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REPORT

BALLISTIC DATA FOR FLAT FIRE

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BALLISTIC DATA FOR FLAT FIRE

Abstract

The purpose of this paper is to exhibit a new form of the Hitchcock-Kent Siacci treatment of flat fire ballistics. This form has proved convenient in certain aircraft-fire computation.

The principal feature of the new form is that time of flight and drop are tabulated directly as functions of distance along the line of fire. ("Ground coordinates" are used). Two "Siacci trajectories" and interpolation suffice to give, with fair accuracy, ballistic data for forward fire from aircraft for all airspeeds and densities.

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Introduction:

In issuing ballistic data for forward fire from pursuit aircraft certain condensed forms appeared desirable. The tabular data, as furnished on this project, consisted of σt, $\sigma^2 y$, listed against σx, where σ is the ratio of density at the muzzle to standard density, t, y, and x are respectively time of flight, drop, and distance along the line of fire. Certain advantages of this form are set forth in the following.

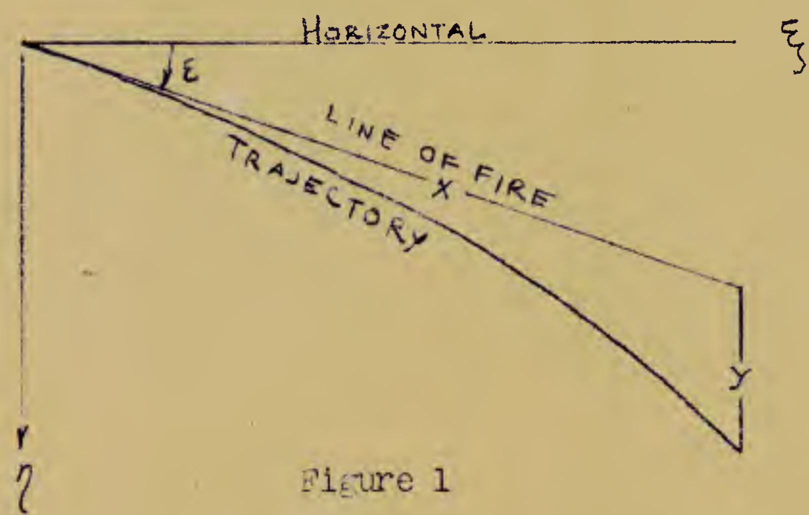

Figure 1

1. Modified Siacci Equations.

Referred to mutually perpendicular axes, $\xi\eta$, in the plane of the trajectory with origin at the muzzle of the gun and the y-axis pointing vertically downward (see Figure 1), the (particle) equations of motion may be expressed as

$$\ddot{\xi} = -E\dot{\xi} \qquad (1.1)$$

$$\ddot{\eta} = -E\dot{\eta} + g$$

where,

$$E = \frac{\rho(\eta)}{\rho_0} G(v, a, \rho_0) \frac{1}{c} \qquad (1.2)$$

and v denotes the velocity of the projectile at time t,
c denotes the ballistic coefficient of the projectile,
$\rho(\eta)$ represents air density at distance η from the horizontal axis,
ρ_0 represents standard density at the muzzle,

a is the velocity of sound in air at temperature T (absolute temperature) and has the form

$$a = a_s \sqrt{T/T_s}$$

where a_s = velocity of sound in air at standard temperature at sea level,
T_s, (T_s = 518.6° F on the absolute scale which corresponds to 59° F)

G(v, a, ρ) is the resistance function which is defined as $\rho v K_D(v/a)$, where K_D is the drag function of the projectile and is experimentally determined.

In this form, the equations present two computational difficulties:

1. The resistance function is tabulated from experiments for standard temperatures and standard density at sea level, ρ_s, i.e.

$$G(v) = G(v, a_s, \rho_s).$$

Therefore, G(v, a, ρ_0) must be related to G(v, a_s, ρ_s).

2. In aircraft gunfire the altitude of the muzzle changes. In the form in which E is taken above, changing the altitude of the muzzle requires recomputing the trajectory.

The second difficulty is easily remedied. Since

(1.3) $$G(v, a, \rho) = \rho v K_D(v/a)$$

we have

$$G(v, a, \rho_0) = \rho_0 v K_D(v/a),$$

$$G(v, a, \rho_s) = \rho_s v K_D(v/a).$$

Therefore, we have

(1.4) $$G(v, a, \rho_0) = \left[\rho_0/\rho_s\right] G(v, a, \rho_s).$$

Denoting ρ_0/ρ_s, the ratio of density at the muzzle to standard density at sea level, by σ, and replacing G(v, a, ρ_0) in (1.2) by (1.4), we obtain

(1.5) $$E = \frac{\rho(\eta)}{\rho_0} \sigma G(v, a, \rho_s) \frac{1}{c}.$$

It will presently be shown that writing Σ as shown in (1.5) enables one to solve the ballistic equations once for quantities equivalent to $\sigma\xi$ and $\sigma\eta$ and then substitute for σ to find the ξ and η appropriate to the altitude of the muzzle.

