur, in median plane	19.5 cm below, 21.5 cm above, 1.5 cm behind linea aspera, toward median
10.	Lower leg	18.7 cm below knee axis, 24.3 cm above ankle axis, 1 cm behind interosseous lig. near tibia	17.7 cm below, 25.2 cm above (as in right)
11.	Lower leg + foot	26.9 cm below knee axis, 16.1 cm above ankle axis, at attachment interosseous lig. to tibia	26.0 cm below, 19.9 cm above (as in right)

Table 4 (continued)

		Right	Left
12.	Foot	11.4 cm from rear edge of foot, 15.1 from tip, at angle lower and center edge cuneiform III near its articulation with navicular	11.8 cm from rear, 15.1 cm from tip, at angle between cuneiform III and cuboid on plantar side, close to joint of III with navicular

(Braune and Fischer, pp. 584-587)

Cadaver # 4 (168.8 cm , 56090 gms , good muscular build): Here wire nails were driven in before freezings, and all markings made. A wire was driven through humeral head, postero-anteriorly. The elbow axis was located via palpation and marked. The "center region" of the capitulum was transfixed by wire, and the anterior superior iliac crests marked. The upper edge of the great trochanter was marked to indicate the center of the femoral head. The knee-joint axis was marked. All lines drawn and /or indicated were also projected to the horizontal surface of the table. The center of gravity of the entire body was located in the cavity of the false pelvis, in the median plane, 2.1 cm vertically below the promontary, 4.5 cm above a connecting line between the centers of the femoral heads at a level with the upper edge of S3, 7 cm in front of S3, and 7 cm above the upper edge of the pubic symphysis. Dimensions of this body are as follows (in cm):

	Part		Right	Left
1.	Total length	168.8		
2.	Head length (chin-crown)	21.3		
3.	Lower edge of chin to line connecting centers of humeral heads	10.1		
4.	Line connecting centers of humeral head to line connecting centers of hip joints,	49		
5.	Limb lengths			
	upper leg		40	40
	lower leg		41.5	41.5
	foot height		6.5	6.5
	foot length		26.5	26.5
	upper arm		32	32
	forearm		27	27

6. Distance between centers of hip joints, 17
7. Distances between centers of humeral heads, (with shoulders drawn forward) 36

(Braune and Fischer, p. 595)

Table 5. Weights of Cadaver #4 (Gms)

	Part		Right	Left
1.	Entire body	55,700		
2.	Head, without neck	3,930		
3.	Torso, without limbs	23,780		
4.	Entire arm		3520	3710
5.	Upper arm		1730	2020
6.	Forearm, less hand		1790	1690
7.	Forearm, with hand		1300	1240
8.	Hand		490	450
9.	Entire leg		10110	10650
10.	Upper leg		6150	6750
11.	Lower leg, with foot		3960	3900
12.	Lower leg, less foot		2970	2900
13.	Foot		990	1000

(Braune and Fischer, ;. 594)

Table 6. The Centers of Gravity of Cadaver # 4

(see Plates I - II)

Pl. I - II 1. Torso alone, at front surface of the upper edge of L1, 0.5 cm to right of median plane, 25.8 cm above the line connecting the femoral heads.

2. Torso plus head plus arms, in median plane at front edge of T11, 29 cm above the line connecting femoral heads.

3. Head, in median plane 0.7 cm behind the slope of the sella turcica in the Fossa Tarini, and in the angle formed by the upper edge to the bridge with the posterior lamina perforata.

		Right	Left
4.	Total arm	0.5 cm below elbow axis 0.3 cm in front of bone	0.5 cm above elbow axis, 0.5 cm in front of humerus
5.	Upper arm	16.3 cm below center humeral head, 15.7 cm above elbow axis, in median plane, near rear edge of humerus, in bone	15.3 cm below center humeral head, 16.7 cm above elbow axis (as in right)
6.	Forearm	11.4 cm below elbow axis, 15.5 cm above center head of capitulum 1.5 cm in front interosseous lig., nearer radius	11.5 cm below elbow axis, 15.1 cm above center head of capitulum (as in right)

Table 6 (continued)

		Right	Left
7.	Forearm + hand	17.7 cm below elbow axis, 9.3 cm above center of head of capitulum, 0.5 cm in front of interosseous lig., nearest radius	17.9 cm below elbow axis, 9.1 cm above center of head of capitulum (as in right)
8.	Hand, fingers half flexed	5.5 cm below center of head of capitulum, between metacarpal III and palm, 1 cm from palm	5 cm below center of head of capitulum (as in right)
9.	Entire leg	35.5 cm below center of femoral head, 4.5 cm above knee axis 1 cm behind femur	33 cm below center of femoral head, 7 cm above knee axis (as in right)
10.	Upper leg	17 cm below center of femoral head, 23 cm above knee axis, 1.5 cm behind linea aspera, near median plane	15.5 cm below center of femoral head, 24.5 cm above knee axis (as in right)
11.	Lower leg	17 cm below knee axis, 24.5 cm above ankle axis, 0.7 cm behind interosseous lig., nearest tibia	17.5 cm below knee axis, 24 cm above ankle axis (as in right)
12.	Lower leg + foot	25 cm below knee axis, 16.5 cm above ankle axis, exactly at attachment interosseous lig. to tibia	25.5 cm below knee axis, 16 cm above ankle axis, (as in right)
13.	Foot	12 cm in front of rear edge, 14.5 cm behind tip, 3 cm above sole, under forward edge of cuneiform III	(exactly as in right)

(Braune and Fischer, pp. 590-594)

A resume of the findings to this point permits the following statements:

1. There is good correlation between the centers of gravity and the joint axes.

2. In the upper arm and the upper leg the center of gravity is on a straight line that passes through the head and joint on the one hand, and the center of elbow and knee joint axes on the other.

3. In the lower leg the center of gravity is in a straight line joining the center of the knee joint axis with the center of the ankle joint axis.

4. In the forearm (average pronation) the center of gravity is in a straight line through the enter of the head of the capitulum and the center of the elbow joint axis.

5. In the torso the vertebral curve was used as the basis. The center of gravity of the torso alone was definitely related to the extremities; it was in a straight line which bisected a line joining the centers of the two hip joints and connecting it with the center of the atlanto-occipital joint.

Development of Systems of Coordinates (pp 595-602)

Braune and Fischer stated as an aim: is it possible for the body to maintain such an erect position so that the centers of all main joints and all sectional centers of gravity that lay between them would fall in a single vertical frontal plane. All dimensions of Cadaver #4 were transferred life-size to squared paper, profile and front views; then a skeleton and soft tissue contours were drawn in according to these dimensions. All centers of gravity found by direct measurement were indicated. (Pl. III). This permitted the drawing-in of the three coordinates upon which a rectangular system could be based: frontal plane equals YZ plane; Z-axis equals line of intersection of median plane with the frontal plane; Y- axis equals line of intersection of frontal plane with the horizontal plane on which the body stands; X-axis equals a perpendicular to these planes at the point of intersection, O of the two axes.

Directions forward, to the right, and upward for X, Y, Z axes measuring from O on the graph, were stated as positive. The profile view equals projection of YZ plane, and the horizontal ground plane equals XY plane. The spinal curve was laid out in a plane through the center point of the two hip joints and the atlanto-occipital joint (the center of gravity of the torso was assumed to be in this plane). The position of the arms in the drawing was changed so that the center of humeral head and the centers of gravity of the separate parts were in the XY plane. The two drawings were photographed and reduced to 1/10 (all mm in Plates equals cm in drawing; Plate III).

The problem of going rom the recumbent to the erect position was considered, for spinal curve is not the same in the two positions. The problem was met as follows: 1) a muscular man had the centers of joints marked on his body, just as in the drawing; 2) the center of gravity of the head was projected to the side of the head and marked; 3) two long plumb lines determined the YZ Plane; 4) the model was then moved until all joint markings were in the YZ Plane.

The model was then photographed, as in Plate IV (left arm bent a bit to show hip markings: consequent displacement of center of gravity is 7 mm upward, 4 mm forward 1 mm. to right). Plates III-IV compared favorably so the method was deemed acceptable. The result is a standard or normal body position (n. b. p.). The centers of gravity for the whole body and the limb systems were calculated for the n. b. p. from the centers of gravity for the separate sections; this is done for the control of the measurements of centers of gravity both of the separate limbs and of the whole body.

If the joint centers in the n. b. p., plus measured centers of the different limbs, are plotted on the system of coordinates, the following coordinate values will be obtained (read off from Plate III):

Table 7. Coordinates for Centers of Joints, Crown, Tip of Foot and Lower Edge of Bent Hand (Fingers Flexed)

		x	y	z
1.	Crown	0	0	168.8
2.	Atl. - Occ. Joint	0	0	154.0
3.	Hip			
	R	0	+ 8.5	88.0
	L	0	- 8.5	88.0
4.	Knee			
	R	0	+ 8.5	48.0
	L	0	- 8.5	48.0
5.	Ankle			
	R	0	+ 8.5	6.5
	L	0	- 8.5	6.5
6.	Tip of foot			
	R	+20.5	+13.0	0
	L	+20.5	-13.0	0
7.	Shoulder			
	R	0	+18.0	137.0
	L	0	-18.0	137.0
8.	Elbow Joint			
	R	0	+18.0	105.0
	L	0	-18.0	105.0
9.	Wrist			
	R	0	+18.0	78.0
	L	0	-18.0	78.0
10.	Lower Edge bent hand			
	R	0	+18.0	67.5
	L	0	-18.0	67.5

(Braune and Fischer, p. 600)

The coordinates for centers of gravity may be given. The following notations are employed, as examples: the upper leg is 3, 4 because it is between joints numbered 3 and 4 respectively. This makes it easy to differentiate between the coordinates of the center points of joints and those of the center of gravity, e.g., coordinates for the center of the hip joint = x_3, y_3, z_3, coordinates for the center of gravity of the upper leg - $x_{3,4}$, $y_{3,4}$, $z_{3,4}$. If right and left are used then the coordinates of the left hip joint are x'_3, y'_3, and the coordinates of the center of gravity of the left upper leg = $x'_{3,4}$, $y'_{3,4}$, $z'_{3,4}$.

Coordinates for the whole body are x_0, y_0, z_0.

Table 8. Coordinates for Centers of Gravity of Entire Body

Coords. for	x	y	z
1, 2 - head	0	0.0	157.8
2, 3 - torso	0	+0.5	113.8
3, 4 - upper leg			
R	0	+8.5	71.0
L	0	-8.5	72.5
4, 5 - lower leg			
R	0	+8.5	31.0
L	0	-8.5	30.5
5, 6 - foot			
R	+ 6.5	+10.3	3.0
L	+ 6.5	-10.3	3.0
7, 8 - upper arm			
R	0	+ 18.0	120.7
L	0	- 18.0	121.7
8, 9 - forearm			
R	0	+ 18.0	93.6
L	0	- 18.0	93.1
9, 10 - Hand			
R	0	+ 18.0	72.5
L	0	- 18.0	73.0
1, 6, 7, 10 - whole body	0	0	92.5
1, 3, 7, 10 - Torso + head + arms	0	0	117.0
3, 6 - whole leg			
R	0	+ 8.5	52.5
L	0	- 8.5	55.0
4, 6 - lower leg and foot			
R	0	+ 8.5	23.0
L	0	- 8.5	22.5
7, 10 - whole arm			
R	0	+18.0	104.5
L	0	-18.0	105.5
8, 10 - forearm + hand			
R	0	+ 18.0	87.3
L	0	- 18.0	87.1

(Braune and Fischer, p. 601)

Since weights are necessary for calculations they are restated below; since ratios are important they are reduced to figures per 10,000, for this is advantageous in the calculation of the common center of gravity.

Table 9. Weights and Weight Ratios

Parts of Body	Wt. (gms) Cad. IV	Wt. ratio per 10,000 whole body
1, 2 - head	3930	705.5
2, 3 - torso	23780	4270
3, 4 - upper leg		
R	6150	1104
L	6750	1212
4, 5 - lower leg		
R	2970	533
L	2900	520.5
5, 6 - foot		
R	990	178
L	1000	179.5
7, 8 - upper arm		
R	1730	310.5
L	2020	362.5
8, 9 - forearm		
R	1300	233
L	1240	222.5
9, 10 - hand		
R	490	88
L	450	81
	55,700	10,000

(Braune and Fischer, p. 602)

As an example the weight, p, for the right upper arm = $p_{3,4}$, and the weight for the left upper leg = $p'_{3,4}$. This corresponds with the designations used for the center of gravity coordinates, and so on.

The Calculation of the Common Center Of Gravity for the Entire Body, and for Whole Sections, from the Centers Of Gravity and Weights of Separate Limbs. (pp 603-625)

The forces of inertia are parallel forces. Hence, it is necessary to show how a resultant can be obtained from two or more parallel forces. In Fig. 1 assume A and B to be points of attachment of two parallel forces, with strengths P_1 and P_2. AC and BD are graphic lines representing by length and direction the size and strength of these two forces. If two equal but opposed forces of any desired size are placed in the direction of a line connecting A and B, so that one force is attached at A, and the other at B, this will not change the original system of forces.

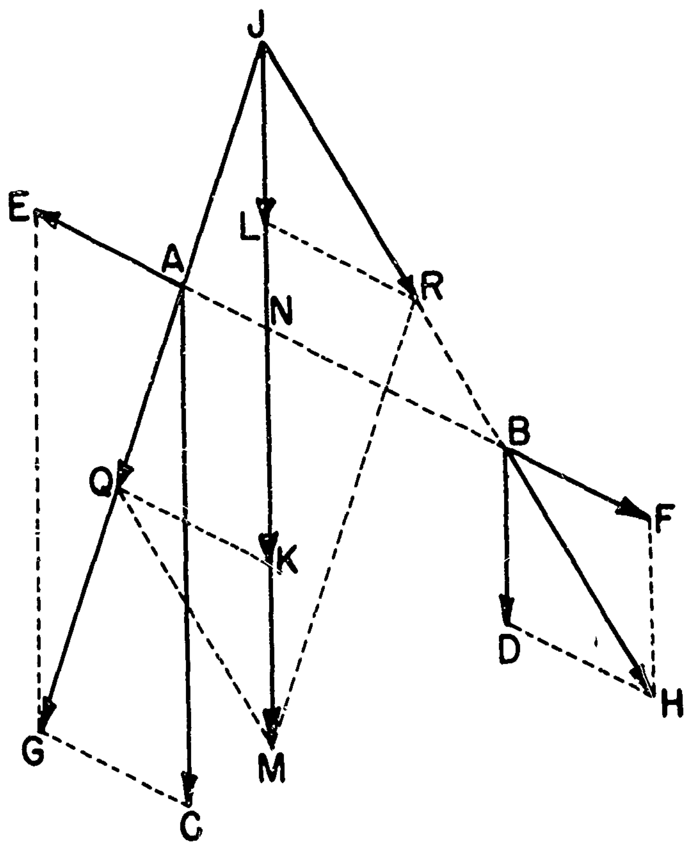


Figure 1. Diagram to Show How a Resultant Can be Obtained from Two or More Parallel Forces

These two new forces (Fig. 1) are shown by lines AE, BF, and their strength by q. If the two forces at A are combined to give AG and those at B to give BH, the total of AG and BH is still the same as p_1 and p_2. The same is true if the two resultants, are moved to a common intersection point, J, of their directions, so that J is the common point of attachment of them both. The two new forces JQ and JR can be combined with the further resultant JM that will represent the resultant equal to the sum of the two parallel forces.

The size of JM equals the sum of the two parallel forces, or $p_1 + p_2$. From Fig. 1 it is noted that $\Delta GCA \simeq \Delta QKJ \simeq \Delta RLM$, and $\Delta HDB \simeq \Delta RLJ$

= $\triangle$QKM. Therefore JK = AC = P and KM = BD = p_2.

Hence, JM = $p_1 + p_2$. From the congruence of the triangles it follows that JM is parallel to the two original forces. JM intersects in point N, the line that connects the points A and B. The distances of N from A and B are related to each other as the reverse of the parallel forces at A and B, for from the similarity of triangles ANJ and QKJ the proportion follows: AN: JN = q: p_1. From triangles BNJ and RLJ it follows:

BN: JN = q: p_2 - therefore

JN: q = AN. p_1 = BN. p_2 - therefore

AN: BN = p_2: p_1

If the two parallel forces are exerted in another direction, but still parallel to each other, and still fixed to A and B, then the direction of the new resultants of parallel forces intersects AB in a point whose distances from A and B are in reverse ratio to p_1, p_2. Hence, N is again the point of intersection.

It is concluded that the direction of all resultants via any rotation of forces p_1, p_2 around points of attachment all pass through N, if forces are parallel after rotation. If p_1, p_2 are forces of inertia N is their center of gravity.

If x_1, y_1, z_1 are coordinates of A of force p_1 and x_2, y_2, z_2 of B of force p_2, and if coordinates of center of gravity N are x_0, y_0, z_0, then the following ratios hold:

$(x_0 - x_1) : (x_2 - x_0) = p_2 : p_1$ this is so because

$(y_0 - y_1) : (y_2 - y_0) = p_2 : p_1$ N is on AB

$(z_0 - z_1) : (z_2 - z_0) = p_2 : p_1$

From this it is found:

$$x_0 = \frac{p_1 x_1 + p_2 x_2}{p_1 + p_2}$$

for coordinates

$$y_0 = \frac{p_1 y_1 + p_2 y_2}{p_1 + p_2}$$

x_0, y_0, z_0 of center of gravity.

$$z_0 = \frac{p_1 z_1 + p_2 z_2}{p_1 + p_2}$$

Assume three parallel forces p_1, p_2, p_3 whose points of attachment have coordinates x_1, x_2, x_3; y_1, y_2, y_3; z_1, z_2, z_3. These may be combined into one resultant by first obtaining p_1 and p_2 and combining their resultant with p_3. Hence, coordinates x'_0, y'_0, and z'_0 of the center of gravity of $(p_1 + p_2)$ and p_3, whose points of attachment have coordinates x_0, y_0, z_0 and x_3, y_3, z_3, may be calculated, to give:

$$x'_0 = \frac{(p_1 + p_2)\, x_0 + p_3 x_3}{(p_1 + p_2) + p_3} = \frac{p_1 x_1 + p_2 x_2 + p_3 x_3}{p_1 + p_2 + p_3}$$

$$y'_0 = \frac{(p_1 + p_2)\, y_0 + p_3 y_3}{(p_1 + p_2) + p_3} = \frac{p_1 y_1 + p_2 y_2 + p_3 y_3}{p_1 + p_2 + p_3}$$

$$z'_0 = \frac{(p_1 + p_2)\, z_0 + p_3 z_3}{(p_1 + p_2) + p_3} = \frac{p_1 z_3 + p_2 z_2 + p_3 z_3}{p_1 + p_2 + p_3}$$

The above three equations formulate a principle: <u>n</u> parallel forces, p_1, p_2, p_3 . . . . p_n, can be combined into a single resultant, according to which coordinates x_0, y_0, z_0 indicate the center of gravity for all <u>n</u> forces of inertia. This may be generalized:

$$x_0 = \frac{p_1 x_1 + p_2 x_2 + p_3 x_3 \cdots\cdot p_n x_n}{p_1 + p_2 + p_3 \cdots\cdot p_n} = \frac{\sum p_i x_i}{\sum p_i}$$

$$y_0 = \frac{p_1 y_i + p_2 y_2 + p_3 y_3 \cdots \quad p_n y_n}{p_1 + p_2 + p_3 \cdots\cdot p_n} = \frac{\sum p_i y_i}{\sum p_i}$$

$$z_0 = \frac{p_1 z_1 + p_2 z_2 + p_3 z_3 \cdots \quad p_n z_n}{p_1 + p_2 + p_3 \cdots\cdot p_n} = \frac{\sum p_i z_i}{\sum p_i}$$

From these three equations it is noted that x_0, y_0, z_0 do not change if we substitute for the real weights (p_i) the ratios of body weight calculated as 10,000. The calculation of the common center of gravity for any position can be done with one divisor (10,000) in all three equations.

From this the values of Cadaver #4 are (for products $p_1\ z_1$ for calculating coordinate z_0 for center of gravity of whole body in normal position):

Head	111327.9
Torso	485926.0
R upper leg	78384.0
L " "	87870.0
R lower leg	16523.0
L " "	15875.25
R foot	534.0
L "	538.5
R upper arm	37477.35
L " "	44116.25
R forearm	21808.8
L "	20714.7
R hand	6380.0
L "	5913.0
	933388.8

(Braune and Fischer, p. 607)

By dividing by $\sum p_i = 10,000$ the result for $z_0 = 93.3$ cm., whereas direct measure of the Z coordinate gives 92.5 cm. (Because of saw cuts and variable positioning absolute accuracy is impossible). If the center of gravity for feet is disregarded and mass is balanced in the ankle axis:

$$\sum p_i\ z_i = 933388.8 - (534 + 538.5)$$
$$= 932316.3$$

Divided by $10,000 - (178 + 179.5) = 9642.5$.

Hence Z coordinate for center of gravity of body sans feet, using coordinates x'_0, y'_0, z'_0 is

$$z'_0 = \frac{932316.3}{9642.5} = 96.7$$

The following values for products $p_i\ y_i$ may be used in calculating y_0 coordinates of the common center of gravity.

Head	0		
Torso	+2135		
R upper leg	+9384	L upper leg	-10302
R lower leg	+4530.5	L lower leg	- 4424.25
R foot	+1833.4	L foot	- 1848.85
R upper arm	+5589	L upper arm	- 6525
R forearm	+4194	L forearm	- 4005
R hand	+1584	L hand	- 1458
	+29249.9		-28563.10

(Braune and Fischer, p. 608)

From this $\sum p_i y_i = 29249.9 - 28563.1$
$= 685.8$, and

$$y_0 = \frac{686.8}{10,000} = +0.07 \text{ cm}$$

In calculating x_0 coordinates for the common center of gravity only products $p_i\, x_i$ that belong to the center of gravity for the two feet have a value other than zero in the normal position.

Accordingly,

$$x_0 = \frac{178\ .\ \ 6.5 + 179.5\ .\ \ 6.5}{10,000}$$

$$= \frac{2323.75}{10,000}$$

$$= 0.2 \text{ cm}$$

Hence, the common center of gravity for the position of the two feet is 2 mm in front of the frontal plane in which lie all the centers of gravity save those for the feet. The X-coordinates for the common center of gravity without feet is zero, because all x_i values are zero.

Braune and Fischer present in detail the calculated coordinates for the centers of gravity of limbs and their combinations - previously done by direct measurements - in addition to the coordinates for the common center of gravity. Only one example is here reproduced.

Table 10. Calculated Coordinates for Centers of Gravity for the Entire Leg.

Right $p_i z_i$		Left $p_i z_i$	
Upper leg	78384	Upper leg	87870
Lower "	16523	Lower "	15875.25
Foot	534	Foot	538.50
Totals	95441		104283.75

Weights		Weights	
Upper leg	1104	Upper leg	1212
Lower "	533	Lower "	520.5
Foot	178	Foot	179.5
Totals	1815		1912

Therefore:

$$z_{3,6} = \frac{95441}{1815} = 52.6 \text{ cm} \qquad z'_{3,6} = \frac{104283.75}{1912} = 54.5 \text{ cm}$$

$$y_{3,6} = +8.7 \text{ cm} \qquad y'_{3,6} = -8.7 \text{ cm}$$

$$x_{3,6} = +0.6 \text{ cm} \qquad x'_{3,6} = -0.6 \text{ cm}$$

(Braune and Fischer, p. 609)

If it is assumed that there is a uniform and symmetrical distribution of body masses it is possible to take the average of the center of gravity coordinates and the weight ratios (previously tabulated). If, further, the center of gravity of the torso is in the median plane, the mean coordinate values of the centers of gravity will be:

Parts of Body	x	y	z
Head	0	0	157.8
Torso	0	0	113.8
Both upper legs	0	± 8.5	71.75
" lower "	0	± 8.5	30.75
" feet	± 6.5	± 10.5	3
" upper arms	0	± 18	121.2
" forearms	0	± 18	93.35
" hands	0	± 18	72.75

(Braune and Fischer, p. 612)

Positive values of y are on the right side, negative values on the left.

If it is assumed that the total weight is 10,000 it follows that average weights and weight ratios may be given as:

Mean weights (gms)		Mean weight ratios (as whole numbers)
whole body	55,700	10,000
Head	3,930	706
Torso	23,780	4,270
Upper leg	6,450	1,158
Lower	2,935	527
Foot	995	179
Upper arm	1,875	336
Forearm	1,270	228
Hand	470	84

(Braune and Fischer, p. 612)

From this, assuming uniform and symmetrical distributions of masses, may be calculated center of gravity coordinates.

Entire Body

	$p_i\ z_i$
Head	111327.9
Torso	485926.0
Both upper legs	166173.0
" lower "	32395.125
" feet	1072.5
" upper arms	81567.6
" forearms	42520.925
" hands	12294.75
Total	933277.8

$$z_0 = \frac{933277.8}{10,000} = 93.3 \text{ cm}$$

$$y_0 = 0.0 \text{ cm}$$

$$x_0 = \frac{2 \cdot 6.5 \cdot 178.75}{10,000} = 0.2 \text{ cm}$$

If feet are not included, from $\sum p_i z_i$ may be derived:

whole body	=	933277.8
both feet	=	- 1072.5
Total		932205.3

For weights:

whole body	=	10,000
both feet	=	- 357.5
Total		9,642.5

$$\text{Therefore } z'_0 = \frac{932205.3}{9642.5} = 96..7 \text{ cm}$$

$$y'_0 = 0 \text{ cm}$$

$$z'_0 = 0 \text{ cm}$$

(Braune and Fischer, p. 613)

[Braune and Fischer perform these calculations for the body and its parts in detail and find great agreement between calculated centers of gravity and those determined by direct measurement. They conclude that this validates going from recumbent body relationships to those of the vertical position]

A distance ratio of 0.609 from above and 0.391 from below in the body position for the center of gravity for torso alone is obtained (atlanto-occipital joint to line connecting hip joints reduced to 1.0). By limiting the distance ratios in the extremities to two decimal points the following is obtained:

Body Part	From Above	From Below
Upper arm	0.47	0.53
Forearm	0.42	0.58
Upper leg	0.44	0.56
Lower "	0.42	0.58
Foot	0.43	0.57

This is a 4/9 - 5/9 ratio. It may be said that the center of gravity of the separate extremities lies above the center of the latter and divides the distance between the two ends (joint axes or center points) as 4:5. To get the center of gravity of an extremity rapidly, measure the distance between its axes, divide by nine, and place the center of gravity 4/9 of the distance below the proximal joint and in a straight line with the center of the joint. The center of gravity is almost at the edge of the bone in the humerus and femur, and about 1 cm from the center of the interosseous lig. (toward the curved side) in the forearm and the lower leg.

In an over-all view the findings of Braune and Fischer are in reasonable agreement with those of Harless, but not with those of Meyer (1853).

Determining the Location of the Center of Gravity on the Living Human Body in Different Positions and With Different Loads. (pp 626-672)

Measuring and weighing of a cadaver permits the location of the center of gravity of the living body, at any one time. The body must be plotted on a graph of space coordinates. Two projections are enough, on one plane each; best are sagittal and frontal planes. The horizontal ground surface always equals XY Plane. The vertical that bisects the line connecting the centers of the hip joints equal Z Axis. If the X Axis is in a sagittal direction, then the Y Axis will have a frontal direction. This in itself determines a system of right-angled coordinates, when it is also decided that the positive direction of:

X axis = forward
Y axis = to the right
Z axis = upward

Two plumb lines, which gave the YZ Plane for the normal position, orient the body for all photography. Joint centers are precisely marked on photos, as indicated by measurements on the living in the same position and as projected on the body surface. Joint-center location also serves to locate the center of gravity of a limb, for the center of gravity lies very nearly in a line connecting the centers of two neighboring joints.

After determining the ratio of the distance of a limb's center of gravity and the center of the neighboring joint, it is possible to calculate from the coordinates of the centers of the joints the coordinates of the center of gravity lying between them. If x_i, y_i, z_i and also x_k, y_k, z_k are the coordinates of the centers of the joints of a limb, and ϵ_i ϵ_k are the ratios of the distances of a center of gravity from the neighboring joint centers, then are derived the following proportions for the center of gravity in calculating the coordinates:

$$(x_i - x_{i,k}) : (x_{i,k} - x_k) = \epsilon_i : \epsilon_k$$

$$(y_i - y_{i,k}) : (y_{i,k} - y_k) = \epsilon_i : \epsilon_k$$

$$(z_i - z_{i,k}) : (z_{i,k} - z_k) = \epsilon_i : \epsilon_k$$

From this it follows that:

$$(\epsilon_i + \epsilon_k)\, x_{i,k} = \epsilon_k x_i + \epsilon_i x_k$$

$$(\epsilon_i + \epsilon_k)\, y_{i,k} = \epsilon_k y_i + \epsilon_i y_k$$

$$(\epsilon_i + \epsilon_k)\, z_{i,k} = \epsilon_k z_i + \epsilon_i z_k$$

Since ratio ϵ_i and ϵ_k are calculated for a total limb length of 1.0, then $\epsilon_i + \epsilon_k = 1.0$.

Therefore:

$$x_{i,k} = \epsilon_k x_i = \epsilon_i x_k$$

$$y_{i,k} = \epsilon_k y_i = \epsilon_i y_k$$

$$z_{i,k} = \epsilon_k z_i = \epsilon_i z_k$$

The quantities to be used as values for the ratios ϵ_i and ϵ_k are averages of their values taken from the three centers of gravity determinations, calculated to two decimals:

Body Part	ϵ_i	ϵ_k
Torso	0.61	0.39
Upper leg	0.44	0.56
Lower leg	0.42	0.58
Foot	0.43	0.57
Upper arm	0.47	0.53
Forearm	0.42	0.58

In this ϵ_i always belongs to the proximal joint. When the distance of the atlanto-occipital joint to the hip axis equals 66 cm , the distance of the center of gravity from the atlanto-occipital joint was 40.2 cm and from the axis of the hip joint was 25.8 cm (the ϵ_i and ϵ_k in the the above tabulation were calculated from this); it was further assumed that the center of gravity of the torso was in the median plane. The hand is too variable for the calculation of ϵ_i and ϵ_k. The authors do not recommend averaging the Harless data and the present data to get the weights of the different parts of the living body. They used a model very similar in size and build to Cadaver #4.

#4 = 55,700 gms

model = 58,700 gms

58,700/55,700 = 1.05386.

The following are weights on the living model which are the bases for further study:

Total	58,700	gms	
Head	4,140	"	
Torso	25,060	"	
Upper leg	6,800	"	
Lower leg	3,890	"	single limb
Foot	1,050	"	
Upper arm	1,980	"	part, in each
Forearm	1,340	"	
Hand	490	"	instance

A number of photographs were made, so that on the photo 1 mm equals 1 cm of body dimension.

1. Plate IV = normal position
2. Plate V = "easy" self-assumed position
3. Plate III = "military" position (nude, with military equipment, sans pack)
4. Plates VII - VIII = "present arms"
5. Plates IX - X = firing
6. Plates XI - XII = holding rifle, with arm outstreached, and nude positions in complete military equipment, with loaded cartridges
7. Plates XIII - XIV = standing and shouldering arms
8. Plates XV - XVI = firing

In the first three positions profile views are given; in the other views frontal and profile views, left arm bent or slightly forward. The coordinates of the joint centers are determined from the photos. On the live model the joint centers were projected to the surface.

Transparent paper (millimeter) was laid on the photograph. The Z Axis was in the direction of the plumb-line, and through the center of a line connecting the two hip centers, so that the point of origin of the coordinates lay at the horizontal ground level. This made it possible to obtain all three coordinates in positions 3-8, above, from both views, allowing for foreshortening and/or lengthening due to perspective. For positions 1-3, above, only coordinates X and Z were obtained from one photo; but since distances for the symmetrically placed limbs had been measured on the model, the Y coordinate could be assumed from bilateral symmetry.

POSITION OF THE BODY WITHOUT A LOAD (pp 631-643)

1. NORMAL POSITION (Plate IV) (pp 631-637)

Table 11. Coordinates of the Centers of the Joints in the Normal Position

Joints	x	y	z
Atl-occip	0	0	152
Hip			
R	0	+ 8.5	87
L	0	- 8.5	87
Knee			
R	0	+ 8.5	47
L	0	- 8.5	47
Tibio-talar			
R	0	+ 8.5	6
L	0	- 8.5	6
Rear Edge Foot			
R	- 4	+ 6	4
L	- 4	- 6	4
Tip of foot			
R	+ 20	+ 16	1.5
L	- 20	- 16	1.5
Humeroscapular			
R	0	+ 18	134
L	0	- 18	134
Elbow			
R	0	+ 18	103
L	0	- 18	103
Wrist			
R	0	+ 18	76
L	+ 19	- 11	108

(Braune and Fischer, p. 631)

From these the cent. rs of gravity (except for head and hand) were calculated directly from the figure, using ϵ_i and ϵ_k, with the aid of the formulae:

$$x_{i,k} = \epsilon_k x_i + \epsilon_i x_k$$

$$y_{i,k} = \epsilon_k y_i + \epsilon_i y_k$$

$$z_{i,k} = \epsilon_k z_i + \epsilon_i z_k$$

Table 12. Coordinates of Center of Gravity in the Normal Position

	x	y	z
Head	0	0	156
Torso	0	0	112.4
Upper leg			
R	0	+ 8.5	69.4
L	0	- 8.5	69.4
Lower leg			
R	0	+ 8.5	29.8
L	0	- 8.5	29.8
Foot			
R	+ 6.3	+ 10.3	3
L	+ 6.3	- 10.3	3
Upper arm			
R	0	+ 18.	119.4
L	0	- 18	119.4
Forearm			
R	0	+ 18	91.7
L	+ 8	- 15.1	105.1
Hand			
R	0	+ 18	71
L	+23	- 8	108.5

(Braune and Fischer, p. 632)

With these coordinates and the weights earlier given there were calculated

Table 13. The Centers of Gravity for x_0, y_0, z_0 Coordinates of the Common Center of Gravity

Parts of Body	$p_i x_i$	$p_i y_i$	$p_i z_i$
Head	0	0	645840
Torso	0	0	2816744
Upper leg			
R	0	+57800	471920
L	0	-57800	471920
Lower leg			
R	0	+26265	92082
L	0	-26265	92082
Foot			
R	+6615	+10815	3150
L	+6615	-10815	3150
Upper arm			
R	0	+35640	236412
L	0	-35640	236412

Table 13 (continued)

Forearm			
R	0	+24120	122878
L	-10720	-20234	140834
Hand			
R	0	+8820	34790
L	+11270	-3920	53165
Total	+35220	+8786	5421379

(Braune and Fischer, p. 632)

From this $x_0 = \frac{35220}{58700}$, $y_0 = \frac{8786}{58700}$ and $z_0 = \frac{5421379}{58700}$, or $x_0 = +0.6$, $y_0 = 1.51$ and $z_0 = 92.4$

If bent left forearm be corrected for symmetry to right the three coordinates for center of gravity are 0, -18, 91.7, and for the left hand 0, -18, and 71 .

The three coordinates for the common center of gravity for normal position are

$$x_0 = \frac{13230}{58700} = +0.2$$

$$y_0 = 0.1$$

$$z_0 = \frac{5385048}{58700} = 91.7$$

So, raising the left forearm displaced the common center of gravity 6.7 cm higher, 0.4 cm farther forward, and 0.1 cm to the right. The common center of gravity in the normal symmetrical position is 4.7 cm above the hip join ; direct measurement gave 4.5 cm.

In the normal position stability of the body is greatest in resisting a push from the rear rather than from in front, because the body leans a bit more backward and hence falls backward more easily. This position is also best for locating center of gravity directly on the body without photographs.

The authors demonstrate how to calculate coordinates for centers of gravity after bending the limb, from its coordinates before bending (i.e., from the normal position). In Figure 2 assume that M_i M_k are the center points of joints of a limb, with M_i M_k is in the normal position it must be parallel to z-axis.

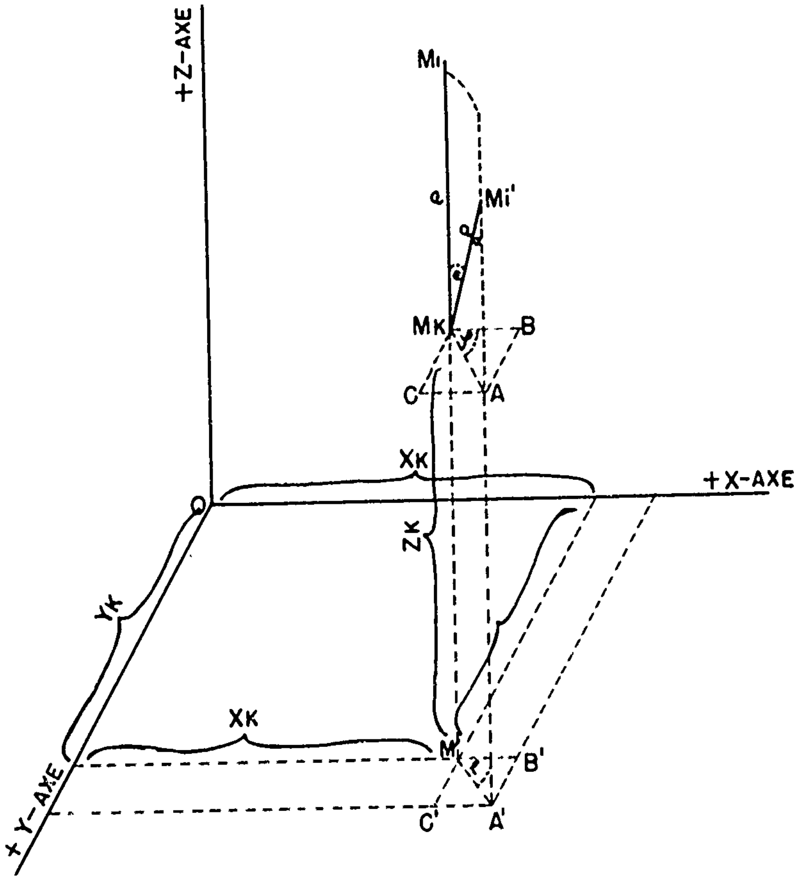


Figure 2. Calculation of Coordinate Centers of Gravity in Flexed Limb, Based on Coordinates in Normal (Unflexed) Position

Now rotate the limb on joint center M_k about an angle α, in a plane that forms angle φ with the YZ plane. Joint center of M_i assumes position M_i. Angle α is the angle which $M_k M_i$ forms with the positive direction of the Z-axis. Coordinates of point M'_i may be calculated from this fixed position M_k as follows: if limb length is called a then $M_k AM'_i$ forms a right-angled triangle, with the right angle at $\bar{A}$, the acute angle α, and the length of the hypotenuse a; the M_k A = sine α However the distance M_k A is also the hypotenuse of a right-angled triangle ABM_k, with right angle at B, and the acute angle φ. In this right angled-triangle the two perpendiculars M_kB and AB are exactly the distance by which coordinate x'_i is greater than x_k, and coordinate y'_i is greater than y_k. Hence, z'_i is greater than z_k by the distance AM'_i. If we consider that AM'_i is a perpendicular in the first right-angled triangle, then:

$$x'_i = x_k + \alpha \text{ sine } \alpha \text{ , } \cos \varphi$$

$$y'_i = y_k + \alpha \text{ sine} \alpha \text{ , sine} \varphi$$

$$z'_i = z_k + \alpha \cos \alpha$$

2. THE EASY NATURAL POSITION

(Plate V) (pp 637-639)

As for the normal position three sets of tabulations are offered.

Table 14. Coordinates for Centers of Joints in the Easy Natural Position

Joints	x	y	z
Atl-occip	- 1.5	0	150.5
Ankle			
R	0	+ 8.5	87
L	0	- 8.5	87
Knee			
R	- 1	+ 7	47
L	- 1	- 7	47
Tibio-talar			
R	- 5	+ 4.5	6
L	- 5	- 4.5	6
Rear Edge Foot			
R	- 9	+ 2	4
L	- 9	- 2	4
Tip of foot			
R	+ 15	+ 12	1.5
L	+ 15	- 12	1.5
Humeroscapular			
R	- 1	+ 18	133
L	- 1	- 18	133
Elbow			
R	- 3	+ 18	102
L	- 3	- 18	102
Wrist			
R	+ 1	+ 18	75.5
L	+ 17	- 13	102.5

(Braune and Fischer, p. 637)

Table 15. Coordinates for Centers of Gravity in the Easy Natural Position

	x	y	z
Head	- 1	0	154.5
Torso	- 0.6	0	111.8
Upper leg			
R	- 0.4	+ 7.8	69.4
L	- 0.4	- 7.8	69.4

Table 15 (continued)

	x	y	z
Lower leg			
R	- 2.7	+ 6	29.8
L	- 2.7	- 6	29.8
Foot			
R	+ 1.3	+ 6.3	3
L	+ 1.3	- 6.3	3
Upper arm			
R	- 1.9	+ 18	118.4
L	- 1.9	- 18	118.4
Forearm			
R	- 1.3	+ 18	90.9
L	+ 5.4	- 15.9	102.2
Hand			
R	0	+ 18	70.5
L	+ 20.5	- 10	102

(Braune and Fischer, p. 638)

Table 16. Coordinates x_0, y_0, z_0 for the Common Center of Gravity

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Head	- 4140	0	639630
Torso	-15036	0	2801708
Upper leg			
R	- 2720	+53040	471920
L	- 2720	-53040	471920
Lower leg			
R	- 8343	+ 18540	92082
L	- 8343	- 18540	92082
Foot			
R	+ 1365	+ 6615	3150
L	+ 1365	- 6615	3150
Upper arm			
R	- 3762	+ 35640	234432
L	- 3762	-35640	234432
Forearm			
R	- 1742	+ 24120	121806
L	+ 7236	- 21306	136948
Hand			
R	0	+ 8820	34545
L	+ 10045	- 4900	49980
Totals	-30557	+ 6734	5387785

Hence, $x_0 = \dfrac{30557}{58700}$, $y_0 = \dfrac{6734}{58700}$, $z_0 = \dfrac{5387785}{58700}$.

$$x_0 = -0.5$$

$$y_0 = 0.1$$

$$z_0 = 91.8$$

(Braune and Fischer, pp. 638-639)

If the bent left forearm be made symmetrical with the right the three coordinates for the centers of gravity will be:

for left forearm	- 1.3	- 18	90.9
for left hand	0	- 18	70.5

As a result, the center of gravity coordinates for the entire body in the easy natural position are:

$$x_0 = \frac{49580}{58700} = -0.8$$

$$y_0 = 0$$

$$z_0 = \frac{5357208}{58700} = 91.3$$

Here raising the left forearm drew the center of gravity 0.3 cm farther forward, 0.1 cm to the right, and 0.5 cm higher up. In the symmetrical condition here the common center of gravity is 7.3 cm above the hip axis, 0.8 cm toward the rear, hence always above the hip socket.

3. Military Position (Plate VI) (pp 640-643)

The three tabulations are given here.

Table 17. Coordinates for Joint Centers in the Military Position

	x	y	z
Atlanto-occip.	+ 5	0	152
Hip			
R	0	+ 8.5	87
L	0	- 8.5	87
Knee			
R	- 4.5	+ 6	47
L	- 4.5	- 6	47

Table 17 (continued)

	x	y	z
Tibio-talar			
R	- 7	+ 4.5	6
L	- 7	- 4.5	6
Rear edge foot			
R	-11	+ 2	4
L	-11	- 2	4
Tip foot			
R	+13	+12	1.5
L	+13	-12	1.5
Humeroscapular			
R	+ 4.5	+18	133
L	+ 4.5	-18	133
Elbow			
R	+ 0.5	+18	102.5
L	+ 0.5	-18	102.5
Wrist			
R	0	+18	76
L	+ 7.5	-18	76.5

(Braune and Fischer, p. 640)

Table 18. Center of Gravity Coordinates in the Military Position

	x	y	z
Head	+6	0	156
Torso	+2	0	112.4
Upper leg			
R	-2	+ 7.4	69.4
L	-2	- 7.4	69.4
Lower leg			
R	-5.6	+ 5.4	29.8
L	-5.6	- 5.4	29.8
Foot			
R	-0.7	+ 6.3	3
L	-0.7	- 6.3	3
Upper arm			
R	+2.6	+18	118.7
L	+2.6	-18	118.7
Forearm			
R	+0.3	+18	91.4
L	+3.4	-18	91.6
Hand			
R	0	+18	70
L	+7.5	-18	70.5

(Braune and Fischer, p. 640)

Table 19. The x_0, y_0, z_0 Coordinates for the Common Center of Gravity

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Head	+24840	0	645840
Torso	+50120	0	2816744
Upper leg			
R	-13600	+50320	471920
L	-13600	-50320	471920
Lower leg			
R	-17304	+16686	92082
L	-17304	-16686	92082
Foot			
R	- 735	+ 6615	3150
L	- 735	+ 6615	3150
Upper arm			
R	+ 5148	+35640	235026
L	+ 5148	-35640	235026
Forearm			
R	+ 402	+24120	122476
L	+ 4556	-24120	122744
Hand			
R	0	+ 8820	34300
L	+ 3675	- 8820	34545
Totals	+30611	0	5381005

Here $x_0 = \frac{30611}{58700}$, $y_0 = \frac{0}{58700}$, $z_0 = \frac{5381005}{58700}$.

$$x_0 = 0.5$$

$$y_0 = 0$$

$$z_0 = 91.7$$

(Braune and Fischer, p. 641)

If the left arm is made symmetrical:

L forearm	+0.3	-18	91.1
R hand	0	-18	70.0

The coordinates for the common center of gravity in the military position are :

$$x_0 = \frac{22782}{58700} = +\ 0.4$$

$$y_0 = 0$$

$$z_0 = \frac{5380492}{58700} = 91.7$$

The common center of gravity is 4.7 cm above the hip axis, and drawn 0.4 cm within the body. The gravity line is 7.2 cm in front of the tibio-talar (ankle) joint, nearer the ball of the foot.

POSITIONS OF BODY WITH A LOAD
(Military Equipment) (p. 643-666)

Clothing is omitted. Helmet center of gravity is not calculated; belt and its lock are omitted. Weight of military equipment M/87 is as follows :

Misc. articles	12 kg , 250 gms
Rifle equipment	7 kg , 300 gms
Rifle	4 kg , 700 gms

[A total weight of 24 kg , 100 gms is used in all calculations the centers of gravity of each item of equipment are calculated.]

4. <u>Military Position, Presenting Arms</u>

(Plates VII - VIII)(pp.648-650)

The three tabulations are here given.

Table 20. <u>Coordinates for Centers of Joints for Military Position Presenting Arms</u>

	x	y	z
Atl-occip.	+ 3	+ 1.5	151
Hip			
R	0	+ 8.5	87
L	0	- 8.5	87

Table 20. (continued)

	x	y	z
Knee			
R	- 5.5	+ 6	47
L	- 5.5	- 6	47
Tibio-talar			
R	-10	+ 4.5	6
L	-10	- 4.5	6
Rear edge foot			
R	-14	+ 2	4
L	-14	- 2	4
Tip of foot			
R	+10	+12	1.5
L	+10	-12	1.5
Humeroscapular			
R	+ 5.5	+18	133
L	+ 5.5	-18	133
Elbow			
R	+15	+13.5	103
L	+ 4	-20	101
Wrist			
R	+23	+ 7.5	81
L	+25.5	- 7.5	102

(Braune and Fischer, p. 648)

Table 21. Coordinates for Centers of Gravity for Military Position Presenting Arms

	x	y	z
Head	+ 4	+ 1.5	155
Torso	+ 1.2	+ 0.6	122
Upper leg			
R	- 2.4	+ 7.4	69.4
L	- 2.4	- 7.4	69.4
Lower leg			
R	- 7.4	+ 5.4	29.8
L	- 7.4	- 5.4	29.9

Table 21 (continued)

	x	y	z
Foot			
R	- 3.7	+ 6.3	3
L	- 3.7	- 6.3	3
Upper arm			
R	+10	+15.9	118.9
L	+ 4.8	-18.9	116.9
Forearm			
R	+18.4	+11	93.8
L	+13	-14.8	101.4
Hand			
R	+27	+ 4.5	77
L	+29.5	- 3.5	103
Rifle	+27	- 0.5	104.5

(Braune and Fischer, p. 648)

(Here, as in all later positions, detd. c.g. coords. for rifle, head, hands, from fotos)

Table 22. The x_0, y_0, z_0 Coordinates for the Common Center of Gravity

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Head	+ 16560	+ 6210	645840
Torso	+ 30072	+ 15036	2806270
Upper leg			
R	- 16320	+ 50320	471920
L	- 16320	- 50320	471920
Lower leg			
R	- 22866	+ 16686	92082
L	- 22866	- 16686	92082
Foot			
R	- 3885	+ 6615	3150
L	- 3885	- 6615	3150
Upper arm			
R	+ 19800	+ 31482	235422
L	+ 9504	- 37422	231462
Forearm			
R	+ 24656	+ 14740	125692
L	+ 17420	- 19832	135876
Hand			
R	+ 13230	+ 2205	37730
L	+ 14455	- 1715	50470
Rifle	+126900	- 2350	491150
Totals	+ 186455	+ 8354	5894666

Here $x_0 = \frac{186455}{63400}$, $y_0 = \frac{8354}{63400}$, $z_0 = \frac{5894666}{63400}$

$$x_0 = + 2.9$$

$$y_0 = + 0.1$$

$$z_0 = 93.0$$

(Braune and Fischer, p. 649)

Here the gravity line is near foot-tip; it is very easy to fall forward. The center of gravity is 12.9 cm in front of the tibio-talar (ankle) joint, and 2.9 cm in front of and 6 cm above the hip joint. If the left forearm position without rifle be assumed, then

$$x_0 = \frac{59555}{58700} = + 1.0$$

$$y_0 = \frac{10704}{58700} = + 0.2$$

$$z_0 = \frac{5403516}{58700} = 92.1$$

Hence, the rifle alone displaces the center of gravity upward 0.9 cm, and forward 1.9 cm.

5. Firing Position Without Pack (Plates IX - X)(pp 651-653)

The three tabulations are here given:

Table 23. Coordinates for Centers of Joints in Firing Position, Without Pack

	x	y	z
Atl-occip.	+ 3.5	+ 3.5	147
Hip			
R	- 5	+ 7.5	86
L	+ 5	- 7.5	86
Knee			
R	- 9.5	+ 11	46.5
L	+ 8	- 13	46.5
Tibio-talar			
R	- 11	+ 16	6
L	+ 7	- 19	6
Rear edge foot			
R	- 11.5	+ 12.5	4
L	+ 2	- 18.5	4

Table 23 (continued)

	x	y	z
Tip foot			
R	+ 5	+29.5	1.5
L	+28	-18.5	1.5
Humerscapular			
R	-16	+15	137
L	+ 9.5	-14	132.5
Elbow			
R	- 2.5	+37	133
L	+38.5	- 3.5	120
Wrist			
R	+16.5	+15.5	141.5
L	+54	+ 8	141

(Braune and Fischer, p. 651)

Table 24. Coordinates for Centers of Gravity in Firing Position, Without Pack

	x	y	z
Head	+ 4.5	+ 5	151
Torso	+ 1.4	+ 1.4	109.8
Upper leg			
R	- 7.	+ 9	68.6
L	+ 6.3	- 9.9	68.6
Lower leg			
R	-10.1	+13.1	29.5
L	+ 7.6	-15.5	29.5
Foot			
R	- 6.1	+19.8	3
L	+13.2	-18.5	3
Upper arm			
R	- 9.7	+25.3	135.1
L	+23.1	- 9.1	126.6
Forearm			
R	+ 5.5	+28	136.6
L	+45	- 1.3	128.8
Hand			
R	+20	+12.5	142.5
L	+57.5	+ 8.5	142.5
Rifle	+48	+ 9.5	146

(Braune and Fischer, p. 651)

Table 25 The x_0, y_0, z_0 Coordinates for the Common Center of Gravity

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Head	+18630	+20700	625140
Torso	+35084	+35084	2751588
Upper leg			
R	-47600	+61200	466480
L	+42840	-67320	466480
Lower leg			
R	-31209	+40479	91155
L	+23484	-47895	91155
Foot			
R	- 6405	+20790	3150
L	+13860	-19425	3150
Upper arm			
R	-1[illegible]6	+50094	267498
L	+45738	-18018	250688
Forearm			
R	+ 7370	+37520	183044
L	+60300	+ 1742	172592
Hand			
R	+ 9800	+ 6125	69825
L	+28175	+ 4165	69825
Rifle	+225600	+44650	686200
Totals	+406461	+169891	6197950

Here, $x_0 = \frac{406461}{63400} = +6.6$. $y_0 = \frac{169891}{63400} = +2.7$, and

$$z_0 = \frac{6197950}{63400} = 97.8 .$$

(Braune and Fischer, p. 652)

The center of gravity is 11.8 cm above the hip axis, or 7.1 cm higher than in the normal position. It is over the promontory, at level of cartilage between L 4-5. It is about 5.5 cm forward, 1 cm to left of the median plane, and 7 cm in front of the center of the line connecting the hip centers. This position is a good and stable one.

6. Position of the Nude Figure, Rifle Extended and Arms Outstreched (Plates XI - XIII) (pp 654-657)

The three tabulations are here given.

Table 26. Coordinates of Joint Centers of the Nude Figure, Rifle Extended, Arms Outstretched

	x	y	z
Atl-occip.	0	+ 3	150
Hip			
R	0	+ 8.5	87
L	0	- 8.5	87
Knee			
R	- 2	+ 9	47
L	- 2	- 9.5	47
Tibio-talar			
R	- 4	+11	6
L	- 4	+12.5	6
Rear edge foot			
R	- 8.5	+ 9	4
L	- 8.5	-10.5	4
Tip foot			
R	+16.5	+16	1.5
L	+16.5	-17.5	1.5
Humeroscapular			
R	+ 2.5	+17.5	137
L	- 5	-16.5	131
Elbow			
R	+29.5	+15.5	131
L	- 5	-18	100
Wrist			
R	+53.5	+12.5	140.5
L	+ 3.5	-21	74

(Braune and Fischer, p. 654)

Table 27. Coordinates for Centers of Gravity of the Nude Figure, Rifle Extended, Arms Outstretched

	x	y	z
Head	+ 1	+ 4	154
Torso	0	+ 1.2	111.6
Upper leg			
R	- 1.1	+ 8.7	69.4
L	- 1.1	- 8.9	69.4
Lower leg			
R	- 2.8	+ 9.8	29.8
L	- 2.8	-10.8	29.8
Foot			
R	+ 2.3	+12	3
L	+ 2.3	-13.5	3
Upper arm			
R	+15.2	+16.6	134.2
L	- 5	-17.2	116.4

Table 27 (continued)

	x	y	z
Forearm			
R	+39.6	+14.2	135
L	- 1.4	-19.3	89.1
Hand			
R	+57	+11	140.5
L	+ 6	-22	69
Rifle			
R	+86.5	+ 7	143.5

(Braune and Fischer, p. 654)

Table 28. Coordinates x_0, y_0, z_0 for the Common Center of Gravity

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Head	+ 4140	+ 16560	637560
Trunk	0	+ 30072	2796696
Upper leg			
R	- 7480	+ 59160	471920
L	- 7480	- 60520	471920
Lower leg			
R	- 8652	+ 30282	92082
L	- 8652	- 33372	92082
Foot			
R	+ 2415	+ 12600	3150
L	+ 2415	- 14175	3150
Upper arm			
R	+30096	+ 32868	265716
L	- 9900	- 34056	230472
Forearm			
R	+53064	+ 19028	180900
L	- 1876	- 25862	119394
Hand			
R	+29370	+ 5390	68845
L	2940	- 10780	33810
Rifle	+406550	+ 32900	674450
Totals	485510	60095	6152147

Here $x_0 = \frac{485510}{63400} = +7.7$, $y_0 = \frac{60095}{63400} = +0.9$, and

$z_0 = \frac{6152147}{63400} = 97.0$.

(Braune and Fischer, p. 655)

The center of gravity is 10 cm. above and 7.7 cm. in front of the hip axis, at the level of the center of L 5. It is in front of the front edge of the pelvis, 0.9 cm to the right of the median plane. Coordinates x_0, y_0, z_0 for the center of gravity were calculated without the rifle, and here $x_0 = \frac{78960}{58700} = +1.3$, $y_0 = \frac{27195}{58700} = +0.4$,

and $z_0 = \frac{5477697}{58700} = 93.3$.

Advancing the rifle displaces the center of gravity 6.6 cm forward, 0.5 cm to the right, and 3.7 cm higher.

7. Military Position, Full Pack, Shoulder Arms. (Plates XIII - XIV) (pp 657-662)

The three tabulations are here given.

Table 29. Coordinates for the Joint Centers in Military Position, Full Pack, Shoulder Arms

	x	y	z
Atl-occip	+ 3	+ 1	151
Hip			
R	0	+ 8.5	87
L	0	- 8.5	87
Knee			
R	- 5.5	+ 6	47
L	- 5.5	- 6	47
Tibio-talar			
R	-11	+ 4.5	6
L	-11	- 4.5	6
Rear Edge foot			
R	-15	+ 2	4
L	-15	- 2	4
Tip foot			
R	+ 9	+12	1.5
L	+ 9	-12	1.5
Humeroscapular			
R	+ 3	+19	132
L	+ 6.5	-18	131
Elbow			
R	- 1	+21	101
L	+ 4	-27.5	102
Wrist			
R	+ 0.5	+24	74
L	+26	-18	91

(Braune and Fischer, p. 656)

Table 30. Coordinates for the Centers of Gravity in Military Position, Full Pack, Shoulder Arms

Table 30 (continued)

	x	y	z
Head	+ 4	+ 1	155
Torso	+ 1.2	+ 0.4	112
Upper leg			
R	- 2.4	+ 7.4	69.4
L	- 2.4	- 7.4	69.4
Lower leg			
R	- 7.8	+ 5.4	29.8
L	- 7.8	- 5.4	29.8
Foot			
R	- 4.7	+ 6.3	3
L	- 4.7	- 6.3	3
Upper arm			
R	+ 1.1	+19.9	117.4
L	+ 5.3	-22.5	117.4
Forearm			
R	- 0.4	+22.3	89.7
L	+13.2	-23.5	97.4
Hand			
R	+ 0.5	+23.5	69
L	+29.5	-15.5	90
Rifle	+ 6	-11.5	143
Knapsack, rear cart. case	-15.5	+ 0.4	114.5
Knapsack, front cart. case R	+16	+12.5	95.5
Knapsack, front cart. case L	+16	-12.5	95.5
Bayonet + spade	- 7.5	-18	75
Bread sack + water bottle	- 7.5	+18	75

(Braune and Fischer, pp. 657-658)

Table 31. <u>Coordinates x_0, y_0, z_0 for the Common Center of Gravity</u>

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Head	+ 16560	+ 4140	641700
Torso	+ 30072	+10024	2806720
Upper leg			
R	+ 16320	+50320	471920
L	- 16320	-50320	471920
Lower leg			
R	- 24102	+16686	92082
L	- 24102	-16686	92082
Foot			
R	- 4935	+ 6615	3150
L	- 4935	- 6615	3150
Upper arm			
R	+ 2178	+39402	232452
L	+ 10494	-44550	232452

Table 31 (continued)

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Forearm			
R	- 536	+29882	120198
L	+ 17688	-31490	130516
Hand			
R	+ 245	+11515	33810
L	+ 14455	- 7595	44100
Rifle	+ 28200	-54050	672100
Knapsack, rear cart case	-189175	+ 4900	1402625
Front cart case			
R	+ 25440	+19875	151845
L	+ 25440	-19875	151845
Bayonet + spade	- 11775	-28260	117750
Bread sack and water bottle	- 11775	+28260	117750
Totals	-133903	-37822	7990167

(Braune and Fischer, pp. 658)

Here $x_0 = \frac{-133903}{81970} = -1.6$, $y_0 = \frac{-37822}{81970} = -6.5$, and $z_0 = \frac{7990167}{81970} = 97.5$. The position of the left arm has drawn the center of gravity 0.5 cm forward, and 0.5 cm higher. The displacement from the median plane is 0.2 cm.

If all equipment is omitted $x_0 = \frac{442}{58700} = 0$, $y_0 = \frac{11328}{58700} = +0.2$, and $z_0 = \frac{5376252}{81970} = 91.6$.

If the left arm is made symmetrical with the right, no pack or rifle, then $x_0 = \frac{-40308}{58700} = -0.7$, $y_0 = \frac{+14164}{58700} = +0.2$ and $z_0 = \frac{5355644}{58700} = 91.2$.

The center of gravity coordinates for the pack (sans rifle) must be omitted to show the effect of the rifle alone. In this case $x_0 = \frac{+28642}{63400} = +0.5$, $y_0 = \frac{-42722}{63400} = -0.7$, and $z_0 = \frac{6048352}{63400} = 95.4$.

In summary form the coordinates of the center of gravity are given as follows:

	Position in Plate			Symmetrical Position		
	x_0	y_0	z_0	x_0	y_0	z_0
Body with full pack, + rifle	-1.6	-0.5	97.5	-	-	-
Body with full pack, no rifle	-2.1	+0.2	94.7	-2.6	+0.2	94.4
Body no pack, + rifle	+0.5	-0.7	95.4	-	-	-
Body alone	0	+0.2	91.6	-0.7	+0.2	91.2

The center of gravity is displaced only a little in pack alone, for it is but 3.2 cm higher and 1.9 cm further back than in symmetrical body position. The pack does not displace the gravity line outside of the hip joint axis. The rifle over the left shoulder causes a displacement of 0.5 cm forward, 0.9 cm to the left, and 3.8 cm. upward.

8. Full Pack, in Firing Position (Plates XV - XVI) (pp 663-666)

The three tabulations are here given.

Table 32. Coordinatates for Joint Centers, Full Pack, Firing Position

	x	y	z
Atl-occip	+1	+7	147
Hip			
R	-5	+7	86
L	+5	-7	86
Knee			
R	-12	+9	46.5
L	+5	-14	46.5
Tibio-talar			
R	-16	+15	6
L	+ 2.5	-21	6
Rear Edge Foot			
R	-19.5	+11	4
L	- 2.5	-20.5	4
Tip of foot			
R	0	+28	1.5
L	+23.5	-20.5	1.5
Humeroscapular			
R	-19	+14.5	135
L	+ 7.5	-12.5	133
Elbow			
R	- 5	+36.5	128
L	+36.5	- 1.5	120
Wrist			
R	+16.5	+18.5	141
L	+49.5	+11.5	141

(Braune and Fischer, p. 663)

Table 33. Coordinates for the Centers of Gravity, Full Pack, Firing Position

	x	y	z
Head	+ 1.5	- 8.5	151
Torso	+ 0.4	+ 2.7	109.8
Upper leg			
R	- 8.1	+ 7.9	68.6
L	+ 5	-10.1	68.6
Lower leg			
R	-13.7	+11.5	29.5
L	+ 4	-16.9	29.5
Foot			
R	-11.1	+18.3	3
L	+ 8.7	-20.5	3
Upper arm			
R	-12.4	+24.8	131.7
L	+21.1	- 7.3	126.9
Forearm			
R	+ 4	+28.9	133.5
L	+42	+ 4	128.8
Hand			
R	+19.5	+16	142.5
L	+53	+12	142.5
Rifle	+47.5	+13	146
Knapsack, rear cart. case	-18	- 8	114
Front cart. case			
R	- 2.5	+15	98
L	+16	- 3.5	98
Bayonet + spade	+ 2	-21	74
Bread sack + water bottle	-12	+12	74

(Braune and Fischer, pp. 663-664)

Table 34. Coordinates x_0, y_0, z_0 for the Common Center of Gravity

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Head	+ 6210	+ 35190	625140
Torso	+ 10024	+ 67662	2751588
Upper leg			
R	- 55080	+ 53720	466480
L	+ 34000	- 68680	466480

Table 34 (continued)

	$p_i x_i$	$p_i y_i$	$p_i z_i$
Lower leg			
R	- 42333	+ 35535	91155
L	+ 12360	- 52221	91155
Foot			
R	- 11655	+ 19215	3150
L	+ 9135	- 21525	3150
Upper Arm			
R	- 24552	+ 49104	260766
L	+ 41778	- 14454	251262
Forearm			
R	+ 5360	+ 38726	178890
L	+ 56280	+ 5360	172592
Hand			
R	+ 9555	+ 7840	69825
L	+ 25970	+ 5880	69825
Rifle	+223250	+ 61110	686200
Knapsack, rear cart. case	-220500	- 98000	1396500
Front cart. cases			
R	- 3975	+ 23850	155820
L	+ 25440	- 5565	155820
Bayonet + spade	+ 3140	- 32970	116180
Bread sack + full water bottle	- 18840	+ 18840	116180
Total	+ 85567	+ 128607	8128158

Here $x_0 = \frac{85567}{81970} = +1.0$, $y_0 = \frac{128607}{81970} = +1.6$ and

$z_0 = \frac{8128158}{81970} = 99.2$.

(Braune and Fischer, pp. 664-665)

The center of gravity is 13.2 cm above the hip axis, at the level of the lower 1/3 of L 4, at its front edge, and about 1 cm to the right of the median plane.

To determine the effect of the pack on the center of gravity, in firing position coordinates x_0, y_0, z_0, omitting all but the rifle, it is found that $x_0 = \frac{300302}{63400} = +4.7$, $y_0 = \frac{222542}{63400} = +3.5$, and

$z_0 = \frac{6187658}{63400} = 97.6$.

Now the center of gravity is at the level of the cartilage between L 4-5, and 5 cm in front of it, 1 cm to the right of the median plane. It is 5 cm in front of the hip axis. Hence, the pack displaces the center of gravity 3.5 cm backward, 1.6 cm upward;

the line of gravity is displaced backward.

The Importance of the Relation of the Location of the Gravity Line to the Supporting Surface. (p. 666 - 668)

This problem focusses on the stability and/or reliability of the several positions. To elucidate the problem all centers of gravity for the individual positions are shown in Plate XVII. As a result the following values for x'_0, y'_0, and z'_0 centers of gravity coordinates are given (as plotted in the normal figure in Plate III):

	Position	x'_0	y'_0	z'_0
1.	Normal, Pl. IV	+0.2	0	92.7
2.	Easy, Pl. V	-0.8	0	92.3
3.	Military, without pack, Pl. VI	+0.4	0	92.7
4.	Military, without pack, presenting arms, Pl. VII-VIII	+2.9	+0.1	94.0
5.	Firing, without pack, Pl. IX-X	+7.0	-1.0	99.8
6.	Rifle advanced, R arm extended, Pl. XI-XII	+7.7	+0.9	98.0
7.	Military, full pack, shoulder arms, Pl. XIII-XIV	-1.6	-0.5	98.5
8.	Firing, full pack, Pl. XV-XVI	+1.5	+1.0	101.2

(Braune and Fischer, p. 667)

The locations for these centers of gravity indicate three things: 1) the position in which the man must stand to place the gravity line as nearly as possible in the center of the supporting surface; 2) which locations of the center of gravity strain the muscles the most because of the flexion that must ensue; and 3) which location of the center of gravity is nearest the norm.

Upon these bases it is concluded that positions 5 and 6 are not good. Position 8 is best. With reference to the height of the center of gravity both firing positions (5 and 6) are equally undesirable.

Effect of the Pliability of the Torso on the Location of the Common Center of Gravity (pp 668 - 670)

The common center of gravity in the various positions is not in the plane determined by the center of the atlanto-occipital joint and by the hip axis. When the torso bends forward the center of gravity remains at the same level, but it moves forward; there is a play of 2.5 cm of the center of gravity in the torso. Displacing the center of gravity of the torso 1 cm forward or backward will give a displacement of $\frac{25060}{58700} = 0.427$ cm forward or backward for the common center of gravity, for whole body sans load. If body is loaded with a rifle the displacement is 0.395 cm (rifle alone) or 0.306 cm (rifle + pack). The maximum amount of play (2.5 cm) for the torso in Pl . IV and VI amounts to only 1.7 cm for the common center of gravity itself.

Effect of Ground Slope on the Position of the Body (p. 670-672)

A wooden board 50 cm wide, 2 m long, was raised and lowered to different angles. At 32° a man with full pack could still stand erect; he could not step forward, and kept his body bent forward; he could walk up without the pack, or with the pack on his head.

A small board was nailed to the larger one and three subjects were placed with their heels resting on it. A could still stand, body bent far forward, with the pack, at 41.5°, without pack up to 47°, and with pack on chest 50°. B could stand with pack up to 41°, without pack to 47° - 49°, and with pack on chest up to 52°. C could stand with pack to 42°, without pack to 48°, and with pack on chest to 48°.

A load on uneven ground will change the position of the lower part of the back, as the body leans forward. The steeper the slope the farther back the center of gravity. Bending occurs in ankle, hip joint, and spine. The bending shifts the pack load to the upper part of the body.

Appendix A

Cadaver Lengths (cm), #2, #3, #4

	2		3		4	
Entire Body	170		166		168.3	
Head	21		20.2		21.3	
	R	L	R	L	R	L
Upper leg	44	43.3	42	41	40	40
Lower leg	41.4	41.8	43	42.9	41.5	41.5
Foot ht.	7.8	7.5	7.7	7.7	6.5	6.5
Foot lgth.	28.5	28.3	26.5	26.9	26.5	26.5
Between centers hip joint	17		17.5		17	
Upper arm	31.7	31.5	30.6	30.2	32	32
Lower arm, without hand	29.5	29.5	26.3	27.1	27	27

Appendix A (continued)

	2	3	4
Line, center hum. head to center hip joint	-	-	49
Dist. between hum. heads	-	-	36

(Summarized from text: W.M.K.)

Appendix B

Cadaver Weights (g), #2, #3, #4

	2	3	4
Entire body	75100	60750	55700
Head	5350	4040	3930
Torso	36020	28850	23780

	R	L	R	L	R	L
Arm	4950	4790	3550	3480	3520	3710
Upper arm	2580	2560	1990	1880	1720	2020
Forearm and hand	2370	2230	1550	1600	1790	1690
Forearm	1700	1600	1050	1120	1300	1240
Hand	600	600	500	470	490	450
Leg	12120	11890	10650	10250	10110	10650
Upper leg	7650	7300	6690	6220	6150	6750
Lower leg and foot	4470	4500	3950	3980	3960	3900
Lower leg	3210	3320	2670	2880	2970	2900
Foot	1100	1160	1060	1090	990	1000

(Summarized from text: W.M.K.)

Appendix C

Averages in Cadavers #2 - #4 of Distances Between the Centers of Gravity and the Joint Axes in Joint Centers for Each Limb

Parts of the Body, Both Sides;		Average of Ratios	
Upper arm			
R 0.4735)	0.470	R 0.5265)	0.530
L 0.466)		L 0.534)	

Appendix C (continued)

Parts of the Body, Both Sides;		Average of Ratios	
Forearm			
R 0.418)	0.421	R 0.582)	0.579
L 0.4235)		L 0.5765)	
Upper leg			
R 0.442)	0.439	R 0.558)	0.561
L 0.4365)		L 0.5635)	
Lower leg			
R 0.422)	0.4195	R 0.578)	0.5805
L 0.417)		L 0.583)	
Foot			
R 0.429)	0.434	R 0.571)	0.566
L 0.439)		L 0.561)	
Forearm + hand			
R 0.4735)	0.472	R 0.5265)	0.528
L 0.470)		L 0.530)	
Lower leg + foot			
R 0.517)	0.519	R 0.483)	0.481
L 0.521)		L 0.479)	

(Braune and Fischer, p. 622)

Appendix D

Ratios of Distances of Centers of Gravity From Joint Axes or Centers of Respective Joints

(here distances between joint axes or axial centers of a limb reduced to 1)

		2		3		4	
		from above	from below	from above	from below	from above	from below
Upper arm[1]	R	-	-	0.438	0.562	0.509	0.491
	L	-	-	0.454	0.546	0.478	0.522
Forearm[1]	R	-	-	0.414	0.586	0.422	0.578
	L	-	-	0.406	0.594	0.441	0.559

Appendix D (continued)

		2		3		4	
		from above	from below	from above	from below	from above	from below
Upper leg	R	0.432	0.568	0.469	0.531	0.425	0.575
	L	0.446	0.544	0.476	0.524	0.3875	0.6125
Lower leg	R	0.420	0.580	0.435	0.565	0.410	0.590
	L	0.416	0.584	0.413	0.587	0.422	0.578
Foot[2]	R	0.404	0.596	0.430	0.570	0.453	0.547
	L	0.424	0.576	0.439	0.561	0.453	0.547
Forearm+ hand[1,3]	R	-	-	0.475	0.525	0.472	0.528
	L	-	-	0.463	0.537	0.477	0.523
Lower leg+ foot[4]	R	0.500	0.500	0.531	0.469	0.521	0.479
	L	0.517	0.483	0.514	0.486	0.531	0.469

1. Ratios for upper extremity 2 not included because saw-cut not thru elbow axis in manner comparable with 3 and 4.

2. Length front to rear reduced to 1.0.

3. Length elbow axis to lower edge flexed fingers reduced to 1.0.

4. Distance knee axis to sole reduced to 1.0.

(Braune and Fischer, p. 621)

Appendix E

Summary: Location of Centers of Gravity of Entire Body and Separate Limbs in Cadavers #2 - #4

	2	3	4
Total length	170 cm	166 cm	168.8 cm
Total Weight	75100 g	60750 g	56090 g (p. 34)
c. g. entire body	Just below promontory	4 cm front lower edge S1, 0.2 cm to R	2.1 cm vert. below promontory, 7 cm front of upper edge S3.
c. g. torso alone, sans head or arms	Front surface lower edge L1, near median plane.	Center L1, 2 cm from front surface, 1.4 cm from rear surface, 0.3 cm to R median plane.	Upper edge L1, at front surface, 0.5 cm to R
Entire leg R	c. g. upper leg 39 cm from above, 5 cm from below, directly on bone center	c. g. upper leg 37.7 cm from above, 4.3 cm from below, directly behind bone center	c. g. upper leg 35.5 cm from above, 4.5 cm from below, 1 cm behind bone
Entire leg L	38.5 cm from above, 4.8 cm from below, same as in R	38.5 cm from above, 2.5 cm from below, in bone itself, 0.7 front of rear edge, and a bit outward; lgth. 50.7 cm.	in upper leg, 33 cm from above, 7 cm from below; same as in R
Upper leg alone, R	Lgth. 44 cm, wt. 7650 g; c. g. 19 cm. from above, and 25 cm from below, 1.5 cm behind linea aspera. to R	Lgth. 42 cm, wt. 6690 g; c. g. 19.7 cm from above and 22.3 cm from below, 2.2 cm behind linea aspera.	Lgth. 40 cm (av.), wt. 6750 g; c. g. 17.0 cm from above, and 23.0 cm from below, 1.5 cm behind linea aspera.
Upper leg alone, L	Lgth. 43.3 cm, wt. 7300 g.; c. g. 19.3 cm from above, and 24 cm from below.	Lgth. 41 cm, wt. 6220 g; c. g. 19.5 cm from above, and 21.5 cm from below, 1.5 cm behind linea aspera.	Lgth. 40 cm, wt. 6750 g; c. g. 15.5 cm from above, and 24.5 cm from below, same as in R.

Appendix E (cont.)

	2	3	4
Lower leg + foot, R	Lgth. 49.2 cm ; c.g. in lower leg 24.6 cm from above, and 16.8 from below, just behind attach. inteross. lig.	Lgth. 50.7 cm ; c.g. in lower leg 26.9 cm from above, and 16.1 cm from below, at attach. inteross. lig. to tibia.	Lgth. 48 cm ; c.g. in lower leg 25 cm from above, and 16.5 from below, at attach. inteross. lig. to tibia.
Lower leg + foot L	Lgth. 49.3 cm ; c.g. 25.5 cm from above, 16.3 cm from below, as in R.	Lgth. 50.6 cm ; c.g. 26 cm from above, 19.9 cm from below, as in R.	Lgth. 48 cm ; c.g. 25.5 cm from above, and 16 cm from below, as in R.
Foot alone R	Lgth. 28.5 cm , ht. 7.0 cm , wt. 1100 g ; c.g. 11 .5 cm from rear, 17 cm from front, 6.5 cm front center of tibio-talar joint, in front surface navic., between cuneiform II - III	Lgth. 26.5 cm , ht. 7.7 cm wt. 1060 g ; c.g. 11.4 cm from rear, 15.1 cm from front, at angle lower and outer edge cuneiform III, near attach. to navic.	Lgth. 26.5 cm , ht. 6.5 cm , wt. 990 g ; c.g. 12 cm. from rear and 14.5 cm from front, under cuneiform III, near its foreward edge, 3 cm above sole.
Foot alone L	Lgth. 28.3 cm , ht. 7.5 cm , wt. 1160 g ; c.g. 12 cm from rear, 16.3 cm from front, as in R.	Lgth. 26.9 cm , ht. 7.7 cm , wt. 1090 g ; c.g. 11.8 cm from rear, and 15.1 cm from front, at front surface of navic, betw. cuneiform III and cuboid.	Lgth. 26.5 cm , ht. 6.5 cm , wt. 1000 g ; c.g. 12 cm from rear, and 14.5 cm. from front, as in R.
Lower leg alone, R	Lgth. 41.4 cm , wt. 3210 g ; c.g. 17.4 cm from above, and 24 cm from below, behind center inteross. lig.	Lgth. 43 cm , wt. 2870 g ; c.g. 18.7 cm from above, and 24.3 cm from below, a bit behind inteross. lig.	Lgth. 41.5 cm , (av.) wt. 2970 g ; c.g. 17 cm from above, and 24.5 cm from below, behind inteross. lig.

Appendix E (cont.)

Lower leg alone, L	Lgth. 41.8 cm , wt. 3320 g ; c.g. 17.4 cm from above, and 24.4 cm from below, as in R.	Lgth. 42.9 cm , wt. 2880 g ; c.g. 17.7 cm from above, and 25.2 cm from below, 1 cm behind inteross. lig.	Lgth. 41.5 cm , wt. 2900 g c.g. 17.5 cm from above, and 24 cm from below, as in R.
Entire arm, R	c.g. in upper arm, a bit below elbow axis, 1.5 cm in front of bone.	c.g. in upper arm, 28 cm from above, and 2.6 cm from below, 1.8 cm in front of humerus.	c.g. in upper arm, 0.5 cm below elbow axis, 0.3 cm in front of bone.
Entire arm, L	As in R.	c.g. 29.1 cm from above, and 1.1 cm from below, 1.4 cm in front center of humerus.	c.g. 0.5 cm above elbow axis, 0.5 cm in front of bone.
Upper arm alone, R	Lgth. to artic surface elbow joint 31.7 cm , wt. 2580 g ; c.g. 14.5 cm from above (center hum. head), 17.2 cm above elbow axis, in med. cavity of hum., near rear edge.	Lgth. 30.6 cm , wt. 1990 g ; c.g. 13.4 cm from above and 17.2 cm from below, at rear surface of humerus.	Lgth. 32 cm , wt. 1730 g ; c.g. 16.3 cm from above, and 15.7 cm from below, in hum. itself, near rear edge.
Upper arm alone, L	Lgth. 31.5 cm , wt. 2560 g ; c.g. 13.3 cm from above, and 18.2 cm from below, in medullary cavity	Lgth. 30.2 cm , wt. 1880 g ; c.g. 13.7 cm from above, and 16.5 cm from below, at rear surface humerus.	Lgth. 32 cm , wt. 2020 g ; c.g. 15.3 cm from above, 16.7 cm from below, as in R.
Forearm + hand, R	Lgth. 41.5 cm , from elbow axis to lower edge flexed fingers; c.g. in forearm, 19.5 cm from elbow axis, 10.5 cm from capitulum, 0.5 cm front attach. inteross. lig. to radius.	Lgth. 37.5 cm ; c.g. in forearm, 17.8 cm from above, and 8.5 cm from below, 0.7 cm. in front of attach. inteross. lig. to radius.	Lgth. 17.5 cm ; c.g. in forearm 17.7 cm from above, and 9.3 cm from below, 0.5 cm front of inteross. lig., nearer radius.

(Braune and Fischer, pp. 617-620, plus additions from text).

DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

OR are
Blank pgs
that have
Been Removed

BEST AVAILABLE COPY

BEST AVAILABLE COPY

THEORETICAL FUNDAMENTALS FOR A MECHANICS OF LIVING BODIES

(Condensed from O. Fischer)

THEORETICAL FUNDAMENTALS FOR A MECHANICS OF LIVING BODIES

(Condensed from O. Fischer)

INTRODUCTION (pp v-vii)

In theoretical mechanics little attention has been paid to freely movable joint systems, termed by Fischer "n - link systems." This book aims: 1) to give the kinetics of joint systems; 2) to give states of motion and equilibrium in Man. He introduces "certain mass systems and fixed points within the individual limbs (main points of the body parts)" which are similar to the center of gravity in the kinetics of a single rigid body. The unit of force is the kilogram weight, not the dyne. The unit of length is the centimeter. The unit of time is the second. The unit of mass is 981,000 grams (the mass number is obtained by dividing the number of grams in a body by 981,000 or the number of kilograms by 981).

In order to investigate the states of motion and equilibrium of the living human body it is necessary to make certain assumptions in the interests of simplicity, even though they do not conform in detail to actual conditions. In general these assumptions concern the composition and behavior of the individual body parts and the connecting joints between them. More specifically the assumptions are as follows:

1) the deformation and mass shifts in body parts due to respiration, blood, muscles, etc., must be neglected; 2) body parts must be conceived as rigid; 3) finger and hand movements must not be considered in locomotion (e.g., finger + hand + forearm equal a single rigid body part); 4) cartilage changes in joints during action must be disregarded; 5) a fixed joint center, let alone a fixed joint axis, cannot really be assumed.

Both bodies (human and animal) and machines have joint systems, more intricate in the former. Individual parts are connected by joints, which are called links. Joint connections, with a fixed joint center, permit neighboring links to have 1, 2, or 3 degrees of freedom. When one link moves all others will perform translatory motions only. ("Motions in which all points of the links have in each movement equally great and parallel velocities and therefore describe congruent paths.") For example, assume that a man stands on one leg, with the entire sole touching the ground; now let him raise himself upon his toes. Here the supporting foot must rotate around a horizontal axis running approximately through the heads of the middle foot bones. Hence, other parts of the body cannot remain in their positions; their motion may be such that they are lifted to the same extent, though with no change of direction in their long axes. As a result these body parts will perform a translatory motion completely defined by the path of one point, the center of the foot joint axis (ankle joint). In substance the entire body, excepting the supporting foot, behaves as a single rigid body, lifted by the pressure exerted from below through the supporting foot; the body, in turn influences the rotation of the foot.

As further example assume that the human body lies on its back on the surface of the water. Now, let the right thigh be rotated in a plane parallel to the median plane of the body: there will be translatory motions around both the knee axis and the hip axis. It must be assumed that masses of lower thigh and leg and foot will be concentrated in the center of the knee joint and that masses of all other body parts will be concentrated in the hip joint. This is the reduced system. In essence this is a concentration of masses at a joint system (Fischer calls it a "fictitious mass") achieved when in each link of a system we suppose to be concentrated the mass of those body parts which would be eliminated were the joint to be severed. In the example given above we may refer to "the reduced foot system", and "the reduced thigh system". Each reduced system must possess the whole mass of the human body. The center of gravity of such a system is far from the center of gravity of the respective body part alone, i. e., it is nearer the center of the reduced system. The center of gravity of a reduced system is "the main point of the link",; the link itself is "the nuclear link of the reduced system."

The mechanics of the human body must: 1) determine the main points of links and moments of inertia of the various reduced systems, apart from a) the masses, b) the centers of gravity, c) the moments of inertia, of individual body parts; 2) learn of reduced systems and their centers of gravity.

GENERAL PART (pp 1-176)

Three-Link Plane Joint System (pp 9-90)

1. Position of the Main Points of the Links and the Magnitude of the Moments of Inertia of the Reduced Systems (pp 9-14)

In three-joint bodies parts 1 and 2 and 2 and 3 will each be connected by a hinge joint. The two joint axes are parallel, and the center of gravity of the middle body will be in one plane with the joint axes. The plane connecting the centers of gravity in any one position of the link system will be perpendicular to the two parallel joint axes; and this will hold in all other positions of the joint links. Assume that the plane defined by the three centers of gravity shall remain fixed in space, then the 3-link system will perform plane movements only. This represents a plane joint system, illustrated in Figure 3.

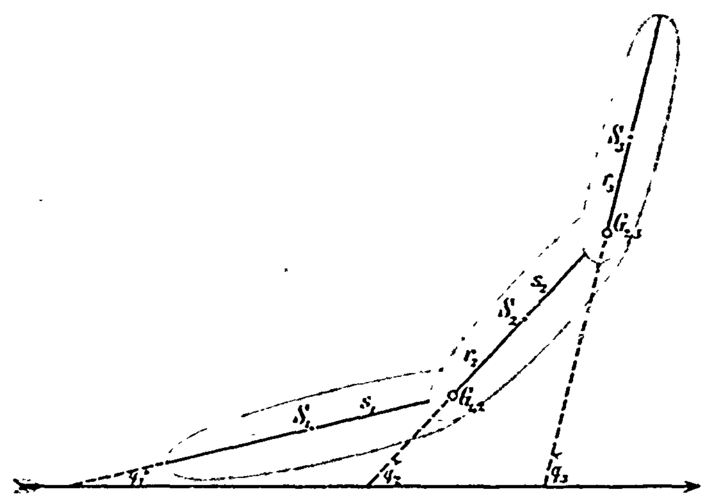


Figure 3. Plane Movements in a 3-Link System

Here the masses of the 3 links = m_1, m_2, m_3; the centers of gravity of the 3 links = S_1, S_2, S_3; the points of intersection of the two joint axes with the fixed plane = the centers of the two joints = $\overline{S_1 G_{1,2}}$, $\overline{G_{1,2}\,G_{2,3}}$, & $\overline{G_{2,3}\,S_3}$ (these or their extensions will always fall within the fixed plane; they are the longitudinal axes of the three links).

It is necessary to establish positive and negative directions: positive = where the longitudinal axis is followed, drawing the (broken) line from S_1 through $S_1G_{1,2}$, $G_{2,3}S_3$. It must be assumed that the long axis of each of the three links represents a main inertia for the center of gravity of the respective link. It must also be assumed that the moments of inertia for all axes perpendicular to the long axis through a center of gravity are of the same magnitude. Hence, center-of-gravity axes parallel to the joint axes of each of the three bodies will also represent a main inertia axis, the inertia radius of which shall = x_1, x_2, x_3, for the three links of the system.

In the most general cases of plane motion of the joint system there are five degrees of freedom. We must be able to determine position in space here by five coordinates. However, if plane motion be restricted so that in it one point of the joint system maintains position in the fixed plane, then only three degrees of freedom exist, and three coordinates suffice. Choose as general coordinates angles φ_1, φ_2, φ_3, formed by the positive action of the three long axes; these three angles are the three coordinates for the joint system. The total center of gravity of the joint system = S_o. We also accept as follows: 1) joint center of gravity S_1 on the long axis of the first link; 2) center of gravity S_2 shall be at distance r_2 from joint center $G_{1,2}$; 3) joint center $G_{2,3}$ shall be at distance s_2 from the same center of gravity; 4) center of gravity S_3 on the long axis of the third link shall be distance r_3 from joint center $G_{2,3}$; 5) if l_2 = distance of the two joint centers from each other then $r_2 + s_2 = l_2$; 6) masses m_2 and m_3 are concentrated in point $G_{1,2}$ and are added to the first link, and in the second link m_1 is concentrated in $G_{1,2}$ and m_3 is concentrated in $G_{2,3}$, and in the third link m_1 and m_2 are concentrated in $G_{2,3}$.

In the foregoing $G_{1,2}$ and $G_{2,3}$ are the fixed points of their respective links, connected by the two joints. As a result there are three reduced systems, of which one has the total mass of $m_1 + m_2 + m_3$ of the whole joint system (for short this = m_o). Centers of gravity here = H_1, H_2, H_3.

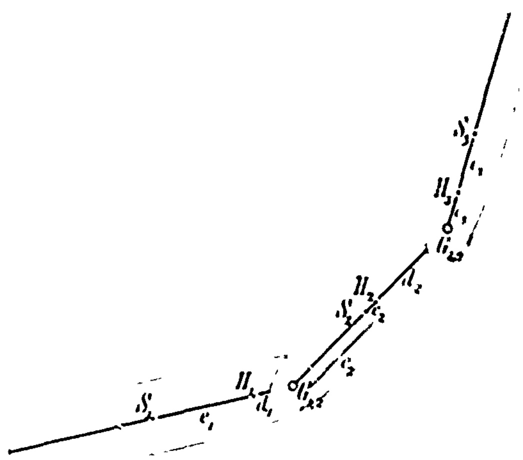


Figure 4. Three Main Points Defined by Centers of Gravity S_1, S_2, S_3 and Ratio of Magnitude of the Three Masses

In Fig. 4 are shown the three main points defined by centers of gravity S_1, S_2, S_3, and the ratio of magnitude of the three masses. Introduced here are the following designations:

$$S_1H_1 = e_1, H_1G_{1,2} = d_1$$

$$S_2H_2 = e_2,\ G_{1,2}H_2 = c_2,\ H_2G_2 = d_2$$

$$S_3H_3 = e_3,\ G_{2,3}H_3 = c_3$$

(individual lengths = positive)

The following lengths are defined:

c_h = distance of main point of the h^{th} link from the joint center of this link

d_h = distance of the other joint centers of the h^{th} link from the main point of this link

e_h = distance of the h^{th} main point from the pertinent center of gravity

The following relations are therefore obtained: *

(1) $-m_1e_1 + (m_2 + m_3)\ d_1 = 0$ (or zero)

$-m_1c_2 - m_2e_2 + m_3d_2 = 0$ (" ")

$(m_1 + m_2)\ c_3 + m_3e_3 = 0$ (" ")

Lengths d_1, c_2, d_2 and c_3 which define the position of the three main points in relation to the two joint centers = main lengths. The formulae for these lengths are:

(2) $m_od_1 = m_1s_1$

$m_oc_2 = m_2r_2 + m_3l_2$

$m_od_2 = m_1l_2 + m_2s_2$

$m_oc_3 = m_3r_3$

---

* The numbers in parentheses, as (1), (2).... refer to the formula number as given by Fischer. Obviously not all formulae will be herein cited. The basis of selection is two fold: 1) introduction of (new) terms; 2) final form of a developed formula. (W.M.K.)

If we use as a reference point the individual center of gravity of each link we get:

(3) $m_o e_1 = (m_2 + m_3) s_1$

$m_o e_2 = -m_1 r_2 + m_3 s_2$

$m_o e_3 = -(m_1 + m_2) r_3$

It is assumed that the long axis of a link = the main axis of inertia, and that the ellipsoid of inertia = a rotation ellipsoid with the long axis of the link as the axis of rotation. Hence, the main point of the axis which is parallel to the joint axes will be the main axis of inertia of the reduced system.

