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DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.
HEAT TRANSFER TO A FLUID FLOWING TURBULENTLY
IN A SMOOTH PIPE WITH WALLS AT CONSTANT TEMPERATURE

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T. SHIMAZAKI

Contract N7-onr-29523 Phase (2)
Project NR 035 324
University of California
Berkeley, California
Sept. 10, 1949
September 8, 1949

Commanding Officer
U.S. Navy Office of Naval Research
301 Donahue St.
San Francisco, Calif.

Reference: ONR: SF/44-3(2) N7-29523
JEL
Serial PL - 176

Dear Sir:

The appended report, entitled, "Heat Transfer to a Fluid Flowing Turbulently in a Smooth Pipe with Walls at Constant Temperature," contains results obtained from analytical work carried out under the contract noted above.

Very truly yours,

R. A. Seban
Project Engineer

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HEAT TRANSFER TO A FLUID FLOWING TURBULENTLY IN A SMOOTH PIPE WITH WALLS AT CONSTANT TEMPERATURE.
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ABSTRACT

The case of heat transfer to an incompressible fluid of constant properties flowing turbulently within a pipe having constant wall temperature has been investigated analytically. Heat transfer coefficients have been evaluated and compared to the coefficients obtained by Martinelli for the same flow conditions under the conditions of the linear variation of wall temperature obtained from a constant heat transfer rate along the pipe length. At low Prandtl Numbers, such as obtain with molten metals, the ratio between the coefficients for these two cases may become as low as 0.72, with the coefficient for the constant wall temperature case being the smaller of the two. A simple equation of the Lyon type is indicated for the correlation of the present results.
Introduction

The use of analogy methods for the calculation of heat transfer coefficients for the flow of all fluids, including those of low Prandtl Number, in circular pipes has been presented by Martinelli (1). This analysis yielded results for the case of a linear variation of wall temperature with pipe length.

The effect of constant wall temperature as contrasted to linear variation of the wall temperature was first noted by Reichardt (2) who, however, confined his attention to fluids of Prandtl Number greater than 0.70 and found only small variation of the heat transfer coefficient due to changes in the wall temperature profile.

This report presents results comparing heat transfer coefficients as obtain under conditions of constant wall temperature and linear variation of wall temperature and it is shown that for fluids of low Prandtl Number the wall temperature effect is significant. Because of the length of the calculations the results are presented only for flow in a circular pipe. The results are to some extent preliminary because the method used is one of successive approximation and for most of the cases presented only one approximation has been made.

Analysis

Variation of Fluid Temperature with Pipe Length

The system considered is that of an incompressible fluid of constant properties flowing turbulently in a circular pipe. It is assumed that both the velocity and temperature distributions are fully developed and that in consequence there exists a generalized temperature distribution which is invariable in the direction of flow.

\[
\frac{d}{dx} \left( \frac{T_w - T}{T_w - T_m} \right) = 0 \tag{1}
\]
Carrying out the indicated differentiation, there is obtained:

\[
\frac{1}{T_w - T_m} \frac{dT}{dx} - \frac{1}{T_w - T_m} \frac{dT}{dx} - \frac{T_w - T}{(T_w - T_m)^2} \left[ \frac{dT_w}{dx} - \frac{dT_m}{dx} \right] = 0 \tag{2}
\]

In virtue of the specified character of flow the heat transfer coefficient will be independent of pipe length and if the rate of heat transfer is constant with respect to length, then since

\[q = h(T_w - T_m)\]

the third term of Eq. (2) is seen to be zero. Further, since for this case \(\frac{dT_m}{dx}\) is a constant the wall temperature varies linearly with \(x\), Eq. (2) becomes

\[\frac{dT}{dx} = \frac{dT_w}{dx}, \text{ which is also } \frac{dT_m}{dx}\]

and \(\frac{dT}{dx}\) is then independent of radial position in the pipe.

If, however, the wall temperature is constant, \((\frac{dT_w}{dx} = 0)\), Eq. 2 yields

\[\frac{dT}{dx} = \frac{T_w - T}{T_w - T_m} \frac{dT_m}{dx} = 0 \tag{3}\]

In this case the axial temperature gradient depends upon the radial position in the pipe.