Now to relate $G(v, a, \rho_s)$ to the tabulated $G(v)$ we note that

(1.6) $$G(v) = G(v, a_s, \rho_s) = \rho_s \, v \, K_D(v/a_s) .$$

Then replacing v in (1.6) by $[a_s/a]v$, we have

$$G([a_s/a]v) = \rho_s [a_s/a] v \, K_D(v/a).$$

But,

$$G(v, a, \rho_s) = \rho_s \, v \, K_D(v/a).$$

Therefore, we have

(1.7) $$G([a_s/a]v) = [a_s/a] \, G(v, a, \rho_s) .$$

Remembering that $a = a_s \sqrt{T/T_s}$ and denoting $\sqrt{T_s/T}$ by λ, we obtain from (1.7)

(1.8) $$\frac{G(\lambda v)}{\lambda} = G(v, a, \rho_s).$$

Finally, since the ratio of standard air density at any altitude R above sea level to standard sea level air density is given by

$$e^{-hR} \quad (h = 0.3158 \times 10^{-4} \text{ if R is given in feet}),$$

we can write

(1.9) $$\frac{\rho(\eta)}{\rho_0} = e^{h\eta} .$$

Substituting (1.8) and (1.9) in (1.5) we arrive at the following form for the ballistic equations

(1.10)
$$\ddot{\xi} = - \frac{\sigma e^{h\eta} G(\lambda v)}{c \lambda} \dot{\xi} ,$$
$$\ddot{\eta} = - \frac{\sigma e^{h\eta} G(\lambda v)}{c \lambda} \dot{\eta} + g.$$

If, instead of rectangular coordinates (ξ, η) we use slant coordinates (x, y), the y-axis pointing vertically downward and the x-axis being taken along the tangent to the trajectory at the initial point, which tangent makes an angle ε with the horizontal (ε measured clockwise), then the equations for the transformation of variables are

$$\xi = x \cos \varepsilon \qquad \eta = \xi \tan \varepsilon + y$$

$$\dot{\xi} = \dot{x} \cos \varepsilon \qquad \dot{\eta} = \dot{\xi} \tan \varepsilon + \dot{y}$$

etc. etc.

and it can readily be shown that the equations (1.10) become

$$(1.11) \qquad \ddot{x} = - \frac{\sigma e^{h\eta} G(\lambda v)}{c \lambda} \dot{x} , \qquad \ddot{y} = - \frac{\sigma e^{h\eta} G(\lambda v)}{c \lambda} \dot{y} + g.$$

The y-coordinate as used here is usually referred to as the drop.

Now, introduce a pseudo velocity u defined as the component of the velocity v along the tangent to the trajectory at the initial point. Clearly,

$$(1.12) \qquad u = \dot{x}, \qquad \dot{u} = \ddot{x}.$$

Approximating* $e^{h\eta}$ by $e^{hx \sin \varepsilon}$ and approximating the actual velocity v by the psuedo velocity u as is done in the usual Siacci method (a satisfactory approximation if the trajectory is fairly flat and we restrict ourselves to short ranges), the equations of motion are approximately given by

$$(1.13) \qquad \dot{u} = - \frac{\sigma e^{hx \sin\varepsilon} G(\lambda u)}{c \lambda} u, \qquad \ddot{y} = - \frac{\sigma e^{hx \sin\varepsilon} G(\lambda u)}{c \lambda} \dot{y} + g.$$

* This approximation is cruder than that used by Hitchcock and Kent (see Ballistic Research Laboratory Report No. 114), but appears adequate, and is simpler computationally.

2. Constant Density Case

If it is assumed that air density along the trajectory does not vary from air density at the muzzle* (i.e. take $e^{hx \sin\epsilon} = 1$ or $h = 0$), the differential equations of motion may be integrated easily in terms of the Siacci Space and Time functions. These are defined

$$S(u) = -\int_{u_0}^{u} \frac{du}{G(u)}$$

$$T(u) = -\int_{u_0}^{u} \frac{du}{uG(u)} \quad .$$

From (1.12) and (1.13) we obtain

$$\sigma\, dx = -\frac{c\,\lambda du}{G(\lambda u)},$$

and, integrating ,

$$\sigma x = c\left[S(\lambda u) - S(\lambda u_0)\right]. \qquad (2.1)$$

This equation enables one to find $S(\lambda u)$ for a given slant range and initial velocity. Then λu may be found from Siacci Space function tables.

From (1.12) we have $dt = \frac{dx}{u}$ and, making use of (2.1), we find

$$\sigma\, dt = \frac{c\, S'(\lambda u)\,\lambda du}{u} \qquad (2.2)$$

which integrates into

$$\sigma t = c\lambda\left[T(\lambda u) - T(\lambda u_0)\right]. \qquad (2.3)$$

Thus σt is found by entering the Siacci Time function table with the value of λu from equation (2.1).

* The treatment of a trajectory in which air density is not considered to be constant will be discussed in Section 4A.