If the radius of inertia of this axis = k_1, k_2, k_3, resp., for the three links, and if each of the reduced systems has a total mass, m_o, we obtain:

(4) $m_o k_1^2 = m_1(x_1^2 + e_1^2) + (m_2 + m_3) d_1^2$

$m_o k_2^2 = m_2(x_2^2 + e_2^2) + m_1 c_2^2 + m_3 d_2^2$

$m_o k_3^2 = m_3(x_3^2 + e_3^2) + (m_1 + m_2) c_3^2$

For any axis parallel to the joint axis the moments of inertia of the reduced system will increase by the amount $m_o a^2$, where a = distance of the new axis from the main point of the respective link.

2. Connections of the Main Points with the Total Center of Gravity of the Joint System and the Centers of Gravity of the Partial System (pp 14-22)

If from any point, O, of the connecting vectors are drawn three individual centers of gravity (S_h) and the total center of gravity (S_o), then:

(5) $m_o \cdot \overline{OS}_o = \sum_1^3 {}^h m_h \cdot \overline{OS}_h$

(where bar over the letters = length as a vector, and the sum symbol = geometric addition).

If the total mass, m_o, is brought to the right side and μ_h is substituted for the ratio $m_h : m_o$, then

$$(6) \quad \overline{OS}_o = \sum_{1}^{3} h \ \mu_h \cdot \overline{OS}_h$$

It follows that the vector from any point O to the total center of gravity S_o = the sum of vectors to the three individual centers of gravity, S_h, which are reduced according to the ratio $\mu_h : 1$.

After considering related point O with the main point H_1 of the first link Fischer comes to the theorem:

> We shall always get to the total center of gravity, S_o, of the joint system if when starting from any main point, H_j, of the three links we form the geometric sum of the main links belonging to the two other links, which within the broken line of the three lingitudinal axes lie nearest to the j^{th} link, and when in this process we use these main links in a direction away from H_j.

Figure 5 illustrates the construction of the total center of gravity, S_o.

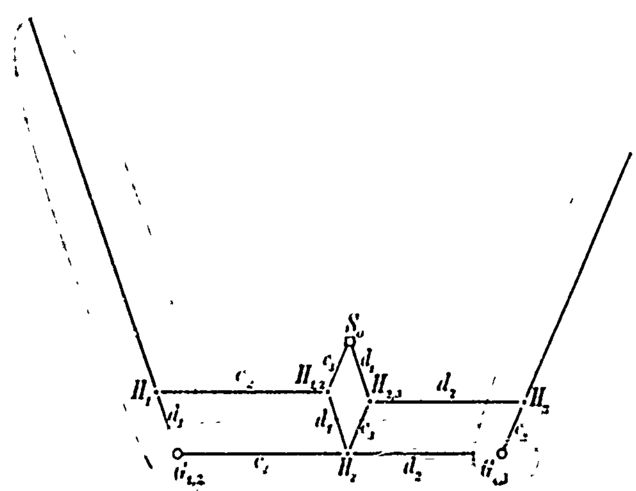


Figure 5. Construction of the Total Center of Gravity, S_o

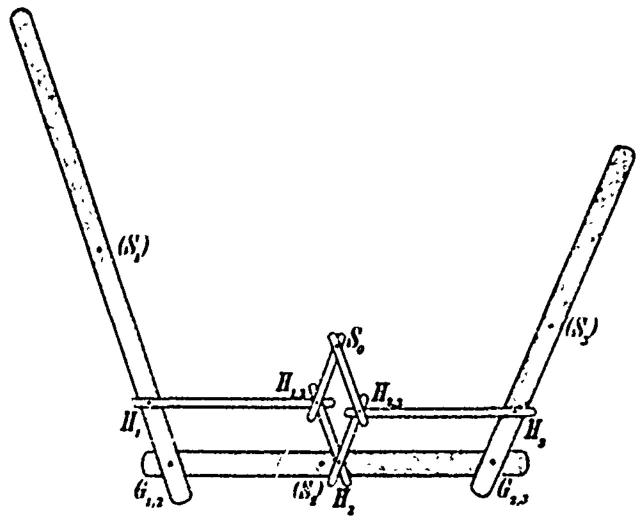


Figure 6. Method of Obtaining Position of S_o of Each Position of Three Links in Relation to One Another.

Figure 6 illustrates the method of obtaining the automatic position of S_o for each position of the three links in relation to one another.

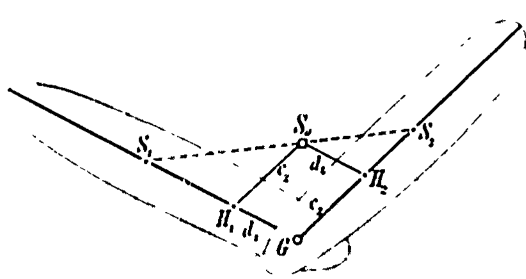


Figure 7. Two - Link System Where S_o, H_1, G , H_2 are Corners of a Parallelogram

Figure 7 illustrates a two link system where the four points S_o, H_1, G and H_2 are the corners of a parallelogram. If here $H_1G = d_1$ and $GH_2 = c_2$ then

(16) $\overline{H_1S_o} = +\overline{c_2}$

$\overline{H_2S_o} = -\overline{d_1}$

For a two-link system there is given the theorem:

We get to the total center of gravity, S_o, of the two link joint system when starting from the main point of the one link we plot the main length belonging to the other link in the direction away from the first.

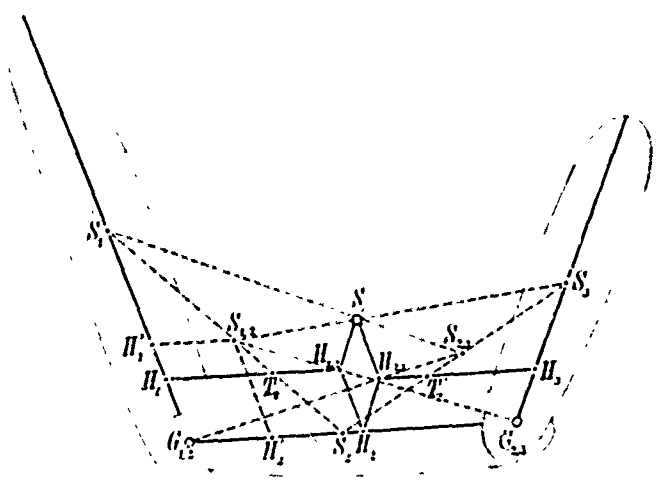


Figure 8. Method of Obtaining S_o of Two-Link System, Starting From Main Point of One Line

In Figure 8 it is seen that:

(17) $G_{2,3}H_{1,2} : H_{1,2}S_{1,2} = (m_1 + m_2) : m_3$

and

(18) $G_{2,3}S_{1,2} : G_{2,3}H_{1,2} = m_o : (m_1 + m_2)$

and

from Fig. 5 it will be seen that length $\overline{H_3S_o} = \overline{G_{2,3}H_{1,2}}$

As long as the first two of the three links are fixed relative to each other $H_{1,2}$ is a fixed point. But if the first two links are move-able then the first connecting joint ($G_{1,2}$) will change relative to $H_{1,2}$ just as the common center of gravity, $S_{1,2}$ of these two links is not fixed. Now it is seen that $H_{2,3}$ in Figure 5 is the main point of the joint system of links 2-3. Hence the following proportion applies:

$$(19) \quad G_{1,2}H_{2,3} : H_{2,3}S_{2,3} = (m_2 + m_3) : m_1$$

and

$$(20) \quad G_{1,2}S_{2,3} : G_{1,2}H_{2,3} = m_o : (m_2 + m_3)$$

In getting the total center of gravity, $S_{2,3}$, of links 2-3, it is not possible to start from H_2 and H_3: one must start from the main points obtained <u>after</u> the first link has been separated. If the third link is removed from a three-joint system, the main point of the first link is H_1' and will move away from $G_{1,2}$; the main point of the second link will be H_2', moving away from $G_{2,3}$. H_1' will divide length $S_1G_{1,2}$ on the long axis of the first link (Fig. 8) in the ratio $m_2 : m_1$, and H_2' will divide $G_{1,2}S_2$ in the same way. Here Fischer presents further data when H_1 is taken as the common center of gravity of two masses $(m_1 + m_2)$ and m_3 concentrated in H_1' and $G_{1,2}$ and when H_2 is taken as the common center of gravity of two masses $(m_1 + m_2)$ and m_3 concentrated in H_2' and $G_{2,3}$.

The connecting line of the two centers of gravity S_1 and S_2 to $H_{1,2}$ at a certain point, T_2 (Fig. 8), the position of which is independent of the link position in $G_{1,2}$. Here we may accept:

$$(24) \quad T_2'H_{1,2} = \frac{m_3}{m_o} s_2$$

Since $l_2 = r_2 + s_2$ the main length, c_2, may be written as:

$$(25) \quad c_2 = \frac{m_2 + m_3}{m_o} r_2 + \frac{m_3}{m_o} s_2$$

Here c_2 will be divided by T_2 into the two parts of the above formulae. In like manner the point T'_2 will be found on main length, d_2.

3. Determination of the Motions of the Total Center of Gravity and of Partial Centers of Gravity with the Help of the Main Points of the Lengths (pp 22-37)

If link 3 is rotated around its axis, $G_{2,3}$, and the other two links are fixed, Figure 5 shows that the total center of gravity will move in a circle around $H_{1,2}$, with a radius $\overline{c_3}$. The angular velocity and the angular acceleration with which S_o moves in its circle are identical with the angular velocity and the angular acceleration of rotation of link 3 around $G_{2,3}$. At the same time the vector $\overline{c_3}$ shows the influence that link 3 has on the total center of gravity. The same is true for link 1, rotated around $G_{1,2}$, with total center of gravity moving in a circle around $H_{2,3}$, with a radius of $\overline{-d_1}$.

The middle link cannot move without imparting motion to links 1 and 3. If link 1 is fixed, link 2 must rotate around $G_{1,2}$; link 3 takes part in the motion, since $G_{2,3}$ moves in a circle around $G_{1,2}$.

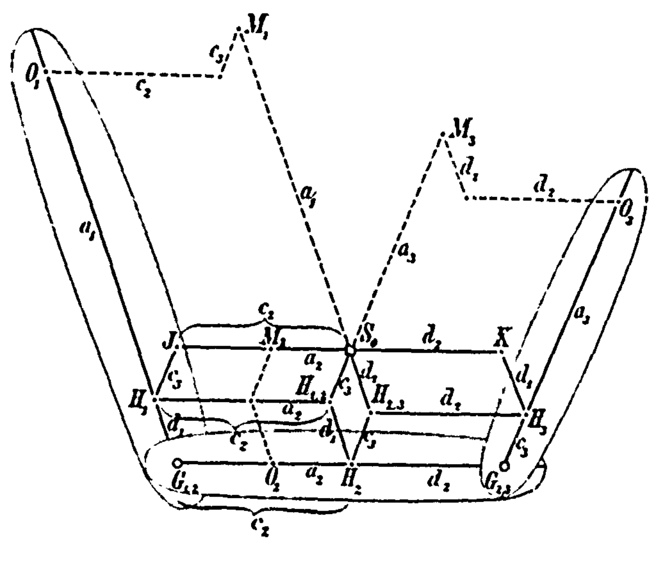


Figure 9. Translatory Motion in Link 3 of a Three-Link System

Figure 9 shows that link 3 has mere translatory motion:

the total center of gravity S_o will move in a circle with radius = vector $\overline{c_2}$. The center of the circle will be on J, which is the distance of vector $\overline{c_3}$ from the main point, H_1, of link 1. If link 2 rotates around joint axis $G_{1,2}$, and if link 3 behaves as though it were firmly connected to link 2, the total center of gravity will describe a circle with its center in main point H_1 of the first link, so that the radius will be $H_1 S_o$ = vector sum $\overline{c_2} + \overline{c_3}$. In like fashion with link 3 fixed link 2 will rotate only around $G_{2,3}$.

If none of the three links are at rest during motion it may be that one link only rotates around a point that is fixed, while the other two links perform translatory movements; e. g. in Figure 9, S_o will describe a circle around M_1, lying in the extension of $H_{2,3}S_o$, removed from O_1 by vector sum $\overline{c_2} + \overline{c_3}$. The radius of the circle = distance a_1 from H_1 from rotation center O_1, which gives:

$$(26) \quad \overline{c}_o = \overline{c}_1 + \overline{c}_2 + \overline{c}_3 = \sum_1^3 {}^h \, \overline{c}_h \, ,$$

The angular velocities with which the three bodies change direction in space, in the motion of the joint system, are φ_1', φ_2' and φ_3', giving angles φ_1, φ_2, φ_3.

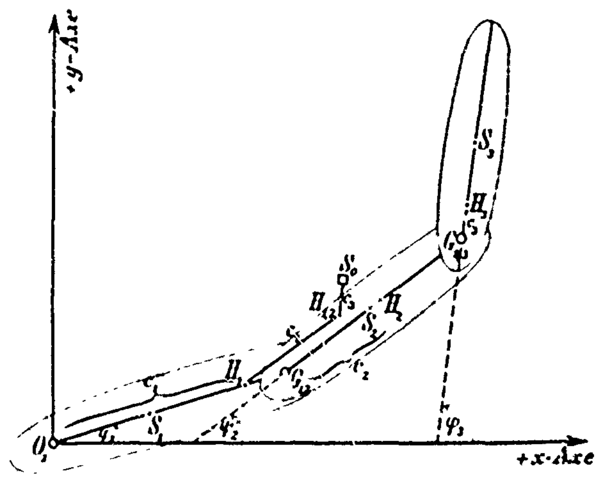


Figure 10. Linear Velocities of Main Point H_1

This will give linear velocities (Figures 3 and 10) of the main point H_1 of $c_1\varphi_1'$, for $H_{1,2}$ relative to H_1 of $c_2\varphi_2'$ and for the total center of gravity, S_o, relative to $H_{1,2}$ of $c_3\varphi_3'$. From this results:

$$(27) \quad v_o = \overline{c_1\varphi_1}' + \overline{c_2\varphi_2}' + \overline{c_3\varphi_3}' = \sum_1^{3} {}^{h}\, \overline{c_h\varphi_h}'$$

We get tangential acceleration for H_1, $H_{1,2}$ and S_o of $c_1\varphi_1''$, $c_2\varphi_2''$ and $c_3\varphi_3''$ to give

$$(28) \quad \gamma_o = \overline{c_1\varphi_1}'' + \overline{c_2\varphi_2}'' + \overline{c_3\varphi_3}'' + \overline{c_1\varphi_1}'^2 + \overline{c_2\varphi_2}'^2 + \overline{c_3\varphi}'^2$$

$$= \sum_1^{3} {}^{h} \left[\overline{c_h\varphi_h}'' + \overline{c_h\varphi_h}'^2 \right]$$

In figure 10 O_1 is the origin of a rectangular coordinate system (xy) within the plane of the three centers of gravity. The positive x-axis has the direction from which are measured angles φ_1, φ_2, φ_3. From this:

$$(29) \quad x_o = c_1 \cos\varphi_1 + c_2 \cos\varphi_2 + c_3 \cos\varphi_3 = \sum_1^{3} {}^{h}\, c_h \cos\varphi_h,$$

$$y_o = c_1 \sin\varphi_1 + c_2 \sin\varphi_2 + c_3 \sin\varphi_3 = \sum_1^{3} {}^{h}\, c_h \sin\varphi_h.$$

If the components of velocity, v_o, and of acceleration, γ_o, of the total center of gravity equal x'_o, y'_o and x''_o, y''_o we derive:

$$(30)(31) \quad x_o'' = -\sum_1^{3} {}^{h} \left[c_h \sin\varphi_h \cdot \varphi_h'' + c_h' \cos\varphi_h'^2 \right],$$

$$y_o'' = \sum_1^{3} {}^{h} \left[c_h \cos\varphi_h \cdot \varphi_h'' - c_h \sin\varphi_h \cdot \varphi_h'^2 \right].$$

The resultant velocity of $H_{1,2}$ relative to $G_{2,3}$ is equal to the geometric sum $-\overline{d_2\varphi'_2} - \overline{d_1\varphi'_1}$. The acceleration is equal to the geometric sum of $-\overline{d_2\varphi''_2} - \overline{d_2\varphi'_2}^2 - \overline{d_1\varphi''_1} - \overline{d_1\varphi'_1}^2$.

From this is derived:

$$\text{(34)}\quad \bar{v}_{r_{1,2}} = -\frac{m_o}{m_1 + m_2}\left(\overline{d_1\varphi'_1} + \overline{d_2\varphi'_2}\right)$$

and

$$\text{(35)}\quad \bar{\gamma}_{r_{1,2}} = -\frac{m_o}{m_1 + m_2}\left(\overline{d_1\varphi''_1} + \overline{d_2\varphi''_2} + \overline{d_1\varphi'_1}^2 + \overline{d_2\varphi'_2}^2\right)$$

(index r is used to indicate the motion relative to $G_{2,3}$, and the components of $v_{r_{1,2}}$ and $\gamma_{r_{1,2}}$ are derived in the direction of the coordinate axes.)

For the components $x'r_{1,2}$ and $y'r_{1,2}$, Fischer gives velocity in (36). For the components x'' and y'' the acceleration is given in (37). Using (35) the force $\overline{Er}_{1,2}$ of the partial center of gravity $S_{1,2}$ for motion relative to $G_{2,3}$ is given in (38) (39).

The motion of $H_{2,3}$ relative to $G_{1,2}$ is a mere circular motion if link 3 is translatory in rotation of link 2 about $G_{1,2}$, or if link 3 is rigidly connected to link 2. Formulae (40) - (43) express velocity and acceleration here.

As in the previous section components x'r, y'r, and $x'r_1$ $y'r_1$ are derived, (48) - (49); also the effect of force E_2 on S_1 relative to $G_{1,2}$ (50); also components x'3, y'3, and $x''_3y''_3$ (52) - (55), and the effect of Er_3 on S_3 relative to $G_{2,3}$ (56). The components of the direction of the coordinate axes are:

$$\text{(57)}\quad X_{r_3} = -m_o\left(c_3 \sin\varphi_3 \cdot \varphi_3'' + c_3 \cos\varphi_3 \cdot \varphi_3'^2\right),$$

$$Y_{r_3} = m_o\left(c_3 \cos\varphi_3 \cdot \varphi_3'' - c_3 \sin\varphi_3 \cdot \varphi_3'^2\right).$$

4. The Kinetic Energy of the Three-Joint System. (pp 37-42)

The kinetic energy of a system equals the sum of external and internal forces. The external force originates in the motion of the total center of gravity, and is equal to the kinetic energy of the total mass, m_o, concentrated in the total center of gravity, S_o; let v_o = magnitude of the velocity of S_o, then the external kinetic energy = $1/2\ m_o\ v_o^2$. The internal force is only related to the motion of the system around its center of gravity, and equals the sum of the kinetic energies of the relative motions of the individual links in relation to the total center of gravity. The motion of each of the three links relative to S_o resolves to the translatory motion of velocity, v_h, of the individual centers of gravity, S_h, of the respective links related to S_o, and a rotation around an axis parallel to the joint axis through S_h, of an angular value φ'_h.

Hence, the kinetic energy of each link relative to the total center of gravity is the sum of two components: 1) the kinetic energy of mass m_h which a link has when the mass moves with velocity, v_h, of the individual center of gravity, S_h, relative to the total center of gravity S_o; 2) the kinetic energy resulting from the angular velocity, φ'_h, of the link in its rotation around the axis through S_h. If the total center of gravity is assumed to be fixed, then consideration of small displacements (movements) of links gives the following theorem:

> Each displacement of the 3-link joint system relative to the total center of gravity, S_o, from the position φ_1, φ_2, φ_3 into the infinitely adjacent system $\varphi_1 + d\varphi_1$, $\varphi_2 + d\varphi_2$, $\varphi_3 + d\varphi_3$ can be divided into three infinitely small rotations about axes throught the three main points, connected with translations of the two other links to which the respective main point does not belong.

The displacements are as follows:

(58) S_1 relative to S_o: $\overline{-e_1 . d\varphi_1} - \overline{c_2 . d\varphi_2} - \overline{c_3 . d\varphi_3}$

S_2 relative to S_o: $\overline{+d_1 . d\varphi_1} - \overline{e_2 . d\varphi_2} - \overline{c_3 . d\varphi_3}$

S_3 relative to S_o: $\overline{+d_1 . d\varphi_1} + \overline{d_2 . d\varphi_2} - \overline{e_3 . d\varphi_3}$

(again, the bars = geometrical addition)

The displacement components of $d\varphi_1$ are perpendicular to the long axis of link 1, those of $d\varphi_2$ and $d\varphi_3$ to the long axes of links 2-3.

In considering the velocity, v_h, of points S_h relative to S_o Fischer employs the division of the time differential, dt, of individual centers of gravity relative to the total center of gravity, occurring at a certain moment.

This gives:

$$(59)\ Tr = 1/2 \sum_{1}^{3} {}^{h} m_h \left[v_h^2 + x_h^2 \cdot \varphi_h'^2 \right]$$

From (58) there is then derived:

$$(60)\ Tr = 1/2\,(m_1 x_1^2 + m_1 e_1^2 + m_2 d_1^2 + m_3 d_1^2) : \varphi_1'^2$$
$$+ 1/2\,(m_2 x_2^2 + m_1 c_2^2 + m_2 e_2^2 + m_3 d_2^2) \cdot \varphi_2'^2$$
$$+ 1/2\,(m_3 x_3^2 + m_1 c_3^2 + m_2 c_3^2 + m_3 e_3^2) \cdot \varphi_3'^2$$
$$+(m_1 e_1 c_2 - m_2 d_1 e_2 + m_3 d_1 d_2) \cos(\varphi_1 - \varphi_2) \cdot \varphi_1' \cdot \varphi_2'$$
$$+(m_1 e_1 c_3 - m_2 d_1 c_3 - m_3 d_1 c_3) \cos(\varphi_1 - \varphi_3) \cdot \varphi_1' \cdot \varphi_3'$$
$$+(m_1 c_2 c_3 + m_2 e_2 c_3 - m_3 d_2 e_3) \cos(\varphi_2 - \varphi_3) \cdot \varphi_2' \cdot \varphi_3'.$$

From Figures 1-2 and from (59) this may be simplified:

$$(61)\ Tr = 1/2\, m_o k_1^2 \cdot \varphi_1'^2 + 1/2\, m_o k_2^2 \cdot \varphi_2'^2 + 1/2\, m_o k_3^2 \cdot \varphi_3'^2$$
$$+ m_o d_1 c_2 \cos(\varphi_1 - \varphi_2) \cdot \varphi_1' \varphi_2' + m_o d_1 c_3 \cos(\varphi_1 - \varphi_3) \cdot$$
$$\varphi_1' \varphi_3' + m_o d_2 c_3 \cos(\varphi_2 - \varphi_3) \cdot \varphi_2' \varphi_3'.$$

If the total center of gravity S_o is not fixed but moves with the velocity, v_o, in space, there must be added to Tr the kinetic energy $1/2\, m_o v^2$ of the motion of the total center of gravity (expressed in (62) - (63)). Finally, if the point O of the first long axis is fixed the expression for total energy is:

$$(67)\ T = 1/2\, m_o \lambda_1^2 \cdot \varphi_1'^2 + 1/2\, m_o \lambda_2^2 \cdot \varphi_2'^2 + m_o l_1 c_2 \cos$$
$$(\varphi_1 - \varphi_2)\, \varphi_1' \varphi_2'.$$

5. <u>The Elementary Work of the Forces</u> (pp 42-43)

In the motion of the 3-link system the kinetic energy of the system will not remain constant. It is necessary to determine the magnitude and direction of the translation performed by the point of application of the force in the displacement of the system. Projection of translation on the direction of force times the intensity of the force will give the value of work effected by force during the displacement. Let the intensity of the force = K, the magnitude of the translation of the point of application to it = dr, and the angle between directions of force and translation = α, then K = cos α. dr. In the 3-link system five displacements are recognized, as $V_{\varphi 1}$, $V_{\varphi 2}$, $V_{\varphi 3}$, V_{x_o} and V_{y_o}.

5a. Elementary Work of External Forces, Especially Gravity (pp 43-48)

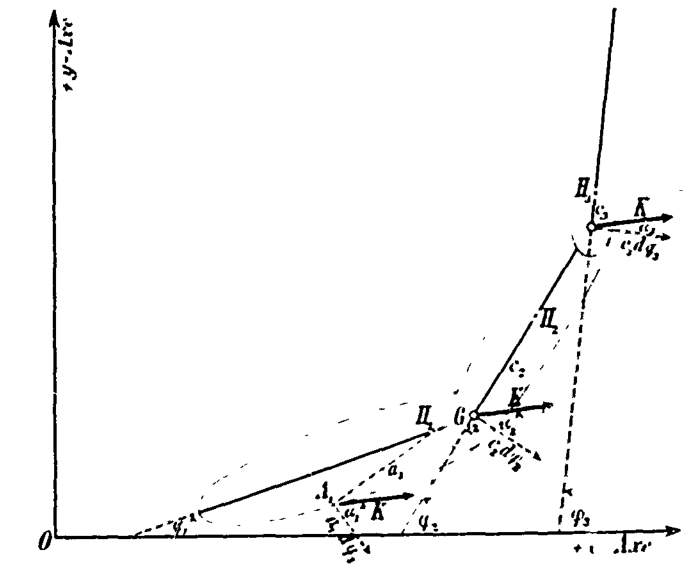


Figure 11. Demonstration of the Work of External Forces, Gravity Especially

Figure 11 serves to illustrate principles here. If force K is applied to any point, A_1, of link 1, which is the distance a_1 from the axis through H_1, parallel to the joint axes, then in displacement, $V_{\varphi 1}$, the point A_1 will have a shift of magnitude $a_1 d_{\varphi 1}$. If this shift forms with the direction of K the angle α_1, then the work of displacement $V_{\varphi 1}$ = K cosα_1. $a_1 d_{\varphi 1}$: If K cosα_1 is the turning moment, $D_{\varphi 1}$. dφ_1. If the point of application of force coincides with the main point H_1 then value of work in displacement $V_{\varphi 1}$ = zero. In similar fashion one may proceed for $V_{\varphi 2} = K \cos\alpha_2 \cdot c_2 d_{\varphi 2}$ and $V_{\varphi 3} = K \cos\alpha_3 \cdot c_3 d_{\varphi 3}$. The work of gravity on the displacement $V_{\varphi 1}$ is given as an example:

$$m_1 g e_1 \cos\varphi_1 \cdot d\varphi_1 - (m_2 g + m_3 g) d_1 \cos\varphi_1 \cdot d\varphi_1.$$

The following two theorems are derived:

I. If force K applies to the $h^{\underline{th}}$ link to which belongs the fixed point O_h then its elementary work in displacement $V\varphi_h$, which alone can change the direction of the long axis of this link, is = to the turning moment of the force relative to the axis through O_h, multiplied by $d\varphi_h$. In this case in the other displacements the force does not perform any work at all.

II. If the force K applies to one of the links to which the fixed point O_h does not belong, then its elementary work in the displacement V_{φ_h} is equal to the turning moment multiplied by d_{φ_h} that the force has relative to the axis through O_h if the point of application of the force is shifted to the joint point of the $h^{\underline{th}}$ link that within the joint system is next to the point of application of the force.

In the displacement that corresponds to an infinitely small rotation of the one link to which the force applies, the force performs an elementary work equal to the product of the corresponding angular change d_φ by the turning moment that the force has for the joint axis of the same link that within the system is next to the point O_h. Finally, as to the third displacement, V_{φ_j}, the question is whether the $j^{\underline{th}}$ link within the system is nearer to or farther from the fixed point O_h than the link to which the force applies. In the first case the elementary work is equal to the turning moment multiplied by $d_{\varphi j}$ that the force has relative to the joint axis of the $j^{\underline{th}}$ nearer to the point O_h, if its point of application is shifted to the joint point of the $j^{\underline{th}}$ link farthest from the point O_h. In the second case, however, the force does not perform any work at all.

5b. Elementary Work of Internal Forces, Particularly of Muscular Forces (pp 48-52)

A muscle at a joint forms a pair of internal forces, giving equal and opposite effects. We must consider two major situations: 1) between 1-joint and several-joint muscles; 2) between muscles acting freely between two bone-points and muscles acting by a "detour" over a bony protrusion.

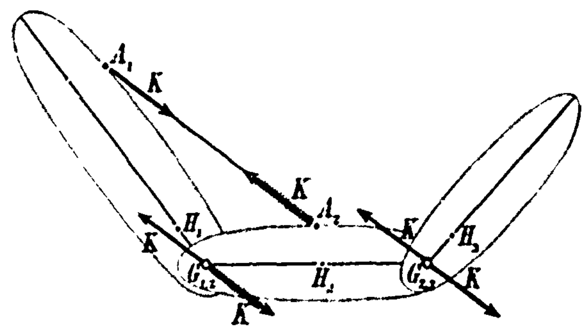


Figure 12. Action of Equal and Opposite Forces in a Three-Link System with Straight Pull-Line

Figure 12 illustrates a first situation in a 3-link joint system. The two equal and opposite forces act between two adjacent links, so that the pull-line is extended straight between the two points of attachment.

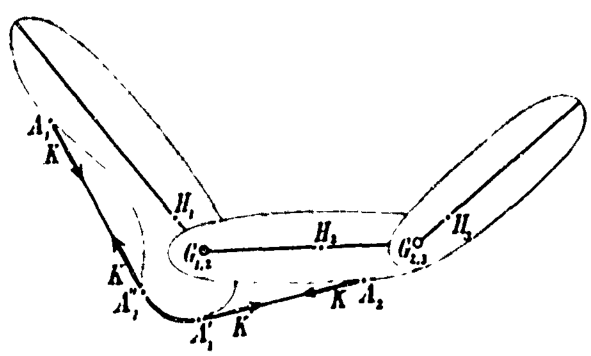


Figure 13. Action of Equal and Opposite Forces in a Three-Link System with Pull-Line Over a Link Protrusion

Figure 13 illustrates a second situation in a 3-link joint system. The two equal and opposite forces act between two adjacent links in such a way that the pull-line runs over a protrusion of the one link.

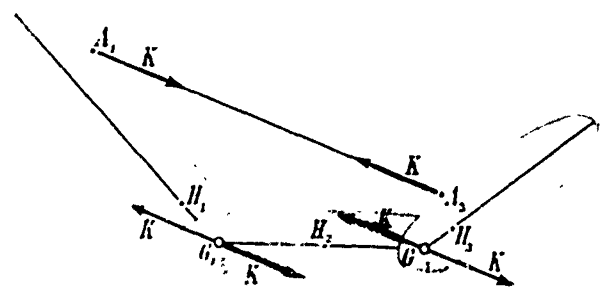


Figure 14. Action of Equal and Opposite Forces in a Three-Link System with Pull-Line Over Two Non-Adjacent Limbs

Figure 14 illustrates a third situation in a 3-link joint system. The two equal and opposite forces act between to non-adjacent links.

6. <u>The Relations Between the Changes in the Kinetic Energy and the Elementary Work of the Acting Forces.</u> (pp 52-55)

The problem here is the relation: 1) between changes in kinetic energy of the moving joint systems; and 2) the elementary work of the total external and internal forces. This may be achieved by differential equations of motion as established by Lagrange (rectangular coordinates replaced by general coordinates defining the position of the system).

In a 3-link system we must consider the angles φ_1, φ_2, φ_3 in relation to the variable coordinates x_o, y_o when the system is freely movable, or in relation to the constant coordinates of the fixed point O_1. The system of five differential equations of movement in free movement are: (h = 1, 2, 3)

$$(68) \quad \frac{d}{dt}\left(\frac{\delta T}{\delta x_o'}\right) - \frac{\delta T}{\delta x_o} = Q_{x_o}$$

$$\frac{d}{dt}\left(\frac{\delta T}{\delta y_o'}\right) - \frac{\delta T}{\delta y_o} = Q_{y_o}$$

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \varphi_h'}\right) - \frac{\delta T}{\delta \varphi_h} = Q_{\varphi_o}$$

The five equations of motion are:

(69)
$$m_o \cdot x_o'' = Q_{x_o},$$
$$m_o \cdot y_o'' = Q_{y_o};$$

$$m_o \Big\{k_1^2 \cdot \varphi_1'' + d_1c_2 \cos(\varphi_1 - \varphi_2) \cdot \varphi_2'' + d_1c_3 \cos(\varphi_1 - \varphi_3) \cdot \varphi_3'' + d_1c_2 \sin(\varphi_1 - \varphi_2) \cdot \varphi_2'^2 + d_1c_3 \sin(\varphi_1 - \varphi_3) \cdot \varphi_3'^2\Big\} = Q_{\varphi_1},$$

$$m_o \Big\{k_2^2 \cdot \varphi_2'' + d_2c_3 \cos(\varphi_2 - \varphi_3) \cdot \varphi_3'' + c_2d_1 \cos(\varphi_2 - \varphi_1) \cdot \varphi_1'' + d_2c_3 \sin(\varphi_2 - \varphi_3) \cdot \varphi_3'^2 + c_2d_1 \sin(\varphi_2 - \varphi_1) \cdot \varphi_1'^2\Big\} = Q_{\varphi_2},$$

$$m_o \Big\{k_3^2 \cdot \varphi_3'' + c_3d_1 \cos(\varphi_3 - \varphi_1) \cdot \varphi_1'' + c_3d_2 \cos(\varphi_3 - \varphi_2) \cdot \varphi_2'' + c_3d_1 \sin(\varphi_3 - \varphi_1) \cdot \varphi_1'^2 + c_3d_2 \sin(\varphi_3 - \varphi_2) \cdot \varphi_2'^2\Big\} = Q_{\varphi_3}.$$

Here the first two equations refer to the motion of the total center of gravity S_o, and the last three to motion of the 3-link joint system and to the S_o (here are contained the angular acceleration and the angular rotation of the individual links). In all equations the total mass m_o of a joint system is given on the left, but the masses m_h of the individual links are not given.

Where O_1 is fixed on the long axis of link 1 at distance l_1 from $G_{1,2}$, there are three equations as follows:

(70)
$$m_o \Big\{\lambda_1^2 \cdot \varphi_1'' + l_1 - c_2 \cos(\varphi_1 - \varphi_2) \cdot \varphi_2'' + l_1c_3 \cos(\varphi_1 - \varphi_3) \cdot \varphi_3'' + l_1c_2 \sin(\varphi_1 - \varphi_2) \cdot \varphi_2'^2 + l_1c_3 \sin(\varphi_1 - \varphi_3) \cdot \varphi_3'^2\Big\} = Q_{\varphi_1},$$

$$m_o \Big\{\lambda_2^2 \cdot \varphi_2'' + l_2c_3 \cos(\varphi_2 - \varphi_3) \cdot \varphi_3'' + c_2l_1 \cos(\varphi_2 - \varphi_1) \cdot \varphi_1'' + l_2c_3 \sin(\varphi_2 - \varphi_3) \cdot \varphi_3'^2 + c_2l_1 \sin(\varphi_2 - \varphi_1) \cdot \varphi_1'^2\Big\} = Q_{\varphi_2},$$

$$m_o \Big\{\lambda_3^2 \cdot \varphi_3'' + c_3l_1 \cos(\varphi_3 - \varphi_1) \cdot \varphi_1'' + c_3l_2 \cos(\varphi_3 - \varphi_2) \cdot \varphi_2'' + c_3l_1 \sin(\varphi_3 - \varphi_1) \cdot \varphi_1'^2 + c_3l_2 \sin(\varphi_3 - \varphi_2) \cdot \varphi_2'^2\Big\} = Q_{\varphi_3}.$$

Here each quantity Q_{φ_h} equals the sum of the turning moments exerted by all external and internal forces acting around the axis through O_1, $G_{1,2}$, or $G_{2,3}$, respectively, after all forces that do not apply directly to the h^{th} link have been shifted, parallel to themselves, to the joint point that is nearest them within the joint system.

In like manner Fischer gives equations (71) for the plane joint system of two links connected with each other by a hinge joint, when there is free mobility, and equations (72) if point O_1, on the long axis of the first of two links, is fixed.

7. Interpretation of the Equations of Motion (pp 55-69)

In general the results of the motion of each of the three links in a 3-link joint system, as a consequence of the joint connection with the others, involve: 1) the inert masses of the other links have to be set in motion; 2) the other links will move relative to the center of gravity through which they are connected with the respective links; 3) the masses will go from the respective link to the reduced system. This is expressed directly by Fischer as follows:

> It can be gathered from the consideration made so far that the motion of each of the three links, as a consequence of the joint connection with the others, will not only be influenced by the fact that the inert masses of the other links generally have also to be set in motion but also by the fact that these other links will as a rule moreover move relative to the center of the joint link either immediately or through the intermediary of another link. The influence that the inert masses of the connected links exert on the motion of a certain link will be accounted for when we suppose these masses to be concentrated in the nearest joint centers, and thereby pass from the respective link to the reduced system. As we have seen, the influence exerted by the relative motions of the connected links is, however, represented by the forces - $\overline{Er}$, which are equal and opposite to the effective forces of the center of gravity of the remaining links. If the value of these effective forces is zero, then in the motion of the reduced system the only forces to matter will be the forces actually present that have been designated above as the forces of the reduced system.
>
> Among other possibilities, the effective forces will disappear when the other links connected with one link perform translatory motions exclusively during the motion of the one link. An in-

stance of disappearing effective forces is given by the lifting up of the human body on the toes. The whole body with only the exception of the foot standing on the floor will then perform a translatory motion in accordance with the motion of the foot-joint axis. Therefore, this part of the body acts on the foot in such a way as if whole mass of this part were concentrated in the axis of the upper ankle joint and all forces applying to it had their point of application in the center of the foot joint.

The disappearance of the effective force will, however, also take place when the center of gravity to which the effective force is related coincides permanently with the joint center relative to which the motion of this center of gravity has to be reckoned.

8. Elementary Derivation of the Equations of Motion (pp 68-90)

This must be based on the theorem that the center of gravity of any body system will move as though the mass of the entire system were concentrated in it, and all external forces acting upon the system were to apply in it as to their direction and magnitude. This applies to a freely movable situation, but also when the motion of individual bodies are subject to various conditions, which are, in general, added to the effect of the external forces.

Also needed is the theorem that a freely movable rigid body will rotate about its center of gravity in the same way as if that center of gravity were not movable, but firm. Hence we must consider only the turning moments of the external forces relative to the axis through the center of gravity.

In a 3-link plane joint system we may consider any one link alone. Here must be noted: 1) its motion is dependent upon forces acting upon it, plus forces on the other links; 2) action and reaction at joints, and at a specific point in the joint, i. e., the joint center.

In the 3-link system the motions of links 2-3 and the forces applying to them will act on link 1 in $G_{1,2}$ only; also, the motions of link 1 will act on link 2 in $G_{1,2}$, and the motion of link 3 will act on it in $G_{2,3}$ etc.

Links 1-2 will act reciprocally upon one another as two pressure forces which may be designated as $\bar{\Gamma}_{1,2}$ and $-\bar{\Gamma}_{1,2}$. The pressure force $-\bar{\Gamma}_{1,2}$, acting in $G_{1,2}$ on a partial system of links 2-3, extends to the center of gravity $S_{2,3}$ and gives acceleration $y_{2,3}$ with mass $m_{2,3}$ (we are here dealing with absolute acceleration). If this be accepted then their vector sum = $\overline{m_{2,3}y_{2,3}} - \sum K_{(2,3)}$, which may be also written as:

$$(89) \quad -\bar{\Gamma}_{1,2} = \overline{m_{2,3}y_{2,3}} + \sum\left(-\bar{K}_{(2,3)}\right)$$

If the effective force of the center of gravity for motion in space is given as $\bar{E}_{2,3}$ then:

$$(90) \quad -\bar{\Gamma}_{1,2} = \bar{E}_{2,3} + \sum\left(-\bar{K}_{(2,3)}\right)$$

From the pressure force $\bar{\Gamma}_{1,2}$ acting on link 1 in $G_{1,2}$ the vector sum is:

$$(91) \quad \bar{\Gamma}_{1,2} = \sum \bar{K}_{(2,3)} + \left(-\bar{E}_{2,3}\right),$$ also expressed as

$$(92) \quad \bar{\Gamma}_{1,2} = \bar{E}_1 + \sum\left(-\bar{K}_{(1)}\right)$$

Fischer then demonstrates that the vector sum is equal to the sum for the pressure force as in (91). This is based on the theorem of the center of gravity for it, i. e. the vector sum of all external forces applying to the joint system is equal to the absolute effective force $\bar{E}_o$ of the total center of gravity, S_o. But not all forces, $\bar{K}_{(1)}$, $\bar{K}_{(2)}$, $\bar{K}_{(3)}$ considered as external forces for each link, respectively, belong to the external forces of the whole joint system, as developed in (93)-(96).

In order to reduce the motion of link 1 to that of a single separate body it will be necessary only to apply in the joint center $G_{1,2}$ all forces applying to links 2-3, and, moreover, a force equal and opposite to the absolute force of the partial center of gravity, $S_{2,3}$. For the regular acceleration of link 1 is derived:

$$(97) \quad \varphi_1'' = \frac{D_1}{m_1 x_1} 2$$ (where D_1 = the turning moment of forces on link 1, and x_1 is the radius of inertia of link 1.).

The acceleration γ_1 of the center of gravity, S_o, becomes:

$$(98) \quad \bar{\gamma}_1 = \frac{\sum \bar{K}_{(1)} + \bar{\Gamma}_{1,2}}{m_1}.$$

In (99) - (107) this is similarly done for links 2 and 3.

It is demonstrated in (108) - (114) that in conditional mobility of the entire joint system that link 2 (joined to links 1 and 3) will rotate around the movable axis $G_{1,2}$ as would a rigid body around an axis at rest. Finally this may be expressed as:

$$(114) \quad \varphi_2'' = \frac{m_2 - D_{1,2}}{m_2(x_2^2 + r_2^2)}$$

Similarly, we have:

$$(121) \quad \varphi_3'' = \frac{M_3 - D_{2,3}}{m_3(x_3^2 + r_3^2)}$$

Finally, Fischer's summary may be quoted:

In the foregoing we have finally dealt with the reduced system in all cases, because it was intended to show that the road taken leads eventually to the form of the equations of motion of the joint system that earlier had been derived from Lagrange's general equations. Besides, as a result we have come in an elementary way to a new interpretaion of the equations of motion, as opposed to the interpretation discussed in section 7. This new interpretation leaves the reduced systems out of discussion and relates to the motion of the actual individual links. Although the form of the equations that results from this interpretation is generally more complicated than when the interpretation is based on the reduced systems, the new interpretation is nevertheless especially important in many applications of the equations of motion to certain problems of the physiology of motion.

This will first of all be true when by suitable experiments we succeed in finding empirically for a certain motion of an organism the paths, velocities, and accelerations of the centers of gravity of all individual body parts. Then we shall also have obtained without further difficulty the absolute effective forces of these centers of gravity, and on the basis of this knowledge it will be possible for us to investigate the motion of each single link separately, without having to take further regard of the connection with the other links than has already been taken by reference to the effective forces.

Indeed, the results obtained up to now relate so far only to a plane three-link system; therefore, for the moment they can only be applied to organisms that behave like such a three-link joint system and certain motions.

The N-Link Plane and Solid Joint Systems (pp 91 - 176)

From a 3-link plane joint system it is possible to go to equations of motion for a plane system of n-links and for a general solid joint system. There will be greater freedom of motion in joints, with multiple hinge joints, with parallel joint axes, and with adequately simple mass distribution in the limbs.

Locomotion involves forward and lateral movement, the former basic (there is some vertical motion). Walking is a plane motion wherein all body parts are more parallel to a vertical plane fixed in space ("walking plane"); motions here are in parallel axes perpendicular to the walking plane in all joints, and involve muscular mechanics. Man performs mostly solid motions in locomotion; the large joints have more than one degree of freedom; 1) shoulder and hip joints have three degrees of freedom; 2) elbow, knee, wrist, ankle have two. Considered here, in order, will be. 1) 2-link systems; 2) differences between motions of solid-and plane-joint systems; 3) n-link solid systems. A plane joint system of n-links has n + 2 degrees of freedom where:
a) plane motion is not subject to any limiting conditions;
b) several links do not form a closed kinetic chain. Such a system is defined by n + 2 coordinates. We must know the two plane coordinates of one point of the system, and an angle of direction for each one of the links, choosing any point desired. The number of equations of motion is equal to the number of the degrees of freedom: 1) there are n + 2 equations in completely free motion of the n-link plane joint

system; 2) there are only n equations where one point of one link is fixed.

A solid joint system of n links has more degrees of freedom than a plane joint system. The number of degrees is determined not so much by the number of links as by the freedom of motion in joints (generally, joints have 3 degrees of freedom). A solid joint system has $3n + 3$ degrees of freedom, and any position in space has $3n + 3$ general coordinates; three of these will give the position of one point of the system and three each will relate one of the n links in space. Hence, three of the determining quantities will be the rectangular solid coordinates of a point, and 3n quantities represent coordinates of direction and hence are angles. This gives $3n + 3$ equations of motion in completely free mobility (three times as many as for a plane system).

In an organic joint system few joints have 3 degrees of freedom. In the leg three parts are assumed as rigid, i. e., thigh, leg and foot. These have 7 degrees of freedom related to the fixed pelvis (3 degrees in knee + ankle); in the mobile pelvis there are 10 degrees of freedom. It is assumed: 1) all joints in the system in each case have a fixed joint center and 3 degrees of freedom; 2) the composition of reduced systems and the location of main points that are identical with the centers of gravity are still useful in the n-link systems.

9. <u>About the Generally Valid Properties of the Reduced System and Main Points.</u> (pp 96 - 99)

After severance of any one joint connection of the system, this system shall be divided into two parts no longer connected with one another (there will be no closed kinematic chain.) In the human trunk one link is in joint connection with more than two links. There are links connected with a joint with only one link - <u>end links</u> (e. g., head, hand, foot, or phalanx III in movable fingers and toes). A <u>nuclear</u> link equals all joints to be severed at the link forming the nucleus of the reduced system; these nuclear links eventually receive the entire mass of the whole joint system. The reduced system will act as a single rigid body, so that the center of gravity has a fixed position within the nuclear link. The mass of the nuclear link is concentrated in its own center of gravity. Moments of inertia of the reduced system must be considered.

9a. Connection Between the Main Points and the Total Center of Gravity of the General Joint System (pp 99 - 111)

Simple plane n-link joint systems must first be considered. Each link in the n-link system is connected via a hinge joint with parallel axes; apart from the end link each is connected with two others (only). The centers of gravity shall be in a single plane perpendicular to the joint axes, and the points of intersection of the hinge axes with this plane shall be the joint center. The connecting line between the two joint centers of the inner link shall be the longitudinal axis (long axis); 1) in each link it passes through the center of gravity of the link; 2) for an end link the long axis goes from one joint center to the end, O_1 or O_n. Hence, the n-links form a continuous open kinematic chain.

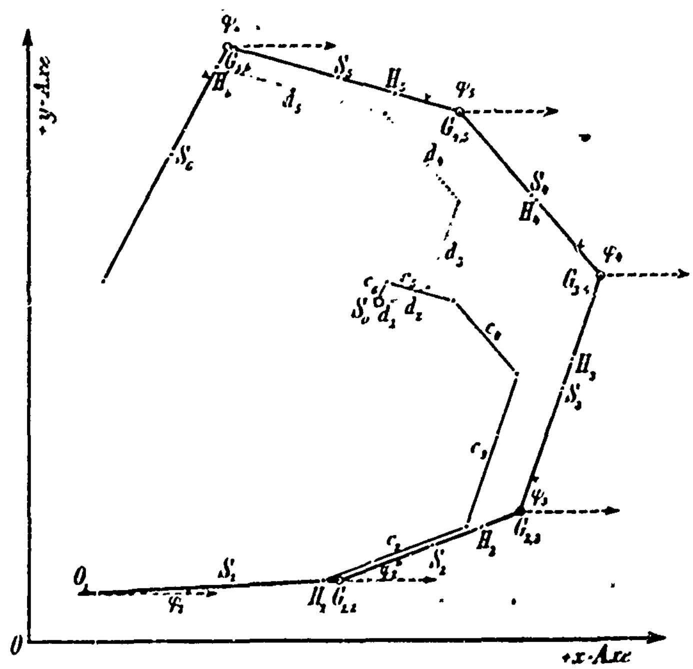


Figure 15. A Six-Link System

In Figure 15, a 6-link joint system, the system is shown by the broken lines of n long axes. Links are numbered from the end link, in a positive direction in each link for its long axis, as well as for individual lengths plotted on the long axis. If j is the number given to one link, all other traits of this link shall be so designated. The length of a link = the distance between two joint centers (in end link it is from the joint center to a terminal point). The length = l_j, the center of gravity = S_j; S_j divides l_j into two parts; 1) the first part in the long axis in a positive direction = r_j; 2) the second part = s_j; hence (122) $r_j + s_j = l_j$. Since S_j of the j^{th} link is on the long axis the main point, H_j, must be on this axis; hence, the length of the link is divided into two parts by H_j: 1) first part c_j; 2) second part = d_j, and (123) $c_j + d_j = l_j$.

If m_o = total mass of the whole joint system and m_j = the mass of the $j^{\underline{th}}$ link then (124) $m_o c_j = m_j r_j + (m_{j+1} + m_{j+2} \ldots m_n)\, l_j$, and $m_o d_j = m_j s_j + (m_1 + m_2 + \ldots m_{j-1})\, l_j$. Now let μ_j = ratio of mass m_j of the $j^{\underline{th}}$ link to the total mass m_o, and also = to the ratio of weight G_j of the $j^{\underline{th}}$ link to the total weight G_o. The sum of the series of these ratio numbers, μ, that belong to the several links in succession shall be designated by giving μ the two indices (separated by a comma) of the first and last links of the sum:

$$(125)\quad c_j = \mu_j r_j + \mu_{j+1,n} l_j$$

$$d_j = \mu_j s_j + \mu_{1,j-1} l_j$$

Using these formulae the main lengths c_j and d_j for all n links can be calculated.

As an example, here is a 6-link joint system (l = length, r and s = distances of centers of gravity of the link, G = weight of the link)

No. of the link	Lgth. of link (cm)	c. g. distance of link (cm)		wt. of link (kg)
	l	r	s	G
1	45, 5	25, 5	20	8, 888
2	34	17, 5	16, 5	9, 672
3	42, 5	22	20, 5	10, 382
4	37	18	19	12, 001
5	43	22, 5	20, 5	12, 627
6	45	21	24	11, 669

(Fischer, p. 101)

For the total weight, G_o, we have here 65, 239 g. The ratio numbers of μ and their sums will be as follows:

μ_1 = 0, 1362		
μ_2 = 0, 1483	$\mu_{1,2}$ = 0, 2845	$\mu_{2,6}$ = 0, 8638
μ_3 = 0, 1591	$\mu_{1,3}$ = 0, 4436	$\mu_{3,6}$ = 0, 7155
μ_4 = 0, 1840	$\mu_{1,4}$ = 0, 6276	$\mu_{4,6}$ = 0, 5564
μ_5 = 0, 1936		
μ_6 = 0, 1788	$\mu_{1,5}$ = 0, 8212	$\mu_{5,6}$ = 0, 3724

From these six pairs of main lengths may be calculated:

c_1 = 42,78 cm	d_1 = 2,72 cm
c_2 = 26,92 cm	d_2 = 7,08 cm
c_3 = 27,15 cm	d_3 = 15,35 cm
c_4 = 17,09 cm	d_4 = 19,91 cm
c_5 = 12,04 cm	d_5 = 30,96 cm
c_6 = 3,75 cm	d_6 = 41,25 cm

In Figure 18 the lines of the long axes = 1/10 natural size. It is possible to get the total center of gravity, S_o, for any position of the long axes plotted against one another. Starting from H_1, form the vector sum of main lengths, c, belonging to the other five links, as in Figure 18; or, instead of H_1, take any other main point from H_3, giving the vector sums $\overline{-d}_1$, $\overline{-d}_2$, $+\overline{c}_4 + \overline{c}_5 + \overline{c}_6$. If we start at H_6 we have links 1-5 in a negative direction. For the n-link system (126) $\overline{OS}_o = \sum_1^n j\mu_j \cdot \overline{OS}_j$ (where line over S = vector sum).

If we combine all terms of the vector sum on the right side of (126) we get (127) $\overline{O}_1S_o = \sum_1^n j\ \overline{c}_j$ The vector system from H_1 to S_o becomes (128) $H_1S_o = \sum_2^n j\ \overline{c}_j$

For Figure 18 the main lengths f_{jh} have values as follows:

h	f_{1h}	f_{2h}	f_{3h}	f_{4h}	f_{5h}	f_{6h}
1	$-e_1$	$-c_2$	$-c_3$	$-c_4$	$-c_5$	$-c_6$
2	$+d_1$	$-c_2$	$-c_3$	$-c_4$	$-c_5$	$-c_6$
3	$+d_1$	$+d_2$	$-e_3$	$-c_4$	$-c_5$	$-c_6$
4	$+d_1$	$+d_2$	$+d_3$	$-e_4$	$-c_5$	$-c_6$
5	$+d_1$	$+d_2$	$+d_3$	$+d_4$	$-e_5$	$-c_6$
6	$+d_1$	$+d_2$	$+d_3$	$+d_4$	$+d_5$	$-e_6$

(In the above e_h = distance to the h^{th} main point from the respective center of gravity, c_h = distance of the main point from the first joint cente , d_h = distance of the second joint center from the main point of the h^{th} link. Also, the six vectors that form the vector line from S_o - S_h are in the same horizontal line).

Fischer illustrates via a 20-link solid joint system that the close connections between the main points and the total center of gravity are valid here also. The following theorem results:

> In any solid joint system we shall always be led to the total center of gravity when, starting from the main point of any one of the n links, we form the vector sum of the n-1 main lengths belonging to the remaining links, which main lengths lie nearest within the system to the link from the main point from which we start. Each main length has to be taken in the direction away from the respective link.

9b. Relation Between the Main Points and the Centers of Gravity of the Partial System. (pp 111-118)

Here is discussed in a plane n-link joint system the relations between the main points of the links and the main points and the centers of gravity of the partial system. This, largely via Figure 18, is applied to a solid joint system. As a result two theorems are formulated:

A. If in any one plane or solid joint system we suppose all links combined to a partial system that would fall off the entire system if one joint were severed, then we get the main point variable as to its position of this partial system when starting from the main point of one link of the partial system we form the vector sum of the main lengths that belong to the remaining links of the partial system and within the system lie nearest to the link from the main point of which we start, this is a di.ection away from the link.

B. The center of gravity of a partial system. . . will always be on the extension of the vector that connects the center of the separation joint with the main point of the partial system. The ratio of the distance of this center of gravity from the center of separation joint to the distance of the main point from the same joint center is the same as the ratio of the mass of the whole joint system to the mass of the partial system.

9c. Relation of the Main Points to the Displacement of the General Solid System (pp 118-120)

The thesis developed here is: "if we impart to any one joint system a displacement in which only one link changes its orientation in space by rotation while all other links perform more translatory motions, and we want the total center of gravity to remain at its place at the same time, then the rotation of the respective link must take place about an axis through its main point."

9d. Inferences Drawn for the Kinetics of the Solid Joint Systems (pp 120-121)

All properties of the reduced systems and main points proved for the plane 3-link joint system "retain their full validity in any one solid joint system."

10. The Kinetic Energy and the Equations of Motion of the N-Link Plane Joint System. (pp 121-137)

10a. Derivations of the Kinetic Energy of the N-Link Plane Joint System (pp 121-128)

In the plane of the long axes of the n-link system we shall fix a rectangular coordinate (xy) system, as in Figure 18. The coordinates to the total center of gravity, $S_o = x_o, y_o$. The positive direction of the long axis of the j^{th} link = angle φ_j, with positive direction of the X-axis. Then a portion of the joint system is defined by x_o, y_o and the n angle φ_j. Let x'_o and y'_o = the components taken in the direction of the two coordinate axes of the velocity, v_o, of the total center of gravity, S_o. Let φ'_j = the angular velocity with which the long axis of the j^{th} link changes its direction in the plane of motion common to all long axes. Then the kinetic energy of the n-link system can be expressed as in a 3-link system:

(145) $1/2\, m_o\, (x'^2_o + y'^2_o)$.

For calculation of the total internal kinetic energy T, of the n-link joint system:

$$(147)\quad T = \frac{1}{2} m_o (x'^2_o + y'^2_o + \frac{1}{2} \sum_1^n h \; M_h \; \left[v_h^2 + x_h^2 \cdot \varphi_h'^2 \right].$$

The velocity, v_h, of the center of gravity, S_h, relative to S_o is made up components $\overline{f_{1h} \cdot \varphi'_1}$, $\overline{f_{2h} \cdot \varphi'_2}$, $\overline{f_{3h} \cdot \varphi'_3}$ . . . . $\overline{f_{nh} \cdot \varphi'_n}$. These are perpendicular to the long axes of links 1, 2, 3 . . . n, in order; so for vector v_h the vector sum is:

$$(148)\quad v_h = \sum_1^n j \; f_{jh} \cdot \varphi'_j,$$ here in the sum the index h, has a constant value; only the index, j, has values of 1 . . . . n.

From (62), for a 3-link plane system, Fischer derives (154) for an n-link plane system:

$$(62)\quad T = \frac{1}{2} m_o \Big\{ x'^2_o + y'^2_o + k_1^2 \cdot \varphi'^2_1 + k_2^2 \cdot \varphi'^2_2 + k_3^2 \cdot \varphi'^2_3$$
$$+ 2d_1c_2 \cos(\varphi_1 - \varphi_2) \cdot \varphi'_1 \varphi'_2 + 2d_1c_3 \cos(\varphi_1 - \varphi_3) \cdot \varphi'_1 \varphi'_3$$
$$+ 2d_2c_3 \cos(\varphi_2 - \varphi_3) \cdot \varphi'_2 \varphi'_3 \Big\} .$$

$$(154)\quad T = \frac{1}{2} m_o \Big[x'^2_o + y'^2_o + \sum_1^n j \; k_j^2 \cdot \varphi'^2_j - 2 \sum_1^{n-1} i \; \sum_2^n k \; d_i c_k \cos$$
$$(\varphi_i - \varphi_k) \cdot \varphi'_i \varphi'_{k'} \Big]$$

It is concluded that "the velocity of the total center of gravity and the external kinetic energy resulting from it can also be represented by the velocity of any one point of the joint system by means of the coordinates of direction, φ'_j, and the angular velocities, φ'_j, on the basis of the relations of the total center of gravity to the main points of the individual links." Here once more the validity between derivations for a 3-link system (64) and those for an n-link system (159) is demonstrated.

10b. <u>The Equations of Motion of the N-Link Plane Joint System</u> (pp 128-137)

Here we must go to n + 2 equation, since the index, h, must be given values 1 - n in succession:

$$(163)\quad m_o x_o'' = Q_{x_o}$$

$$m_o y_o'' = Q_{y_o}$$

$$m_o \left\{ k_h^2 \cdot \varphi_h'' - \sum_1^n {}_j f_{hj} f_{jh} \left[\varphi_j'' \cos(\varphi_h - \varphi_j) + \varphi_j'^2 \sin (\varphi_h - \varphi_j) \right] \right\} = Q_{\varphi h},$$

$$(h = 1, 2, 3. \ldots n)$$

Each of the last n equations of motion of (163) corresponds to that of the three equations of motion of (59) for the 3-link system.

In general all considerations lead to the result that "in an elementary way" there applies here the mathematics used in the 3-link system. Two theorems are presented:

A. In the free motions of the n-link plane joint system each of the n reduced systems behaves in every respect, such as if apart from the forces belonging to the system there were to apply a force in each joint center of its nuclear link that is equal and opposite to the effective force of the center of gravity of the particular system dependent, or relative to the joint center.

B. In the free motion of this n-link plane joint system we can transform the motion of each individual link into the motion of a rigid body separated from the remaining links when, apart from the forces directly applying to it, we suppose to be shifted to each joint center of this link the forces applying to the partial system dependent on it without modification of their magnitude and their direction, and we further add a force that is equal and opposite to the absolute effective force of the center of gravity of the partial system dependent on the link.

11. The Two-Link Solid Joint Systems. (pp 137-172)

Here links 1-2 shall be connected via a ball joint, each moving against the other about one point. Hence there exists 3 degrees of freedom. The long axis = line $G_{1,2}$ to the center of gravity S_j; and the long axis of each joint = the main axis of the ellipsoid of inertia belonging to its center of gravity.

11a. General Coordinates of the System. (pp 137-143)

The 2-link joint system freely moving in space has 9 degrees of freedom, and its shape and position in space are determined by nine general coordinates. It is best here to use three solid coordinates that give the total center of gravity, and also to use for each link three angles that give its position in space. Here x, y, z coordinates will be used, each perpendicular to the other two. In Figure 16 the basic principles are illustrated.

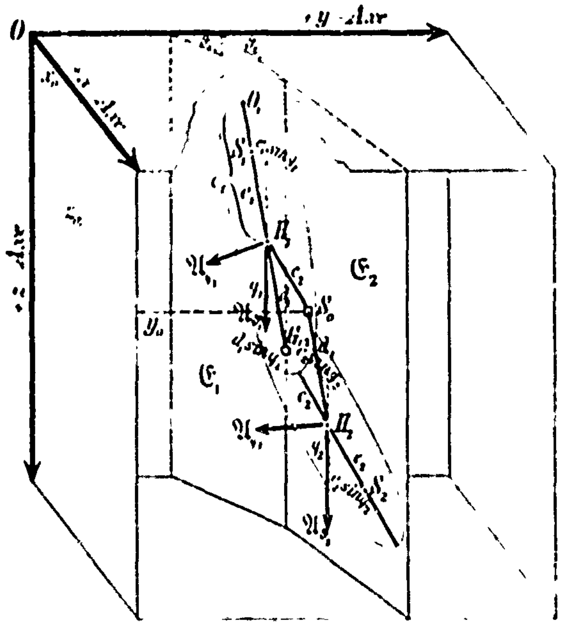


Figure. 16. Basic Principles of the Coordinate System

The x-axis and the y-axis are in the horizontal plane. The z-axis is vertical, and is positive starting downward from O. Coordinates of the total center of gravity = x_o, y_o, z_o. The orientation of each of the two links is determined by the "so-called unsymmetrical angles of Euler". For link 1 there are $\varphi_1, \delta_1, \rho_1$, for link 2 there are $\varphi_2, \delta_2, \rho_2$. For each link, φ_j, there is an angle determined by the positive direction of its long axis related to the vertical direction downward, i. e., with the positive z-axis. The positive direction of the long axis is the direction in which this axis is run through when it goes from the free end of link 1 past the joint center $G_{1,2}$ to the free end of link 2. The vertical plane through each long axis (Figure 21, planes E_1, E_2) will be inclined against the vertical yz-plane coordinate system. The angle of the two planes will = δ_j. The perpendicular on the yz-plane may be erected in the direction of the positive x-axis. The perpendicular on the plane E_j may be erected on the side from which the positive section of the long axis of the link seems rotated counter-clockwise by the angle φ_j, against the positive z-axis, directed downward. This halfway perpendicular to the plane E_j is called the knotline of the main point. The angle δ_j is positive if the knotline of the main point, seen from the side toward which the positive z-axis points (from below), seems rotated counter-clockwise around this angle against the positive x-axis. The angle φ_j has only a positive value. The two angles, φ_j and δ_j will determine the direction of each long axis and also the position of each relative to S_o. From Figure 21 the following are derived:

$$(175)\quad \begin{cases} m_o d_1 = m_1 s_1 \\ m_2 d_1 = m_1 e_1 \\ m_o e_1 = m_2 s_1 \end{cases} \quad \text{and } (176)\quad \begin{cases} m_o c_2 = m_2 r_2 \\ m_1 c_2 = -m_2 e_2 \\ m_o e_2 = -m_1 r_2 \end{cases}$$

Once S_o is determined it will be easy to locate the two main points by angles φ_1, δ_1 and φ_2, δ_2. In order to determine the orientation of a link in space it will be necessary to fix two straight lines within it. Fischer chooses the long axis and a half-ray perpendicular to it, starting from the main point (this half-ray is the first cross axis of the link). The zero position of a link is the position in which the positive direction of the long axis of the link has the direction of the positive z-axis, and the first cross axis the direction of the positive x-axis. Fischer similarly sets up a half-ray in the direction of the positive y-axis (this half-ray is the second cross axis of the link).

11b. The Derivation of the Kinetic Energy. (pp 144-157)

This section is based, in principle, upon the 2-link joint system. Fischer states that for the solid system the kinetic energy at any one time shall be determined. It will be necessary to separate from a translatory motion of the whole system (related to S_o) the motion of the two links relative to S_o, and to determine the kinetic energy for the two kinds of motion of the system. "The total kinetic energy will then again simply be the sum of the two parts."

$$(199)\quad T = 1/2m_o(x_o'^2 + z_o'^2)$$
$$+ 1/2m_o\left[p_1^2\cos^2\rho_1 + q_1^2\sin^2\rho_1\right]\cdot\varphi_1'^2$$
$$+ 1/2m_o\left[p_2^2\cos^2\rho_2 + q_2^2\sin^2\rho_2\right]\cdot\varphi_2'^2$$
$$+ 1/2m_o\left[(p_1^2\sin^2\rho_1 + q_1^2\cos^2\rho_1)\sin^2\varphi_1 + r_1^2\cos^2\varphi_1\right]\cdot\delta_1'^2$$
$$+ 1/2m_o\left[(p_2^2\sin^2\rho_2 = q_2^2\cos^2\rho_2)\sin^2\varphi_2 + r_2^2\cos^2\varphi_2\right]\cdot\delta_2'^2$$
$$+ 1/2m_o r_1^2\cdot\rho_1'^2 + 1/2m_o r_2^2\cdot\rho_2'^2$$
$$+ m_o(p_1^2 - q_1^2)\sin\rho_1\cos\rho_1\sin\varphi_1\cdot\varphi_1'\delta_1'$$
$$+ m_o(p_2^2 - q_2^2)\sin\rho_2\cos\rho_2\sin\varphi_2\cdot\varphi_2'\delta_2'$$
$$+ m_o r_1^2\cos\varphi_1\cdot\delta_1' q_1' + m_o r_2^2\cos\varphi_2\cdot\delta_2'\rho_2'$$
$$+ m_o d_1 c_2\left[\sin\varphi_1 + \sin\varphi_2 + \cos\varphi_1\cos\varphi_2\cos(\delta_2 - \delta_1)\right]\cdot\varphi_1'\varphi_2'$$
$$- m_o d_1 c_2\cos\varphi_1\sin\varphi_2\sin(\delta_2 - \delta_1)\cdot\varphi_1'\delta_2'$$
$$+ m_o d_1 c_2\sin\varphi_1\cos\varphi_2\sin(\delta_2 - \delta_1)\cdot\varphi_2'\delta_1'$$
$$+ m_o d_1 c_2\sin\varphi_1\sin\varphi_2\cos(\delta_2 - \delta_1)\cdot\delta_1'\delta_2'.$$

In the above terms for T only the first relates to the kinetic energy that is derived from the motion of S_o. From (199) it is possible to get values for the kinetic energy of a single rigid body moving freely in space. Accordingly, the following equations are derived for the total kinetic energy, T, of the joint system where that system moves about a fixed point, O_1, of the long axis of link 1:

$$(206)\quad T = 1/2m_o\left[p_{o1}^2\cos^2\rho_1 + q_{o1}^2\sin^2\rho_1\right]\cdot\varphi_1'^2$$
$$+ 1/2m_o\left[p_{o2}^2\cos^2\rho_2 + q_{o2}^2\sin^2\rho_2\right]\cdot\varphi_2'^2$$

(206) continued -

$$+ 1/2m_o \left[(p_{01}^2 \sin^2 \rho_1 + q_{01}^2 \cos^2 \rho_1)\sin^2 \varphi_1 + r_{01}^2 \cos^2 \varphi_1\right] . \delta_1'^2$$

$$+ 1/2m_o \left[(p_{02}^2 \sin^2 \rho_2 + q_{02}^2 \cos^2 \rho_2)\sin^2 \varphi_2 + r_{02}^2 \cos^2 \varphi_2\right] . \delta_2'^2$$

$$+ 1/2m_o r_{01}^2 \rho_1'^2 + 1/2m_o r_{02}^2 . \rho_2'^2$$

$$+ m_o(p_{01}^2 - q_{01}^2)\sin\rho_1 \cos\rho_1 \sin\varphi_1 . \varphi_1'\delta_1'$$

$$+ m_o(p_{02}^2 - q_{02}^2)\sin\rho_2 \cos\rho_2 \sin\varphi_2 . \varphi_2'\delta_2'$$

$$+ m_o r_{01}^2 \cos\varphi_1 . \delta_1'\rho_1' + m_o r_{02}^2 \cos\varphi_2 . \delta_2'\rho_2'$$

$$+ m_o l_1 c_2 \left[\sin\varphi_1 \sin\varphi_2 + \cos\varphi_1 \cos\varphi_2 \cos(\delta_2 - \delta_1)\right] . \varphi_1' \varphi_2'$$

$$- m_o l_1 c_2 \cos\varphi_1 \sin\varphi_2 \sin(\delta_2 - \delta_1) . \varphi_1' \delta_2'$$

$$+ m_o l_1 c_2 \sin\varphi_1 \cos\varphi_2 \sin(\delta_2 - \delta_1) . \varphi_2' \delta_1'$$

$$+ m_o l_1 c_2 \sin\varphi_1 \sin\varphi_2 \cos(\delta_2 - \delta_1) . \delta_1' \delta_2' .$$

11c. The Equations of Motion (pp 157 - 172)

Nine equations of motion will correspond to 9 degrees of freedom, and may be derived from values of T by use of the differential equations of Lagrange, of the second type, relating to the coordinates x_o, y_o, z_o, φ_1, δ_1, ρ_1, φ_2, δ_2, ρ_2; partial differentiations must be done, in order, for these coordinates. When this is accomplished the complete system of the equations of motion (by Lagrange becomes here:

$$(219)\quad m_o p_{01}^2 . \varphi_1'' - m_o(p_{01}^2 - r_{01}^2)\sin\varphi_1 \cos\varphi_1 . \delta_1'^2$$

$$+ m_o r_{01}^2 \sin\varphi . \delta_1' \rho_1' = D\varphi_{01}$$

$$m_o(p_{01}^2 \sin^2\varphi_1 + r_{01}^2 \cos^2\varphi_1) . \delta_1'' + m_o r_{01} \cos\varphi_1 . \rho_1''$$

$$+ 2m_o(p_{01}^2 - r_{01}^2)\sin\varphi_1 \cos\varphi_1 . \varphi_1' \delta_1'$$

$$- m\ r_{01}^2 \sin\varphi_1 . \varphi_1' \delta_1' = D_{\delta 01}$$

(219) continued

$$m_o r_{01}^2 \cdot \rho_1'' + m_o r_{01}^2 \cos\varphi_1 \cdot \delta_1'' - m_o r_{01}^2 \sin\varphi_1 \cdot \varphi_1' \delta_1' = D_{\rho 01}$$

$$m_o p_{02}^2 \cdot \varphi_2'' - m_o (p_{02}^2 - r_{02}^2) \sin\varphi_2 \cos\varphi_2 \cdot \delta_2'^2$$

$$+ m_o r_{02}^2 \sin\varphi_2 \cdot \delta_2' \rho_2' = D_{\varphi 02}$$

$$m_o (p_{02}^2 \sin^2\varphi_2 + r_{02}^2 \cos^2\varphi_2) \cdot \delta_2'' + m_o r_{02}^2 \cos\varphi_2 \cdot \rho_2''$$

$$+ 2m_o (p_{02}^2 - r_{02}^2) \sin\varphi_2 \cos\varphi_2 \cdot \varphi_2' \delta_2'$$

$$- m_o r_{02}^2 \sin\varphi_2 \cdot \varphi_2' \rho_2' = D_{\delta 02}$$

$$m_o r_{02}^2 \cdot \rho_2'' + m_o r_{02}^2 \cos\varphi_2 \cdot \delta_2'' - m_o r_{02}^2 \sin\varphi_2 \cdot \varphi_2' \delta_2' = D_{\rho 02}.$$

12. The General Solid Joint System (pp 172-176)

The kinetics of the 2-link system have been reduced to the kinetics of a single rigid body by the use of the reduced systems. In the n-link system the same principles apply, though the formulae are more complex. Each of the equations of motion can be interpreted as an equation of motion of a reduced system, i. e., as an isolated rigid body. Fischer summarizes as follows:

> The only difference between the equations of motion of a rigid body and a reduced system consists in the fact that for the latter there have to be added to the forces applying directly to it the relative effective forces to be taken in the opposite direction that have but to be derived from the motion of the joint system. According to the comprehensive discussions in the 9th section, this derivation is, however, not subject to any difficulties. Since we get to the main point of a partial system of the $j^{\underline{th}}$ link starting from the center of the connecting link simply by forming the vector sum of the main lengths belonging to the links of the partial system that within the joint system lie nearest to the $j^{\underline{th}}$ link (of the earlier theorem), we can also easily obtain the acceleration of the main point of the partial system belonging to the respective joint center. This acceleration will be composed of as many components as the partial system possesses links. Each acceleration component

originating from a certain link will in turn be resolved into a series of components that depend upon the coordinates of direction, the angular velocities, and the angular accelerations of the respective link, such as has already been shown under the simple conditions in the two-link solid joint system.

Hereby the way is clearly prescribed by which the equations of motion in the simplest form can be established without difficulty for all joint systems, be they as complex as it may, such as they exist in the human and animal bodies or in some machines. For the special cases in which some of the joints between the links of the system have only one or two degrees of freedom are, as have been emphasized earlier, contained in the assumed general case of joints that all have three degrees of freedom. These particular cases need therefore no investigation that is basically new, and they can best be dealt with not generally but on hand of special examples. Joints of more than three degrees of freedom of motion will, however, appear only as rare exceptions, and they can then as a rule be considered as a combination of joints of three degrees of freedom and less.

If, however, the equations of motion have been established for a joint system, we have thereby obtained the basis on which we can build up the kinetic investigation of the process of motion in the system.

SPECIAL PART (pp 177-364)

Applications to the Mechanics of the Human Body (pp 177-348)

It is necessary to know the mechanical properties of the animal (human) body. The focus is on muscle activity: muscular statics and dynamics.

13. Determination of Masses and Centers of Gravity in the Human Body. (pp 176-198)

In locomotion the human body is a 12-link joint system: 1) trunk + head are assumed to be rigid; 2) each leg is divided into three rigid links connected by knee and ankle joints: 3) each arm is divided into two rigid links connected by the elbow joint (wrist joint is not considered). The discussion is based on the earlier cadaver -section studies of frozen cadavera, by Braune and Fischer.

13a. Determination of the Masses of the Individual Body Parts. Establishment of the System of Measurements to be Used. (pp 178-180)

Here Fischer first presents a tabulation:

Values of the Ratio, μ, of the Masses of Individual Sections of the Human Body to the Total Mass of the Body.

Head	0.0706	Head + Trunk	0.4976
Trunk	0.4270	Leg + Foot	0.0706
Thigh	0.1158	Total Leg	0.1864
Leg	0.0527	Forearm + Hand	0.0312
Foot	0.0179	Total Arm	0.0648
Upper Arm	0.0336	Both Legs	0.3728
Forearm	0.0228	Both Arms	0.1296
Hand	0.0084	Head + Trunk + Arms	0.6272

(Based on man of 58.7 kgs.) (Fischer, p 279)

The units of measurement employed are those stated in the INTRODUCTION.

From the tabulation above given the following data are derived:

Table 35. Weight Numbers, G, and Mass Numbers, m, for the Individual Sections of a Living Human Being*

Body Part	Weight, G	Mass, m
Head	4.14	0.00422
Trunk	25.06	0.02554
Thigh	6.80	0.00693
Leg	3.09	0.00315
Foot	1.05	0.00107
Upper Arm	1.98	0.00202
Forearm	1.34	0.00136
Hand	0.49	0.00050
Head + Trunk	29.20	0.02976
Leg + Foot	4.14	0.00422
Total Leg	10.94	0.01115
Forearm + Hand	1.83	0.00186
Total Arm	3.81	0.00388
Both Legs	21.88	0.02230
Both Arms	7.62	0.00777
Head + Trunk + Arms	36.82	0.03753
Total Body	58.70	0.05983

* Kg is unit of weight, and mass of weight of 981.11 kgs is unit of mass.

(Fischer, p. 180)

13b. Determination of the Centers of Gravity of the Individual Body Parts. (pp 181-184)

The center of gravity of the head is in the median plane 0.7 cm back of the sella turcica in the fossa Tarini (at the angle formed by the upper edge of the pons with the lamina perforata superioris). That for the erect trunk is at the anterior upper edge of first sacral vertebra, S1, in the line of the atlanto-occipital joint and the middle of the hipline (line connecting the centers of R and L hip-joints). The head-joint to hip-line axis is the long trunk axis, passing through the shoulder-line. The center of gravity on this trunk axis divides it in the ratio of 0.61: 0.39. Correspondingly the shoulder-line: hip-line ratio is 0.47: 0.53.

In the living body the head-joint (atlanto-occipital) bisects a line from the ear-hole to "about where the upper part of the auricle is attached to the head." The hip-line is established by R and L major trochanters, and the shoulder-line by R and L acromial processes. Because of vertebral curves the location of the center of gravity of the trunk is variable.

In upper/lower arm and thigh/leg the centers of gravity will fall in the long axis between the two joints, nearer the proximal than the distal. If ϵ_1 and ϵ_2 are, respectively the ratios of the distance of the center of gravity from the proximal and distal joint centers, then we get:

Body Part	ϵ_1	ϵ_2
Upper arm	0.47	0.53
Forearm	0.42	0.58
Thigh	0.44	0.56
Leg	0.42	0.58

In the foot the back-to-front ratio is 0.43: 0.57. In the hand, with fingers slightly flexed, the center of gravity is between head of metacarpal III and palmar surface, 1 cm. from the bone.

As a rule the ratio in the proximal extremity parts is ϵ_1: ϵ_2 : : 0.44: 0.56 or 4:5. If l is the distance between two joint centers then the ratio is 4/9 l and 5/9 l; this holds also for the shoulder-line: hip-line ratio. In the lower leg the knee-joint: ankle-joint ratio is 0.61: 0.39 or 3:2. In the forearm or elbow-joint: hand-joint the ratio is 0.66: 0.34 or 2:1.

13c. Derivation of the Center of Gravity of Various Partial Systems Composed of Several Limbs and of the Total Center of Gravity of the Living Human Body. (pp 184-193)

The common center of gravity of two masses lies on the connecting line of the individual centers of gravity and divides this line in the inverse ratio of the masses. From the table earlier given the upper arm and lower arm + hand have center-of-gravity ratios of 0.0312: 0.0336 (0.481: 0.519). Similarly, trunk + head = 0.142: 0.853, lower leg + foot = 0.253: 0.747, one entire leg = 0.379: 0.621; trunk + head = 0.207: 0.793. The total center of gravity of the entire body is on a line connecting the center of gravity of the system head + trunk + both arms and the system both legs, in a ratio of 0.373: 0.627.

It will be best in the living body to relate the foregoing to a rectangular solid coordinate system. For the calculation of the coordinates of the centers of gravity of the partial systems and of the total center of gravity from the coordinates of the individual center of gravity (126) is employed: $\overline{OS_0} = \sum_1^n \mu_j . \overline{OS_j}$ The vector line is to be projected in succession on each of the three axes of the rectangular coordinate system. If the projections of the vector leading from the coordinate origin to any point, P, on the three coordinate axes, and these are the three rectangular coordinates of P, then we get the theorem:

> Each of the three coordinates of the center of gravity of a component system is equal to the algebraic sum of the respective coordinates of the individual centers of gravity reduced at the ratio μ, when by μ we understand the ratio of the mass of the limb (to which the individual center of gravity belongs) to the total mass.

Consider the body to consist of rigid sections numbered 1....n, and designate by x_h, y_h, z_h the coordinates of the center of gravity of the n^{th} limb for which the ratio of its mass to the total mass equals μ_h, then we get for the calculation of coordinates x_o, y_o, z_o to the total center of gravity:

$$(220)\quad x_o = \mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3 + \dots \mu_n x_n = \sum_1^n{}^h \mu_h x_h$$

$$y_o = \mu_1 y_1 + \mu_2 y_2 + \mu_3 y_3 + \dots \mu_n y_n = \sum_1^n{}^h \mu_h y_h$$

$$z_o = \mu_1 z_1 + \mu_2 z_2 + \mu_3 z_3 + \dots \mu_n z_n = \sum_1^n{}^h \mu_h z_h$$

(Here the values of μ are from the earlier tabulation)

From (220) it is possible to calculate the coordinates of the center of gravity of a partial system, e. g., the leg. Now μ = the ratio of the individual masses to the mass of the partial system and the n = the number of limbs of the partial system only. It is also possible to construct a joint mechanism that automatically indicates the location of the total center of gravity for any body position, as in Figure 17.

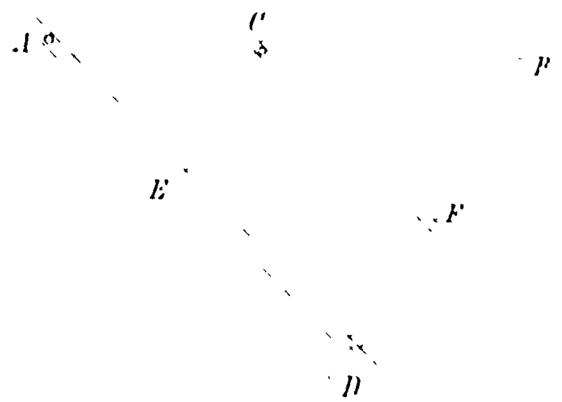


Figure 17. Joint Mechanism to Show Total Center of Gravity for Any Body Position

The four rods AD, DB, EC, and FC are connected at points D, E, F, and C by hinge joints with parallel lines (parallelogram). Here AE:ED= DF: FB and EC = DF, FC = DE. Points A, C, B will lie in a straight line so that for each joint position the proportion will hold: AC: CB = AE: ED = DF: FB. Lengths AD and DF may be arbitrarily chosen, but their sum must equal the greatest distance and their difference must equal the smallest distance that can be assumed by points A and B; if A and B coincide then AD and DB will be of equal length.

If we consider the projection of motion of the human body on any plane there will appear only the part of each joint motion that consists in a rotation of the projections of the two body parts connected by the joint about the joint axis perpendicular to the plane of the projection.

In Plate I Fischer offers a model of the projection of the human body in the median plane; it is movable in the main joints. In all, 12 sections of the body are numbered, with individual centers of gravity, S of trunk + 1: the 3 parts of the R leg = 2, 4, 6: of the L leg = 3, 5, 7;

the 2 parts of the R arm = 8, 10; of the L arm = 9, 11; the head = 12. The joints, G, have the two numbers of the parts connected by the joint. The total centers of gravity of the parts are given several indices, i. e., with the numbers of all body parts that belong to the respective center of gravity. The total center of gravity of the body = S_o. In this Plate the ratios previously determined are observed. Examples are:

$$S_5S_{5,7} : S_{5,7}S_7 = 0.253 : 0.747$$

$$S_3S_{3,5,7} : S_{3,5,7}S_{5,7} = 0.379 : 0.621$$

$$S_1S_{1,12} : S_{1,12}S_{12} = 0.142 : 0.858$$

$$S_{1,8-12}S_0 : S_oS_{2-7} = 0.373 : 0.627$$

If we want to determine not only a projection of the center of gravity but also its position in space it is best to get a solid model of the human body and attach it to a center-of-gravity mechanism according to the same principle.

14. The Main Points of the Human Body. (pp 193 - 205)

Here Fischer refers to the reduced system: here is presumed to be concentrated in the center of each joint of the limb the masses of all links of the joint system in immediate or intermediate connection. For example, we get the main point of the R upper arm when in the center of the R elbow joint we suppose to be concentrated the mass of the R forearm + Hand; in the shoulder will be concentrated the masses of all other body masses to the R upper arm: the center of gravity of this reduced R upper arm system represents the main point of the R upper arm. Everywhere the main point will be nearer to the proximal joint than to the center of gravity of the respective limb.

In Plate II the approximate position of the individual main points is shown. Needed only is the ratio, μ, of the individual masses to total mass.

14a. Determination of the Main Points of the Individual Body Parts. (pp 194 - 199)

The main point of the thigh equals the total center of gravity of the three main masses concentrated in the hip-joint center, knee-joint center, and center of gravity of the thigh: ratio numbers 0.8136, 0.0706, and 0.1158, respectively. The main point of the thigh will divide the long axis as 0.122: 0.878; if ℓ = thigh length then the thigh main point e.g. 0.122 . ℓ from the hip-joint and 0.878 . ℓ from knee-joint. In like manner lower leg is 0.040 . ℓ and 0.960 . ℓ, respectively, from centers of the knee- and foot-joints. If in the foot we designate r the distance between the center of the foot-joint and the center of gravity of the foot, then the main point of the foot is 0.018 . r and 0.982 . r, respectively, from these two points. In the upper arm the main point equals 0.047 .ℓ and 0.953 .ℓ, respectively, from the centers of shoulder-and-elbow joints. In lower arm + hand the main point equals 0.021 .ℓ from the elbow-joint center and 0.979 . from the hand-joint center.

In the trunk six mass points are concentrated (two hip-joints, two shoulder-joints, the head-joint, the center of gravity of the trunk). Each mass in hip-joint center has a ratio = 0.1864, in shoulder-joint center = 0.648, in head-joint center = 0.766, and in center of gravity of trunk = 0.4270. If we replace the two hip-joint masses by 0.3728, and the two shoulder-joint masses by 0.1296 (both mid-line) only four masses are concentrated in the trunk. Let h = distance between the middle of the hip-line and shoulder-line, and l = distance of hip-line and center of head-joint, then the main point = 0.451 . h or 0.333 .ℓ from the middle of the hip-line, or 0.549 . h from the middle of the shoulder-line and 0.667 .ℓ from the center of the head-joint. Since ℓ = 1.35 greater than h, the two distances are equal only when 0.451 is also 1.35 times greater than 0.333. This is actually the case.

If b = 1/2 the distance of the two hip-line centers from each other, and if a = 1/2 the distance of the shoulder-line centers from each other, then in the erect posture the following holds:

1) distance of the trunk main line from each hip-joint center = $\sqrt{0.203 . h^2 + b^2}$ or $\sqrt{0.111 . l^2 + b^2}$

2) distance of the trunk main line from each shoulder-point center = $\sqrt{0.301 . h^2 + a^2}$

If we assume the mass of the head to have a ratio number of 0.706 in the middle of the shoulder-line then the distance of the main point from the middle of the hip-line = 0.427 . h and from the shoulder-line = 0.573 . h. Hence, the main point of the trunk would be from the hip-joint and shoulder-joint center $\sqrt{0.182 . h^2 + b^2}$ and $\sqrt{0.328 . h^2 + a^2}$, respectively.

For the main point of the head there must be noted two different positions: 1) where rotation center is in the head-joint; and 2) where rotation center is in the shoulder-line.

In 1) if l = head-joint center to top of head, then distance of the main point from head-joint center = $0.018.l$ and from top of head = $0.982.l$. In 2) if h = length from mid-shoulder-line to top of head, then distance of the main point = $0.049.h$ and $0.951.h$, (near the shoulder-line).

The lengths between the joints defining a body part and the main point of the part are called main lengths; two each for thigh, lower leg, upper arm; one each for forearm + hand, foot, head; and five for the trunk.

14b. Use of the Main Points for the Determination of the Total Center of Gravity and of the Centers of Gravity of the Partial Systems of the Human Body (pp 199-204)

It is possible to locate the total center of gravity of the human body by means of main points and main lengths. It is necessary only to start from the main point of one of the n body parts, adding in any succession the n-1 body parts that are facing the n^{th} part. As an example from Plate II: start from H_1 of the trunk, which will give two connecting lengths $\bar{c}_2$ and $\bar{c}_3$ of a hip-joint center, $G_{1,2}$ or $G_{1,3}$ (respectively); with the main points H_2 and H_3 in the thigh, and $\bar{c}_4$ and $\bar{c}_5$ of the knee-joint center $G_{2,4}$ or $G_{3,5}$; with main points H_4 and H_5; lower leg $\bar{c}_6$ and $\bar{c}_7$ of the foot-joint center $G_{4,6}$ or $G_{5,7}$; main point H_6 or H_7, $\bar{c}_8$ or $\bar{c}_9$; and so on.

Main lengths are important because here we have measurements for the action that the motion of the various body parts and body part system exert on the position and displacement of the total center of gravity. Assume that all other body parts are fixed and then rotate the entire R leg in the hip-joint, with no motion at knee-and ankle-joints. The total center of gravity of the body will move along a circle with a radius equal to the distance of the main point of the entire R leg ($H_{2,6}$) from the center of the hip-joint. The length $\overline{G_{1,2}H_{2,6}}$ (designated $\bar{c}_{2,6}$) is equal to the geometrical sum of the three main lengths thigh, leg, and foot that within the leg lie nearest the hip-joint, or:

$$(221) \qquad \bar{c}_{2,6} = \bar{c}_2 + \bar{c}_4 + \bar{c}_6$$

14c. The Action of Gravity on the Individual Sections of the Human Body. (pp 204-205)

If the foot is pressed on the floor the reaction will produce pressure force at the joint of each body part facing the pressure spot. The pressure force will be equal and opposite to the weight of the entire body. Hence, it will constitute a couple with the force of weight applying in the main point of the respective body part. The axial moment of this couple represents the turning motion exerted on the body part by gravity.

If only a partial system of the body is presumed to be mobile and the remaining body is fixed with regard to the action of gravity, this system may be considered as an independent joint system.

15. Determination of the Moments of Inertia of the Various Body Parts. (pp 205-214)

This is based on the cadaver studies of Braune and Fischer. Steel axes were inserted into each severed body part. For each part there was determined the duration of oscillation for the two axes parallel to each other, the plane of which contained the center of gravity, so that the axes were on different sides of the center of gravity, as shown in Figure 18.