Calculation of the Temperature Distribution

For the fully developed flow considered, the equations of momentum and energy can be written (with flow dissipation effects neglected) as:

\[\frac{1}{\rho} \frac{dP}{dx} = \frac{1}{r} \frac{dr}{dr} \left[ r (\epsilon_m + \nu) \frac{du}{dr} \right] \tag{4}\]

\[u \frac{dT}{dx} = \frac{1}{r} \frac{dr}{dr} \left[ r (\epsilon_m + \nu) \frac{dT}{dr} \right] \tag{5}\]

The temperature distribution satisfying Eq. (5) is the solution desired.
The radial distribution of the heat rate is given by one integration of Eq. (5)

\[ \int_0^r u \frac{dT}{dx} r \, dr = \int_0^r d \left[ r (\varepsilon_m + a) \frac{dT}{dr} \right] \]

since \( q \) is zero at \( r = 0 \).

In the case that the wall temperature varies linearly, \( \frac{dT}{dx} \) is not a function of \( r \). Martinelli (1) made the assumption that the velocity \( u \) equalled the mean velocity and by this further assumption of invariance of velocity with radius removed the product \( u \frac{dT}{dx} \) from within the integral sign. Formal integration of the left side of Eq. 6 is then possible and it is seen that those conditions dictate a linear variation of the heat rate, \( q \), with the pipe radius, \( r \).

Lyon (3), retaining the assumption of linear variation of the wall temperature, accounted for the actual variation of the velocity by performing this and subsequent integrations graphically. His results demonstrated that the heat transfer coefficient is reduced by an average of \( \frac{1}{8} \) for Reynolds numbers in the range 20,000 to 1,000,000.

For the case of constant wall temperature, the use of Eq. 3 to eliminate \( \frac{dT}{dx} \) in Eq. (6) yields the expression

\[ \frac{dT_m}{dx} \int_0^r u \left( \frac{T_w - T_m}{T_w - T_m} \right) r \, dr = \int_0^r d \left[ r (\varepsilon_m + a) \frac{dT}{dr} \right] \]

OR

\[ \frac{dT_m}{dx} = \frac{r u_m}{T_w - T_m} \left( T_w - T_k \right) \int_0^r \frac{u}{u_m} \frac{T_w - T_k}{T_m - T_k} \, d\left( \frac{r}{r_o} \right) \]

For subsequent simplification let

\[ \phi \left( \frac{r}{r_o} \right) = \int_0^{r/r_o} \frac{u}{u_m} \frac{T_w - T_k}{T_m - T_k} \, d\left( \frac{r}{r_o} \right) = f \left( \frac{u m D}{v}, \frac{v}{a} \right) \]

The evaluation of \( \phi \left( \frac{r}{r_o} \right) \) is seen to require knowledge of the very temperature distribution being sought. Thus some approximate procedure
is necessary and that procedure is here based on the use of the temperature distribution, \( \frac{T_w - T}{T_w - T_\text{k}} \), obtained by Martinelli (for the case of constant heat rate). Thus the factor \((T_w - T_\text{k})\) occurring before the integral on the left side of Eq. 7 must also be evaluated on the basis of the Martinelli results.

The function \( \phi \left( \frac{r}{r_\text{o}} \right) \) is seen to depend upon the Reynolds and Prandtl numbers of the flow.

Eq. 7 is now integrated again, yielding the expression:

\[
\frac{r^2}{r^2} \frac{D T_m}{T_w - T_\text{k}} \left( T_w - T_\text{k} \right) \int_0^{r/r_\text{o}} \frac{\phi \left( \frac{r}{r_\text{o}} \right) d \left( \frac{r}{r_\text{o}} \right)}{r/r_\text{o} (\varepsilon_n + a)} = T - T_\text{k} \tag{9}
\]

This specifies the temperature distribution. To evaluate the integral the radial variation of the eddy diffusively for heat, \( \varepsilon_M \), is required and this is obtained by assuming it equal to the eddy diffusively for momentum transfer \( \varepsilon_M \). The diffusivity for momentum transfer is obtained for the turbulent core from the Prandtl Nikuradse velocity distribution and the specified linear variation of shear with pipe radius. Thus

\[
u^+ = 5.5 + 2.5 \ln y^+
\]

\[
\varepsilon_M = \frac{2 \nu}{\nu} \left( 1 - \frac{y}{r_\text{o}} \right) = \nabla
\]

and with \( \nabla \) neglected in the turbulent region

\[
\varepsilon_M = \frac{u_\tau r_\text{o}}{2.5} \left( \frac{r}{r_\text{o}} \right) \left( 1 - \frac{r}{r_\text{o}} \right) \tag{11}
\]