Finally, using Dederick's identity*

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = g/u^2$$

we find

$$(2.4) \qquad \sigma^2 y = g \int_0^x \int_0^x \frac{\sigma^2 dx\, dx}{u^2} .$$

These equations are those used in making the fundamental table of σt and $\sigma^2 y$ against σx, for a fixed initial velocity.

In Table 1 below, time of flight and drop corresponding to slant ranges up to 4500 yards have been computed in two ways for the 37 mm. Mark 9 HE 54 shell, fired horizontally (ε = 0°) at sea level with standard density (σ = 1) at temperature 59°F (λ = 1).

The 2nd and 3rd columns respectively show time of flight and drop computed by numerical integration of the normal equations of motion given in (1) and (2). Columns (4) and (5) show time of flight and drop computed from equations (1.13), (2.1),(2.3) and (2.4) which make use of the Siacci approximations mentioned in Section 1. Notice that through 3000 yards in slant range, drop and time of flight agree exactly for the two methods of computation. Even at a slant range of 4500 yards, the discrepancy in drop is under 1 %.

37 mm Mark 9 HE M54

Muzzle Velocity = 2550 fps   Airplane Velocity = 300 mph u_o = 2990 fps

ε = 0°   T = 59°F σ = 1   c = 1

	Normal Equations with h = h		Siacci with h = 0	
σ x yds.	σ t sec.	σ^2 y yds.	σ t sec.	σ^2 y yds.
500	0.549	1.5	0.550	1.5
1000	1.208	6.9	1.208	7.0
1500	2.012	18.1	2.013	18.1
2000	3.010	37.8	3.010	37.9
2500	4.268	71.3	4.265	71.3
3000	5.777	125.6	5.769	125.4
3500	7.442	207.0	7.419	206.3
4000	9.244	320.8	9.203	318.9
4500	11.197	472.4	11.122	468.2

Table 1

* This identity may be arrived easily from equations (1.1) and (1.2) of Section 1.

3. Interpolation for Time of Flight and Drop.

In order for tabulation of σt and $\sigma^2 y$ against σx to be computationally successful it is necessary that this tabulation be done for as few velocities as possible. For many cases, as will be seen, tables need be constructed for only two velocities, provided interpolation is done in a special way.

A consideration of the trajectory in vacuo suggests a possible method of interpolation for time of flight and drop. The equations of motion for the trajectory in vacuo are:

$$\ddot{x} = 0 ,$$

$$\ddot{y} = g ,$$

and the initial conditions are:

$$x = 0 \qquad \dot{x} = u_0$$

$$y = 0 \qquad \dot{y} = 0 .$$

The solution of the equations is

(3.1)
$$t = \frac{x}{u_0}$$

$$y = 1/2\, g x^2 \left(\frac{1}{u_0}\right)^2 .$$

Equations (3.1) above lead one to suspect that, for a given slant range, it may be possible to compute time of flight by a linear interpolation on $1/u_0$ between two known times of flight corresponding to two initial velocities u_{o_1} and u_{o_2}.

In Table 2 a harmonic linear interpolation based on times of flight corresponding to initial velocities of 2550 fps and 3430 fps was used to find time of flight for σx through 4500 yards.

The interpolation formula for finding σt corresponding to initial velocity u_0 is clearly

(3.2)
$$\sigma t = \left(1 - \frac{\frac{1}{u_0} - \frac{1}{u_{o_1}}}{\frac{1}{u_{o_2}} - \frac{1}{u_{o_1}}} \right) \sigma t_1 + \left(\frac{\frac{1}{u_0} - \frac{1}{u_{o_1}}}{\frac{1}{u_{o_2}} - \frac{1}{u_{o_1}}} \right) \sigma t_2$$

The results for this interpolation are shown in column 4 below. In column 3 are listed times of flight for an initial velocity of 2990 fps computed directly by means of equation (2.7). Notice that the results are the same to the nearest hundredth of a second through 2500 yard slant range. The discrepancy beyond 2500 yard range, however, may be due not to method but to a lack of smoothness in the neighborhood of sound in the G tables on which the Siacci computations for the 37 mm Mark 9 HE shell M54 were based.

TIME OF FLIGHT

By Siacci Formula and by Linear Interpolation on $(1/u_o)$

37 mm Mark 9 HE Shell M54, $c = 1$, $\varepsilon = 0°$, $T = 59°$, Constant Density

σx yds.	for $u_{o_1} = 2550$ fps	for $u_o = 2990$ fps		for $u_{o_2} = 3430$ fps
	SIACCI σt_1 sec.	SIACCI σt sec.	LINEAR INTERPOLATION ON $(1/u_o)$ $\sigma t = \sigma t_1 + \Delta\sigma t$ sec.	SIACCI σt_2 sec.
500	0.646	0.550	0.549	0.477
1000	1.433	1.203	1.211	1.046
1500	2.409	2.013	2.021	1.732
2000	3.633	3.010	3.024	2.571
2500	5.116	4.265	4.256	3.616
3000	6.753	5.769	5.736	4.928
3500	8.524	7.419	7.345	6.469
4000	10.432	9.203	9.113	8.141

Table 2

In support of this view similar computations for the 75 mm HE shell M48 whose Siacci tables are smooth are offered in the footnote below.*

The approximation of drop by a quadratic function of $(1/u_0)$ is suggested by the solution for drop in the vacuum case given in (3.1). Such an approximation was used to find $\sigma^2 y$ by interpolation for the 37 mm shell.