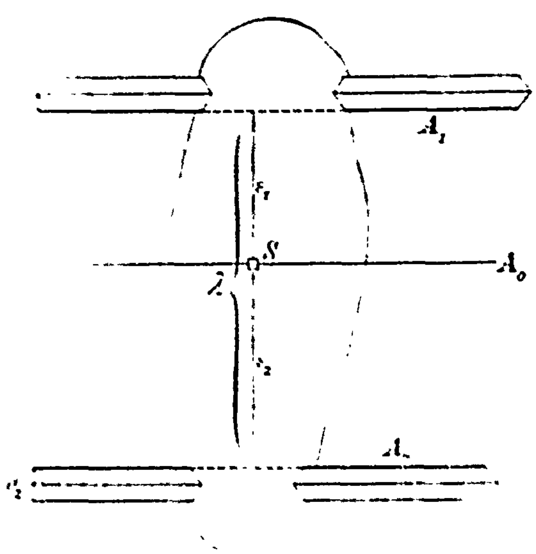


Figure 18. Duration of Oscillation for Two Axes Parallel to Each Other, With Axes on Different Sides of the Center of Gravity.

Here the steel rods = A_1 and A_2; S = center of gravity of the body part alone; ε_1 and ε_2 = distance of this center of gravity from the edges A_1 and A_2 of the steel rods nearest to it (these rods were axes of rotation). S lies exactly in the plane determined by the two parallel axes A_1, A_2. Then if the distance of these axes is λ we get:

$$\varepsilon_1 + \varepsilon_2 = \lambda \tag{222}$$

Further designations are as follows:

m = mass of limb (without rods)

x_o = radius of inertia of the limb alone relative to axis A_o that goes through S and is parallel to A_1, A_2

μ_1 = mass of steel rod with axis A_1

δ_1 = 1/2 the diagonal of its cross-section

μ_2 = mass of steel rod with axis A_2

δ_2 = 1/2 the diagonal of its cross-section

τ_1 = duration of oscillation of the mass system (limb + rods) at axis of rotation A_1.

τ_2 = the same for A_2

g = the acceleration of gravity

First in relation to axis A_1 there will be the moment of inertia in the limb alone:

$m(x_0^2 + \epsilon_1^2)$; and for the upper steel rod $\mu_1\left(\frac{\delta_1^2}{3} + \delta_1^2\right) = \frac{4}{3}\mu_1 \delta_1^2$,

and for the lower rod $\mu_2\left(\frac{\delta_2^2}{3} + (\lambda + \delta_2)^2\right) = \frac{1}{3}\mu_2\left(\delta_2^2 + 3(\lambda + \delta_2^2)^2\right)$,

After deriving the same for axis A_2, there is obtained according to the laws of the pendulum:

$$\tau_1 = \pi\sqrt{\frac{m(x_0^2 + \epsilon_1^2) + \frac{4}{3}\mu_1\delta_1^2 + \frac{1}{3}\mu_2\left\{\delta_2^2 + 3(\lambda+\delta_2)^2\right\},}{g\left\{m\,\epsilon_1 + \mu_1\delta_1 + \mu_2(\lambda+\delta_2)\right\}}} \tag{223}$$

$$\tau_2 = \pi\sqrt{\frac{m(x_0^2 + \epsilon_2^2) + \frac{1}{3}\mu_1\left\{\delta_1^2 + 3(\lambda+\delta_1)^2\right\} + \frac{4}{3}\mu_2\delta_2^2.}{g\left\{m\,\epsilon_2 + \mu_1(\lambda+\delta_1) + \mu_2\delta_2\right\}}}$$

From (222) and (223) the following is derived:

Table 36. Weights, Masses, Locations of the Centers of Gravity and Moments of Inertia of Individual Parts of the Human Body (Fischer, p 209)

Body Part		Weight in kg	Mass No.	Lgth. (1) in cm	Distance of Center of Gravity: ϵ_1 from above in cm	Distance of Center of Gravity: ϵ_2 from below in cm	Radii of inertia x and moments of inertia mx^2 for axes through center of gravity — Axis perp. to long axis of limb: x in cm	Axis perp. to long axis of limb: mx^2	Axis parallel to long axis of limb: x in cm	Axis parallel to long axis of limb: mx^2
Trunk + head		23,790	0,02425	72,75	42,61	30,14	21,08	10,7750		
Trunk		19,910	0,02029	56,75 1)	34,50	22,25	16,73	5,6819		
Head		3,880	0,00396	16,0 2)	11,93	4,07	6,81	0,1834		
Total leg	r	7,840	0,00799	78,9	32,74	46,16	25,10	5,0344		
	l	7,640	0,00779	79,2	32,42	46,78	25,08	4,8981		
Thigh	r	4,860	0,00495	35,9	15,72	20,18	11,01	0,6004	4,55	0,1025
	l	4,810	0,00490	36,65	15,90	20,75	11,43	0,6405	4,56	0,1019
Leg + foot	r	2,980	0,00304	43,9	(24,77	19,13	14,41	0,6305) 6)		
	l	2,800	0,00285	43,1	21,94	21,16	15,10	0,6507		
Leg	r	2,070	0,00211	37,9	16,13	21,77	9,16	0,1770	3,12	0,0205
	l	1,890	0,00193	37,1	16,30	20,80	9,66	0,1798	3,05	0,0179
Foot	r	0,910	0,00093	Height 6,0 Lgth. 20,0 3)	6,38	13,62	5,91	0,0324	6,21	0,0358
	l	0,910	0,00093	Height 6,0 Lgth. 20,0 3)	6,95	13,05	5,97	0,0331	6,24	0,0361
Total arm	r	2,360	0,00245	59,0	25,18	33,82	18,38	0,8127		
	l	2,470	0.00252	58,5	27,17	31,33	17,60	0,7798		
Upper arm	r	1,243	0,00127	25,5	11,37	14,13	7,95	0,0801	2,79	0,0099
	l	1,252	0,00128	27,1	12,31	14,79	7,79	0,0774	2,74	0,0096
Forearm + Hand	r	1,117	0,00114	36,0 4)	15,99	20,01	10,43	0,1238	2,75	0,0086
	l	1,205	0,00123	35,5 5)	17,02	18,48	11,24	0,1551	2,71	0,0090
Total body		44,057	0,04491	150,5						

(Fischer, p 209)

Notes:
1) Trunk length here is the distance between the middle of the Atlanto-occipital joint and the line connecting the mid-points of the two hip joints.
2) The length of the head is the distance from the crown to the middle of the Atlanto-occipital joint.
3) The length of the foot is the distance from the tip of the foot (big toe) to the axis of the upper ankle-joint.
4) Of this distance, 24 cm was the length of the forearm and 12 cm the distance from the first interphalangeal joint to the wrist joint.
5) Of this distance, 25 cm was the length of the forearm and 10.5 cm was the distance from the first interphalangeal joint to the wrist joint.
6) Comma (,) is decimal (.).

From (222) and (223) the x_o/l ratios follow:

Head + trunk		0. 29	Leg	R	0. 24
Trunk		0. 29		L	0. 26
Head		0. 43	Foot	R	0. 30
Total leg	R	0. 32		L	0. 30
	L	0. 32	Total arm	R	0. 31
Thigh	R	0. 31		L	0. 30
	L	0. 31	Upper arm	R	0. 31
Leg + Foot	R	0. 33		L	0. 29
	L	0. 35	Forearm +	R	0. 29
			Hand	L	0. 32

(Fischer, p. 210)

For all axes through the center of gravity perpendicular to the long axes of the greater extremity sections, and for all axes through the center of gravity of the head and trunk parallel to the hip axes, the radii of inertia to length of limb is 3:10.

In the <u>body</u> we must measure length, l, and the average thickness, d, of the limbs. Let x_o' = radius of inertia for axis through the center of gravity perpendicular to the long axis, and x_o'' = radius of inertia for the long axis, then:

(224) $x_o' = 0,3\,l$ and $x_o'' = 0,35\,d$

Using the μ values already tabulated we get values of corresponding moments of inertia via:

(225) $mx_o'^2 = 0,09\mu m_o l^2$ and $mx_o''^2 = 0,12\mu m_o d^2$

From this is derived:

(226) $x_o = \sqrt{x_o'^2 \sin^2\gamma + x_o''^2 \cos^2\gamma}$.

As examples here are the calculated radii of inertia for:

Part	x_o'	x_o''
Thigh	11. 22 cm	4. 56 cm
Leg	9. 41 "	3. 09 "
Upper Arm	7. 87 "	2. 77 "
Forearm	10. 84 "	2. 73 "

With the aid of (226) we get (in cm):

Table 37. The Values of the Radii of Inertia, x_o, for Axes Through the Center of Gravity *

Angle γ	Thigh	Leg	Upper Arm	Forearm + Hand
0^o	4, 56	3, 09	2, 77	2, 73
5^o	4, 65	3, 19	2, 84	2, 88
10^o	4, 89	3, 45	3, 05	3, 28
15^o	5, 28	3, 85	3, 36	3, 85
20^o	5, 75	4, 34	3, 74	4, 51
25^o	6, 29	4, 86	4, 17	5, 21
30^o	6, 86	5, 41	4, 61	5, 91
35^o	7, 44	5, 96	5, 05	6, 61
40^o	8, 01	6, 50	5, 49	7, 28
45^o	8, 56	7, 00	5, 90	7, 90
50^o	9, 08	7, 48	6, 29	8, 49
55^o	9, 56	7, 91	6, 64	9, 02
60^o	9, 98	8, 29	6, 95	9, 49
65^o	10, 35	8, 63	7, 23	9, 89
70^o	10, 66	8, 91	7, 46	10, 23
75^o	10, 90	9, 12	7, 64	10, 49
80^o	11, 08	9, 28	7, 77	10, 69
85^o	11, 18	9, 38	7, 84	10, 80
90^o	11, 22	9, 41	7, 87	10, 84

* With the long axis of the limb angle γ is formed (in cm)

(Fischer, p. 212)

In Plate III Fischer illustrates the dependence of the radius of inertia, x_o , upon the angle γ in the upper arm.

If the relation between the radii of inertia x_o' and x_o'' be considered, and the length, l, and thickness, d, also be considered, then (222) and (226) will lead to the calculation of the remaining radii of inertia:

$$x_o = \sqrt{0,09\ l^2 \sin^2 \gamma + 0,12\ d^2 \cos^2 \gamma} \tag{227}$$

And the moment of inertia becomes:

$$mx_o^2 = \mu m_o \sqrt{0,09\ l^2 \sin^2 \gamma + 0,12\ d^2 \cos^2 \gamma}$$

16. The Turning Moments of the Muscles (p. 214 - 239)

16a. Definition of the Turning Moments. Static Measurement of a Muscle. (pp 214-221)

In contraction or during passive tension a muscle acts on two points (origin and insertion) and with forces on various body parts. If there is a bony protrusion between origin and insertion the muscle acts as though it were two muscles, though both "parts" are stretched at the same time; hence, they behave as one. Each of the two equal and opposite forces that arise in contraction or passive tension exerts an external force for the body part to which it applies: these are forces of origin and forces of insertion. Both represent a couple of internal forces for the entire joint system; two of such forces will act with a couple of forces upon each limb within the immediate sphere of action. The primary effect of a muscle is its tendency to set in rotary motion a number of body parts.

A one-joint muscle will have first the forces of insertion, then those of origin; both forces will be forces at respective joint centers. The moments of the two couples of force are equal in magnitude. A two-joint muscle is similar to a one-joint muscle in principle. One force of the couple is the muscle force at the point of insertion, the other at a second point of insertion on the joint facing the application point of this force. As a result a muscle acts on the middle of the body part with a couple of forces caused by: 1) force of insertion and force of origin exert pressure on the joint of the body part nearest them; 2) this pressure is equal in magnitude and direction to these forces. The two turning moments, especially for a one-joint muscle, are equal and opposite.

If a muscle deviates from a straight-line pull then for the purpose of the three turning moments we must assume the muscle to be a combination of two one-joint muscles. In a multi-joint muscle, if only one joint remains free while the others are fixed, then the muscle will act as a one-joint muscle.

The turning moments of individual body parts can be determined by the moments of a one-joint muscle. Let the turning moment = 1 . . . . . n; let turning moment D_1 be determined by fixing all joints except the one connecting first and second body parts. If the joint between the second and third body parts is left free then the turning moment will be obtained that is the resultant of $D_1 + D_2$, and D_2 can be derived from D_1. If the joint between the third and fourth body parts is left free then we get $D_1 + D_2 + D_3$; from this one can go on to $D_1 \ldots . D_n$. To the turning moments of the muscle must be added or considered: 1) masses of the body parts; 2) mass distribution within each body part: 3) especially the particular kind of joint connection.

16b. The Derivation of the Turning Moment as Illustrated by Special Examples. (pp 221 - 239)

Illustrated will be the M. iliacus, M. biceps femoris and M. semimembranosus, which are in a plane perpendicular to the middle axis of the knee-joint; hence, the plane of the couples of force will also be perpendicular to the middle axis of the knee-joint. The muscles will tend to rotate body parts about axes parallel to this knee-joint axis.

Here the body is considered as a 3-link system: link 1 = entire body less one leg; link 2 = thigh; link 3 = leg + foot. At the same time the joint system may be considered as a plane system.

16bα. The Method of Derivation. (pp 222-228)

In the hip or knee, alone, the dependence of the turning moments upon the position of the joints can be shown by moment curves, and in knee + hip by moment surfaces. In plotting these it shall be assumed that the tension of the muscle as related to the unit of physiological cross-section is the same in all positions of the 2-link system. There the turning moments will be proportional to the product of the arm of the couple of force to which belongs the number of surface units of the physiological cross-section.

If we take as a unit of turning moment 1 cm -kg , and if this be represented by 1 mm , then the number of mm for each ordinate will give the number of cm -kgs of the turning moment for a muscle with a tension of 1 kg as related to the unit of the physiological cross-section. It is necessary to measure directly only one turning moment for a 1-joint muscle and two turning moments for a 2-joint muscle.

In the anatomical model it is necessary to measure only the arms of the various couples of force and the cross-section of the muscles. The determination of insertion points is arbitrary, since insertion involves a large area. In practice Fischer connects insertion (attachment) points by a thread; pelvis, femur, tibia + fibula are projected on heavy carboard, parallel to the median plane; these carboard units are then cut out, oriented, and rotated as are the real parts and their joints.

16bβ. The Arms of the Couples of Forces and the Values of the Turning Moments (pp 228-239)

The M.iliacus will serve as an illustration. Here are couples of force on trunk and thigh. The arms of both couples of force are equal in any position of the hip-joint. The sense of rotation of the one couple of force is opposite to the sense of rotation in the other. The arm of a couple of force and the pertaining turning moment are positive when the respective couple of force rotates the body part counter-clockwise as seen from the R side, and negative if clockwise. The M. iliacus will act with a negative turning moment on the trunk, positive on the thigh.

On the model there is derived for the arm of the two couples of force a length of 3.0 cm (both extension and flexion). The values of the resulting arms of the couples of force with which the M. iliacus acts on the pelvis and thigh at a total tension of 1 kg may be calculated. As in Plate II the joint positions in the hip have been designated by the angle $\psi_{1,2}$ by which the long axis of the thigh deviates from its direction in the normal position; the extreme backward extension of the thigh goes to -10^o; flexion is positive.

Flexion of the thigh on the pelvis is called "the rectangular bent position", and hence $\psi_{1,2} = 100^o$. According to Plate II a turning moment on the trunk = D_{m_1} and the arm of its couple of force = h_1; similarly for the thigh we have D_{m_2} and h_2; and for the leg D_{m_4} and h_4. The M. iliacus concerns h_1 and h_2, as follows:

Arms of the Couples of Forces of the M. Iliacus (in cms)

Hip-joint angle $\psi_{1,2}$	-10^o	0^o	$+10^o$	$+20^o$	$+30^o$	$+40^o$	$+50^o$	$+60^o$	$+70^o$	$+80^o$	$+90^o$	$+100^o$
h_1	-3,0	-3,0	-3,0	-3,0	-3,0	-3,0	-3,0	-3,0	-3,0	-3,6	-4,4	-4,9
h_2	+3,0	+3,0	+3,0	+3,0	+3,0	+3,0	+3,0	+3,0	+3,0	+3,6	+4,4	+4,9

(Fischer, p 230)

The M. iliacus has a physiological cross-section of 8 cm^2. The tension of this muscle per cm^2 ("the specific tension of the muscle") = 1 cm. From this there emerges as follows:

Values of the Turning Moments, D_{m_1} and D_{m_2} with which the M. Iliacus Acts on the Trunk and Thigh in the Various Positions of the Leg, with 1 kg of Specific tension, Expressed in cm-kgs.

Hip-joint angle $\psi_{1,2}$	-10°	0°	$+10^{\circ}$	$+20^{\circ}$	$+30^{\circ}$	$+40^{\circ}$	$+50^{\circ}$	$+60^{\circ}$	$+70^{\circ}$	$+80^{\circ}$	$+90^{\circ}$	$+100^{\circ}$
D_{m1}	-24	-24	-24	-24	-24	-24	-24	-24	-24	-29	-35	-39
D_{m2}	+24	+24	+24	+24	+24	+24	+24	+24	+24	+29	+35	+39

(Fischer, p 230)

In like manner data for the M. biceps femoris focus on D_{m2} and D_{m4} and $\psi_{2,4}$. The physiological cross-section is 6 cm^2. Data for the M. semimembranosus focus on D_{m1} and D_{m4} and $\psi_{1,2}$ and $\psi_{2,4}$. The physiological cross-section is 15 cm^2. In Plate IV the dependence of this muscle upon joint postures is seen by the corresponding moment surfaces: 1) those of D_{m1} are farthest to the left; 2) those of D_{m4} are farthest to the right; 3) those of D_{m2} are in the middle.

17. The Problem of Equilibrium (pp 239-268)

Only in lying or sitting are the muscles relatively inactive. The erect posture demands muscle action. To get absolute equilibrium "it will be necessary and sufficient for each individual body part that the geometric sum of all active turning moments shall cease." This relates to the principle of virtual displacement if regard is taken to the elementary work performed by the external and internal forces applying to a joint system during the various posture displacements of the system.

17a. General Conditions of Equilibrium for the Two-Link Plane Joint System (pp 240-244)

A 2-link plane joint system is used, where link 1 rotates around a fixed axis in space, and the two links are connected via a hinge joint with an axis parallel to the former (see Figure 19, right arm).

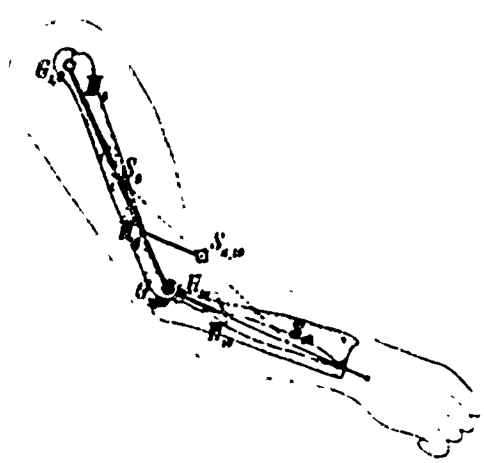


Figure 19. Two-Link System Where Link 1 Rotates Around a Fixed Point.

Here must be noted:

1) the plane of movement is perpendicular to joint axes $G_{1,8}$ and $G_{8,10}$ (Plate II), which are R. shoulder-joint and R. elbow-joint resp.
2) S_8 = center of gravity of upper arm
3) S_{10} = " " " " rigid system, forearm + hand
4) H_8 and H_{10} = main points of upper arm and of forearm + hand, resp., which points are near to $G_{1,8}$ and $G_{9,10}$
5) M_o = mass of entire body
6) M_8 and M_{10} = masses of two arm sections
7) r_8 and c_8 = distance of S_8 and H_8 from shoulder-joint center, G_1
8) r_{10} and c_{10} = distances of S_{10} and H_{10} from elbow-joint center $G_{8,10}$
9) l_8 = distance between elbow-joint and shoulder-joint centers

The first relations here are:

(227) $m_o c_8 = m_8 r_8 + mo_{10} l_8$ and $m_o c_{10} = m_{10} r_{10}$

If the main lengths of c_8 and c_{10} are increased in the ratio of $m_o : (m_8 + m_{10})$ we get to points H'_8 and H'_{10} on the long axis of the two links that are now main points on upper arm and on forearm + hand; these are partial main points and they validate partial main lengths c'_8 and c'_{10}, so that:

(228) $(m_8 + m_{10})\, c'_8 = m_8 r_8 + m_{10} l_8$ and

$$(m_8 + m_{10})\, c'_{10} = m_{10} r_{10}$$

If the fixed axes of the shoulder- and elbow-joints are horizontal then the plane of motion of the 2-link system will be vertical. From this it is easy to calculate the turnin g moments; as was noted in 14c gravity will create on the upper arm a couple of forces, one at H'_8, the other at $G_{1,8}$; both are equal to the weight of $G_8 + G_{10}$ of the entire arm.

The arms of the two couples of force are shown in Figure 20.

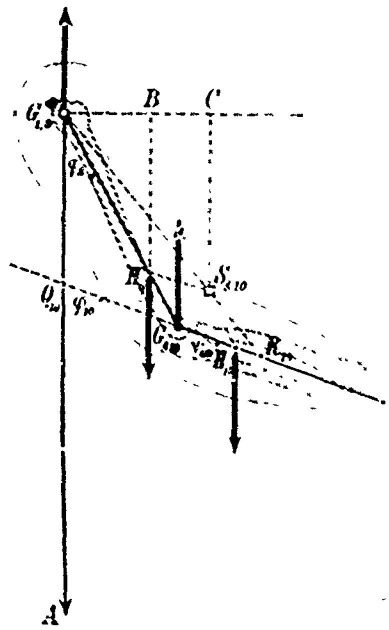


Figure 20. Arms of Two Couples of Forces

(Here c'_8 and c'_{10} are projected in the horizontal; φ_8 and φ_{10} are angles formed by the long axes of the two sections of the arm; the vertical directed downward has length $c'_8 \sin \varphi_8$; and the other vertical has length $c'_{10} \sin \varphi_{10}$; D_{s_8} and $D_{s_{10}}$ are turning moments with which gravity acts on upper arm and forearm + hand; with positive angles φ the turning movement gives a clockwise or negative direction.)

For the turning moments we derive:

$$(229) \quad D_{s_8} = -(G_8 + G_{10})\, c'_8 \sin \varphi_8$$

$$D_{s_{10}} = -(G_8 + G_{10})\, c'_{10} \sin \varphi_{10}$$

Finally, we can write:

$$(232) \quad D_{s_8} + -(G_8 + G_{10})\, c'_8 \sin \psi_{1,8}$$

$$D_{s_{10}} = -(G_8 + G_{10})\, c' \sin(\psi_{1,8} + \psi_{8,10})$$

$$D_{s_{8,10}} = (G_8 + G_{10})\, [c'_8 \sin \psi_{1,8} + c'_{10} \sin(\psi_{1,8} + \psi_{8,10})] .$$

If $D_{m_8}+D_{m_{10}}$ are the turning moments on upper arm and forearm + hand, resp., then conditions of equilibrium are:

(235) $D_{m8} + D_{s8} = 0$ and $D_{m10} + D_{s10} = 0$

For the entire arm it is:

(236) $D_{m8,10} + D_{s8,10} = 0.$

17b. Equilibrium Between Gravity and Muscles that Pass Over Only the Intermediate Joint (p 244 - 258)

17bα. Equilibrium of the Arm (pp 244-251)

In the shoulder angle, $\psi_{1,8}$, there are two sets of values, one positive (above 90°), the other negative (below 90°):

$\psi_{8,10}$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°
$\psi_{1,8}$	0°	-3°	-6°	-9°	-12°	$-14\frac{1}{2}$	-17°	-19°	-21°	-23°	-24°	-25°	-25°	-24°

or

$\psi_{8,10}$	0°	10°	20°	30°	40°	50°	60°
$\psi_{1,8}$	+180°	+177°	+174°	+171°	+168°	$+165\frac{1}{2}$°	+163°

$\psi_{8,10}$	70°	80°	90°	100°	110°	12°	130°
$\psi_{1,8}$	+161°	+159°	+157°	+156°	+155°	+155°	+156°

(Fischer, p 245)

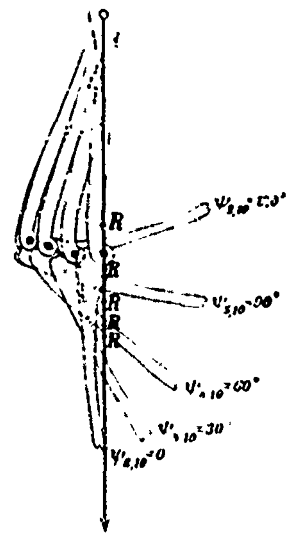


Fig. 21 Flexion Positions

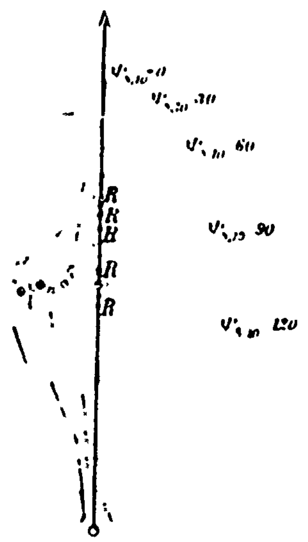


Fig. 22 Extension Positions

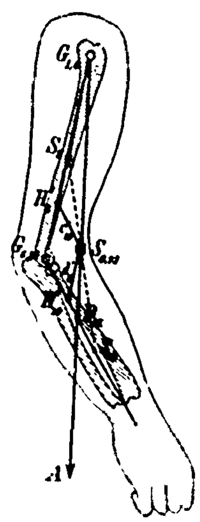


Fig. 23 Shoulder-Joint ($G_{1,8}$) Related to $S_{8,10}$ of Arm.

In Figure 21 there is a series of flexion positions shown: here the center of gravity lies vertically below the shoulder joint.

In Figure 22 there is a series of extension positions shown: here the center of gravity lies vertically above the shoulder joint.

In Figure 23 it is assumed that the shoulder-joint center $G_{1,8}$ is connected with the total center of gravity $S_{8,10}$ of the arm, and this connecting line extends to intersection point R_{10} with the long axis. For any flexed posture of the elbow-joint we always get the same point R_{10}. In Figure 23 the triangle $R_{10}\ G_{8,10}\ G_{1,8}$ is similar to triangle $S_{8,10}\ H'_8\ G_{1,8}$ for all elbow flexion. Hence, the distance ρ_{10} of point R_{10} from center $G_{8.10}$ of the elbow joint has the constant value of:

$$(239) \quad \rho_{10} = \frac{c'_{10}}{c'_8} l_8 .$$

The main point H'_8 of the arm divides l_8 of the upper arm in the ratio of 5:2.

In a single muscle let K equal the total tension and k equal the distance of its direction of pull from the axis of the elbow-joint. Then the turning moment, $D_{m_{10}}$, for the lower arm will be:

$$(240) \quad D_{m_{10}} = \pm Kk \text{ (the + or - depends upon whether the muscle is a flexor or an extensor)}$$

In the M. brachialis, as an example, we get total tension, as expressed in (240) (235), (232) as follows:

$$(241) \quad K = \frac{(G_8 + G_{10})\, c'_{10} \sin(\psi_{1,8} + \psi_{8,10})}{k}$$

$$= \frac{33.736 \sin(\psi_{1,8} + \psi_{8,10})}{k} \text{ kgs}$$

From (241) we obtain:

$\psi_{8,10}$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°
K in kg	0	3,33	5,91	7,90	9,32	10,12	10,00	9,86	9,64	9,27	8,89	9,36	10,06	10,33

(Fischer, p 249)

To get the specific tension of the M. brachialis divide K by the cm^2 of the physiological cross-section which is 12 cm^2:

Bending angles in the elbow joint	0^o	10^o	20^o	30^o	40^o	50^o	60^o
Specific tension in kgs per cm^2	0	0, 28	0, 49	0, 66	0, 78	0, 84	0, 83

Bending angles in the elbow joint	70^o	80^o	90^o	100^o	110^o	120^o	130^o
Specific tension in kgs per cm^2	0, 82	0, 80	0, 77	0, 74	0, 78	0, 84	0, 86

(Fischer, p 249)

The maximum tension of a muscle per cm^2 which can keep the unloaded arm in its position is 0. 8 kgs. For a 1-joint muscle of the elbow-joint the turning moments D_{m_8} and $D_{m_{10}}$ are equal and opposite. Hence, from (240) we get:

(242) $D_{m_8} = \pm Kk$ (flexion = $-$, extension = +)

There emerges the <u>theorem</u>:

In the case of equilibrium between a 1-joint muscle of the elbow joint and the gravity of the arm, the total tension of the muscle to the total weight of the arm will be at the same proportion as the distance of the vertical line passing through the center of the shoulder joint from the main point of the upper arm to the distance of the resulting direction of pull of the muscle from the center of the elbow joint.

If we consider several muscles, 1.... n, and i_h, q_h, k_h as the specific tension, physiological cross-section, and the arm of the couples of force of the n^{th} muscle, resp., then from (232) we derive:

(244) $\pm i_1 q_1 k_1 \pm i_2 q_2 k_2 \pm \dots i_n q_n k_n = (G_8 + G_{10}) c'_{10} \sin (\psi_{1,8} + \psi_{8,10})$

17bβ. The Equilibrium of the Leg (pp 251-254)

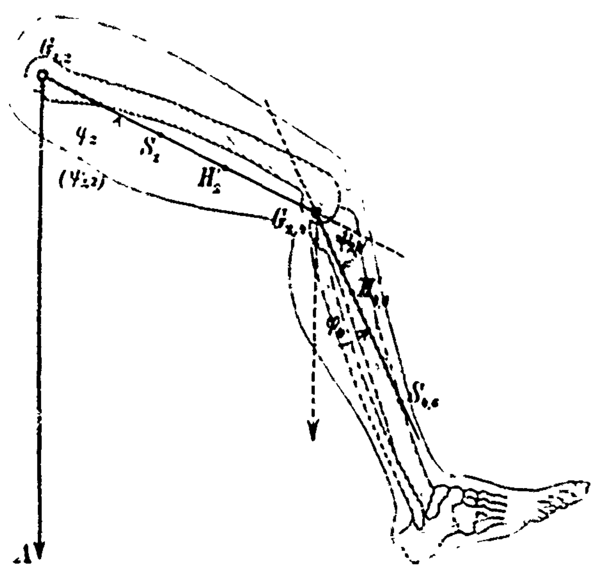


Figure 24. Long Axes of Thigh Between Hip-Joint Center ($G_{1,2}$) and Knee-Joint Center ($G_{2,4}$)

The leg is articulated in the knee-joint with pelvis fixed. As in the arm it is assumed that lower leg + foot are combined as a single rigid mass, that the knee-joint has a fixed axis, and that in the hip there is only rotation around an axis parallel to the knee axis.

In Figure 24 the long axis of the thigh is between hip-joint center $G_{1,2}$ and knee-joint center $G_{2,4}$ (thigh center of gravity S_2 is in here). Correspondingly, the long axis of the lower leg is from $G_{2,4}$ to the common center of gravity of the foot $S_{4,6}$. The long axis of thigh contains the main point H'_2, that of the lower leg $H'_{4,6}$; $\varphi_{2,4}$ is the angle formed by the long axis of the thigh and a vertical line through hip-joint center $G_{1,2}$ (+ when the thigh is rotated forward, - when it is rotated backward).

Since the pelvis is fixed its angle is the joint angle $\varphi_{1,2}$ of the hip-joint. The knee-bending angle $\varphi_{2,4}$ that the long axis of leg + foot forms with the extension of the long axis of the thigh, has only negative values (compared to elbow, which has only positive values). Weight of thigh = G_2, of lower leg = G_4, of foot = G_6, with corresponding masses m_2, m_4, m_6. Length of leg = l_2. The distance of the center of gravity, S_2, from the hip-joint center = r_2, and that of leg + foot from knee-joint center = $r_{4,6}$.

For the calculation of the two main lengths c'_2 and $c'_{4,6}$ that correspond to leg attached to trunk we get:

(245) $$(m_2 + m_4 + m_6)\, c'_2 = m_2 r_2 + (m_4 + m_6)\, l_2$$

$$(m_2 + m_4 + m_6)\, c'_{4,6} = (m_4 + m_6)\, r_{4,6}$$

17bγ. <u>Equilibrium When the Mass of the First Link is to Dissappear: Standing on the Toes.</u> (pp 254-256)

Here, in a 2-link system, it is necessary only to have the weight, G, of the entire system, plus distance l_1 of the two joint centers from each other, plus distance r_2 of the center of gravity of link 2 from the center of the intermediate joint. The main length, c_1, of link 1 will equal l_1 and the main length, c_2, of link 2 will equal r_2. If we call the two joint angles ψ_1 and ψ_2 there will be the relation:

(249) $$\sin(\psi_1 + \psi_2) = \frac{r_2}{l_1} \sin \psi_1$$

The tension K of the muscle may be derived from:

(250) $$K = \pm \frac{G r_2 \sin(\psi_1 + \psi_2)}{k}$$ (where k = the arm of the two couples of force)

When the entire body is raised on the toes a 2-link system ensues: link 1 = R and L feet as a single rigid body; link 2 = rest of the body as a single rigid unit; link 2 rotates on link 1 and R and L ankle joints. The common axis of the ankle joint runs parallel to the axis through the metatarsal heads; both axes are perpendicular to the median plane of the body. There emerges the <u>theorem</u>:

> The total tension of the calf muscles of both legs in the case of equilibrium in raising on the toes will be in the same proportion to the total weight of the human body as the distance h of the foot joint center from the vertical passing through the head of the first metatarsal bone to the distance k of the same foot joint center from the direction of the resulting muscle pull.

Since h and k are at the ratio 3:1 the total tension of the calf muscle = 3 x body weight. If the body weight is 60 kgs , then total tension = 180 kgs (one leg = 90 kgs). The average value of the cross-section of the calf muscles of one leg = 40 cm^2; then, the tension of a bundle of muscle fibers of the cross-section, i. e., the specific tension, will be about 2-1/4 kgs.

17bδ. Equilibrium of the Loaded Arm (pp 257-258)

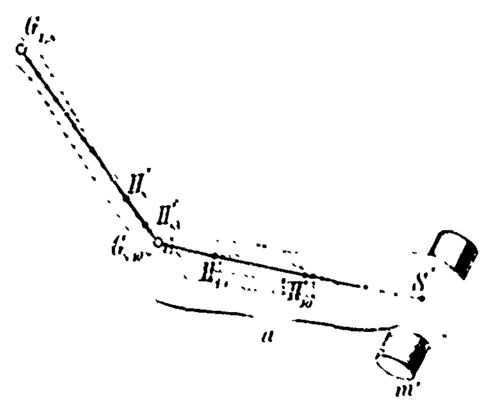


Figure 25. One-Joint Muscle in Equilibrium to Gravity, with Weight, G', Held in Hand.

Here, as seen in Figure 25, a one-joint muscle shall maintain equilibrium to gravity when a weight, G', is held in the hand, against which the weight of the upper arm cannot be neglected. The weight in the hand is added to the rigid system forearm + hand. Here it is necessary to determine a new main mass (m'), a new setting point $[\rho_{10}]$ and two new main lengths, and we may proceed as in the unloaded arm. Instead of the total weight of the arm $(G_8 + G_{10})$ we now must consider $(G_8 + G_{10} + G')$.

17c. Equilibrium Between Gravity and Two-Joint Muscles (p 258-268)

17cα. General Methods of Investigation (pp 258-264)

Conditions of equilibrium as given in (235) and (236) for the unloaded arm are valid for any muscle that acts with turning moments on the two sections of the arm.

Since the muscle now goes over both shoulder- and elbow-joints the two turning moments D_{m_8} and $D_{m_{10}}$ are no longer equal and opposite. Instead, (256) $D_{m_{8,10}} = D_{m_8} + D_{m_{10}}$ and we proceed to (258) $D_{m_8} : D_{m_{10}} : D_{m_{8,10}} = \overline{G_{8,10}Q_{10}} : \overline{R_{10}G_{8,10}} : \overline{R_{10}Q_{10}}$.

The following theorem is derived:

Each muscle can keep equilibrium to gravity only in the posture of the arm in which on the longitudinal axis of the lower arm the distance of the point of equilibrium from the center of the elbow joint, the distance of the elbow joint center from the setting point, and the distance of the point of equilibrium from the setting point are in the proportion of the three turning moments D_{m_8}, $D_{m_{10}}$ and $D_{m_{8,10}}$ with which the muscles act on the upper arm, the lower arm, and the entire arm.

In a "general case" where the muscle passes only over the shoulder-joint and the turning moment $D_{m_{10}}$ = zero, the determination of posture of equilibrium becomes:

$$(259)\quad \frac{\overline{R_{10}Q_{10}}}{\overline{R_{10}G_{8,10}}} = \frac{c'_8 \sin \psi_{1,8} + c'_{10} \sin(\psi_{1,8} + \psi_{8,10})}{c'_{10} \sin(\psi_{1,8} + \psi_{8,10})}$$

17c β. Special Example: Two-Joint Muscle (Long Head of M. Biceps Brachii) (pp 265-268)

Here the muscle is divided into two parts, the first over the shoulder-joint, the second over the elbow-joint. Both have the same tension. Turning moment $D_{m_{8,10}}$ deals with the first part, $D_{m_{10}}$ with the second. The arm of the couples of force is exerted on the entire arm and the moment $D_{m_{8,10}}$ is constant, and is equal to the radius ρ of the head of the humerus, while that on the lower arm and it moment, $D_{m_{10}}$, will depend on the angle $\psi_{8,10}$ of the elbow-joint.

Here the total tension of the muscle also equals K, and hence:

(260) $D_{m_{8,10}} = K\rho$, $D_{m_{10}} = Kk$; and it follows that

$$(261)\quad \frac{D_{m_{8,10}}}{D_{m_{10}}} = \frac{\rho}{k} \text{ or resp. } \frac{D_{m_{10}}}{D_{m_{8,10}}} = \frac{k}{\rho}$$

Values for k are given as follows:

$\psi_{8,10}$	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°
k in cm	1,15	1,24	1,37	1,52	1,68	1,90	2,14	2,40	2,69	2,97	3,24	3,54	3,74	3,93

$\psi_{8,10}$	70°	75°	80°	85°	90°	95°	100°	105°	110°	115°	120°	125°	130°
k in cm	4,08	4,23	4,35	4,44	4,54	4,58	4,55	4,45	4,27	4,08	3,92	3,75	3,58

(Fischer, p 266)

In general here the determination of the tensions of the different postures in this special case will be in accordance with (232), (235), (236), with (234) giving values in cmkg.

18. The Joint Movement Setting in at the Beginning of the Contraction of a Muscle (pp 269-321)

The focus here is on the movement, rather than on equilibrium, that occurs in muscle action. Equations of motion apply here, as for example the $n + 2$ equations (163) where there is an n-link plane joint system with free mobility, or in the n-equation (170) where a point on the .g. axis of a link is fixed, so that this link rotates only around an axis that in this point is perpendicular to the plane of motion. In the case of movement of an extremity with reference to a fixed trunk (171) is more directly applicable.

If we assume the joint system to be at rest we want to get the initial moment when the muscle contracts. Here Q_{x_0} and Q_{y_0} of (163) will have a value of zero. Hence, only n equations of motion remain. The angular velocities φ'_j in the equations of motion n will retain a value of zero. The rotations of individual links are in direct proportion to angular accelerations φ''_j.

18a. The Investigation of the Initial Movements at the Two-Link Plane Joint System: Kinetic Measurements of a Muscle (pp 271-276)

Assume that the first of two links can rotate only around an axis passing through a fixed point, 0_1, on the long axis of link 1 that runs parallel to the axis of the intermediate joint, as in Figure 26. As a result there will be two equations of motion (72):

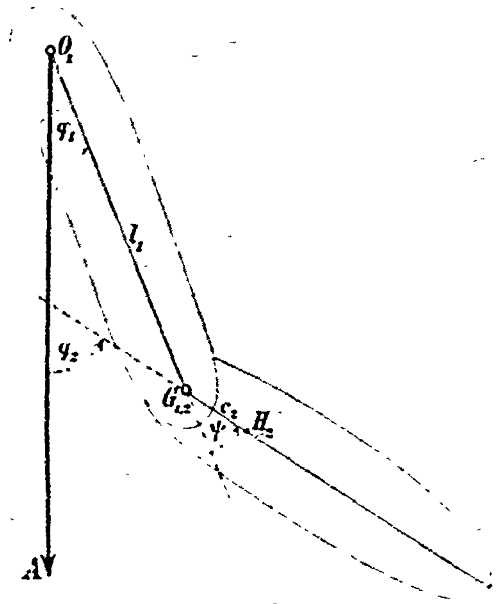


Figure 26. Rotation of First of Two Links Through Fixed Point 0_1

Here the ratio of the accelerations is:

$$(266)\quad \frac{\varphi_1''}{\varphi''} = \frac{\lambda_2^2 \frac{D_1}{D_2} - l_1 c_2 \cos \psi}{(\lambda_1^2 + l_1 c_2 \cos \psi) - (\lambda_2^2 + l_1 c_2 \cos \psi) \frac{D_1}{D_2}} .$$

Hence, the ratio of angular acceleration in the two joints, 0_1 and $G_{1,2}$, and consequently the ratio of the initial rotation of these joints, depends only on a ratio of the turning moments. If turning moments D_1 and D_2 are caused by a single muscle then their ratio is independent of the tension of the muscle and is equal to the ratio of the arms of the two couples of force with which the muscle acts on the two limbs.

From (266) by dividing the numerator and denominator on the right side by $l_1 c_2$ is derived:

$$(267)\quad \frac{\varphi_1''}{\varphi''} = \frac{\dfrac{\lambda_2^{\,2}}{l_1 c_2} \cdot \dfrac{D_1}{D_2} - \cos\psi}{\left(\dfrac{\lambda_1^{\,2}}{l_1 c_2} + \cos\psi\right) - \left(\dfrac{\lambda_2^{\,2}}{l_1 c_2} + \cos\psi\right) \cdot \dfrac{D_1}{D_2}}.$$

Where σ_1 is the length of the mathematical pendulum that has the same time of swinging as the first reduced system around the fixed axis set horizontally, and σ_2 similarly swings around the intermediate joint, then the kinetic measure for the action of all muscles applying to the 2-link system is expressed as:

$$(268)\quad \frac{\varphi_1''}{\varphi''} = \frac{\dfrac{\sigma_2}{l_1} \cdot \dfrac{D_1}{D_2} - \cos\psi}{\left(\dfrac{\sigma_1}{\rho_2} + \cos\psi\right) - \left(\dfrac{\sigma_2}{l_1} + \cos\psi\right) \cdot \dfrac{D_1}{D_2}}.$$

The preceding refers to a hinge-joint. A ball-joint with 3 degrees of freedom is also discussed. It is observed:

> .... Each increase of the freedom of motion in one joint or several joints, as well as an increase in the number of the system, will also bring about an increase in the number of ratios of two angular accelerations each....

18b. <u>Kinetic Measure for the Muscles of the Arm.</u> (p 276 - 287)

18bα. <u>General Expressions for the Kinetic Measure of the Muscles of the Arm.</u> (pp 276-277)

Here certain measurements are given: upper arm wt. = 1.908 kg : forearm + hand = 1.775 kg ; entire arm = 3.683 kg ; upper arm lgth. = 30.3 cm ; distance r_8 of upper arm from mid-point of shoulder-joint = 13.6 cm ; distance r_{10} of center of gravity of forearm + hand from mid-point of elbow-joint = 19.0 cm. In working out the problem the index 8 shall be for upper arm, and 10 for forearm + hand. The main lengths will be c'_8, c'_{10}, belonging to the main points H'_8, H'_{10}. For the radii of inertia λ_8, λ_{10} and for σ_8, σ_{10}, and for ρ_{10} the prime (') will be omitted.

By dividing the weight of upper arm (1.908 kg), and of forearm + hand (1.775 kg) by 981.11 the mass numbers are:

(269) $m_8 = 0{,}01945$; $m_{10} = 0{,}001809$; $m_{8,10} = 0{,}003754$

From the anatomical model the radius of inertia, x_8, of upper arm = 9.1 cm ; of x_{10}, forearm + hand, = 12.4 cm ; the length l_8 = 30.3 cm and the distances r_8, r_{10}, of the center of gravity from the resp. proximal joints = 13.6 cm and 19.0 cm. After calculating (270-271) λ_8 = 24.11 cm and λ_{10} = 15.75 cm , there is finally derived (274) the formula for the calculation of the kinetic measure of the arm muscles based on (256) $D_{m_{8,10}} = D_{m_8} + D_{m_{10}}$) and on (273)

$$(273)\quad \frac{\psi''_{1,8}}{\psi''_{8,10}} = \frac{0{,}894\,\dfrac{D_{m_8}}{D_{m_{10}}} - \cos\psi_{8,10}}{(2{,}095 + \cos\psi_{8,10}) - (0{,}894 + \cos\psi_{8,10})\dfrac{D_{m_8}}{D_{m_{10}}}}$$

$$(274)\quad \frac{\psi''_{1,8}}{\psi_{8,10}} = \frac{0{,}894\,\dfrac{D_{m_{8,10}}}{D_{m_{10}}} - (0{,}894 + \cos\psi_{8,10})}{(2{,}989 + 2\cos\psi_{8,10}) - (0{,}894 + \cos\psi_{8,10})\dfrac{D_{m_{8,10}}}{D_{m_{10}}}}$$

18b β. <u>One-Joint Muscles of the Elbow-Joint</u> (pp 278-281)

Here the ratio of the turning moments $D_{m_{8,10}}$ and $D_{m_{10}}$ is always zero. From (273) and (274) there becomes:

$$(275)\quad \frac{\psi''_{1,8}}{\psi''_{8,10}} = -\frac{0.894 + \cos\psi_{8,10}}{2.989 + 2\cos\psi_{8,10}}$$

This is valid for all muscles passing over the elbow-joint alone, regardless of extension or flexion. All one-joint muscles of the elbow-joint will cause initial rotation in the shoulder and elbow-joints solely during contraction, starting from rest. From (275) there is calculated the magnitude of the ratio of rotation that changes with the flexion angle $\psi_{8,10}$.

Flexion angle $\psi_{8,10}$ of elbow-joint	0°	10°	20°	30°	40°	50°	60°
Ratio $\frac{\psi''_{1,8}}{\psi''_{8,10}}$ of initial rotation	-0, 38	-0, 38	-0, 38	-0, 37	-0, 37	-0, 36	-0, 35

Flexion angle $\psi_{8,10}$ of elbow-joint	70°	80°	90°	100°	110°	120°	130°
Ratio $\frac{\psi''_{1,8}}{\psi''_{8,10}}$ of initial rotation	-0, 34	-0, 32	-0, 30	-0, 27	-0, 24	-0, 20	-0, 15

(Fischer, p 279)

A one-joint flexion of the elbow is also an extensor of the shoulder-joint and a one-joint elbow extensor is a flexor of the shoulder-joint. The value of the ratio of rotation $\psi_{8,10}$ increases with extension. When angle $\psi_{8,10} = 130^{\circ}$ the absolute value of the ratio is considerably below the initial value.

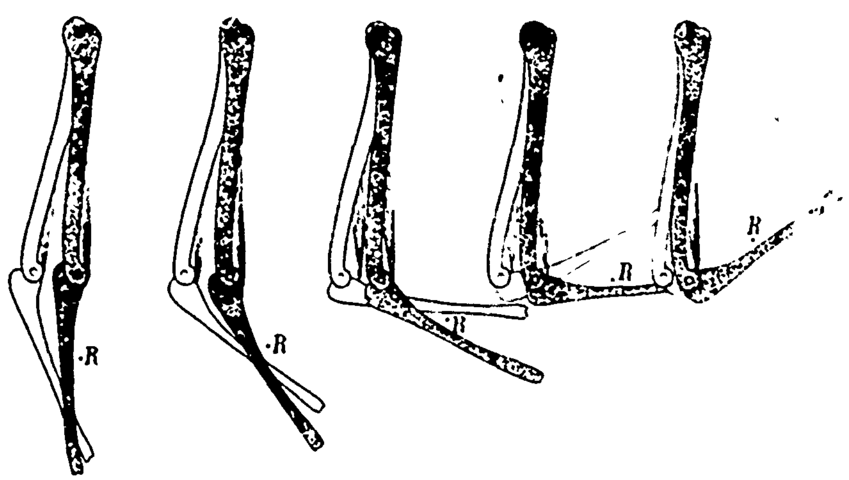


Figure 27. Initial Movement of Arm in Flexion of One-Joint Muscle (M. brachialis)

Figure 27 shows the initial movement of the arm in the contraction (flexion) of a one-joint muscle (M. brachialis)

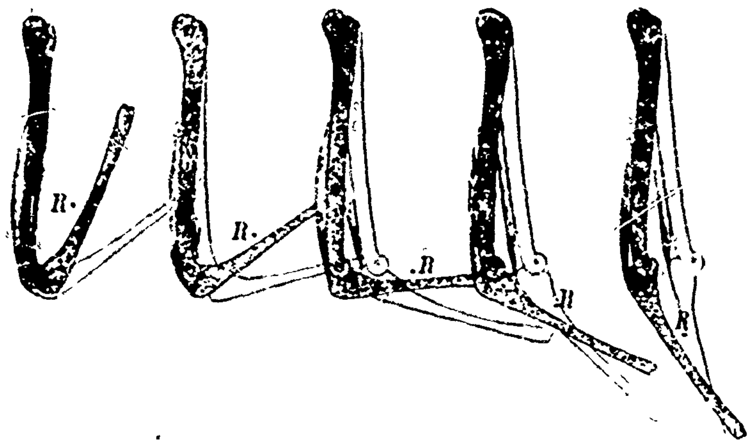


Figure 28. Initial Movement of Arm in Extension of One-Joint Muscle (head of M. triceps brachii)

Figure 28 shows the initial movement of the arm in the extension of a one-joint muscle (e. g., one of the heads of the M. triceps)

18bγ. <u>One-Joint Muscles of the Shoulder</u> (pp 281-284)

Here there is no turning moment in the forearm, so $D_{m_{10}}$ = zero. The reciprocal value of the two turning moments also = zero. Hence:

(276) $$\frac{\psi''_{1,8}}{\psi''_{8,10}} = - \frac{0,894}{0,894 + \cos \psi_{8,10}}$$

From here we may tabulate:

Flexion angle $\psi_{8,10}$ of the elbow-joint	0^o	10^o	20^o	30^o	40^o	50^o	60^o
Ratio $\frac{\psi''_{1,8}}{\psi''_{8,10}}$ or $\left[\frac{\psi''_{8,10}}{\psi''_{1,8}}\right]$ of the initial rotation	-0,47	-0,48	-0,49	-0,51	-0,54	-0,58	-0,64

Flexion angle $\psi_{8,10}$ of the elbow joint	70°	80°	90°	100°	110°	120°	130°
Ratio $\frac{\psi''_{1,8}}{\psi''_{8,10}}$ or $\left[\frac{\psi''_{8,10}}{\psi''_{1,8}}\right]$ of the initial rotation	-0,72	-0,84	-1	[-0, 80]	[-0, 62]	[-0, 44]	[-0, 28]

(Fischer, pp 282-283)

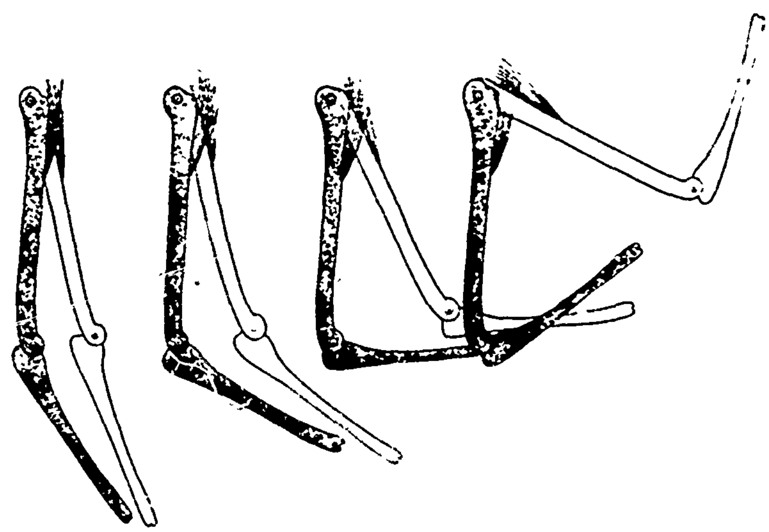


Figure 29. Initial Movement of Arm in Contraction of One-Joint Flexor, of the Shoulder-Joint (Ant. Part M. deltoideus)

Figure 29 shows the initial movements of the arm as a result of the contraction of a one-joint flexing muscle of the shoulder joint (e. g., anterior part of M. deltoideus)

Figure 30 shows a number of initial movements that a one-joint extensor muscle of the shoulder-joint e. g., M. coracobrachialis) would cause via contraction, starting from rest.

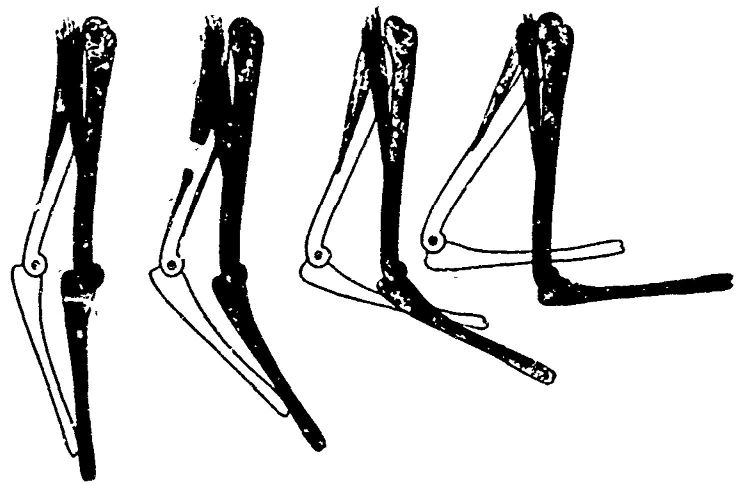


Fig. 30. Initial Movement of Arm in Contraction of One-Joint Extensor of the Shoulder-Joint (M. coracobrachialis)

18b 6. <u>Two-Joint Muscles of the Arm</u> (pp 284-288)

Here, using the long head of the M. biceps brachii again, Fischer uses the $Dm_{8,10}$ and Dm_{10} ratios previously given by means of (273) (274) . It is necessary in (274) to insert the values of ratio $\frac{Dm_{8,10}}{Dm_{10}}$ of the angle $\psi_{8,10}$. This will give:

Flexion angle $\psi_{8,10}$ of the elbow-joint	0°	10°	20°	30°	40°	50°	60°
Ratio $\frac{\psi''_{1,8}}{\psi''_{8,10}}$ of the initial rotation	-0, 09	-0, 21	-0, 26	-0, 28	-0, 29	-0, 28	-0, 27

Flexion angle $\psi_{8,10}$ of the elbow-joint	70°	80°	90°	100°	110°	120°	130°
Ratio $\frac{\psi''_{1,8}}{\psi''_{8,10}}$ of the initial rotation	-, 025	-0, 21	-018	-0, 12	-0, 04	+0, 07	+0, 21

(Fischer, p 285)

The foregoing ratios of rotation are valid for any value of the shoulder-joint angle $\psi_{1,8}$. For the long head of the Biceps the ratio of rotation has a negative value; the ratio will become positive only at the very end. The long head of the Biceps, with radio-ulnar joint fixed, will cause, during contraction, flexion in the elbow and simultaneous (backward rotation) in the shoulder, until beyond the rectangular flexion of the elbow.

18c. Kinetic Measure for the Muscles of the Leg (p 288 - 295)

18cα. Comparison of the Ratios of the Leg with Those of the Arm. (pp 288-291)

Here (268) is used, except that the constants $\frac{\sigma_1}{\rho_2}$ and $\frac{\sigma_1}{l_1}$ will have different values. For the calculation of these (271) is used: λ_8 = 24.11 cm , λ_{10} = 15.75 cm , and (272): σ_8 = 26.85 cm , σ_{10} = 27.08 cm , and ρ_{10} = 12.82 cm. Also, here replace index_8 by index_2 and index_{10} by $\text{index}_{4,6}$, and use $m_{4,6}$ = the mass of the system. The following data are emergent, as seen in Table 38.

The data compare favorably to the constants 2.095 and 0.894 found for the arm. For the four segments above there may be calculated the weights, the various lengths, and the ratios $\frac{\sigma_4}{\rho_6}$ and $\frac{\sigma_6}{l_4}$; the index_4 applies to the lower leg, index_6 to the foot, as seen in Table 39.

Here comparability with the arm is not nearly so approximate.

By using D_{m_2} as the turning moment of muscles acting on the thigh, D_{m_4} on leg + foot, $\psi_{1,2}$ as the hip-joint angle, $\psi_{2,4}$ as the knee-joint angle, then (273) gives the ratio of angular accelerations $\psi_{1,2}$ and $\psi_{2,4}$ in hip- and knee-joints, resp.:

Table 38. Direct Measurements on the Lower Extremity with Values $\frac{\sigma_2}{\rho_{4,6}}$ and $\frac{\sigma_{4,6}}{l_2}$

Body Part		G in kg	m	l in cm	r in cm	x in cm	c in cm	λ in cm	σ in cm	ρ in cm	$\frac{\sigma_2}{\rho_{4,6}}$	$\frac{\sigma_{4,6}}{l_2}$
Thigh	R	6,45	0,006574	40,0	17,7	10,3	25,79	29,12	32,88	-	2,335	0,853
Leg + Foot	R	3,68	0,003751	-	25,0	15,1	9,08	17,60	34,10	14,08		
Thigh	L	6,99	0,007125	40,0	16,7	10,8	24,72	28,46	32,76	-	2,353	0,847
Leg + Foot	L	3,67	0,003741	-	25,0	14,9	8,60	17,07	33,88	13,92		
Thigh	R	4,86	0,004954	35,9	15,72	11,01	23,39	26,80	30,70	-	2,126	0,924
Leg + Foot	R	2,98	0,003037	-	24,77	14,41	9,41	17,67	33,16	14,44		
Thigh	L	4,81	0,004903	36,65	15,90	11,43	23,54	27,14	31,29	-	2,491	0,883
Leg + Foot	L	2,80	0,002854	-	21,94	15,10	8,07	16,16	32,35	12,56		

(Fischer, p 288)

Table 39. Direct Measurements on the Lower Extremity with Values $\frac{\sigma_4}{\rho_6}$ and $\frac{\sigma_6}{l_4}$

Body Parts		G in kg	m	l in cm	r in cm	x in cm	c in cm	λ in cm	σ in cm	ρ in cm	$\frac{\sigma_4}{\rho_6}$	$\frac{\sigma_6}{l_4}$
Leg	R	2,69	0,002742	39,0	16,5	9,8	22,58	26,07	30,09	-	10,746	0,308
Foot	R	0,99	0,001009	-	6,0	6,0	1,62	4,41	12,00	2,80		
Leg	L	2,67	0,002721	39,0	16,0	10,1	22,28	25,99	30,32	-	10,564	0,318
Foot	L	1,00	0,001011	-	6,0	6,2	1,64	4,51	12,39	2,87		
Leg	R	2,07	0,002110	37,9	16,13	9,16	22,77	26,02	29,74	-	9,182	0,312
Foot	R	0,91	0,000928	-	6,38	5,91	1,95	4,80	11,83	3,25		
Leg	L	1,89	0,001927	37,1	16,30	9,66	23,06	26,26	29,91	-	8,217	0,325
Foot	L	0,91	0,000928	-	6,95	5,97	2,26	5,22	12,07	3,64		

(Fischer, p 290)

(277) $$\frac{\psi''_{1,2}}{\psi''_{2,4}} = \frac{0,894 \frac{D_{m_2}}{D_{m_4}} - \cos \psi_{2,4}}{(2095 + \cos \psi_{2,4}) - (0894 + \cos \psi_{2,4}) \frac{D_{m_2}}{D_{m_4}}}$$

18c β. Special Examples for the Determination of the Kinetic Measures of the Muscle (pp 291-295)

The kinetic measure of muscles passing over the knee-joint only (e. g., M. vastus lateralis, M. vastus intermedius, M. vastus medialis, M. biceps femoris) is the same as for one-joint muscles. Here the ratio of turning moments D_{m_2} and D_{m_4} has a constant value of -1: hence, the kinetic measure may be determined by the right side of (275). For muscles passing over the hip-joint only the turning moment D_{m_4} = zero. The kinetic measure of these muscles is as in the right side of (276). The calculation for multi-typed muscles is based on (277). Fischer gives the M. semimembranosus as a prime example here. He concludes that: 1) when hip-and knee-joints are freely movable this muscle is a bending (flexing) muscle of knee and hip, in medium flexion of the knee, when the knee is not too strongly bent; 2) near extreme stretch (extension) and the rectangular bent position of the knee this muscle does not act on the hip; 3) the knee will be bent by this muscle in all initial postures of the leg.

18d. Initial Movements Under Simultaneous Action of Muscle and Gravity (pp 295-321)

When on the 2-link system several different moving forces act at the same time (268) is still valid for the determination of the ratio of initial rotations caused in both joints. Here D_1 is the resulting turning moment with which all forces act jointly on the link nearest the fixed axis; D_2 = resulting turning moment of the second link; D_s = the turning moment of gravity. This will give (if we give proper index numbers):

(278) $D_1 \; D_{m_8} + D_{s_8}; \quad D_2 = D_{m_{10}} + D_{s_{10}}.$

If for the leg we assume the ankle-joint to be fixed we get:

(279) $D_1 = D_{m_2} + D_{s_2}; \quad D_2 = D_{m_4} + D_{s_{4,6}}.$

18dα. The Action of Gravity Alone (pp 296-309)

From (273) we derive for the ratio of the initial rotation of shoulder- and elbow-joints:

$$(280) \qquad \frac{\psi''_{1,8}}{\psi''_{8,10}} = \frac{0.894\,\dfrac{D_{s8}}{D_{s10}} - \cos\psi_{8,10}}{(2.095 + \cos\psi_{8,10}) - (0.894 + \cos\psi_{8,10})\,\dfrac{D_{s8}}{D_{s10}}}$$

From (274) we get $D_{s8,\,10}$ and D_{s10}. If the long axis of the upper arm is vertical then the D_{s8} and the ratio of the two turning moments in (280) = zero; hence, (280) becomes:

$$(281) \qquad \frac{\psi''_{1,8}}{\psi''_{8,10}} = -\frac{\cos\psi_{8,10}}{2.095 + \cos\psi_{8,10}}$$

From the various postures of the elbow-joint we get:

Flexion angle $\psi_{8,10}$ of the elbow joint	0°	10°	20°	30°	40°	50°	60°
Relation $\frac{\psi''_8}{\psi''_{10}}$ of the initial rotation	-0, 32	-0, 32	-0, 31	-0, 29	-0, 27	-0, 24	-0, 19

Flexion angle $\psi_{8,10}$ of the elbow joint	70°	80°	90°	100°	110°	120°	130°
Relation $\frac{\psi''_8}{\psi''_{10}}$ of the initial rotation	-0, 14	-0, 08	0	+0, 09	+0, 20	+0, 31	+0, 44

(Fischer, p 297)

The data show that in the vertical position of the upper arm gravity will cause an initial rotation of the shoulder-joint, but gravity does not exert a turning moment on the upper arm, for forearm is the cause. When the long axis of the upper arm is vertically downward gravity will lower the raised forearm in any one flexed position and will cause extension of the elbow-joint. As long as the elbow-joint is not at a right angle the shoulder-joint will be flexed and the upper arm rotated forward.

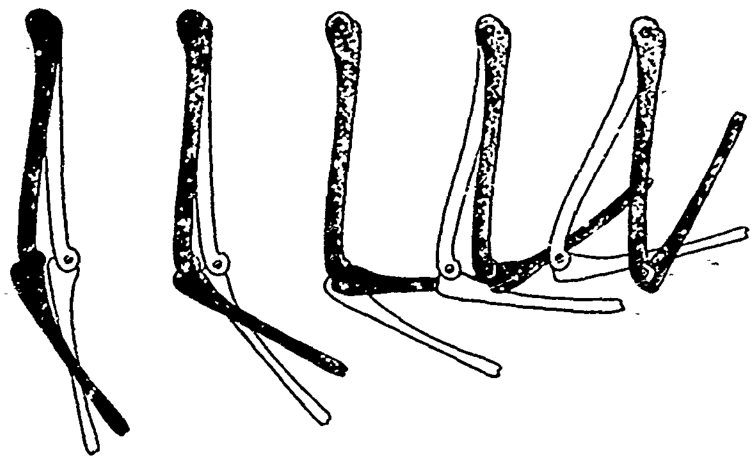


Figure 31. Initial Movement of Arm Due to Gravity When Upper Arm is Vertically Downward

Figure 31 shows the initial movement of the arm caused by gravity when the upper arm is positioned vertically downward. Figures 27-28 are comparable here.

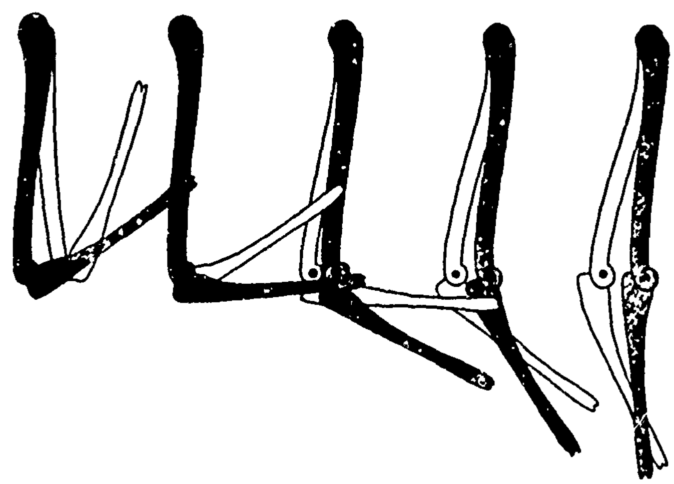


Figure 32. Initial Movement of Arm Under Action of Gravity Alone.

Figure 32 shows the initial movement of the arm under the action of gravity alone. Figures 25-26 are comparable here.

In any position of the arm we must calculate the ratio of the two turning moments of gravity from the values of the two joint angles before (280) can be used to determine the ratio of initial rotations. Using (232) and (233) where $c'_8 = 21.65$ cm and $c'_{10} = 9.16$ cm, we get:

(282) $$\frac{D_{s_8}}{D_{s_{10}}} = \frac{21.65 \sin \psi_{1,8}}{9.16 \sin(\psi_{1,8} + \psi_{8,10})}$$

Employing (265) we derive:

$$\varphi''_1 = \frac{\lambda_2^2 \cdot D_1 - l_1 c_2 \cos\psi \cdot D_2}{m_o \left[(\lambda_1 \lambda_2)^2 - (l_1 c_2 \cos\psi)^2\right]}$$

$$\psi'' = \frac{(\lambda_1^2 + l_1 c_2 \psi) \cdot D_2 - (\lambda_2^2 + l_1 c_2 \cos\psi) \cdot D_1}{m_o \left[(\lambda_1 \lambda_2)^2 - (l_1 c_2 \cos\psi)^2\right]}$$

This will permit calculation of the angular accelerations in the two joints of a two-joint system (insert here values constants λ_1, λ_2, l, and c_2 found for the arm). If the numerator is divided by $l_1 c_2$, (266) becomes (268). Then with reference to (280) we find that $\psi''_{1,8}$ has the same sign as

$$\left[0.894 \frac{D_{s_8}}{D_{s_{10}}} - \cos\psi_{8,10}\right] D_{s_{10}}$$ and $\psi''_{8,10}$ the same sign as $$\left[(2.095 + \cos\psi_{8,10}) - (0.894 + \cos\psi_{8,10})\frac{D_{s_8}}{D_{s_{10}}}\right] D_{s_{10}}$$

It follows that the shoulder-joint is extended and the elbow-joint is flexed.

Rotation in the elbow-joint will occur only when:

(283) $$\frac{D_{s_8}}{D_{s_{10}}} = \frac{\cos\psi_{8,10}}{0.894}$$

Rotation in the shoulder-joint will occur only when:

(284) $$\frac{D_{s_8}}{D_{s_{10}}} = \frac{2.095 + \cos\psi_{8,10}}{0.894 + \cos\psi_{8,10}}$$

The values of joint angles $\psi_{1,8}$ and $\psi_{8,10}$ belonging to each other can be calculated if we set equal the right sides of (283) and (282) amd the right sides of (284) and (282). From these two combinations will be seen the reciprocal action of the two joints (shoulder and elbow) as illustrated in Figure 33.

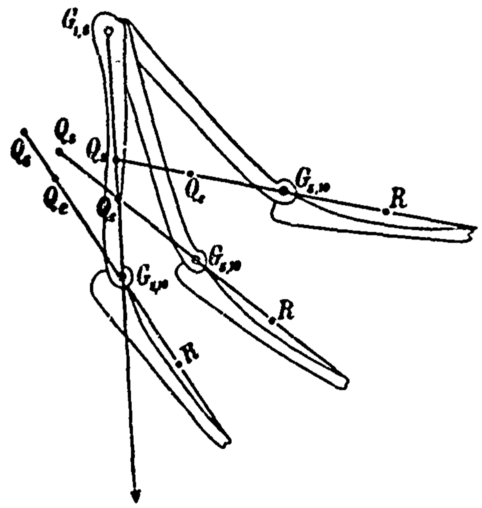


Figure 33. Reciprocal Action of Shoulder- and Elbow-Joints

By means of stated ratios of the turning moments we may obtain exact position, Q_e and Q_s of the point of equilibrium belonging to the two special postures in Figure 33. This gives a simple way to decide for any other position of the arm, with the same elbow angle, the way in which the rotations in the two joints will take place. The position of Q_e and Q_s in Figure 33 is valid only where angle $\psi_{8,10}$ of the elbow = 30°

From (283) the following calculations emerge:

Flexion angle $\psi_{8,10}$ of the elbow-joint	0°	10°	20°	30°	40°	50°	60°
Ratio $\frac{D_{s_8}}{D_{s_{10}}}$ or resp. $\left[\frac{D_{s_{10}}}{D_{s_8}}\right]$	(+0, 894)	(+0, 908)	(+0, 951)	+0, 969	+0, 857	+0, 719	+0, 559

Flexion angle $\psi_{8,10}$ of the elbow-joint	70^o	80^o	90^o	100^o	120^o	130^o	
Ratio $\frac{D_{s_8}}{D_{s_{10}}}$ or resp. $\left[\frac{D_{s_{10}}}{D_{s_8}}\right]$	+0, 383	+0, 194	0	-0, 194	-0, 383	-0, 559	-0, 719

(Fischer, p. 306)

Here the value of the reciprocal ratio is given in brackets where D_{s_8} is greater than $D_{s_{10}}$.

Figure 34 shows some positions of the arm where gravity causes only extension in the elbow joint.

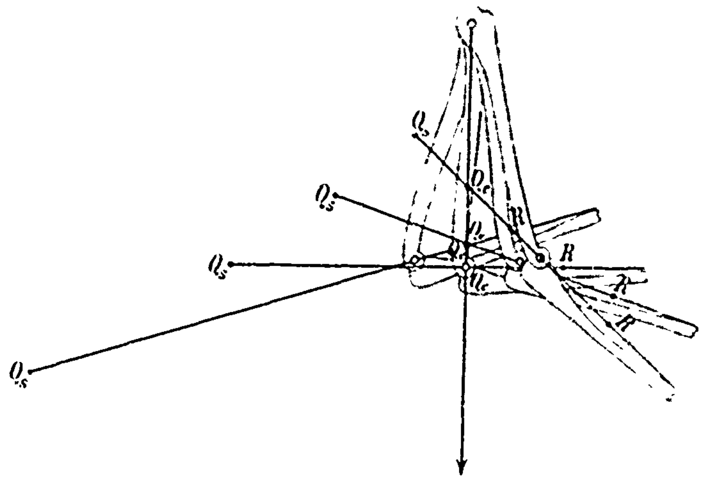


Figure 34. Position of the Arm Where Gravity Causes Only Extension at the Elbow-Joint

Figure 34 along with Figure 35, shows the position of Q_s and Q_e, resp., on the long axis of the forearm.

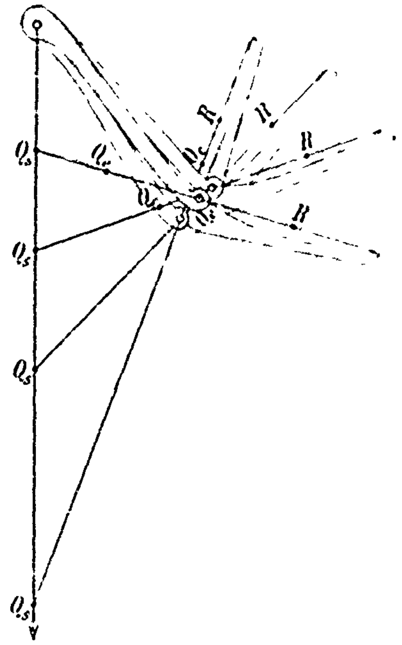


Figure 35. Position of Q_s and Q_e on the Long Axis of the Forearm

18dβ. Special Examples of the Initial Movement as a Consequence of the Muscles and Gravity: The Detachment of the Heels from the Ground (pp 309-321)

Section 17b presented the conditions under which the muscles and gravity remain in equilibrium with heels raised so that a person can stand on the toes. Here Fischer considers lifting the heels from the ground and the initial rotation in joints to be considered when starting from various positions. Figure 36 (which should be compared to Figure 27) shows both feet parallel, rotating around a fixed axis M, through the heads of the R and L metatarsals I.

The rest of the body (a rigid link 2 of the 2-link plane system) rotates at point F of the two ankle joints. It is assumed that all forces acting on the two sections of the body run parallel to the median plane and also that movements of the detachment of the feet run parallel to this plane. Then, it may be assumed that the entire body with all forces will be projected in this plane (as in Figure 26). The following may be established: MF = long axis of link 1, and its extension = l_1; the length from F to the center of gravity of link 2 = long axis of link 2, and is r_2.

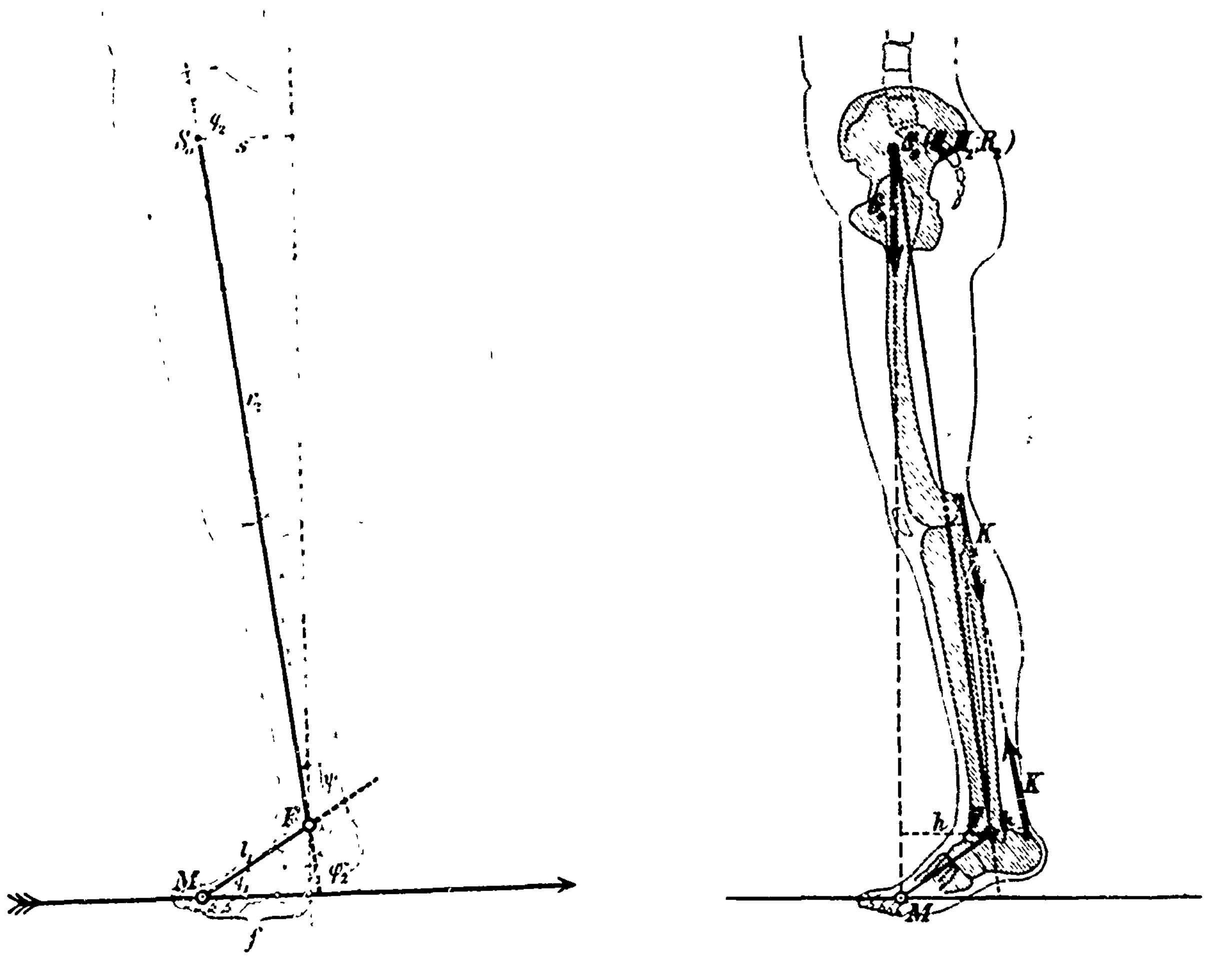


Figure 36 and 37. Two Views Showing Distribution of Forces When Heel is Raised from the Ground

Assume the mass of the feet to be neglected in relation to the mass of the rest of the body, then the center of gravity will coincide with S_o ; the main point of link 2 will also coincide with S_o, and that of link 1 with F. Hence, l = main length, c_1, of link 1, and r_2 = main length c_2 ; the mass of link 2 = mass of the entire body. The long axes MF and FS_o will form angles φ_1 and φ_2, resp., with the horizontal directed backward. Then the difference $\varphi_2 - \varphi_1$ represents the angle ψ of the intermediate joint (foot-joint), whereas φ_1 is the angle in the common metatarsophalangeal-joint, M.

Here the muscular behavior is essentially that of a one-joint muscle. Muscles behind the axis of the ankle joint tend to increase angle φ_1, those in front decrease angle φ_1. The turning moment of the former on the link is positive, of the latter negative. Calf muscles behind the axis are basic for lifting: the turning moment of muscles will be positive for link 1 and negative for link 2 (if the first turning moment = D_m, the second will = $-D_m$).

The main point of link 1 falls into the axis F of the foot-joint. Hence, the turning moment with which gravity acts on link 1 = the product of weight G of the entire body by the horizontal distance, f, of the vertical line through F from point of rotation, M, (f is shown to = $l \cos\varphi_1$). There will result:

$$(287) \qquad m_o \lambda_1^2 \cdot \varphi_1'' + m_o l_1 r_2 \cos\psi \cdot \varphi_2'' = D_m - Gf$$

$$m_o \lambda_2^2 \cdot \varphi_2'' + m_o l_1 r_2 \cos\psi \cdot \varphi_1'' = D_m + Gs$$

The first and secon d reduced systems are derived from (287) as follows:

$$(288) \qquad \varphi_1'' = \frac{D_m (M_2 + M_{1,2} \cos\psi) - G(f \cdot M_2 + s \cdot M_{1,2} \cos\psi)}{M_1 M_2 - M_{1,2}^2 \cos^2\psi}$$

$$\varphi_2'' = \frac{-D_m (M_1 + M_{1,2} \cos\psi) + G(s \cdot M_1 + f \cdot M_{1,2} \cos\psi)}{M_1 M_2 - M_{1,2}^2 \cos^2\psi}$$

(here M_2 is equated with $m_o \lambda_1^2$ and $m_o l^2$; M_2 is equated with $m_o \lambda_2^2$; $M_{1,2}$ is equated with $m_o l_1 r_2$)

For calculation of the moments of inertia, M_2, we may employ the following:

Body Parts	Wt. (kg)	Mass No.	Radius of inertia, x, in relation to axis through c. of g. of body part perpendic. to med. pl. (cm)	Distance of c. g. of body part from axis of ankle-joint (cm)	Moment of inertia of body part related to axis of ankle-joint
Head+trunk	27.710	0.02824	23.4	113.5	379.258
Entire Arm	3.615	0.00368	21.2	98.5	37.258
Thigh	6.450	0.00657	12.4	65.3	29.025
Leg	2.935	0.00299	10.4	24.3	2.089

(Fischer p312)

From this emerges as follows:

1) The angular acceleration ψ''_1 of the feet is positive, zero, or negative, depending on whether the turning moment D_m of the muscles is greater to, equal to, or smaller than:

$$(289) \quad G \frac{f \cdot M_2 + s \cdot M_{1,2} \cos \psi}{M_2 + M_{1,2} \cos \psi}$$

2) The angular acceleration ψ''_2 of the rest of the body is positive, zero, or negative, depending on whether the turning moment D_m of the muscles is smaller, equal to, or greater than:

$$(290) \quad G \frac{s \cdot M_1 + f \cdot M_{1,2} \cos \psi}{M_1 + M_{1,2} \cos \psi}$$

From (288) after inserting the values for the magnitudes M, we get the ratio of magnitude of the two angular accelerations φ''_1 and φ''_2, which gives the ratio of the initial rotation of both sections of the body in space:

$$(293) \quad \frac{\varphi''_1}{\varphi''_2} = \frac{D_m (7,312 + \cos \psi) - G(f \cdot 7,312 + s \quad \cos \psi)}{-D_m (0,174 + \cos \psi) + G(s \cdot 0,174 + f \quad \cos \psi)}$$

Fischer then considers and develops two cases:

1) where the center of gravity is vertically above the metatarsal axis;
2) where the center of gravity lies vertically above the common axis of the ankle-joint.

19. <u>On the Entire Course of the Joint Movements During Continued Contraction of a Muscle.</u> (pp 321-333)

In Section 18 it was assumed that before contraction of a muscle the links of the 2-link system were at rest. We must now consider continuous contractions with continuous new impulses. It is no longer enough to set angular velocities φ'_1, or φ'_2 or ψ' equal to zero, resp., and then to derive angular accelerations φ''_1, φ''_2 and ψ'', resp.

In the General Part I it was shown how equations of mention could be derived from the value of the kinetic energy T of the joint system: (67) $T = 1/2 m_o \lambda_1^2 . \varphi'_1{}^2 + 1/2 m_o \lambda_2^2 . \varphi'^2_2 + m_o l_1 c_2 \cos (\varphi_1 - \varphi_2) \varphi'_1 \varphi'_2$ now rewritten as:

$$(306) \quad T = 1/2\, m_o \lambda_1^2 . \varphi'^2_1 + 1/2\, m_o \lambda_2^2 . \varphi'^2_2 + m_o l_1 c_2 \cos(\varphi_2 - \varphi_1) . \varphi'_1 \varphi'_2 .$$

From (68) for the equations of motion we may derive:

$$(307) \quad \begin{aligned} \frac{\delta T}{\delta \varphi_1} &= m_o l_1 c_2 \sin(\varphi_2 - \varphi_1) . \varphi'_1 \varphi'_2 , \\ \frac{\delta T}{\delta \varphi_2} &= - m_o l_1 c_2 \sin(\varphi_2 - \varphi_1) . \varphi'_1 \varphi'_2 . \end{aligned}$$

These two partial equations are equal and opposite; remove $\frac{\delta T}{\delta \varphi_1}$ and $\frac{\delta T}{\delta \varphi_2}$ from (68) and use D_1 and D_2 for $Q\varphi_1$ and $Q\varphi_2$, to derive:

$$(308) \quad \frac{d}{dt}\left(\frac{\delta T}{\delta \varphi'_1}\right) + \frac{d}{dt}\left(\frac{\delta T}{\delta \varphi'_2}\right) = D_1 + D_2 .$$

By multiplying both numbers with the differential dt and by integration we get:

$$(309) \quad \frac{\delta T}{\delta \varphi'_1} + \frac{\delta T}{\delta \varphi'_2} = \int (D_1 + D_2)\, dt + C, \text{ (where C = the constant of integration)}$$

From (306) we can calculate the two partial differential quotients $\frac{\delta T}{\delta \varphi'_1}$ and $\frac{\delta T}{\delta \varphi'_2}$, so that (309) becomes:

$$(310) \quad m_o \left[\lambda_1^2 + l_1 c_2 \cos(\varphi_2 - \varphi_1) . \varphi'_1\right] + m_o \left[\lambda_2^2 + l_1 c_2 \cos(\varphi_2 - \varphi_1)\right] . \varphi'_2 = \int (D_1 + D_2)\, dt + C.$$

In a one-joint muscle that goes over only one intermediate joint the sum of the turning points D_1 and D_2 = zero.

If it be considered that the two links of the joint system are at rest before contraction we have (excluding gravity):

$$(311) \quad \frac{\varphi'_1}{\varphi'_2} = - \frac{\lambda_2^2 + l_1 c_2 \cos(\varphi_2 - \varphi_1)}{\lambda_1^2 + l_1 c_2 \cos(\varphi_2 - \varphi_1)}$$

The angular velocity in an intermediate joint will be equal to the difference of the two angular velocities so far considered. If in (311) we replace $\varphi_2 - \varphi_1$ by ψ we derive:

$$(312) \quad \frac{\varphi'_1}{\psi'} = - \frac{\lambda_2^2 + l_1 c_2 \cos\psi}{\lambda_1^2 + \lambda_2^2 + 2 l_1 c_2 \cos\psi}$$

If we divide the numerator and the denominator of (312) by $l_1 c_2$ we can substitute $\frac{\sigma_1}{\rho_2}$ and $\frac{\sigma_1}{l_1}$ for $\frac{\lambda_1^2}{l_1 c_2}$ and $\frac{\lambda_2^2}{l_1 c_2}$, resp., to give:

$$(313) \quad \frac{\varphi'_1}{\psi'} = - \frac{\frac{\sigma_2}{l_1} + \cos\psi}{\frac{\sigma_1}{\rho_2} + \frac{\sigma_2}{l_1} + 2\cos\psi}$$

(Here σ_1 = reduced length of the pendulum of first reduced system that can be rotated around a fixed axis; σ_2 = same of second reduced system around axis of an intermediate point; ρ_2 = distance of setting point of link 2 from the center of the intermediate joint).

The angular velocities φ'_1 and ψ are identical with the differential quotients $\frac{d\varphi_1}{dt}$ and $\frac{d\psi}{dt}$. If for the first of these we insert the product $\frac{d\rho_1}{d\psi} \; \frac{d\psi}{dt}$ that is equal to it, then the common factor $\frac{d\psi}{dt}$ on the left side of (313) will cancel out to give:

$$(314) \quad \frac{d\varphi_1}{d\psi} = - \frac{\frac{\sigma_2}{l_1} + \cos\psi}{\frac{\sigma_1}{\rho_2} + \frac{\sigma_2}{l_1} + 2\cos\psi}$$

In this differential equation φ_1 and ψ can be separated by multiplying by $d\psi$, and can be at once integrated, to give the following relation between φ_1 and ψ:

$$(315)\quad \varphi_1 = \frac{\frac{\sigma_1}{\rho_2} - \frac{\sigma_2}{l_1}}{\sqrt{\left(\frac{\sigma_1}{\rho_2} + \frac{\sigma_2}{l_1}\right)^2 - 4}} \operatorname{arctg}\left\{\sqrt{\frac{\frac{\sigma_1}{\rho_2} + \frac{\sigma_2}{l_1} - 2}{\frac{\sigma_1}{\rho_2} + \frac{\sigma_2}{l_1} + 2}}\ \operatorname{tg}\frac{\psi}{2}\right\} - \frac{\psi}{2} + \alpha,$$

(where α = the integration constant)

From this may be calculated all values of the two joint angles in all movements. In examples Fischer uses the elbow-joint as an illustration. The elbow is considered in terms of a one-joint flexing muscle: elbow extended, upper arm with vertical axis downward; here shoulder-joint angle $\psi_{1,8}$ and α will have values of zero. From (317) $\psi_{1,8} = 0.54054 \operatorname{arctg}\left[0.4452 \operatorname{tg}\frac{\psi_{8,10}}{2}\right] - \frac{\psi_{8,10}}{2} + \alpha$, the values of the two joint angles are calculated:

Shoulder angle $\psi_{1,8}$	0°	$-5^\circ 41'$	$-11^\circ 19'$	$-16^\circ 51'$	$-22^\circ 13'$	$-27^\circ 18'$
Elbow angle $\psi_{8,10}$	0°	15°	30°	45°	60°	75°

Shoulder angle $\psi_{1,8}$	$-32^\circ 2'$	$-36^\circ 13'$	$-39^\circ 39'$	$-42^\circ 4'$	$-43^\circ 8'$
Elbow angle $\psi_{8,10}$	90°	105°	120°	135°	150°

(Fischer, p 326)

Figure 38 shows positions of the arm according to the foregoing tabulation. There is involved only the contraction of a one-joint flexing muscle of the elbow (M. brachialis).

The above tabulation and Figure 38 apply to a case in which the arm is transferred into an extended from a flexed position, caused by

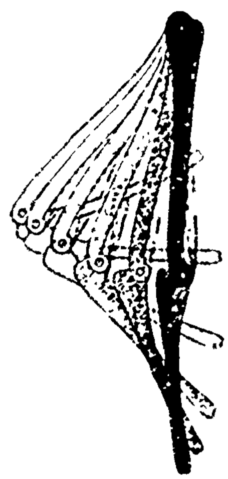


Figure 38. Positions of the Arm in Varying Shoulder and Elbow Angles

the contraction of a one-joint extensor muscle of the elbow (e. g. one-joint heads of the M. triceps brachii). The kind of movement in a contraction of one-joint muscles depends upon: 1) the ratio of the masses of the two arm segments; 2) the position of the resp. centers of gravity; 3) the distribution of the masses within the individual parts.

From Section 17aδ Fischer considers the problem of a weight held in the hand (weight = G'; mass = m'; it is in long axis of forearm at distance a from the elbow axis). Different weights are employed, but that of 15 kgs is most fully discussed. In a comparison of loaded with unloaded arms certain resemblances and differences were found: 1) the signs agree in both (rotation in the shoulder-joint will be opposed to that in the elbow-joint); 2) the absolute values of the ratio of angular velocity increase in the loaded arm, the more so with increased flexion (the reverse holds for the unloaded arm; 3) in the unloaded arm the absolute values are below 0. 5, while in the loaded arm they are above 0. 5 ; 4) in general in the loaded arm there is a greater change of direction of the long axis of the upper arm than of the forearm.

With a load of 15 kgs in the hand movement will occur so that the center of gravity of the object will approach the shoulder-joint on a straight line (flexion) or move away from the shoulder-joint in a straight line (extension).