In the buffer layer \( y^+=5 \) to \( y^+=30 \) the velocity distribution is taken as:

\[
u^+ = -3.05 + 5.0 \ln y^+
\]

and this yields for the diffusivity:

\[
\varepsilon_M = \frac{u_\tau r_\text{o}}{5} \left( \frac{r}{r_\text{o}} \right) \left( 1 - \frac{r}{r_\text{o}} \right) - \nabla \tag{12}
\]
In the laminar region from the pipe wall, \( y^+ = 0 \) to \( y^+ = 5 \) the eddy diffusivity is zero.

The temperature distribution with regard to radius can now be obtained from Eq. 9, since the specification of the diffusivities enables the evaluation of the integral in that equation. That integral can be stated as

\[
\psi \left( \frac{r}{r_0} \right) = \int_{r_0}^{r_0} \frac{d \left( \frac{r}{r_0} \right)}{\eta r_0 \left( \frac{\rho u}{\eta} + a \right)} d \left( \frac{r}{r_0} \right)
\]  

(13)

and with this definition Eq. 9 can be re-stated as:

\[
\frac{r^2 U_m}{T_w - T_m} \frac{d \psi}{d \theta} = \frac{1}{V} \left( T_w - T_e \right) \psi \left( \frac{r}{r_0} \right) = T - T_e
\]  

(14)

The temperature distribution across the pipe can be conveniently stated in the form:

\[
\frac{T_w - T_m}{T_w - T_e} = 1 - \frac{T - T_e}{T_w - T_e} = 1 - \frac{\psi \left( \frac{r}{r_0} \right)}{\psi (1)}
\]  

(15)

The determination of the heat transfer coefficient requires the specification of the mixed mean fluid temperature in the pipe. This mixed mean temperature is defined as

\[
T_m = \int_{r_0}^{r_0} \frac{2 U T \theta r^2 dr}{U_m \theta r^4}
\]  

(16)

The temperature \( T \) is specified by Eq. 14.

Before proceeding to this final calculation it can be noted that, since

\[
\frac{U_m}{T_w - T_m} \frac{d \psi}{d \theta} = \frac{h}{\frac{D}{4} \gamma C_p}
\]

Equation 14 can be stated in the form:

\[
\left( \frac{h}{\frac{D}{4} \gamma C_p} \right) \frac{D}{4} V \left( T_w - T_e \right) \psi \left( \frac{r}{r_0} \right) = T - T_e
\]  

(17)
It is assumed that the heat transfer coefficient in Eq. 17 is the true heat transfer coefficient and that it will be satisfactorily approximated by the calculation in process. The factor \((T_w - T_m)\) of Eq. 17 is the wall to centerline temperature difference calculated by the method of Martinelli for the case of constant heat rate, which case was used for the first approximation to the temperature distribution necessary to the evaluation of the function \(\phi \left(\frac{r}{R} \right)\), as defined in Eq. 8. Re-examination of Eq. 8 reveals that it would also have been possible to assume that the heat transfer coefficient as contained in Eq. 17 is the heat transfer coefficient of the first approximation since, with the wall heat rate \(q_0\) assumed the same throughout it is possible to argue that the difference \((T_w - T_m)\) in Eq. 7 derives logically from the first approximation. However, it seems more logical to assume this difference as the true value and this is borne out by the more rapid convergence of the results when the original assumption is made.