The interpolation formula is

$$(3.3) \qquad \sigma^2 y = \left(1 - \frac{\frac{1}{u_0^2} - \frac{1}{u_{0_1}^2}}{\frac{1}{u_{0_2}^2} - \frac{1}{u_{0_1}^2}} \right) \sigma^2 y_1 + \left(\frac{\frac{1}{u_0^2} - \frac{1}{u_{0_1}^2}}{\frac{1}{u_{0_2}^2} - \frac{1}{u_{0_1}^2}} \right) \sigma^2 y_2 .$$

The results of this interpolation are listed below in Table 3, column 7. Drop as computed by (2.4) is shown in column 4. The discrepancy between the Siacci computation and the interpolated drop to σx is shown in column 8. It is only at σx = 4000 yards that the discrepancy exceeds 1 % and here but slightly.

* TIME OF FLIGHT

75 mm HE Shell M48

c = 1.686   ε = 0°   T = 59°   Constant density

	for u_{0_1} = 1950 fps	for u_0 = 2390 fps		for u_0 = 2830 fps
	SIACCI	SIACCI	HARMONIC LINEAR INT.	SIACCI
σx	σt_1	σt	σt	σt_2
yds.	sec.	sec.	sec.	sec.
1000	1.649	1.340	1.341	1.128
2000	3.552	2.871	2.876	2.411
3000	5.751	4.630	4.638	3.871
4000	8.287	6.660	6.664	5.545
5000	11.128	9.007	8.966	7.477

For ease of computation a harmonic linear interpolation for drop would be preferable to a harmonic quadratic for then the same coefficients could be used in interpolating for drop as are used in the time of flight interpolation. In column 5 below the results of harmonic linear interpolation are listed. The harmonic linear results are not quite as accurate as those of the harmonic quadratic interpolation except for $\sigma x \geq 3000$ yards where the linear interpolation appears to be more accurate. This reversal, however, appears to be an accident again due to lack of smoothness in the shell's Siacci tables.

Similar computations for the 75 mm shell show the harmonic quadratic interpolation to be distinctly more accurate at all ranges than the harmonic linear interpolation (Table 4).

The choice between a harmonic linear interpolation for drop and a harmonic quadratic interpolation depends upon the accuracy required in the results. In any case the use of harmonic interpolation on u_0 can produce a great saving in time and labor since it enables one to find time of flight and drop for intermediate airplane speeds simply from a tabulation of these quantities first for initial shell velocity with the airplane at rest and then with the airplane having another extreme velocity, say 600 mph (880 fps).

DROP

By Siacci Formulae and by Interpolation

37 mm Mark 9 HE Shell M54 c = 1, ε = 0°, T = 59°, Constant density

σ x yds.	u_{o_1} = 2550 fps σ $^2y_1$ yds.	u_{o_2} = 3430 fps σ $^2y_2$ yds.	u_o = 2990 fps SIACCI σ ^2y yds.	HARMONIC LINEAR σ ^2y yds.	DISCREPANCY %	HARMONIC QUAD. σ ^2y yds.	DISCREPANCY %
500	2.11	1.15	1.53	1.56	.1	1.53	0
1000	9.71	5.24	6.95	7.15	.2	6.93	0
1500	25.60	13.54	18.11	18.68	.4	18.25	.1
2000	54.45	28.06	37.92	39.31	.7	38.36	.2
2500	103.27	52.06	71.31	73.90	1.0	72.05	.3
3000	178.43	90.99	125.38	128.28	1.0	125.13	.1
3500	284.72	152.11	206.32	208.66	.7	203.88	1.2
4000	427.22	241.13	318.89	320.48	.4	313.78	1.7

Table 3

DROP

By Siacci Formulae and by Interpolation

75 mm HE Shell M48 c = 1.686, ε = 0°, T = 59° Constant density

σ x yds.	u_{o_1} = 1950 fps σ $^2y_1$ yds.	u_{o_2} = 2830 fps σ $^2y_2$ yds.	u_o = 2390 fps SIACCI σ ^2y yds.	HARMONIC LINEAR σ ^2y yds.	DISCREPANCY %	HARMONIC QUAD. σ ^2y yds.	DISCREPANCY %
1000	13.9	6.6	9.2	9.6	.4	9.3	.1
2000	61.7	28.7	40.5	42.2	.8	40.7	.1
3000	154.5	70.9	100.7	105.0	1.4	101.3	.2
4000	307.8	139.4	199.3	208.1	2.2	200.6	.4
5000	542.6	242.7	349.3	365.0	3.1	351.7	.5

Table 4

4. Differential Corrections.

A. Variable Density Along the Trajectory.

The equations of motion have so far been solved for the case in which air density at all altitudes was assumed to remain the same as standard density at the muzzle. This was accomplished by setting $h = 0$ in the equations of motion. Next we will attempt to consider the effect of variable air density.