20. Use of Equations of Motion for the Determination of the Muscle Forces with the Movement of the Human Body Being Known.
(pp 333-348)

Here will be considered as group 1 of problems the movement of body parts that one muscle, or several, cause under given conditions during their contraction (see Secs. 18-19). Group 2 of problems assumes the state of motion of the human body to be known for the entire course of a movement; sought are the muscle and muscular forces that effect this movement, together with external forces. In general, locomotion is involved.

In a first approximation Fischer considers the oscillations of the leg as a plane movement parallel to the median plane: 1) foot is movable referrable to lower leg at ankle, giving a plane 3-link joint system; 2) leg is joined to hip, so the concern is not with the free mobility of the hip. Section 7 applies here (equations of motion), as well as Section 8 (derivations of such equations).

Fischer here uses the equations of motion as follows: the rotation of each one of the three sections of the leg about its axis through the center of gravity that is perpendicular to the plane of motion, is represented in its dependence upon 1) the turning moment of muscles acting on the leg sections, 2) on gravity, and 3) on effective forces to be taken in the opposite direction of the associated or attached body parts. The following forces for the three sections of the R leg are considered: 1) gravity = weight G_2 of thigh, pulling vertically down, at S_2, center of gravity of the thigh (Figure 39); 2) muscles inserting on the thigh will set up tension forces (point of force of muscle does not always equal point of insertion). The over-all problem is to dimension the intensity and direction of forces that apply to the center of gravity of the thigh in such a way that the pressure + external forces applying to the entire leg will be able to impart to the center of gravity of the entire leg just its acceleration present in the respective moment as to magnitude and direction.

The effective force of the entire leg is the force evenly directed with the acceleration of the total center of gravity of the leg, the intensity of which is measured by the product of the mass of the entire leg ($m_{2,4,6}$) by the acceleration of its center of gravity. Let the mass of the entire leg equal $m_{2,4,6}$; let the acceleration of $S_{2,4,6}$ equal $\gamma_{2,4,6}$; then the magnitude of the effective force of the entire leg will be $m_{2,4,6}\,\gamma_{2,4,6}$.

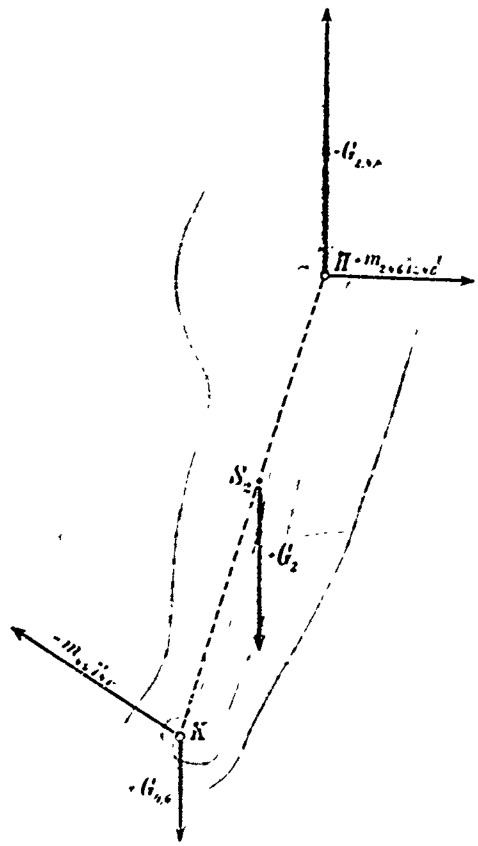


Figure 39. Centers H, K, F of the Three Joints of the Leg. Due to External Force

External forces are of two kinds: 1) the weight $G_{2,4,6}$ of the intire leg is vertically downward in the center of gravity $S_{2,4,6}$; 2) muscles starting at the leg will be forces during contraction (in Figure 39 the centers of the three joints are not $G_{2,4,6}$ but are HKF). It is necessary to consider only muscles the origin of which is outside of the leg (muscles with both insertions on the leg set up only internal forces); e. g., in the hip-joint will be found one pressure component that is directed vertically upward and is equal to the weight of the leg ($-G_{2,4,6}$ in Figure 39).

The pressure force exerted on the thigh in the knee-joint has the following components: 1) a component that is equal to the effective force $m_{4,6}\gamma_{4,6}$ of the system lower leg + foot, but that has the opposite direction as the latter; 2) , component that is equal to the weight $G_{4,6}$ of lower leg + foot and is also directed vertically downward like this weight (Figure 39); 3) a number of components that are equal to the external muscular forces applying directly to lower leg + foot and that have the same direction as these forces.

Under the above forces the thigh will move as though it were free.

In Figure 39 summed as $m_2 \gamma_2$, the m_2 = mass, the γ_2 is the acceleration of the center of gravity, and their product therefore is the effective force of the thigh. Two pressure components in the hip, + $m_{2,4,6} \gamma_{2,4,6}$ and - $m_{4,6} \gamma_{4,6}$, are in the hip center, H, and knee center, K, and will give $m_2 \gamma_2$ as a resultant. This is because the effective forces $m_{2,4,6} \gamma_{2,4,6}$ of the entire leg are the resultant of three effective forces, $m_2 \gamma_2$, $m_2 \gamma_4$, and $m_6 \gamma_6$, of thigh, leg, foot, resp.; the effective force $m_{4,6} \gamma_{4,6}$ of leg + foot is the resultant of $m_4 \gamma_4$, $m_6 \gamma_6$. Hence, $m_{2,4,6} \gamma_{2,4,6}$ plus - $m_{4,6} \gamma_{4,6}$ will equal effective force $m_2 \gamma_2$.

Three forces of weight apply to H_1, S_2, and K. It is possible to combine S_2K to give $G_{2,4,6}$, equal and opposite to $-G_{2,4,6}$. The point of application of this resultant is H_2 in Figure 40, which lies on the connecting length S_2K and divides this in the inverse proportion of weight (it is near the center of gravity of the thigh plus the leg + foot).

The force of weight, $+G_{2,4,6}$, applied to H_2 forms with the pressure component, $-G_{2,4,6}$, a couple of force acting on the thigh in the sense of rotation, but it cannot accelerate the center of gravity. The moment of this couple represents the turning moment of gravity exerted on the thigh in the period of oscillation (it is designated D_{s_2}). The muscle forces plus the pressure components in the hip and knee form couples of force in such a way that each muscle has a special couple of force (each muscle has an equal and opposite force). In muscles with origin and insertion outside the thigh there will be a couple of force in the hip-joint, with equal and opposite force in the knee-joint; these couples of force are different from those with which gravity acts on the thigh.

In the assumption of plane movement only those components of muscle couples of force will act, the planes of which are parallel to the median plane of the body. We must distinguish between couples of force which rotate the thigh clockwise (negative or -) or counter-clockwise (positive or +). The moment for all couples of force from muscles = the algebraic sum of moments for individual couples of force = D_{m_2}.

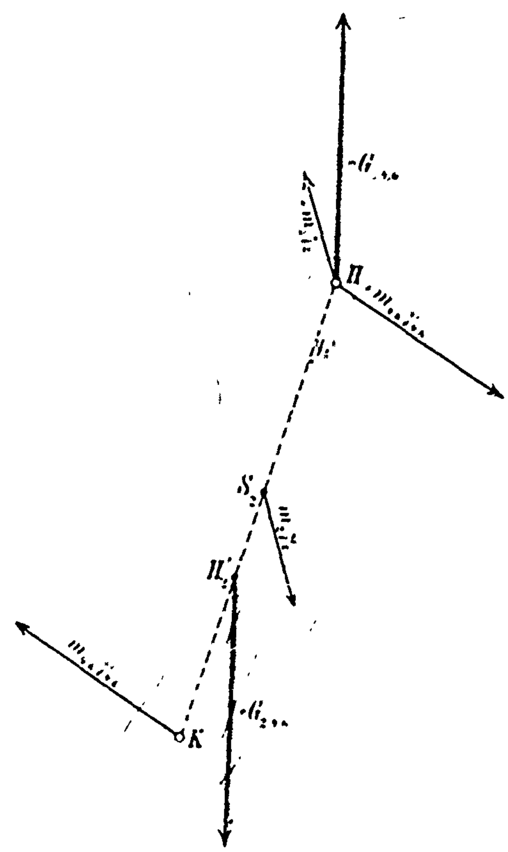


Figure 40. Resultant, H_2, of Three Forces of Weight, H_1, S_2, and K, in the Leg

All muscle forces are opposite in pairs, and cannot effect movement of the total center of gravity of the thigh. Hence, only two components of pressure will remain: 1) hip-joint center = effective force of the entire leg; 2) knee-joint center = equal and opposite to the effective force of leg + foot. These two pressure forces will exert a rotatory influence on the thigh, shown in Figure 40 by $-m_2\gamma_2$. This $m_2\gamma_2$ may be resolved into two components (see Fig. 39):

1) $-m_{2,4}\gamma_{2,4}$ (equal and opposite to force in hip)
2) $+m_{4,6}\gamma_{4,6}$ (" " " " " " knee)

This will now give two couples of force:

1) $+m_{2,4,6}\,\gamma_{2,4,6}$, $-m_{2,4,6}\,\gamma_{2,4,6}$

2) $+m_{4,6}\,\gamma_{4,6}$, $-m_{4,6}\,\gamma_{4,6}$

We can resolve force $m_{2,4,6}\,\gamma_{2,4,6}$ in the hip (Figure 60) into $+m_{4,6}\,\gamma_{4,6}$ and $+m_{2,}\,\gamma_2$.

In the two couples of force one has two forces:

1) $+m_{4,6}\,\lambda_{4,6}$ and $-m_{4,6}\,\lambda_{4,6}$, app'ying to hip and knee-joint centers;

2) $+m_2\,\lambda_2$ (hip) and $-m_2\,\lambda_2$ (thigh ... of gravity, resp. Here γ_2 = acceleration of center of gravity ..., $\lambda_{4,6}$ = that of leg + foot.

The resultant turning moment of the effective forces for the thigh equals D_{e_2} which is equal to the algebraic sum of moments of two couples of force under the assumption of plane movement, and where counter-clockwise is -, clockwise is +.

Fischer also does for the lower leg what is above outlined for the thigh. Here D_{m_4} equals the turning moment of the muscles of the lower leg and D_{e_4} equals the algebraic sum of the moments of the two couples of force. Corresponding values for the foot become D_{m_6} and D_{e_6}.

Now designate by φ''_2, φ''_4, and φ''_6 as the angular accelerations of entire leg, thigh, and leg + foot which will rotate around S_2, S_4, S_6 in the period of oscillation. Also, x_2, x_4, x_6 are the radii of inertia in relation to axes through their centers of gravity, perpendicular to the plane of motion (cross axes)(of Sec. 15). There emerges:

(328) $m_h\, x_h\, \varphi_h^2 = D_{m_h} + D_{s_h} + D_{e_h}$ (Where for the three sections of the R leg - thigh, leg, foot - the h has values of 2, 4, 6, and for L leg values of 3, 5, 7).

This formula is the equation of motion of the three sections of the leg in a form that is best for muscle forces.

For the four variable quantities φ''_h, D_{m_h}, D_{sh}, and D_{eh} three can be determined empirically for each moment of the oscillatory movement. These are: the angular acceleration φ''_h ; the turning moment of gravity D_{s_h} ; and the resulting turning moment of the effective forces D_{e_h}.

From (328) we can calculate D_{m_h}:

$$(329) \quad D_{m_h} = m_h x_h^2 \varphi''_h - D_{s_h} - D_{e_h}$$

To determine the values that the three magnitudes x_h, D_{s_h}, and D_{e_h} possess in the three various phases of movement of the oscillating leg Fischer says that it is necessary to obtain an exact kinematic analysis of the oscillating movement (s) during walking (he refers to studies by Fischer and Braune employing photographic analysis). The muscles act on the three sections of the leg with far greater turning moments than does gravity. The oscillation of the leg in walking is not the mere swinging of a pendulum; again, muscles act more than does gravity. Fischer concludes that in his muscle studies he has gone from the field of kinetics into the field of statics.

B. Some Applications to Processes of Motion in Machines (pp 349-364)

21. The Resulting Mass Pressure at the Crank Mechanism and Its Balance (pp 349-359)

22. The Motions of a Physical Pendulum with Rotary Bob (pp359-364)

These two Sections are not included in the resumé.

DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

OR are
Blank pgs
that have
Been Removed

BEST
AVAILABLE COPY

BEST
AVAILABLE COPY

THE HUMAN MOTOR

(Condensed from J. Amar)

THE HUMAN MOTOR

(Condensed from J. Amar)

The book is divided into six major sections: I The General Principles of Mechanics; II The Human Machine; III Human Energy; IV Man and His Environment; V Experimental Methods; VI Industrial Labor.

I <u>The General Principles of Mechanics</u>. (pp 1-84)
I <u>Statics and Kinetics</u>. (pp 1-30)

All movement is relative, e. g., a walking man moves relative to the Earth. A moving body is a point which traces a path, either straight or curved. Movement is uniform or variable, i. e., space is passes over at equal intervals of time or unequal.

The unit of time equals a second. Let speed equal V. At end of t seconds a body will pass through space, so that $s = vt$ (equation for body moving at a constant speed). Increase of speed equals acceleration equals f. Speed of a moving body is $V = \frac{ds}{dt}$ (speed equals differential of space with regard to time). An accelerated rate of change of velocity, $f = \frac{dv}{dt}$.

In the speed of falling the speed at moment of release = 0; f at 1 second, ft at end of t seconds. Average speed is $Vm = \frac{0 + ft}{2} = 1/2\ ft$. At an even rate of space covered $s = vm\ xt = 1/2\ ft xt - 1/2\ ft^2$. To illustrate these principles Amar presents curvilinear motion of a path around the circumference of a circle and the movements of a pendulum.

There are two types of movement, rectilineal (R) and curvilineal (C)

Figures 41 and 42. Rectilinear and Curvilinear Trajectories

Figure 43. Vector MM', Speed per Second

<u>Rectilireal</u> movement has a path xx' or x'x (either direction). If speed is MM' in a second, it can be represented by a straight line MM';

if speed from M to M' is uniform, MM' is a vector (M equals origin, M' equals extremity). This is true also of M'M.

Curvilineal movement shows M to M', with time t (moving along chord . . . .). The vector will be the tangent MV.

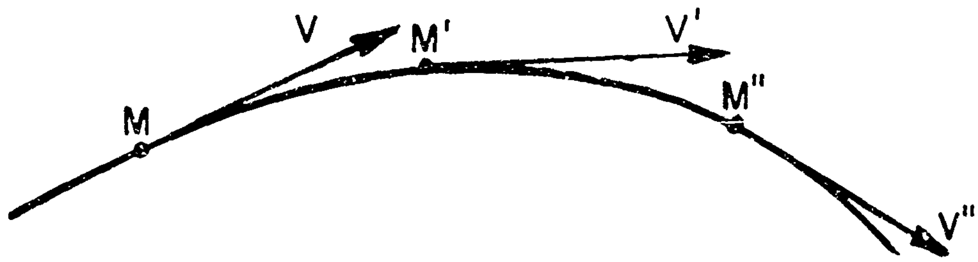


Figure 44. Vectors MV and M"V" with Variable Velocity

Velocity at M" will be M"V". If velocity is uniform vector MV = M"V". In variable velocity vectors MV and M"V" will be of different lengths, proportionate to the velocities of M and M", respectively. Acceleration can be considered a vector. Any movement can be defined by an equation.

1) simple harmonic motion $s = a.\ \sin 2\pi \frac{t}{T}$,

2) rectilineal movement with uniform acceleration is: $s = 1/2\ ft^2$,

3) movement at constant velocity is: $s = vt$.

Any motion can be graphed:

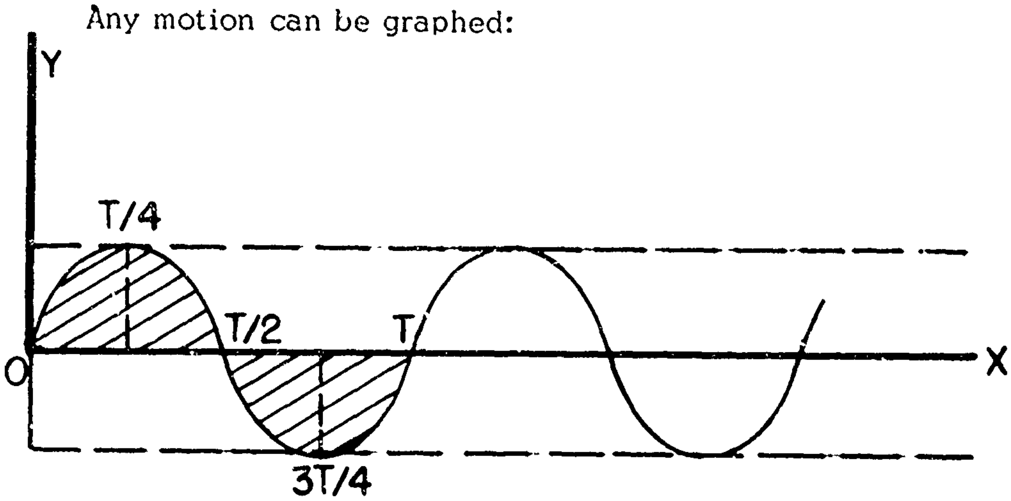


Figure 45. Diagram of Motion

OX and OY are axes of coordinates, with O as origin. Time is the abscissa with values of t from zero to T (the period) and corresponding values of the displacement as the ordinate. Then we have the curve OT. If $t = O$, then $s = O$. If $t = \frac{T}{4}$ then $s = a$. If $t = 1/2\,T$ then $s = O$. The curve is repeated periodically (sinusoidal). Thus, the movement has the equation $s = a.\ \sin 2\pi \frac{t}{T}$.

Movements of bodies may be:

1) Translation - moved without turning, so that each of the straight lines remains parallel.

2) Rotation - revolved on a straight line or axis; each point has the same angular speed of rotation (axis either horizontal or vertical).

3) Helicoidal - turning on axis, with displacement along the length of the axis. There is both translation and rotation here, with movement to right or to left. The vertical distance between two revolutions of a helix is the pitch.

Jointed systems involve the transformation of one nature into another, e. g., a crank, which transforms rectilineal movement into continuous circular movement.

Time and space convey the idea of speed. The unit is a second or 1/1000 thereof. Movement is a resultant of force. An example is the force of gravity (g). The acceleration of a falling body = 9.81 m at the end of a second: $s = 1/2\, g^2$.

Force is a calculated quantity, a vector. It has sense, direction, magnitude.

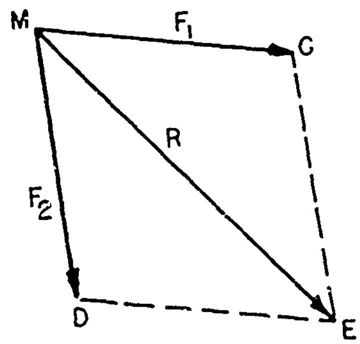


Figure 46. Diagram of Composition and Resolution of Forces

In Figure 46 the composition and resolution of forces F_1 and F_2 and R measure composing and resultant intensities. In the triangle MCE, $CE = F_2$ and $R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos M\hat{C}E$. Therefore, angles at C and M are supplementary (together 180°). Hence, the + cosine of one = the cosine of the other: $- \cos M\hat{C}E = + \cos$. CMD = $+ \cos F_1F_2$. Finally, $R^2 = F_1^2 + F_2^2 + 2F_1F^2 \cos F_1\hat{\ }F^2$. Knowing the two forces and the angle which they form it is possible to deduce the intensity of the resultant.

The general method of the decomposition of force is that of three rectangular axes.

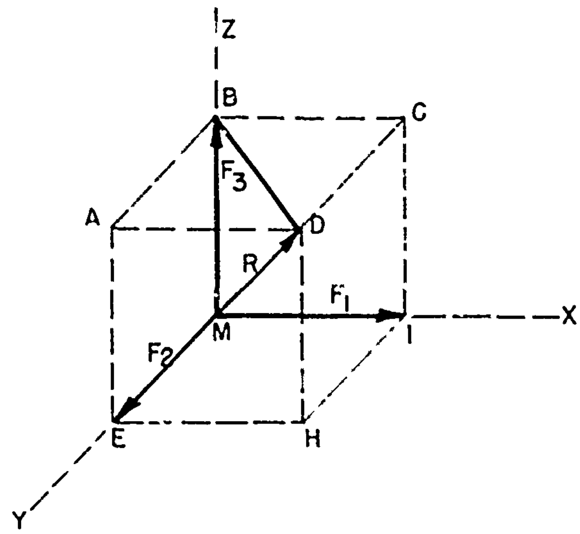


Figure 47. Diagram of Decomposition of Forces in Three Rectangular Axes

In Figure 47 R is the known force, and F_1, F_2, F_3 are forces to be determined along axes X, Y, Z. By projecting the planes the parallelopiped ABCDEMHI is formed, which will give the desired forces the intensities F_1, F_2, F_3. In the right-angled BMD it is found that $\overline{MD}^2 = \overline{BM}^2 + \overline{BD}^2$, and in the right-angled triangle BDC, $\overline{BD}^2 = \overline{BC}^2 + \overline{CD}^2$. Hence, $\overline{MD}^2 = \overline{BM}^2 + \overline{BC}^2 + \overline{CD}^2$, i. e., the square of the resultant = sum of the squares of the components. Thus, $R^2 = F_1^2 + F_2^2 + F_3^2$ or $R^2 = X^2 + Y^2 + Z^2$. The directing cosines here may be written $F_1 = R \cos \alpha$; $F_2 = R \cos \beta$; $F_3 = R \cos \gamma$ "Forces always act upon a point or a material body as if they were independent. Their resultant effect is their algebraic sum." In equilibrium two opposite and equal forces of the same intensity acting on one point cause no displacement. In parallel forces if two divergent forces, F_1 and F_2, change direction and become parallel, the point of convergence is infinity, and the resultant will be a straight line joining the points of application of F_1 and F_2.

Two bodies have the same mass when under the action of equal forces they take the same acceleration. If acceleration differs, then masses differ. In general mass is proportionate to acceleration:

$\frac{F}{m} = f$; $\frac{F}{m'} = f'$; $\frac{F}{m''} = f''$ . . . .; or $\frac{m}{m'} = \frac{f'}{f}$; $m' = \frac{f''}{f'}$. . . . For forces F and F' producing the same acceleration f on masses m and m' $\frac{F}{F'} = f$, $\frac{F'}{m'} = f$, from which $\frac{F}{F'} = \frac{m'}{m}$ . . . . It follows that $F = mf$ (force = product of mass times acceleration), and can be measured by the product.

The intensity of gravity is $P = mg$ (where P = weight of a body). The acceleration of gravity is constant, so $P' = m'g$, $p'' = m''g$. Thus, the weights of bodies are in proportion to their masses $\frac{P}{P'} = \frac{m}{m'}$ . . . . where $P = mg$. If the unit of mass $m = 1$, then $P = g$, and the intensity of gravity will be that of the acceleration $\underline{g}$.

At Paris $\underline{g}$ = 9.81m per sec.

The <u>unit of mass</u> = mass of cu. cm of H_2O at 4^oC (the <u>gramme</u>). <u>Unit of length</u> is the <u>centimeter</u>.

Hence force of weight $p = mg$ will be p = 1gr x 980.97 cm. = 980.97 gr cm. This is the <u>dyne</u>, which is $\frac{1}{g}$ or $\frac{1}{981}$ of a gr. Since $\underline{g}$ is constant the gramme becomes the unit of force or weight.

The <u>center of mass or gravity</u> is the point in a body through which the resultant of all weights of its parts passes, in every position the body can assume. If all forces representing the effects of gravity on the molecules of the body are composed, the resultant is at the center of bulk, which is the center of gravity. A vertical line traversing a body at its center of gravity is its <u>axis of gravity</u>. In a composite body the center of gravity of each part can be found, and forces of gravity composed from these (e.g., a dumbbell).

Conditions of <u>equilibrium</u> of a body depend on the effects of gravity and the reaction on a body of its support. To produce a state of equilibrium a body on a surface must have its axis of gravity passing through a <u>base of support</u> (either a point upon which it rests, or within the base upon which it rests).

II <u>Dynamics and Energetics</u> (pp 31-59)

Forces are either <u>external</u>, as gravity, reaction, pressure, or <u>internal</u>, equal and opposite, acting between various points of the system, (these latter have a zero resultant). It is assumed that the mass of the system is concentrated at the center of gravity, or G. Movement of the system will be the same as that of point G, acting under external forces only. The body will describe a parabola in space, influenced by its initial speed and by G (it is the center of gravity of the body which will describe the parabola).

In movement of a material system the product mv (mass x velocity) must be considered. This is momentum. The force giving mass a speed, dv in space of time dt gives $F = m\frac{dv}{dt}$ or F = mf for the duration of one second. In the element of time dt the force has an "impulse" F x dt which is equivalent to M x dv. From the instant zero to instant t the equation F dt = mdv will be the sum of several similar products. In integral calculus this sum has as its sign.

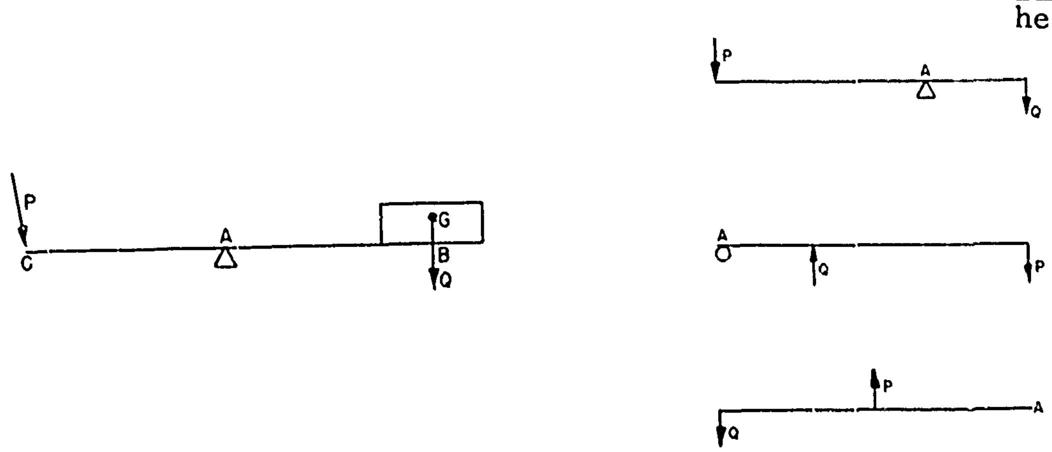
$\int_0 Fdt = MV - MV_0$. This equation states that in rectilineal movement "impulse " Ft develops momentum mv. Example, to stop a wagon of 60 kgs , at speed 5 m, in 3 seconds, a force of 10.2 Kgs is necessary, because $F = \frac{MV}{3} = \frac{60}{9.81} \times \frac{5}{3} = 10.2$ Kgs.

When a force acts to produce or retard the displacement of a body (a point or system of points) it performs work, which is the product of force by displacement, ℓ, in its own direction: $T = F \times \ell$. A force of 1 kg displacing a body 1 m on a path in its direction performs work = 1 kilogrammeter (kgm). Unit of work done by 1 dyne for 1 cm of displacement = erg. Since a dyne = $\frac{1\ gr}{981}$ the erg will $= \frac{1}{981} \times 1$ cm , or $\frac{.001\ kg}{981} \times .01$ m $= \frac{1}{981 \times 10^5}$ of a kgm , i.e. a kgm = 9.81×10^7 ergs or nearly 100 million ergs.

In general work done may be expressed as the integral

$$T = \int_0^t Fdl \cos \alpha \quad .$$

Live power, arising from stored-up work is $1/2\,mv^2$ or energy, which is the capacity for work (either potential or active and kinetic). Heat = work, and vice versa. "A constant relation of equivalence exists between work and heat." The heat needed to raise 1 kg of H_2O from O°C to 1°C = "Grande Calorie", or "kilo calorie" = C. An amount of work of 426.4 kgm. = 1 C, or inversely 1 kgm $= \frac{1}{425}$ C. This defines the mechanical equivalent of heat as $\frac{work}{heat} = 425 = E$



Figures 48-49. Diagrams of Principles of Levers

III Resistance of Materials - Elasticity - Machines (pp 60-84)

A lever (Figures 48-49) is a rigid bar, movable around a fixed point, the fulcrum (A). Resistance (Q) to be overcome is a force or weight at (B) of the lever. Power is applied at the extremity (C).

This is a lever of the first order

If the resistance is at the center of the lever, this is a lever of the second order

If the power is at the center of the lever, this is a lever of the third order

Amar presents a summary of the systems of units: unit of time = second; unit of length = centimeter (meter, kilometer; English mile = 1,609.315 m , knot or nautical mile = 1.855 m). Unit of mass = gramme (kilogram)(metric ton = 1000 kgs). The cm-gr-sec system = C.G.S. system, adopted in 1881. Unit of force = force which gives the mass the gramme an acceleration of 1 centimeter (this is the dyne). Acceleration of gravity at Paris = 981 cm ; the dyne is smaller than the weight of 1 gramme ($\frac{1}{981}$ gr) or 1 gr = 981 dynes. The unit of work = erg, which is work of the dyne through a distance of 1 cm. As 1 kg = 981,000 dynes and 1 m = 100 cm , it follows that the kilogrameter, the practical unit = 981,000 x 100 = 981 x 10^5 ergs. Also useful is a joule (10^7 ergs); $\frac{10^7}{981 \times 10^5} = \frac{1}{9.1}$ = .1019 kgm (.102 kgm).

The power of a motor = the number of units of work per second. A watt is the unit of power which gives 1 joule per second or 0.102 kgm per sec. In industry a poncelet = 100 kgm ($\frac{100}{.102}$ = 981 watts), a horsepower = 75 kgms ($\frac{75}{.102}$ = 735.75 watts).

In the equivalence between mechanical and thermal energy, 426.4 kgm is transformed into 1 grand calorie (C), which is the amount of heat able to raise 1kg of H_2O from O° to 1°C. A grand calorie = 426.4 x 981 x 10^5 = 418.3 x 10^8 ergs, or 4183 joules. If only 1 gr of H_2O is to be raised 1°C this will give a small calorie (c): 418.3 x 10^5 ergs = 4.183 joules.

Here are summarized formulae of dimensions. Length x length = area; length has one dimension, l , while area has two dimensions, l^2, and volume has three dimensions = l^3. Speed is the quotient of length x time, expressed as the dimension $\frac{l}{t}$ or lt^{-1}.

Acceleration is the quotient of speed x time, with the dimension $= \frac{lt^{-1}}{t} = lt^{-2}$. Force, f = m will have the dimension mlt^{-2}, hence work = fl = ml^2t^{-2}, etc. Work done may be given by the equation T = fxl = 1/2 mv^2 (in 1/2 mv^2 the speed, v, is that of the moving body; but it must not be multiplied by time and written 1/2 mv^{2-t}. The formulae of dimension are ml^2t^{-2} for 1/2 mv^2. Hence, if time were introduced the formulae would no longer be homogeneous).

II. The Human Machine (pp 85-164)

I. The Human Structure (pp 85-116)

The internal activity of protoplasm to maintain a continuous dynamic state demands water, food, oxygen, and heat (37.5°C for Man). In the vertebrate body bones = levers, muscles = powers. Bone density shifts from 1.87 to 2.00 in the first 30 years in Man. The heads of the long bones reveal the curve of pressures in the arches.

The breaking stress of bone is derived from $R = \frac{Ks^2}{h^2}$,* the section being squared, the length equal at the most to 15 x the side of the section; the coefficient K = 2.565 is in kgs , s in sq. cm , and h in decimeters.

From Young's modulus $\frac{1}{\alpha}$ = E (where E = the elastic force or "souplesses") the following is given:

Table 40. Breaking Stress in the Femur and Fibula

Bone	Sex	Age	Density	Young's Modulus	Breaking Stress per mm^2
Femur					
	M	30	1.984	E = 1,819	10,500 kg
	M	74	1.987	2,638	7.300
	F	21	1.968	2,181	6,870
	F	60	1.849	2,421	6,400
Fibula					
	M	30	1.997	2,059	15,030
	M	74	1.947	?	4,335
	F	21	1.940	2,710	10,260
	F	60	0.799	?	3,300

(Amar, p. 91)

---

* This is from Rondelet and Hodgkinson on different woods. The breaking stress increases as the square of the section, and, inversely, diminishes as the square of the length.

Young's modulus averages 2,300 kgs per mm^2. The average resistance to fracture by traction equals 12 kg in adult M, 6 kg in an old M; it is less in F, and also decreases with age. Here a modifocation of Hodgkinson's formula is used in fracture by compression (1/6 higher than in traction: R = 14 kgs on the average): $R = 2,700 \times \frac{s^2}{h^2}$, where h cannot exceed 1/2 decimeter and prisms must approach a cube. Under compression for a cu e of 3 mm R = 16 kg per mm^2, and for a cube of 5 mm, R = 15 mm^2. On prisms only 10-12 kgs is obtainable.

Rate of application of compression stress is very important, since pressure in shock has a high value : the impulse is $Ft = mv$, and hence $F = \frac{mv}{t}$; for a short duration tF may have a very high value.

In flexion and torsion the resistance of the long bones is increased because thay are hollow. Amar cites Lesshaft and Messerer on the weights (kgs) producing the initial fracture (H = humerus, R = radius, U = Ulna, F = femur, T = tibia, Fi = fibula) :

	H	R	U	F	T	Fi
Male, age 31	850	535	550	1300	600	300
Female, age 24	600	390	310	1100	650	310

To produce a complete fracture one needs 2900 kgs for the femur. 4100 kgs for the tibia.

The weights (kgs) needed to produce rupture by flexion and shearing are :

Table 41. Breaking (Weight) in Flexion and Shearing

Bone	Flexion	Shearing	Zone of Rupture
H	120-300	250-505	ends
R	55-140	105-334	mid
U	70-140	9[illegible]-235	everywhere
F	230-475	400-810	neck
T	135-500	450-1060	lower end
Fi	21-55	20-61	mid

(Amar, p. 93)

Torsion, acting at the end of a lever of 16 cm , produces a spiral fracture (values in kgs): H = 40, R = 12, U = 8, F = 89, T = 48, Fi = 6. Other values of interest are: 1) lumbar vertebrae compressed vertically break in M of 30 years at 1000 kg , F of 80 at 2400 kg ; 2) thorax compressed transversely (ribs broken), M of 30 at 200 kg , F of 82 at 40 kg ; 3) thorax compressed sagittally. M or 40 = 60 kg , F of 82 = 40 kg ; 4) pelvis crushed laterally at 180 kg ; 5) sacrum crushed sagitally at 170-250 kg.

Resistance of bone increases up to old age and is greater in M than in F. Health and diet are also modifying factors.

Striated <u>muscles</u> are agents of movement. The more fibers in a muscle, the greater its resistance, i.e., resistance is proportionate to the mass of a muscle. For the M. sartorius Amar quotes Wertheim:

Sex	Age	Density	Young's Modulus	R per mm^2
M	1	1.071	E = 1.271	0.070 kg
M	30	1.058	0 [illegible]2	0.026
M	74	1.045	0.[illegible]1	0.017
F	21	1.049	0.857	0.040
F	60	1.040	?	?

A fresh cadaver muscle has an E = 0.95, and an R of 40 g per mm^2. Values decrease with age, and the E is higher (resistance greater) in a contracted muscle. Tonicity is a variable factor.

For <u>tendons</u> and <u>cartilage</u> the data are as follows (the Plantar tendon):

Sex	Age	Density	Young's Modulus	R per mm^2
M	35	1.125	E = 139.42	4.910 kg
M	40	1.124	134.78	7.100
M	74	1.105	200.50	5.390
F	21	1.115	164.71	10.380
F	70	1.114	169.21	5.610

On the average a tendon has a modulus of 146 and a force of cohesion of 7 kg. On resistance to traction Triepel found R = 5, about 1/2 that of bone. The resistance of cartilage to fracture is greater in compression than in traction, i.e., 1.5 kg compared to 0.18 kg (Triepel). Young's Modulus is about 1.50, or about that of muscle.

The resistance of nerves is great: 25 kg to break the ulnar, 55 kg to break the sciatic. R = 2-3 kg per mm^2, Young's Modulus = 10.9. Nerves are the best conductors of electricity. If 100 is taken as conductivity, K, in muscle, then muscle = 100, nerve = 588, tendon = 30, bone = 7.

Here are comparative data:

Table 42. Elastic Properties of Various Materials

Substance	Density	Young's Modulus	R per mm^2
Bone	1.95	E = 2,300	12 kg
Muscle	1.12	146	7
Tendon	1.05	0.95	0.4
Nerve	1.04	10.9	2.5
India Rubber	0.92	variable	0.6
Silk Thread	1.33	650	27.5
Spider Thread	1.58	306	18.35

(Amar, p. 101)

The viscous nature of muscles and tendons renders them very susceptible to deformation. Fatigue is a very potent factor.

Joints are bones in contact at "jointed surfaces", which are flat or curved (spherical, cylindrical, oval). Opposing joint surfaces are covered by cartilage. Joint movement varies from synarthrosis or fixed (suture as example) to diarthrosis or freely movable (e.g. bones of limbs or fingers, which act like levers). The shape of joints is so variable that there may be 5-6 classes, according to degrees of freedom. The freest movement = 6 degrees of freedom (3 translations, 3 rotations). Since at joints the contact surfaces are curved they often have 1, 2, 3 axes and 1, 2, 3 degrees of freedom (found in cylindrical, oval, or spherical joint surface). Systems with one degree of freedom are said to have "complete connection".

A spherical joint surface has one axis and permits one degree of freedom. As a rule the surface has a groove which forms part of the arc of a circle. Movement is in one plane. It is therefore a hinge joint (e.g., the fingers). A pivot joint has a longitudinal rotation around a single axis (e.g., the radius). If the bone has a helical joint (the furrow S, is oblique) then there is translation + rotation: the rotation is at maximum 180° and the translation is limited by its connections. Hence, there is really only 1 degree of freedom, i.e., rotation (e.g., elbow joint, tibio-talar, knee joint).

Movement with two degrees of freedom occurs in oval joints with two very unequal axes (ellipsoid) or nearly equal (spheroid). The larger axis is perpendicular to the limb, i.e. transverse.

If the surfaces fit tightly the only movement possible (Fig. 50) is around the larger axis, A B; e.g. the radio-carpal joint has basically only one degree of freedom, but there is slight movement around

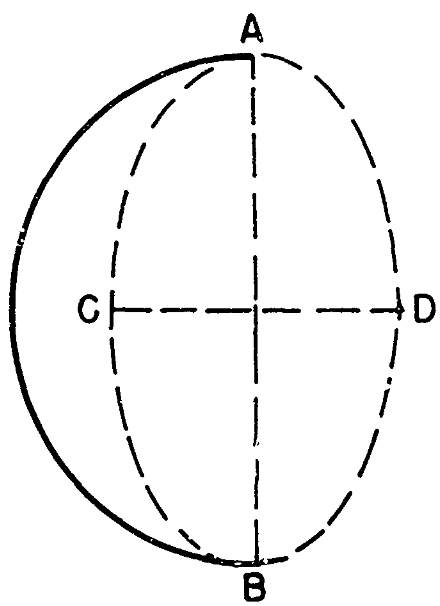


Fig. 50. Oval Joint with Two Degrees of Freedom

axis C D also. Rotation, or torsion with flexion or extension is here produced.

Movement with three degrees of freedom occurs in "ball and socket joints" of shoulder and hips. Here there are possible three rotations around three rectangular axes. These three axes are instantaneous axes, but a bone considered in repose, the femur for example, presents a frontal axis around which the limb bends and a sagittal axis for lateral movement.

When two or more joints act together the degree of freedom increases, as in the radio-carpal joint, the vertebral column, etc.

Amar feels that body shape as a whole is that of a prism. The upper 1/2 of the body is called the bust. Basic body measurements are height, weight, sitting height, thoracic circumference (at nipple), and span. The "thoracic coefficient" = SH/H.

There are two main body types: lower limbs long relative to SH (bust); and lower limbs short relative to SH. Vital capacity, via spirometer, averages 3.75 liters of air for male, 2.75 liters for female. Vital capacity increases 0.42 liters per annum up to 30 years, and increases 0.05 and 0.04 liters per cm. increase in stature, in males and females, respectively.

There is a relation between the surface area and the volume of the body, which is expressed as the relation between volume, weight and density: $V = \frac{P}{D}$ (where P = wt.). If the density (D) is known, volume (V) can be calculated from weight (P). In practice D is determined by the volume, from displacement in water. D is generally taken as 1.035. The surface area of the body is theoretic: assume a cube, with volume V; the side of the cube = $\sqrt[3]{V}$, and the total surface of the 6 faces is $6(\sqrt[3]{V})2 = 6\sqrt[3]{V2}$. It is possible to substitute weight for the volume and write $S = K\sqrt[3]{p^2}$; K changes with body shape, which is really not cubic. K for Man = 12.312. The surface is therefore $S = 12.312\sqrt[3]{P^2}$ (where P is in grammes and S is in cm^2 (Meek). Example: adult male of 65 kg has a surface $S = 12.312\sqrt[3]{65,000^2} = 19.896\ cm^2$ or $1.99\ m^2$ (really $2\ m^2$).

If two individuals of different weight are considered (e.g., an adult and a child) their surfaces will be to each other as the cube root of the square of their weights; hence surface area diminishes less rapidly than weight.

$$\frac{S'}{S} = \frac{12.312\sqrt[3]{P'^2}}{12.312\sqrt[3]{P^2}} = \frac{\sqrt[3]{P'^2}}{\sqrt[3]{P^2}} = \sqrt[3]{\frac{P'^2}{P^2}}$$

Let $P' = 1/8P$, then $\frac{S'}{S} = 1/4$. Thus the weights vary from 8 to 1, but the surfaces will vary only from 4 to 1. Therefore, children have a larger surface area relative to their weight than do adults.

In a discussion of weight Amar presents Bouchard's "segment anthropometrique" (A), where $A = \frac{\text{wt. in kg}}{\text{ht. in dm*}}$. The following is given in terms of values for A

Normal nutrition	3.9 F, 4.0 M
Emaciation	3.6
Obesity	5.4
Marasmus	2.9
Extreme Marasmus	2.0

Example: Adult male = 65 kg , 16.8 dm.

$$A = \frac{65}{16.8} = 3.87 \text{ which is near to 4.0 normal.}$$

II The Muscular Motor and Alimentation: (pp 117-138)

The "Muscular Motor", according to Amar, is where the muscle = the motor, muscular contraction = the force. A muscle functions like a heat engine. (Figure 51)

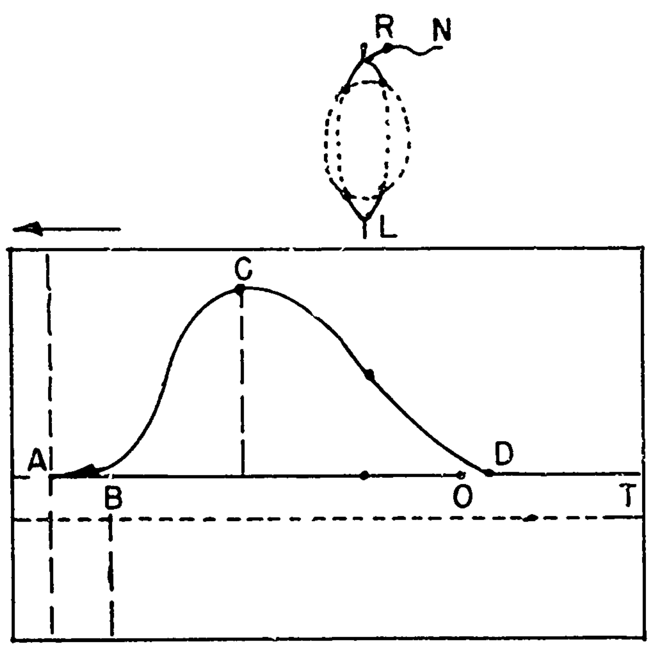


Fig. 51 Graph of a Muscle Jerk

The muscle is fixed at R, with the other end attached to an indicating style, AO, jointed to point O, and whose point moves on prepared paper. If the nerve N is stimulated electrically a trace ACD is produced, the duration of the stimulus being read in line T (12-16/100 sec.) AB is the "latent period" of 3/1000 second; this is the period between the moment of excitation and the moment of response. A single nerve impulse gives (at the rate of c. each 15/100 sec.) a curve as above. When the impulses occur at shorter intervals they tend to blend in more nearly a straight line ("physiological tetanus"). The number of impulses producing a sustained tetanic contraction pattern varies with age, temperature,

* decimeter

weight under which muscle contracts, and its initial state. In man it corresponds to 20-30 excitations per second; when there are up to 60 per sec. the muscle remains tetanized for 4-5 minutes. Above a frequency of 60 fatigue is very rapid.

The duration of a reflex decreases as the intensity of the excitation increases. The following are average values:

Tactile	reaction	14/100 sec.
Visual	"	19/100 "
Auditory	"	15/100 "

The laws of muscle contraction show that the length is reduced, the width increased, but that volume is about the same. The law of elasticity suggests that a muscle sustaining weight P will be lengthened by a quantity $L = \frac{PL}{ES}$. If by contraction it resists elongation its interval force will be equal and opposite to P. If it shortens by a quantity, r, while sustaining weight the interval force will exceed P by the effort necessary to restore the muscle to its original length. This supplementary effort will be proportionate to the shortening, so it will be P x r. The force of static contraction, to balance weight P with a shortening r will be $F = P + Pr = P(1 + r)$.

Besides the work of the muscle in displacing the weight P, the shortening takes two limiting values, r and r', to which forces of contraction F and F' will correspond, so that force has an average value of $\frac{F + F'}{2} = F_m$. Now replace F by $P(1 + r)$ and F' by $P(1 + r')$ and the result is $F_m = P(1 + \frac{r + r'}{2})$

The dynamic contraction accomplishes work via the average shortening and the average forces, F_m.

According to the laws of elasticity muscular deformation is proportionate to the length of the muscle. A bone capable of extensive movement must be operated by a muscle capable of much deformation; hence, long muscles are here best. Short, thick muscles can develop much effort because they have so many fibers.

Muscles occupy space minimally and economically; hence, as a rule, muscles are parallel to the bones they move. Muscle action (Figure 52) is a function of mass, degree of contraction, and the angle which its direction makes with the bone to be moved. A muscle has a definite moment of rotation in relation to the axis, for each of its positions. The moment of rotation = the product of force F of the muscle by the distance d (from Braune and Fischer, on cadavera, with F deduced from the size of the muscle section). There results M = F x d. The moment varies with flexion up to a certain value:

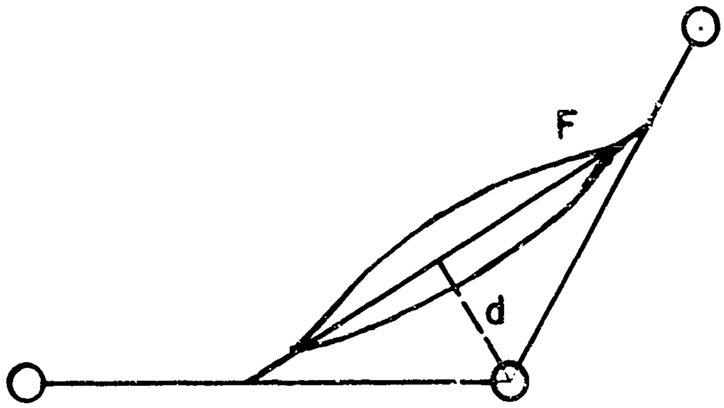


Fig. 52. Diagram of Muscle Action

when the arm is fully extended the product of F and d is not zero, for in a human body muscle power generally acts upon levers of the third order.

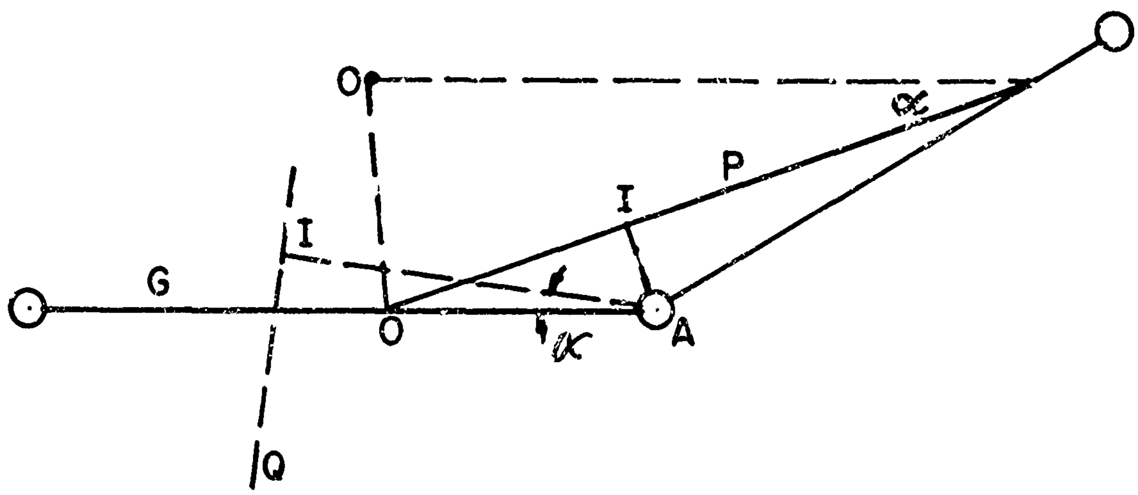


Fig. 53. Diagram of Stress Applied at the Center of Gravity of a Mobile Limb.

If stress is applied (Figure 53) at the center of gravity (G) of a mobile limb, the useful component of power will be OO' = P sin α , i.e., effort of flexion varies as the sine of the angle of the inclination of the muscle to the mobile limb. The arm of the lever A1' of the power is smaller than the A1 of the resistance. The amplitude of movement of this resistance is increased in the relation of $\frac{A1}{A1'}$ and produces a large number of useful actions.

Muscular force: absolute force is based on the fact that force of a muscle increases in proportion to its contraction. If it is loaded to prevent shortening the "absolute" effort can be measured. On the living subject a particular muscle is chosen with a known section; the kind of a lever to which it is attached is noted, and the value of the maximum static effort is calculated.

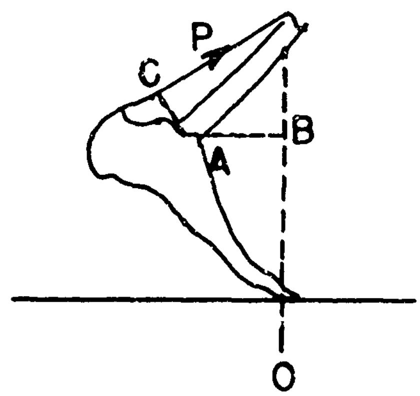


Fig. 54. Forces Involved in Lifting the Seated Body

As an example a man of 70 kg weight loaded with 70 kg cannot lift himself when seated. The resistance is 140 kg, and

$$P \times Ac = Q \times AB \text{ ; whence}$$

$$P = Q \frac{AB}{Ac} \qquad \text{(Figure 52)}$$

Hermann (1898) calculated P by determining the relation $\frac{AB}{Ac'}$; he obtained a force of mm^2 of the calf muscles of 62.4 grammes, for arm flexors of 50-80 g, and for the Masseter 90-100 g. The average in Man equals 75 g. Via dynamometers great individual variability can be shown. In traction both hands give an average of 45 kg. The back pull (called "renal pull" by Quetelet) is greater in M than in F, and increases up to about 40 years, when it decreases. In the prime of vigor an adult M = 150 kg, an adult F = 78 kg. For races in the "renal pull" Gould reported as follows: American white 144.4 kg, American Negro 146.7 kg, Mulatto 158.3 kg, American Indian 159.2 kg.

The periodicity of voluntary muscle contractions shows a variability: impulses in Biceps brachii equals 50 per sec., in Masseter equals 60-65, in the flexors of the fingers equals 8-11. Voluntary contraction is similar, in principle, to "psysiological tetanus". Static effort leads to real vibratory movement, i.e., internal work. There must be an expenditure of work to produce an effort. The elastic work of traction is $T = 1/2 \frac{ESl^2}{L} = \frac{Fl}{2}$, and the differential of the work in respect to the deformation will be $\frac{dT}{dl} = \frac{ESl}{L} = F$.(6)

The contraction of a muscle and its tetanic character, where voluntary, alternates like the stroke of a piston. The maximum rhythm is illustrated to vary: fingers, 8-9 strokes per sec. (480-540 per min.); jaw 360; foot 210; great toe 250; forearm 230-240; leg 120 (against the thigh). The work of the fingers is the most rapid; there is probably a special rhythm for each muscle or group of muscles.

Joint movement speed also varies; phalanges 300-400 strokes per min., wrist 600, elbow 530, shoulder 310. Values for "unloaded" muscles are: forearm 30-35; masticatory 90-100; fingers 150; heart 70.

Short people are relatively stronger than tall people, and quicker, because weight decreases as the cube of size, while "force" decreases as the square of the size, being proportionate to the section of the muscles.

Amar discusses alimentation, classification of food stuffs, and the composition of foods: certain tabulations may be noted.

Food	Caloric Power	Heat of Combustion
1 g carbohydrate	4.10 cal	4.20 cal
1 g fat	9.10 "	9.40 "
1 g proteid	4.10 "	4.75 "

	Carbohydrate	Fats	Proteids
Heats of combustion	4.15 cal	9.40 cal	4.40 cal
Calorific power	4.10 "	9.10 "	4.10 "
Coeff. of digestibility of normal alimentation	0.98 "	0.95 "	0.91 "

III Alimentation and the Expenditure of Energy (pp 139-164)

An "alimentary ration" is given to a determined weight of aliments. For example, 100 g bread and 100 g dried beans gives:

carbohydrate	107.25 g	or 439.77 cal	(107.26 x 4.10).
fats	1.60 g	or 14.56 cal	(1.60 x 9.10).
proteids	28.23 g	or 115.75 cal	(28.23 x 4.10).
Total =		570.08 cal	

For maintenance rations 1 g proteid is needed for each kg of body weight. To maintain body temperature intake must be greater in Winter.

The evaluation of the ration involves the method of nutritive evaluation, which is to adjust food intake to body weight (from tables of caloric equivalents). The oxygen method is to study the O_2 balance in expired air (over a given period of time). An adult male in repose consumes 26 liters of O_2, measured at $0^oC.$, at normal pressure of 760 mm. It is necessary to determine the ratio between the amount of energy expended, and the corresponding volume of O_2 consumed. Energy is proportioned to the O_2 and depends on the nature of the combustible. "Mixed alimentation" has a value of 4.90 cal for the energy developed by the consumption of a liter of O_2 at $0^oC.$ and 760 mm. pressure. Here are examples:

	Carbohydrates	Fats	Proteids
Calorific power of a liter of O_2 at 0^o and 760 mm.	5.05 Cal	4.70 Cal	4.53 Cal
Respiratory quotient CO_2/O_2	1.00	0.71	0.82

Some foods act on living matter directly, i.e., on muscle fibers and nerves: coffee, tea, mate, cocoa, kola, pimento, butyric acid, sodium chloride, ordinary alcohol. These are "aliments of economy", or "dynamophores", or "nervine aliments".

Rations and energy expenditure (with calorimeter chamber at 20^oC) may be illustrated as follows:

Table 43. Caloric Expenditure and Weight

Body Weight (kg)	Static Expenditure (Cal)		Gross Dynamic Expenditure (Cal)	
	calc.	measured	calc.	measured
65	2, 119	2, 133	3, 559	3, 544
70	2, 279	2, 283	3, 892	3, 861
70	2, 305	2, 337	-	-
70	2, 357	2, 397	5, 143	5, 135
Error	+0. 96%		-0. 43%	

(Amar, p 151)

For a man at rest 24 hours p. d. the average is 32. 56 Cal per kg of body weight at 20°C. Such an adult male of 65 kg expends statically 32. 56 x 65 = 2, 120 Cal p. d.

In an adult male, age 22, robust, with weight 76 kg, at 20°C, on a mixed diet, the data on static energy expenditure were as follows (24 hr period): static expenditure 2, 397 Cal; H_2O exhaled and perspired 881 g ; CO_2 eliminated 8. 12 g ; total O_2 consumed 689 g ; urine weight 1421. 8 g ; volume of CO_2 $\frac{812.10}{1.9769}$ liters; volume of O_2 $\frac{689}{1.429}$ liters; respiratory quotient $\frac{CO_2}{O_2} = 0.853$.

In deep sleep the expenditure per kg of body weight is 7/10 of waking.

The same subject for gross dynamic expenditure (static + dynamic), with 8 hrs of work, or 603. 8 cal on a bicycle, showed as follows: 5, 176, 62 Cal of gross dynamic expenditure; 3297. 6 g of H_2O exhaled and perspired; 1759. 7 g of CO_2 eliminated; total O_2 consumed 1558. 8 g ; urine weight 2401. 8 g ; $\frac{CO_2}{O_2} = 0.815$. It is seen that a ration producing 5176. 2 Cal is needed for this subject when work = 603. 8 Cal or 260, 000 kgm.

Energy expenditure is influenced by the amount of mechanical work, the mass of the body, and the external environment.

III Human Energy (pp 165-214)

I The Laws of Energetic Expenditure (pp 165-186)

Static muscular work of tonic contraction is the equivalent of frictional or vibratory work. There is some force, with variable duration and speed, with fatigue resulting. The basic studies are those of Chauveau (1899), who stated that F = P (1 + r); for the same effort F and the same time the expenditure is: 1) proportional to the weight lifted, P, and 2) proportional to the degree of contraction, r. In absolute mechanical work all energy is lost as heat. The only result of a static effort is heat production, raising the temperature of the contracted muscle. An example is a weight lifted by the Biceps:

Wt. Lifted	1 kg	2 kg	5 kg
Temp. rise in 2'	0.17°C	0.32°C	0.98°C
" " " 4'	0.25°C	0.58°C	1.15°C

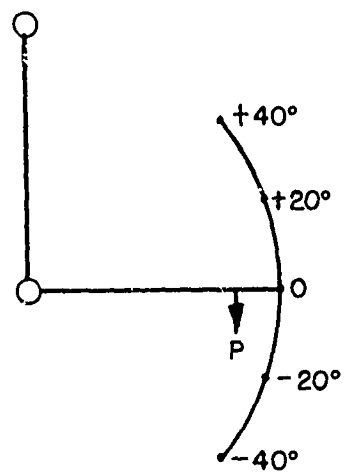


Figure 55. Diagram of Movement of Forearm Through Arc -40° to + 40°

In Figure 55 at P, a constant weight of 2 kg was lifted with the forearm flexed from -40° to +40° for two minutes. The results are:

Angle	-40°	-20°	0°	+20°	+40°
0°C	0.28°C	0.50°C	0.67°C	0.78°C	0.88°C

The effect of load and muscle contraction on the use of O_2 is demonstrated in several situations.

1. Sustained variable load by constant muscle contraction:

Weight	(a) 1 2/3 kg	(b) 3 1/3 kg	(c) 5 kg
Additional O_2	119 cc	204 cc	319 cc

Here, calories may be assumed to be 4.60 per liter. Hence, expenditure (a) = 0.547, (b) = 0.938, and (c) = 1.467 calories, respectively.

2. Effort of five kilograms, with variable angles of flexion

Angle	-20°	0°	+20°
Additional O_2	212 cc	344 cc	360 cc

The expenditure is proportionate to the static load and the duration of muscle action:

$D_e = K + P(1+r)$, where D_e = the expenditure of static effort (in excess of repose) and K = the coefficient of proportion.

In dynamic contraction the work must be considered. If a load is displaced the work of a muscle is either motive or resistant. If D_d = dynamic expenditu.:e, it wi'l be related to static effort, D_e, and to work accompolished, T:

$D_d = D_e + T$ (e. g., the quantities being expressed in calories).

The expenditure which is equal to T comprises (P x h) of the mechanical work (weight, P, lifted to height, h), and the value of friction, R, which opposes muscle movement. Also there must be included a fraction, V, which is equivalent to expenditure in starting the action. R(= to passive resistance) is included in both static contraction D_c and in the fraction V; hence, R figures twice in the expenditure of D_d and remains to be known in D_e and V:

$$D_d = (D_e - R) + Ph + R + (V - R)$$

or

$D_d = D_e + Ph + V - R$ (where Ph = work)

All values may be calories, liters of O_2, or in kgm. In an example of a man on a treadmill the expenditure, via O_2 consumed, = 257 Cal for a work of 68 Cal , and 198 Cal are registered on the calorimeter. Hence, D_d = 193 + 68 = 261 Cal. In descending the treadmill he spends 125 Cal , the calorimeter registering 164 Cal. This is written:

$$U = Q - T \text{ or } 125 = 164 - 68 = 96 \text{ Cal}$$

The practical results of Chauveau's formula are shown in the following experiments:

1. A man of 50 kg on the treadmill at a speed of 431 m per hour, constant speed, with progressive increase of load by 10 kg to modify the speed of ascension:

Weight kg	Expend. of O. cu. cm	Expend. per kgm cu. cm
50	53,700	2.49 or 0.011454
60	63,550	2.45 or 0.011270
70	82,650	2.73 or 0.012558

2. Same man, with load constant of 50 kg, but with speed varied.

Hourly speed	Expend of O	Expend per kgm	
m	cu. cm	cu.cm	Cal
302	44,900	2.97 or	0.013662
431	53,400	2.47 or	0.011362
454	60,300	2.17 or	0.009982

Economy increases with speed. The expenditure of O_2 per kgm is as given:

	cu. cm	cu. cm	cu. cm
Inc weight	1.41	1.21	1.47
Inc speed	1.68	1.32	1.12

In comparing motor and resistant work it is seen that the latter benefits more than the former from the effects of speed. If movement is gradually reduced a point is reached where expenditure is about the same, whether, for example, a man of 70 kg is ascending or descending a stair.

Hourly speed	Expend. per kgm		Ratio
	motive work	resistant work	
m	cu. cm	cu. cm	
136.80	4.30	3.32	1.29
176.50	3.76	2.18	1.72
248.70	3.70	1.82	2.03

The expenditure for motor and resistant work, resp., is about equal for a speed of 100 m per hr. Speed is an important factor in economy: increase of speed = less expenditure; increase the load = more expenditure.

Chauveau's formula ($D_d = D_e + Ph + V$) was analyzed by several experiments: the arm lifted different weights at variable heights and speeds, each lift followed by a rest.

1. Study of D_e: increased weights can be sustained by contracting the flexor muscles 13 times per minute. No work is produced.

Weights	1.5 kg	3 kg	4.5 kg	6 kg
Expend. of O_2	40 cu. cm	79 cu. cm	133 cu. cm	197 cu. cm

If the number of cor ctions is increased:

Contractions	13 cu. cm	26 cu. cm	39 cu. cm	52 cu. cm
Expend. O_2 for 1.5 kg	36 " "	44 " "	63 " "	76 " "
" " " 4.5 kg	98 " "	125 " "	163 " "	187 " "

Frequent starts and stops increase the expenditure. There is more expenditure of energy in setting heavy weights in motion.

2. Study of Ph + V. This is work itself, estimated in quantity (Ph) and in quality (V): Ph was varied, leaving h constant at 4.42 m, and also the speed, while P varied from 1.5-6 kg. In 13 contractions per min. a weight was lifted 4.42 m, i.e. 0.34 m per flexor connection: here D_d is measured, and knowing D_e the Ph + V can be found.

Weight Displaced	D_d	D_e	Ph + V
1.5 kg	99 cu. cm	40 cu. cm	59 cu.cm or 1.00
3 "	158 " "	79 " "	79 " " or 1.34
4.5 "	241 " "	133 " "	108 " " or 1.83
6 "	324 " "	197 " "	127 " " or 2.15

Variation of Ph + V is less rapid than that of the weight lifted.

D_d varies in proportion to the weight. Hence, D_e, the static effort, is most felt in the expenditure.

Now take a constant weight of 1.5 kg, increase the number of contractions, and carry it to 4.48 m, 8.84 m, 13.26 m, 17.68 m in the same time. The speed will increase as 1, 2, 3, 4.

Contractions	D_d	D_e	Ph + V
13	91 cu. cm	36 cu. cm	55 cu. cm or 1.00
26	151 " "	44 " "	107 " " or 1.94
39	199 " "	63 " "	136 " " or 2.47
52	243 " "	76 " "	167 " " or 3.03

There is increased expenditure, but there is economy also, for the same amount of work is produced with a small load and great speed. For example, a 6 kg weight is carried to the 6th floor (17.50 m.). In a single load the expenditure = 324 x 4 = 1296 cu. cm; in fractions of 1.5 kg it is 243 x 4 = 972 cu. cm, a saving of 324 cu. cm or $\frac{324}{1296} = 25\%$

By giving the value of D_d for increased weights and speeds the expenditure per kgm. may be found (in cu. cm):

For inc. weight	14.90	11.90	12.10	13.20
" " speed	13.70	11.30	10.00	9.10

The economy increases with the speed, reaching a maximum value at 3 kg, which is an expenditure of 11.90 cu. cm.

If the flexions of the arm succeed each other in a growing number, in the same proportion so that in the end the weight is carried at the same height, then:

Expenditure per kgm	cu. cm	cu. cm	cu. cm	cu. cm
3 kg weight	8.10	8.80	10.10	10.70
6 " "	9.50	9.50	12.20	13.30

The expenditure increases.

<u>In resistant work</u> the weight instead of being lifted, descends, The variations of D_d and V - Ph were :

Weights	D'_d	D_e	V - Ph
1.5 kg	66 cu. cm	40 cu. cm	26 cu. cm or 1
3.0 "	131 " "	79 " "	52 " " or 2
4.5 "	206 " "	133 " "	73 " " or 2.80
6.0 "	277 " "	197 " "	80 " " or 3.07

The increase of work with variations in speed was :

Contractions	D'_d	D_e	V - Ph
13	68 cu. cm	36 cu. cm	32 cu. cm or 1.00
26	114 " "	44 " "	70 " " or 2.18
39	161 " "	62 " "	99 " " or 3.09*
522	201 " "	70 " "	131 " " or 3.90*

From the above speed once more is the dominating factor in the economy of energy.

From the foregoing the expenditure per kgm can be deduced.

Inc. load	9.90 cu. cm	9.80 cu. cm	10.30 cu. cm	10.40 cu. cm
" speed	10.20 " "	8.50 " "	8.09 " "	7.50 " "

Again, there is a saving of 27% in expenditure, if the load of 6 kg is carried in fractions of 1.5 kg.

<u>Voluntary work</u> may be shown via the "ergograph" or "ergogram". Voluntary work is accomplished with a minimum of expenditure. If h = the vertical height of a trace of muscle contraction and P = the weight then work is one contraction = Ph. If, before fatigue sets in, there are n contractions, the total work = nPh.

The ergogram shows that each muscle group produces maximum work for a definite value of the load. If the latter is modified, then the rhythm of contraction must also be modified. If the rhythm = 6 per min., weight = 3 kg , the flexors of finger III "could work almost indefinitely." If the weight is double then the interval of repose must also be doubled. If the rest periods increase fourfold the preceding maximum work will be exceeded. Frequent short intervals of repose are more efficient than one long rest.

<u>Maximum work</u> = resultant of an effort and rhythm suitable for "practically continuous action." It is important to work <u>below</u> the limit of the effort which would have produced the maximum amount of work.

---

* Here Amar had 98 and 125, resp. I have indicated the correct D'_d - D_e values. (W.M.K.)

The maximum work of man, from a mechanical point of view, must relate to the power of the subject, the product of his effort, and his speed in the unit of time (1 sec.), and by multiplying power ($F \times v$) by the effective duration of labor in seconds; thus gives $T = F \times v \times t$. Values of these can be determined to that $F \times v \times t$ is at a maximum; T will then represent man's maximum daily output. In principle a worker without a weight could climb to height, H, in a working day, with the work done = PH. With a load, Q, he could climb only to H', or work = $(P+Q)\ H'$. This is a diminution $PH - (P+Q)\ H'$ caused by the load, Q. Via calculation Coulomb found a value of Q which gave a minimum diminution, i.e., a maximum daily output.

As an example, take a man of 65 kg , ascending a mountain in Tenerife (2,923 m), doing 189,915 kgm of work in that day. With a load of 68 kg he did only 105,336 kgm of work, a decrease of 84,579 kg or $\frac{84{,}579}{68}$ per kgm , or $\frac{84{,}579}{68} \times Q'$ for the load, Q'. Let $\frac{84{,}579}{68} = b$, and work without the load = a. Then $a - bQ'$ is the possible work that can be done. If height is reduced to h then $(65 + Q')h$ will be the new expression for the same amount of work.

From the foregoing the equation is:

$$a - bQ' = (65 + Q')h \text{ and } h = \frac{a - bQ'}{65 + Q'}$$

The useful work is that of load $Q'h$

$$Q'h = Q' \frac{a - bQ'}{65 + Q'} \text{ or } T = Q' \frac{a - bQ'}{P + Q'}$$

For T to be maximum the value of Q' will be

$$Q' = P\left[\left(1 + \frac{a}{bP}\right)^{1/2} - 1.\right]$$

Now give values so that $Q' = 53$ kg, and $Q'h = 55{,}350$ kgm. This gives $h = \frac{55{,}350}{53}$ and the total amount of work done:

$$(65 + 53)h = \frac{(65 + 53)\ 55{,}350}{53} = 123{,}232 \text{ kgm}$$

The maximum work with load is $\frac{123{,}232}{189{,}915}$, or 65% of the work without load.

Amar feels that a carbohydrate diet is more economical than a nitrogenous, for the former is more quickly utilized. A day's work with a carbohydrate diet is 4.5% more economical.

Certain conclusions may be formulated.

1. Work, compared to simple support of a load, necessitates a higher energy expenditure in the same time ; motive work is more expensive than resistant.

2. Economy of expenditure is achieved by working quickly, i.e., divide the load into fractions, and take short and frequent rests.

3. Movements of great amplitude are more economical than those of small amplitude, more often repeated; a rhythm gives the best results.

4. In a given time the maximum amount of work will be done if the resistance to be overcome decreases progressively, i.e. if fatigue regulates muscle effort.

5. Carbohydrate intake is 4.5% more economical per day in the expenditure of energy, compared to nitrogenous.

6. The law of repose must regulate the duration of intervals of rest in a working day, related to type of work.

II The Yield of the Human Machine (pp 186-198)

The yield of the human machine is the relation of mechanical energy produced to energy expended. Industrial yield, r , is the gross value deducted from the relation of useful energy to total energy (static + dynamic). The net yield, R, includes only dynamic expenditure. This can be expressed as

$$r = \frac{T}{D_s + D_d} \quad \text{and} \quad R = \frac{T}{D_d} .$$

For example, an adult male spends 100 cal per hr in repose; doing work in the amount of 25,500 kgm (60 cal per hr) he expends 340 cal. Hence:

$$D_s = 100, \; T = 60, \; D_d = 240 \text{ and}$$

$$r = \frac{60}{340} = 17.6\% \qquad R = \frac{60}{240} = 25\%$$

If the value of the net yield of a subject is definitely determined, the amount of expenditure will give the value of work done:

$$T = R \times D_d .$$

To get D_d, the O_2 consumption before the trial, and when normally working, is calculated. From the ratio 4.90 Cal per liter (at 0^oC and 760 mm) can be calculated the energy, D_s and D_d, in a period of time (e.g. 30 min.) Comparisons are possible only within the same occupation.

Now consider the net yield of the muscles in the equation $R = \frac{T}{D_d}$, where T = the product Ph of weight, P, of the subject and of height, h, of the staircase mounted. D_d is not equal solely to this amount of work, because the amount of heart and respiratory effort increases (heart-rate goes from 72 to 110 per min., and vol. of air breathed from 500 to 1200 liters per hr). The heart pumps 150 g of blood per cycle, at a pressure 1/10 of the surrounding atmosphere, equivalent to a height of 1.25 m ; this is 0.150 x 1.25 = 0.1875 kgm of work done. In repose 0.1875 x 72 = 13.75 kgm per min. ; at work 0.1875 x 110 = 20.625 kgm per min., air increase of 6.875 kg per min., or 412.50 per hr. In respiration the pressure of the air expired varies from 0.06 m to 0.30 m of water in repose and work, resp., which gives 500 x 0.06 = 30 kgm in one case and 2000 x 0.30 = 600 kgm , in the other, both per hr The increase of work done = 570 kgm per hr.

As the volume of air is both inspired and expired the muscle work is doubled, so that 570 x 2 = 1140 kgm. Hence, D_d covers an increase of organic work = 412 + 1140 = 1552 kgm per hr.

In all this R = yield of both motor and resistant work.

In a series of experiments, mostly bicycle work, Amar showed that yield was variable. Increase in speed increased the yield up to a certain limit, then decreased it. Other consideration involve the factors of fatigue and the passive resistance of the experimental tools and/or machines used.

In an overall statement Chauveau's Ratio may be expressed as

$$R = \frac{T:\ 425}{D_d} = 0.25$$

This is an average net yield of 25%, and is the maximum in the case of the arms.

III Physiological Effects of Labor - Fatigue (pp 199-214)

The effects of work on physiological functions was considered.

There are 15-18 respirations per min. in repose, though it varies with age: 40-25 at 1-5 yrs., 25-21 at 5-15 yrs., 20-18 at 15-25 yrs., 18-17 at 25-50 yrs. In work the rate is doubled at an external temperature of 12°C - 15°C. In very heavy work, under bad conditions, the rate of $\frac{CO_2}{O_2}$ diminishes and the subject becomes "breathless."

An example is given of respiration per hr. in 45,000 kgm work, in five consecutive hours, registered by a bicycle dynamometer: (Hourly flow of the respiration): -

Days	1st hr.	2nd hr.	3rd hr.	4th hr.	5th hr.
1st	842.1	1051.1	1074.1	107[illegible].1	1080.1
2nd	878	1139	1167	1187	1188
3rd	896	1063	1122	1128	1133
Av.	872.1	1084.1	1121.1	1131.1	1133.1

In the <u>circulation</u> in the heart, systole = contraction and diastole = repose. Cardiac pulsation is measured via ventricular systole (72-75 in repose). In a tracing a systole is represented as the elevation of the curve. There is an age factor in <u>pulse-rate</u> : 98 at 5-6 yrs., 93 at 6-8 yrs., 89-87 at 8-14 yrs., 82-77 at 14-19 yrs., 75-70 at 19-80 yrs., and 79 at 80 yrs. The relation between the durations of systole and diastole is 3/4:1/4 = approximately 3.

There is also an age-factor in arterial pressure: 13-14 cm mercury at 10-20 yrs., and 20 + at 50 yrs. and older.

In work <u>pulse-rate</u> goes to 112-114 for 200,000 - 250,000 kgm in 8 hours; and extreme limit = 167. An example is given of a walk of 38 m. at increasing speed:

Pulse	= 84	85	88	90	90
Increase	= 9	10	13	15	15
Duration of walk	= 30	27	26	19	18 (seconds)

In other words, duration is inversely proportionate to the rhythm. In heavy work the systolic phase is prolonged and the relation $\frac{\text{diastole}}{\text{systole}}$ decreases, while pressure increases.

In the regular performance of work muscles get "trained", and the starting-time is reduced. Work increases the intensity of all cellular functions. When the body returns to repose after a period of work its internal reactions and its expenditure decrease according to Newton's law for the cooling of bodies (law of repose).

The general metabolic effects of work are: 1) muscles more acid; 2) in urine the elimination of phosphorus is increased; 3) cellular oxidation increases; 4) there is greater loss of H_2O, mainly in sweat; 5) proteids are drawn on, to the extent of 60-65 g per day; 6) body temperature goes up, on the average, $1^{o}C$.

The amount of work is limited by <u>fatigue</u>, which limits duration also. The curve of muscular effort shows a descent fast or slow, according to the rhythm of contraction and the intensity of the effort. The fatigue curve is fairly constant for an individual. There are different fatigue curves for different sets of muscles. In a walk of 10 km it was noted that the forearm muscles tired most rapidly, and gave a less amount of mechanical work. Fatigue in muscles is attributable to an alteration of the elastic properties and of the cohesion of muscle fibers; the matter or substance of the muscle changes its chemical properties.

Endurance is resistance to fatigue. Resistance = F x t, where F = effort expended, weight lifted, and t = time occupied in the effort. Haughton and Niper gave the relation $F^2xt = C^{te}$. The t varies 20-78 seconds, when a weight of 5 kg is held in each hand with arm extended, until fatigue forces a lowering of the arm; t decreases quicker the shorter the intervals of repose. The product F^2xt has great individual variability.

The problem of nervous fatigue and intellectual activity is a difficult one. Thurnburg (1905) found that filaments of nerves consume in pure air, at a temperature of 20°C, 22.20 mm^3 of O_2 per g per hr., and exhale 22 mm^3 of CO_2. For the 1700 g of an adult "nervous mass", in 24 hours: 0.045 cc x 1700 x 24 = 1,836 cu. cm. This equals 1836 x 5.05 Cal = 9271.80 Cal.* This is about 4/1000 of the entire organism in repose. There is no appreciable rise in temperature.

Benedict and Carpenter (1909) noted the caloric production in a man "performing difficult intellectual operations." They obtained an excess of 1.32 cal in four hours, just about 4 per 1000.

Table 44. Caloric Production in Intellectual Work

Av. of 22 Experiments	Repose	Intellectual Activity
Heat expended per hr.	98.43 Cal	98.80 Cal
Vaporised H_2O	37.80 g	39.23 g
O_2 consumed	25.86 g	27.30 g
CO_2 exhaled	32.76 g	33.42 g
Temperature of body (variation)	0.980°C	0.989°C
Pulsations per min.	74	79

(Amar, p. 212)

It was concluded that cerebral effort does not "exert any positive influence on the metabolic activity."

IV Man and His Environment (pp 215-260)

I The Internal Environment (pp 215-226)

Amar observes that Claude Bernard defined blood as the internal environment, and that he, Amar, feels that the quantity and quality of the aliment (diet) must be considered. The influence of alcohol is discussed. Ethyl alcohol = C_2H_6O = 46 g, with heat of combustion reaching 7.069 Cal per g. An intake of 1 g per kg of body weight is not toxic. It is concluded that in a ration adapted

---

* On p. 211 Amar gives 1,836 x 5.05 Cal = 9.27 Cal. This has been changed as noted (W. M. K.).

to work by a subject alcohol cannot replace carbohydrates in the diet. "Alcohol is a thermogenous aliment and not a dynamogenous aliment." In general what are termed "aliments of economy" are : alcohol in small doses, tea, coffee, cocoa, pimento, etc.

In fasting and inanition there is no appreciable reduction of expenditure in the first 24 hours, i.e., it amounts to only 5-6%. The organism shows a general independence in regard to amount of food taken in. But general forces diminish progressively, e.g., muscles lose contractile power (the main sufferers in inanition). Intellectual faculties, based on nervous tissue, are least affected.

Experiments on pigeon, cat, and dog showed the following percentages of utilization at the end of long fasts.

1.	Brain, cord	2.0 to 1.1%	5.	Heart	45 to 55%
2.	Skeleton	21. " 19 %	6.	Pancreas	64 to 39%
3.	Lungs	22 " 30 %	7.	Spleen	71 to 75%
4.	Muscles	42 " 70 %			

Starved persons regain weight very quickly by eating ternary substances rather than proteids, providing the latter remain in ratio of 2 g per kg of body weight. As an example, a man of 54 kg fasted 40 hours, during which time he produced 36,089 kgm (85 Cal) of work on a bicycle dynamometer in a four-hour work-period. He was then given a ration of 2600 Cal of which 1750 Cal were carbohydrates. Results were as follows:

Diet	Cal	Cal	Cal	
Proteid	93	652	330	averages
Fats	750	191	513	of several
Wt. gain	433 g	707 g	1218 g	days

The fats assure weight conservation better than the proteids, but carbohydrates are even better.

Certain "physiological troubles" may interfere with work efficiency. The compression of tight clothing or a poor working position (or posture) will affect the circulation of the blood. There must be a free flow of air, i.e., O_2 must be in excess of CO_2. The auto-intoxication of fatigue will lead to exhaustion, affecting muscles the earliest and/or the most. As far as age, per se, is concerned, between 20-50 years it "only causes insignificant modifications in the organism." Miscellaneous factors in functional efficiency are: the senses of sight, hearing, touch, which are trainable; the restorative qualities of sleep, rest, or massage (in the latter "kneading is better than percussion or friction"); moral and mental influences, such as love of work, interest in work, freedom from worry, etc.; body-build or "dynamic morphology" (Amar recognizes the digestive, muscular, respiratory, and nervous types); social factors are both personal (hygiene of body and mind) and cultural (justice, concord, and the common prosperity of the workers).

II The External Environment (pp 227-236)

The external environment focuses first on the atmosphere, which is basically 79% N, 21% O_2 in volume (trace of CO_2 = 0.3%). Temperature variation ranges from 3.6°C in January to 9.3°C in July, in a single day. The weight of air on the body is the barometric pressure. At sea level the normal pressure is 760 mm mercury.

The weight of one cu. cm of mercury = 13.6 g ; hence, the weight of the barometric column = 1033 g. The average body surface = 2 sq. m ; therefore the total atmospheric pressure = 20,000 x 1033 = 20,660 kg.

Cold stimulates vital combustion, while heat relaxes these processes. Bodies cool by radiation. At lower external temperatures more heat is lost at the surface. The speed of cooling of hot bodies increases proportionate to the excess of their own temperature over that of the external environment.

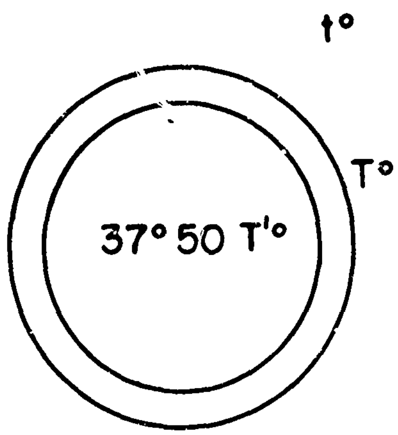


Fig. 56. Diagram of Skin Temperatures, T and T', and Outer Air (t)

Let the body (Figure 56) be represented by a circle of hot water: radiation is measured by the number of small calories emitted at 1 sq. cm surface. Here, T' and T represent the internal and external temperature of the skin; t is that of the surrounding air. Emission (Newton) is proportionate to the difference of temperature between the surface and the external environment; the emissive power, K, is the number of cal per sec. per degree difference. In a difference of temperature (T - t) the loss per sq. cm of radiating surface will be:

$$q_1 = K_1 \ (T - t) \text{ sm. cal}$$

Radiation lowers the body temperature, so that heat must be produced by the body to hold at 37.5°C. The value of K varies according to the nature and color of the radiating surface. In addition to radiation air carries away heat by convection. The loss via convection depends on the differences of temperature (T - t) on the surface area, but also on the shape of the surface area. According to Peclet this factor can be represented by the coefficient K_2 = .000066 c per sq. cm per sec., so that:

$q_2 = K_2 (T - t)$ in sm. cal Hence, cal loss per sq. cm per sec. is:

$$q_1 + q_2 = (K_1 + K_2)(T - t).$$

The temperature, T, of the skin surface is neither easy to measure nor is it constant. The skin receives on its internal surface as much heat as it loses on the external surface, i.e.,

$q_3 = (K_1 + K_2)(37.5^oC - T')$, (Where T' = internal cutaneous temp.).

Hence, $q_3 = q_1 + q_2$

The conductivity of the skin, c must be determined for the number of sm. cal able to traverse a surface of 1 sq.cm with a thickness 1 cm in 1 sec. for a difference in temperature of 1°C between the internal and external walls. Here the heat transmitted is:

$$q_4 = c \frac{(T' - T)}{e} \text{ in sm. cal and}$$

$$q_4 = q_3 = q_1 + q_2 = q \text{ (generally)}.$$

It is now possible to eliminate T and T' and solve, using 37.5°C. This results in:

$$q = \frac{cK_2(K_1 + K_2)(37.5^oC - t)}{c(K_1 + 2K_2) + eK_2(K_1 + K_2)}$$

The average thickness of the skin, e = 0.2 cm. The coefficients are: c = 0.00060 c ; K_1 = 0.00015 c ; K_2 = 0.000066 c.

This gives $q = 497 \quad 10^{-7} . (37.5^oC - t^o)$.

For 24 hours (3600 x 24 sec.) a sq. cm of skin will lose as heat: $Q_1 = 0.034294 \; (37.5^oC \text{ -- } t^o$ in great calories.

Since adult surface area = 19,900 sq. cm the loss will be:

$Q = 85.540 \text{ Cal } (37.5^oC - t^o)$, in an external environment of t^o.

As an example, let there be an environment of 9°C; then $t^o = 9^oC$ in the expression for q. This will give Q = 2435 Cal. At this temperature Lefèvre measured a total of 3216 Cal. This yields a physiological expenditure of 781 Cal, independent of the thermal influences of the exterior. In another example let t = 20°C; Q = 1496 Cal; this gives 1496 + 781 = 2277 Cal, as the total static expenditure, of which 2/3 is expended in the regulation of the temperature.

At 20°C muscle contraction is very good, and resistance to fatigue is at a maximum. Cellular renovation is accelerated between + 37°C and 40°C. The excitability of nerves increases with temperature rise from 0°C to 40°C. The respiratory rate increases with increase in temperature. Extremes of heat are, in general, inefficient.

<u>Clothing</u> as a protective device is an important heat regulator. The best heat insulator (Rubner) is air, with a coefficient of conductivity (c) of 0.0000532. Example: a man, wearing a woolen garment (c = 0.0000686), with a total surface of 19,000 sq. cm, a thickness of 0.75 cm, with the difference in temperature between inside and outside = 10°C; what will be the heat loss by conduction in 24 hours?

$$Q = \frac{0.0000686 \times 19,000 \times 3600 \times 24 \times 10}{0.75 \times 1,000} = 1528 \text{ Cal}$$

(division is by 1,000 to give Q in large cal)

As regards emissive powers black is the most radiant. It is possible to establish radiant heat via Stefan's Law: radiation from the surface is proportionate to the fourth power of its absolute temperature. Let T = ordinary temperature of the surface, t = temperature of the external environment, then:

$$q_1 = K'_i \quad [(T + 273)^4 - (t + 273)^4]$$

The factor of proportionality, K'_i, $= 1.355 \times 10^{-12}$, according to A. Shakespeare, 1.27×10^{-12}, according to Bauer, for black surfaces. In the human skin $K'_i = 1.02 \times 10^{-12}$, approximately.

In determining the coefficient of utility of clothing Coulier used a cylindrical brass vessel filled with hot water, and covered with the material to be tested. Bergonie used a copper bust in which the water temperature = 37°C. He noted the time, t, when the temperature of the bust (covered by a given material) fell 1°C. The external temperature = 12°C. The time taken in a fall of 1°C. when the bust was bare was **O**. The ratio $\frac{t}{O}$ = c' is the coefficient of utility of the garment. Bergonie found a range of c' = 1.10 in a close-fitting cyclists' costume to 4.50 in a fur-lined Winter overcoat.

III The External Environment (cont.) (pp 237-49)

The hygrometric state (humidity) is the ratio of the weight of water vapor contained in air to the maximum weight it could contain in the same volume. Let p and P be these two weights, then the hygrometric state will be:

$$e = \frac{p}{P} \text{ (the relative humidity)}$$

The state of dryness increases as p is reduced.

In practice the weights of p and P of vapor are not considered, but its tensions ("elastic force") f and F are given, so that:

$$e = \frac{f}{F}$$

The values of F may be found in Regnault's Tables; the value of f is determined by a hygrometer. For example, in a locality of 15°C the dew point of the hygrometer is found when t' = 5°C. For this temperature the tables give f = 6.55 mm ; for t = 15°C , F = 12.70 mm. Hence:

$$e = \frac{f}{F} = \frac{6.55}{12.7} = 0.514 \text{ or } 51.4\%$$

Knowing f or e, the weight of water vapor per cu. m at t°C can be calculated:

$$p = \frac{290.2 \times f}{273 + t} \text{ g} \qquad \text{or} \qquad p = \frac{290.2 \times F \times e}{273 + t} \text{ g}$$

In perspiration the value of elimination of water vapor by the body varies inversely with the hygrometric state. At 20°C an adult male loses 900 g H_2O in repose, 3000 g in an average amount of work of 200,000 kg ; in very hard work 7-8 kg of H_2O are lost. For work in the dry state an average temperature of 15°C - 16°C is best.

Garments absorb water in varying amounts. Rubner found that the maximum quantity of H_2O retained, per g of material was:

flannel	10.30 g	silk tricot	3.80 g
flannelette	6.00 "	linen	2.10 "
wool tricot	4.80 "	cotton	0.80 "
cotton	4.20 "		

Wool is equally valuable, Summer and Winter.

Speed of air current is important. For a man walking 1.50 m per sec., the air resistance is $R = K \times S \times V^2$.

(R = kg , V = m , S = surface in sq. m); coefficient of resistance, K, is 0.079 up to a speed of 40 m , a man's effective surface, S, when walking, is 0.75 sq. m. Hence:

$$R = 0.079 \times \overline{1.50}^2 \times 0.75 = 0.133 \text{ kg}$$

This represents, per sec., a useless labor of:

$$0.133 \times 1.5 = 0.20 \text{ kgm}$$

Lefevre studied the influence of the speed of air current in a subject by placing him in a "physiological calorimeter", in a draft of air of known mass and speed, noting temperature of air at entry and exit. Let M = total mass of air circulated in t minutes and raised to temperature at 0^o. Multiply excess of temperature, 0^o, by specific heat of air 0.237, the number of calories eliminated by the subject is obtained, i.e., per hour:

$$Q = \frac{0.237 \times 0 \times M}{t} \times 60 \text{ cal}$$

Here are data on an adult:

Velocity of Air	Temp. of Air	Per Hr in great cal: Naked	Per Hr in great cal: Clothed	Ratios: A	Ratios: B
1.20 m	9°C	134	98		
				$\frac{201}{134} = 1.50$	$\frac{134}{98} = 1.36$
3.80 m	9°C	201	130		
				$\frac{130}{98} = 1.32$	$\frac{201}{130} = 1.54$
1.50 m	5°C	185	143		
				$\frac{277.5}{185} = 1.50$	$\frac{185}{143*} = 1.29$
3.80 m	5°C	277.5	172		
				$\frac{172}{143} = 1.20$	$\frac{277.5}{172} = 1.61$
4.00 m	4°C	313	170	-	$\frac{313}{170} = 1.84$

(Amar, p 241)

Thermogenesis increases with the fall of temperature and with the increase of wind velocity. The increase is less for a man clothed than naked (ratio A). The protection is the greater the colder the current (ratio B).

In swimming the expenditure of calories is modified by the force of the current and the temperature of the water. Resistance is $R = K \times S \times V^2$; where K = 73 and S = .034 sq. m. If the velocity of the current is 4 m we have:

$$R = 73 \times .034 \times 16 = 41 \text{ kg}$$

* Amar gave $\frac{185}{148}$, in error. (W. M. K.)