From the first approximation (Martinelli analysis)

\[
T_w - T_k = \frac{q_0}{\gamma c_p u_T} 5 \left[ \left( \frac{\nu}{\alpha} \right) + \ln \left( 5 \frac{\nu}{\alpha} + 1 \right) + \frac{E}{2} \ln \frac{R^2}{a^2} \right]
\]

or, defining the symbol \(\psi\) by

\[
T_w - T_k = \frac{q_0}{\gamma c_p u_T} 5 \psi
\]

substitution into Eq. 17 yields

\[
\left( \frac{hD}{k} \right) \left( \frac{a}{\nu} \right) \left( \frac{q_0}{\gamma c_p u_T} \right) 5 \psi \left( \frac{r}{R} \right) = T - T_k \tag{18}
\]

Integration of this equation for the specification of the mixed mean fluid temperature yields

\[
\left( \frac{hD}{k} \right) \left( \frac{a}{\nu} \right) \left( \frac{q_0}{\gamma c_p u_T} \right) 5 \psi \left( \frac{r}{R} \right) \left( \frac{r}{R} \right) d \left( \frac{r}{R} \right) = T_m - T_k \tag{19}
\]
For simplification let
\[ X = \int_0^1 \frac{u}{u_m} \psi \left( \frac{r}{r_a} \right) \left( \frac{r}{r_a} \right) d\left( \frac{r}{r_a} \right) \]

Equation (19) can be modified to give
\[ \left( \frac{hD}{k} \right) \left( \frac{\nu}{a} \right) \left( \frac{q^*}{\gamma C_p u_T} \right) 5 \mathcal{U} X = T_w - T_f - (T_w - T_m) \]

And finally, by division of both sides by \((T_w - T_m)\)
\[ \left( \frac{hD}{k} \right)^2 \left( \frac{q^*}{\nu} \right)^2 5 \mathcal{U} X = \frac{T_w - T_f}{T_w - T_m} - 1 \]  

From Eqs. 18 and 19
\[ \frac{T_w - T_f}{T_w - T_m} - 1 = \frac{1}{\psi(X) - 1} \]

Substitution into Eq. 20 yields the final expression for the Nusselt Number
\[ \frac{hD}{k} = \frac{\nu}{a} \sqrt{\frac{u_m D}{\gamma} \frac{C_f}{\mathcal{U}^2} 2 \frac{(\psi(X) - X)}{5 \mathcal{U}}} \]  

(21)

If the Martinelli solution for constant heat transfer rate is visualized as the first approximation to the case of constant wall temperature then the heat transfer coefficient given by Eq. 21 is a second approximation. This is what has been done for the majority of the cases considered.

A third approximation can be made by repeating the procedure, using in Eq. 8 the temperature distribution obtained from Eq. 17. This has been done for three cases.

**Results**

Table I gives the results of the calculations made according to the procedure indicated in the preceding section. Because of the substantial amount of numerical work involved only a limited number of conditions have been considered but these are adequate to demonstrate the effect. A third
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<th>$\frac{V}{a}$</th>
<th>$\frac{U_m D}{a}$</th>
<th>$\frac{(hD)}{k}$</th>
<th>$\frac{(hD)}{k^2}$</th>
<th>$\frac{(hD)}{k^3}$</th>
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approximation, made for three conditions, reveals fair correspondence with the second approximation.

Figure 1 reveals the predicted Nusselt Numbers for a Prandtl Number of 0.01 in comparison to the constant heat rate solution.

**Discussion**

Figure 2 reveals the comparison of the present solution for constant wall temperature to that of Martinelli for constant heat rate (linear variation of wall temperature), the basis of comparison being the ratio of the Nusselt Numbers for the two solutions for the same flow conditions.

As the Prandtl Number becomes large, the Nusselt Number values tend to become equal. Since, for the same heat transfer rate, the heat transfer coefficient depends on the difference between wall and mean temperature values, the difference in the heat transfer coefficients for the cases considered depends upon the magnitude of this temperature difference. For fluids of large Prandtl Number, the change in temperature between wall and fluid is localized in the sublayer region immediately adjacent to the wall and the temperature is almost constant across the entire turbulent core. Thus the magnitude of the mean temperature is but insignificantly affected by variation in the temperature of the wall.