Taking $\lambda = 1$ for simplicity (a correction for temperature will be discussed later), equation (1.13) leads to

$$(4.1) \quad \int_0^x e^{hx} \sin \epsilon \, dx = \frac{c}{\sigma}\left[S(u) - S(u_0)\right].$$

For fixed values of initial velocity, u_0, and angle of elevation, ϵ, equation (4.1) determines u as a function of σx and of h. Then σt and $\sigma^2 y$ given respectively by

$$(4.2) \qquad \sigma t = \int_0^x \frac{\sigma \, dx}{u(\sigma x, h)}$$

and

$$(4.3) \qquad \sigma^2 y = g \int_0^x \int_0^x \frac{\sigma^2 \, dx}{u^2(\sigma x, h)}$$

are also functions of σx and h.

At a given slant range σx, then, the effect of a change in density on σt and $\sigma^2 y$ respectively, $\Delta_h \sigma t$ and $\Delta_h \sigma^2 y$, may be thought of as

$$\Delta_h \sigma t = \sigma t(\sigma x, h) - \sigma t(\sigma x, 0)$$

$$\Delta_h \sigma^2 y = \sigma^2 y(\sigma x, h) - \sigma^2 y(\sigma x, 0).$$

We approximate $\Delta_h \sigma t$ and $\Delta_h \sigma^2 y$ by the first order derivative terms in the Taylor expansions of $\sigma t(\sigma x, h)$ and $\sigma^2 y(\sigma x, h)$ respectively about $h = 0$.

Finding $\frac{\partial u}{\partial h}$ from (4.1) and using (4.2) we obtain

$$\sigma^2 \left.\frac{\partial t}{\partial h}\right|_{h=0} = -\frac{\sin \varepsilon}{2c}\int_0^x \frac{\sigma^2 x^2 \sigma\, dx}{u^2(\sigma x,0)\, S'(u)}$$

or, using $\sigma x = c\left[S(u) - S(u_0)\right]$ for $h = 0$

$$\sigma^2 \left.\frac{\partial t}{\partial h}\right|_{h=0} = -c^2 \frac{\sin \varepsilon}{2}\int_{u_0}^u \left[\frac{S(u) - S(u_0)}{u(\sigma x,0)}\right]^2 du.$$

This latter form has the advantage of eliminating $S'(u)$ from the computation.

From (4.3) we obtain

$$\sigma \left.\frac{\partial \sigma^2 y}{\partial h}\right|_{h=0} = -\frac{g}{c}\sin \varepsilon \int_0^x\int_0^x \frac{\sigma^2 x^2 \sigma^2 dx\, dx}{u^3 S'(u)} .$$

Hence, the differential corrections for density on time of flight and drop respectively are given by

$$(4.4) \qquad \sigma \cdot \Delta_h \sigma t = -\frac{h \sin \varepsilon}{2c}\int_0^x \frac{\sigma^2 x^2 \sigma\, dx}{u^2 S'(u)}$$

and

$$(4.5) \qquad \sigma \cdot \Delta_h \sigma^2 y = -\frac{hg \sin \varepsilon}{c}\int_0^x\int_0^x \frac{\sigma^2 x^2 \sigma^2\, dx\, dx}{u^3 S'(u)} .$$

In order to observe how well the first order approximation for the correction in time of flight works, we have tabulated below in Table 5, for the 37 mm Mark 9 HE M54 shell, time of flight computed from the normal differential equations, with density varying with altitude, compared with time of flight computed for constant density and corrected by (4.4). The initial angle was taken as 60° upward ($\varepsilon = -60°$), an extreme angle of fire and one for which extreme variation in density is to be expected. Through 2500 yards slant range there is exact agreement in time of flight. Beyond 2500 yards in slant range the differential correction seems very inadequate. The table also contains a comparison of drop. The y's in column 3 were computed from the normal equations with variable density. The y's in column 5 were computed with the Siacci equations with constant density and are not corrected for variable density. Through 2500 yards slant ranges it seems to be unnecessary to make the complicated correction on drop since the discrepancy between actual drop/range and uncorrected drop/range is under 2%. Nor is there any point in correcting drop beyond 2500 yards since even the corrected time of flight beyond this range is quite inaccurate.