IV The External Environment (cont.) (pp 253-260)

Barometric pressure decreases with a rise in altitude, as does also the air temperature. Determine pressure at base and at summit, H and H'; the temperature likewise, t and t'; and knowing the latitude, λ ; it is possible to deduce altitude, A, from Laplace's formula:

$$A = 18,405 \text{ m} \quad (1 + 0.002552 \cos 2\lambda) \left(1 + \frac{t + t'}{500}\right) \quad \log \frac{H}{H'}$$

To the height A, calculated, there must be added the height of the foot of the mountain above sea level; if this is B meters, the true height will = A + B. This is "the reduced altitude" (reduced to sea level).

A decrease in barometric pressure causes a decrease in the normal proportion of O_2. Up to 115 g per cu. m the respiration is not troubled, (this is pressure of 290 mm at an average height of 7,680 m). At altitude of 1500 m and up the number of red corpuscles increases. At a pressure of about 400 mm "mountain sickness" ("altitude sickness") occurs: nausea, accelerated heart and respiration, difficulty of movement, etc. There are other environmental factors of working: pressure below ground level ("caisson sickness"); work in mines with deleterious gases, vapors, and dust; radiation and solar action (light vs. dark); electric and magnetic fields ; sounds and noises ; design and type of tools.

V Experimental Methods (pp. 261-332)

I Measurements and Instruments (pp 261-288)

A measurement is never absolutely correct; there is always an error, no matter how small. The difference between the value found and the real value is the absolute error. For example, weigh a 65 kg man. If the scale shows 64.9 kg , this is a negative error of 0.1 kg , if 65.1 kg , a positive error of 0.1 kg. The relation of absolute error to the total dimension to be measured is the relative error. In the data just given it is $\frac{0.1}{65} = \frac{1}{650}$. The relative error decreases in proportion as the total quantity increases ; for 130 kg it would be $\frac{0.1}{130} = \frac{1}{1300}$.

In experiments the permissible relative error depends on the end in view. If a gas meter had a constant error of 1/2 liter (minus) then 100 would be read 99.5; this is an error of .5% (1/2 of 1%), which is acceptable; the same error for 10 (= 5%) would not be acceptable. Measurements may be expressed statistically, in examples given by Amar, by the mean or average, the average error, and the probable error. Methods of graphical registration are the cardiograph, pneumograph, metronome, chronograph, stop watch, tuning fork, tachometer (measurement of speed), various counters, including anemometer (measuring linear speed of gases).

The linear measurement of the human body focuses upon stature, sitting height, span, and chest circumference. Vital capacity (spirometer) and surface area (Meeh's formula) are also included. Body volume is taken with the body immersed in water up to the ears, with water temp. 20° - 25°C. The water displaced and weighed will give volume of the immersed body at the rate of (1 cu. cm. + α) per g. Following are values of $1 + \alpha$ for various temperatures:

Table 45. Volume of the Body at Different Temperatures

T	$1 + \alpha$	T	$1 + \alpha$
+ 10°C	1.000273 cu. cm	+ 19°C	1.001571 cu. cm
11°	1.000368 "	20°	1.001773 "
12°	1.000476 "	21°	1.001985 "
13°	1.000596 "	22°	1.002208 "
14°	1.000729 "	23°	1.002441 "
15°	1.000874 "	24°	1.002685 "
16°	1.001031 "	25°	1.002938 "
17°	1.001200 "	26°	1.003201 "
18°	1.001380 "	27°	1.003473 "

(Amar, p. 275)

Body weight should be taken in the nude.

Strength is registered via dynamometers of various types. They should have springs whose deformations are proportionate to the forces which produce them. In various occupational situations strength or muscular effort can be measured via the "cabrouet"* and the wheelbarrow. There are also experimental shoes to register the pressures of walking, and tools with various recording devices (springs, tension gauges, etc.).

II. Measurements - Dynamic Elements (pp 289-307)

The evaluation of work is facilitated by the use of a dynamograph, arranged to indicate the effort and the displacement at the point of application of that effort. The graph shows the value of the effort and the amount of displacement at any given moment.

Muscles produce a constant effort, as seen below, (Figures 57-59) ordinates $Y_1 \ldots Y_n$ in n seconds, and displace resistance opposing them by distances ox_1, x_1x_2 ...., $x_n - 1^{x}n$. The total work done by the muscles will be the sum of the products $ox_1 \times y_1 + x_1x_2 \times y_2 + \ldots$, or $ox \times y$. (Figure 57).

---

* Two wheeled cart or truck, drawn by a man.

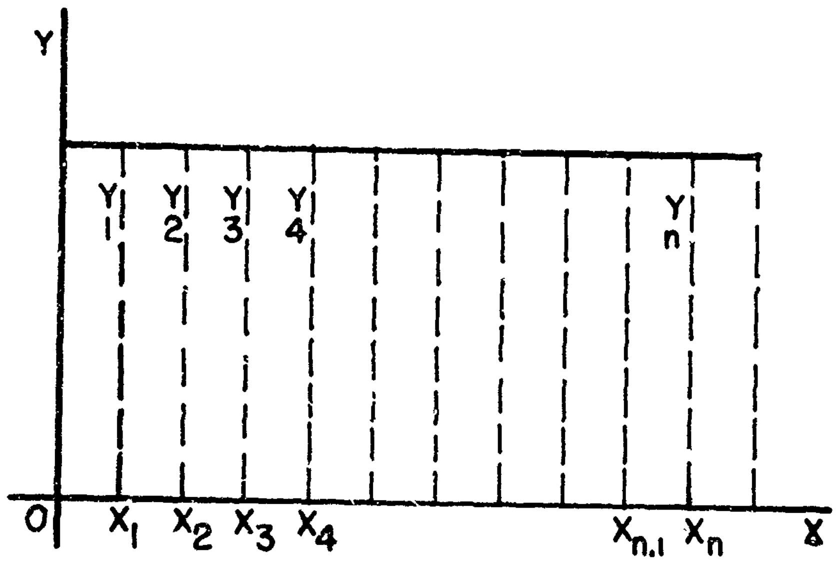


Figure 57 Scheme of Dynamograph : Work of Muscle.

If the effort were variable the total area would be as below: (Figure 58).

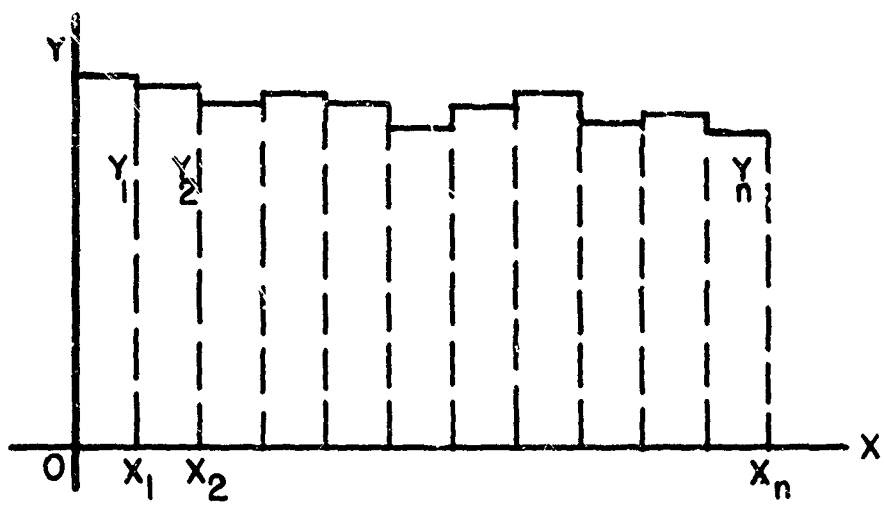


Figure 58 Variable Work Registered by Dynamograph.

If the variation of the effort is continuous the graph would take the form of a curve, C, and the area to be found would be OABx (Figure 59).

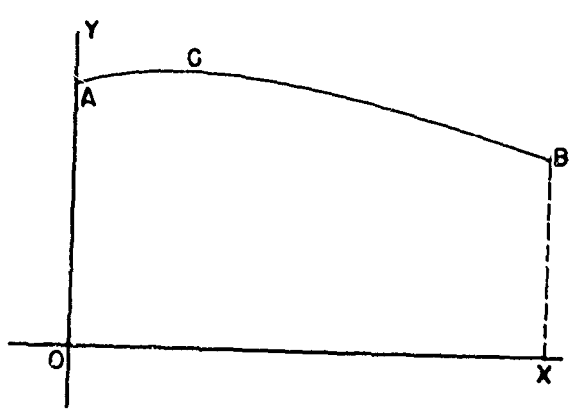


Figure 59 Curve of Continuous Variation in Work.

When displacements are large they can be proportionately reduced and "scaled down", e.g. on the scale 1mm = 10 cm. Areas can be measured with a planimeter. In essence "the area of a graph is equal to the sum, either of a series of small equal rectangles,.... or more exactly as a series of small trapeziums". In this case divide the base or the line of the abscissae, Ox, into a sufficiently large number of equal parts, d.

The value of S is given by Poncelet's formula:

$$S = d \left[2 \, (y_1 + y_3 + \ldots . \; y_n - 1) + \frac{y_o + y_n}{4} - \frac{y_i + y_{n-1}}{4} \right]$$

The highest limit of error is:

$$\frac{d}{4} \, (y_1 + y_n - 1 - y_o - y_n)$$

Ergometry and ergography are, respectively, measurement of work done and the graphic representation of work done. In various work or industrial situations they may measure hand, arm, leg, foot work outputs.

In considering the mechanical work done by man a classification of active muscles should be based upon whether they are of arms, legs, or fingers, or of several groups at once, or if the whole weight of the body is put to work at once. The use of the arm muscles is the most variable, for they come into play wherever manual strength or dexterity is a factor.

Amar focuses upon the work of the work of the muscles of the leg, as noted in bicycling: F = uniform pressure on each pedal; d = the diameter of the circle of the pedal; hence, work done = F x d. Per stroke of the pedal (both legs) it is T = 2Fd. This is the work of both legs + the work done in advancing the loaded bicycle. With each stroke of the pedal the bicycle advances the distance, D, depending on the gear. Let R = all resistance involved:

$$R \times D = T = 2Fd$$

Knowing T or R we can deduce F. A value of R is acceptable as:

$R = 0.012\,P + 0.0738\;SV^2$ { P = total wt. of bicycle + rider, S = surface area of bicycle, (about 0.60 sq. m.), and V = speed in m. per sec. }

If slope is inclined i per meter:

$$R = P(0.012 \pm i) + 0.73\;SV^2$$

As an example: speed = 18 km per hr, on slope where i = 0.02 m, bicycle wt. = 15 kg, rider wt. = 65 kg:
$P = 65 + 15 = 80$ kg ; $V = \frac{18000}{3600} = 5$ m per sec. Therefore,

$$R = 80 \times 0.032 + 0.073 \times 0.6 \times 5 \times 5 = 3.655 \text{ kg}$$

The work done per hour will be $3.655 \times 18{,}000 = 65{,}790$ kgm, and per meter will be $\frac{65{,}970}{18{,}000} = 3.655$ kgm.

In descent the value of $R = 80 \times (-0.008) + 0.073 \times 0.6 \times 5 \times 5 = 0.455$ kg (The nature of the ground and the state of the tires will be variable factors).

Legs are basically used in locomotion but must also support the body in all different movements; they must resist carried loads, while displacing this total weight on the level or on a slope. Therefore, useful work is "the product of the weight displaced by the sum of the amplitudes of the vertical oscillation of the body walking on level ground, or by the sum increased by the total slope that has been climbed." The new unit now defined is the product of 1 kg of wt. displaced per m covered, horizontally or on a slope: this is the meterkilogram (Mkg).

Lifting a weight to the top of a stairs, H, will be simply P x H. In mounting a ladder the H = vertical distance to the ground, not the length of the ladder. According to the speed of ascent a man will expend an effort greater than P, alone; hence, muscular work exceeds mechanical work.

III Measurement of Energy (pp 308-332)

In the measurement of energy there are two forms concerned in the human machine: 1) that manifested externally in work or heat (physiological work or physiological energy); 2) that which acts from outside the organism (natural or meteorological). The expenditure of the human motor indicates the quantity of energy a certain amount of work entails.

The measurement of physiological energy may be done via the method of maintenance rations. The usual diet is noted; then a given diet is set up so that every day at the same hour the subject's weight is the same; also, the diet is related to a given constant amount of work. For example a subject in repose weighed 80.2 kg. The variation for nine days of work was as follows:

Day 1	80.2 kg	Day 6	80.2 kg
" 2	79.98 "	" 7	80.2 "
" 3	80.1 "	" 8	80.28 "
" 4	80.18 "	" 9	80.3 "
" 5	80.2 "		

The next stage of the inquiry was to note the quantities of proteids, fats, carbohydrates and the corresponding energy values from available tables. Then age and sex were equated and Atwater's coefficients were used:

The consumption of an adult male	1.00
" " " " " female	0.80
Boy 14-16 years	0.80
Girl 14-16 years	0.70
Child 10-13 years	0.60
" 6-9 years	0.50
" 2-5 years	0.40
" below 2 years	0.30

Physiological energy can also be measured via the oxygen method. The test is run in the morning, after a fast of 10 hours. The caloric power of a liter of O_2 at O^o and 760 mm will be 4.6 cal. Respiratory exchange is measured via a spirometer. The gas is analyzed via Laulanie's eudiometer or that of Bonnier and Mangin. An example of the use of the latter is given. Let it be assumed that there are, as determined by readings, 450 divisions or volumes of air to be analyzed; this will be N. Samples of air are passed through potassium, and CO_2 will be absorbed; bring the reading to 600, so that the division to the left in the apparatus will be 165; hence, N' = 600 - 165 = 435 vols.; here CO_2 = 450 - 435 = 15 vols. Now take a sample of pyrogallate and absorb O_2, letting N'' = 600 - 240 = 360 vols.; the O_2 content is therefore 435 - 360 = 75 vols. in 450.

Proportions will be:

$$CO_2 = \frac{15}{450} \times 100 = 3.33\%$$

$$O_2 = \frac{75}{450} \times 100 = 16.66\%$$

With N, N', N'' attained, calculations are:

$$CO_2\% = \frac{N - N'}{N} \times 100$$

$$O_2\% = \frac{N' - N''}{N} \times 100$$

The study is made at temperature, t, and atmospheric pressure, H, The volume, V_t, thus measured must be reduced to O^oC. and 760 mm pressure. The volume of a gas increases with the temperature by $\alpha = 1/273$ per 1^oC and diminishes as the pressure rises. The volume at O^oC and 760 mm is

$$V_o = \frac{V_t}{1 + \alpha t} \times \frac{H}{760}, \text{ or approximately}$$

$$V_o = \frac{0.36 \times V_t \times H}{273 + t}.$$

In Laulanie's eudiometer H is a measure of dry air and the tension, F, of the water vapor at temperature t (F is found from Regnault's tables). Therefore:

$$V_o = \frac{0.36 \times V_t (H - F)}{273 + t}, \text{ where}$$

H = atmospheric pressure at the moment,

F = tension of water vapor at temperature t^o.

The volume of the dry gases is thus determined at O^o and 760 mm.

Knowing the volume of the gases, V_o, for the duration of the experiment, and CO_2 and O_2 having been determined, it is easy to deduce:

1. The respiratory quotient $\frac{CO_2}{O_2}$

2. The total O_2 consumed. In pure air taken in O_2 = 21%; if analysis = 17%, then 4% has been utilized. Hence, the proportion, r, in the expired gases having been calculated, the quantity left in the body will be:

$$\frac{(21 - r) \times V_o}{100}$$

3. The total CO_2 eliminated. Here all the CO_2 in the expired gases is taken. For example let v_o be the consumption of O_2 in 10 min., of a man in repose, and $\frac{CO_2}{O_2} = 0.98$. A liter of O_2 = 5.05 Cal, and v_o liters represents static expenditure. Then:

$$q_s = v_o \times 5.05 \text{ Cal}$$

Let v'_o be the amount of O_2 absorbed during work, T, in 10 min., and let $\frac{CO_2}{O_2} = 0.97$. The coefficient 5.05 Cal will be applicable here:

$$q = v'_o \times 5.05 \text{ Cal}$$

Hence, the dynamic expenditure is:

$$q_d = (v'_o - v_o)\ 5.05\ \text{Cal}\ , \text{ or } q - q_s$$

Measurements of tactile sensibility is secured via Weber's "esthesiometer" (a caliper gauge with two pointed arms, touching the skin simultaneously). The degree of sensibility is the smallest distance between the two points at which both are distinctly felt. Here are pertinent data:

Table 46. Tactile Sensibility

Locale	Adults	Children
Tip of tongue	1.10 mm	1.10 mm
" " nose	6.80 "	4.50 "
Palm of hand	8.90 "	4.50 "
Eyelids	11.30 "	9.00 "
Back of hand	31.60 "	22.60 "
Sternum	45.10 "	33.80 "
Middle of back, arms, thighs	67.70 "	40.60 "

(Amar, p. 322)

Sensitivity is greater in the blind, higher in children.

The measurement of thermal energy comprises the energy of physical waste and that of the physiological minimum. Waste is calculated from the theoretical relations of radiation and convection in air at a temperature of t^o. Applying Newton's Law $K_1 = 0.00015c$ for the emissive power. In Stefan's Law $K'_1 = 1.02 \times 10^{-12}$ in the formula

$$q_1 = K'_1 \quad (T + 273)^4 - (t + 273)^4$$

An example of the use and calculation of Lefèvre's "physiological calorimeter" is given. The subject is seated in the apparatus, head exposed. The outlet chamber has a fan, S, which can exhaust 600 m^3 of air per hour. The inlet chamber, N, has 3 intakes from the outside, with an anomometer fixed to each opening. The sections s_1, s_2, s_3 of the intakes are known, and the average speeds v_1, v_2, v_3 of the air current are also known; from this the volume of air can be calculated by the sum:

$$s_1 v_1 + s_2 v_2 + s_3 v_3 \dots\dots = SV$$

Thermometers correct to 1/10 degree are placed in front and behind the subject to give the amount θ, by which the air is heated.

Let V = av. speed of air during the experiment, and SV = vol. in liters. The weight of a liter of air = 1.293 g , and so the total mass, M, will be:

$$M = S \times V \times 1.293 \text{ g}$$

Correcting for air pressure, H, and air temperature, t, the impression is:

$$M = S \times V \times \frac{0.36 \times H}{273 + t}$$

The increase in temperature, θ, will be:

$Q = M \times \theta \times 0.237$ cal (where 0.237 cal = specific heat of the air)

Tables of thermal conductivity range from 0.0000330 for compressed cotton to 1.0400000 for copper. In an adult male subject Lefèvre reported a value of 11.70 Cal for caloric loss of water. Also given here are tables of tension of vapor in mm. of mecury (Regnault, Thiesen and Scheel), and tables of sines and tangents.

VI Industrial Labor (pp 333-466)

I Body in Equilibrium and Movement - Locomotion (pp 333-358)

There are two main areas here: 1) the human body in equilibrium and in motion; 2) the influences on the human body of the nature and quality of the tools and appliances used. The human body is an articulated or jointed system which is never at complete rest, being subject to external forces (the most important of which is gravity) and to internal forces (wherein muscular actions are basic). The human body is heterogeneous whose density varies in different parts; it is irregular in shape; it is not isolated in space; for it is posed on the Earth's surface, supported by a base, i.e., the feet. The laws of mechanics apply to the human body, in both static and dynamic states.

The body has a center of gravity, so that in the erect position the muscles must aid and act to establish equilibrium. Instability is present, for the base (feet) is small and the center of gravity is relatively high.

In this discussion the work of Braune and Fischer is basic. They found the center of gravity of the erect body at the level of the top of the third sacral vertebra (S3), roughly at navel level. There are two basic positions: 1) symmetrical ("attention" or "at ease"); 2) asymmetrical "haunched" or "station hanchee"). In the "attention" position the line of gravity passes through the heels; in the "at ease" it is in front of the heel. In the symmetrical positions body weight is evenly distributed in the two legs; in the asymmetrical it is carried mainly in one leg.

In the symmetrical position there is a frontal plane passing through the hip joints, at right angles to a median plane (sagittal). In a 58.4 kg male cadaver, in "rest position" the center of gravity of trunk, head, arms was found 18 cm above the hip axis and 8.6 mm in front of the frontal plane. For the whole bust the moment of rotation M = P x .0086. The weight of P = 36.82 kg, distributed as follows:

Part	Weight (kg)	% Total Body Weight	
Head	4.140	7.1 %	
Trunk	25.060	42.92	
Arm	3.810	6.52	(3.14% is in
Arm	3.810	6.52	forearm)
Bust Totals	36.820	63.06%	

The trunk is held erect by back and abdominal muscles.

Since its center of gravity is in front of the hip axis the weight tends to thrust the thighs back at their lower end, and the knees take up this thrust. In the "easy" position the joint between hip and thigh is 1 cm behind the frontal plane. The foot receives the thrust of the entire weight. In the "rest" position the ankle joint is 5 cm behind the frontal plane, or 4 cm behind the plane of the knee joint. In the leg it may be noted:

	Weight (kg)	% Total Body Weight
Thigh	6.80	11.64%
Leg + foot	3.99	6.83
	10.79	18.47
R and L leg	21.58	36.94%

The muscles of the sole of the foot have to balance a moment of rotation produced by a weight of 36.82 + 2 x 6.8 = 50.42 kg. The arm of the lever is about 4 cm. in length. The Achilles tendon pulls up the heel, so that the front of the foot is pressed on the ground, transmitting the weight, the reaction of the ground on the foot being equal and opposite.

Body stability is not great in either of the two symmetrical positions. It is best in the asymmetrical position, for then the center of gravity is directly above the leg of support. In the sitting position, with a larger base, stability is increased. This is seen in the measuring of O_2 consumption:

Sitting	100	
"Haunched"	103	(the relative
"Easy"	106	expenditure
"Attention"	125	of energy)

The recumbent position gives maximum stability and efficiency, base is maximum, expenditure of energy is 7-8% less than in sitting. Men with heavy loads, obese men, hunchbacks, men bent with age, and pregnant women have the center of gravity displaced.

In summary to this point: 1) the center of gravity is at about 57/100 of total height; 2) the center of gravity of the bust is 18 cm above the transverse axis of the hips; 3) in an adult male of 65 kg the bust is 41 kg. (63%) and the leg 24 kg (37%) of the total weight; 4) lower leg is 6.83% of the total body weight; 5) total leg length varies from .87-.90 m, with a weight of 12 kg (lower leg = 4.4 kg.); 6) Atwater showed for an adult male in a calorimeter at 20°C an expenditure of energy = 2,120 Cal, or $\frac{2120}{65 \times 24}$ = 1.36 Cal per kg of body wt. per hr. In open air, at 14°-15°C, this may reach 1.5 Cal.

When the mechanical equilibrium of the body is disturbed <u>movement</u> begins. As a result there are both external and internal forces operative, focusing on the center of gravity. The internal forces do not affect the center of gravity. An erect body on a smooth surface can only rotate around a vertical axis on which the center of gravity remains unchanged. In the external forces the major factor is gravity, with friction and air resistance having minor roles.

An acrobat in a leap can turn himself around a transverse axis. Main factors are height, speed, and duration of the leap, governed and connected by:

$$v = \sqrt{2gh}; \quad h = 1/2\, gt^2; \quad t = \frac{v}{g}$$

If the angle of inclination of the initial speed is α the time $t = \frac{v \sin \alpha}{g}$. The body reaches its maximum height via a parabolic curve. "Laws which govern the movement of projectiles are equally applicable in the case of the human body." Hence, to get the most length in a jump, for example, the angle of the jump should = 45^{o}.

Gravity acts on each component of the body, balanced or overcome by muscle action. The result is motion (muscle dynamics). Motion is conditioned as to form by degrees of liberty (freedom) and as to speed by the magnitude of applied stresses.

Movements, as in the arm for example, are "infinite": flexion, extension, abduction, adduction, pronation, supination, circumduction, etc. (seen in shoulder and hip joints). The degrees of liberty or movement of a limb depend upon the type of joint. "For joints of the same type the speed and amplitude of movement are determined by the length of the arm of the lever, the power, and the resistance." Levers of locomotion are basically of the third order. (Figure 60)

Planes of movement are:

1) frontal X'Z XZ'
2) horizontal XY'X'Y
3) sagittal XZY'Z'

All movements are referrable to these three planes, though abduction and adduction are referred to the median (sagittal) plane.

The following basic assumptions are in order: 1) the longitudinal axis of a limb passes through its center of gravity and through the center of its joint; 2) any joint has one or more definite and fixed axes, and these axes are slightly displaced, in movement, from their initial positions; 3) muscle insertions may be considered as joints, so that a muscle may be represented as a straight line (a vector) in the same plane as the longitudinal axis of the limb; 4) the movement of any articulated bony segment may be studied as though the remainder of the body were " a solid and invariable mass."

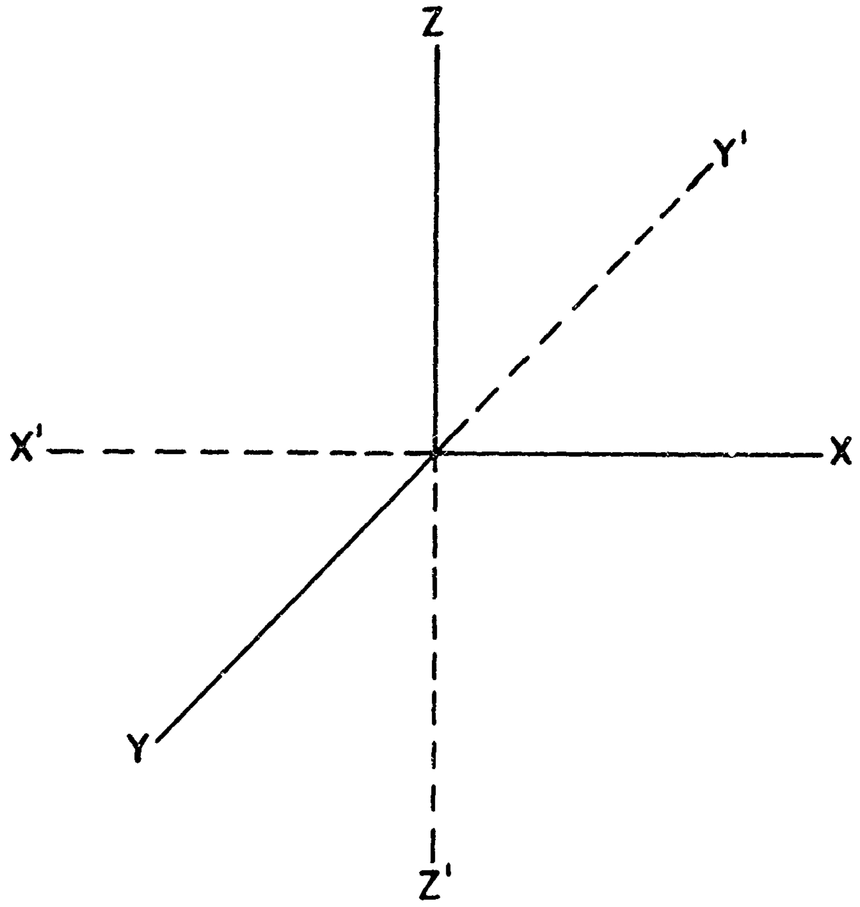


Figure 60. Plan of the Orientation of the Limbs of Man

As an example, move the forearm relative to the upper arm (here there is only one degree of liberty); assume the upper arm fixed, in a vertical position: (Figure 61).

The Biceps = AB, with tension at A and at B; A is fixed, and B turns around an axis, o, at the elbow; the moment of rotation of the force F (or AB) in reference to o is:

$$M = F \times od$$

The movement of forearm + hand is attracted downward by gravity. If the weight of the forearm (the resistance) is P, acting at its center of gravity, G, then the weight has a motion:

$$M' = P \times od'$$

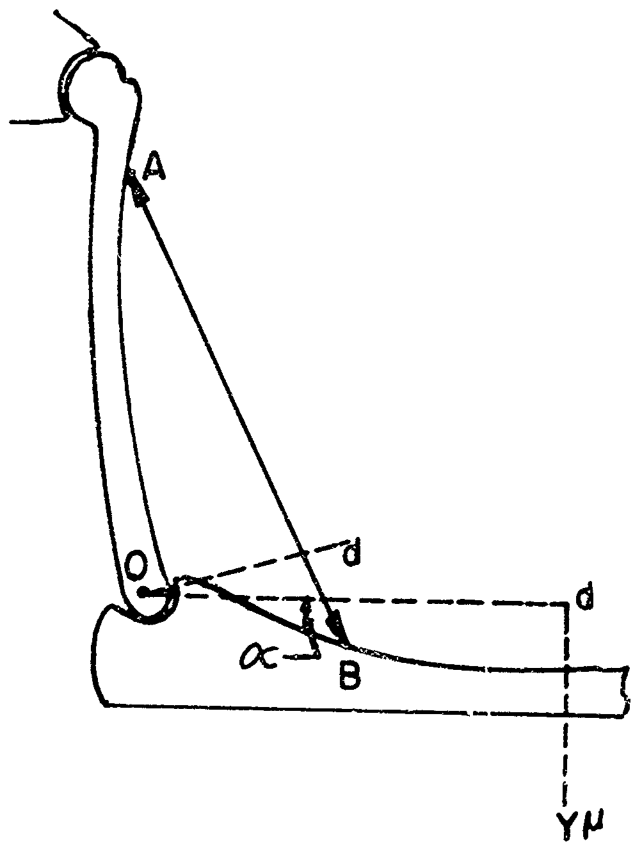


Figure 61. Moment of Rotation of the Forearm

Flexion occurs as the "motor" moment, M, exceeds the "resistant" moment, M'.

The necessary muscular effort varies inversely as the length of the lever od. Hence, in any given position the shorter the distance from B to the joint the greater the force which must be developed. Hence: od = oB sin α.

$$M = F \times oB \sin \alpha$$

The " motor" moment varies, according to a sinusoidal law, with the inclination of the muscle to the segment being displaced (Figure 62).

In the complete movement of the forearm the angle α varies from 0°-180° (extension to flexion). When angle $\alpha = 30^{\circ}$ $\sin \alpha = 1/2$. The position of maximum moment is when muscle AB is at right angles to the movable segment.

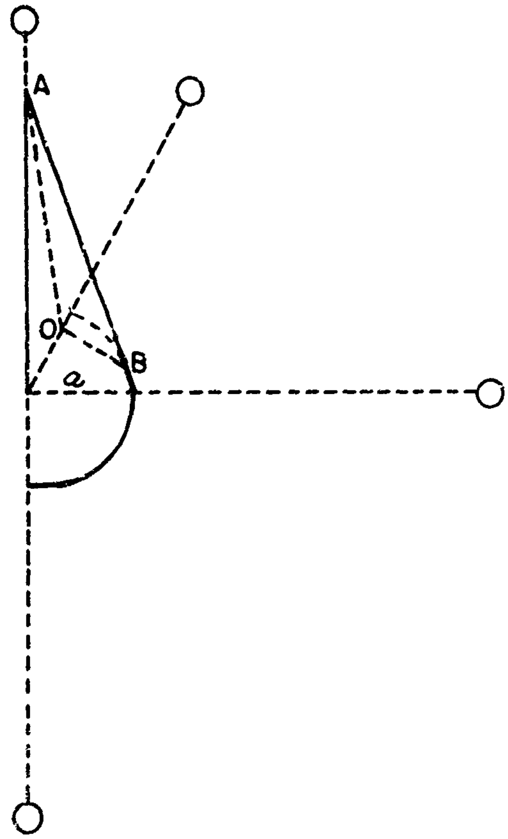


Figure 62. Different Degrees of Flexion of the Forearm

In practice the "motor" movement is never zero in any one position at any given moment. The muscle, even when reduced to a straight line, is not always in the same plane as the bone. The effective component of its force will be its projection on the plane of the bone. (Figure 63).

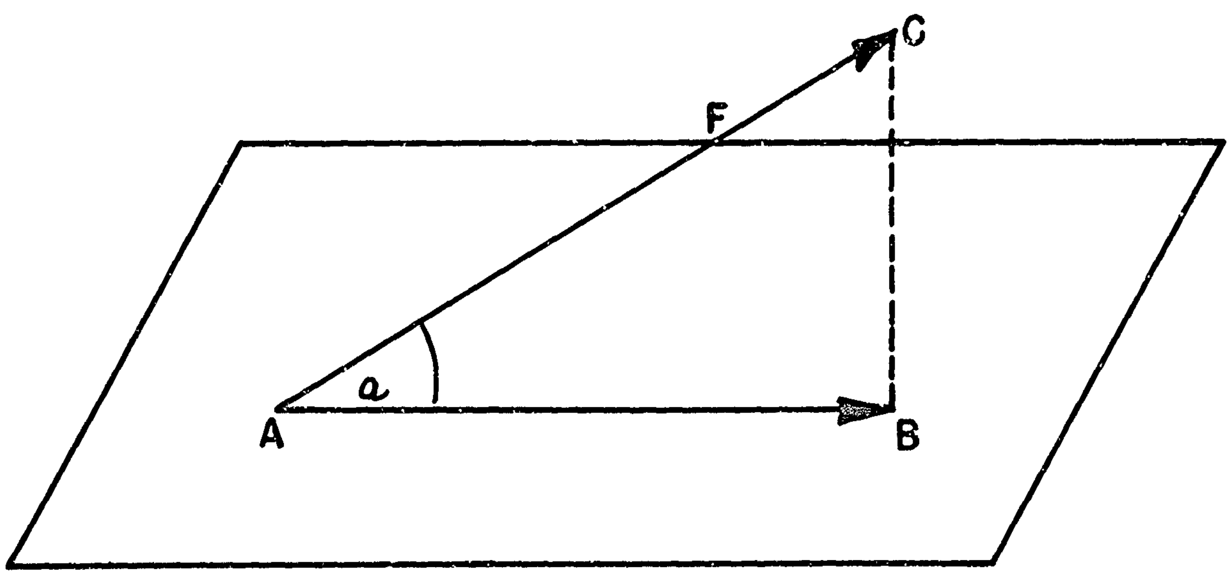


Figure 63. AB = Component of Force (F) in Plane of Limb Rotation

The magnitude of this component is proportionate to the cosine of angle α . In the above AB is the component of muscle force, F, in the plane of limb rotation.

A movement with one degree of liberty may be due to the action of one muscle or of several. It is necessary to consider here only the resultant. Any system of force can be reduced to an equivilent single force and a "couple." This is only true for a single definite position, since forces change during displacement. Movements with only one degree of liberty are larger joints, e.g. forearm and lower leg, with no rotational movement. Hence, as in the foregoing Figure, the effect of a lateral component, BC, is practically nil in flexion and extension. But the forearm can rotate in supination. Finger joints are the only ones with a single axis. The elbow joint has two degrees of liberty.

Flexor muscles operate more economically than extensors (Chauveau, Fischer).

Spherical joints have three degrees of liberty (flexion or extension in the sagittal plane, adduction in the frontal plane, plus circumduction). Motion is more often in a single direction, around a determined axis, e.g., the legs usually move in the sagittal plane.

Muscles may control two or more joints, e.g., the extensors control the wrist and the fingers. The same member may call into play in its movements a whole group of muscles. Muscle movement has its antagonists which control or regulate speed of movement. In principle, however muscles act in combination, more than in opposition.

The summation of energy expended in various muscle actions is necessary to determine the "degree of fatigue."

In body movements centers of gravity and moments of inertia can be studied, for various parts of the body. Fischer showed that the limbs and the head and trunk have centers of gravity referrable to a common origin, the line of the shoulder and hip joints.

If the trunk can be considered as a cylinder and the limbs as frustra of cones the movements of inertia can be calculated.

$I \quad M\rho^2$ where the mass M equals $\frac{P}{g.}$, and ρ equals the radius of gyration. Here are data on an adult male of 65 kg.

1) Bust (head + trunk), with weight equals 50% of the whole or 32.5 kg. Center of gravity equals 0.32 m from line of hips. Dimensions of the cylinder are: height equals 0.88 m, radius equals 0.13 m. Moment of inertia is:

$$I \quad \frac{M}{12}(3r^2 + 4h^2) = 0.86$$

2. Upper arm: weight equals 2.20 kg, center of gravity equals 0.145 m from center of shoulder joint. Height equals 0.35 m; and compare the segment to a truncated cone: $r = 0.047$ m, $r' = 0.04$ m. This gives:

$$I = M\left[\frac{h^2}{g}\left(1+\frac{d}{r+r'}\right)+\frac{(r+r')^2-2d^2}{16}\right]$$

3. Forearm: $M = \frac{2.04 \text{ kg}}{g}$, h equals 0.35 m, r equals 0.045 m, r' equals 0.027 m, center of gravity equals 0.54 m, from center of the elbow joint. Hence I equals 0.0037.

4. For the fingers I is as follows:

Thumb	0.000006	Ring	0.000012
Index	0.000012	Little	0.000004
Middle	0.000014		

5. Entire arm: center of gravity equals 45/100 from shoulder joint, i.e. 0.32 m. With closed fist equals 0.70 m. P equals 4.20 kg, r equals 0.047 m, r' equals 0.27 m. Hence I equals 0.03, approximately.

6. Lower leg: P equals 4.4 kg (foot included); h equals 0.44 m, r equals 0.062 m, r' equals 0.038 m. Hence, I equals 0.013.

7. Entire leg: P equals 12 kg, h equals 0.88 m, r equals 0.086 m, r' equals 0.038 m. Hence, I equals 0.146. Center of gravity equals 0.38 m from hip joint. Radius of gyration is calculated from I equals $M\rho^2$, so that ρ equals 0.34 m.

From the moments of inertia the work of oscillation may be calculated, the angular speed being ω:

$$T = 1/2 \ I \ \omega^2$$

In the foregoing no load is carried by the subject.

Locomotion is terrestrial or aquatic, the same laws of muscular action applying in each. Terrestrial locomotion involves walking, crawling, running, jumping, climbing, etc. Walking has been studied by Weber and Weber, Marey, and Braune and Fischer (Figure 64).

Walking involves a disturbance of body equilibrium, with displacement of the legs, which alternately carry the weight of the body. The center of gravity oscillates regularly above the leg carrying the weight. The leg on the ground is "the carrying leg"; the leg in the air is the "oscillating leg." A pace is the distance between the center of the feet when walking. A double pace is the complete cycle, after which legs are in the same relative position as before. The double pace has three phases, A, B, C, as below:

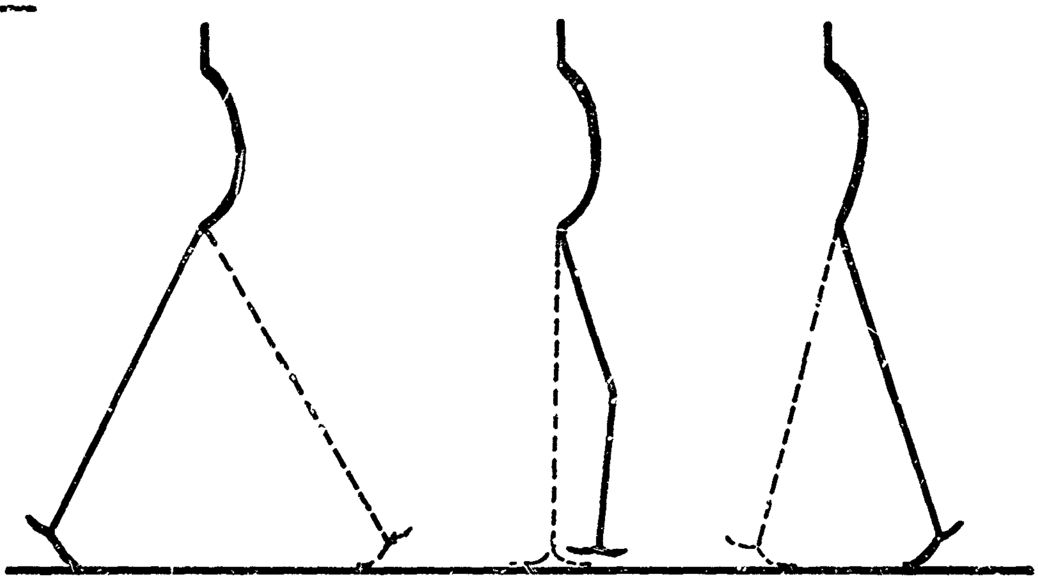


Figure 64. Different Phases of a Single Step, A to C

A = 4/10 sec., with hinder leg leaving ground by the toe, swinging forward, meeting the ground with the heel; B = 1/10 sec., with both feet on the ground. There is first a period of oscillation, then a period of "double support." In ordinary walking (121 paces, 0.75 m long, per min.) the time of a double pace = 1 sec. The oscillating leg rests on the ground 5/10 sec., supporting the body.

The double pace was analyzed by Fischer on an adult male 1.87 m tall, 58.7 kg, with leg length = 0.87 m. The average per sec. number of double paces = 121, and the average length of pace = 0.75 m. The average time of a single pace = .495 sec., and the rate of walking = 5.445 km per hr.

Beginning when the right foot leaves the ground and is in the air, with left leg in front and the pace made, trunk is over that leg, then period of "support" and "propulsion" begins. Right leg is flexed at the knee (150°), so that it can oscillate. The knee then stiffens, the thigh moves forward to an angle of 25° with the vertical, and the right heel makes contact with the ground. This is done slowly, without acceleration, i.e., it is like a pendulum of a period:

$$t = \pi\sqrt{\frac{I}{Mg\,l}} \quad \text{(where I = moment of inertia)}$$

This leads to:

$$t = \pi\sqrt{\frac{0.146}{12 \times 0.38}} = 0.56\text{s., which is too high a value.}$$

There is (Fischer) muscle action greater than gravity, due mainly to the flexor muscles.

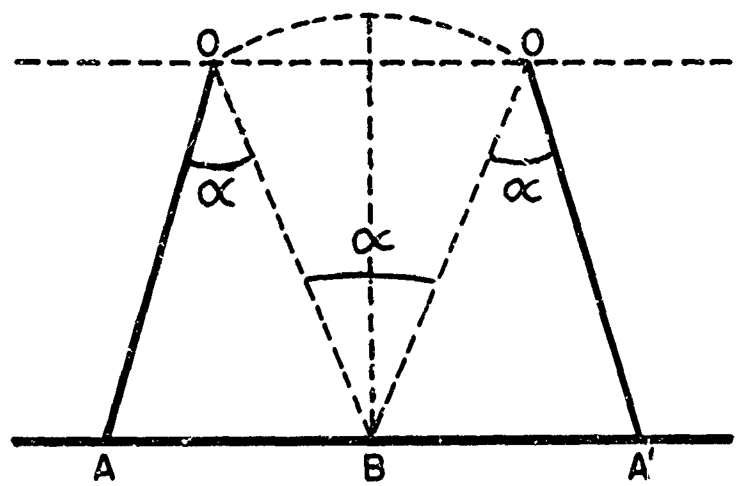


Figure 65. Diagram of Hip and Leg Movement

In Figure 65, the hip has moved from O to O', and the leg from OA to O'A'. The duration of oscillation = distance BA', and the angle of oscillation = α . Braune and Fischer found t = $\frac{415}{1000}$ of a second, AB = 0.778 m. and OA = 0.870 m. If triangle AOB were equilateral then α = 60°. Hence, $\alpha = \frac{60 \times 778}{870} = 53.60°$, which gives an average angular speed of $\frac{53.60 \times 1000}{415} = 129°$. In radians, $\omega = \frac{\pi \times 129}{180} = \frac{43\pi}{60}$.

In walking the rear leg transmits pressure to the front leg via propulsion. In an adult male 58.4 kg Fischer found pressure to be 70-77 kg , an increase of 25%. In general the increase in weight does not exceed 20 kg.

In walking there is a reaction from the ground, a tangential force represented by a negative value. Fischer found this to be 7 kg at the moment the rear leg is lifting, increasing to 16 kg at the time and duration of support. The period of support = $\frac{494}{1000}$ of a sec., which added to that of "double support" gives $\frac{494}{1000} + \frac{81}{1000} = \frac{575}{1000}$.

In walking the body and limbs oscillate. The legs move in a sagittal plane. There are side differences (possibly related to handedness). The period of oscillation in the hipline is the same as for a step, with an average amplification of 4 cm. It swings forward toward the leg being placed vertically on the ground, and swings back to the median plane of the body when the foot is lifted. The shoulder and hip lines are in the same vertical position in rest, but in walking are 1.5 cm out of line. Arms and legs tend to swing together. The head moves in walking: at the moment the foot is on the ground the center line of the head is 2.5 cm from the median plane; at the moment of "double support" the difference is zero.

In summary (121 steps per min., av. speed = 5.45 km per hr); center of gravity has a vertical oscillation of 4 cm in amplitude, a lateral of 1.3 cm , and a displacement from front to back of 2.5 cm.

II Industrial Labor and Locomotion (pp 359-391)

The work of the muscles in walking is based on Marey and on Braune and Fischer. First there is a vertical rise of the body of 0.04 m , followed by a corresponding fall under restraint (the one = 52% of the other, in terms of energy).

In an adult male of 65 kg. the muscle work is:

$$T_1 = 65 \times 0.04 \times \frac{152}{100} = 3.952 \text{ kgm}$$

The oscillation of the leg produces work:

$$T_2 = 1/2\ I\ \omega^2.$$

Since $I = 0.146$ and $\omega = \frac{43\pi}{60}$, the result is:

$$1/2\ I\ \omega^2 = 1/2 \times 0.146 \times \frac{43^2\ \pi^2}{3,600} = 0.370 \text{ kgm}$$

Taking the restraint into account:

$$T_2 = \frac{0.370}{2} \times \frac{152}{100} = 0.281 \text{ kgm}$$

The general center of gravity is subjected to a variation of vis viva in its translation; the speed varies by about 0.60 m. Hence,

$$1/2\ mv^2 = 1/2 \times \frac{65}{9.81} \times \overline{0.60^2} = 1.192 \text{ kgm.}$$

To include the work of restraint of the muscles:

$$T_3 = 1.192 \times \frac{152}{100} = 1.812 \text{ kgm.}$$

Small muscle movements of other oscillations of the trunk are negligible. So, finally, $T = T_1 + T_2 + T_3 = 3.952 + 0.281 + 1.812 = 6.045$ kgm, or on the average 6 kgm per step. For a pace .778 m long the muscle work per km will be $\frac{6 \times 1000}{.778} = 7,712$ kgm.

Amar feels that since Man is adapted to walking the value of work at each step does not exceed 4 kgm.

There is a rhythm to walking. In each man 1.67 m tall the maximum pace length = 0.85 m. Each person has his own rhythm.

Démeny found the period of "double support" to be 175/1000 sec. : at 80 paces per min., and 50/1000 sec. at 200 paces per min. The lateral oscillation of the trunk decreases with the rapidity of the cadence because the feet tend to come nearer the line of progression. A given distance is covered more economically by fast walking, but "beyond a certain point walking becomes so tiring that it is preferable to run."

In summary an adult male of 65 kg expends 7.712 kgm, covering 1 km at a speed of 5,450 m per hr. The expenditure is therefore $\frac{7712}{65 \times 1000}$ = .119 kgm per Mkg (meterkilogram). The corresponding work per Mkg at 80, 120, 180 paces per min. = .088 kgm., .119 kgm, and .176 kgm, respectively.

In walking with a load the length of the step is decreased, the period of support (especially double support) is prolonged, the foot is flat on the ground, the muscle contractions in the "carrying leg" increase, and the vertical oscillation is decreased. If the load is placed on the head the center of gravity is raised; it is better to place the load on the shoulder and/or back. Most economical is a load of about 20 kg on the shoulder.

In walking with displacement of resistance the load may be pushed or pulled. The periods of support and double support increase; the body is inclined so that the center of gravity is thrown forward; the legs are slightly bent (flexed) and vertical oscillation is reduced. The effort of propulsion increases proportionate to the resistance.

The ascending walk involves the flexion of one leg and the movement of the body and the center of gravity forward. The work = weight of the body x the height ascended. If h = height of the stairs, $T' = P \times h$; if the load is Q then $T' = (P + Q)h$. Walking on an inclined plane is a combination of walking on the level plus climbing a stair.

The descending walk shows an upright body, with the "carrying leg" bent, the other leg extended (the latter then becomes the "carrying leg", and so on). In rapid descent the body moves forward so that the center of gravity is advanced and the amount of work is reduced. The best step height is 7-10 cm. In descending an inclined plane the body is inclined forward. The pace length is greater in ascent than in descent; steps are shorter, but more in number, and the descent is accomplished with greater speed.

In running the period of "double support" virtually disappears; there is a period of "suspension" during the alternate movement of the legs. Maximum speed = 10 m per sec., i.e. 3-4 paces, varying 2.5-3.0 m per sec. Time of pace = .20-.35 sec., as the length goes from 1.5-3.4 m. The period of suspension increases with the increase in length of pace. Various oscillations occur in running. Beyond 160-170 paces per min. it is more economical to run than to walk. Maximum economy in running is at 220 paces per min.

In jumping "the man is flung like a projectile". Initial speed = v, and the work done is $T_1 = 1/2\ mv^2$. For example if a man of 65 kg leaves the ground at a speed of 8 m per sec. $T_1 = 1/2 \times \frac{65}{9.81} \times 8 \times 8 = 212$ kgm. If the angle of spring = 45°, the space covered, at the above speed, = 6.52 m. It is then necessary to compare 6.52 x 65 = 424 Mkg and 300 kgm , which gives about .77 kgm per Mkg.

The expenditure of energy in walking is measured by the consumption of O_2 in calories. Dynamic expenditure is the excess over that consumed at rest. Zuntz studied five adult male subjects of 1.70-1.81 m height, 65-80 kg weight. Chosen was one subject of 1.69 m , 65 kg , and leg length = 0.86 m. When he walked at a speed of 4,588 m per hr the dynamic expenditure was .574 small cal per kg of body weight per meter covered (this is the meter-kilogram = 1/2 small cal). In the entire sample the average was .518 small cal per Mkg. Speed influences the expenditure of energy, as here seen. (Figure 66).

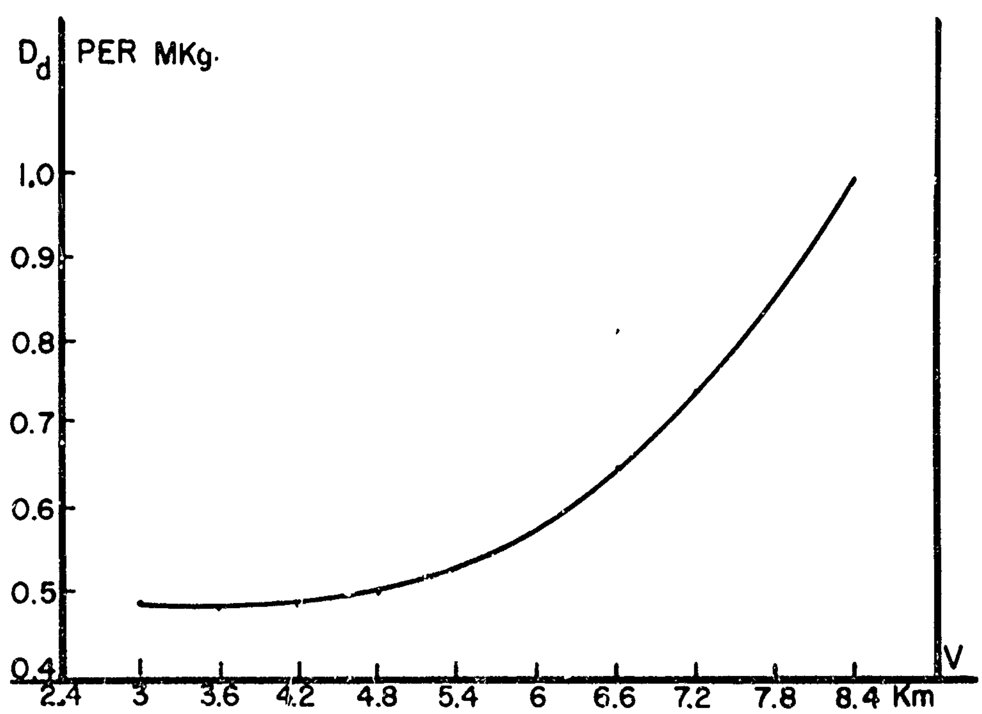


Figure 66. Dynamic Expenditure per Mkg at Different Speeds

Between 2.9 km and 8.9 km per hr it varies from .4 to .9 small cal. The minimum expenditure is seen above to be at a speed of 4.5 km per hr.

As an example, if a man of 65 kg walks at 5.4 km per hr the expenditure per Mkg is .56 small cal per kg of body weight. The expenditure for a single pace .78 m long is .56 x 65 x .78 = 28.392 small cal.

Braune and Fischer found the muscle work to be 6 kgm. If the 28.392 small cal be converted into kgm. the net effidency is

$\frac{6}{28.392 \times .425}$ = 49%, which Amar says is too high. The most economical speed is 4.5 km per hr, without a load.

Walking <u>with a load</u> causes an increase in the expenditure of energy. At a speed of 4.5 km per hr with a load of 7.3 kg the increase is 20%.

Table 47. Energy Expenditure Per M-Kgm

Speed (m per hr)	Weight of Subject + Load: 70 Kgm (no load)	70 + 11	70 + 21	70 + 33	70 + 43	70 + 53
3,000	0.486c	0.463c	0.490c	0.485c	0.542c	0.547c
3,600	0.508	0.466	0.497	0.484	0.545	0.547
4,200	0.508*	0.464*	0.454*	0.506	0.575	0.673
4,800	0.517	0.474	0.498	0.523*	0.600*	0.673
5,400	0.537	0.531	0.534	0.575	0.690	0.695
6,000	0.566	0.602	0.605	0.651	0.842	0.695
6,600	0.653c	0.688c	0.724c	0.773c	0.842c	0.695c

Expenditure per Mkg (Amar, p. 376)

* = most economical

In over-all the most economical is a load of 21 kg at 4.2 km per hr; here .454 small cal are consumed.

Amar found a maximum of daily activity for a man of 71 kg with a load of 45 kg walking at 4.8 km per hr to be (71 + 45) 25930 = 3,007,880 Mkg.

In "marking time" let n = cadence of motion, the number of flexions and extensions per minute, in an adult male of 65 kg. The consumption of O_2 per step for various cadences is (in cu. cm):

Consumption	2.25	2.521	2.708	2.911	3.115
Cadence	76	85	94	103	113

The expenditure is d = an + b, where a and b, as constants, are such that a = .02 cu. cm and b = .83 cu. cm. The equivalent caloric expenditure for the above cadences is .16, .178, .191, .206, and .220 small cal , respectively, per step, per kg. Marking time equals 1/2 the energy of walking and increases with speed and the height to which the legs are lifted (in the above h = .13 m).

A summary of walking on the level shows a maximum efficiency for a working day, with 26 km traversed (13 km with a load of 42 kg , 13 without a load). Average speed of walking = 3 km per hr. The average expenditure = 2-1/2 million Mkg per diem, equivalent to 1348 Cal. Hence, expenditure per kg of body weight per hr $= \frac{1348}{75 \times 24} = .75$ Cal. Physiological changes observed were: pulse rate increased 30-50%; duration of systole increased 20-25%; diastole diminished in duration by 1/3 to 1/2; respiratory rate increased from 16 per min. to 25 per min.; the temperature increased 1/4 - 1/2°C.

For the ascending walk Amar offers his own data.

Table 48. Energy Expended in the Ascending Walk

Weight (kg) (Man + Load)	Height Asc. (m)	Total Trips	Time hr. min.	Expend. per kgm
80.2 + 45	4.25	128	4 00	5.00 c
74.5 + 45	4.25	131	4 30	5.20
74.5 + 40	4.80	46	1 00	5.70
71.3 + 45	4.25	141	5 00	6.10
56.6 + 40	4.80	45	1 00	6.60
67.7 + 50	3.78	158	4 00	8.30
70.5 + 50	3.78	124	4 30	8.40
61.3 + 40	3.78	52	1 25	9.50
60.4 + 50	4.80	112	4 00	9.50
69.2 + 50	3.78	50	1 30	9.60
68.0 + 50	4.80	100	3 00	10.20
62.7 + 50	3.78	105	3 30	10.30
62.3 + 50	3.78	120	4 00	10.30
Av. expenditure per kgm				8.05 c

(Amar, p. 386)

The best result obtained was 31716 kgm per hr with a subject of weight 57 kg and load 40 kg , speed .12 m per sec. In eight hours the work output = 31716 x 8 = 253728 kgm. This can be averaged to 250,000 x .008 = 2000 Cal.

The inclined plane is important for the relations between height, load, speed, and inclination. Amar used a plane 12 m long, with a slope varying 8-13 cm per meter. The speed was 3.7 km per hr , at 100 paces per min. The subject weighed 66 kg , with a load of 7.3kg.

Slope of the Plane	Man + Load	Expenditure per Mkg Ascent	Descent	Level
0.08 m	66 kg	1.00 c	0.85 c	0.41 c
	66 + 7.3	1.50	1.13	0.49
0.13 m	66	1.80	1.00	0.49
	66 + 7.3	2.20	0.84	0.49

In ascending the expenditure of energy increases proportionate to the slope of the plane. The work done in raising 1 kg to a height of 8 cm (.08 kgm) involves an expenditure of energy of .59 c + 7.4 sm. cal per kgm. In the 13 cm slope expenditure = 10.7 small cal per kgm. In descending Amar found for the 8 cm slope 5.5 small cal per kgm, and for the 13 cm slope 4.53 small cal per kgm.

Chauveau found that the expenditure of energy is less in the (resistant) work of descent than in the (motor) work of ascent, with a ratio of about 52: 100 at ordinary speed. Amar found a ratio of 85: 100 in the unbaded descent down a slope of 8 cm per m; with a load on the same slope the ratio is 75: 100. For a slope of 13 cm per m the ratio without load is 52: 100, but with a load it is 38: 100.

When horizontal and vertical walking are compared it is found that the expenditure per kg per m traversed, horizontally = .51 c and in ascending stairs = 8 c. Hence, the ratio between the kgm and the Mkg is $\frac{8}{.51}$ = 16. A man of 65 kg does 65 Mkg per m or $\frac{65}{16}$ = 4 kgm. Hence, a pace of .8 m long = 4 x.8 = 3.2 kgm. By this method the efficiency of ordinary walking is about 32%.

In terms of human strength a man can do 300,000 kgm of work in eight hours. Power per second = 300,000/8 x 3600 = 10.4 kgm per sec., which is about 1/7 of a horsepower (p.459, Amar).

Amar concludes the book by two Chapters on the use of tools, e.g., vertical hauling with ropes as in a pile driver, use of the hammer, file, saw pruning shears, glass polishing, spade work, wheelbarrow trundling, typing, bricklaying, etc. A "philosophy" of labor is given.

II Industrial Labor - Tools (pp 392-426)

IV Industrial Work (cont.) (pp 427-461)

V General Conclusions (pp 462-466)

SPACE REQUIREMENTS OF THE SEATED OPERATOR

(Condensed from W.T. Dempster)

DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

OR are
Blank pgs
that have
Been Removed

BEST AVAILABLE COPY

BEST AVAILABLE COPY

SPACE REQUIREMENTS OF THE SEATED OPERATOR

(Condensed from W.T. Dempster)

ANGIOCARDIOGRAPH--A technique for making multiple x-ray exposures of a region in sequence and at intervals of a second, more or less; designed for following the course through the heart of radiopaque materials added to the blood — here used for making sequential x-rays of wrist movement.

ANGULAR MOTION--Movement of a body in rotating about some center of reference; two points on the moving body change their angular positions relative to one another during the motion (cf. translatory motion).

ASTHENIC--Characterized by weakness, feebleness or loss of strength; here used to refer to the thin, linear type of body physique.

AXIS OF ROTATION--For angular movements of one body relative to another in a plane, all points in the moving member describe arcs about a center axis, this is the axis of rotation; it may have a stable position or may shift from moment to moment relative to the non-moving body.

CARDINAL PLANES--In anatomical descriptions the body is conventionally considered as if it were in the upright standing posture, and positions and shapes of parts are differentiated as up vs. down, fore vs. aft and mid-plane vs. lateral: three classes of mutually perpendicular cardinal planes related to the standing posture are: (1) sagittal (front-to-back and up-and-down), (2) transverse or horizontal (front-to-back and side-to-side), and (3) coronal (up-and-down and side-to-side).

CENTER OF CURVATURE--The center or focus of a radius which moves to describe a curve; there is one center for a circle and a changing pattern of instantaneous centers for other curvatures for which the radial length changes.

CENTER OF GRAVITY--The point or center of mass through which, for all orientations of a body, the resultant of the gravitational forces acting upon particles in the body pass.

CENTROID--A point in any geometrical figure (volume, area, line) analogous to the center of gravity of a body with weight; the moments of the figure (the masses of minute segments multiplied by their distance from the centroid) are in balance relative to the centroid.

CINEFLOUROSCOPIC TECHNIQUE--A technique of making motion pictures of x-ray images by photographing a fluoroscopic screen.

CLAVISCAPULAR JOINT (Orig. term)--The functional composite consisting of acromiclavicular joint plus coracoclavicular joint (i. e., coracoclavicular ligaments or trapezoid plus conoid ligaments).

CLOSED CHAIN--A sequence of several links so interconnected by joint elements with limited degrees of freedom that only determinate or predictable movements of the parts are possible.

COMPONENTS--According to Sheldon's system of typing body build of each individual physique shows in different degree characteristics of softness, roundness, and breadth (called component I), of muscularity (component II) and of linearity (component III); the amount of each of the three components may be graded by inspection and measurements of photographs on a seven-point scale (or on a 13-point scale counting half points).

CONGRUENCE--If the opposed articular faces of a joint were in simultaneous contact with no areas that did not coincide the joint would be congruent; congruent position - a position of a joint where the contacts are congruent.

CONTINGENT MOVEMENTS (Orig. term)--In joints of one degree of freedom of motion (viz. elbow; ankle) the relative movement of bones forming the joint do not move exclusively in a plane on one axis of rotation alone, rather the contracting curvatures of the joint induce secondary, concomitant motion on axes perpendicular to that for the principal movement; these accessory and invariate motions are called contingent movements.

COUPLE--A pair of equal forces acting in parallel but opposite directions, not in the same line, and tending to produce rotation.

DEGREE OF FREEDOM (OF MOVEMENT)--A free body may rotate in any angular direction about a point, i. e., it rotates in any of the three dimensions of space; thus it has three degrees of freedom. When its motion is so restrained that it may rotate about two perpendicular axes only, it has two degrees of freedom. When it is limited to movement about one axis only, it has one degree of freedom.

DEGREE OF RESTRAINT (OF MOVEMENT)--When an unrestrained rotating body moves about a point in any of its three degrees of freedom, there is a zero degree of restraint. When it may rotate about any two axes, movement about the third axis is restrained, i. e., one degree of restraint. With two degrees of restraint, movement is limited to rotation in one plane only, i. e., it has one degree of restraint. With three degrees of restraint all angular motion is prevented.

DESMO-ARTHROSES (Orig. term)--A functional joint system consisting of a gliding joint (arthrosis) with, at an appreciable distance, a ligamentous binding together (desmoses) of the bones concerned; examples: sternoclavicular joint, claviscapular joint.

EFFECTIVE JOINT CENTER (Orig. term)--In a composite chain of three or more links with interconnecting joints, the rotation of a distal link over two or more joints may be related to some point in space as center of rotation; this is the effective joint center.

EFFECTIVE SEAT CONTACT (Orig. term)--The mean reaction vector at a seat (or foot rest) which opposes the gravity vector of a seated individual to form a couple; its position in the sagittal plane is located by balancing moments involving forces which act at the front and rear of the supporting surface.

END MEMBER--The terminal segment, such as the hand, foot, head or finger, beyond the last joint in a body joint chain.

ERGOSPHERE (Orig. term)--The total work space available to the end member of a link system, i.e., to a hand or foot, relative to some fixed point of reference; the term may in addition include the whole body of the individual and the augmented space required for facilitating movements of other body parts.

EVOLUTE--For every system of curvature, called involute, a second curve, called evolute, may be constructed; tangents to the latter are normal to points on the involute curve; the evolute is the focus of the center of curvature of the involute arc.

EXCURSION CONE OF A JOINT (same as joint sinus)--The total range of angular motion permitted a moving member of a joint when the other member is rigidly fixed.

GLOBOGRAPHIC PRESENTATION (OR RECORD, ETC)--A method of showing the angular range of joint movement upon a globe with meridians and parallels; one member of a joint is regarded as rigidly fixed with the functional center of the joint at the center of the globe while a point on the other member describes a circuit upon the surface of the globe which encloses all possible positions of the moving part.

GRAVITY LINE (LINE OF GRAVITY)--A downward vertical force vector through the center of gravity of a body.

GREAT CIRCLE--The intersections with the surface of a sphere of a plane which passes through the center of the sphere.

INCONGRUENCE--When the opposed surfaces of the members of a joint articulate in partial or ill-fitting contacts, the joint is incongruent.

INSTANTANEOUS AXIS OF MOVEMENT--When two points on a body in motion, relative to another body or to the space of the observer, change their angular relations during the motion, this angular change for any instant occurs in a plane, and the movement for that instant may be described as a rotation about a stationary perpendicular axis called an instantaneous axis.

INSTANTANEOUS JOINT CENTER--The momentary center about which a body moves in rotating on a plane; a point on an instantaneous axis of rotation. In a circular motion the center has a constant position, for other angular motion in a plane it shifts to a new locus; successive loci describe a path of instantaneous centers.

INVOLUTE--A curve traced by a point on a taut thread as it winds upon a fixed curve called an evolute; perpendiculars to an involute curve are tangent to points on the evolute curve.

JOINT SINUS--When one member of a joint permitting at least two degrees of freedom of movement is held fast and the other member link is rotated to its limits in all directions, the moving member sweeps out a conical concavity which includes within its range all possible movements allowed by the joint structures.

KINEMATICS (N), -IC (Adj.)--The science which is concerned with the motions of bodies and systems without regard to forces producing them; the geometry of moving systems.

KINETOSPHERE (Orig. term)--The space or space envelope which encloses all possible translatory movements of the end segment of a link system held in some constant orientation; it is defined relative to some fixed reference point of the body or contact point in the environment (seat "R" point, etc.).

LEVERAGE--The straight line distance from the center of rotation of a body to the point of application of a perpendicularly acting force which tends to rotate the body, cf. lever arm; moment arm.

LINK--The straight line which interconnects two adjacent joint centers; the axial core line of a body segment between two joints or between a terminal joint and the center of gravity of the end member (or between the terminal joint center and an external body contact).

MEAN DEVIATION--A measure of the variability about an average; the deviations on either side of an average, all considered as positive values, are summated and divided by 4.

MOMENT--The effectiveness of a force in tending to twist or cause motion around a central point.

MOMENT OF INERTIA--A measure of the effectiveness of a mass in rotation. The change in angular velocity in a rigid body which rotates through the action of a given torque depends upon both the mass of the body and the distribution of that mass about the axis of rotation. Moment of inertia is the summation of the point masses of a body times the square of their radial distance from the axis of rotation.

MOMENT OF ROTATIONS--See TORQUE.

PERCENTILE--Any of the points which may be used to divide a series of quantities or values arranged in order of magnitude into 100 equal groups; thus the 50th percentile divides the sample equally while the 95th divides the upper five percent from the remainder.

PYKNIC--The thick set, short, stocky body build.

"R" POINT--Fixed reference point; as seat "R" point, the midpoint of the junction of the seat back and the seating surface.

RADIUS OF CURVATURE--The radius of a circle drawn through three or more consecutive points in a curve at the region of reference.

REULEAUX METHOD--A method developed by the German engineer Reuleaux in 1875, for locating instantaneous centers of rotation; when any body moves in an arc, instantaneous centers for a short phase of the motion are located at the intersections of normals to midpoints on the mean paths of two points on a rotating body.

SEAT "R" POINT--The fixed reference point in considerations of hand and foot movements, etc.; the midpoint of the junction of a seating surface and the back of the seat.

SINUS--(See joint sinus or excursion cone).

SOMATOTYPE--An overall classification of an individual's physique based upon an evaluation of the relative rotundity, muscularity, and linearity shown; these three components are separately evaluated on a seven-point scale (frequently half-points are used also). The evaluations of an individual are designated by a formula indicating the quantity of each components as 1-2-7 (e.g., 1 - low in rotundity; 2 - slight development of muscularity; 7 - extreme linearity).

SPIROMETER--An instrument for measuring the air capacity of the lungs.

STANDARD DEVIATION--A measure, calculated by statistical methods, of the dispersion of values about their mean; one standard deviation (symbol: sigma) includes about 68 percent of the observations above or below mean, two standard deviations include about 95-1/2 percent.

STROPHOSPHERE (Orig. term)--The space which incloses the total range of movement of a point on a terminal link segment through all possible translatory movements and for the possible end member positions as the part rotates about one axis only of the terminal joint; the space envelope for the movement is related to coordinates through a fixed reference point in space.

TORQUE--A measure of the effectiveness of a twisting force or couple; force times the distance from a perpendicular to the line of action of the force to the center of rotation. Alternate term: moment of rotation.

TRANSLATION--TRANSLATORY MOTION--Straight or curved motion in which all points of a moving body have at each instant the same direction and velocity. Contrast with rotation.

INTRODUCTION (Dempster, pp. 1-5)

The function of this report, based largely upon original research, is to report data essential for three areas of knowledge in biomechanics. These are:

1) to permit the synthesizing of realistic manikins;
2) to understand the body kinematics of a seated operator in his work space;
3) to define the dimensions of the work space.

An understanding of body kinematics is essential in the proper design of the work space; it must be remembered that the limitations of anatomy are immutable, and that efficiency may therefore be increased only through changes in the relationships of the work space itself.

There are several important factors, therefore, that must be considered in planning the work space:

1) the operation of one control must not hinder that of another;
2) control placement must be in terms of operational sequence;
3) routinely-operated controls must be separated from those rarely used;
4) body comfort, particularly when operation is carried out over long periods, must be near optimum;
5) emergency and safety factors must be considered.

Basic research, as in this study, must furnish information to the design engineer which will enable him to meet the above requirements.

Utilization of manikins, reduced or simplified scale models, becomes extremely valuable in design engineering. However, a suitable manikin must meet several requirements. It must be comparable to the mean build of the population to be placed within the workspace. (In this study, the emphasis will be upon the pilot, and hence, the builds of U.S. Air Force personnel will be considered.) In addition, a manikin must represent a seated figure correctly, in terms of joint and segment relations. And finally, these joints and segments must move in a fashion comparable to actual individuals.

The postures into which a manikin is placed must not only represent human movement ranges, but also functioning body postures. As the actual end members involved in the operation, the hand and foot require special operational significance, particularly the relationships to functional joint centers. Thus, how does the movement of a particular joint affect the next member, and especially the effective end member?

Consideration must also be made of the points at which the manikin begins to differ from the actual body system; a knowledge of manikin limitations permits maximum efficient utilization of the work space.

These factors are all considered and discussed in this study, utilizing, whenever possible, actual experimental data. Other factors are also analyzed, for example, the effects of body weight and the roles of muscle tension and external resistances, such as the inertia of body mass.

MATERIALS AND METHODS[1] (Dempster, pp. 6-67)

This study involves living individuals, cadavers, and skeletal material. The skeletal material, numbering more than 400 specimens of each major bone, came from persons of unknown age, sex, or physical status ; it was a part of the University of Michigan osteological collection. All measurements were carried out utilizing standard anthropometric instruments.

The articular curvatures of joint surfaces were studied by means of two approaches. The first was the dial gauge method, using a Lensometer (Figures 67 and 68). The reference pins were 10 mm apart, with the middle pin depression read to the nearest hundredth of a millimeter ; this depression represented the depth of the arc defined by the three contact pins. The average radius of curvature was computed from the formulae in Figure 68.

The second method used was the evolute method, which indicated the changing radius of curvature which existed rather than merely the average. Any regular curve, in geometry, is called an involute. The evolute is the locus of centers of curvature of the involute. (Figure 69). It may be visualized by observing a taut cord unwound from round a circular rod ; a point at the moving end of the cord describes a spiral, the involute.

The surface curvature, or cross section, of the rod is the evolute. At all times, the taut cord is tangent to the evolute and normal (perpendicular) to the involute. The straight line portion of the cord is the radius of curvature.

The evolute method was applied by rotating a joint surface upon a chalked surface through an arc. A saw cut was made in the plane of the resulting chalk mark and the contours studied.

---

1. This chapter is heavily abstracted. Anyone interested in the methodological details presented by the author should refer to the original. - FEJ.

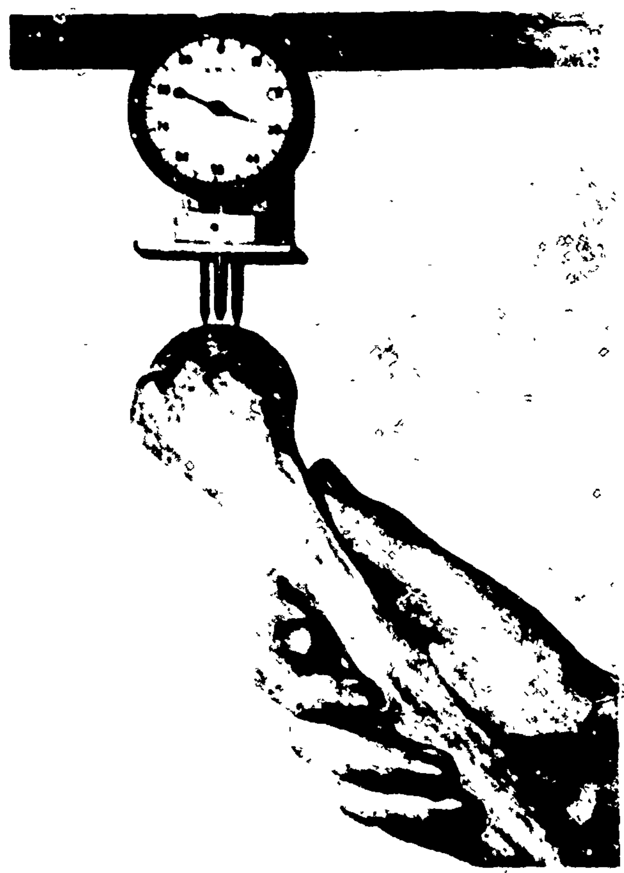


Figure 67. Dial Gauge (Lensometer) Method of Determining Average Radius of Curvature

Only joints free from pathology were studied. Dissected joint specimens were also prepared. The joints analyzed were the hip, knee, ankle, shoulder, elbow, wrist, sterno-clavicular, and coraco-acromio-clavicular. Joint centers were located by the Reuleaux Method for locating instantaneous centers of rotation. This is demonstrated in Figure 70. Instantaneous centers were located at 5-10 degree intervals. In addition, the areas of joint contact at differing angular positions were studied through the transfer of paint from one member to another.

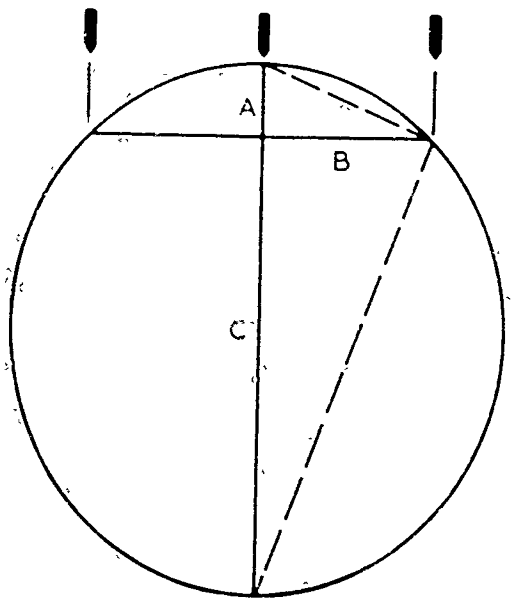


Figure 68. Method of Calculating Average Radius of Curvature. A and B are known values; the depth at the heavy arrows is obtained from the Lensometer.

Thirty-eight living subjects were selected on the basis of physique, Sheldonian somatotypes being used. The final sample was broken down as follows:

thin	10
median	11
muscular	11
rotund	6
Total	38

The median group consisted of individuals whose builds were commonly found among Air Force fighter pilots.

Sixty-nine anthropometric measurements were taken, as shown in Figures 71, 72, 73, 74, together with the values for the mean and standard deviation of each physique group. These subjects were analyzed for joint and limb movement, and thoracic and pelvic tilting at various postures.

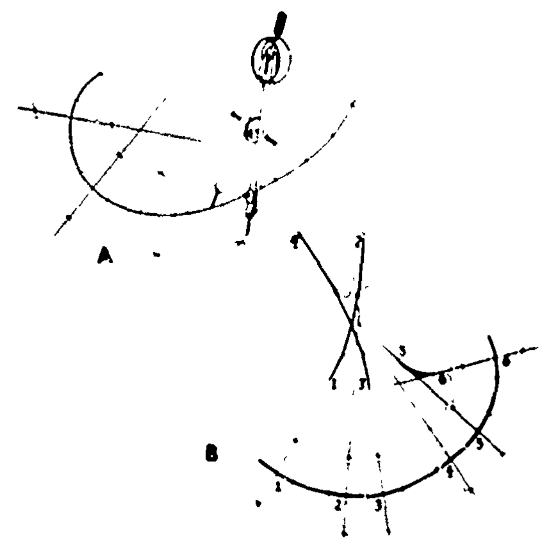


Figure 69. Curves Explanatory of Evolute Analysis.

A. Construction of normals to a test curve through use of compass arcs.

B. A composite curve is broken into component involute curvatures 1-2, 3-4, and 5-6 by analysis; normals to the involutes are tangent to evolute curves designated by primes.

Ten cadavers (three embalmed) were studied as a group. They were dismembered into five parts: head and trunk, right and left upper limbs, right and left lower limbs. Centers of gravity were located on a balance plate, and measurements taken to the extremities of each mass.

The average of 10 five-degree oscillation periods was determined for each mass on a free-swinging pendulum. The moment of inertia was computed from the following:

$$I_o = \frac{WL}{4\pi^2 f^2} \text{ ; where,}$$

$\underline{W}$ = mass times $\underline{g}$ (i. e. 980 cm / sec.^{2})
$\underline{L}$ = distance from the C. G. to the point of suspension
$\underline{f}$ = the frequency of oscillation
The moment of inertia at the center of gravity was, therefore,

$$I_{c.g.} = I_o - \frac{WL^2}{g}$$

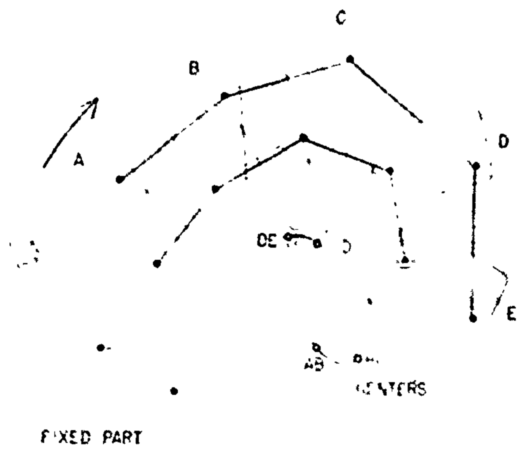


Figure 70. Method for Locating Instantaneous Centers, after Reuleaux (1875).

Heavy lines show relative motion of two points on the moving part.

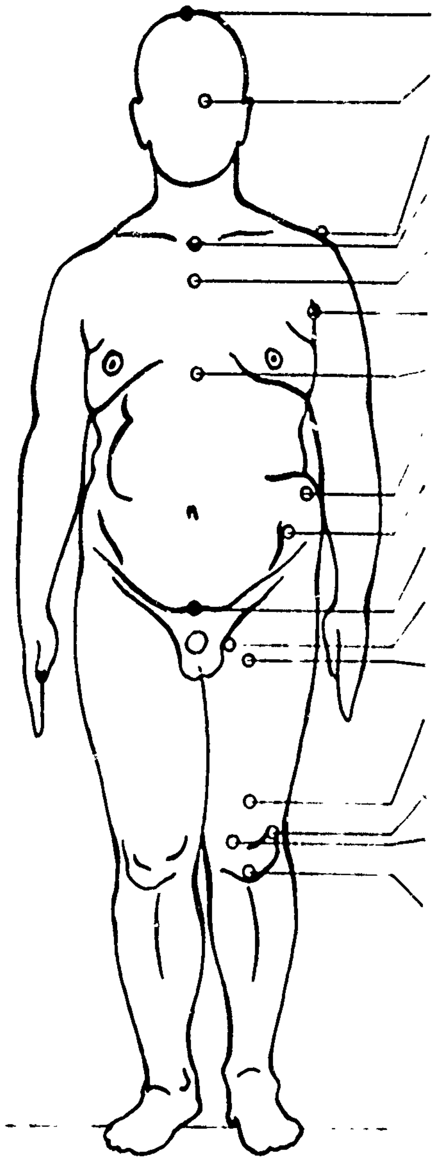
Mid-interval normals intersect at points (AB, BC, etc.) which, in sequence, are momentary centers about which angular motion occurs.

Volumes were computed by weighing the part in air and in water. The difference became the volume equivalent.

The trunk segments were subdivided into head and neck, thoracic, and abdomino-pelvic units, with similar operations performed.

Volumes of living subjects were obtained by water displacement, both on whole and partial limb segments, and on cross-sectional areas, to be related to height. These areas were obtained by pantograph tracings of body contours at 20 recorded heights, and graphed against the heights themselves. The same data were gathered from cadavers, by sawing one specimen into 1-inch transverse segments, and plotting area against height.

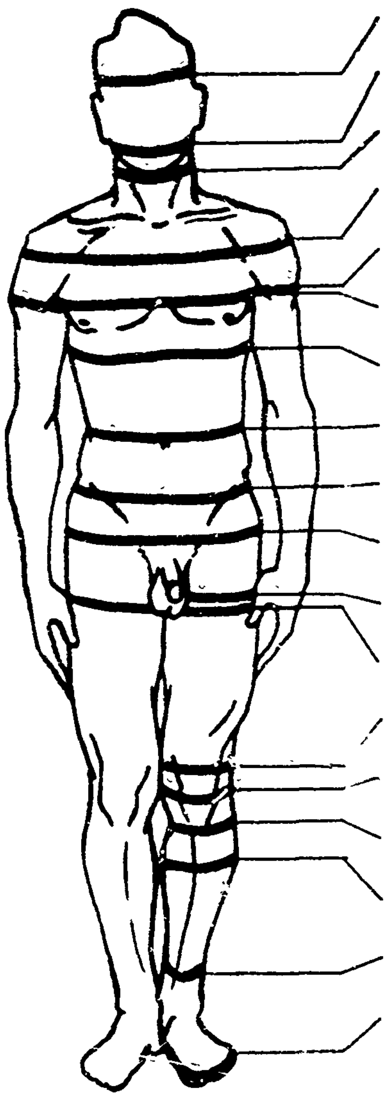
Data were also gathered on horizontal pushes and pulls in sagittal planes. Six different postures were studied, though the six were not necessarily the same for pushing and pulling due to subject adaptation in order to gain maximum force. These postures are



HEIGHT FROM FLOOR: (in cm.)	MEDIAN Av.(8)	S.D.	ROTUND Av.(7)	S.D.	MUSCULAR Av.(8)	S.D.	THIN Av.(10)	S.D.
STATURE	175.1	4.3	173.3	5.7	176.3	5.3	186.2	5.1
INNER CANTHUS	163.1	3.8	162.4	5.1	164.5	6.8	174.4	5.2
ACROMION	143.5	3.7	144.1	4.6	144.9	5.0	153.8	4.9
SUPRASTERNALE	142.6	3.8	141.5	5.1	144.1	4.8	151.7	4.8
STERNAL ANGLE	139.4	3.8	139.4	5.0	140.8	5.2	147.8	5.1
ANT AXILLA	132.8	3.0	133.7	5.4	133.6	5.0	143.5	4.8
XIPHISTERNUM	124.3	2.7	124.3	5.6	127.0	4.2	133.0	4.7
ILIAC CREST	105.0	2.8	103.2	4.1	[illegible]	2.9	111.8	3.6
ANT SUP SPINE	99.4	3.2	96.6	4.0	100.4	3.1	106.8	3.7
PUBIS	89.3	3.8	85.7	3.2	90.5	3.2	95.9	2.9
GLUTEAL SULCUS	79.9	3.3	78.5	2.7	80.4	3.2	87.2	2.6
CROTCH	83.2	4.2	79.2	2.5	84.0	4.1	89.6	4.0
SUPRA-PATELLARE	52.[illegible]	2.1	52.8	1.8	53.1	2.1	56.0	2.1
LATERAL TIBIALE	47.5	2.[illegible]	47.3	2.5	47.7	3.1	51.1	3.4
MEDIAL TIBIALE	47.0	2.6	45.5	2.3	46.8	2.9	49.8	3.2
TIBIAL TUBEROSITY	42.7	1.8	42.4	1.6	44.5	2.5	46.4	1.9

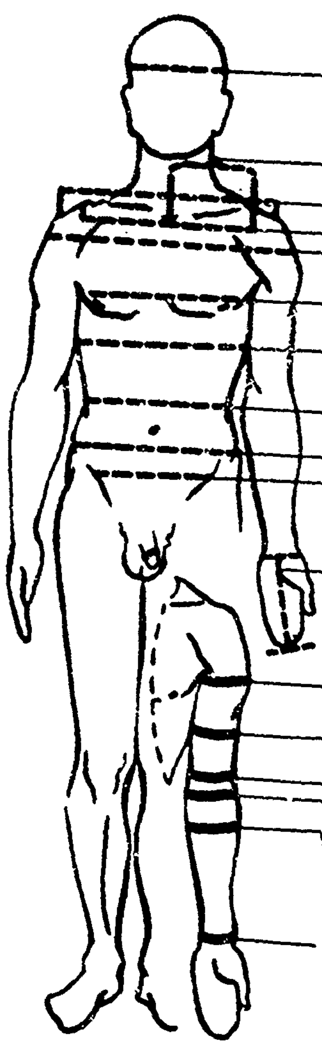
SUPINE MEASUREMENTS: (in cm.)	MEDIAN Av.(10)	SD	ROTUND Av.(7)	SD	MUSCULAR Av.(11)	SD	THIN Av.(10)	SD
BODY LENGTH	176.6	4.9	174.0	7.1	177.5	5.7	186.6	5.2
VERTEX TO PUBIS	85.8	2.9	86.5	5.3	86.8	2.6	90.6	5.6
VERTEX TO C G	76.1	5.0	76.2	4.6	78.7	3.2	84.1	3.3

Figure 71. Measurements of the Study Sample.



CIRCUMFERENCES: (in cm.)	MEDIAN Av.(8)	MEDIAN S.D.	ROTUND Av.(7)	ROTUND S.D.	MUSCULAR Av.(8)	MUSCULAR S.D.	THIN Av.(8)	THIN S.D.
HEAD (euryon)	58.0	2.2	58.1	2.0	57.7	1.6	57.2	1.7
HEAD (gonion)	47.3	2.2	51.2	3.2	48.4	2.0	44.1	3.1
NECK (thyroid cartilage)	38.0	1.5	41.0	2.8	38.5	1.6	35.2	2.0
SHOULDER (sternal angle)	106.0	4.6	116.0	9.0	107.5	5.0	102.9	4.4
SHOULDER (axilla)	116.6	3.9	126.6	9.0	118.9	4.2	104.8	5.2
CHEST (axilla)	97.2	3.9	109.0	6.5	98.3	5.5	87.2	5.4
CHEST (xiphisternum)	91.3	4.0	104.7	7.4	89.8	4.2	79.3	5.2
WAIST (min.)	79.8	3.9	102.9	10.6	78.2	4.1	71.2	3.0
ILIAC CREST	83.9	6.2	112.1	10.6	82.8	5.9	77.8	2.2
PUBIC SYMPHYSIS	95.4	3.1	112.3	9.2	96.1	5.1	89.3	3.3
TRUNK (gluteal sulcus)	94.8	3.7	106.7	7.1	95.0	5.2	84.8	2.7
THIGH (crotch)	57.1	2.4	66.9	5.0	59.3	4.1	49.5	3.7
SUPRA-PATELLARE	38.8	1.5	45.9	4.1	39.7	2.4	34.8	1.6
MID-PATELLA	37.5	1.5	42.4	3.3	37.9	2.3	35.1	1.6
TIBIAL TUBEROSITY	34.0	1.2	39.6	3.2	34.7	2.1	32.0	1.7
CALF (max.)	38.3	2.0	41.9	3.9	38.6	1.9	33.7	2.1
ANKLE (min.)	23.6	1.2	25.1	2.7	23.5	1.2	21.3	1.5
FOOT AT FLOOR	62.2	2.3	62.4	3.0	63.5	3.3	63.1	3.0

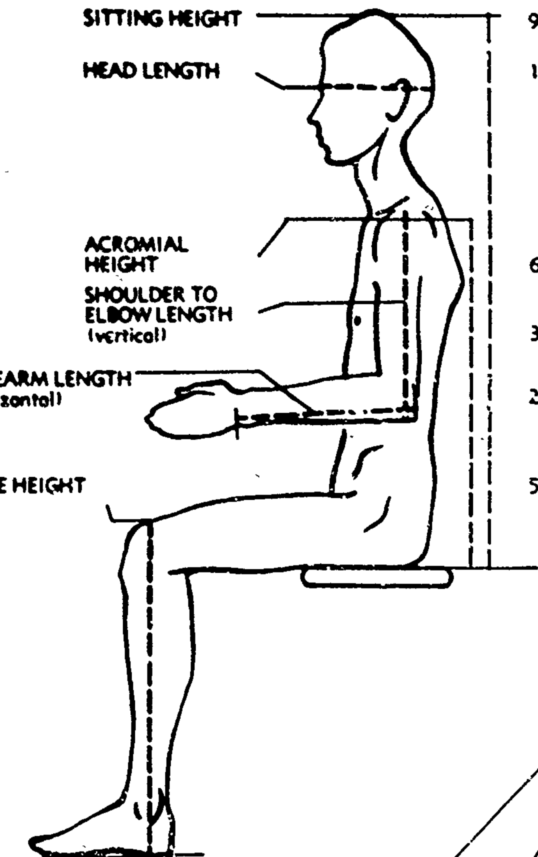
Figure 72. Measurements of the Study Sample.