At Prandtl Numbers below unity the difference between the two cases becomes appreciable and is dependent in magnitude on the Reynolds Number. For very low Prandtl Numbers, the ratio of the Nusselt Numbers tends to a limit as conduction becomes the predominant transfer mechanism in the turbulent core. This lower limit is difficult to specify briefly since it depends on the velocity distribution and so upon the effective Reynolds Number. Considering conduction only, the following typical magnitudes obtain.
Nusselt Numbers

<table>
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<th>Const. Wall Temp.</th>
<th>Const. Heat Rate</th>
<th>Ratio</th>
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<td>Parabolic Velocity Distribution</td>
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<td>0.84</td>
</tr>
<tr>
<td>Const. Velocity (Plug Flow)</td>
<td>5.78</td>
<td>8</td>
<td>0.72</td>
</tr>
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</table>

Since for turbulent flow the constant velocity case is a somewhat better approximation to the actual velocity distribution than is the parabolic case, the lowest ordinates on Figure 2 correspond to about 0.72.

The difference in temperature distributions indicated by the analysis of Martinelli for constant heat rate and by the present analysis for constant wall temperature is shown on Figure 3. It is seen that for the same wall to centerline temperature difference the heat transfer rate is substantially less for the case of constant wall temperature. This does not exactly reflect the difference in the heat transfer coefficients since the mean temperature magnitudes are also different for the two solutions.

It has been demonstrated by Lyon (3) that for low Prandtl Numbers, an equation of the type

\[
\frac{hD}{k} = A + B \left( \frac{U_m D}{\alpha} \right)^n
\]

with \(A\), \(B\), and \(n\) arbitrary constants, serves as a fair correlation with the analytical results. Because of its simple form, it is a preferred working equation. Figure 4 reveals the predicted Nusselt Numbers of the present analysis contrasted to the equation

\[
\frac{hD}{k} = 4.8 + 0.0248 \left( \frac{U_m D}{\alpha} \right)^{0.8}
\]

The use of this equation for Prandtl Numbers of less than 1/10 provides Nusselt Numbers agreeing with the theory within \(\pm 10\%\).
Conclusions

1. The case of heat transfer to a fluid flowing turbulently in a smooth pipe has been considered on an approximate basis, and heat transfer coefficients for this case have been shown to be significantly different from the case of constant heat rate as presented by Martinelli.

2. The predicted values of Nusselt Numbers presented from the approximation are probably correct to 10% of the ultimate exact values. The exact Nusselt Numbers will be greater than the values presented herein.

3. The Nusselt Numbers obtained for the case considered can be obtained with an accuracy of ±10% from the empirical equation

\[
\frac{hD}{k} = 4.8 + 0.0248 \left( \frac{\mu m D}{\alpha} \right)^{0.8}
\]

for Prandtl Numbers less than 1/10.

4. The presently available data on heat transfer to fluids of low Prandtl Number, which enable the evaluation of heat transfer coefficients, correspond to the constant heat rate case. No data for the constant wall temperature case are available to the authors at the present time and hence no comparison of the new theory and experiment are possible.
REFERENCES


The following nomenclature is used in the report:

$a$ = thermal diffusivity of fluid, sq. ft./sec.

$C_p$ = heat capacity of fluid, Btu/1b. °F

$D$ = diameter of pipe = $2r_o$

$h$ = heat transfer coefficient between pipe well and fluid, Btu/hr ft$^2$ °F

$g$ = gravitational force = 32.2 ft./sec.$^2$

$k$ = thermal conductivity of fluid, Btu/hr ft°F

$l_n$ = natural logarithm

$p$ = mean pressure, lb/ft$^2$

$q_f$ = rate of heat flow through unit area of pipe wall, Btu/ft$^2$hr

$r$ = radius, ft.

$r_o$ = radius of pipe, ft.

$T$ = temperature at any point, $r$, °F

$T_f$ = temperature of fluid at $r = 0$, °F

$T_m$ = mean mixed temperature, °F

$T_w$ = temperature of pipe wall, °F

$U$ = velocity in x-direction at any point $r$, ft./sec.

$U_m$ = mean bulk velocity, ft./sec.

$U_f = \sqrt{\frac{\gamma^2}{k}}$ = friction velocity, ft./sec.

$y = r_o - r$ = distance from pipe wall, ft.

$x$ = distance along pipe axis, ft.

$\gamma$ = weight density of fluid, lb./ft$^3$

$\varepsilon_h$ = eddy diffusivity for heat, ft.$^2$/sec.

$\varepsilon_m$ = eddy diffusivity for momentum, ft.$^2$/sec.