TIME OF FLIGHT AND DROP WITH VARIABLE AIR DENSITY

37 mm Mark 9 HE shell M54, c= 1, σ = 1, ε = -60°, airplane velocity=300 mph, muzzle velocity=2550 fps } u_0=2990 fps

x yds.	NORMAL EQUATIONS t(lx,h) secs.	NORMAL EQUATIONS y(lx,h) yds.	SIACCI EQUATIONS t(lx,h) = t(lx,0) + Δt	SIACCI EQUATIONS y(lx,0) yds.
500	0.548	1.4	0.549	1.5
1000	1.201	6.7	1.201	7.0
1500	1.983	17.7	1.984	18.1
2000	2.920	36.5	2.923	37.9
2500	4.056	66.7	4.061	71.3
3000	5.383	113.5	5.449	125.4
3500	6.822	181.0	6.986	206.3
4000	8.335	271.8	8.616	318.9
4500	9.920	383.1	10.323	468.2

TABLE 5

B. Non-Standard Temperature.

In a somewhat analogous fashion to that of 4A we can arrive at an approximation for the effect of a deviation from standard temperature on time of flight and drop. This time, considering air density to be constant along the trajectory, we have, for a given range and a fixed initial velocity, from equations (2.1), (2.3), (2.4), σt and $\sigma^2 y$ defined as functions of slant range, σx, and λ, the square root of the ratio of standard temperature to actual temperature.

The effect of a deviation in the ratio $\lambda = \sqrt{T_s/T}$ on time of flight and drop will be denoted by $\Delta_\lambda \sigma t$ and $\Delta_\lambda \sigma^2 y$ respectively and defined as follows:

$$(4.6) \qquad \Delta_\lambda \sigma t = \sigma t(\sigma x, \lambda) - \sigma t(\sigma x, 1)$$

and

$$(4.7) \qquad \Delta_\lambda \sigma^2 y = \sigma^2 y(\sigma x, \lambda) - \sigma^2 y(\sigma x, 1).$$

Again $\Delta_\lambda \sigma t$ and $\Delta_\lambda \sigma^2 y$ will be approximated by the first order terms in their respective Taylor expansions about $\lambda = 1$. Differentiation with respect to λ in (2.1), (2.3), and (2.4) leads to

$$(4.8) \qquad \Delta_\lambda \sigma t = (\lambda - 1) \left\{ \sigma t(\sigma x, 1) + c\, S'(u_0) \left(\frac{u_0}{u} - 1 \right) \right\},$$

$$(4.9) \qquad \Delta_\lambda \sigma^2 y = 2(\lambda - 1) \left[\sigma^2 y(\sigma x, 1) - g u_0 S'(u_0) \int_0^x \int_0^x \frac{\sigma^2\, dx\, dx}{u^3 S'(u)} \right]$$

For the 37 mm Mark 9 HE M54, we have tabulated below in column 4 time of flight for $\lambda = 1.125$* as corrected by the first order approximation given in (4.8). Time of flight computed directly from formula (2.3) with λ set equal to 1.125 appears in column 5. Through 2700 yards, the discrepancy between the entries in columns 4 and 5 is no more than one hundredth of a second. Beyond 2700 yards, the discrepancy becomes considerably larger.

* $\lambda = 1.125$ corresponds to a temperature of -49.8° F, as extreme a deviation from the standard temperature of 59° F as is likely to be encountered.

TIME OF FLIGHT CORRECTED FOR NON-STANDARD TEMPERATURE

37 mm Mark 9 HE shell M54, $c = 1$, $\varepsilon = 0^\circ$, $u_o = 2990$ fps, constant density

σx yds.	σt for $\lambda = 1$	$\Delta_\lambda \sigma t$ for $\lambda = 1.125$	$\sigma t(\sigma x, 1.125)$: $\sigma t(\sigma x,1) + \Delta_\lambda \sigma t$	$\sigma t(\sigma x, 1.125)$: $c\lambda[T(\lambda u) - T(\lambda u_o)]$
1000	1.208	-.010	1.198	1.193
1500	2.013	-.207	1.986	1.991
2000	3.010	-.064	2.946	2.958
2500	4.265	-.108	4.157	4.166
2700	4.843	-.102	4.741	4.731
2900	5.454	-.075	5.379	5.348
3100	6.088	-.036	6.052	6.014
3300	6.745	+.009	6.754	6.708
3500	7.419	+.056	7.475	7.432
3700	8.115	.106	8.221	8.173
3900	8.834	.157	8.991	8.942
4100	9.574	.210	9.784	9.735
4300	10.339	.265	10.604	10.549
4500	11.122	.321	11.443	11.392

TABLE 6

Here again, a similar computation for the 75 mm shell suggests that roughness in the 37 mm Siacci Tables may be partial cause of the discrepancy beyond 2700 yards. Even as far out as $\sigma x = 5500$ yards, there is exact agreement to hundredths of a second between time of flight corrected for temperature and time of flight exactly computed for the actual temperature.

TIME OF FLIGHT CORRECTED FOR NON-STANDARD TEMPERATURE

75 mm HE Shell M48, $c = 1.686$, $\varepsilon = 0^{\circ}$, $u_0 = 2390$ fps, constant density

σx yds.	σt for $\lambda = 1$ secs.	σt for $\lambda = 1.125$	
		$\sigma t(\sigma x,1) + \Delta_{\lambda}\sigma t$	$c\lambda[T(\lambda u) - T(\lambda u_0)]$
1000	1.340	1.337	1.336
1500	2.079	2.071	2.073
2000	2.871	2.856	2.859
2500	3.718	3.692	3.697
3000	4.630	4.589	4.596
3500	5.606	5.548	5.558
4000	6.660	6.584	6.591
4500	7.789	7.693	7.703
5000	9.007	8.894	8.896
5500	10.308	10.184	10.182
6000	11.696	11.607	11.580

TABLE 7

No correction on drop has been made since for σx less than 2700 yards, the drop correction is under 1 ⌀.