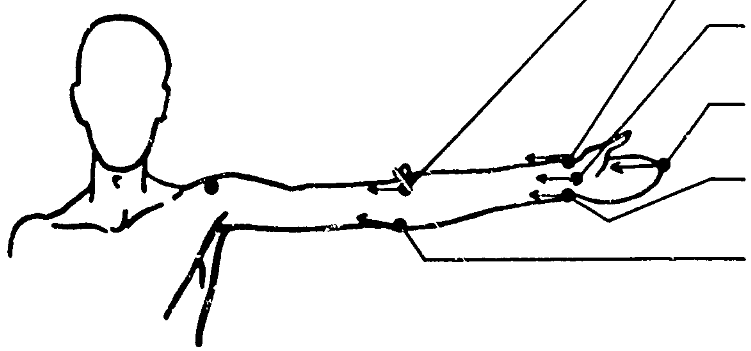
BREADTHS: (in cm.)	MEDIAN Av (11)	S.D.	ROTUND Av (7)	S.D.	MUSCULAR Av (11)	S.D.	THIN Av. (10)	S.D.
HEAD	15.8	0.5	16.1	0.4	15.8	0.5	15.4	1.0
CLAVICLE LENGTH	16.8	0.6	16.7	1.3	16.6	1.1	16.1	0.9
BIACROMIAL	40.4	1.4	40.7	3.2	41.7	1.7	39.9	2.3
BICLAVICULAR WIDTH	33.4	1.0	34.1	2.4	32.9	2.0	32.4	1.7
BIDELTOID (level of sternal angle)	45.3	1.1	48.2	3.4	46.7	3.0	43.3	2.5
CHEST WIDTH (axilla)	33.0	1.8	36.6	2.7	34.1	2.1	31.5	2.3
CHEST WIDTH (xiphoid)	30.7	1.8	35.9	3.0	31.3	1.4	27.9	1.8
WAIST WIDTH	28.7	1.6	34.1	2.8	28.3	1	25.5	1.6
INTERCRISTAL	29.3	1.3	37.1	3.9	29.3	2.3	29.4	1.8
INTERSPINOUS	24.7	1.3	32.2	3.2	24.8	2.7	25.9	2.0
HAND LENGTH	19.1	1.0	18.9	0.6	19.7	0.8	19.5	1.1

UPPER LIMB CIRCUMFERENCES: (in cm.)	MEDIAN Av (11)	S.D.	ROTUND Av (7)	S.D.	MUSCULAR Av (11)	S.D.	THIN Av. (10)	S.D.
ARM AT AXILLA	31.6	1.6	36.2	4.7	32.7	2.0	26.2	1.4
MID-ARM	28.6	1.3	32.1	4.0	29.3	1.3	24.1	1.6
ARM ABOVE MED EPICONDYLE	25.3	0.8	29.0	2.5	26.2	1.5	22.1	1.0
FOREARM BELOW MED. EPICONDYLE	26.8	1.1	29.6	2.5	27.8	1.4	23.8	1.0
UPPER THIRD OF FOREARM	27.0	1.1	29.0	2.7	28.4	1.2	23.3	1.8
WRIST (styloids)	17.4	0.6	18.2	1.2	17.8	0.9	16.6	1.0

Figure 73. Measurements of the Study Sample.



SEATED MEASUREMENTS: (in cm.)	MEDIAN Av.	MEDIAN SD	ROTUND Av.	ROTUND SD	MUSCULAR Av.	MUSCULAR SD	THIN Av.	THIN SD
SITTING HEIGHT	91.5	3.3	89.3	2.3	91.4	3.1	95.0	3.7
HEAD LENGTH	19.9	0.6	19.8	0.6	20.2	0.8	19.9	0.9
ACROMIAL HEIGHT	60.8	2.7	59.3	2.3	61.2	3.5	64.1	2.9
SHOULDER TO ELBOW LENGTH (vertical)	35.3	1.7	35.7	1.1	37.0	2.4	37.3	2.0
FOREARM LENGTH (horizontal)	27.3	1.1	28.2	1.4	30.0	3.5	29.2	1.1
KNEE HEIGHT	53.6	1.6	54.5	2.7	54.6	2.0	57.8	1.9



ARM LENGTHS (in cm.) (arm supported) ACROMION TO:	MEDIAN Av.	MEDIAN SD	ROTUND Av.	ROTUND SD	MUSCULAR Av.	MUSCULAR SD	THIN Av.	THIN SD
RADIALE	29.1	2.5	29.3	1.8	29.3	1.8	32.1	2.0
RADIAL STYLOID	53.5	2.6	52.4	3.3	54.4	2.5	58.3	2.2
DISTAL FLEXURE LINE OF WRIST	55.1	3.1	52.8	3.4	55.6	2.4	58.9	2.0
END OF FINGER TIP III	73.0	2.9	71.8	3.9	74.1	2.6	77.9	2.8
ULNAR STYLOID	53.9	2.8	52.9	2.3	54.6	2.3	58.3	2.5
INFERIOR POINT OF MEDIAL EPICONDYLE	30.3	2.5	30.8	1.3	31.0	1.6	31.3	1.4

Figure 74. Measurements of the Study Sample.

shown in Figures 75 and 76. For each test, records were obtained on the hand force exerted, the horizontal counterforce at the seat, and the distribution of the subject's weight at the front and rear of the seat.

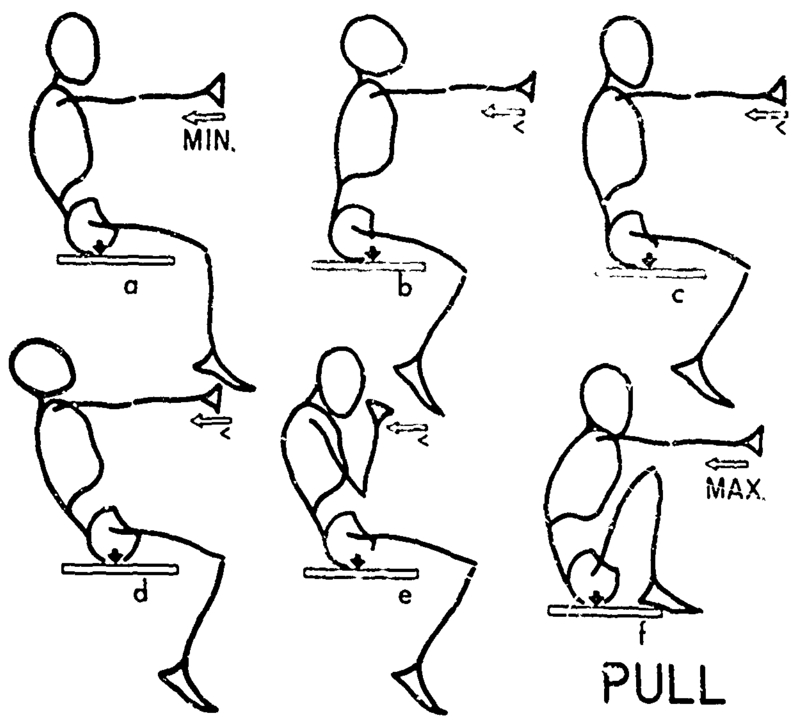


Figure 75. Six Standard Postures Used in Pull Experiments. They are arranged in order of increased magnitude of horizontal force applications.

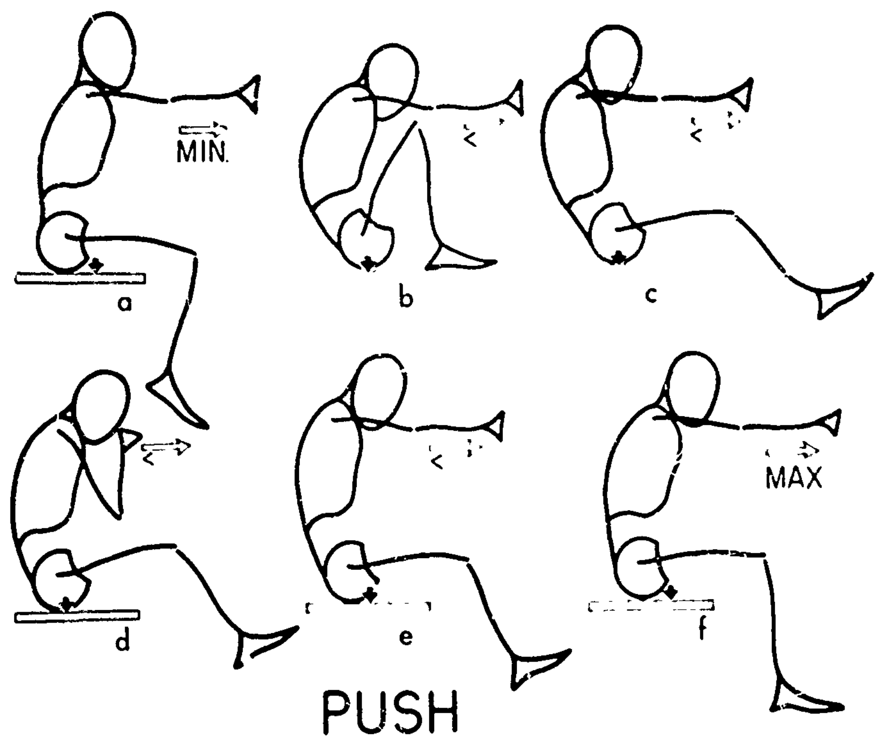


Figure 76. Six Standard Push Postures Used in Experiments. They are arranged in order of magnitude of horizontal force produced.

THE LINK SYSTEM OF THE BODY. (Dempster, pp. 68-79)

In a mechanical and kinematic treatment of the human body, one must think of masses, levers, and forces, rather than of bones and muscle. Traditional anthropometric measurements of bone and other tissue, though of scientific and practical value, are not functional; their applicability is limited to those rather standard conditions which exist when they are taken and they may not be transferred to other postures. Postural and kinematic problems may only be solved by a functional system of measurements.

Bones are not actually rigid bodies, but they may routinely be treated as such. Forces may be transmitted along a bone or from one bony member to another across a joint. Positions of body equilibrium are never devoid of force, but instead are maintained by either low forces across joints or opposing muscles under considerable tension.

Bone masses are surrounded by non-rigid tissues, which move with the bones; body segments, therefore, become composite units involving rigid and soft tissues. The boundaries between these segments are difficult to locate and may often become quite arbitrary.

Due to the spiralling curvatures of some bones (e.g., the clavicle), effective force vectors from one segment to another may fall outside the limits of the bony material. In all cases, however, these vectors follow straight lines passing through areas of joint contact and through the joint centers. The spanning distances between the joints are the functional equivalents of the engineer's <u>links</u>.

In general, links are usually considered to constitute a two-dimensional system, with the articulating members overlapping and joined by pins acting as axes of rotation. At first though, bone may be considered as a link (through its basic rigidity) but this is not strictly true. Some differences lie in the fact that body joints rarely overlap and have no pin-centered axes; in addition, movement occurs in any one of several directions rather than in a predictable plane. Nevertheless, the convenience of this approach warrants its use, with some modifications due to the nature of the biological structures with which this study deals.

The body link can be visualized as the central straight line or core-line which extends longitudinally through a body segment and terminates at both ends in axes (hinge points), about which the adjacent members rotate. From this conceptualization it becomes obvious that the bony structure itself is not the link. The flaring-away from the core line, found in bones, is due to many forces, both genetic and environmental, to which the bone is submitted during growth. They are eccentric specializations which have functions irrelevant to, but compatible with, the link system of the body. Some of these functions result in motion, such as rotations about the long axes of links, which may be important functionally, but which make up a special category, and which will not receive full treatment here. In any case, the bones provide areas of attachment for action in link movement.

The body links are connected by joints and the range of movement possible depends upon the anatomical structure of the articulation. Though all movement between adjacent links is purely rotational, translatory movement may be effected by reciprocal rotations of three or more links at two or more joints.

The link system of the engineer also differs from that of the body in that engineering links are of a <u>closed chain type</u>, while the <u>overall</u> body mechanism is basically an <u>open chain system</u>. This means that, in the body, there is a range of non-determinate movements, in contrast to a closed system. Many body positions become possible, limited mainly by three sets of factors:

1) the range of movement of the joints;

2) the character of the joint action;

3) intrinsic mechanical factors imposed by body bulkiness or muscular control.

The satisfactory application to human engineering of mechanical principles necessitates a knowledge of these factors.

For practical purposes, many minor link movements (e. g. vertebral) may be ignored. The major links are shown in Figure 75; note that segment mass is regarded as concentrated at the center of gravity. These major links are:

1) <u>trunk links</u>

   a) <u>head-end member</u>; from the atlanto-occipital joint to the head center of gravity, plus a cervical chain of seven links

   b) <u>thoracic link</u>; the thoracic column and the thorax

   c) <u>lumbar chain</u>; five minor links plus a pelvic link

2) lower limb links

   a) femoral link

   b) leg link

   c) foot link

3) upper limb links

   a) upper rib link; connects the limb to the thoracic column

   b) clavicular link

   c) scapular link

   d) humeral link

   e) forearm link

   f) hand

As mentioned above, minor links may be ignored here. However, they must be considered for specialized movements.

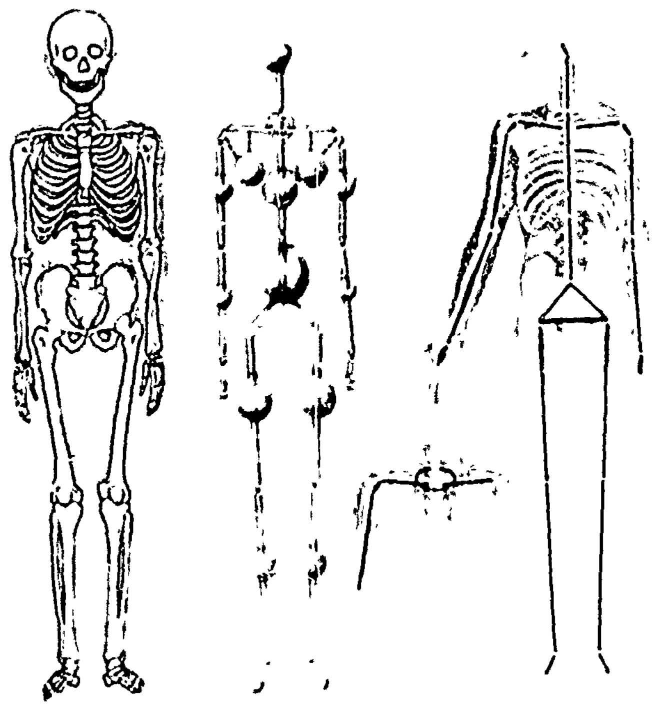


Figure 77. Plan of Body Links. The pattern of links at the right is shown by heavy black lines. The center figure shows the loading of the links by segment masses located at appropriate centers of gravity.

Joints, about which links rotate, are of several types, of which is a unique mechanical system. The extremities of the bones, which provide distinctive articular surfaces, are generally thickened to insure broad joint contacts. These articular surfaces both permit and facilitate movement.

Muscle systems do not form limits to joint range; rather they are superimposed force systems which may produce or stop movement at any point along the joint range of movement.

There are only three possible reciprocal contours which permit sliding motion while the surfaces remain in mutual contact. These are:

1) plane surfaces;

2) spherical, conical or cylindrical surfaces of circular cross section;

3) spherical helix.

If the nature of the contacting surfaces is to control and guide movement in specific ways, all other types of movement must be eliminated. In machines, this is accomplished by flanges or locking pins.

In the body, the elbow and ankle joints come the closest to the elimination of these accessory movements. The humerus and talus are the male members, the ulna and tibia the female. The closeness of fit and the collateral ligaments (hinges) virtually eliminate sidewise sliding and rotation while permitting flexion-extension along a predetermined path. This has been described as one <u>degree of freedom</u> (of rotational motion) and two of restraint at these points; i. e., free movement is permitted in only one plane.[1]

Ball-and socket joints (the hip and shoulder) permit three degrees of freedom with restraint at only the terminal ranges. Thus abduction-adduction, flexion-extension, and medial-lateral movements, as well as various combinations of the three, are possible.

---

1 The distinction between rotational (i. e. angular) and translatory motion makes possible three degrees of freedom for <u>each</u> type, or a total of six for a given joint.

The wrist and the knee have two degrees of freedom. With the wrist, flexion-extension and abduction-adduction are possible, while the knee joint permits flexion-extension and medial-lateral rotation. It should be noted that the restraints here are not absolute.

The sternoclavicular joint possesses three degree of rotational freedom; protraction-retraction, elevation-depression, and upward and downward rotation. At least one degree of translatory motion is also found.

Thus, relative to the pelvis, the foot as an end member has six degrees of freedom: three at the hip, two at the knee, and one at the true ankle, i. e. the supratalar, joint. Additional degrees of freedom, however, are permitted beyond this point in the foot via the subtalar joints. (Other members may be treated in this manner.) (Limitations are imposed upon this movement by ligaments.)

The open-chain link system of the human body makes possible certain advantages. An end member may be placed at a variety of point positions relative to the trunk. In addition, temporary closed-chain systems may be developed, as e. g., the interlocking of the fingers utilizes the limit segments plus the intervening trunk region. Non-rigid soft-tissue link systems are also involved in this example.

KINEMATIC ASPECTS OF EXTREMITY JOINTS
(Dempster, pp. 80-133)

Though the links are the basic units for consideration the type and range of the movement depends upon the joint structure. Individual variation in joint movement is more quantitative than qualitative, aside from pathological variants.

The method of locating instantaneous centers has been outlined in Chapter II. However, it may be added that movement is circular when successive centers fall on a point. On the other hand, when movement is not circular, the instantaneous centers lie on a path which correlates with the angular pattern of movements.

In this section, the ankle, knee (flexion-extension), hip (abduction-adduction and flexion-extension), elbow, and gleno-humeral (abduction-adduction parallel to scapular blade and flexion-extension at right angles to the scapular blade) joints were studied. Data from other authors are presented for additional joints. In one cadaver, sternoclavicular and claviscapular joints were analyzed for movements in each of three planes. Since findings at each joint present certain common characteristics, joint-by-joint analyses are unnecessary and the ankle joint is presented in detail.

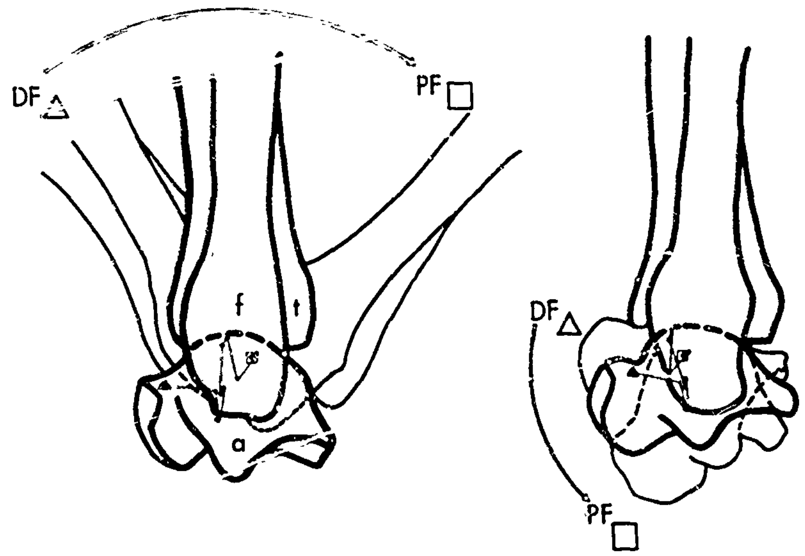


Figure 78. Instantaneous Joint Centers of the Ankle Joint. At the left, the talus bone (a) is stationary, with the tibia (t) and fibula (f) in midrange (standing) position. At the right, the leg bones are fixed, with the talus rotating.

Figure 78 presents the analysis of the instantaneous joint centers of the ankle joint; the centers were located at successive 10^{o} rotations. It can be seen that, though the centers move erratically, the relative movement of the members proceeds smoothly. The paths of instantaneous centers were not identical in different ankle joints, but they were still generally similar.

Contingent movement during dorsi and plantar-flexion were also recorded; the amount is graphed in Figure 79, as measured by a protractor, for a typical ankle joint. These movements cannot be separated and one component is invariably correlated with the other components.

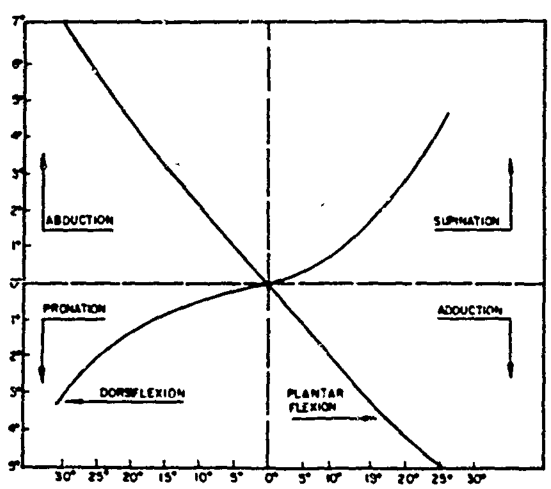


Figure 79. Contingent Movement at the Ankle.

The implication from joint-center data is that the link dimensions change in a reciprocal fashion as the parts rotate, amounting to a half-inch or more. For practical purposes, mean joint centers and link dimensions appear as the only way that kinematic analyses can be carried out practically.

The shifting of these axes is related to changes in the actual contact area: articular congruence is not permitted along a section for any joint position. For example, evolutes of the tibia and talus (the ankle) are at differing distances from the articular surfaces; in addition the curvatures and slopes of these evolutes differ. This change in contacting area was also demonstrated by paint impressions transferred from one area to another.

The characteristic of shifting was also found for other joints. The ankle, hip, and shoulder joints showed data generally similar to those for the ankle. The knee joint however, was analyzed differently from the others. Elastic bands were used with the cadavers to attempt to simulate the cruciate ligaments.

There is some doubt concerning the applicability of these data to real life. However, the results were not qualitatively different from other joint data: erratic shifts in axial position, and a generally similar size for the cluster of centers.

The structural distinctness of the shoulder joints (sterno-clavicular and claviscapular) necessitate a separate mention. They are gliding joints with also a ligamentous binding-together of the bones. The erratic shifting for humeral abduction is seen in Figure 80, as taken from a cinefluorographic film. This pattern is to be expected because of the composite nature of the joint (three bones, each with three degrees of freedom) and the effect of distant ligaments. Though the instantaneous centers for the claviscapular joint showed patterns to lie in the span between the syndesmosis and the arthrosis, the joint itself ranged over a distance equal to or larger than the scapular link itself.

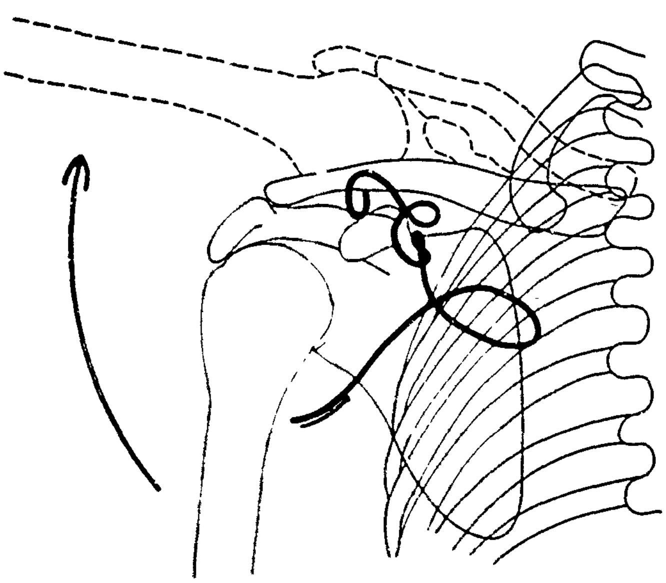


Figure 80. Path of instantaneous Center of Rotation During Shoulder Abduction. (Reuleaux analysis of cinefluorographic film.)

For the following descriptions of the range of joint movement, the frame of reference will be a sphere. For example if one member is fixed firmly and the other is allowed to rotate (or is rotated) to all its extremes, the distal end of the member will describe the base of a cone. Concomitant movements may be studied by referring to a globe; this has the additional advantage of freeing the observations from the artificiality of the standard anatomical planes. The locus of these extremes, first called extension cone, is referred to, for globographic representation, as a joint sinus.

Gleno-humeral joint data are given in Figure 81. For this representation, the findings of Shino (1913) and Pfuhl (1934), grouped for the left side, are presented globographically. The scapular blade is fixed at 0° meridian. This makes the diagram somewhat schematic as, in life, the glenoid fossa lies 30-45° forward of the coronal plane, while the entire scapula tilts forward about 10° to fit thoracic curvature.

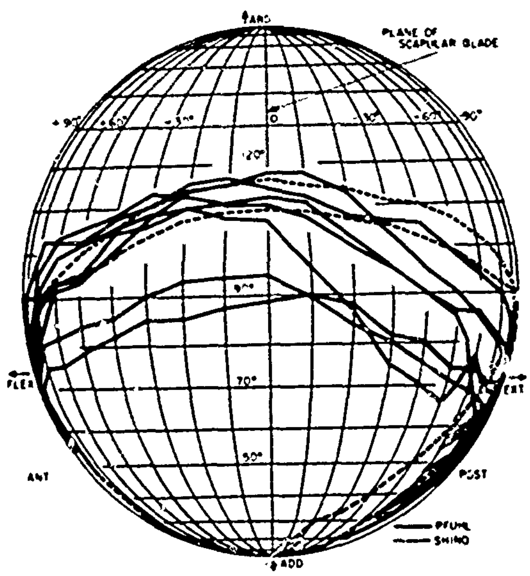


Figure 81. Globographic Presentation of the Range of Movement of the Humerus at the Gleno-humeral Joint (left side, cadavers). From Shino (1913) and Pfuhl (1934).

The data representing nine specimens show individual variation within a similar pattern. For example, maximal abduction in the plane of the scapula varies from 90-120°. Cadaver data in the present study clearly indicate that, during the full 360° movement (with clavicle and scapula unrestricted) the "front face" (anterior distal portion) of the humerus turns forward in the vertical downward position, medial in the horizontal, forward, up and inward in the anterior superior quadrant, up, forward, and inward in the abducted position, and outward and upward in the horizontal posterior position. In addition, the humeral head has a marked amplitude of movement.

Figure 82 A and B gives the joint sinus for the sterno-clavicular joint, a composite based upon four living subjects and a ligament-skeleton preparation. The excursion cone for the link axis of the clavicle (distal end) is as follows:

from protraction to retraction	35°
from elevation to depression	44°

The rest position of the distal end of the clavicle, relative to the proximal, in seated individuals, averages 8° upward from full depression, and 15° behind full protrusion.

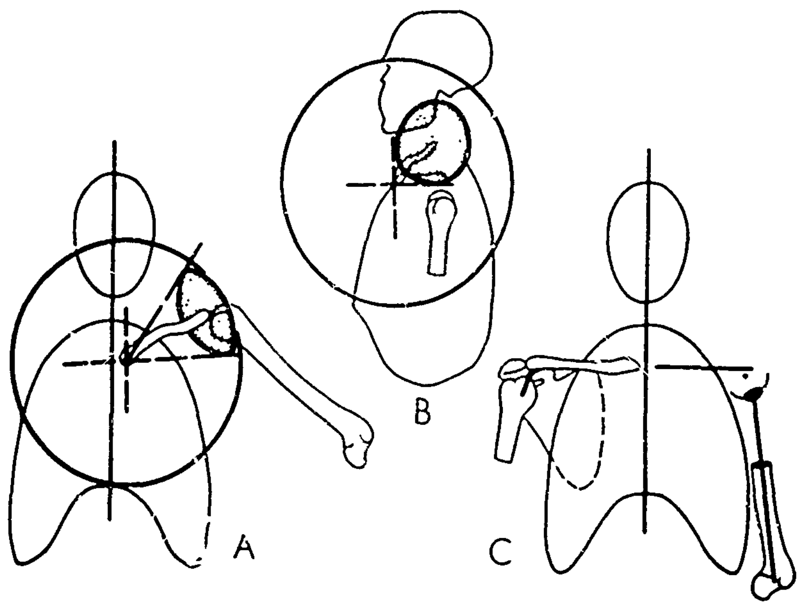


Figure 82. Range of Sternoclavicular and Claviscapular Joint Movement. A and B, front and side views of sternoclavicular joint sinus. The radius of the circular outline is the clavicular link. At C, scapular link length is shown on the left; on the right, the stippled area shows the range of movement of its distal end.

Figure 82 C shows the small concavo-convex area swept by the claviscapular link; this link may be defined as the mean claviscapular to mean glenohumeral joint center distance.

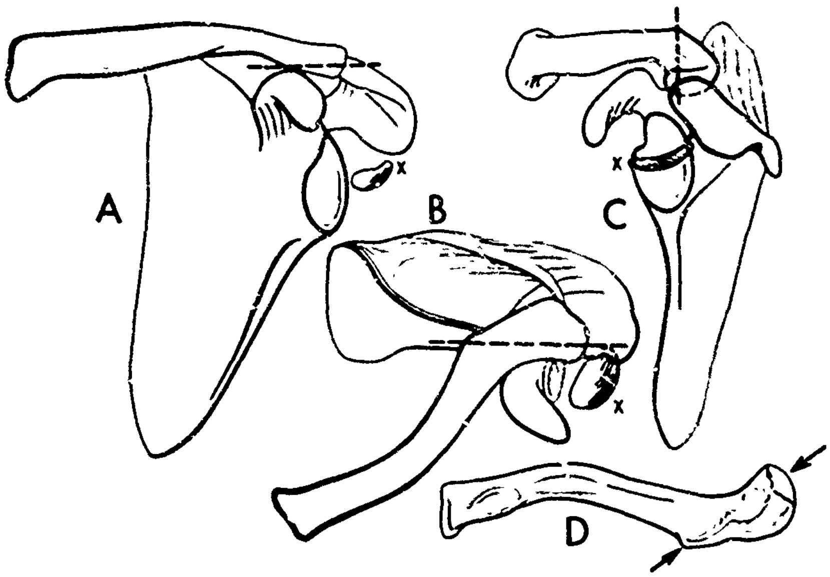


Figure 83. Structural Relations at the Claviscapular Joint. The heavy dashed line in A, B, and C indicate the line between the arrows in D: the attachment inferiorly on the clavicle of the coracoclavicular ligaments.

Figure 83 shows the complex relations of the claviscapular joint. The heavy dashed line corresponds generally to the conoid and trapezoid (coracoclavicular) ligaments, and provides an axis for the inferior surface of the scapula about which movements can be analyzed. Major claviscapular movement occurs as the scapula swings fore and aft on the ligaments; it may tilt endwise along the axis, or it may twist on them. Three degrees of freedom are permitted. (The plane is somewhat obliquely oriented relative to a frontal plane.)

Scapular rotation is a special situation that cannot be analyzed via simple scapular link rotation. For this study, a stylus was driven into the glenoid fossa with a projection equal to average radius of curvature of the humeral head. With the clavicle fixed this stylus gouged out a surface in a block of clay shown in Figure 83 as x.

These problems may be met in manikin construction in three ways:

1) by using a short pin-centered link with a suitable template to limit the range of movement;

2) by ignoring the link, per se and providing a suitable surface, properly oriented relative to the clavicular link, on which the upper humeral link may range;

3) by ignoring both the clavicle and scapula links, per se, and providing a suitable template relative to the trunk for humeral movement.

Simple globographic presentations, used by earlier authors, are inadequate for this joint, due to a lack of constancy in the locus of the center of rotation. However, they do represent gross values and have some value. Figure 84 A and B (von Lanz and Wachsmuth, 1938) show the overall range of the composite girdle-joint systems in comparison to the simpler movement of the humerus on the scapula.

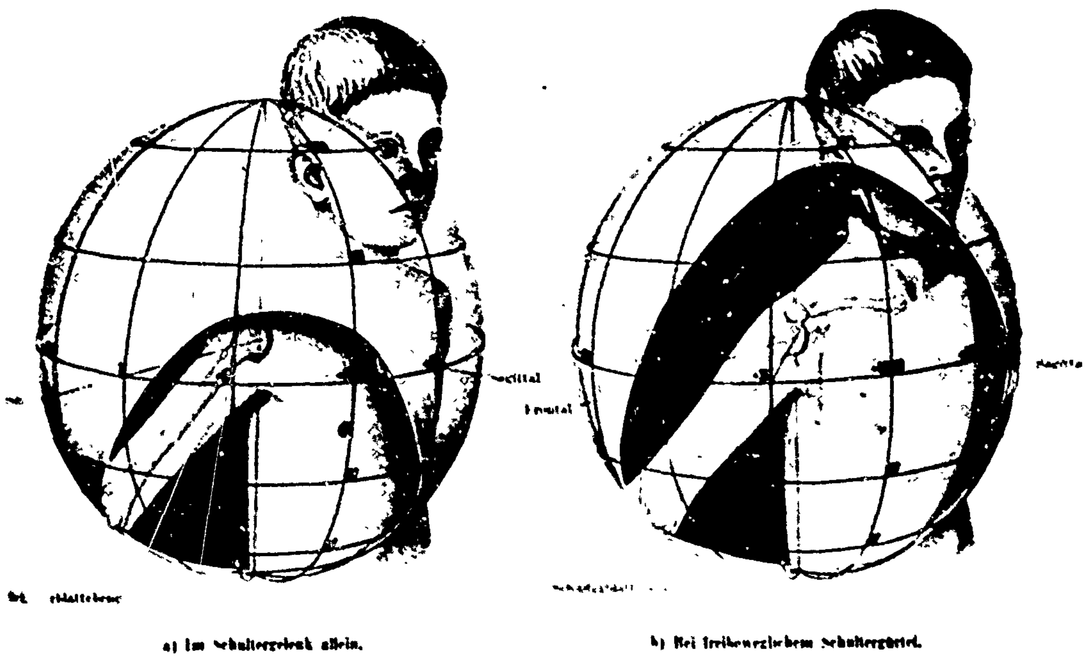


Figure 84. Globographic Illustrations of Shoulder Movement (after von Lanz and Wachsmuth, 1938). A, scapular position fixed; B, movement permitted by glenohumeral, claviscapular, and sternoclavicular joints.

The elbow is a much simpler mechanism than the shoulder. Figure 85 is a globograph of the permissible range of movement; the humerus is fixed. The heavy solid line shows movement of the styloid process of the ulna as some 140° of flexion is accomplished. Contingent lateral-medial movement may be summarized as follows:

Degree of Flexion	Lateral-Medial Movement
0° - 45°	medial and back to zero
45° - 90°	lateral and back to zero
90° - 140°	medial

However, since all deviations were found to be less than 3°, planar movement becomes a reasonable approximation.

From Figure 85, it can also be noted that lines P and S, representing the margins of the distal ends, closely parallel the styloid curve; hence, the radio-ulnar joint range of pronation-supination is fairly constant for the entire flexion - extension range.

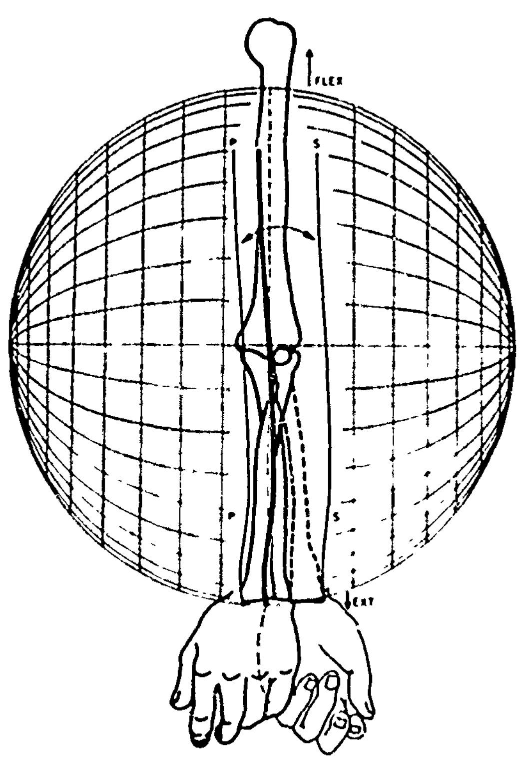


Figure 85. Globographic Plot of Elbow Flexion. P and S show movement of radial styloid process in extremes of pronation and supination.

The movement range of the wrist, as modified from Braune and Fischer (1887), is given in Figure 86. The outer dashed line represents a point on the 3rd metacarpo-phalangeal knuckle, showing a flexion range of nearly 180° with abduction and adduction each some 30°. The inner curves represent radiocarpal and intercarpal joints.

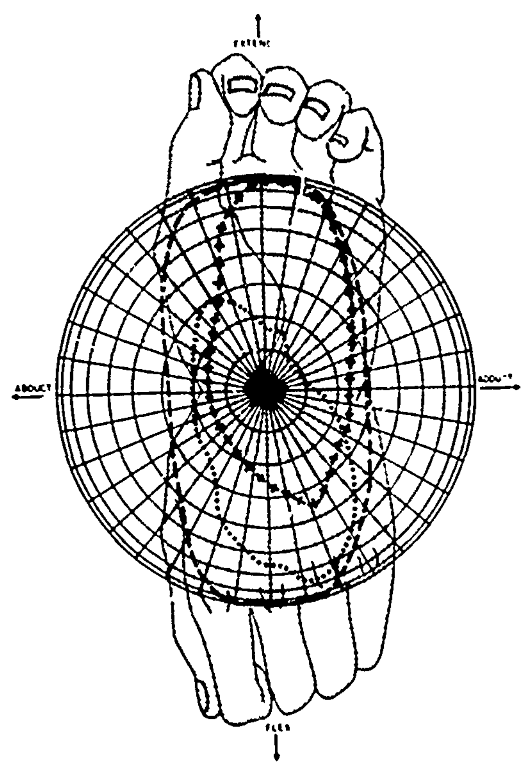


Figure 86. Ranges of Wrist Movement. Outer dashed line, maximum range; dotted line, range of proximal wrist joints; crosses, range of distal joints (modified from Braune and Fischer, 1887).

Hip joint data are taken from Strasser and Gassman (1893) and Strasser (1917). The joint sinus is given in Figure 87. The pelvis is situated as in the standing position with the anterior superior spines and the pubic symphysis in a vertical plane. Movement may be summarized as follows:

flexion	115°
abduction	$50 - 55^{\circ}$
adduction	$30 - 35^{\circ}$

The highest point is above and lateral to the hip joint, the greatest sidewise range is some 30° below hip level. It has also been found that movement, as shown, is more restrained in the upper medial quadrant than in the upper lateral.

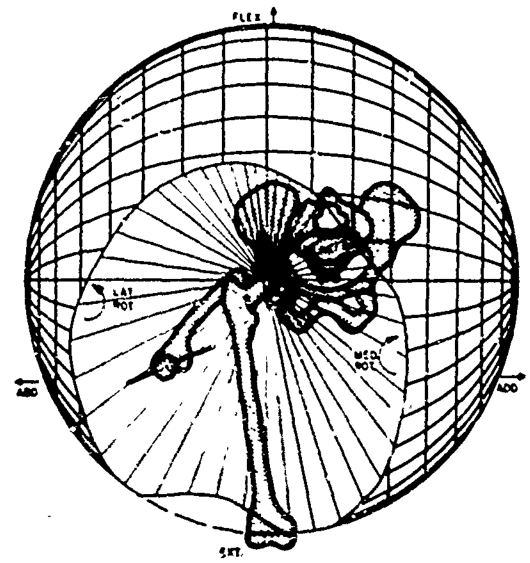


Figure 87. Strasser's (1917) Globographic Presentation of Hip Joint Movements

The knee globograph given in Figure 88; the femur was moved on fixed and vertical leg bones. Flexion was found to be about 160° in the 0° meridian. However, there is also lateral and medial rotation as follows:

Flexion Range	Lateral Rotation	Medial Rotation
mid - 90°	20-25°	<20°*
first and last 30°	markedly reduced**	markedly reduced**
0°	none	none

*- except for 115-145° span

**- especially last 10° nearest full extension, due to the locking mechanism of the knee joint.

Ankle and foot ranges are given in Figure 89, with the foot on a horizontal plane and the mean ankle center at the sphere's center. The indicated movement is both sub-and supra-talar. The greatest range from the pole position is 60-70° backward and 65° medially (plantar flexion-supination).

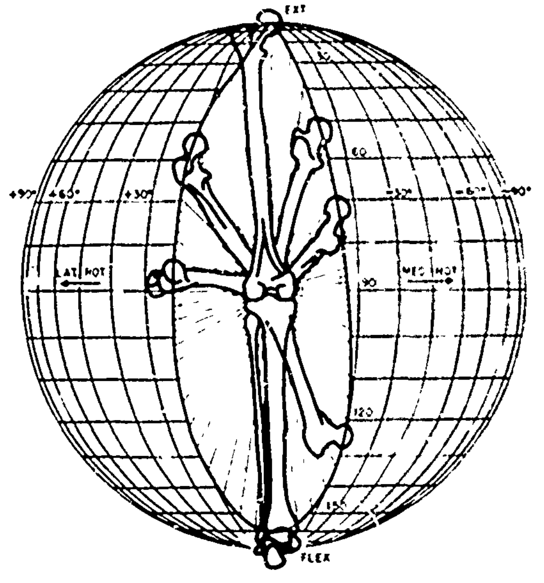


Figure 88. Globographic Plot of Knee Joint Range. Leg bones are fixed, the foot axis is perpendicular to the page, and the femur is flexed and rotated to its limits.

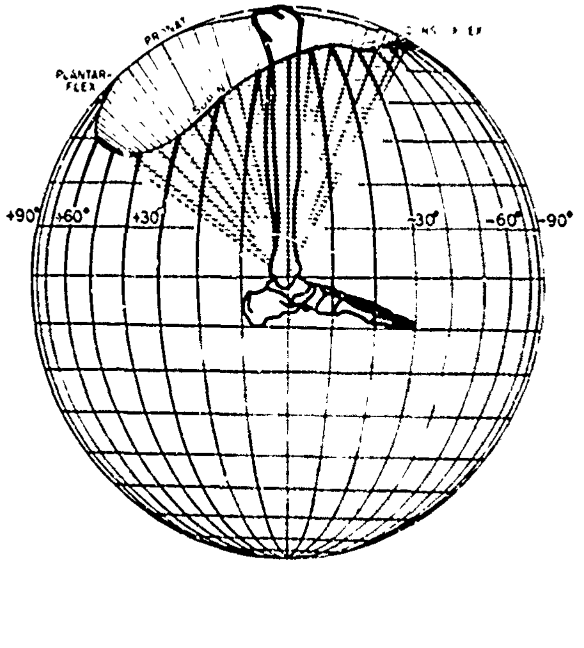


Figure 89. Globographic Data on Ankle and Foot Joints of the Left Foot.

A study of foot positions at varying radii from the knee, as viewed from directly above, showed an amount of associated abduction or adduction, as well as dorsi- or plantar-flexion, when the feet are a given distance apart, i. e., medial or lateral to the knee centers.

The above data, on cadaver material, clearly shows the advantage in fixing one joint member in a vise so that all potential positions of the other may be determined. Though it should be possible to obtain comparable data on living subjects, this has not been done due mainly to the oversimplification of theory (e. g., the consideration of movements as purely planar). An attempt has been made to obtain data from the 39 subjects in this study; means and standard deviations are given in Table 49, broken down by physique type. It can be seen that differences in joint range exist which correlate with body type. Thin men have the greatest range, with the order being thin, median, muscular, rotund.

Pelvic and thoracic tilt angles were measured to aid in accurately locating the hip and shoulder joints for the seated individual. Pelvic and manubrial guides were fixed to anatomical landmarks on 41 men, and lateral photographs made of them as they stood, lay supine, and sat.

Though the same order of angular values was found to exist with body type as previously (thin > median and muscular > rotund) the differences were not great and results may be pooled. Manubrial tilt was greatest (30^{o} relative to the vertical) when standing and supine (the 90^{o} postural change in the latter was allowed for). Intermediate positions were somewhat less. Pelvic tilt was the greatest when the subject sat on the floor. It was found to decrease, at varying rates, as the seat height increased.

The importance of this tilt lies in the orientation of the reference globes for joint sinuses of shoulder and hip areas. Sinuses derived for the standing subject may be adapted to a variety of postures.

It is important to note that a study of asymmetrical leg movements while sitting (i. e., pelvic tilt during unilateral leg movement) suggests that joint range data based upon a standing posture should be adjusted 5^{o} or so beyond the 40^{o} found for pelvic tilt when planning seated manikins.

Table 49

Joint Range of Study Subjects

Joint and Type of Movement	Median (n=11)		Muscular (n=11)		Thin (n=10)		Rotund (n=7)	
WRIST:								
Flexion	94.6°	± 5.2	91.9°	± 9.4	95.6°	± 9.0	79.7°	±14.8
Extension	102.0°	± 8.9	97.0°	± 9.0	100.0°	± 8.3	86.8°	±17.8
	196.6°	±12.2	188.9°	±14.5	195.6°	±14.1	166.5°	±26.7
Abduction	25.1°	±10.2	27.1°	± 7.6	28.8°	± 7.9	27.5°	± 8.0
Adduction	46.3°	± 5.1	47.4°	± 6.4	47.1°	± 6.7	46.1°	± 6.4
	71.4°	±13.6	74.5°	±12.4	75.9°	± 9.8	73.6°	±12.9
FOREARM:								
Supination	100.6°	±22.1	113.3°	±15.8	123.8°	±20.0	114.8°	±19.2
Pronation	74.0°	±17.0	69.1°	±14.8	75.0°	±18.0	89.0°	±37.9
	174.6°	±26.0	182.4°	±18.6	198.8°	±22.9	203.8°	±42.3
ELBOW:								
Flexion	141.0°	± 9.8	140.3°	± 7.6	144.6°	±10.2	143.2°	± 8.6
SHOULDER:								
Flexion	193.2°	± 9.6	190.2°	± 9.9	186.0°	±11.7	184.0°	±12.2
Extension	63.0°	±14.3	58.1°	± 9.1	67.0°	±11.1	54.5°	±14.8
	256.2°	±17.7	248.3°	±10.3	253.0°	±17.3	238.5°	±23.9
Abduction	132.1°	±16.5	135.0°	±10.4	142.5°	±19.0	127.5°	±12.3
Adduction	50.8°	± 6.4	44.3°	± 6.7	53.5°	± 8.9	43.6°	± 5.2
	182.9°	±17.1	179.3°	±12.8	196.0°	±23.3	171.1°	±16.2
Medial Rotation	95.7°	±25.4	95.2°	±20.9	97.0°	±13.8	100.1°	±24.6
Laterial Rotation	30.7°	±13.1	33.0°	±13.7	39.5°	±10.1	32.5°	± 8.5
	126.4°	±24.4	128.2°	±28.5	136.5°	±16.7	132.6°	±15.6

Table 49 (continued)

Joint and Type of Movement	Median (n=11)		Muscular (n=11)		Thin (n=10)		Rotund (n=7)	
HIP:								
Flexion	117.1°	± 12.7	118.2°	± 12.2	116.5°	± 8.3	99.9°	± 7.7
Abduction	58.0°	± 10.1	54.4°	± 10.3	53.0°	± 10.2	47.5°	± 11.9
Adduction	27.7°	± 8.4	30.2°	± 11.6	32.0°	± 9.4	34.2°	± 15.7
	85.7		84.6		85.0		81.7	
PRONE								
Medial Rotation	38.7°	± 5.6	39.0°	± 11.0	41.0°	± 10.0	39.0°	± 11.6
Lateral Rotation	34.7°	± 8.6	33.1°	± 7.3	40.5°	± 9.3	28.2°	± 9.1
	73.4°	± 8.5	72.1°	± 16.0	81.5°	± 14.6	67.2°	± 19.7
SITTING								
Medial Rotation	31.4°	± 7.6	29.3°	± 5.8	34.5°	± 8.4	28.2°	± 11.2
Lateral Rotation	31.9°	± 6.2	31.0°	± 7.9	33.0°	± 6.5	22.5°	± 8.5
	63.3°	± 5.9	60.3°	± 13.0	67.5°	± 11.2	50.7°	± 17.4
KNEE:								
PRONE								
Flexion (voluntary)	123.8°	± 7.8	127.0°	± 6.7	135.5°	± 6.4	114.0°	± 6.4
Flexion (forced)	142.1°	± 5.8	147.8°	± 4.9	152.0°	± 7.3	136.1°	± 8.8
STANDING								
Flexion (voluntary)	112.2°	± 13.3	119.6°	± 10.8	117.5°	± 10.7	102.5°	± 9.3
Flexion (forced)	162.4°	± 5.0	160.3°	± 5.6	165.0°	± 6.0	146.8°	± 6.1
KNEELING								
Medial Rotation	40.9°	± 8.7	38.9°	± 12.4	32.0°	± 10.1	29.0°	± 6.7
Laterial Rotation	43.3°	± 8.5	39.3°	± 9.1	46.4°	± 10.5	41.7°	± 15.1
	84.2°	± 11.5	78.2°	± 19.8	78.4°	± 13.6	70.7°	± 11.8

Table 49 (continued)

	Median (n=11)	Muscular (n=11)	Thin (n=10)	Rotund (n=7)
ANKLE:				
Flexion	39.0° ± 7.8	34.2° ± 4.7	39.3° ± 4.7	29.0° ± 4.0
Extension	41.6° ± 10.6	44.9° ± 9.5	35.5° ± 8.4	28.9° ± 10.0
	80.6° ± 9.4	79.1° ± 10.6	74.8° ± 11.1	57.9° ± 11.0
FOOT:				
Inversion	23.4° ± 6.7	20.9° ± 7.5	29.5° ± 10.8	22.1° ± 6.2
Eversion	23.2° ± 5.2	23.5° ± 4.9	26.5° ± 8.3	20.7° ± 4.8
	46.6° ± 10.8	44.4° ± 10.8	56.0° ± 16.0	42.8° ± 4.3
GRIP ANGLE:	100.7° ± 5.4	99.7° ± 4.9	102.6° ± 7.0	104.1° ± 6.9

(Dempster, Table 5)

The living subjects were studied to determine the degree of accuracy at which shoulder and hip joint centers could be marked. The lateral projection of the hip joint center was found to lie within a 1.2 x 1.5 mm ellipse around the femoral trochanter, for a given degree of accuracy in palpation. An easier method was determined, whereby a line dropped vertically from the anterior superior spine of the ilium (assuming a standard seated 40° pelvic tilt) fell within 3° of the mean joint center. Half of the distance from the spine to the seat formed a reasonable estimate of the mid-acetabulum. In either case, accuracy is possible only to within a 1.2 x 1.5 mm ellipse.

Reasonable approximations were possible for the shoulder without the detailed analysis necessary for the hip.

Before the link dimensions themselves could be determined, it was necessary to gather information relative to bone length and stature. Consequently, from the data of Trotter and Gleser (1952), stature was plotted against long bone length for 710 U.S. Army personnel. From the osteological collection, bone lengths were obtained which correspon ded to the 5th, 50th, and 95th percentiles for stature of U.S. Air Force flying personnel (Hertzberg, Daniels, and Churchill, 1954). It is felt that the two military populations, selected by similar standards, correspond closely in bone length.

The bone lengths were next plotted against each other. Although correlations were positive, the variability was considerable.

Prior to measuring the link dimensions, the individual bones were corrected to the actual link size. This involved the determination of mean joint centers, as determined previously. Even though the radii of curvature of larger specimens of a given bone were smaller, and vice versa, the errors introduced were negligible, and these radii were determined by a Lensometer. Final corrections were worked out, and the mean percentage values are given in Table 50. It must be realized that these are only averages, about which individuals may vary; in addition, there will be instantaneous variation within the bones. However, they do have a utility in manikin construction.

TABLE 50

Estimation of Link Dimensions of Air Force Flying Personnel Based on Ratios from Cadaver Measurements

	95th Percentile cm	50th Percentile cm	5th Percentile cm
Clavicle length	17.6	16.3	15.1
(40.7% of biacromial width)			
Biacromial width	43.1	40.1	37.0
Clavicle Link	15.2	14.1	13.1
(86.4% of clavicle length)		(sternal end 26 mm from mid-line)	
Scapula Link		±3.5	
Humerus length	35.9	33.9	32.1
Humerus Link	32.0	30.2	28.6
(89.0% of humerus length)			
Radius length	26.6	25.4	24.0
Radius Link	28.5	27.2	25.7
(107.0% of radius length)			
Hand length	20.4	19.0	17.6
Hand Link (wrist center to center of gravity)	7.4	7.0	6.7
(20.6% of humerus length)			
Transpelvic Link		17.1	
(37.2% of femur length)			
Femur length	50.3	47.5	44.3
Femur Link	46.0	43.4	40.5
(91.4% of femur length)			
Tibial length	39.9	37.2	34.5
Tibial Link	43.9	40.9	38.0
(110.0% of tibial length)			
Foot length (heel to toe I)	28.6	26.7	24.8
Foot Link (talus center point to center of gravity)	8.8	8.2	7.6
(30.6% of foot length)			
Vertical distance from midtalus to floor level		8.2	

(Dempster, Table 7)

On the living, a suggested pattern for locating joint centers is as follows:

Sternoclavicular joint center. --Midpoint position of the palpable junction between the proximal end of the clavicle and the sternum at the upper border (jugular notch) of the sternum.

Claviscapular joint center. --Mid-point of a line between the coracoid tuberosity of the clavicle (at the posterior border of the bone) and the acromioclavicular articulation (or the tubercle) at the lateral end of the clavicle; the point, however, should be visualized as on the underside of the clavicle.

Clavicular link. --The direct distance between the two joint centers listed above.

Glenohumeral joint center.--Mid-region of the palpable bony mass of the head and tuberosities of the humerus; with the arm abducted about 45^{o}, relative to the vertebral margin of the scapula; a line dropped perpendicular to the long axis of the arm from the outermost margin of the acromion will approximately bisect the joint.

Scapular link. --The distance between the centers of the foregoing mean claviscapular and glenohumeral joints - an unsatisfactory measurement - approximately 3. 5 cm.

Elbow joint center. --Mid-point of a line between (1) the lowest palpable point of the medial epicondyle of the humerus, and (2) a point 8 mm above the radiale (radio-humeral junction).

Humeral link. --The distance between the glenohumeral and elbow joint centers.

Wrist joint center. --On the palmar side of the hand, the distal wrist crease at the palmaris longus tendon, or the mid-point of a line between the radial styloid and the center of the pisiform bone; on the dorsal side of the hand, the palpable grove between the lunate and capitate bones, on a line with metacarpal bone III.

Radial link. --The spanning distance between the wrist and elbow joint centers.

Center of gravity of the hand (position of rest). -- A point of the skin surface midway between the proximal transverse palmar crease and the radial longitudinal crease in line with the third digit; flattening or cupping the hand changes the relative location of this point very little, except to change the position normal to the skin surface.

Hand link. --The slightly oblique line from the wrist center to center of gravity of the hand.

Hip joint center. --(Lateral aspect of the hip). A point at the tip of the femoral trochanter 0. 4 inch anterior to the most laterally projecting part of the femoral trochanter.

Knee joint center. --Mid-point of a line between the centers of the posterior convexities of the femoral condyles.

Femoral link. --Distance between the foregoing centers.

Ankle joint center. --Level of a line between the tip of the lateral malleolus of the fibula and a point 5 mm distal to the tibial malleolus.

Leg link. --The distance between knee and ankle centers.

Center of gravity of the foot. --Halfway along an oblique line between the ankle joint center and the ball of the foot, at the head of metatarsal II.

Foot link. --The distance between the ankle joint center and the center of gravity of the foot.

Table 51 gives the ratio of one body link to another. Note that the hand and foot links terminate at the respective segmental centers of gravity.

The spatial orientation of one link relative to another is also necessary information and was studied here. For the lower limb the femur was selected as the basis of reference, a joint preparation being placed on a table with the longitudinal axis normal to the table edge and the leg overhanging. It was noted that the leg averaged 5° medial to the vertical, while the foot toed inward some 8°.

The upper limb was treated as two segments, proximal and distal to the elbow. Certain relationships were also determined which were keyed into the globographic data presented earlier. In all instances, ranges of movement were related to these interrelationships.

TABLE 51

Relative Dimensions of Extremity Links
Expressed as Ratios of One Dimension to Another*

	Thigh	Leg	Foot	Arm	Fore-arm	Hand	Clavic-ular	Trans-pelvic
Thigh		(16:15) 107%	(16:3) 533%	(10:7) 144%	(8:5) 161%	(6:1) 625%	(3:1) 310%	(5:2) 255%
Leg	(15:16) 94%		(5:1) 500%	(4:3) 135%	(3:2) 150%	(6:1) 585%	(3:1) 290%	(7:3) 239%
Foot	(3:16) 19%	(1:5) 20%		(3:8) 27%	(3:10) 30%	(7:6) 117%	(4:7) 58%	(1:2) 48%
Arm	(7:10) 69%	(3:4) 74%	(8:3) 369%		(10:9) 111%	(13:3) 432%	(15:7) 214%	(7:4) 176%
Forearm	(5:8) 62%	(2:3) 66%	(10:3) 332%	(9:10) 90%		(4:1) 388%	(2:1) 193%	(8:5) 159%
Hand	(1:6) 16%	(1:6) 17%	(6:7) 85%	(3:13) 23%	(1:4) 26%		(1:2) 50%	(2:5) 41%
Clavicular	(1:3) 32%	(1:3) 34%	(7:4) 172%	(7:15) 47%	(1:2) 52%	(2:1) 201%		(5:6) 82%
Transpelvic	(2:5) 39%	(3:7) 42%	(2:1) 209%	(4:7) 57%	(5:8) 63%	(5:2) 244%	(6:5) 121%	

*Figures in parentheses represent simple proportions, which figure to the approximate percentage values immediately below. Links listed in the column to the left are numerators; those of the transverse row are denominators. Hand-and-foot links terminate in centers of gravity of the segments.

(Dempster, Table 8)

APPLICATION TO MANIKIN DESIGN (Dempster, pp. 134-158)

The purpose of this chapter is to synthesize the preceding data into constructions to duplicate the dimensions and movements of the average individual, as nearly as possible. In this respect, limb mechanisms are emphasized more than trunk. In addition, the emphasis will be upon the seated position, and will not be strictly applicable to standing, prone, or kneeling postures.

For this reason, a manikin constructed from these data will not be universal, though the data should be capable of transfer to such a mechanism. What will be possible will be the construction of a three-dimensional manikin that is kinematically correct, as far as the limbs are concerned.

McFarland, et al. (1953) has shown that "average" concepts are of little value when dealing with individuals and it becomes necessary to consider control placement relative to smaller, as well as larger, individuals. A given percentile range can effectively be used, depending upon the designer's aims. Individuals outside this range, forming only a small part of the population, can be assigned alternate duties, where body size is of no importance.

Two types of manikins will be presented. First, a three dimensional model, emphasizing the limbs, with accurate link dimensions, as well as all movements found at major points. Plans for such a model are presented, which have been tested for functional accuracy. It must be remembered that the purpose is to present a type of standard construction which will closely approximate body movements, uniquely controlled in life by bones, joints, and ligaments.

Second, a two dimensional manikin is presented which can be placed on a drawing board, and which will permit adjustments of segments into a number of different postures. The models are very general in their scope; however, the component parts are suitable for modification by the engineer for his specific purpose.

A. The Three-Dimensional Model

Limb links were constructed of square section aluminum rod with machined interconnecting joints. These joints were centered mechanisms (hinges, rod and sleeve joints, and universals) with a range of movement limited by end stops or metal guides or templates to specific angular ranges. The general plan of assembly is given in Appendix F. Though the 50th percentile values were used, larger or smaller dimensions could have been substituted.

Movement ranges were geared to the earlier globographic data, modified by quantitative findings of Table 49, with the final orientation of the links relative to each other.

Appendix F presents detailed plans for the construction of the model, left side only. Note that the joints are designed to give more than the range of movement with a guide or template as a limit, defining a joint sinus, and permitting a free movement within the limits. Additional notes on construction follow:

(1) Sternoclavicular joint: A universal joint was set transversely with its inner or sternal end normal to the body mid-plane; its support represented the upper portion of the manubrium of the sternum and the joint center was one inch from the side of the mid-plane. Between the center of the universal and the sheet metal guide a rod and sleeve joint permitted additional rotation about the axis of the clavicular link. Suitable stops corresponding with the range of upward and downward rotation of the clavicle were built in.

(2) Claviscapular joint, scapular link, and glenohumeral joint: These three components were planned as an integral unit. The claviscapular joint permits three degrees of freedom of movement of the scapula with respect to clavicle; a universal joint duplicated these movements in two axes normal to the scapular link. The additional rotation about the longitudinal axis of the link was built into a telescoping rod and sleeve joint. The distal end of the telescoping link moved over a flat surface placed obliquely in the joint unit and its range of motion was controlled by a template. The socket in which the humerus sits is a part of the scapula. If one considers the movement of the geometrical center of this socket with respect to the clavicle the point actually moves on a surface which has a very slight curvature (Figure 83). Since the error introduced is small, the slight curvature was replaced with a plane and the center of the gleno-humeral (humerus-scapula) joint traveled over this. The construction of a telescoping scapular link was a necessary compensation because of this plane. Since, as intimated, the link also rotated about its axis, this feature was also included.

At the end of the scapular link was the center of the gleno-humeral joint which represented the movement of the humerus with respect to its socket on the scapula. This movement was provided for by a universal joint and its angular limits were assured by an attached sheet metal guide with a central cut-out. In this instance, the universal joint was a gimbal to permit an adequate range of motion. Beyond the gimbal was a rod and sleeve joint to permit an axial rotation of the humeral link. Each joint (claviscapular and gleno-humeral) thus had three degrees of freedom.

(3) Elbow joint: This joint was a hinge with terminal stops to limit motion at each end of the range of movement.

(4) Forearm joints: Beyond the elbow joint proper, a rod and sleeve joint with suitable stops provided for pronation-supination movement of the forearm link and regions beyond.

(5) Wrist joint and hand assembly: This joint was a universal joint; it was provided with a sheet metal guide which was bent into a half cylinder; on its longitudinal axis was the center of the universal joint. In this instance the guide was a part of a larger sheet metal piece which was shaped and bent as indicated in Appendix F to give a general form that suggested hand, thumb, and finger shape. Into this hand shape was built a projecting knobbed rod that represented the grip angle relative to the axis of wrist motion; the knob end corresponded with a rod gripped in the hand and projecting from the thumb side of the hand.

(6) Hip joint: The center of the hip joint was mounted on a link rod, representing half of the transpelvic link, which projected 86 mm. as a normal to the mid-sagittal plane; its surfaces represented fore-and-aft and upper-and-lower directions relative to the standing position of a man. The joint itself consisted of a universal joint with a rod and sleeve joint just beyond; the universal joint was specially constructed to permit an adequate range of movement and the sheet metal guide which limited the range of movement was bent to an angle to provide sufficient clearance for the femoral link in its movement.

(7) Knee joint: The knee joint in the constructed model permitted axial rotation of the shank link as well as flexion-extension movements; its movements corresponded with the globographic data (Figure 88) as corrected for data on living subjects. The range of shank rotation was actually increased to correspond with knee rotation plus foot abduction or adduction. Accordingly, values of Table 49 and other data on knee rotation were utilized. In the detail drawings the joint is shown to be a combined hinge and axial joint; it was provided with a guide and template system to limit motion to a normal range. The joint was locked in the extended position with no rotation possible except straight flexion; with further flexion, however, increasing amounts of axial shank rotation were permitted.

(8) Ankle joint and foot assembly: The shape of the foot was represented simply as a flat plate of sheet metal, suggesting the foot sole from heel to toe, and an oblique piece from the ball of the foot upward toward the ankle. (The foot form might have been more elaborately worked out like the hand section.) From the foot sole piece a vertical pillar, at a suitable distance between the heel and toe, extended upward to the ankle joint level. This joint was represented by a universal joint and the movement of the lower section of it was guided by a sheet metal guide plate attached to the lower end of the shank section.

(9) Though the femoral link was vertical in the standing position, the shank section was bent inward so that a line from the knee center made an angle of 5° with the vertical.

(10) The foot abduction-adduction range was simply added to the axial rotation range of the shank. A rod and sleeve joint below the ankle would provide for a separate foot range and more perfectly restrict the knee movement.

In addition to (10) above, inadequacies were found with rotations of the hip and shoulder. In the hip, there is no provision, in the model, for varying amounts of shank rotation as the hip ranges through varying degrees of flexion, as when the legs are crossed, or if one ankle is placed on the opposite knee.

In the shoulder, the humeral range of motion, with the elbow bent 90°, is only about half as great when the humerus is vertically upward as when it is vertically downward. The model shows identical axial movement for the humeral link at all positions.

These movements are, however, very specialized and have generally negligible effects on the end members.

B. Drafting Board Manikins

These manikins were constructed in relation to the 5th, 50th, and 95th percentiles of the dimensions of Air Force flying personnel. In planning a work space, the small manikin can be placed on the drawing board; after its positions are noted, the large one can be substituted and equivilant positions marked. The designer may then determine the best functional balance between the two locations, allowing clearance for the large man, but placing assemblies within the reach of the smaller.

In manikin construction, median and muscular subjects of the sample were screened to determine which ones most nearly duplicated the above percentile levels. Four anthropometric criteria were used: stature, crotch height, acromial sitting height, and upper limb length.

The selected group was photographed nude in the standard cockpit seat (heel-eye height equals 39.4"), side view. Tracings of the men in each percentile group were superimposed and joint centers for the shoulder, hip, knee, and ankle were marked. The lower limb segments were aligned so that these contours corresponded as closely as possible, after which average joint centers, link lines, and contours were drawn in; average link dimensions from Table 50 were substituted and the profile contours modified. The same was done for upper limb contours. The mean trunk contour was estimated from hip and resting shoulder joint centers, enlarged to include the variability discussed earlier. (Note-analysis of motion pictures of arm movement while seated necessitated an enlargement of the globographic range of the humeral head for dynamic behavior). The tracings were then cut out and adjacent parts united by a 5/32" pin.

Appendix G presents the plans for the contours of the three manikins on a scale of 1:4. These may be cut out as desired. However, any enlargement or reduction of the scale necessitates an equivalent modification in size of the assembly pins.

The artifical nature of this manikin must be emphasized, with caution exercised to insure that manikin postures truly refiect the positions of choice which a subject would assume.

WORK SPACE REQUIREMENTS OF THE SEATED INDIVIDUAL (Dempster, pp. 159-182)

This chapter considers the range of active movement permitted in the hand and foot, utilizing living subjects. In the analysis of this movement, two assumptions are made:

1) the seated worker most often uses his hands within the field of his vision;

2) his feet are more or less toe-forward or toe-upward, with the foot sole not slanted to the side.

Thus transverse movements of the hands are considered as miscellaneous while toeing-in or toeing-out, or slanting of the sole, are less common as functional postures, and not considered here. The forearm will always have a forward component while the shank will always have a downward or forward component.

The resultant patterns of translatory motion can be enclosed within a space envelope called a kinetosphere. These spheres will be defined by two points: first, the central axis of the grip at the third knuckle, and, second, the mid-posterior point of the heel at the foot sole level. Movement of these points is defined in relation to a fixed reference at the seat (R). An example, from one subject with a constant horizontal and transverse grip angle, is given in Figure 90. Since each kinetosphere involves only the translatory motion by an end member, it has a distinctive shape, volume, and position relative to the seat. Changes in end member orientation result in changes in position and shape.

The study of a kinetosphere series representing different degrees of hand orientation permits the study of end member movement patterns of one degree of rotatory motion in addition to three classes of translatory movement. This grouping of systematically changing orientations may be done for any of the three perpendicular planes in space as well as for the sagittal, frontal, or transverse planes of the body. The new pattern resulting is called a strophosphere. It can be further widened by including other strophospheres involving different planes of rotation. The widest possible grouping of strophospheres and kinetospheres defines the total work space.

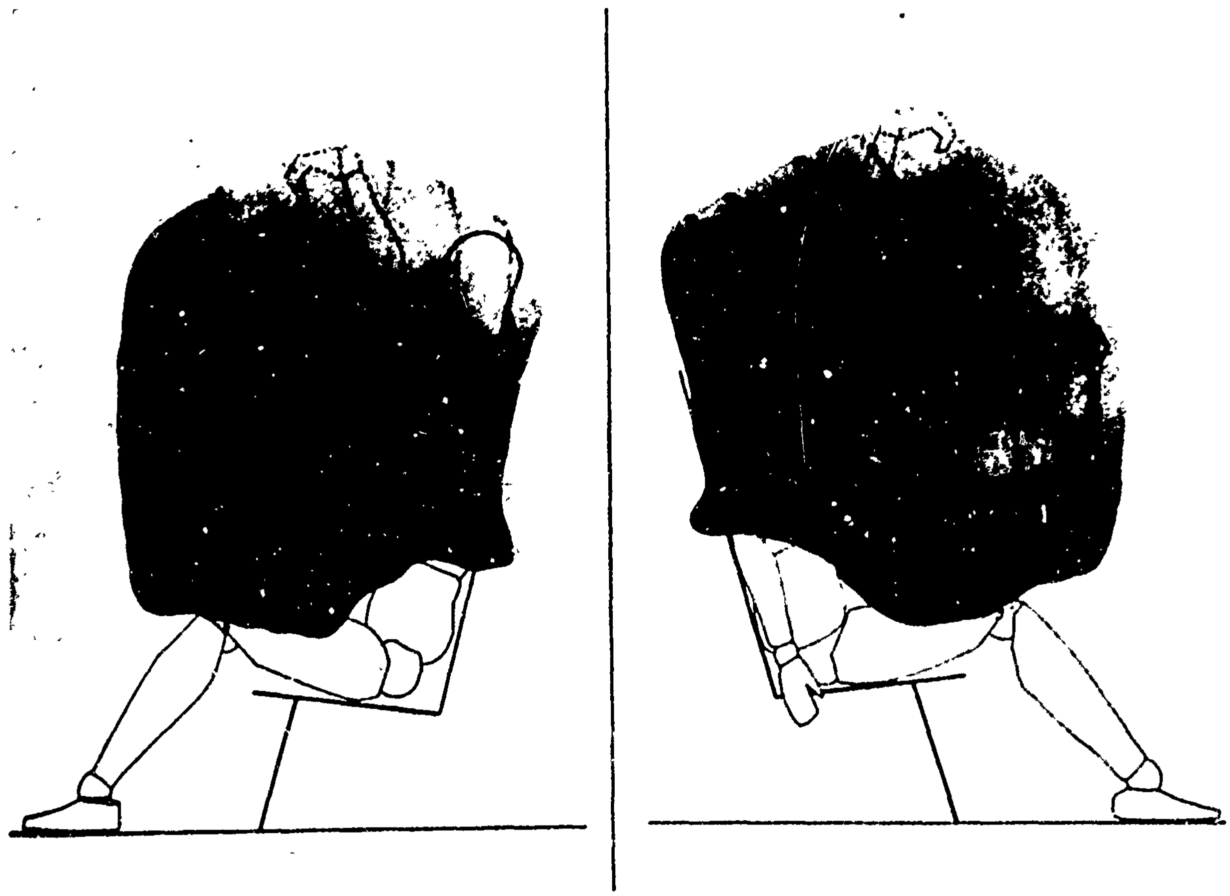


Figure 90. Lateral and Medial Views of Reconstruction of a Hand Kinetosphere. Range of movement for the prone left hand for one subject is represented.

Of all the analyses carried out, the most value is gained from frontal plane serial sections through the kinetosphere for the hand and horizontal sections through that for the foot. In addition valuable information can be gained from replots of the data to represent contours in planes which pass through the centroids, or centers of gravity of the shapes. The kinetospheres are projected from the centroids in frontal, sagittal, and horizontal planes. Standard methods such as this permit the comparison of different kinetospheres.

A. Characteristics of the Hand Space

The different hand orientations used in making this study are shown in Figure 91. Figure 92 presents the average frontal plane cross-sectional area available for the hand at both varying distances from the R (reference) point of the seat and different hand orientations. The data are taken from 22 subjects of median and muscular builds. The areas can be seen to be greatest at distances of 12-24" from R. The 0° vertical, $\pm 30^{\circ}$, and the prone orientations had their greatest area at 15-18" from R. This therefore, becomes an important area in control placement for the prone or the near vertical hand. On the other hand, tilting the angle through 60° toward 90° extends the maximum area to some 24" in front of R. For the supine, 90° and invert orientations, the areas are quite close to the value for 0°.

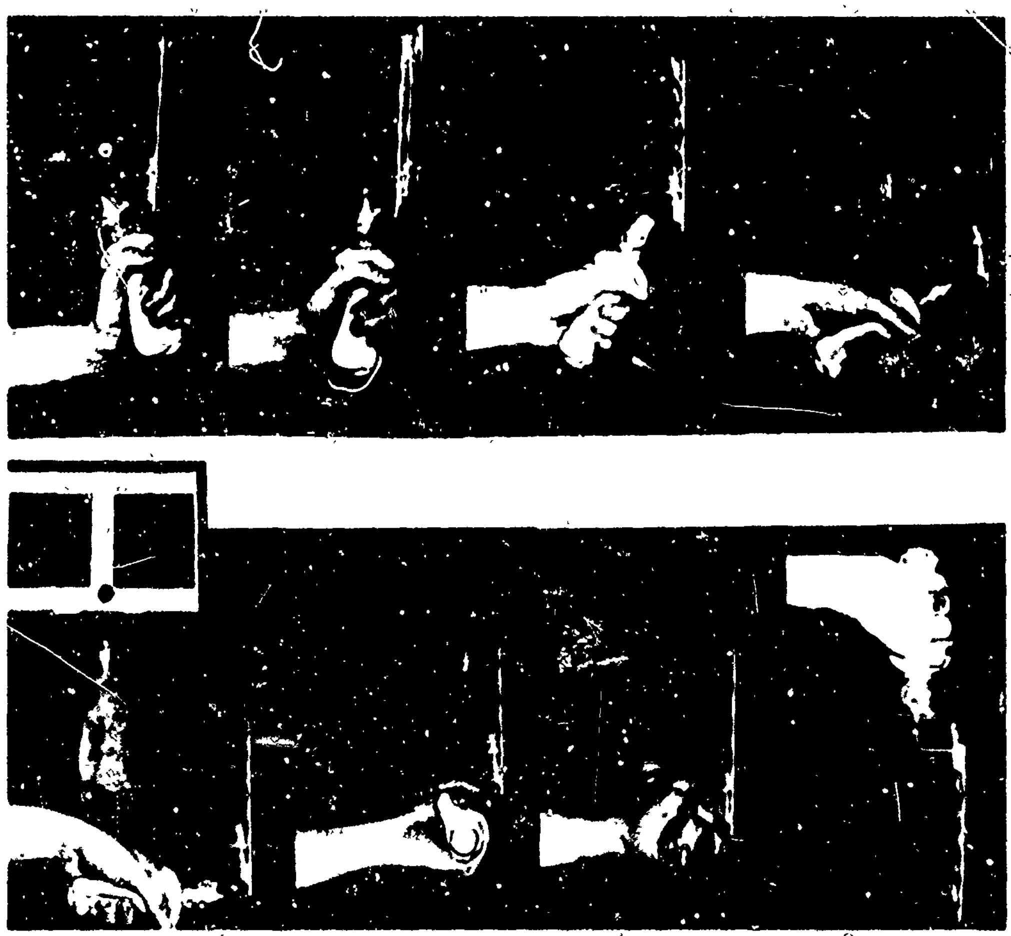


Figure 91. Various Adjustments of Hand Grip Used in Acquiring Data on the Manual Work Space. Upper row: -30°, 0°, $+30^{\circ}$, $+60^{\circ}$; lower row: 90°, supine, prone, invert.

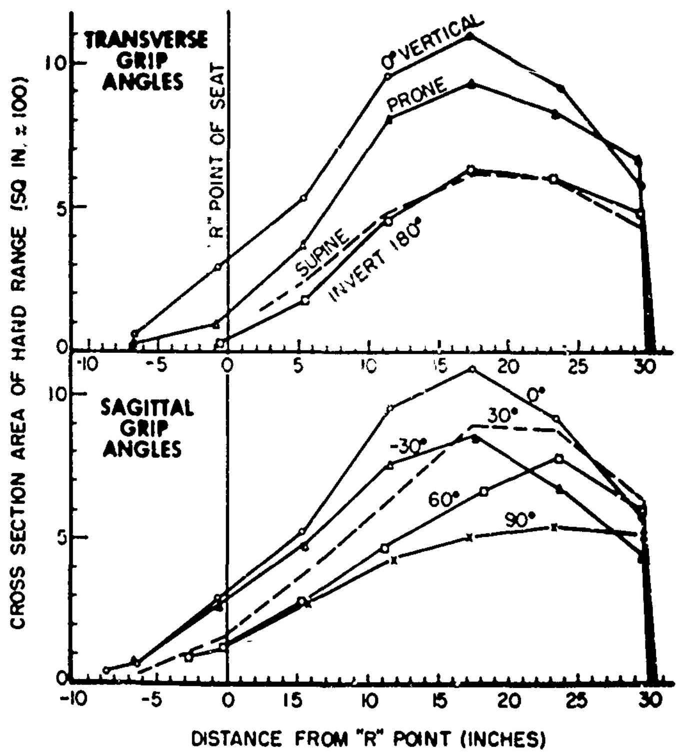


Figure 92. Plots Showing Frontal Plane Areas Available to Different Orientations of the Hand at Various Distances from the "R" Point of the Seat.

The areas under the curves represent kinetosphere volumes, the larger volumes denoting a wider range for the hand in particular hand orientations. This is summarized below.

Orientation	Vol. (cu. in.)	Orientation	Vol. (cu. in.)
-30	19,990	+90	13,040
0	24,630	Supine	13,180
+30	19,940	Prone	20,670
+60	15,850	Invert	12,920

Kinetosphere volume also varied with body build, as follows:

thin > muscular > median > rotund

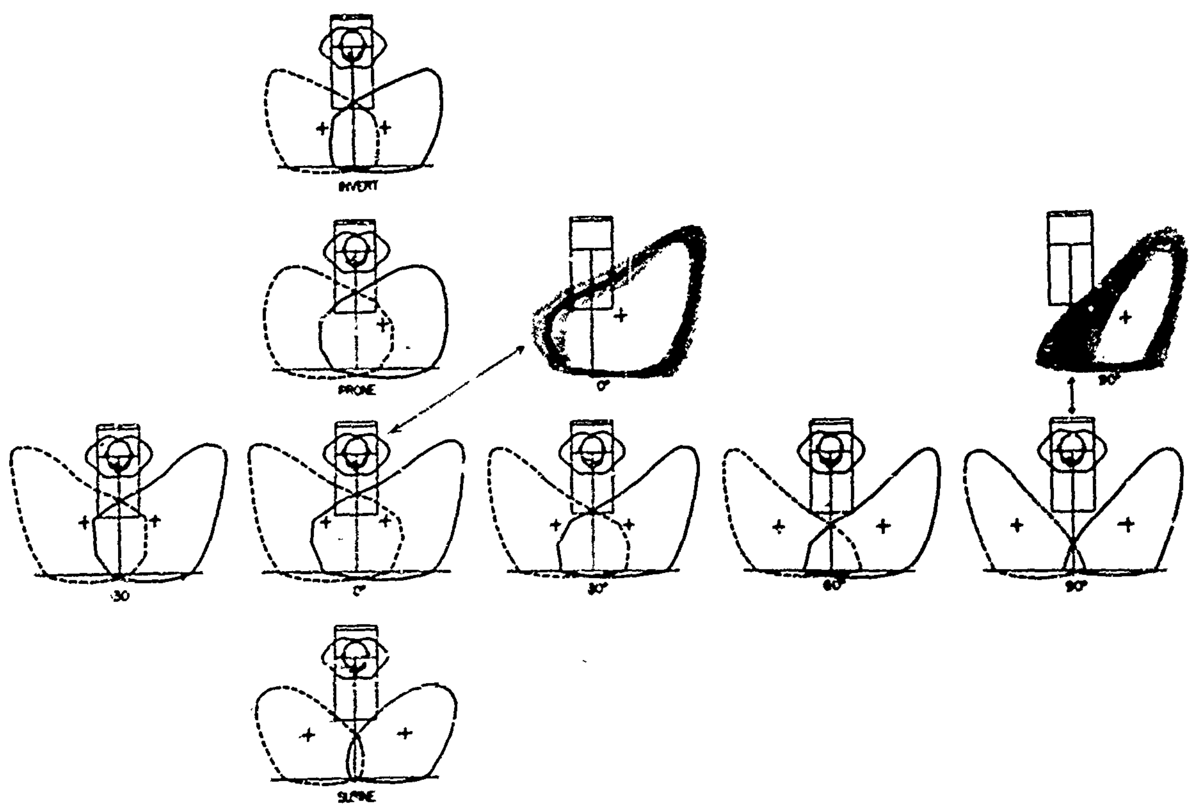


Figure 93. Mean Shapes of Eight Hand Kinetospheres for Muscular and Median Subjects as Seen in Horizontal Sections Through Centroids. Shaded figures show mean deviations of the contours for two representative hand postures.

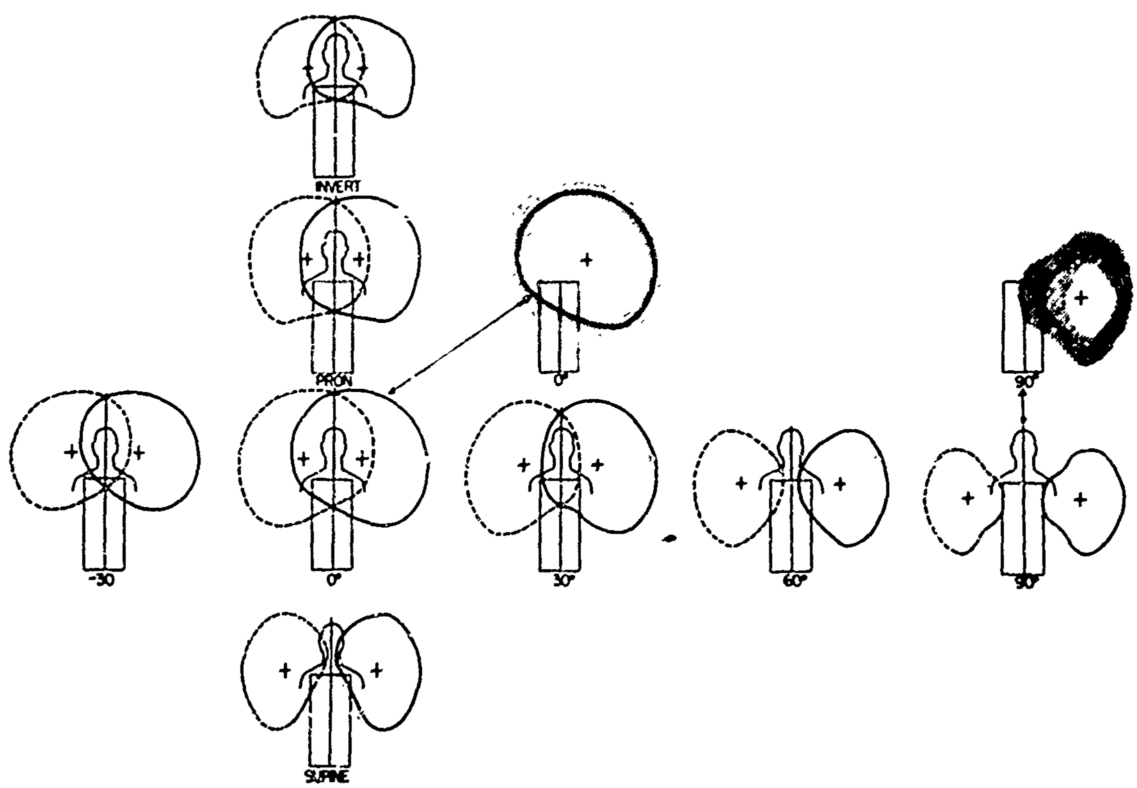


Figure 94. Frontal Sections Through the Centroids of Eight Hand Kinetospheres. Shaded figures show mean deviations of contours for two different hand orientations.

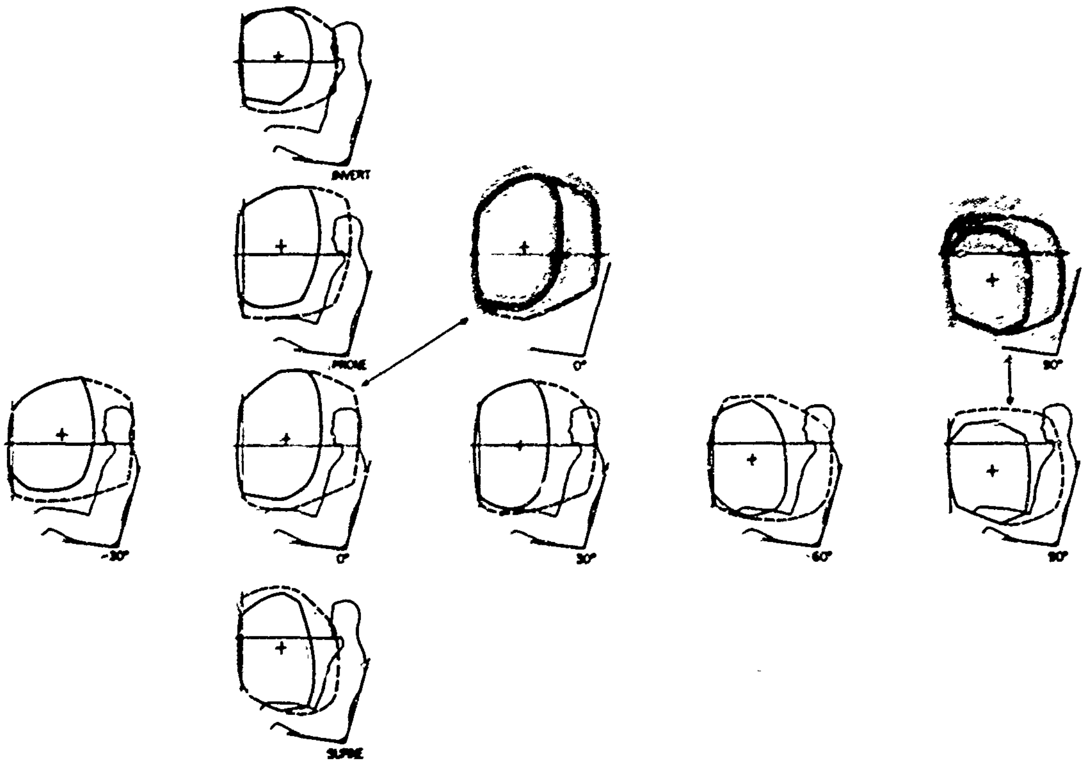


Figure 95. Sagittal Sections Through the Centroids of Different Hand Kinetospheres. Shaded figures show mean deviations of two contours.

Since data from combined muscular and median builds should approximate those for military populations, average kinetosphere patterns from these groups are given in Figures 93, 94, and 95. Note that the sections are made <u>through centroids</u> in each of the three standard perpendicular planes. As the hand moves from 0° to a 90° orientation, the following is noted:

1) a decrease in kinetosphere size;
2) a decrease in right-left overlap;
3) a forward and downward shift in the centroid.

Size, and hence range of movement, also decreased as the transverse orientations move from 0° toward supine or through prone to invert.

The shaded portions of the figures indicate mean deviations of the sample; they become a measure of variability. It can be seen that forward reach is markedly less variable than either up or down. The posterior range and the area near the body are the most variable. Quantitatively, the standard deviations ranged from a $\pm 2.8 - \pm 4.7$" for various orientations.

Two factors contributing to this variability, other than intrinsic anatomic limitations, are, first, the percentage of full exertion accomplished by an individual in reaching extremes, and, second, the fact that some individuals utilized more thigh clearance than others, and therefore more clearance than necessary.

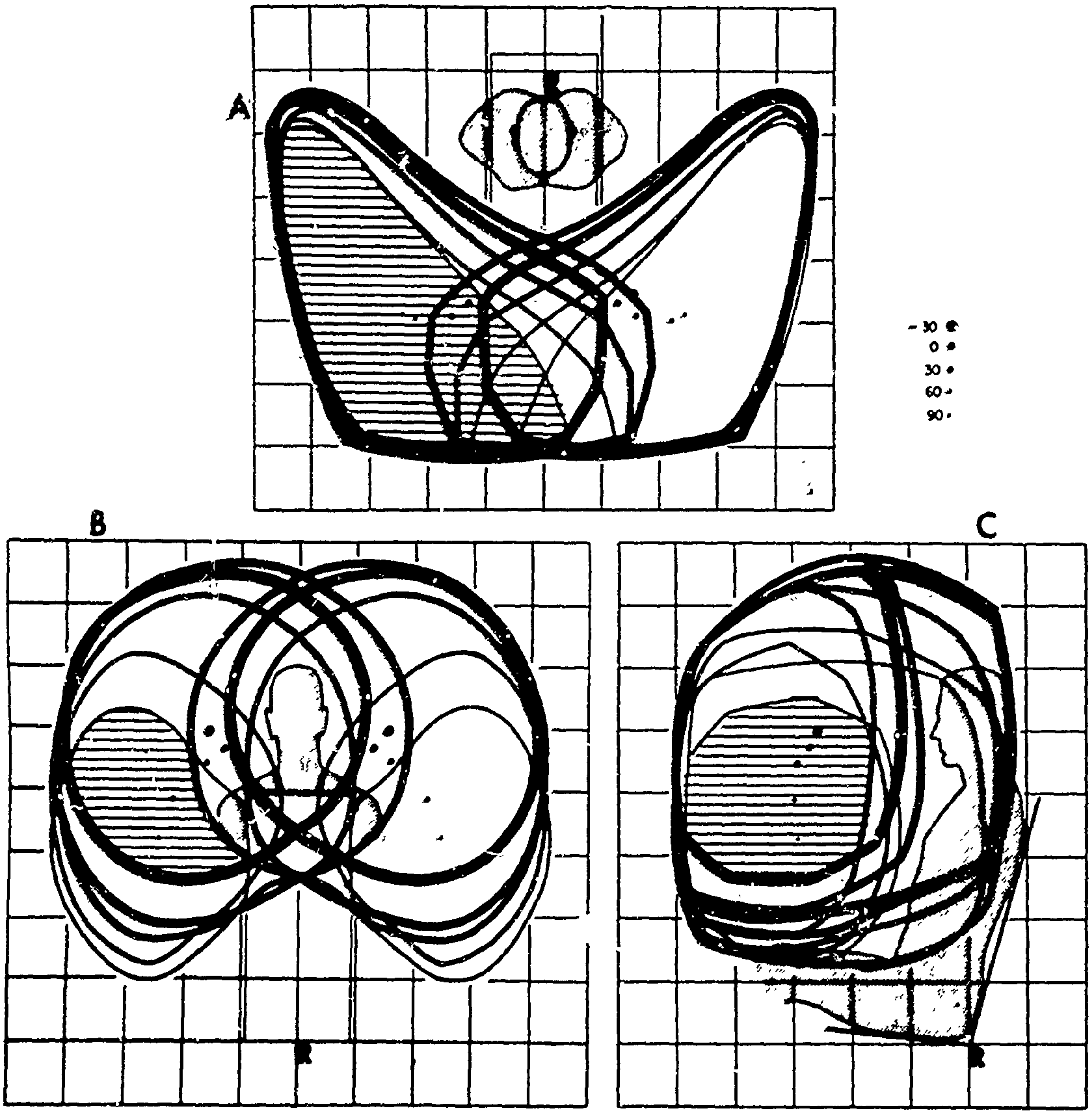


Figure 96. Hand Strophospheres Showing Five Superimposed Kinetospheres Representing Different Sagittal Orientations of the Hand Grip. Shaded areas show the common region where all sagittal orientations of the hand are possible. Dots indicate centroid locations, grid represents 6-inch intervals.