$\psi$ = variable defined in text
\[ \nu = \text{kinematic viscosity of fluid, } \text{ft}^2/\text{sec} \]
\[ \rho = \text{mass density of fluid, } \text{lb} \cdot \text{sec}^2/\text{ft}^4 \]
\[ \tau = \text{shear between wall and fluid, } \text{lb/ft}^2 \]
\[ \phi \left( \frac{r}{r_0} \right) = \text{integral defined in text} \]
\[ \psi \left( \frac{r}{r_0} \right) = \text{integral defined in text} \]
\[ \psi (l) = \psi \left( \frac{r}{r_0} \right) \text{ for the limits } r = 0 \text{ to } r = r_0 \]
\[ \chi = \text{integral defined in text} \]

**Dimensionless Groups**

\[ C_f = \text{friction factor for pipe flow defined by} \quad \frac{dP}{dx} = C_f \cdot \frac{V^2}{2g} \]
\[ \lambda = \frac{\frac{1}{2} \frac{D}{a} \sqrt{C_f}}{u_m \frac{D}{a}} \]
\[ F = \ln \left[ \frac{5 \lambda + \frac{y_2}{r_0} \left( 1 - \frac{y_2}{r_0} \right)}{5 \lambda} \right] + \frac{1}{\sqrt{1 + 20 \lambda}} \ln \left[ \frac{1 + \sqrt{1 + 20 \lambda} \left( \frac{2 y_2}{r_0} - 1 \right) - \sqrt{1 + 20 \lambda}}{1 - \sqrt{1 + 20 \lambda} \left( \frac{2 y_2}{r_0} - 1 \right) + \sqrt{1 + 20 \lambda}} \right] \]
\[ h_D = \frac{h_D}{k} = \text{Nusselt Number} \]
\[ \left( \frac{h_D}{k} \right)_2 = \text{Nusselt Number for constant heat rate} \]
\[ \left( \frac{h_D}{k} \right)_2 = \text{Nusselt Number for constant wall temperature, 2nd approximation} \]
\[ \left( \frac{h_D}{k} \right)_3 = \text{Nusselt Number for constant wall temperature, 3rd approximation} \]
\[ \frac{u_m D}{a} = \left( \frac{V}{a} \right) \left( \frac{u_m D}{V} \right) = \text{Peclet Number} \]
\[ \frac{u_m D}{V} = \text{Reynolds Number} \]
\[ \frac{V}{a} = \text{Prandtl Number} \]
\[ u^+ = \frac{u}{\sqrt{\nu}} = \text{dimensionless velocity parameter} \]
\[ y^+ = \frac{y \sqrt{\frac{\nu}{V}}}{V} = \text{dimensionless distance parameter} \]
FIGURES

Figure 1. Nusselt Number as a function of Reynolds Number, Prandtl Number 0.01 Turbulent Flow in a Smooth Pipe.
   a. For constant heat rate. Martinelli.
   b. For constant wall temperature, second approximation
   c. Position of next approximation for constant wall temperature.

Figure 2. Ratio of Nusselt Number for Constant Wall Temperature, \( \left( \frac{hD}{K} \right)_1 \), to the Nusselt Number for Constant Heat Rate, \( \left( \frac{hD}{K} \right)_2 \), as a function of Prandtl Number.

Figure 3. Temperature Distribution in a Pipe for Constant Wall Temp. and Constant Heat Rate. Reynolds Number, 10,000; Prandtl Number, 0.01.
   a. Constant Heat Rate
   b. Constant Wall Temp.

Figure 4. Nusselt Number as a Function of the Peclet Number. The curve represents Eq. 22; the points are the results from Table I.
\[ \left( \frac{h D}{K} \right)^2 \]

\[ \frac{\nu D}{a} \]

\[ \frac{u m D}{V} = 10^6 \]

\[ 10^5 \]

\[ 10^4 \]

FIGURE 2
FIGURE 3

Graph showing the relationship between $\frac{t_{w} - t}{t_{w} - t_{e}}$ and $1 - \frac{r}{r_{o}}$ for $\frac{u_{m}D}{\nu} = 10,000$ and $\frac{\nu}{\sigma} = 0.01$. Graphs labeled (a) and (b) are plotted on the same scale.