SUMMARY

It is suggested from the preceding study on the 37 mm projectile that the equations of motion be solved for σt and $\sigma^2 y$ as functions of σx first for constant air density along the trajectory, for standard air temperature and for standard muzzle velocity by the equations:

$$\sigma x = c\,[S(u) - S(u_0)]$$

$$\sigma t = c\,[T(u) - T(u_0)]$$

$$\sigma^2 y = g \int_0^x \int_0^x \frac{\sigma^2\, dx\, dx}{u^2} \quad .$$

These results may be tabulated with initial velocity adjusted for two airplane speeds: 0 and 600 mph. Corrections may be added to these results for conditions different from the assumed ones.

The effect on time of flight of variable air density along the trajectory can be computed by the formula

$$\sigma \cdot \Delta_h \sigma t = - \frac{h \sin \varepsilon}{2c} \int_0^x \frac{(\sigma x)^2\, \sigma\, dx}{u^2 S'(u)} \quad .$$

Even in the extreme case of 60° upward fire this correction yields accurate results through σx = 2500 yards. Density corrections on drop appear to be unnecessary. Even in the 60° upward angle of fire case, the discrepancy between corrected and uncorrected drop compared with σx is under 2 ¢ as far out as 2500 yards.

The correction on time of flight for non-standard temperature is made by the formula

$$\Delta_\lambda \sigma t = (\lambda - 1) \left\{ \sigma t(\sigma x, 1) + cS'(u_0) \left(\frac{u_0}{u} - 1\right) \right\} \quad .$$

The corrections so obtained will be good to approximately 2700 yards in σx. The temperature correction on drop may very well be omitted since through σx = 2700 yards $\Delta_\lambda \sigma^2 y / \sigma x$ is under 1 ¢.

From the tabulations of σt and $\sigma^2 y$ for initial velocities obtained when the airspeed of the plane is 0 and 600 mph, one can find σt and $\sigma^2 y$ when the airplane has intermediate speeds.

The time of flight and drop are given by

$$\sigma t = \left(1 - \frac{1/u_o - 1/u_{o_1}}{1/u_{o_2} - 1/u_{o_1}}\right)\sigma t_1 + \left(\frac{1/u_o - 1/u_{o_1}}{1/u_{o_2} - 1/u_{o_1}}\right)\sigma t_2$$

and

$$\sigma^2 y = \left(1 - \frac{1/u_o - 1/u_{o_1}}{1/u_{o_2} - 1/u_{o_1}}\right)\sigma^2 y_1 + \left(\frac{1/u_o - 1/u_{o_1}}{1/u_{o_2} - 1/u_{o_1}}\right)\sigma^2 y_2$$

or, where greater accuracy is required,

$$\sigma^2 y = \left(1 - \frac{1/u_o^2 - 1/u_{o_1}^2}{1/u_{o_2}^2 - 1/u_{o_1}^2}\right)\sigma^2 y_1 + \left(\frac{1/u_o^2 - 1/u_{o_1}^2}{1/u_{o_2}^2 - 1/u_{o_1}^2}\right)\sigma^2 y_2 \quad .$$

A. K. Goldstine

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J. L. Kelley

J. L. Kelley

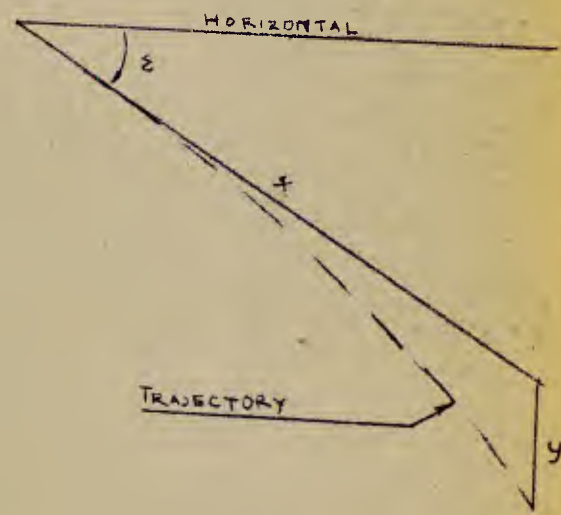
APPENDIX

Ordnance Research Center
Project No. 4005

BALLISTIC DATA FOR CAL. 0.50 API MARK 8 FIRED FORWARD FROM AN AIRPLANE

1. Definitions:

The gun is fired along the line of flight of an airplane flying with speed V_A, at dive angle ε.



x = distance along line of fire

y = drop from line of fire

$t_f = t_{f_o} + \Delta t_f \sin\varepsilon$ = time of flight

t_{f_o} = time of flight for $\varepsilon = 0$

$\Delta t_f \sin\varepsilon$ = correction for variation of density

σ = density ratio = density $\div$ standard density

2. Assumptions:

Temperature = 59°F.