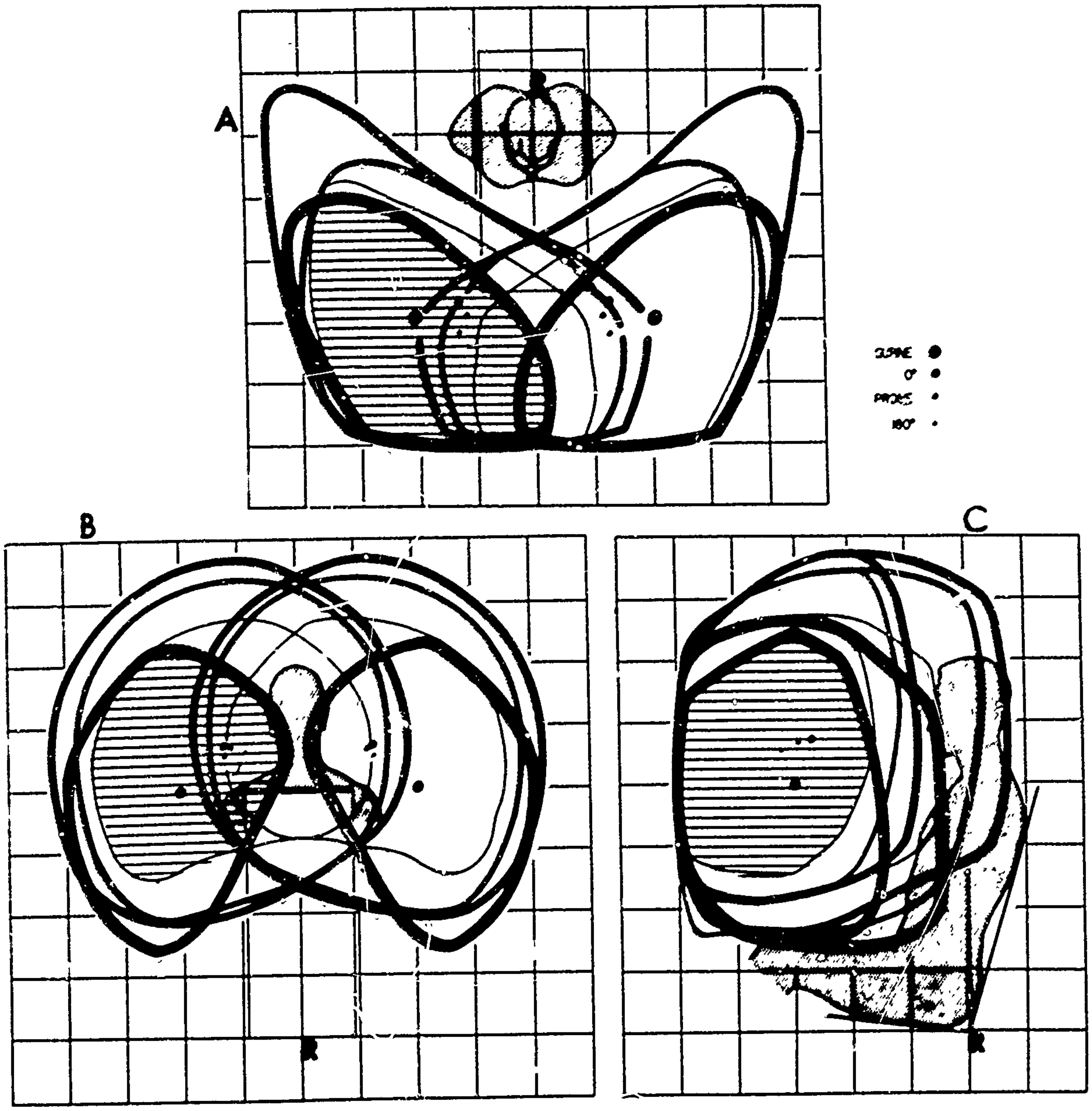


Figure 97. Hand Strophosphere Showing Superimposed Kinetospheres Representing Transverse Orientations of the Hand Grip. Shaded areas show the common region of the strophosphere.

Figures 96 and 97 are strophospheres of hand movement; in Figure 96, all sagittal orientations are superimposed relative to R, whereas, Figure 97, presents all transverse orientations, again relative to R. Each drawing therefore represents the space required for one degree of rotational, plus three of translatory, movement. It is apparent that forward and lateral contours are generally similar while kinetosphere differences generally occur in postero-medial regions.

As gauged from the 6-inch reference grid on the figures, the common region for all orientations has the following dimensions:

Height	16-18"
Width	15-20"
Depth	18-24"

This space lies obliquely to the seat and subject generally lateral to the shoulder, and extends from the nose to the waist level. It becomes the preferred one for control placement calling for miscellaneous orientations; more specific orientations allow for placement beyond this common region.

At this point, it is well to note that other factors are important in control placement, such as, the strength and direction of hand pull and push, the speed of movement from one control to another, and the relationships of the posterior strophosphere limits to the field of vision.

All centroids were enclosed in an area 15-19" forward, 7-15" lateral, and 19-30" above R. (The 60° and 90° centroids fell within the region common to all orientations). Since the centroid, in effect, is the farthest possible mean distance from the limits of its kinetosphere in all directions, a hand, at a given centroid, will be the greatest average distance from the kinetosphere limits. This implies a mean joint position for the whole chain of limb joints, correlating with muscle function. At this point, the upper limb muscles will have, in general, average lengths; this position becomes the one where the largest potential range is possible. In addition, hand forces should be more powerful as the hand moves toward, rather than away from, the centroid for a given grip orientation.

B. Characteristics of the Foot Space

In general, the flat foot (i. e., 90° to the vertical) kinetosphere is dome-shaped, compressed in an antero-posterior direction. It lays across the midline with the greatest area on the test side of the mid-sagittal plane.

The 15° kinetosphere (i. e., the angle of the foot to the vertical) increases in size as the foot is raised, its principle mass lying above the seat; it is much farther forward than the 0° figure.

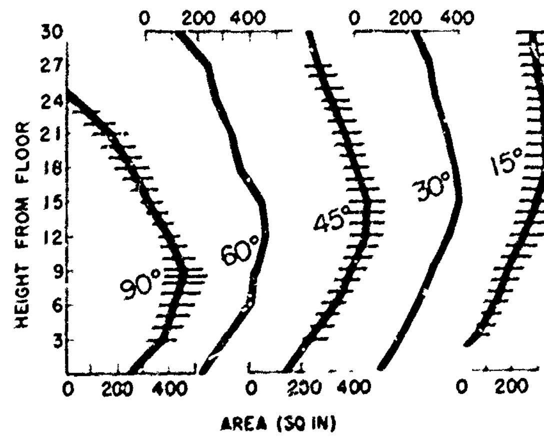


Figure 98. Area-to-Height Plots for Mean Foot Kinetospheres of Muscular and Median Men. Horizontal Shading shows the mean deviation of individual contours.

Figure 98 presents the kinetosphere areas for various heights above the floor at different foot orientations. Notice that maximum area height increases as the foot angle decreases. As the angle to the vertical decreases (foot inclination increases) the horizontal projection of the foot upon the floor surface naturally decreases; this amounted to from 100% to 26% of its length. This change has been incorporated in planning the work space; however, the measurements were made on the nude foot and about 1-1/2" forward extension should be provided to allow for shoes.

Figure 99 presents superimposed kinetospheres (strophospheres) of the median-muscular subjects; the sections were made, as for the hand, through centroids. The common sagittal region (shaded in 99A) is very small. Centroids, as seen in the sagittal sections of 99A, move up and away at a 45-50° angle as the foot inclination increases (angle to the vertical decreases). Hence, movement in forward and downward directions, relative to the centroids, should result in stronger thrusts. It should also be noted that the dashed lines in 99A show the addition of average foot length dimensions.

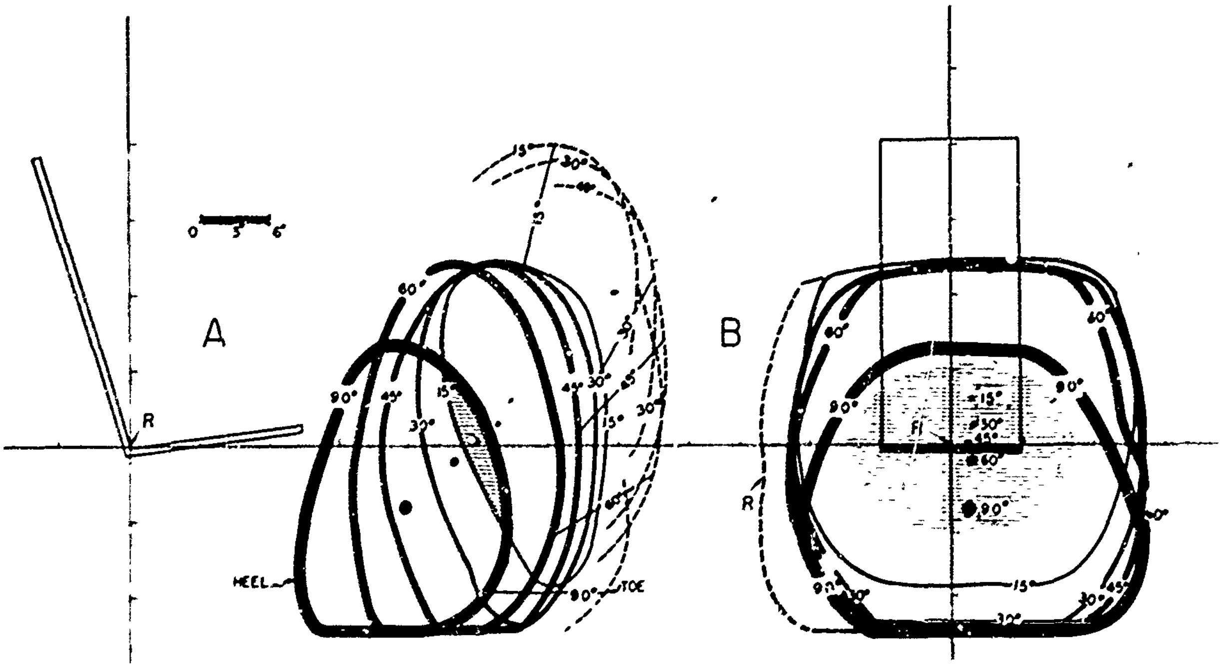


Figure 99. Sagittal (A) and Frontal (B) Sections at Centroidal Levels Through the Mean Foot Strophosphere of Median and Muscular Subjects. Dots represent centroids, shading shows the common Region, dashed lines show additional span required for different foot angulations, stippled area in B shows on one side the region of no right-left overlap.

The amount of right-left overlap area is greater in the foot than the hand. However, it should be noted that, with the foot, the whole strophosphere is quite often not available because of the interference of objects with the thigh or leg. This assumes significance as, often, the amount of knee lift is hampered by a table, steering wheel, etc. There must be a compromise between space alloted to the hand and the foot. Unrestrained foot movement demands a clear space between the knee and trunk.

C. The Overall Work Space

The floor plan of the overall work space is shown in Figure 100. The vertical point of reference is the standard

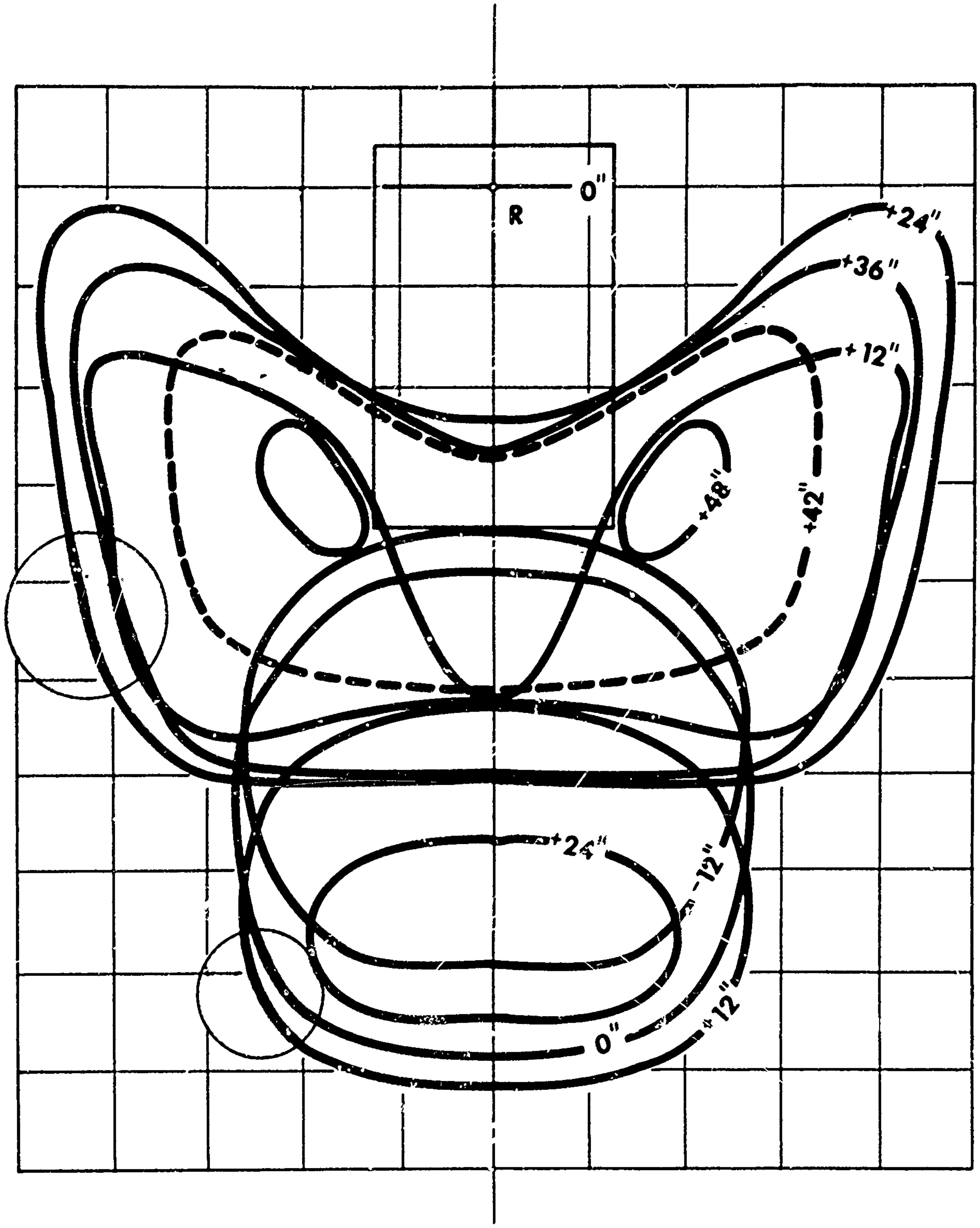


Figure 100. Floor Plan of Work Space Relative to the Standard Seat Shown By 12-inch Contours. Grid squares are 6-inches; the radius of the shaded circles repr ents width to be added or subtracted to include the 5th and 95th p centiles of movement.

seat, while the horizontal is R. Note that the shaded circles provide data on the variation between the 5th and 95th percentiles. A side view of these modified spaces is shown in Figure 101. If the area of 101B is to be used, the data should be critically balanced against factors such as common regions, amount of right-left overlap. etc.

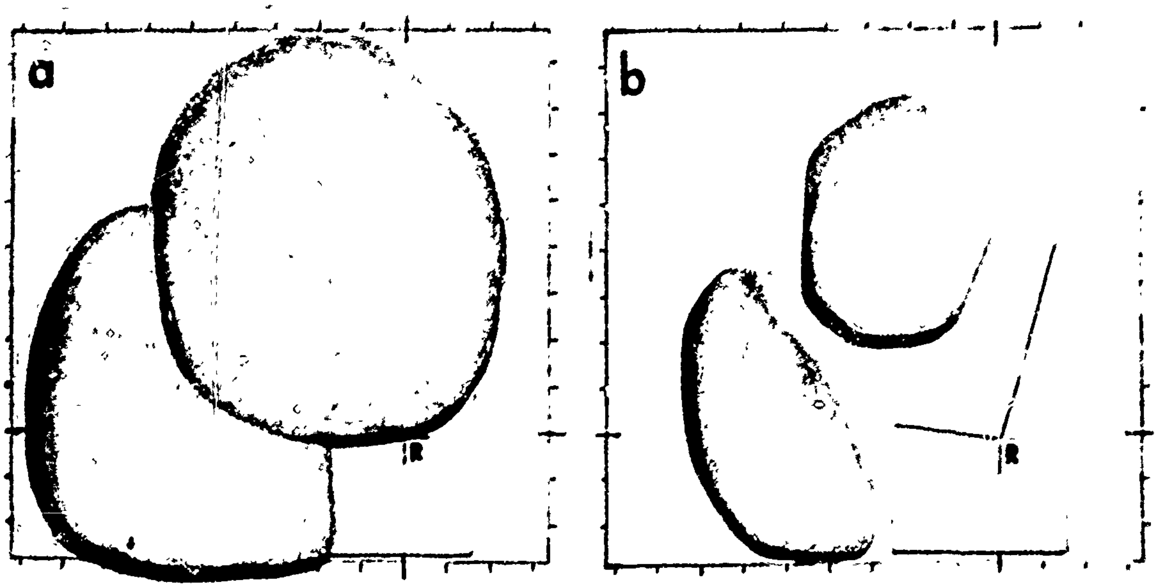


Figure 101. Models of Side View of Work Space. A - 95th percentile, B - 5th percentile. Scale is 6-inch intervals relative to seat "R" point.

Figure 100 is an overall work space, which can be utilized if no data on the nature of the operation are known. The reduction of the work space below maximum limits will necessitate specific planning.

One further note is necessary. In this presentation, the subject was erect with back against the seat rest and buttocks well back. Adequate provision for hand range with forward bending could be allowed if the anterior contours of Figure 101A were to be adjusted so that there was a continuation of hand and foot limits.

MASS RELATIONS OF CADAVER SEGMENTS
(Dempster, pp. 183-201)

Previously-published data on segmental mass has been limited to reports based on only a few cadavers. (The classic data are from Braune and Fischer, 1890). The total number, from all sources, of adults studied has been only five. Data will be presented here on an additional eight. This presentation will be concerned with segments, per se, since basic data on segment masses, on centers of gravity, on moments of inertia, and of parts relative to joint centers are considered more relevant in the analysis of static postures or instantaneous phases of body movement than a coordinate system of reference for the whole body.

In attempting to gather data in mid-range joint positions, rather than outstretched positions, limb joints were divided in planes that sought to separate segment masses into units of mean size, representing, more correctly, values for a variety of postures. The trunk mass, as described earlier, was divided into shoulder, neck, thorax, and abdomino-pelvis units.

The subjects were white adult males, from middle to old age, with no obvious physical defects.

The masses of various body segments for each cadaver, together with segmental relationships in percent, are given in Tables 52, 53, and 54. Certainly, errors crept into data due to factors such as blood loss while handling, mass changes due to water immersion, etc., but no marked discrepancies could be found due to them. Other discrepancies in percentage values can be attributed to recording errors; calculated values, mathematically correct, were substituted in only the most obvious cases; otherwise the recorded value was retained.

Centers of gravity were obtained by balancing a segment longitudinally with a variety of surfaces uppermost. A dowel was inserted into the balance point on each surface. When the established plane was cut transversly, the dowels pointed to the anatomical center of gravity.

TABLE 52

Mass of Body Parts

Weights in grams; percentages are ratios to total body weight.

Cadaver Number	Body Weight	Trunk Minus Limbs	%	Trunk Minus Shoulders	%	Both Shoulders	%
14815	51364	31363	61.1	26818	52.2	4310	8.4
15059	58409	32955	56.4	26705	45.7	6535	11.2
15062	58409	(34558)	59.1	(27670)	47.3	6888	11.8
15095	49886	29300	58.7	24431	49.0	5743	11.5
15097	72500	40568	56.0	33409	46.1	8039	11.1
15168	71364	38369	53.8	33377	46.8	7229	10.1
15250	60455	31558	52.2	25909	42.9	5708	9.4
15251	55909	30341	54.3	25341	45.3	4942	8.8
Mean %			56.5		46.9		10.3

Cadaver Number	Head and Neck	%	Thorax	%	Abdomen Plus Pelvis	%
14815	----	---	----	----	-----	----
15059	3797	6.5	4803	8.2	18182	31.1
15062	5227	8.9	6136	10.5	16364	28.0
15095	4348	8.7	5341	10.7	14515	29.1
15097	5337	7.4	8754	12.1	19187	22.0
15168	4850	6.8	9053	12.7	17237	24.1
15250	4371	7.2	6620	10.9	(14918)	24.7
15251	4340	7.8	6637	11.9	(14364)	25.7
Mean %		7.9		11.0		26.4

(Dempster, Table 10)

TABLE 53

Mass, Upper Extremity

Weights are in grams; percentages represent ratios to total body weight.

Cadaver Number	Entire Upper Extremity	%	Arm		Forearm and Hand	%	Forearm	%	Hand	%
				Left Side						
14815	2720	5.3	1157	2.3	1290	2.5	850	1.7	445	0.9
15059	2770	4.7	1541	2.6	1256	2.2	934	1.6	325	0.6
15062	2485	4.3	1373	2.4	1080	1.8	747	1.3	332	0.6
15095	2132	4.3	1133	2.3	1003	2.0	703	1.4	317	0.6
15097	3899	5.4	2199	3.0	1691	2.3	1191	1.6	500	0.7
15168	3453	4.8	1909	2.7	1515	2.1	1104	1.5	417	0.6
15250	3080	5.1	1663	2.8	1400	2.3	1002	1.7	390	0.6
15251	2459	4.4	1315	2.4	1140	2.0	780	1.4	339	0.6
Mean %		4.8		2.6		2.1		1.5		0.6
				Right Side						
14815	2641	5.1	1212	2.4	1342	2.6	865	1.7	457	0.9
15059	3277	5.6	1920	3.3	1340	2.3	995	1.7	352	0.6
15062	2695	4.6	1528	2.6	1134	1.9	815	1.4	311	0.5
15095	2125	4.3	1123	2.3	1024	2.1	710	1.4	317	0.6
15097	3947	5.4	2171	3.0	1777	2.5	1250	1.7	517	0.7
15168	3673	5.1	1970	2.8	1699	2.4	1265	1.8	452	0.6
15250	3035	5.0	1614	2.7	1414	2.3	1021	1.7	400	0.7
15251	2394	4.3	1372	2.5	1017	1.8	713	1.3	295	0.5
Mean %		4.9		2.7		2.2		1.6		0.6

(Dempster, Table 11)

TABLE 54

Mass Lower Extremity

Weights are in grams; percentages represent ratios to total body weight

Cadaver Number	Entire Lower Extremity	%	Thigh	%	Leg and Foot	%	Leg	%	Foot	%
				Left Side						
14815	6255	12.1	3495	6.8	2602	5.1	1961	3.8	725	1.4
15059	9855	16.9	6482	11.1	3384	5.8	2629	4.5	760	1.3
15062	8390	14.4	5520	9.5	2835	4.9	2080	3.6	754	1.3
15095	8313	16.7	5285	10.5	3041	6.1	2218	4.4	814	1.6
15097	11907	16.4	7093	9.8	4846	6.7	3860	5.3	967	1.3
15168	11111	15.6	6258	8.8	4812	6.7	3552	5.0	1209	1.7
15250	11337	18.8	7700	12.7	4045	6.7	2991	4.9	949	1.6
15251	(8092)	15.0	4660	8.3	3432	6.1	2564	4.6	796	1.4
Mean %		15.7		9.7		6.0		4.5		1.4
				Right Side						
14815	6176	12.0	3385	6.6	2613	5.1	1963	3.8	655	1.3
15059	9580	16.4	6115	10.5	3472	5.9	2674	4.6	800	1.4
15062	8303	14.2	5370	9.2	2907	5.0	2165	3.7	746	1.3
15095	7715	15.5	4770	9.6	2878	5.8	2205	4.4	767	1.5
15097	11920	16.4	7155	9.9	4825	6.7	3899	5.4	924	1.3
15168	11904	16.7	6902	9.7	4765	6.7	3606	5.1	1095	1.5
15250	11791	19.5	7215	11.9	3955	6.5	2954	4.9	865	1.4
15251	(8457)	15.1	5135	9.2	3322	5.9	2459	4.4	808	1.4
Mean %		15.7		9.6		5.9		4.5		1.4

(Dempster, Table 12)

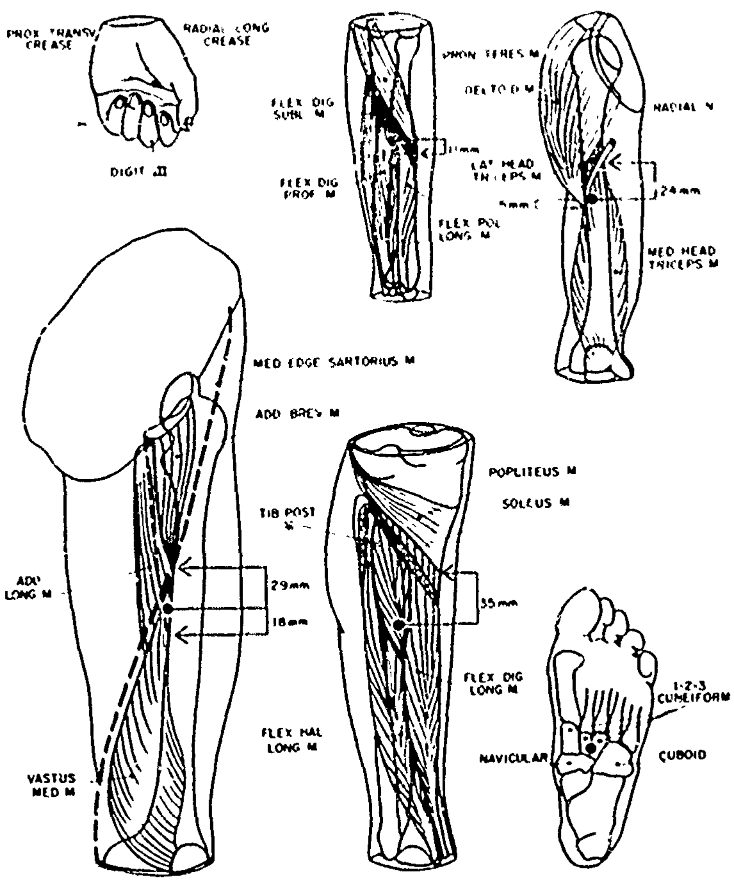


Figure 102. The Anatomical Location of Centers of Gravity of Limb Segments, as Shown By Dots in Each Segment.

Distances from centers of gravity to anatomical landmarks were measured on several cadavers, and the most common or average sites were determined. These are shown in Figure 102, and described in Tables 55 and 56. The results verified Braune and Fischer's conclusion that centers of gravity are characteristically aligned between adjacent joint centers; this, however, was not found to be true for the shoulder.

TABLE 55
ANATOMICAL LOCATION OF SEGMENT CENTERS OF GRAVITY

Segment	Proximo-Distal Location	Position in Cross Section
Arm	5 mm proximal to distal end of deltoid M insertion and 24 mm distal to most proximal fibers of medial head of triceps.	In medial head of triceps adjacent to radial nerve and radial groove of humerus.
Forearm	11 mm proximal to most distal part of insertion of pronator teres M.	9 mm anterior to interosseous membrane, usually between flexor digitorum profundus and flexor pollicis longus MM or more toward flexor pollicis longus M or toward flexor digitorum sublimus M.
Hand	(In position of rest) 2 mm proximal to proximal transverse palmar crease in angle between the proximal transverse and the radial longitudinal creases.	On axis of III metacarpal, usually 2 mm deep to skin surface.
Thigh	29 mm below apex of femoral triangle and 18 mm proximal to the most distal fibers of adductor brevis M.	Deep to adductor canal and 13 mm medial to the linea aspera in the adductor brevis M (or in adductor magnus M or vastus medialis M).
Leg	35 mm below popliteus M and 16 mm above the proximal extremity of Achilles tendon.	At posterior part of tibialis posterior M (between flexor digitorum longus and flexor hallucis longus MM) · 8 mm posterior to interosseous membrane.
Foot	66 mm from the center of the body of the talus; below the proximal halves of the second and third cuneiform bones.	In plantar ligaments or just superficial in the adjacent layer of deep foot muscles.

(Dempster, Table 13)

TABLE 56

ANATOMICAL LOCATION OF SEGMENT CENTERS OF GRAVITY

Body Part	Location
Shoulder mass	On a line perpendicular to the anterior f ˙ the outer quarter of the blade of the scapula . near its axillary border 20.5 $\pm$ 9.0 mm from the bone and 78.0 $\pm$ 5.0 mm above the inferior angle; it falls within the axilla or in the adjacent thoracic wall.
Head and neck	8 mm anterior to basior on t'c inferior surface of the basioccipital bone or within the bone 24.0 $\pm$ 5.0 mm from the crest of the dorsum sellae; on the surface of the head a point 10 mm anterior to the supratragic notch above t · head of the mandible is directly lateral.
Head alone	A point in the sphenoid s' s averaging 4 mm beyond the antero-inferior margin of the sella; on the surface, its projections lay over the temporal fossa on or near the nasion-inion line at a point about 32 percent back from the nasion; it was equally distant above the zygomatic arch and behind the malar frontosphenoid process.
Thorax	At the level of the disc between the ninth and tenth thoracic vertebrae or of either of the adjacent vertebral bodies at the anterior border of the column (anterior longitudinal ligament) or in the adjacent posterior mediastinum; on the surface, level below the nipples and above the transverse line between pectoral and abdominal muscles, spine of the eighth thoracic vertebra.
Abdomino-pelvic mass	Level of or below disc between L4 and L5 in the posterior region of vertebral body; between umbilicus and crest of ilium.

(Dempster, Table 14)

TABLE 57

RELATIVE DISTANCE BETWEEN CENTER OF GRAVITY AND JOINT AXES OR OTHER LANDMARKS

	Segment or Part and Reference Landmarks	No. Observed	Distance from Center of Gravity to Reference Dimension Stated as %
1.	Hand (position of rest) wrist axis to knuckle III	16	50.6% to wrist axis 49.4% to knuckle III
2.	Forearm, elbow axis to wrist axis	16	43.0% to elbow axis 57.0% to wrist axis
3.	Upper arm, gleno-humeral axis to elbow axis	16	43.6% to gleno-humeral axis 56.4% to elbow axis
4.	Forearm plus hand, elbow axis to ulnar styloid	16	67.7% to elbow axis 32.3% to ulnar styloid
5.	Whole upper limb, gleno-humeral axis to ulnar styloid	16	51.2% to gleno-humeral axis 48.8% to ulnar styloid
6.	Shoulder mass, sternal end of clavicle to gleno-humeral axis	14	84.0% of clavicular link dimension to sternal end of clavicle (oblique) 71.2% of clavicular link dimension to gleno-humeral axis (oblique)
7.	Foot, heel to toe II	16	*24.9% of foot link dimension to ankle axis (oblique) *43.8% of foot link dimension to heel (oblique) *59.4% of foot link dimension to toe II (oblique)
8.	Lower leg, knee axis to ankle axis	16	43.3% to knee axis 56.7% to ankle axis
9.	Thigh, hip axis to knee axis	16	43.3% to hip axis 56.7% to knee axis
10.	Leg plus foot, knee axis to medial malleolus	16	43.4% to knee axis 56.6% to medial malleolus
11.	Whole lower limb, hip axis to medial malleolus	16	43.4% to hip axis 56.6% to medial malleolus

* Alternately, a ratio of 42.9 to 57.1 along the heel to toe distance establishes a point above which the center of gravity lies; the latter lies on a line between ankle axis and ball of foot.

TABLE 57
(continued)

	Segment or Part and Reference Landmarks	No Observed	Distance from Center of Gravity to Reference Dimension Stated as %
12.	Head and trunk minus limbs, vertex to transverse line through hip axes	7	60.4% to vertex 39.6% to hip axes
13.	Head and trunk minus limb and shoulders, vertex to line through hip axes	7	64.3% to vertex 35.7% to hip axes
14.	Head alone	2	See Table 56
15.	Head and neck, vertex to seventh cervical centrum	6	43.3% to vertex 56.7% to centrum
16.	Thorax, first thoracic to twelfth thoracic centrum	6	62.7% to first thoracic centrum 37.3% to twelfth thoracic centrum
17.	Abdomino-pelvic mass, centrum first lumbar to hip axes	5	59.9% to centrum first lumbar 40.1% to hip axes

(Dempster, Table 15)

Table 57 lists centers of gravity relative to the extremities of the links. Since they tend to be aligned between adjacent joint centers, they may be grossly located this way. Individual variation in the sample was greatest for the hand, reflecting differing degrees of finger stiffness. Corrections have been made for differences between measuring along a surface to the center of gravity projection, and measuring directly to the CG itself.

Volumes, recorded from water displacement, were so close to the figures of Tables 53 and 54 that they are not presented separately. Mean specific gravities $(\frac{\text{Mass}}{\text{Volume}})$ are summarized below.

Segment	Spec. Grav.	Segment	Spec. Grav.
Trunk minus limbs	1.03	Trunk minus shoulders	1.03
Shoulders	1.04	Head and Neck	1.11
Thorax	0.92	Abdomino-Pelvic	1.01
Right Side		Left Side	
Ent. Lower Ext.	1.06	Ent. Lower Ext.	1.06
Thigh	1.05	Thigh	1.05
Leg and Foot	1.08	Leg and Foot	1.09
Leg	1.09	Leg	1.09
Foot	1.09	Foot	1.10
Ent. Upper Ext.	1.11	Ent. Upper Ext.	1.10
Arm	1.07	Arm	1.07
Forearm and Hand	1.11	Forearm and Hand	1.12
Forearm	1.13	Forearm	1.12
Hand	1.17	Hand	1.14

High densities are indicated for the areas of low fat content. Upper limb density is greater than lower. There was increased variability in the smaller parts, probably a reflection of the limits of balance sensitivity for low weight magnitude water immersion.

Moments of inertia for the body segments are given in Tables 58, 59, 60, and 61.

One cadaver was frozen and divided by sawing 1-inch transverse body sections. This specimen was male, well-proportioned, not emaciated but of light weight, and age 90 years. The metric weights of the sections are plotted against the height above the foot soles in the supine position in Figure 103. The greatest mass is indicated for the shoulder regions, with a large hip mass, also; waist and thoracic sections can be seen to be less.

TABLE 58

MOMENTS OF INERTIA ABOUT THE CENTER OF GRAVITY (I_{cg}) OF BODY SEGMENTS

Cadaver Number	Entire Upper Extremity	Arm	Forearm and Hand	Forearm	Hand	Entire Lower Extremity	Thigh	Leg and Foot	Leg	Foot
					Left Side					
14815	1.10×10^{6}	$.122\times10^{6}$	$.187\times10^{6}$	$.059\times10^{6}$	$.005\times10^{6}$	4.90×10^{6}	0.26×10^{6}	0.82×10^{6}	$.321\times10^{6}$	$.021\times10^{6}$
15059	0.78×10^{6}	$.118\times10^{6}$	-----	$.051\times10^{6}$	$.004\times10^{6}$	5.70×10^{6}	0.64×10^{6}	0.73×10^{6}	$.330\times10^{6}$	$.025\times10^{6}$
15062	0.96×10^{6}	$.115\times10^{6}$	$.155\times10^{6}$	$.043\times10^{6}$	$.005\times10^{6}$	6.05×10^{6}	0.82×10^{6}	0.55×10^{6}	$.308\times10^{6}$	$.029\times10^{6}$
15095	0.79×10^{6}	$.079\times10^{6}$	$.128\times10^{6}$	$.035\times10^{6}$	$.005\times10^{6}$	4.60×10^{6}	0.65×10^{6}	0.73×10^{6}	$.260\times10^{6}$	$.026\times10^{6}$
15097	0.98×10^{6}	$.222\times10^{6}$	$.287\times10^{6}$	$.055\times10^{6}$	$.009\times10^{6}$	9.10×10^{6}	1.14×10^{6}	1.58×10^{6}	$.650\times10^{6}$	$.037\times10^{6}$
15168	1.42×10^{6}	$.191\times10^{6}$	$.188\times10^{6}$	$.072\times10^{6}$	$.002\times10^{6}$	10.00×10^{6}	3.29×10^{6}	1.66×10^{6}	$.560\times10^{6}$	$.043\times10^{6}$
15250	1.35×10^{6}	$.155\times10^{6}$	$.218\times10^{6}$	$.074\times10^{6}$	$.003\times10^{6}$	9.80×10^{6}	1.22×10^{6}	1.29×10^{6}	$.560\times10^{6}$	$.035\times10^{6}$
15251	1.02×10^{6}	$.112\times10^{6}$	$.146\times10^{6}$	$.050\times10^{6}$	$.003\times10^{6}$	5.60×10^{6}	0.61×10^{6}	0.94×10^{6}	$.340\times10^{6}$	$.033\times10^{6}$
					Right Side					
14815	0.90×10^{6}	$.190\times10^{6}$	$.220\times10^{6}$	$.058\times10^{6}$	$.007\times10^{6}$	4.60×10^{6}	0.21×10^{6}	0.81×10^{6}	$.307\times10^{6}$	$.018\times10^{6}$
15059	0.78×10^{6}	$.130\times10^{6}$	$.137\times10^{6}$	$.041\times10^{6}$	$.005\times10^{6}$	6.70×10^{6}	0.70×10^{6}	0.85×10^{6}	$.340\times10^{6}$	$.025\times10^{6}$
15062	0.99×10^{6}	$.102\times10^{6}$	$.180\times10^{6}$	$.055\times10^{6}$	$.004\times10^{6}$	5.67×10^{6}	0.76×10^{6}	0.86×10^{6}	$.298\times10^{6}$	$.028\times10^{6}$
15095	0.58×10^{6}	$.062\times10^{6}$	$.128\times10^{6}$	$.039\times10^{6}$	$.003\times10^{6}$	5.20×10^{6}	0.69×10^{6}	0.75×10^{6}	$.275\times10^{6}$	$.013\times10^{6}$
15097	1.10×10^{6}	$.220\times10^{6}$	$.298\times10^{6}$	$.072\times10^{6}$	$.011\times10^{6}$	9.20×10^{6}	1.27×10^{6}	1.65×10^{6}	$.620\times10^{6}$	$.040\times10^{6}$
15168	1.40×10^{6}	$.145\times10^{6}$	$.197\times10^{6}$	$.061\times10^{6}$	$.003\times10^{6}$	8.60×10^{6}	3.12×10^{6}	1.64×10^{6}	$.620\times10^{6}$	$.038\times10^{6}$
15250	1.57×10^{6}	$.166\times10^{6}$	$.232\times10^{6}$	$.068\times10^{6}$	$.004\times10^{6}$	9.50×10^{6}	1.44×10^{6}	1.40×10^{6}	$.620\times10^{6}$	$.035\times10^{6}$
15251	0.91×10^{6}	$.120\times10^{6}$	$.152\times10^{6}$	$.054\times10^{6}$	$.003\times10^{6}$	5.20×10^{6}	0.61×10^{6}	0.96×10^{6}	$.360\times10^{6}$	$.032\times10^{6}$

(Dempster, Table 18)

TABLE 59

MOMENTS OF INERTIA ABOUT THE PROXIMAL JOINT CENTER (I_o) OF BODY SEGMENTS

Cadaver Number	Entire Upper Extremity	Arm	Forearm and Hand	Forearm	Hand	Entire Lower Extremity	Thigh	Leg and Foot	Leg	Foot
					Left Side					
14815	3.82×10^{6}	$.334\times10^{6}$	$.675\times10^{6}$	$.188\times10^{6}$	$.028\times10^{6}$	13.3×10^{6}	2.16×10^{6}	2.46×10^{6}	0.928×10^{6}	$.057\times10^{6}$
15059	2.60×10^{6}	$.372\times10^{6}$	----	$.165\times10^{6}$	$.011\times10^{6}$	13.7×10^{6}	2.33×10^{6}	2.67×10^{6}	1.140×10^{6}	$.047\times10^{6}$
15062	2.66×10^{6}	$.252\times10^{6}$	$.456\times10^{6}$	$.121\times10^{6}$	$.015\times10^{6}$	14.0×10^{6}	2.25×10^{6}	2.00×10^{6}	0.770×10^{6}	$.053\times10^{6}$
15095	2.14×10^{6}	$.239\times10^{6}$	$.406\times10^{6}$	$.123\times10^{6}$	$.014\times10^{6}$	12.3×10^{6}	1.78×10^{6}	2.12×10^{6}	0.805×10^{6}	$.046\times10^{6}$
15097	4.91×10^{6}	$.629\times10^{6}$	$.845\times10^{6}$	$.279\times10^{6}$	$.032\times10^{6}$	24.6×10^{6}	3.44×10^{6}	5.23×10^{6}	2.530×10^{6}	$.063\times10^{6}$
15168	4.36×10^{6}	$.566\times10^{6}$	$.715\times10^{6}$	$.236\times10^{6}$	$.025\times10^{6}$	26.8×10^{6}	5.53×10^{6}	4.89×10^{6}	1.770×10^{6}	$.115\times10^{6}$
15250	4.52×10^{6}	$.593\times10^{6}$	$.655\times10^{6}$	$.221\times10^{6}$	$.015\times10^{6}$	26.7×10^{6}	4.35×10^{6}	3.95×10^{6}	1.950×10^{6}	$.064\times10^{6}$
15251	2.53×10^{6}	$.323\times10^{6}$	$.466\times10^{6}$	$.153\times10^{6}$	$.009\times10^{6}$	15.5×10^{6}	2.27×10^{6}	2.84×10^{6}	1.150×10^{6}	$.057\times10^{6}$
					Right Side					
14815	3.50×10^{6}	$.375\times10^{6}$	$.670\times10^{6}$	$.183\times10^{6}$	$.027\times10^{6}$	12.6×10^{6}	2.08×10^{6}	2.29×10^{6}	0.895×10^{6}	$.076\times10^{6}$
15059	2.23×10^{6}	$.278\times10^{6}$	$.443\times10^{6}$	$.155\times10^{6}$	$.016\times10^{6}$	13.4×10^{6}	2.30×10^{6}	2.64×10^{6}	1.130×10^{6}	$.047\times10^{6}$
15062	2.59×10^{6}	$.319\times10^{6}$	$.449\times10^{6}$	$.136\times10^{6}$	$.013\times10^{6}$	13.6×10^{6}	2.30×10^{6}	2.48×10^{6}	0.882×10^{6}	$.051\times10^{6}$
15095	2.04×10^{6}	$.212\times10^{6}$	$.397\times10^{6}$	$.124\times10^{6}$	$.013\times10^{6}$	13.2×10^{6}	2.08×10^{6}	2.58×10^{6}	0.902×10^{6}	$.013\times10^{6}$
15097	5.07×10^{6}	$.657\times10^{6}$	$.852\times10^{6}$	$.249\times10^{6}$	$.031\times10^{6}$	26.0×10^{6}	3.54×10^{6}	5.09×10^{6}	2.660×10^{6}	$.086\times10^{6}$
15168	4.14×10^{6}	$.541\times10^{6}$	$.652\times10^{6}$	$.212\times10^{6}$	$.027\times10^{6}$	25.5×10^{6}	5.26×10^{6}	5.02×10^{6}	1.810×10^{6}	$.136\times10^{6}$
15250	4.51×10^{6}	$.506\times10^{6}$	$.697\times10^{6}$	$.224\times10^{6}$	$.016\times10^{6}$	24.5×10^{6}	4.28×10^{6}	4.20×10^{6}	1.970×10^{6}	$.057\times10^{6}$
15251	2.71×10^{6}	$.320\times10^{6}$	$.455\times10^{6}$	$.145\times10^{6}$	$.011\times10^{6}$	15.0×10^{6}	1.99×10^{6}	2.84×10^{6}	1.180×10^{6}	$.065\times10^{6}$

(Dempster, Table 19)

TABLE 60

MOMENTS OF INERTIA OF TRUNK SEGMENTS ABOUT THEIR CENTERS OF GRAVITY ($\bar{I}_{cg}$)

Cadaver Number	Trunk Minus Limbs	Trunk Minus Shoulders	Shoulders Left	Shoulders Right	Head and Neck	Thorax	Abdomino-Pelvic Region
15059	15.5×10^6	14.0×10^6	0.355×10^6	0.378×10^6	0.22×10^6	0.45×10^6	-----
15062	23.7×10^6	15.9×10^6	0.500×10^6	0.324×10^6	-----	-----	-----
15095	13.4×10^6	13.0×10^6	0.417×10^6	0.421×10^6	-----	-----	-----
15097	22.1×10^6	21.0×10^6	0.800×10^6	0.700×10^6	0.31×10^6	1.19×10^6	3.24×10^6
15168	24.3×10^6	23.1×10^6	0.520×10^6	0.800×10^6	0.23×10^6	2.18×10^6	9.70×10^6
15250	14.9×10^6	16.4×10^6	0.425×10^6	0.425×10^6	0.32×10^6	0.96×10^6	2.44×10^6
15251	14.9×10^6	14.0×10^6	0.480×10^6	0.420×10^6	0.39×10^6	0.99×10^6	1.96×10^6

(Dempster, Table 20)

TABLE 61

MOMENTS OF INERTIA OF TRUNK SEGMENTS

ABOUT SUSPENSION POINTS (I_o)

Cadaver Number	Trunk Minus Limbs[1]	Trunk Minus Shoulders[1]	Shoulders[2] Left	Shoulders[2] Right	Head and Neck[3]	Thorax[4]	Abdomino-Pelvic Region[1]
15059	42.7×10^6	30.4×10^6	0.539×10^6	0.607×10^6	1.31×10^6	1.51×10^6	------
15062	60.5×10^6	50.4×10^6	0.836×10^6	0.735×10^6	-----	-----	------
15095	39.6×10^6	32.8×10^6	0.937×10^6	0.886×10^6	-----	-----	------
15097	64.5×10^6	50.4×10^6	1.800×10^6	1.530×10^6	1.87×10^6	4.73×10^6	8.72×10^6
15168	78.0×10^6	62.8×10^6	1.090×10^6	1.300×10^6	1.43×10^6	6.56×10^6	12.60×10^6
15250	60.3×10^6	37.7×10^6	0.799×10^6	0.827×10^6	1.44×10^6	3.48×10^6	3.96×10^6
15251	50.9×10^6	34.6×10^6	1.080×10^6	0.961×10^6	1.04×10^6	3.12×10^6	3.77×10^6

(Dempster, Table 21)

1. Suspension from hip joints.
2. Suspension from sternoclavicular joint.
3. Suspension from 7C vertebral body.
4. Suspension from 12th vertebral body.

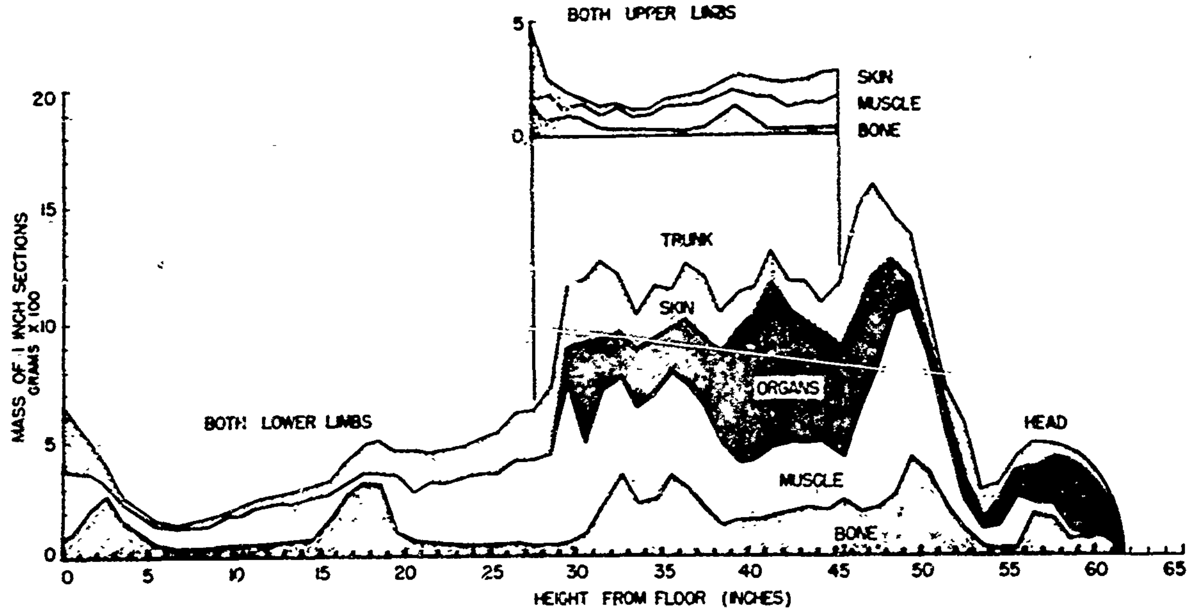


Figure 103. Distribution of the Body Mass of a Cadaver Relative to Its Height

The constituent tissues, for each section, were dissected out and weighed; relative weights are also given in the Figure. It should be noted that the tissues were grouped as follows:

1) skeletal tissue; bone, cartilage and ligaments;

2) muscle; tendon, intermuscular fat, associated nerves and blood vessels, in addition to muscular tissue;

3) viscera; brain, spinal cord, meninges, tongue, throat structures, viscera and blood vessels of the neck, thorax, abdomen, and pelvis;

4) skin and subcutaneous fat.

Overall weight ratios of the four body constituents are as follows:

bone	24.5%	viscera	22.6%
muscle	39.9%	skin and sub. fat	13.6%

BODY BULK DISTRIBUTION IN LIVING SUBJECTS

The practical utilization of the data of the preceding chapter requires its extrapolation to the living. The general height and weight increases that have occurred in our population necessitate up-to-date data that actually conform to present day values.

Though mass, center of gravity, and moment of inertia cannot be obtained from the living, many segmental volumes can.[1] The use of these volumes with density figures from cadavers should permit a more accurate conversion to body mass, than the mere assumption of a 1.0 density for the parts. Due to the predictibility of anatomical relationships, differences between individuals being largely quantitative, it can be assumed that the centers of gravity of different subjects will fall at or near the same locus.

Segmental volumes were obtained by previously-mentioned techniques. When grouped by body build, the mean values for segmental values ranked uniformly as follows:

rotund > median-muscular > thin

These volumes may also be expressed as a percentage of total body weight. If the density for each segment is assumed to be 1.00, the ratios will refer to either percentage mass or percentage volume,[2] as in Table 62. Many ratios differ but little between physique types, but, in general, for the upper limb, muscular and median show higher values than rotund and thin. Thin physique types, however, also have high ratios for forearm and hand. For the lower limb the rotund, thin, and median groups have relatively high volume ratios, with the muscular group consistently low. In the foot, however, rotund individuals have the lowest percentage. Trunk minus limbs for all builds averaged between 49-52%, with the rotund and muscular physiques having the higher values.

---

1. Theoretically, it is true that these values cannot be obtained from the living. However, they can be calculated at a level of accuracy which is suitable for most purposes. These are summarized and presented by Steindler (1955). - FEJ.

2. This assumption introduces a degree of error. Mean specific gravity values cited earlier indicate inter-segmental variation in density. However, the percentage values associated with different physiques can be seen. - FEJ.

TABLE 62

RATIO OF MEAN VOLUME OF LIMB SEGMENTS TO BODY VOLUME* (In percent)

Segment	Rotund	Muscular	Thin	Median
Upper limb	5.28	5.60	5.20	5.65
Arm	3.32	3.35	2.99	3.46
Forearm	1.52	1.70	1.63	1.61
Hand	0.42	0.53	0.58	0.54
Lower limb	20.27	18.49	19.08	19.55
Thigh	14.78	12.85	12.90	13.65
Leg	4.50	4.35	4.81	4.65
Foot	1.10	1.30	1.46	1.25

* Body volume considered as body weight for assumed density of 1.0.

(Dempster, Table 26)

Individual values, showing variation about the mean, are given in Tables 63, 64, 65, and 66. When these values are compared to the cadaver data of Tables 53 and 56, it can be seen that living subjects have markedly higher ratios for the lower limb, while upper limb values are also generally higher. A conversion of the figures by considering density would be more accurate, but, for genral purposes of comparison, need not be considered. The differences may be attributed to the increased number of older cadavers when compared to the ages of the living subjects.

Area-to-height plots using pantograph-planimeter methods were made. Twenty-one landmarks were selected as being at critical levels, i.e., mainly, at sharp contour changes. These landmarks were:

1. Vertex of skull
2. Euryon (widest part of head above the ears)
3. Gonion (angle of the jaw)
4. Thyroid notch (Adam's apple) of thyroid cartilage of larynx
5. Acromion
6. Shoulder at level of sternal angle (joint between manubrium and body of sternum)
7. Shoulder at anterior axillary fold (with arm at side)
8. Trunk alone at the same level (a thin, wooden slat was clamped between the arms and thorax, and the pantograph tracer arms were moved over the back and anterior thoracic

TABLE 63

Volume Of Limb Segments in Cubic Centimeters

and Percentages Of Body Volume*

Rotund Physique (Male)

Subject No.	Body lb	Weight, kg	Whole Upper Limbs	%	Arm	%	Forearm Plus Hand	%	Forearm	%	Hand	%
1	145	65.9	3496	5.3	2217	3.4	1279	1.9	931	1.4	348	0.53
2	185	84.1	4287	5.1	2551	3.0	1736	2.2	1386	1.6	350	0.42
3	224	101.5	5062	5.0	3180	3.1	1882	1.9	1446	1.4	436	0.43
4	244	110.9	5162	4.7	3122	2.8	2040	1.8	1650	1.5	390	0.35
5	261	118.6	7113	6.0	4697	3.9	2418	2.0	1982	1.7	436	0.37
6	272	123.6	6964	5.6	4590	3.7	2374	1.9	1844	1.5	530	0.43

Subject No.	Body lb	Weight, kg	Whole Lower Limbs	%	Thigh	%	Leg Plus Foot	%	Leg	%	Foot	%
1	-	-	15422	23.4	11645	17.7	3777	5.7	2947	4.5	830	1.30
2	-	-	16103	19.2	11655	13.9	4448	5.3	4010	4.8	914	1.20
3	-	-	20236	19.8	14798	14.6	5438	5.4	4605	4.5	833	0.80
4	-	-	20916	18.9	15434	13.9	5482	5.0	4303	3.9	1179	1.10
5	-	-	24368	20.6	17288	14.6	7080	6.0	5724	4.8	1356	1.10
6	-	-	24242	19.7	17248	14.0	6994	5.7	5602	4.5	1392	1.10

(Dempster, Table 22)

* Body volume equals body weight for an assumed density of 1.0.

TABLE 64

Volume of Limb Segments in Cubic Centimeters
and Percentages of Body Volume*

Muscular Physique (Male)

Subject No.	Body lb	Weight, kg	Whole Upper Limbs	%	Arm	%	Forearm Plus Hand	%	Forearm	%	Hand	%
1a	150	68.4	4094	6.0	2427	3.5	1667	2.4	1253	1.8	414	0.60
2a	151	68.7	3757	5.5	2129	3.1	1628	2.4	1306	1.9	322	0.47
3a	153	69.6	4202	6.0	2630	3.8	1572	2.3	1200	1.7	372	0.54
4a	160	72.7	4342	6.0	2537	3.5	1507	2.1	1100	1.5	407	0.56
5a	167	76.0	4254	5.6	2505	3.3	1749	2.3	1282	1.7	467	0.6[illegible]
6a	168	76.4	4249	5.5	2501	3.3	1748	2.3	1361	1.8	387	0.51
7a	171	77.8	3821	4.9	2239	2.9	1582	2.0	1184	1.5	398	0.51
8a	172	78.2	3802	4.9	2211	2.8	1591	2.0	1247	1.6	344	0.44
9a	186	84.6	4935	5.8	3000	3.6	1926	2.3	1535	1.8	391	0.46
10a	186	84.6	4952	5.9	3100	3.7	1852	2.2	1400	1.7	452	0.53
11a	211	95.9	5301	5.5	3106	3.3	2195	2.3	1610	1.7	585	0.61

Subject No.	Body lb	Weight kg	Whole Lower Limbs	%	Thigh	%	Leg Plus Foot	%	Leg	%	Foot	%
1a	-	-	13900	20.4	9788	14.3	4112	6.0	3248	4.8	864	1.30
2a	-	-	11571	16.9	8224	12.0	3347	4.9	2515	3.7	832	1.20
3a	-	-	13026	18.8	9412	13.6	3614	5.2	2805	4.0	809	1.20
4a	-	-	13188	18.1	8994	12.4	4194	5.8	3286	4.5	908	1.20
5a	-	-	13901	18.3	9249	12.2	4652	6.1	3533	4.7	1119	1.50
6a	-	-	13399	17.6	9253	12.1	4146	5.4	3133	4.1	1013	1.30
7a	-	-	14798	18.9	9892	12.7	4906	6.3	3929	5.0	977	1.30
8a	-	-	12286	15.8	8128	10.4	4158	5.3	3258	4.2	900	1.20
9a	-	-	16814	19.9	11972	14.2	4842	5.7	3606	4.3	1236	1.50
10a	-	-	16202	19.2	11788	13.9	4414	5.2	3430	4.1	984	1.20
11a	-	-	18640	19.5	13116	13.5	5524	5.8	4183	4.4	1341	1.40

(Dempster, Table 23)

* Body volume equals body weight for an assumed density of 1.0.

TABLE 65

Volume of Limb Segments in Cubic Centimeters

and Percentages of Body Volume*

Thin Physique (Male)

Subject No.	Body lb	Weight, kg	Whole Upper Limbs	%	Arm	%	Forearm Plus Hand	%	Forearm	%	Hand	%
1b	114	51.7	2652	4.9	1487	2.9	1165	2.3	874	1.7	291	0.56
2b	125	56.9	2494	4.4	1392	2.5	1102	1.9	850	1.5	252	0.44
3b	128	58.2	2884	5.0	[illegible]	2.8	1242	2.1	881	1.5	361	0.62
4b	132	60.0	3803	6.4	2359	3.9	1444	2.4	1153	1.9	291	0.48
5b	133	60.4	2904	4.8	1600	2.7	1304	2.2	932	1.5	372	0.62
6b	134	60.9	3415	5.6	2053	3.4	1362	2.2	1004	1.6	358	0.60
7b	134	60.9	3080	5.1	1682	2.8	1398	2.3	1047	1.7	351	0.58
8b	140	63.6	3712	5.8	2225	3.5	1487	2.3	1103	1.7	384	0.61
9b	154	70.0	3872	5.5	2101	3.0	1771	2.5	1271	1.8	500	0.71
10b	172	78.2	3497	4.5	1897	2.4	1610	2.1	1184	1.5	426	0.54

Subject No.	Body lb	Weight, kg	Whole Lower Limbs	%	Thigh	%	Leg Plus Foot	%	Leg	%	Foot	%
1b	-	-	10274	19.9	7167	13.9	3107	6.0	2341	4.6	766	1.50
2b	-	-	11076	19.5	7333	12.9	3743	6.6	2903	5.1	840	1.50
3b	-	-	10578	18.2	7336	12.6	3242	5.6	2465	4.2	777	1.30
4b	-	-	12214	20.4	8422	14.1	3792	6.3	3001	5.0	791	1.30
5b	-	-	11660	19.3	7346	12.2	4314	7.2	3428	5.7	886	1.50
6b	-	-	11334	18.7	7824	12.9	3510	5.8	2593	4.3	917	1.50
7b	-	-	11453	18.9	7680	12.6	3773	6.2	2839	4.7	934	1.50
8b	-	-	13803	21.7	9171	14.4	4632	7.3	3608	5.7	1024	1.60
9b	-	-	13332	19.0	8860	12.7	4472	6.4	3378	4.8	1094	1.60
10b	-	-	12156	15.2	8006	10.2	4150	5.3	3140	4.0	1010	1.30

(Dempster, Table 24)

* Body volume equals body weight for an assumed density of 1.0.

TABLE 66

Volume of Limb Segments in Cubic Centimeters

and Percentages of Body Volume*

Median Physique (Male)

Subject No.	Body lb	Weight, kg	Whole Upper Limbs	%	Arm	%	Forearm Plus Hand	%	Forearm	%	Hand	%
1c	140	63.7	3494	5.5	2053	3.2	1441	2.3	1081	1.7	359	0.56
2c	152	69.1	3862	5.6	2433	3.5	1429	2.1	1077	1.6	352	0.51
3c	152	69.1	4196	6.1	2659	3.8	1537	2.2	1192	1.7	345	0.50
4c	153	69.6	3909	5.6	2379	3.3	1530	2.2	1205	1.7	323	0.46
5c	156	71.0	3652	5.1	2193	3.1	1459	2.1	1119	1.6	340	0.48
6c	162	73.6	4473	6.1	2727	3.6	1746	2.4	1314	1.8	432	0.59
7c	163	74.1	3991	5.4	2491	3.4	1500	2.0	1070	1.4	430	0.58
8c	168	76.3	4370	5.7	2693	3.5	1677	2.2	1217	1.6	460	0.60
9c	170	77.3	4299	5.8	3171	4.1	1128	1.5	875	1.1	353	0.46
10c	175	79.5	4438	5.6	2586	3.3	1852	2.3	1368	1.7	484	0.60
11c	181	82.3	4662	5.7	2729	3.3	1933	2.4	1450	1.8	487	0.59

Subject No.	Body lb	Weight, kg	Whole Lower Limbs	%	Thigh	%	Leg Plus Foot	%	Leg	%	Foot	%
1c	-	-	11520	18.0	7957	12.5	3678	5.8	2910	4.6	787	1.2
2c	-	-	12002	17.5	9207	13.3	3654	5.3	2867	4.2	787	1.1
3c	-	-	12861	18.7	7835	11.4	4167	6.0	3305	4.8	862	1.2
4c	-	-	14878	21.4	10754	15.5	4124	5.9	3250	4.7	874	1.3
5c	-	-	15413	21.8	11682	16.5	5731	6.0	2815	4.0	816	1.2
6c	-	-	15080	20.5	10768	14.7	4262	5.8	3392	4.6	870	1.2
7c	-	-	12705	17.2	7858	10.6	4847	6.5	3928	5.3	919	1.2
8c	-	-	15636	20.5	11122	14.6	4514	5.9	3525	4.6	989	1.3
9c	-	-	14908	19.3	10296	13.3	4612	6.0	3584	4.6	1028	1.3
10c	-	-	16776	21.1	11703	14.7	5073	6.4	4029	5.1	1044	1.3
11c	-	-	15701	19.1	10721	13.1	4980	6.1	3840	4.7	1140	1.4

(Dempster, Table 25)

* Body volume equals body weight for an assumed density of 1.0.

skin between the slats; two straight lines corresponding with the location of the slats completed the the contour)

9. Xiphisternum (junction of xiphoid process and body of sternum.)

10. Minimum waist level

11. Uppermost palpable part of iliac crest

12. Top of the pubic symphysis

13. Trunk at the level of the subgluteal crease

14. Left thigh at crotch height (a wooden slat clamped high between the thighs - cf. measure no. 8 - provided a medial contour)

15. Left thigh at uppermost margin of patella

16. Left knee at midpatella level

17. Left upper leg at level of tibial tuberosity

18. Left calf at level of maximum circumference

19. Left ankle at level of minimum circumference

20. Tip of left medial malleolus

21. Contour of the left foot standing (1/2 inch above floor level)

Heights above the floor were measured to 0.1" at each landmark, area contours were drawn in, by a pantograph, and measured, by a planimeter.

The contours for each physique group are shown in Figure 104. The "first-choice" group, for each body build, contains the five strongest somatotypes. Since there were only six rotund subjects, this left only one for the second choice group, and no range of variation is expressed.

Figure 105 shows the plot of body volume for the first choice median subjects. When allowances are made for mean densities the data are comparable to those of Figure 103 for the cadaver.

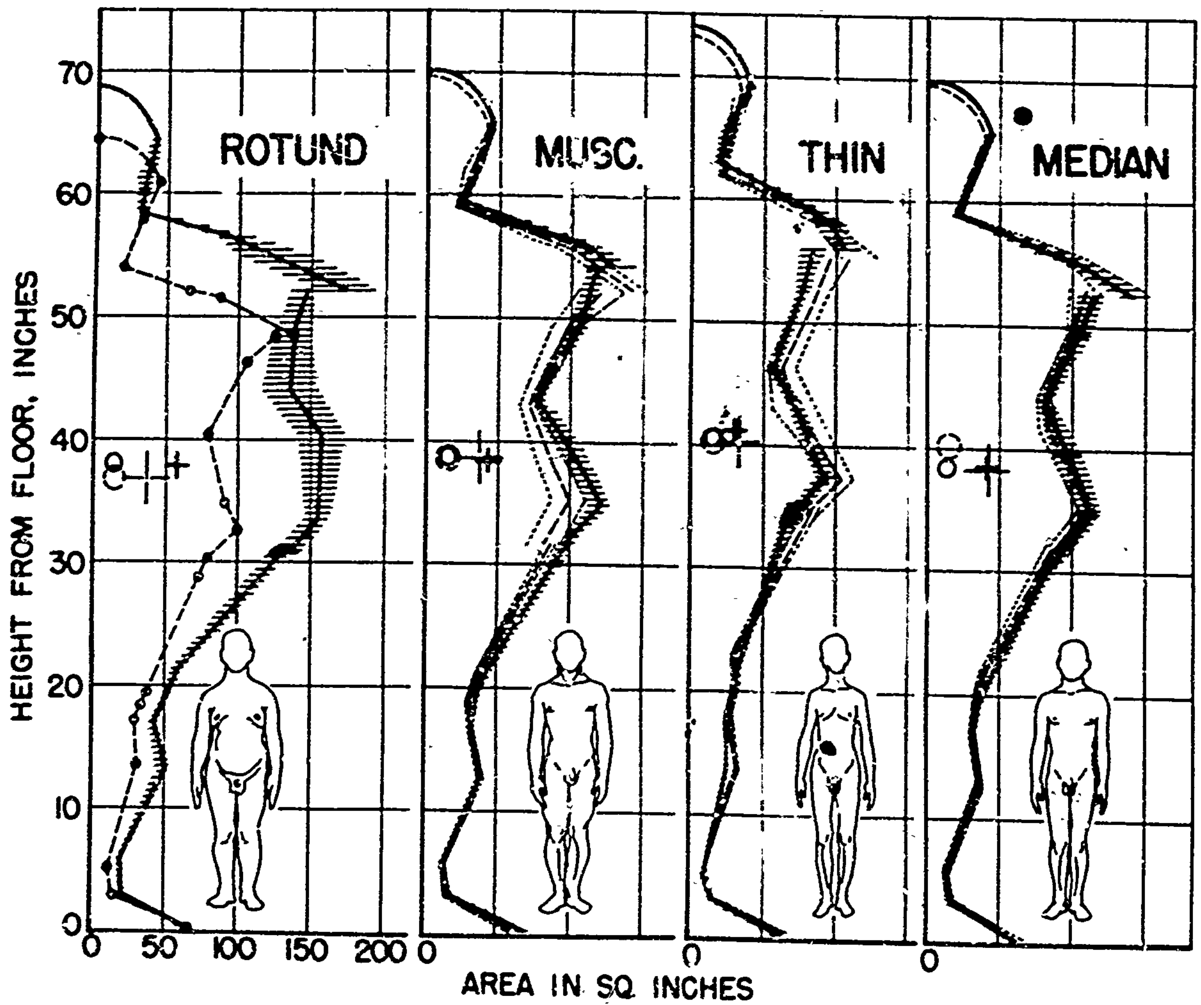


Figure 104. Area-to-Height (or Volume) Contours of the Body Apart from the Upper Limbs for Different Body Types. Heavy outlines show mean cross-sectional areas for the more extreme builds in the sample; shaded halo represents the one-sigma variation in contour. Heavy dashed outlines refer to the mean of the second choice subjects and the one-sigma halo is indicated by light dashed lines. Crosses show the height of the volume centroid (without arms) and the circles show the height of the whole body center of gravity (measured on supine subjects).

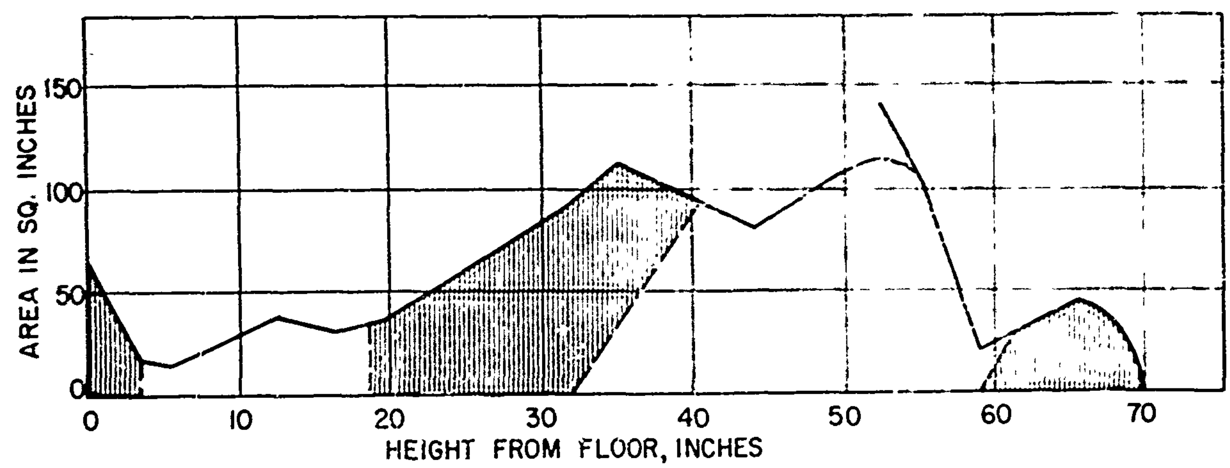


Figure 105. Plot of Body Volume (without upper limbs) of First Choice Median Subjects Expressed as Area-to-Height. Lines separate regions according to external landmarks.

Figure 106 shows a variety of area-height plots. In sketch A, the three phases of respiration are the extremes of inspiration and expiration and a mean position. Note that the whole trunk is involved in forceful breathing. Sketches C, D, E, and F can be taken as a graphical illustration of the effects of body inertia under various accelerations in some horizontal directions. Since the postures show clearly the different distributions of body bulk, different inertia effects should be expected in each instance.

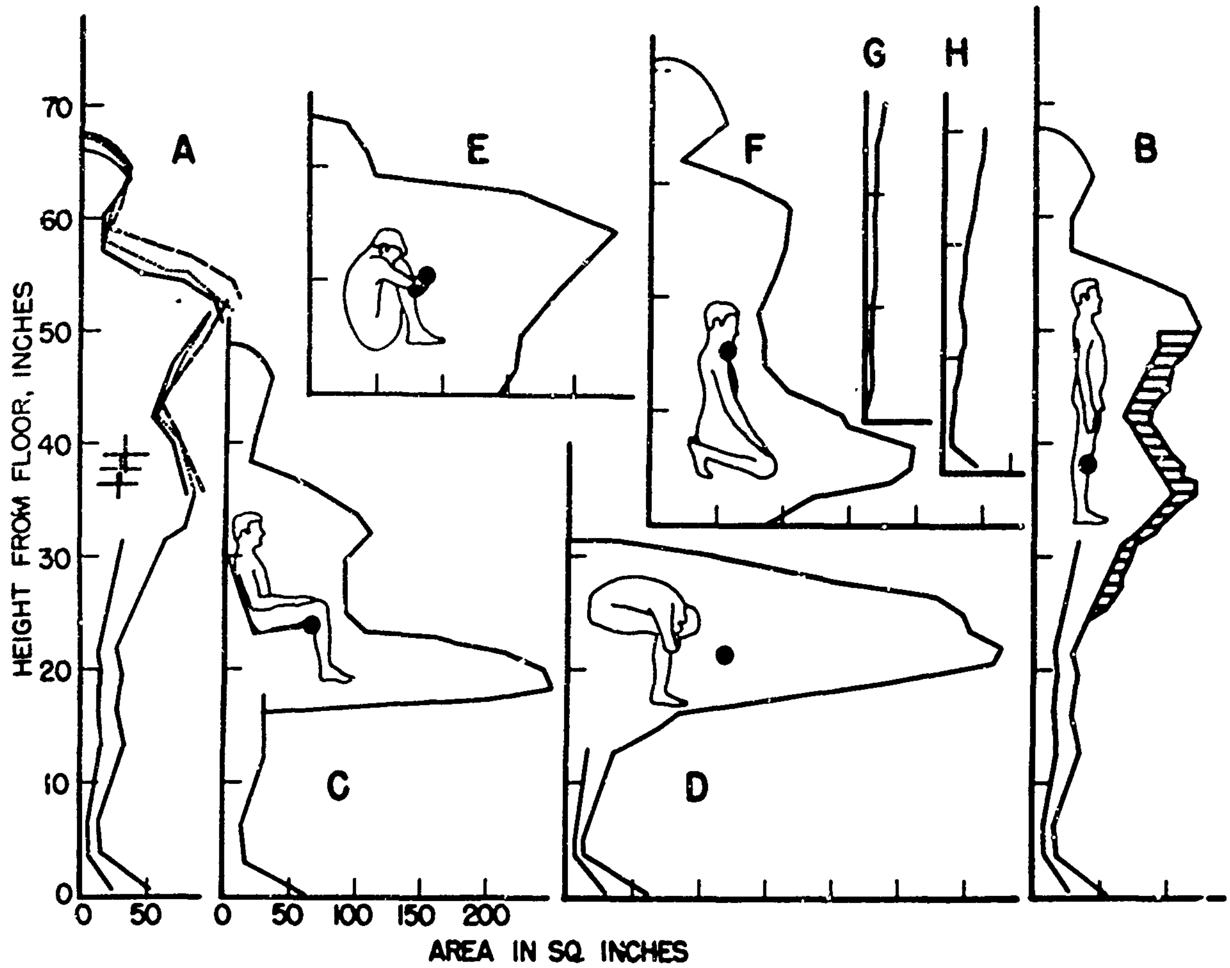


Figure 106. Area-to-Height Plots of One Subject Showing Different Postures. A, phases of respiration; B, standing posture, including upper limbs; C, D, E and F, plots for postures shown; G and H, upper and lower limb values based on water immersion. Crosses in A show centroid positions for inspiration, quiet respiration, and expiratory phases; black dots on other figures are centroid locations.

HOW BODY MASS AFFECTS PUSH AND PULL FORCES
(Dempster, pp. 217-234)

From the preceding chapters, it is clear that body bulk is distributed on the body links in a reasonably predictable way. These segmental masses, continually subjected to the acceleration of gravity, may be visualized as a related system of vertical force vectors. Other forces may be involved, due to muscular tension or to external factors, producing either balanced torques or movements along paths consistent with the kinematics of the linkages involved.

For any static posture, the several masses are balanced in the sense that the torque involving both mass and leverage distances to the centers of gravity of the individual parts are in equilibrium. A true balance over a period of time is only approximated and involves irregular volleys of muscular activity and the fluctuation of forces. Though this maintenance of a posture involves dynamic muscular behavior, instantaneous posture recorded photographically may be analyzed statically.

In this chapter data are presented on how the body mass, exerting a vertical component, is utilized by the seated subject to effect horizontal push and pull forces in sagittal planes.

The effect of body inertia and body dead weight, forming an anchorage and defining the actual maximum produced by the body, may be improved upon by designers who understand the principles involved in the operation of dead weight in addition to the advantages and disadvantages of effective body supports and leverages.

All horizontal and vertical forces within the plane of action were recorded from a hand dynamometer and six force gauges at the seat. Side view photographs were taken to freeze body positions.

Figures 75 and 76 show the postures studied. Constant heights above seat level were used and the postures selected to represent different patterns of body mass distribution. The vertical line of action of the center of gravity is shown by the arrow above the seat in each posture. Each position is based upon the site of the center of gravity and may be summarized as follows:

Posture	Center of Gravity	Description
a	most backward	subject leans back passively holding hand grip
f	almost identical to a	Buttocks and feet in contact with seat; strained
e	forward of a	trunk tensed; arms and knees flexed
d	little forward of a	trunk tensed and strongly inclined backward; head back, knees sharply bent
c	3 cm forward of d	subject upright in strained position; legs flexed
b	most forward	subject as far forward as possible for positive values.

Posture differences between push and pull positions are due mainly to tensed leg and trunk positions.

At each center of gravity vertically downward force vectors with magnitudes equal to segment mass were assumed to operate. Additional data from photographs on the horizontal distances of successive C.G.'s from some arbitrary point permitted the calculation of a common center of gravity, about which moments of the segment masses were in equilibrium. Centers of gravity calculated in this manner were in no instances more than one cm. from the C.G.'s derived by balancing.

In the actual push-pull situation, the reaction vector to body weight (called effective seat contact) is not in the same vertical line as the whole body center of gravity; it was calculated as a distance from the rear pressure gauge. This effective seat contact was shifted by the subject, *via* shifts in muscle tension, to produce a maximum push (contact moved to the rear) or pull (contact moved forward).

The subject's mass, operating vertically downward at the C. G., and the equal and opposite vector at the effective seat contact form a couple; these opposed coplanar forces, separated by a measureable perpendicular distance, act upon a rigid body to cause rotation in their plane of action. This is diagrammed in Figure 107 for pull position d.

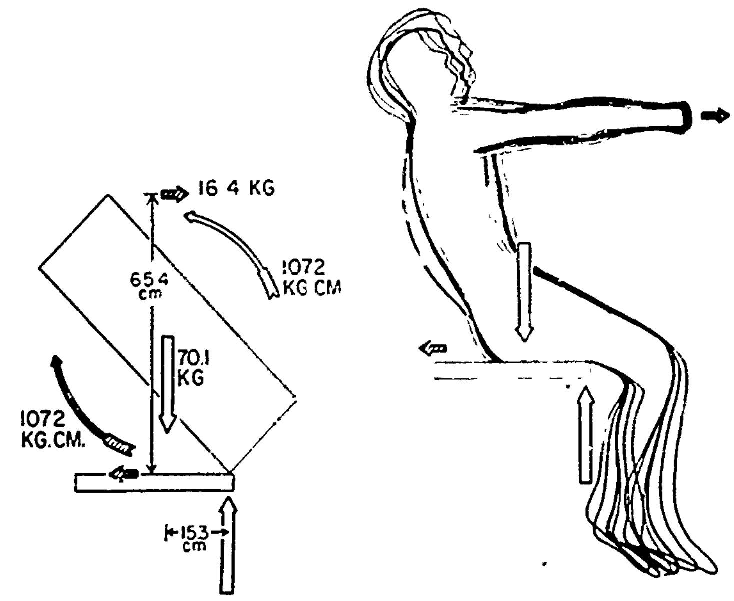
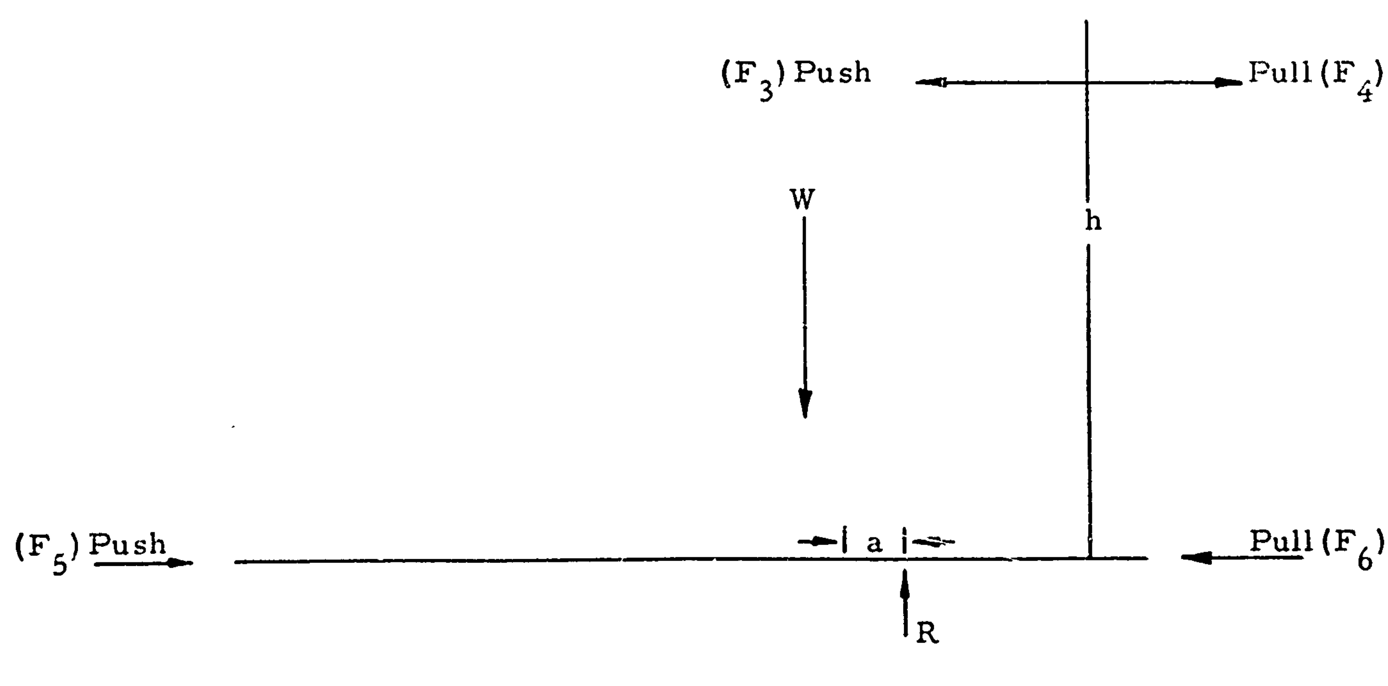


Figure 107. Superimposed Tracings (right) showing Postures at Instant of Ten Maximum Horizontal Pulls. Analysis in terms of vertical force Vectors (white arrows) and horizontal force vectors (shaded arrows). On the left is seen an analogy involving vertical couples with forces and lever distances indicated.

With the subject pulling backward, the dynamometer recorded an equal contrary <u>horizontal</u> force acting at the shoulder level; this formed a couple with a force recorded at the seat; see Figure 107. The tendency here is for rotation in a clockwise direction. The forces of the last two paragraphs are diagrammed below:



The tendency to produce rotation is called moment of rotation, or torque, and is obtained by multiplying the magnitude of one of the forces by the perpendicular distance between them. Thus the force recorded at the dynamometer (F_4) or its reciprocal at the seat (F_6), multiplied by the distance h will be the moment of rotation produced by the horizontal couple. For a push force the value is either (F_3)h or (F_5)h. Likewise for the moment produced by the vertical couple, multiply W (or R) by a.

Any body action which lengthens the moment arm of the horizontal couple for a constant force, will increase the torque produced. Therefore, the straining and adjustment of the effective seat contact by the subject, increasing the moment produced by the vertical couple, is associated with a simultaneous increase in the pull at a fixed height above the seat. This is necessary in the maintenance of a balanced system. Since, in this study, the distance between the horizontal forces was experimentally fixed, it became necessary for the subject to increase the magnitude of the horizontal push or pull force due to an increase in the vertical moment through muscular action.

This was verified by measuring the moment arms of the vertical couples in different postures. When the horizontal force was low, as in posture a, the moment arm was correspondingly short; in addition, the horizontal force was also low. The relationship between the increase in force and in vertical moment arm was found to be a straight line.

A further analysis was done utilizing a foot rest. At maximum forces, the buttocks were raised from the seat and all body weight applied at the foot rest. This increased the moment arm of the vertical couple. However, the horizontal force that could be accommodated did not increase directly with the increase in the vertical moment arm, as the location of the foot rest below seat level also increased the moment arm of the horizontal couple. For more effective use of dead weight, the foot rest should be located close to the level of the hand pull.

Forces were also found to be greater when a back rest was utilized. The same body moment torques existed, but the back rest supplied a counterforce against which muscles could operate to increase the magnitude of the horizontal pushes.

One handed sagittal pulls were also studied, using a dynamometer. In addition to the midsagittal position, pulls were made in the following planes: 6, 12, 18, and 24" to the test limb side, and 6" beyond the subject's plane of symmetry. The dynamometer is free to rotate about its attachment point, allowing the subject to select the ber direction; likewise, the hand grip also rotated about the axis of the pull. The subject could then select both grip orientation and direction of pull, as long as they were directed in one of the sagittal planes.

Although complete results were available on only one subject, less complete records on two others in no way contradicted the findings. It was found that downward and forward pulls were the strongest, utilizing the subject's dead weight. As far as the planes were concerned, pulls were strongest in the midsagittal plane, and 6" to the right or left, decreasing further out.

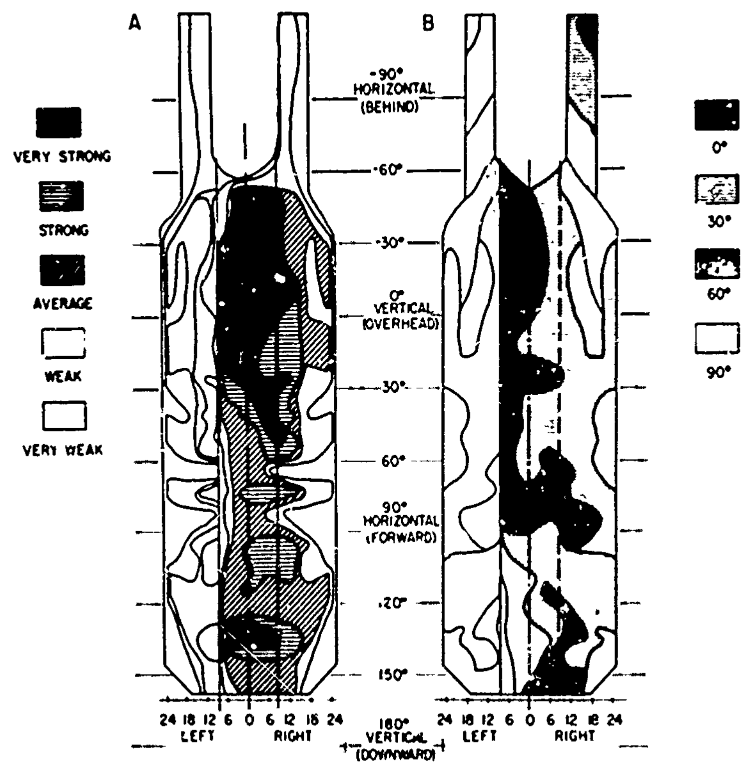


Figure 108. Plots of Hand Force Magnitude and Grip Orientations for Different Regions of the Work Space for the Seated Subject. A, magnitudes of sagittal hand pull; B, hand grip orientation. Both are plotted against angle of pull and distance from the mid-line of the seat.

Results are shown in Figure 108, and summarized as follows:

1) highest values are overhead, utilizing dead weight;

2) upward and backward pulls at angulations 50°-60° below the horizontal are also quite high;

3) pulls more than 12" from the midsagittal plane are variable, but generally low.

Muscular effort is ordinarily a limiting factor only when bracing, body support, and dead weight utilization are maximally effective. Where body leverage and general stability is poor, they are usually limiting factors rather than muscles.

For studying preferred hand grip orientations, four classes were set up:

1) parallel to sagittal plane ($\pm 15^{\circ}$); thumb counterclockwise;

2) perpendicular to sagittal plane ($\pm 15^{\circ}$); thumb medially;

3) 30° ($\pm 15^{\circ}$)

4) 60° ($\pm 15^{\circ}$)

Right handed fore-aft pulls have a most common orientation of 30°, within 12" right, 6" left of the midsagittal plane. At 24" lateral to the midsagittal plane, oblique pulls favor a 90° orientation. Forward pulls from behind the subject tend to have 45° (sometimes flatter) orientations. In general, 30° angulations predominate below the horizontal though there is variation.

CONCLUSION - ASPECTS OF PRACTICAL CONCERN
(Dempster, pp. 235-241)

Kinematic and mechanical information on the body system supplies form and defines limits to the overlying phsiological and psychological knowledge. Pursued far enough, it contributes useful data to the field of human engineering. This study has considered the seated subject oriented in the direction he faces, and has determined a range of space necessary to include specific classes of hand and foot movements. Space envelopes presented are maximal because the hampering effects of clothing have not been considered.

In summarizing the data relative to practical problems, eight points need to be considered.

1) The overall size and shap of the work space for all possible hand orientations. This can be seen in Figure 100, while Figure 101A shows additional tolerance to include the 95th percentile.

2) The relative volumes available for various hand and foot orientations are given in Figures 92 and 98, along with transverse areas in relation to R.

3) The regions of use for special orientations of the hand. This becomes important where the space is not available for unrestricted hand or foot movement. This information is given in Figures 96, 97, and 99.

4) Common regions where combinations of the eight orientations may be used can also be seen in the figures of 3) above.

5) Regions of right-left overlap are also shown in these figures; it is pertinent to know that the region of overlap varies considerably with the hand orientation.

6) Kinetosphere centroids are important in defining mean positions from which the end member may move in any direction to the kinetosphere limits. In addition, the centroids pin down the locus of the longest straight mean path of end member motion; these paths within the kinetosphere will, on the average, be longer if they pass through the centroid.

7) Limb segment weight and associated rest positions. The rest positions have not been studied here and specifications cannot be made relative to R. However, these positions require less energy than active phases; if primary controls on which the hand may rest passively are placed at or below centroid levels and nearer to the body, they will supply support for the upper limb and obviate tiring. The hands will also be in a ready position for properly placed secondary controls.

8) Preferred orientations of the hand grip are summarized in Figure 108, suggesting correct angling of hand controls for optimum efficiency. The observed relationship of finger postures suggests that finger control orientation may be as important as the placement and orientation of hand grip controls.

In all cases, the emphasis has been on underlying principles pertinent to work space designs. Much work remains to be done for specific operational situations.

Further studies are suggested not only along the lines of this investigation but in four related areas.

(1) There should be an exhaustive, well-planned study on the total range of movement at each body joint for well-defined population groups--male, female, special military groups, etc. This should be correlated with standard anthropometric data so that we may learn the relative importance to the individual of body dimensions versus the range of joint movement.

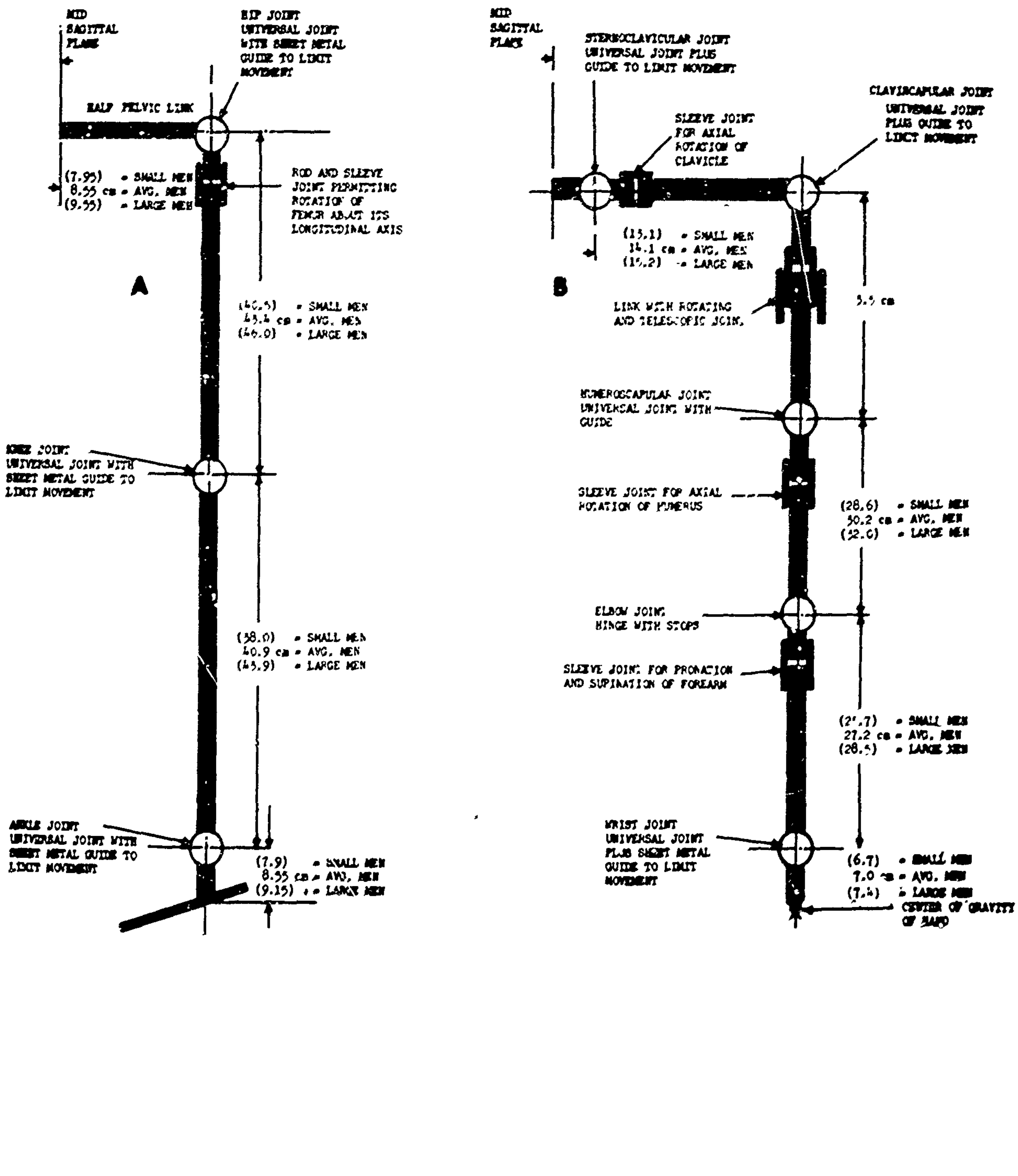
(2) There should be studies involving the kinematics and spatial requirements of specific types of work operations. Our studies have dealt with overall requirements, but how are the total body operations of desk workers, drivers of land vehicles, pilots, machine-operators, etc., effected, how do they differ, and how much space does each require? It should be important to explore different methods of getting this information so that new operations may be analyzed later to supplement basic data on key operations.

(3) More should be known about hand and foot forces and body reactions under a large variety of operative conditions. Body stability, bracing, leverages, the action of couples and the importance of dead weight should be correlated and a number of body positions should be compared. A search for the principles which underly all possible conditions should be sought. Force duration as well as maxima should be studied. The preferred orientations and posturing of parts and of the facilitating joints should be understood.

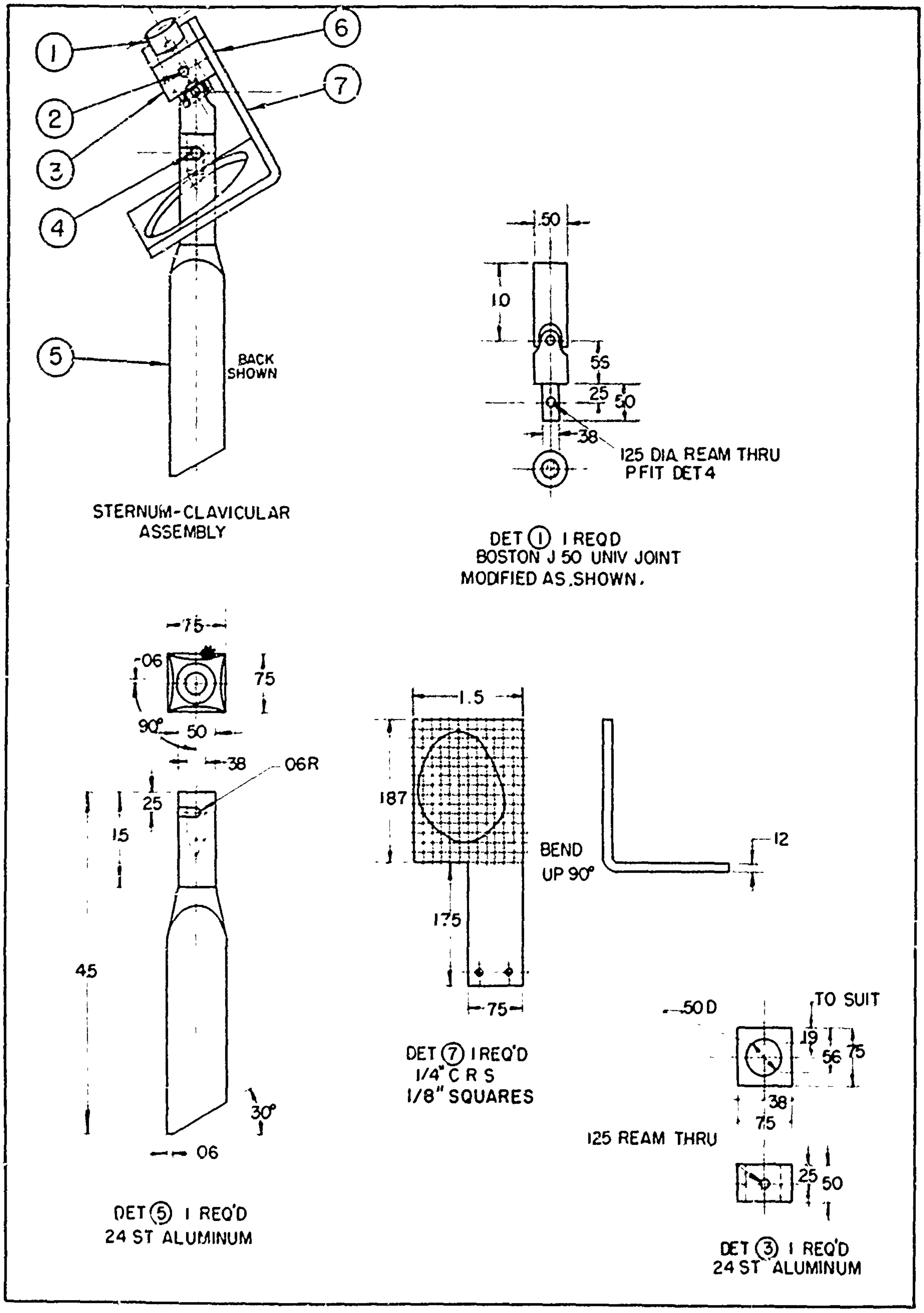
(4) Knowledge on the contribution of different body muscles to purposeful, static and dynamic activities is necessary. Muscle torques and leverages should be studied, and the cooperative actions at different joints should be known. The speed and duration of actions and fatigue relations are also pertinent.

APPENDIX I

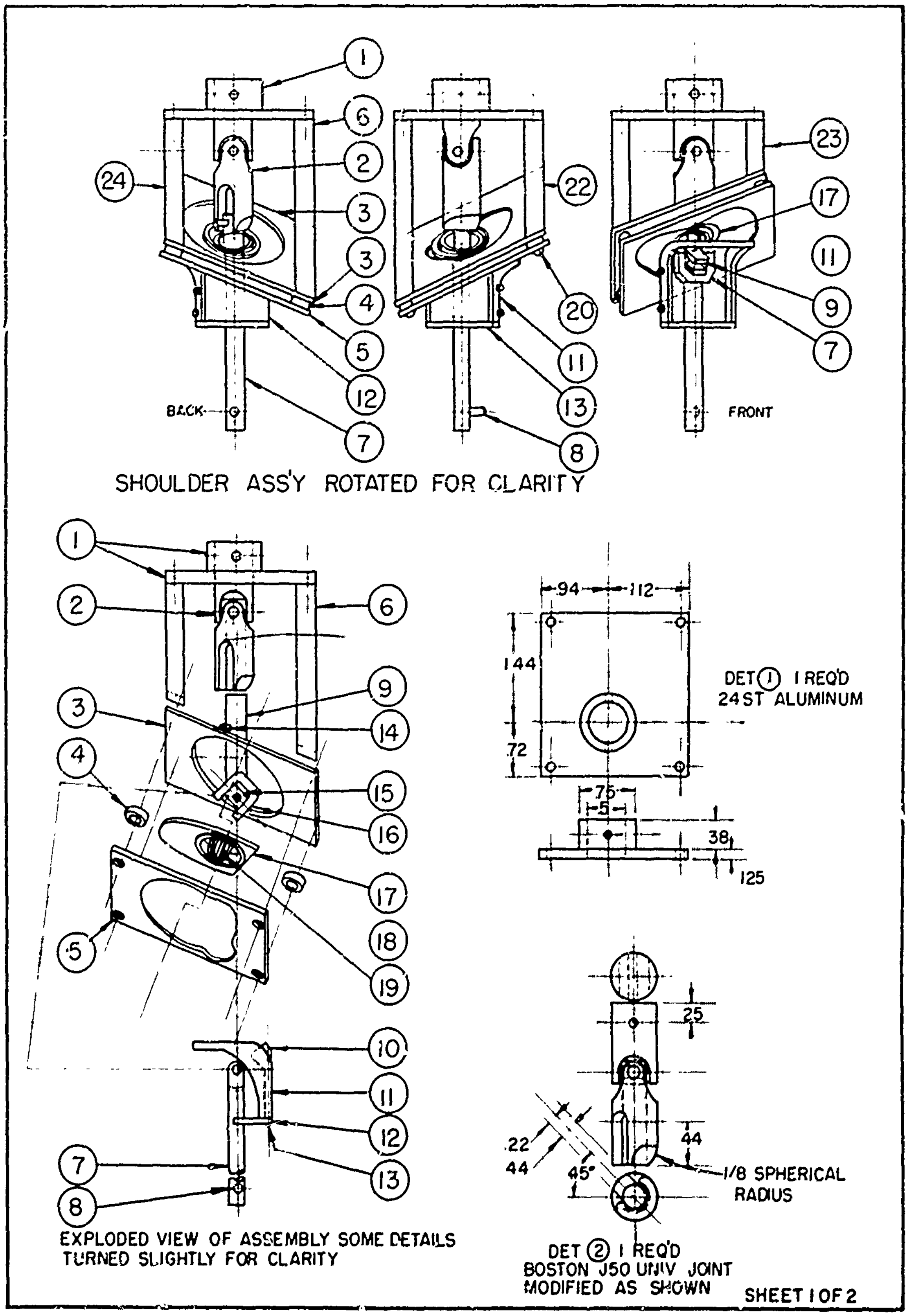
PLANS FOR THREE - DIMENSIONAL MANIKIN



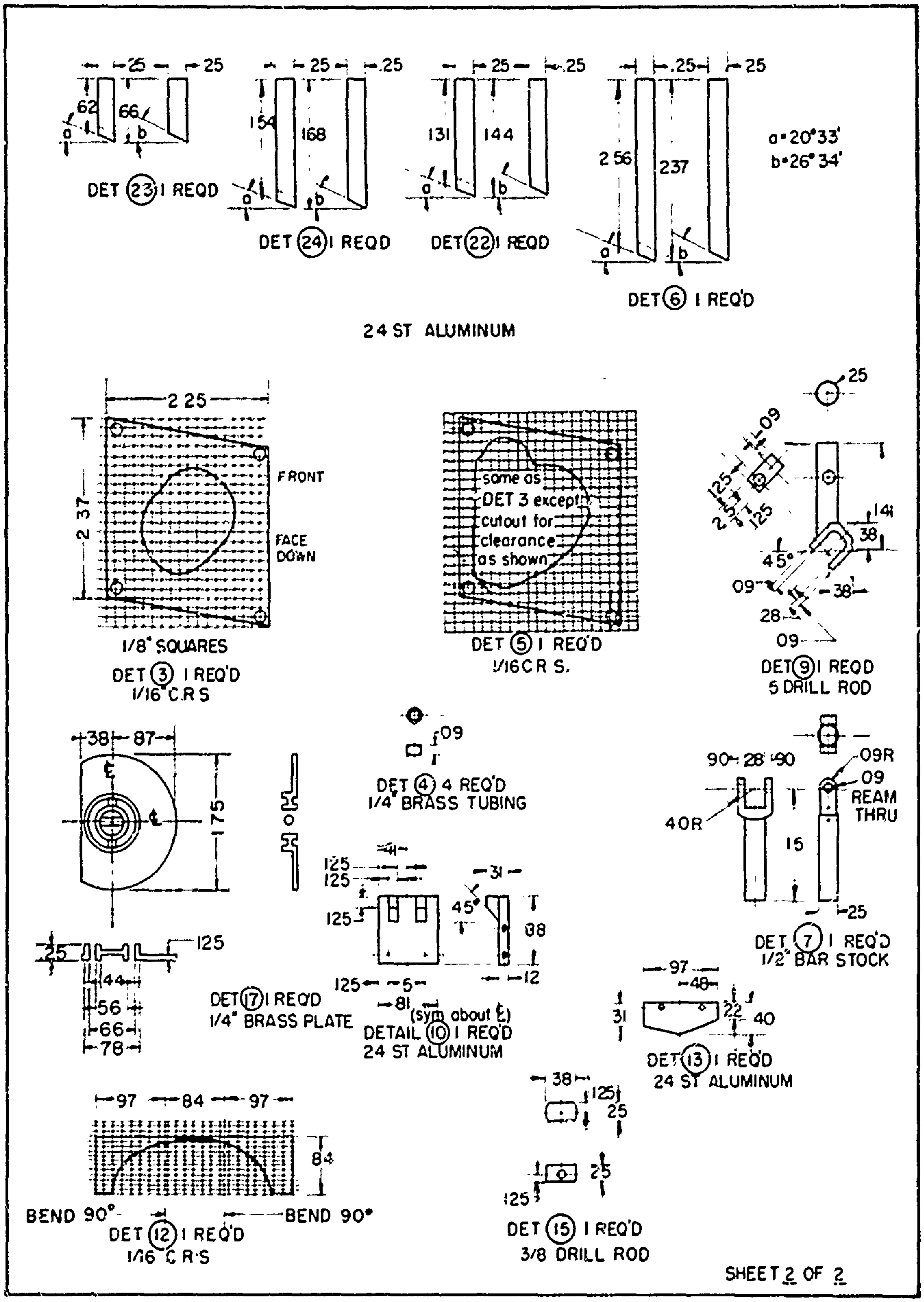
General plan of linkage assembly for the lower (A) and upper (B) Limb Systems.



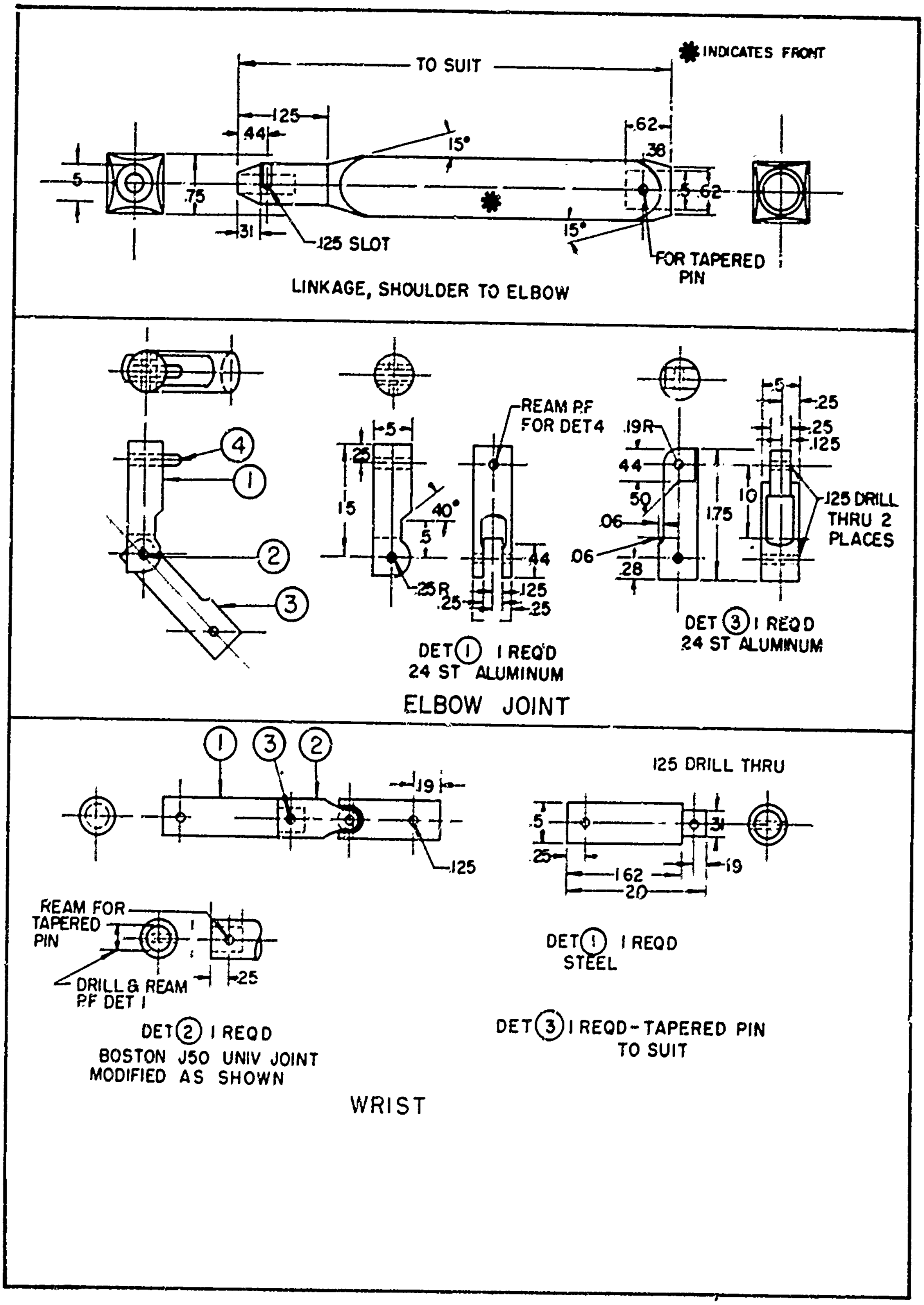
Plan for the sternoclavicular joint.



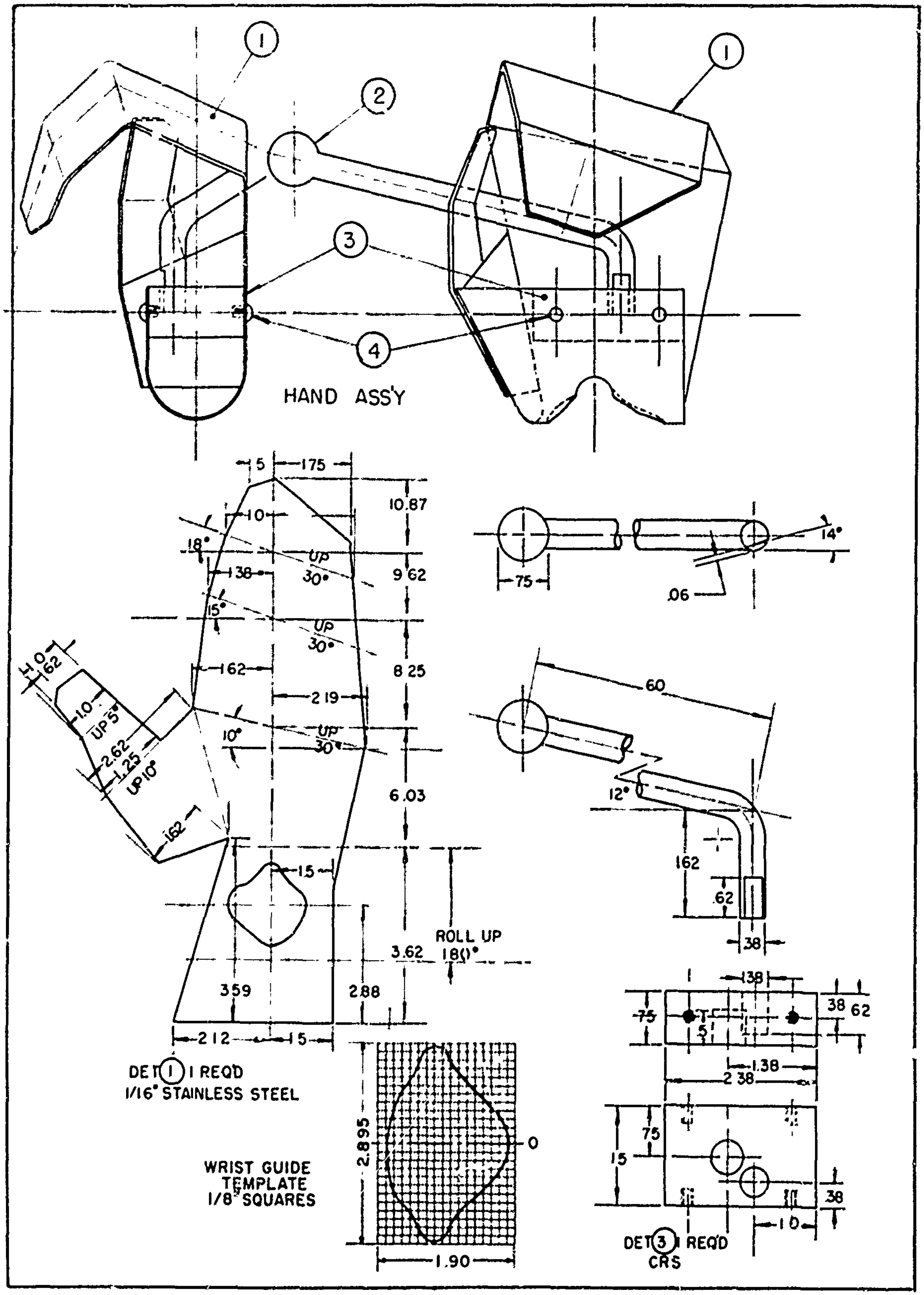
Plan for the combined shoulder joints.



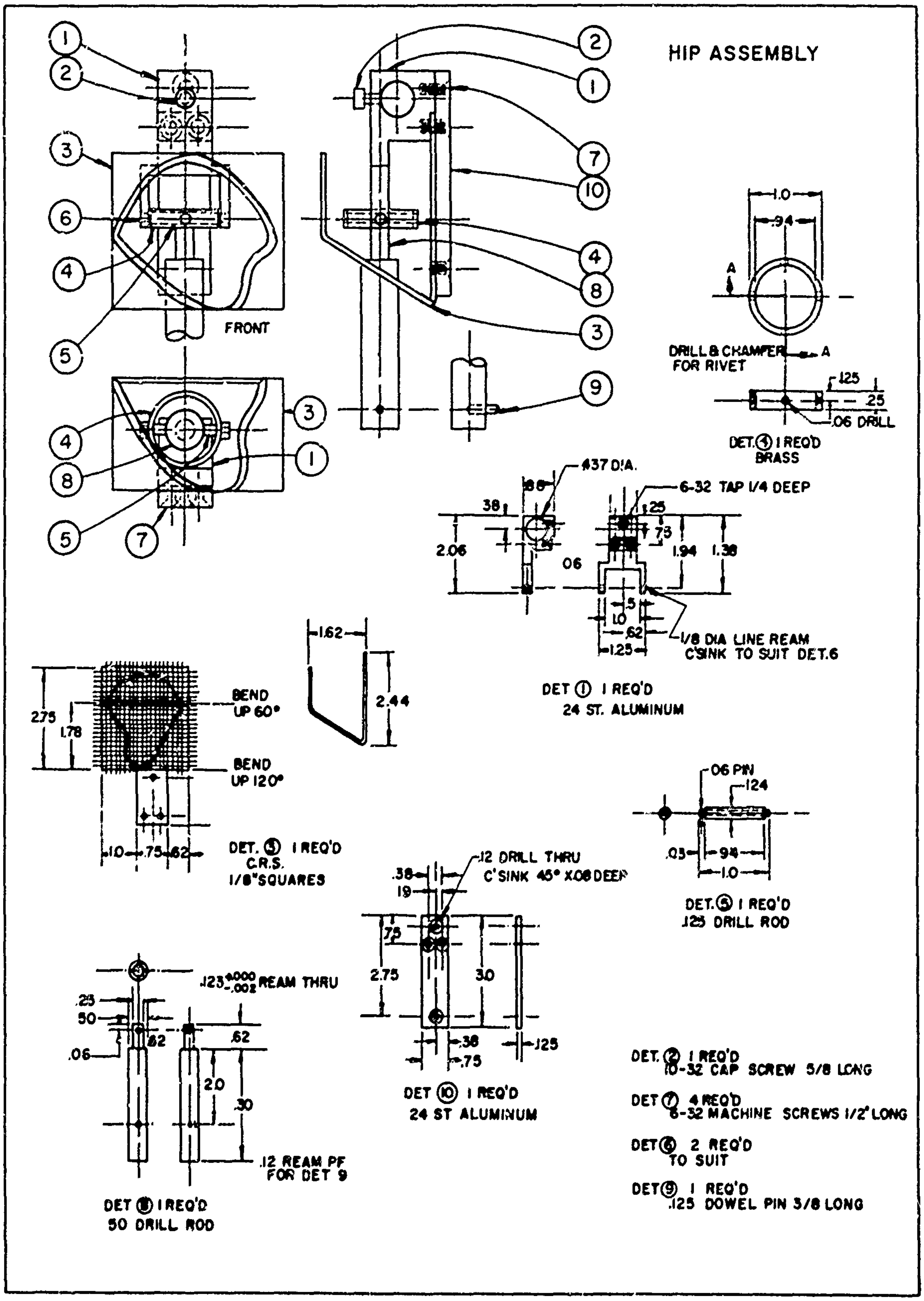
Details of plan for shoulder joint.



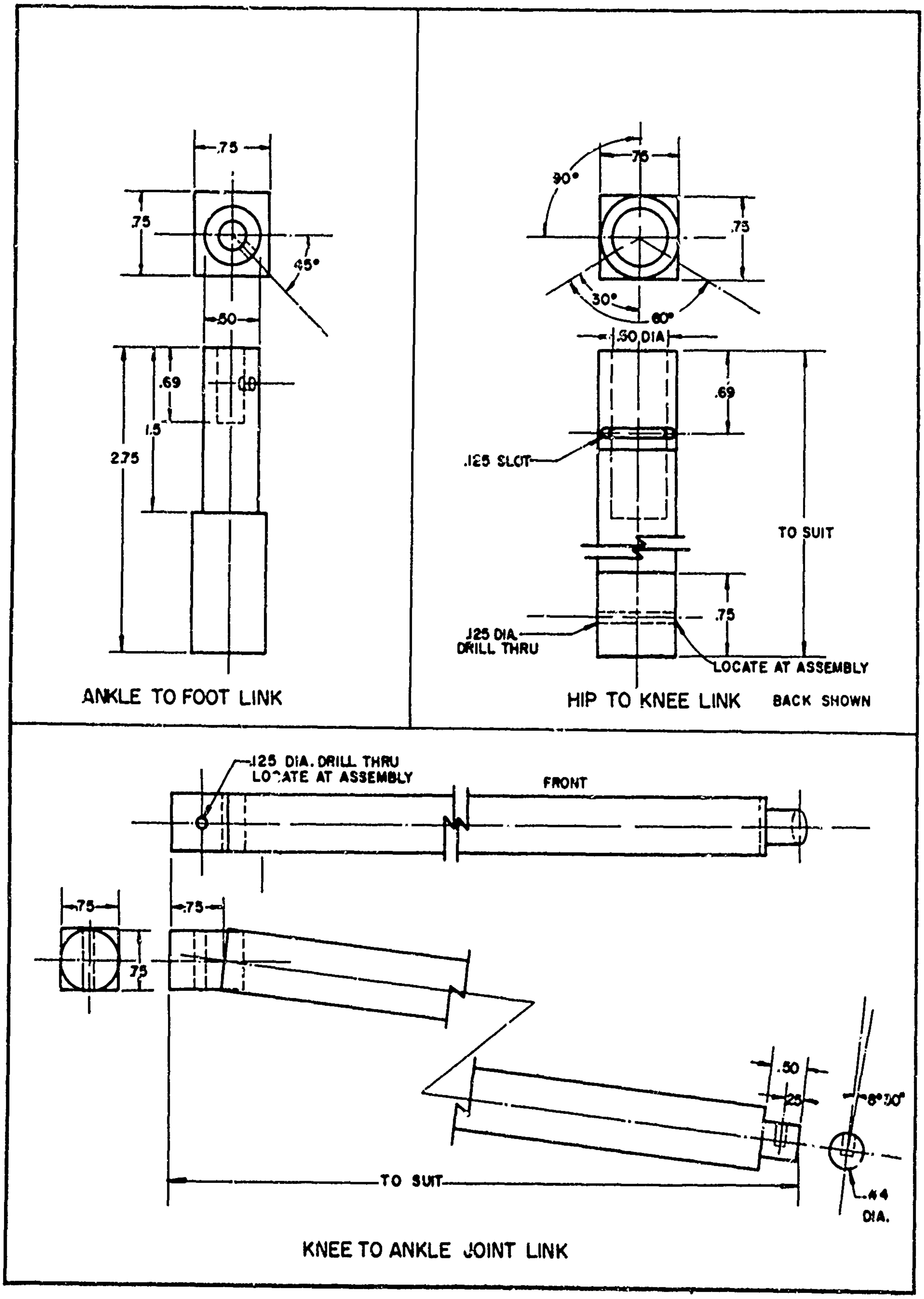
Plans for arm link and for elbow, forearm, and wrist joints.



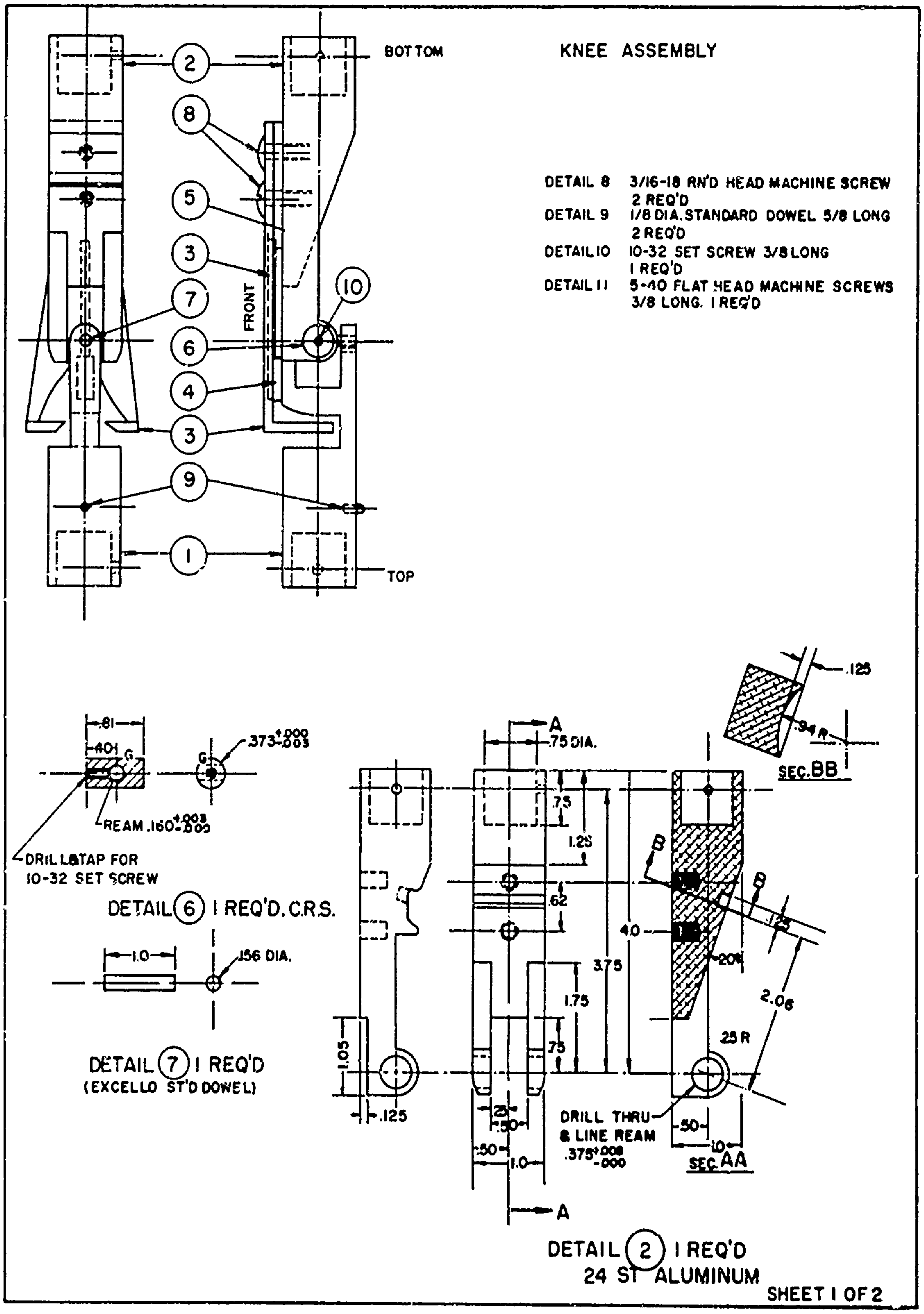
Details of plans for hand and wrist joint assemblies.



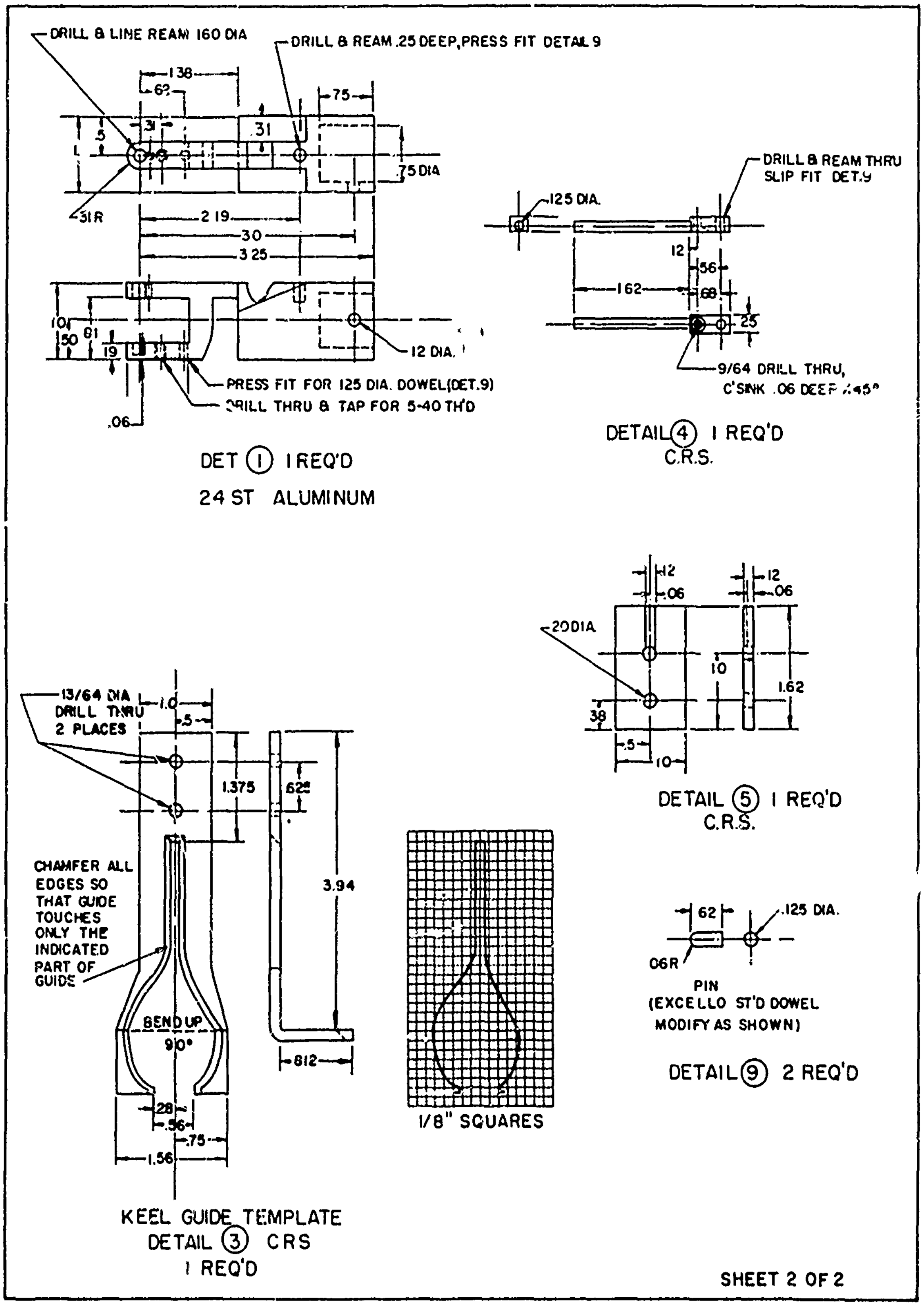
Plan for hip joint.



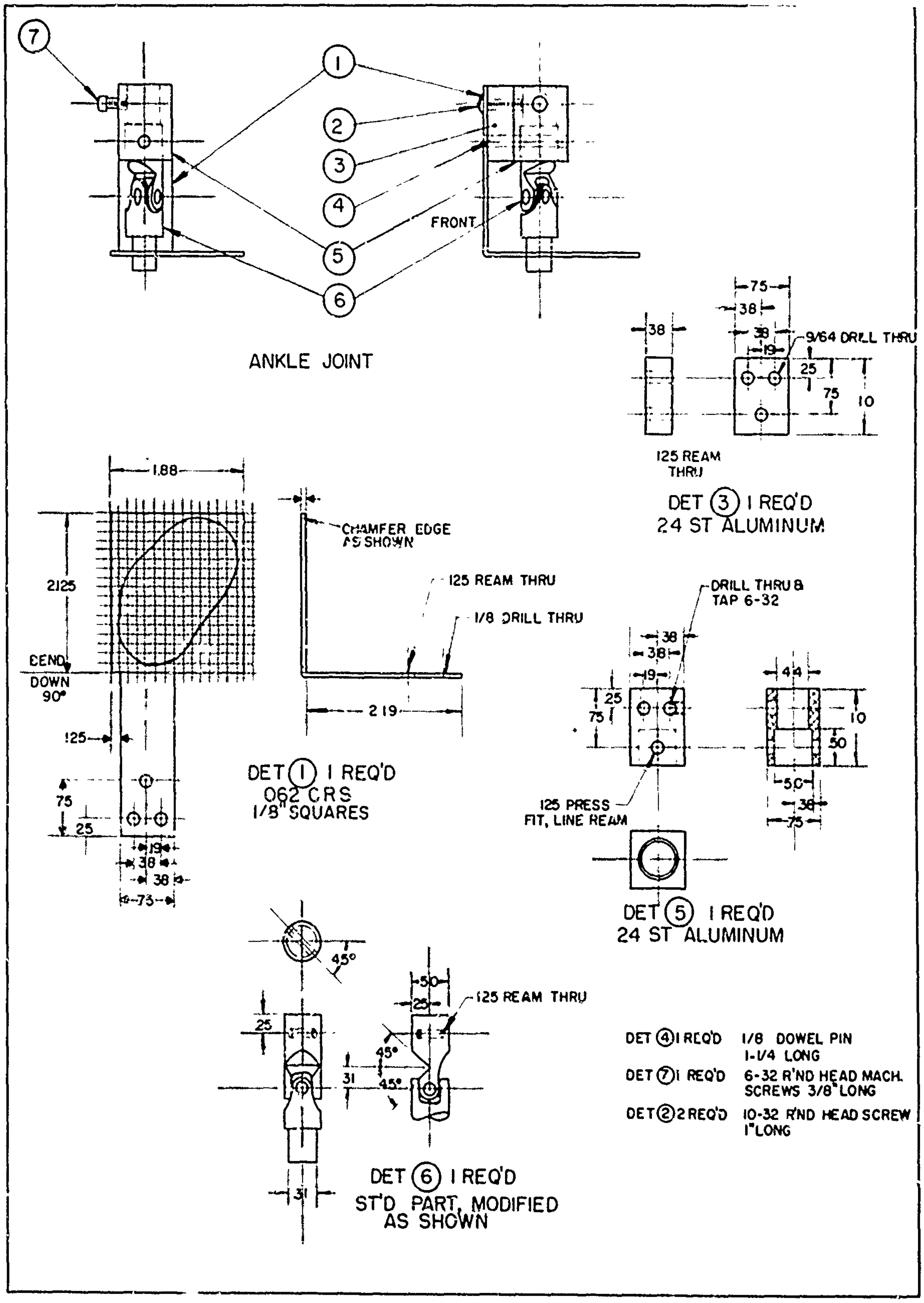
Plan for lower limb links.



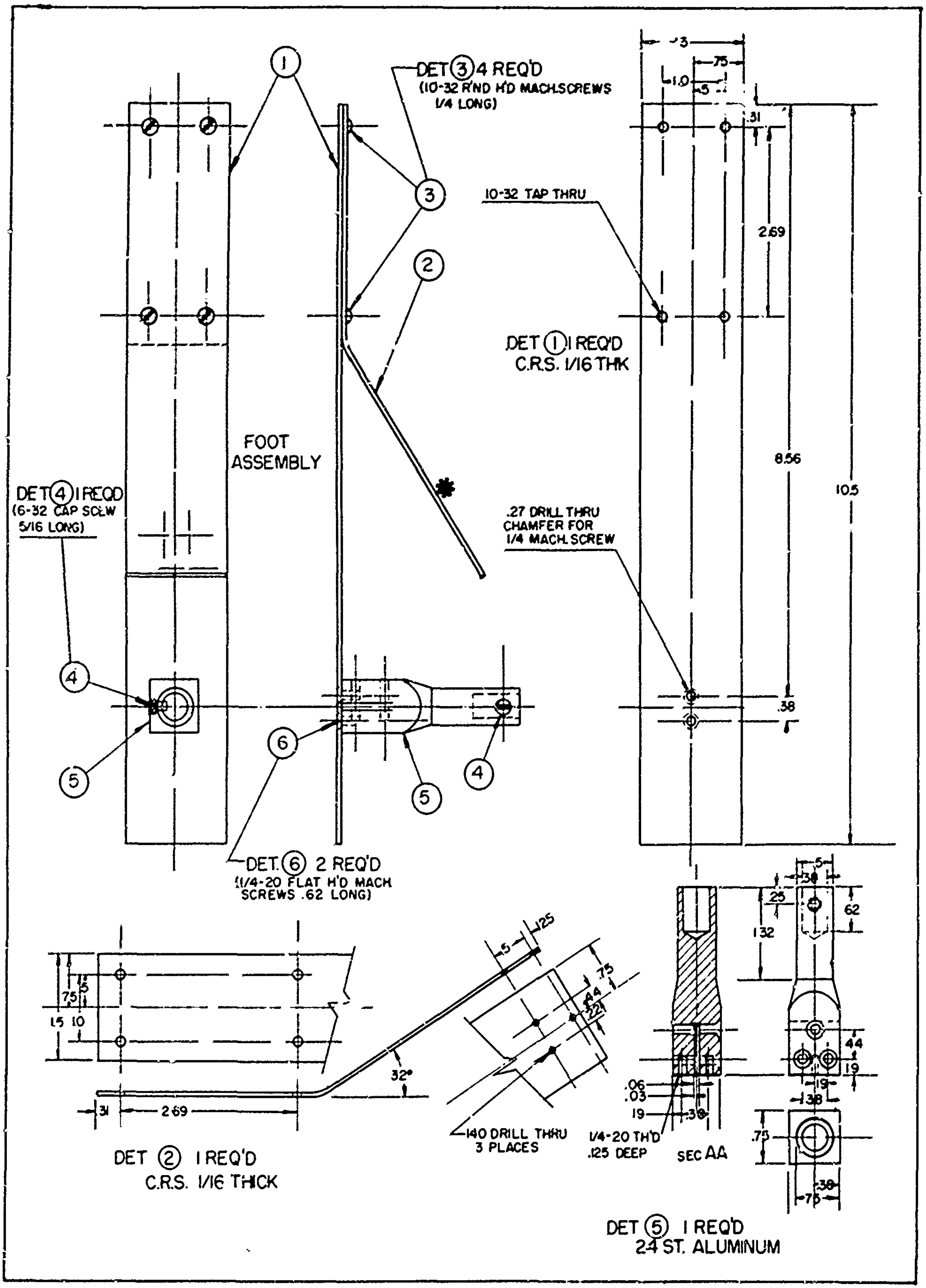
Plan for knee joint.



Details of plan for knee joint.



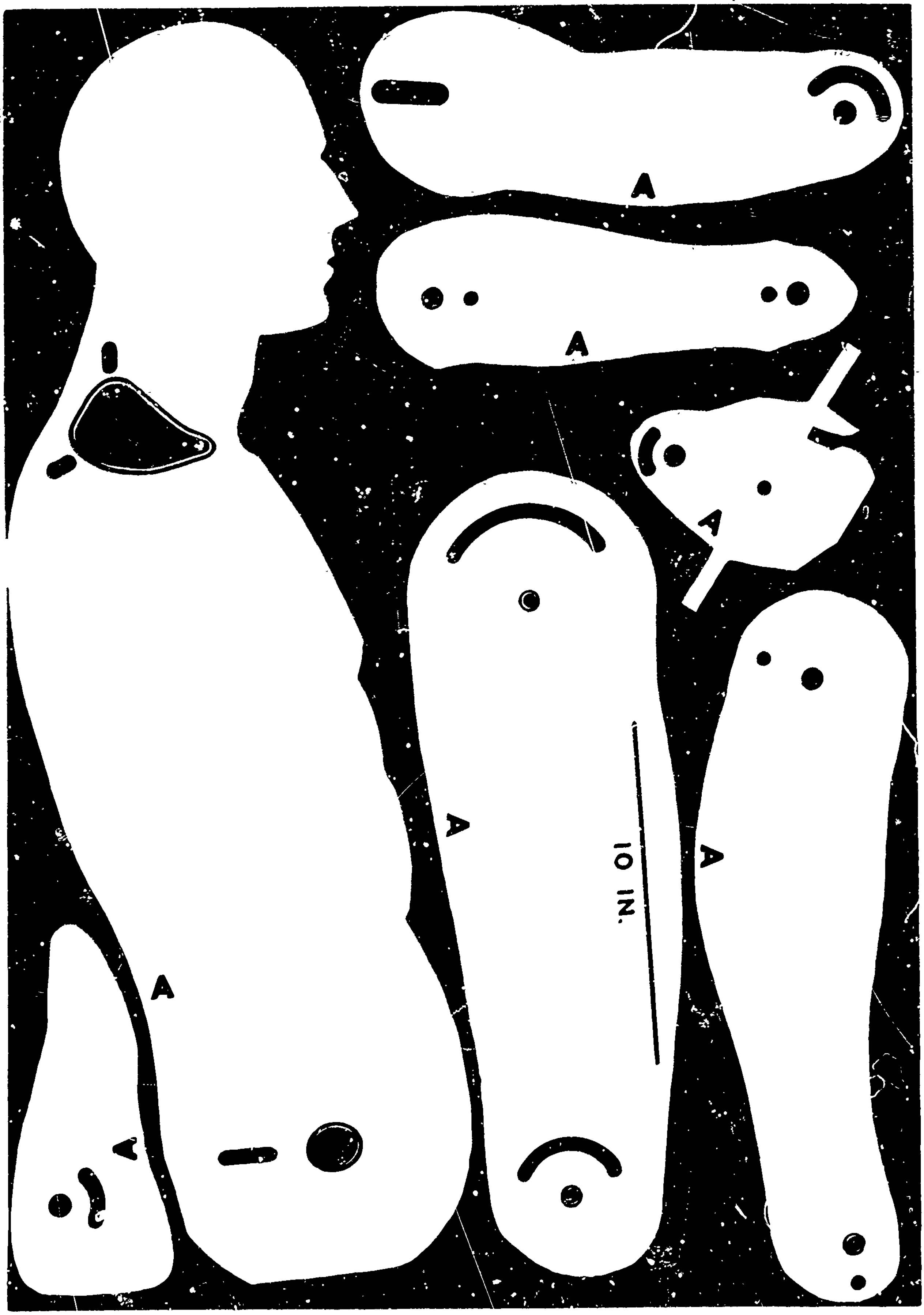
Plan for ankle joint.



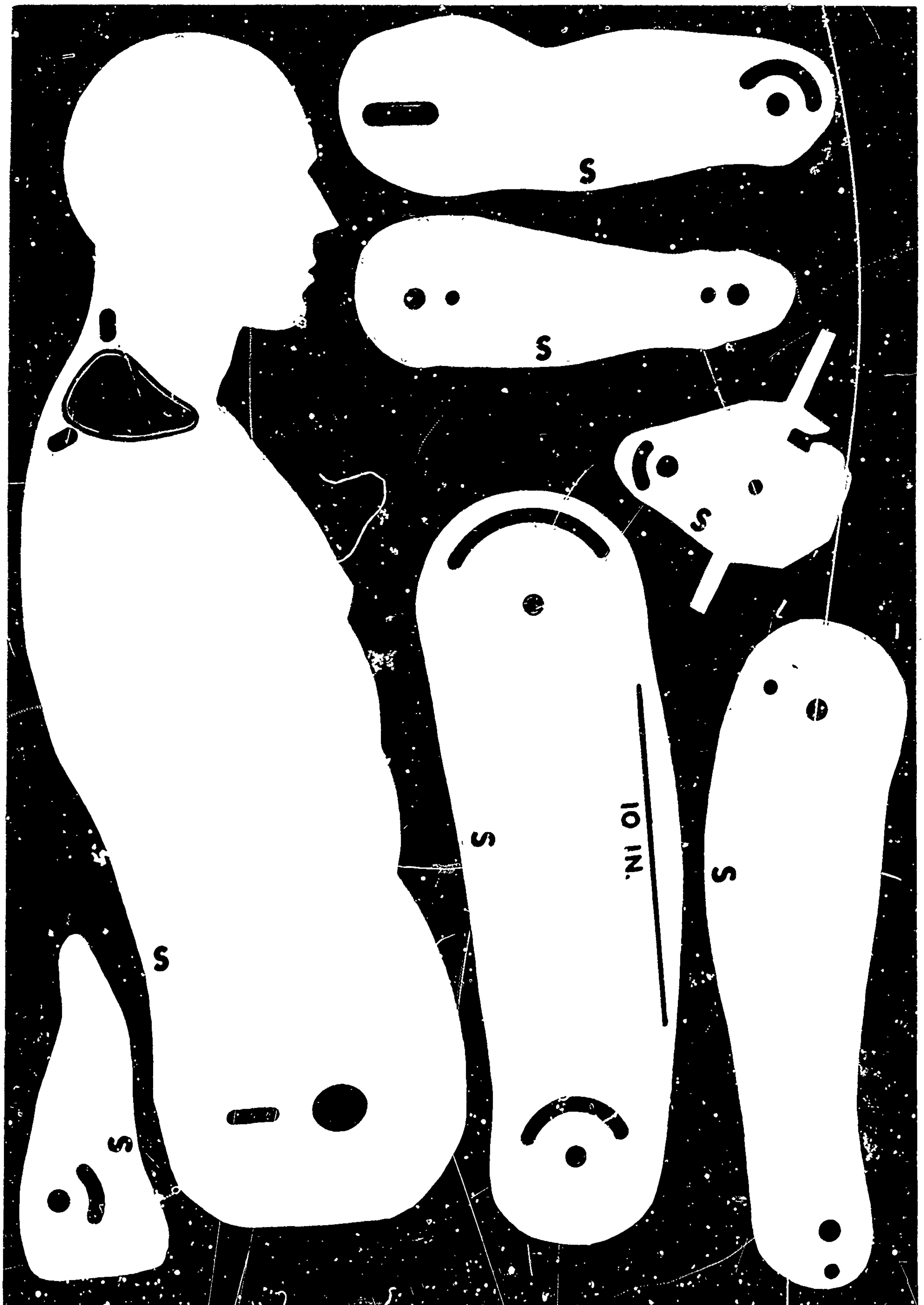
Plan for foot assembly and ankle joint.

APPENDIX II

PLANS FOR DRAFTING BOARD MANIKIN

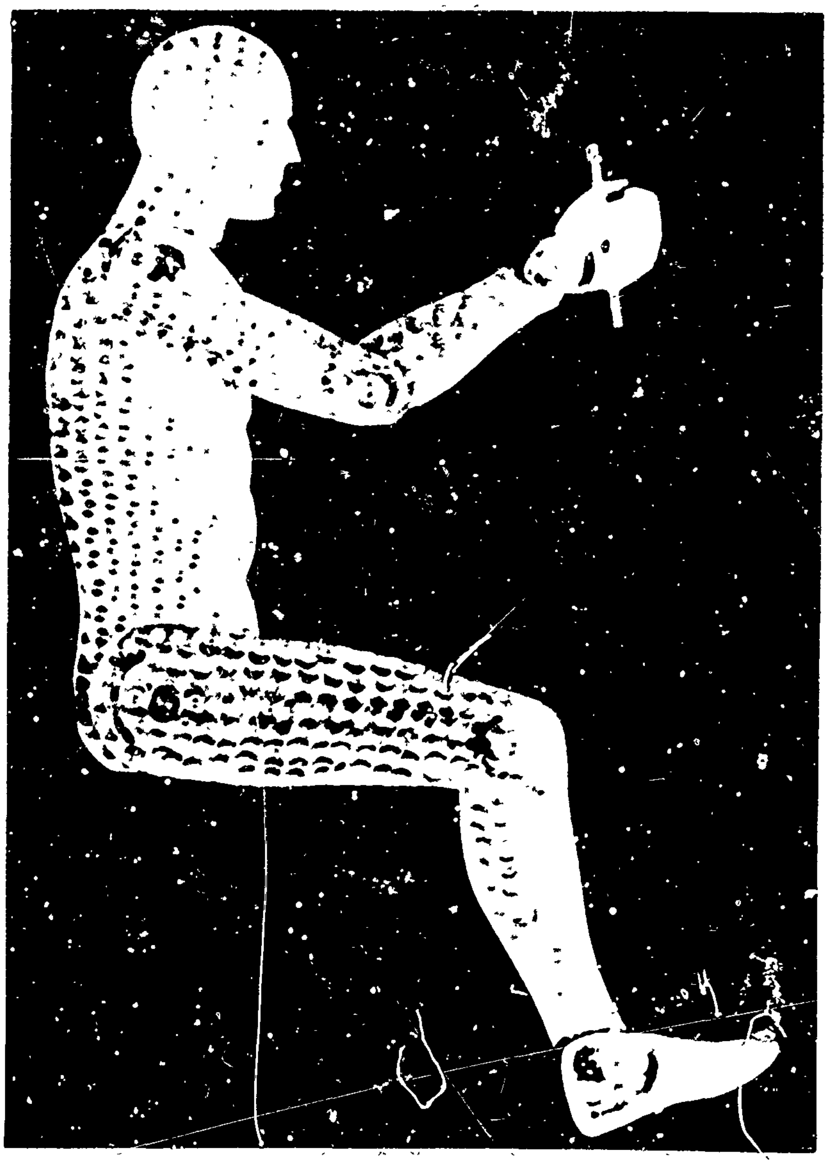


Pattern of body segments for drafting board manikin of average build. Note line on thigh segment which indicates scale.



Segments for small - sized manikin.

Segments for large - sized manikin.



Metai model of drawing board manikin for average build of Air Force flying personnel.

BIBLIOGRAPHY

1. Albert, E. 1876. Zur Mechanik des Hüftgelenkes. Med. Jahrb. k. k. Gesellsch. d. Arzte (Wien). pp. 107-129.

2. Alderson Research Laboratories. 1954. Anthropomorphic Test Dummy—Mark III. Alderson Res. Lab., Inc., New York. 11 pp.

3. Anglesworth, R. D., ed. 1952. Manual of Upper Extremity Prosthetics. Artificial Limbs Research Project, University of California. 240 pp. (unnumbered).

4. Barnes, R. M. 1949. Motion and Time Study. Third Edition. John Wiley and Sons, New York. 559 pp.

5. Barnett, C. H. 1953. Locking at the Knee Joint. J. Anat. Volume 87. pp. 91-95.

6. Barnett, C. H. and Napier, J. R. 1952. The Axis of Rotation of the Ankle Joint in Man. Its Influence upon the Form of the Talus and the Mobility of the Fibula. J. Anat. Volume 86 (1). pp. 1-9.

7. Bedford, T. and Warner, C. G. 1937. Strength Test: Observations on the Effects of Posture on Strength of Pull. Lancet. Volume 2. pp. 1328-1329.

8. Bennett, N. G. 1908. A Contribution to the Study of the Movements of the Mandible. Proc. Roy. Soc. Med. (Odont. Sect.). Volume 1 (3). pp. 79-98.

9. Bernstein, Nik. 1935. Untersuchungen über die Biodynamik der Lokomotion. Bd. 1. Biodynamik des Ganges des normalen erwachsenen Mannes. Institute for Exper. Med. d. Soviet Union, Moscow. 243 pp. (In Russian; German summary.)

10. Bowles, G. T. 1932. New Types of Old Americans at Harvard and at Eastern Women's Colleges. Harvard University Press, Cambridge. 144 pp.

11. Boyd, E. 1933. The Specific Gravity of the Human Body. Human Biol. Volume 5, pp. 646-672.

12. Brash, J. C., ed. 1951. Cunningham's Textbook of Anatomy. Ninth Edition. Oxford, New York. 1604 pp.

13. Braune, W. and Fischer, O. 1885. Die bei der Untersuchung von Gelenkbewegungen anzuwendende Methode, erlautert am Gelenkmechanismus des Vorderarms beim Menschen. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 13 (3). pp. 315-336.

• 14. Braune, W. and Fischer, O. 1887a. Untersuchungen über die Gelenke des menschlichen Armes; 1. Das Ellenbogengelenk (O. Fischer); 2. Das Handgelenk (W. Braune and O. Fischer). Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 14 (2). pp. 29-150.

15. Braune, W. and Fischer, O. 1887b. Das Gesetz der Bewegungen in den Gelenken and der Basis der mittleren Finger und im Handgelenk des Menschen. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 14 (4). pp. 203-227.

16. Braune, W. and Fischer, O. 1888. Über den Antheil, den die einzelnen Gelenke des Schultergürtels am der Beweglichkeit des menschlichen Humerus haben. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 14 (8). pp. 393-410.

• 17. Braune, W. and Fischer, O. 1890. Über den Schwerpunkt des menschlichen Körpers mit Rücksicht auf die Ausrüstung des deutschen Infanteristen. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 15. pp. 561-572.

18. Braune, W. and Fischer, O. 1891. Die Bewegungen des Kniegelenks, nach einer neuen Methode am lebenden Menschen gemessen. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 17 (2). pp. 75-150.

19. Braune, W. and Fischer, O. 1892. Bestimmung der Trägheitsmomente des menschlichen Körpers und seiner Glieder. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 18 (8). pp. 409-492.

20. Braus, H. 1921. Anatomie des Menschen. Bewegungsapparat. J. Springer, Berlin. Volume 1. 835 pp.

21. Breitinger, E. 1937. Zur Berechnung der Körperhöhe aus den langen Gliedmassen Knochen. Anthrop. Anz., 14 (3/4). pp. 249-274.

22. Bridgman, G. B. 1920. Constructive Anatomy. Bridgman Publishing Company, Pelham, New York. 216 pp.

23. Brozek, J. and Keys, A. 1952. Body Build and Body Compositions. Science, Volume 116. pp. 140-142.

24. Bugnion, E. 1892. Le Méchanisme du genou. Recueil inaug. de l'université Lausanne. Viret-Genton, Lausanne.

25. Chissin, Chaim. 1906. Über die Öffnungsbewegung des Unterkiefers und die Beteiligung der äusseren Pterygoidmuskeln bei derselben. Arch. f. Anat. u. Physiol. (Anat. Abt.). pp. 41-67.

26. Churchill, E. 1951. Anthropometric Survey—Statistical Reduction of Data. Contract AF 33(038)-11815. Antioch College, Yellow Springs, Ohio.

27. Churchill, E. 1952. Personal communication.

28. Clark, W. A. 1920. A System of Joint Measurements. J. Ortho. Surg., 2 (12). pp. 687-700.

• Indicates references cited in the condensed text.

29. Clark, L. G. and Weddell, G. M. 1944. The Pressure Which Can Be Exerted by the Foot of a Seated Operator with the Leg in Various Positions. R. N. P. Volume 44. Medical Research Council, Royal Naval Personnel Committee. 153 pp.

30. Cureton, Thomas K. 1939. Elementary Principles and Techniques of Cinematographic Analysis as Aids in Athletic Research. Research Quarterly, Am. Ass'n. Health, Phys. Ed. and Recreation. Volume 10 (2). pp. 3-24.

31. Dally, J. F. H. 1908. An Inquiry into the Physiological Mechanism of Respiration, with Especial Reference to the Movements of the Vertebral Column and Diaphragm. J. Anat. Volume 43. pp. 93-114.

32. Daniels, G. S., Meyers, H. C., and Churchill, E. 1953. Anthropometry of Male Basic Trainees. WADC Technical Report 53-49, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio.

33. Darcus, H. D. 1951. The Maximum Torques Developed in Pronation and Supination of the Right Hand. J. Anat. Volume 85 (1). pp. 55-67.

34. Darcus, H. D. and Salter, Nancy. 1953. The Amplitude of Pronation and Supination with the Elbow Flexed to a Right Angle. J. Anat. Volume 87 (2). pp. 169-184.

35. Darcus, H. D. and Weddell, H. G. 1947. Some Anatomical and Physiological Principles Concerned in the Design of Seats for Naval War Weapons. Brit. Med. Bull., 5 (1). pp. 31-37.

36. Davis, L. E. 1949. Human Factors in Design of Manual Machine Controls. Mech. Engin. Volume 71. pp. 811-816.

37. Dempster, W. T. 1953. Joint Axes and Contours of the Major Extremity Joints. 31 pp. Quarterly Report No. 4. Study of the Hinge Points of the Human Body. Wright Air Development Center, United States Air Force contract no. AF 18(600)-43.

38. Dempster, W. T. and Liddicoat, R. T. 1952. Compact Bone as a Non-isotropic Material. Am. J. Anat. Volume 91 (3). pp. 331-362.

39. Dupertuis, C. W. 1950. Anthropometry of Extreme Somatotypes. Am. J. Phys. Anthrop. Volume 8 n.s. (3). pp. 367-383.

40. Dupertuis, C. W. and Hadden, J. A., Jr. 1951. On the Reconstruction of Stature from Long Bones. Am. J. Phys. Anthrop. Volume 9 n.s. pp. 15-54.

41. Dupertuis, C. W., Pitts, G. C., Osserman, E. F., Welham, W. C., and Behnke, A. R. 1951. Relation of Specific Gravity to Body Build in a Group of Healthy Men. J. Appl. Physiol. Volume 3 (11). pp. 676-680.

42. Dupertuis, C. W. and Tanner, J. M. 1950. The Pose of the Subject for Photogrammetric Anthropometry with Especial Reference to Somatotyping. Am. J. Phys. Anthrop. Volume 8 n.s. (1). pp. 27-47.

43. Eberhart, H. D. and Inman, V. T. 1951. An Evaluation of Experimental Procedures Used in a Fundamental Study of Locomotion. Ann. N. Y. Acad. Sci. Volume 51. pp. 1213-1228.

44. Elftman, Herbert. 1939. The Rotation of the Body in Walking. Arbeitsphysiol. Volume 10 (5). pp. 477-484.

45. Elftman, H. 1943. On Dynamics of Human Walking. Tr. N. Y. Acad. Sci. Volume 6. pp. 1-4.

46. Elftman, H. 1945. Torsion of the Lower Extremity. Am. J. Phys. Anthrop. Volume 3 (3). pp. 255-265.

47. Elftman, H. 1951. The Basic Pattern of Human Locomotion. Ann. N. Y. Acad. Sci. Volume 51. pp. 1207-1212.

48. Evans, F. G. 1953. Methods of Studying the Biomechanical Significance of Bone Form. Am. J. Phys. Anthrop. Volume 11 n.s. (3). pp. 413-436.

49. Fick, R. 1904-1911. Handbuch der Anatomie und Mechanik der Gelenke. Part 1 - Anatomie der Gelenke. 512 pp. 1904. Part 2 - Allgemeine Gelenk- und Muskelmechanik. 376 pp. 1910. Part 3 - Spezielle Gelenk- und Muskelmechanik. 688 pp. 1911. (in Bardeleben: Handbuch der Anatomie des Menschen. Volume 2, Section 1. G. Fischer, Jena.)

50. Fischer, O. 1893. Die Arbeit der Muskeln und die lebendige Kraft des menschlichen Körpers. Abh. d. math.-phys. Cl. d. k. Sachs. Gesellsch. d. Wiss. Volume 20 (1). pp. 4-84.

51. Fischer, O. 1895. Beiträge zu einer Muskeldynamik: Über die Wirkungsweise enggelenkiger Muskeln. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 22. pp. 49-197.

52. Fischer, O. 1896. Beiträge zur Muskelstatik: Über das Gleichgewicht zwischen Schwere und Muskeln am zweigliedrigen System. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 23 (4). pp. 267-368.

53. Fischer, O. 1904. Der Gang des Menschen. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volumes 21 - 28. Leipzig. This extensive study included the following parts:

Braune, W. and Fischer, O. 1895. Der Gang des Menschen. I.Theil: Versuche am unbelasteten und belasteten Menschen. Volume 21. pp. 151-322.

Fischer, O. 1899. Der Gang des Menschen. II.Theil: die Bewegung des Gesammtschwerpunktes und die äusseren Kräfte. Volume 25. pp. 1-130.

Fischer, O. 1901. Der Gang des Menschen. III.Theil: Betrachtungen über die weiteren Zele der Untersuchung und Überblick über die Bewegungen der unteren Extremitäten. Volume 26. pp. 85-170.

Fischer, O. 1901. Der Gang des Menschen. IV.Theil: Über die Bewegung des Fusses und die auf denselben einwirkenden Kräfte. Volume 26. pp. 469-556.

Fischer, O. 1904. Der Gang des Menschen. V.Theil: die Kinematik des Beinschwingens. Volume 28. pp. 319-418.

Fischer, O. 1904. Der Gang des Menschen. VI.Theil: Über den Einfluss der Schwere und der Muskeln auf die Schwingurgsbewegung des Beins. Volume 28. pp. 531-617.

54. Fischer, O. 1906. Theoretische Grundlagen für eine Mechanik der lebenden Körper. B. G. Teubner, Berlin. 327 pp.

55. Fischer, O. 1907. Kinematik organischer Gelenke. F. Vieweg and Sohn, Braunschweig. 261 pp., 77 figs.

56. Fischer, O. 1909. Zur Kinematik der Gelenke vom Typus des humero-radial Gelenkes. Abh. d. math.-phys. Cl. d. k. Sächs. Gesellsch. d. Wiss. Volume 32. pp. 3-77.

57. Franseen, E. B. and Hellebrandt, F. A. 1943. Postural Changes in Respiration. Am. J. Physiol. Volume 138. pp. 364-369.

58. Fries, E. C. and Hellebrandt, F. A. 1943. The Influence of Pregnancy on the Location of the Center of Gravity, Postural Stability, and Body Alignment. Am. J. Obstet. and Gynecol. Volume 46 (3). pp. 374-380.

59. Gardner, E. 1950a. Reflex Muscular Responses to Stimulation of Muscular Nerves in the Cat. Am. J. Physiol. Volume 161 (1). pp. 133-141.

60. Gardner, E. 1950b. Physiology of Movable Joints. Physiol. Rev. Volume 30 (2). pp. 127-176.

61. Garry, R. C. 1930. The Factors Determining the Most Effective Push or Pull Which Can Be Exerted by a Human Being on a Straight Lever Moving in a Vertical Plane. Arbeitsphysiol. Volume 3 (4). pp. 330-346.

62. Gilliland, A. R. 1921. Norms for Amplitude of Voluntary Movement. J. Am. Med. Ass'n. p. 1357.

63. Glanville, A. D. and Kreezer, G. 1937. The Maximum Amplitude and Velocity of Joint Movements in Normal Male Human Adults. Human Biol. Volume 9. pp. 197-211.

64. Goss, C. M., ed. 1948. Gray's Anatomy of the Human Body. Twenty-fifth edition. Lea and Febiger, Philadelphia. 1478 pp.

65. Haines, R. W. 1941. A Note on the Actions of the Cruciate Ligaments of the Knee Joint. J. Anat. Volume 75. pp. 373-375.

66. Harless, E. 1860. Die Statischen Momente der menschlichen Gliedmassen. Abh. d. math.-phys. Cl. d. königl. Bayer. Akad. d. Wiss. Volume 8. pp. 69-96 and 257-294.

67. Harrower, G. 1924. Mechanical Considerations in the Scapulo-humeral Articulation. J. Anat. Volume 58. pp. 222-227.

68. Hellebrandt, F. A. 1938. Standing as a Geotropic Reflex. The Mechanism of the Asynchronous Rotation of Motor Units. Am. J. Physiol. Volume 121. pp. 471-474.

69. Hellebrandt, F. A., Duvall, E. N., and Moore, M. L. 1949. The Measurement of Joint Motion, Part 3, Reliability of Goniometry. Physiotherap. Rev. Volume 29 (6). pp. 302-307.

70. Hellebrandt, Frances A., Braun, Genevieve, and Tepper, Rubye H. 1937. The Relation of the Center of Gravity to the Base of Support in Stance. Am. J. Physiol. Volume 119. p. 113.

71. Hellebrandt, F. A. and Franseen, E. B. 1943. Physiological Study of the Vertical Stance of Man. Physiol. Rev. Volume 23. pp. 220-255.

72. Hellebrandt, F. A. and Fries, E. C. 1942. The Eccentricity of the Mean Vertical Projection of the Center of Gravity during Standing. Physiotherap. Rev. Volume 22. pp. 186-192.

• 73. Hertzberg, H. T. E., Daniels, G. S., and Churchill, E. 1954. Anthropometry of Flying Personnel - 1950 . WADC Technical Report 52-321, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio. Actually, a preliminary summary of these data was used in the earlier phases of this work (Churchill, E., 1951 Anthropometric Survey).

74. Hicks, J. H. 1953. The Mechanics of the Foot. I. The Joints. J. Anat. Volume 87 (4). pp. 345-357.

75. Hooton, E. A. 1947. Up from the Ape. Second edition. MacMillan, New York. 788 pp.

76. Hrdlička, A. 1947. Practical Anthropometry. Third edition. Edited by T. D. Stewart. Wistar Institute, Philadelphia. 230 pp.

77. Hugh-Jones, P. 1944. Some Physiological Aspects of Tank Driving Controls, Part I, The Maximum Force Exertable on Hand Controls and how this Varies with Different Positions of the Control Relative to the Seat. BPC 44/341, PL 137, Gt. Brit., MRC-MPRC.

78. Hugh-Jones, P. 1945. Some Physiological Aspects of Tank Driving Controls, Part III, The Maximum Force Exertable on Hand Controls and how this Varies with Different Positions of the Control Relative to the Seat. BPC 45/410, PL 158, Gt. Brit., MRC-MPRC.

79. Hugh-Jones, P. 1947. The Effect of Limb Position in Seated Subjects on their Ability to Utilize the Maximum Contractile Force of the Limb Muscles. J. Physiol. Volume 104 (1). pp. 332-344.

80. Hultkrantz, J. W. 1897. Das Ellenbogengelenk und seine Mechanik. Fischer, Jena. 148 pp.

81. Hunsicker, Paul A. 1954. Arm Strength at Selected Degrees of Elbow Flexion. Air Force Project No. 7214-71727, United States Air Force Air Materiel Command, Wright-Patterson Air Force Base, Dayton, Ohio.

82. Ingalls, N. W. 1927. Studies on the Femur. IV. Some Relation of the Head and Condyles in the White and Negro. Am. J. Phys. Anthrop. Volume 10 (3). pp. 393-405.

83. Inman, V. T., Saunders, J. B. de C., and Abbott, L. C. 1944. Observations on the Function of the Shoulder Joint. J. Bone and Joint Surg. Volume 26 (1). pp. 1-30.

84. Kennedy, J. L. 1949. Handbook of Human Engineering Data for Design Engineers. Tufts College Institute for Applied Experimental Psychology. Technical Report SDC 199-1-1, Nav Expos P-643, Special Devices Center, Office of Naval Research, Department of the Navy.

85. King, B. G. 1948. Measurements of Man for Making Machinery. Am. J. Phys. Anthrop. Volume 6. pp. 341-351.

86. King, B. G. 1952. Functional Cockpit Design. Am. Engin. Rev. Volume 11 (6).

87. Kingsley, P. C. and Olmsted, K. L. 1948. A Study of the Angle of Anteversion of the Neck of the Femur. J. Bone and Joint Surg. Volume 30-A (3). pp. 745-751.

88. Klopsteg, P. E. and Wilson, P. D. 1954. Human Limbs and Their Substitutes. McGraw-Hill. 844 pp.

89. Knese, K. H. 1950a. Die Bewegungen auf einer Führungsfläche als Grundlage der Gelenkbewegungen, dargestellt am Schultergelenk. Zts. f. Anat. u. Entwgesch. Volume 115. pp. 115-161.

90. Knese, K. H. 1950b. Kinematik der Gliederbewegungen dargestellt am Hüft- und Ellenbogengelenk. Zts. f. Anat. u. Entwgesch. Volume 115. pp. 162-223.

91. Knese, K. H. 1950c. Kinematik des Kniegelenkes. Zts. f. Anat. u. Entwgesch. Volume 115. pp. 287-332.

92. Koch, H. 1941. Fussbediente Arbeitsmaschinen und Ermüdung. Arbeitsschutz. Volume 3. pp. 52-59.

93. Krahl, V. E. 1947. The Torsion of the Humerus: Its Localization Cause and Direction in Man. Am. J. Anat. Volume 80 (3). pp. 275-319.

94. Langer, C. 1858. Das Kniegelenk des Menschen. Sitzungsber. d. k. Akad. d. Wiss. math.-naturw. Cl. (Wien). Volume 32. pp. 99-142.

95. Langer, C. 1865. Lehrbuch der Anatomie des Menschen. Wilhelm Braumuller, Wien. 757 pp.

96. Last, R. J. 1948. Some Anatomical Details of the Knee Joint. J. Bone and Joint Surg. Volume 30-B. pp. 683-688.

97. Lay, W. E. and Fisher, L. C. 1940. Riding Comfort and Cushions. S. A. E. Journal. Volume 47. pp. 482-496. Also mimeograph prepublication of paper with same title.

98. Lehmann, G. 1927. Arbeitsphysiologische studien IV. Pflügers Arch. ges. Physiol. Volume 215. pp. 329-364.

99. Ludkewitsch, A. 1900. L'Articulation de l'épaule. Bull. Soc. Vaud. Sci. Nat. Volume 34 (134). pp. 350-371.

100. MacConaill, M. A. 1941. The Mechanical Anatomy of the Carpus and Its Bearings on Some Surgical Problems. J. Anat. Volume 75. pp. 166-175.

101. MacConaill, M. A. 1946. Studies in the Mechanics of Synovial Joints. I. Fundamental Principles and Diadochal Movements. Irish J. Med. Sci. Volume 6. pp. 190-199.

102. MacConaill, M. A. 1946. Studies in the Mechanics of Synovial Joints. II. Displacement of Articular Surfaces and the Significance of Saddle Joints. Irish J. Med. Sci. Volume 6. pp. 223-235.

103. MacConaill, M. A. 1946. Studies in the Mechanics of Synovial Joints. III. Hinge Joints and the Nature of Intra-articular Displacements. Irish J. Med. Sci. Volume 6. pp. 620-626.

104. MacConaill, M. A. 1948. The Movements of Bones and Joints. I. Fundamental Principles with Particular Reference to Rotation Movement. J. Bone and Joint Surg. Volume 30-B. pp. 322-326.

105. MacConaill, M. A. 1951. The Movements of Bones and Joints. IV. The Mechanical Structure of Articulating Cartilage. J. Bone and Joint Surg. Volume 33-B (2). pp. 251-257.

106. Manouvrier, L. 1892. Determination de la taille d'àpres les grandes os des membres. Rev. mens. de l'école d. Anthrop (Paris). Volume 2. pp. 226-233.

107. Manter, J. T. 1941. Movements of the Subtalar and Transverse Tarsal Joints. Anat. Rec. Volume 80. pp. 397-410.

108. Marey, E. J. 1874. Animal Mechanism. Appleton-Century, New York. 283 pp.

109. Marey, E. J. 1895. Movement (transl., E. Pritchard). W. Heinemann, London. 323 pp.

110. Martin, R. 1914. Lehrbuch der Anthropologie. G. Fischer, Jena. 1181 pp. Also 1928 edition.

111. Martin, W. R. and Johnson, E. E. 1952. An Optimum Range of Seat Positions as Determined by Exertion of Pressure upon a Foot Pedal. Army Med. Res. Lab. Rep. No. 86. p. 9.

• 112. McFarland, Ross A. 1953a. Human Engineering. Aviation Age. Volume 19 (6). pp. 214-219.

113. McFarland, Ross A. 1953b. Human Factors in Air Transportation: Occupational Health and Safety. McGraw-Hill, New York. 830 pp.

114. McFarland, R. A., Damon, A., Stoudt, H. W., Moseley, A. L., Dunlap, H. W., and Hall, W. A. 1953. Human Body Size and Capabilities in the Design and Operation of Vehicular Equipment. Harvard School of Public Health, Boston, Mass. p. 239.

115. Mech, Carl. 1895. Volummessungen des menschlichen Körpers und seiner einzelnen Theile in den verschiedenen Altersstufen. Zts. f. Biol. Volume 13. pp. 125-147.

116. Meredith, H. V. 1941. Stature and Weight of Children of the United States with Reference to the Influence of Racial, Regional, Socio-economic and Secular Factors. Am. J. Dis. of Children. Volume 62. pp. 909-932.

117. Meyer, H. 1853. Die Mechanik des Kniegelenks. Arch. f. Anat. u. Physiol. (Anat. Abt.). pp. 497-547.

118. Meyer, G. H. 1873. Die Statik und Mechanik des menschlichen Knochengerüstes. Wm. Engelmann, Leipzig. 402 pp.

119. Mollier, S. 1938. Plastische Anatomie: Die Konstruktive Form des menschlichen Körpers. Second Edition. Bergmann, Munich. 280 pp.

120. Moore, Margaret L. 1949a. The Measurement of Joint Motion, Part I, Introductory Review of the Literature. Physiotherap. Rev. Volume 29 (6). pp. 195-205.

121. Moore, Margaret L. 1949b. The Measurement of Joint Motion, Part 2, The Technic of Goniometry. Physiotherap. Rev. Volume 29 (6). pp. 256-264.

122. Morant, G. M. 1947. Anthropometric Problems in the Royal Air Force. Brit. Med. Bull. Volume 5. pp. 25-30.

123. Morant, G. M. 1951. Surveys of the Heights and Weights of Royal Air Force Personnel. Flying Personnel Research Committee. FPRC 711.

124. Morton, D. J. 1935. The Human Foot, Its Evolution, Physiology and Functional Disorders. Columbia University Press. 244 pp.

125. Müller, E. A. 1936. Die günstige Anordnung im Sitzen betätigter Fusshebel. Arbeitsphysiol. Volume 9 (2). pp. 125-137.

126. Müller, E. A. and Müller, A. 1949. Die günstigste Grösse und Anordnung von Handrädern. Arbeitsphysiol. Volume 14 (1). pp. 27-44.

127. National Research Council on Artificial Limbs. 1947. Fundamental Studies of Human Locomotion and Other Information Relating to Design of Artificial Limbs. Nat. Res. Council Comm. on Artificial Limbs, Prosthetic Devices Res. Project, University of California, Berkeley, California. 19 chapters (no continuous pagination).

128. Newman, R. W. and White, R. M. 1951. Reference Anthropometry of Army Men. Report No. 180, Environmental Protection Section, Quartermaster Climatic Research Laboratory.

129. Orlansky, J. and Dunlap, J. W. 1948. The Human Factor in the Design of Stick and Rudder Controls for Aircraft. The Psychological Corporation, Division of Bio-mechanics, Office of Naval Research. 77 pp.

130. Pan, N. 1924. Length of Long Bones and Their Proportion to Body Height in Hindus. J. Anat. Volume 58. pp. 374-378.

131. Partridge, E. J. 1924. Joints, the Limitations of Their Range of Movement, and an Explanation of Certain Surgical Conditions. J. Anat. Volume 58. pp. 346-354.

132. Pearson, K. 1899. IV. Mathematical Contributions to the Theory of Evolution. V. On the Reconstruction of the Stature of Prehistoric Races. Philos. Tr. Roy. Soc. Series A, Volume 192. pp. 169-244.

• 133. Pfuhl, W. 1934. Der Bewegungsumfang im Schultergelenk und der Anteil des subacromialen Nebengelenks an den Schultergelenkbewegungen. Morphol. Jahrb. Volume 74. pp. 670-696.

134. Ramsey, R. W. 1944. Muscle Physics. Medical Physics (O. Glaser, ed.). Year Book Publishers, Chicago. Volume 1. pp. 784-798.

135. Randall, F. E., Damon, A., Benson, R. S., and Patt, D. I. 1946. Human Body Size in Military Aircraft and Personal Equipment. United States Air Force Technical Report No. 5501, United States Air Force, Air Materiel Command.

136. Raphael, D. L. 1953. An Analysis of Short Reaches and Moves. Methods-Time Measurement Research Studies, Report 106. M. T. M. Ass'n for Standards and Research, Ann Arbor. 56 pp.

137. Raphael, D. L. and Clapper, G. C. 1952. A Study of Simultaneous Motions. Methods-Time Measurement Research Studies, Report 105. M. T. M. Ass'n for Standards and Research, Pittsburgh. 98 pp.

138. Rees, J. E. and Graham, N. E. 1952. The Effect of Backrest Position on the Push Which Can Be Exerted on an Isometric Foot-pedal. J. Anat. Volume 86 (3). pp. 310-319.

139. Reuleaux, F. 1875. Theoretische Kinematik: Grundzüge einer Theorie des Maschinenwesens. F. Vieweg und sohn, Braunschweig, Germany. (German) Also translated by A. B. W. Kennedy, 1876. The Kinematics of Machinery: Outline of a Theory of Machines. Macmillan Publishing Company, London.

140. Rollet, E. 1888. De la Mensuration des os longs des membres et de sés applications anthropologique et medico-legale. Compt. Rendus. Volume 107. pp. 957-960.

141. Rollet, E. 1889. La Mensuration des os longs des membres. Arch. de l'Anthropol. Crim. Volume 4. pp. 137-161.

142. Roschdestwenski, J. and Fick, R. 1913. Über die Bewegungen im Hüftgelenk und die Arbeitsleistung der Hüftmuskeln. Arch. f. Anat. u. Physiol. (Anat. Abt.). pp. 365-456.

143. Rosen, Neil G. 1922. A Simplified Method of Measuring Amplitude of Motion in Joints. J. Bone and Joint Surg. Volume 20 (3). pp. 570-579.

144. Salter, Nancy, and Darcus, H. D. 1952. The Effect of the Degree of Elbow Flexion on the Maximum Torques Developed in Pronation and Supination of the Right Hand. J. Anat. Volume 86 (2). pp. 197-202.

145. Salter, N., and Darcus, H. D. 1953. The Amplitude of Forearm and of Humeral Rotation. J. Anat. Volume 87 (4). pp. 407-418.

146. Schaeffer, J. P., ed. 1953. Morris' Human Anatomy. Eleventh Edition. Blakiston, New York. 1718 pp.

147. Schwarz, W. 1950. Die Lateralbewegungen im Kniegelenk. Anat. Anz. Volume 97 (18/20). pp. 329-340.

148. Seely, F. B. and Ensign, N. E. 1945. Analytical Mechanics for Engineers. Third Edition. John Wiley, New York. 450 pp. (Fourth Edition, 1952, 443 pp.)

149. Sheldon, W. H., Stevens, S. S., and Tucker, W. B. 1940. The Varieties of Human Physique. Harper and Bros. 347 pp.

150. Sheldon, W. H., Dupertuis, C. W., and McDermott, E. 1954. Atlas of Men: A Guide for Somatotyping the Adult Male at All Ages. Harper, New York. 357 pp.

• 151. Shiino, K. 1913. Über die Bewegungen im Schultergelenk und die Arbeitsleistung der Schultermuskeln. Arch. f. Anat. u. Physiol. (Anat. Abt.). Supplemental volume. pp. 1-88.

152. Sinelnikoff, E. and Grigorowitsch, M. 1931. Die Beweglichkeit der Gelenke als sekundäres geschlechtliches und konstitutionelles Merkmal. Zts. f. Konstitutionslehre. Volume 15 (6). pp. 679-693.

153. Smith, J. W. 1953. The Act of Standing. Acta. Orthopaed. Scand. Volume 23 (2). pp. 158-168.

154. Steindler, Arthur. 1935. Mechanics of Normal and Pathological Locomotion in Man. Charles C. Thomas, Springfield, Ill., and Baltimore, Maryland. 424 pp.

• 155. Strasser, H. 1917. Lehrbuch der Muskel- und Gelenkmechanik. Four Volumes, 1913-1917. J. Springer, Berlin.

• 156. Strasser, H. and Gassmann, A. 1893. Hülfsmittel und Normen zur Bestimmung und Veranschaulichung der Stellungen, Bewegungen und Kraftwirkungen am Kugelgelenk, insbesondere am Hüft- und Schultergelenke des Menschen. Anat. Hefte. Volume 2 (6/7). pp. 389-434.

157. Swearingen, J. J. 1951. Design and Construction of a Crash Dummy for Testing Shoulder Harness and Safety Belts. Civil Aeronautics Medical Research Laboratory, Department of Commerce, Civil Aeronautics Administration Aeronautical Center, Oklahoma City, Oklahoma. 3 pp. and figures and tables.

158. Taylor, C. L. and Blaschke, H. C. 1951. A Method for Kinematic Analysis of Motions of the Shoulder, Arm, and Hand Complex. Ann. N. Y. Acad. Sci. Volume 51 (7). pp. 1251-1265.

159. Telkkä, A. 1950. On the Prediction of Human Stature from the Long Bones. Acta Anat. Volume 9. pp. 103-117. Reprinted Yearbook of Physiol. Anthrop. 1950. Volume 6. pp. 206-219.

160. Timoshenko, S. and Young, D. H. 1940. Engineering Mechanics. McGraw-Hill, New York. 523 pp.

161. Trotter, Mildred and Gleser, G. C. 1952. Estimation of Stature from Long Bones of American Whites and Negroes. Am. J. Phys. Anthrop. Volume 10 n.s. (4). pp. 463-514.

162. Vierordt, H. 1893. Anatomische, physiologische und physikalische Daten und Tabellen zum Gebrauche für Mediciner. G. Fischer, Jena. 400 pp.

163. von Lanz, T. and Wachsmuth, W. 1935, 1938. Praktische Anatomie, ein Lehr- und Hilfsbuch der anatomischen Grundlagen ärztlichen Handelns. Volume 1. Part 3, 276 pp. Part 4, 485 pp. J. Springer, Berlin.

164. Wakim, K. G, Gersten, J. W., Elkins, E. C., and Martin, G. M. 1950. Objective Recording of Muscle Strength. Arch. Phys. Med. Volume 31 (2). pp. 90-99.

165. Walmsley, Thomas. 1928. The Articular Mechanism of the Diarthroses. Bone and Joint Surg. Volume 10. pp. 40-45.

166. Weinbach, A. P. 1938. Contour Maps, Center of Gravity, Moment of Inertia, and Surface Area of the Human Body. Human Biol. Volume 10 (3). pp. 356-371.

167. Wiggers, C. J. 1944. Physiology in Health and Disease. Fourth Edition. Lea and Febiger, Philadelphia. 1174 pp.

168. Wilson, G. D. and Stasch, W. H. 1945. Photographic Record of Joint Motion. Arch. Phys. Med. Volume 26. pp. 361-362.

169. Wiechec, F. and Krusen, F. 1939. A New Method of Joint Measurement and a Review of the Literature. Amer. J. Surg. Volume 43 (3). pp. 659-668.

170. Wollichs, E. W. A. and Hulverscheidt, F. 1935. Die Bedienbarkeit von Handgriffen an Vorrichtungen. Werkstattstechnik und Werksleiter. Volume 29 (10). pp. 193-197.

171. Woodson, E. E. 1954. Human Engineering: Guide for Equipment Designers. Univ. of California Press. 813 pp.

172. Zuppinger, Hermann. 1904. Die aktive Flexion im unbelasteten Kniegelenk. Habilitationsschrift vorgelegt der Hohen Med. Fak. zu Zürich. J. F. Bergmann, Wiesbaden. pp. 3-64.

INDEX

A

B

C

D

E

F

G

H

I

J

K

L

M

N

O

P

R

T

U

V

Y