Drift = 0

3. Accuracy:

At $(\sigma x) \leq 5000$ ft., the error in $(\sigma^2 y)$ is <1 ft., the error in $(\sigma t_f) < .03$ secs. Beyond $\sigma x = 5000$ ft., accuracy falls off badly, and the table cannot be considered to represent the facts adequately.

BALLISTIC DATA FOR CAL. 0.50 API MARK 8 FIRED FORWARD FROM AN AIRPLANE (Cont'd)

V_A = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2870 f.p.s.

σx ft.	$\sigma^2 y$ ft.	σt_{f_o} sec.	$\sigma^2 \Delta t_f$ sec.
100	.0	.027	
200	.0	.054	
300	.1	.082	
400	.2	.110	
500	.3	.138	.000
600	.4	.167	
700	.6	.196	
800	.8	.225	
900	1.0	.255	
1000	1.2	.286	.000
1100	1.5	.316	
1200	1.8	.348	
1300	2.2	.380	
1400	2.6	.412	
1500	3.0	.444	.001
1600	3.4	.478	
1700	3.9	.511	
1800	4.4	.546	
1900	4.9	.580	
2000	5.5	.616	.002
2100	6.2	.652	
2200	6.9	.688	
2300	7.6	.725	
2400	8.3	.763	
2500	9.1	.801	.005
2600	10.0	.840	
2700	10.9	.880	
2800	11.9	.920	
2900	12.9	.961	
3000	14.0	1.002	.009
3100	15.1	1.045	
3200	16.3	1.088	
3300	17.5	1.132	
3400	18.8	1.176	
3500	20.2	1.222	.015
3600	21.6	1.268	
3700	23.1	1.315	
3800	24.7	1.363	
3900	26.4	1.412	
4000	28.1	1.462	.025

BALLISTIC DATA FOR CAL. 0.50 APl MARK 8 FIRED FORWARD FROM AN AIRPLANE (Cont'd)

V_A = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2870 f.p.s.

σx ft.	$\sigma^2 y$ ft.	σt_{f_o} sec.	$\sigma^2 \Delta t_f$ sec.
4100	29.9	1.513	
4200	31.8	1.565	
4300	33.8	1.617	
4400	35.9	1.671	
4500	38.1	1.726	.039
4600	40.4	1.782	
4700	42.7	1.838	
4800	45.2	1.896	
4900	47.8	1.956	
5000	50.5	2.016	.060
5100	53.4	2.078	
5200	56.3	2.141	
5300	59.4	2.205	
5400	62.6	2.270	
5500	66.0	2.337	.088
5600	69.5	2.406	
5700	73.1	2.475	
5800	77.0	2.546	
5900	80.9	2.619	
6000	85.1	2.693	.128
6100	89.4	2.769	
6200	94.0	2.847	
6300	98.7	2.926	
6400	103.6	3.007	
6500	108.8	3.090	.181
6600	114.2	3.175	
6700	119.7	3.261	
6800	125.6	3.348	
6900	131.7	3.439	
7000	138.1	3.530	.243
7100	144.7	3.623	
7200	151.7	3.717	
7300	158.9	3.812	
7400	166.4	3.909	
7500	174.2	4.006	.288

BALLISTIC DATA FOR CAL. 0.50 AP1 MARK 8 FIRED
FORWARD FROM AN AIRPLANE (Cont'd)

V_A = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2870 f.p.s.

σx ft.	$\sigma^2 y$ ft.	σt_{f_o} sec.	$\sigma^2 \Delta t_f$ sec.
100	.0	.030	
200	.1	.061	
300	.1	.093	
400	.2	.124	
500	.4	.157	.000
600	.6	.190	
700	.8	.223	
800	1.0	.256	
900	1.3	.291	
1000	1.6	.326	.000
1100	2.0	.361	
1200	2.4	.397	
1300	2.8	.434	
1400	3.3	.471	
1500	3.8	.508	.001
1600	4.4	.547	
1700	5.0	.586	
1800	5.7	.625	
1900	6.5	.666	
2000	7.3	.707	.003
2100	8.1	.748	
2200	9.0	.791	
2300	9.9	.834	
2400	11.0	.878	
2500	12.0	.923	.006
2600	13.2	.968	
2700	14.4	1.015	
2800	15.7	1.062	
2900	17.0	1.110	
3000	18.5	1.159	.011
3100	20.0	1.209	
3200	21.6	1.260	
3300	23.3	1.312	
3400	25.0	1.364	
3500	26.8	1.418	.019
3600	28.8	1.473	
3700	30.8	1.529	
3800	33.0	1.586	
[illegible]	[illegible]	[illegible]	
4000	[illegible]	[illegible]	.032

BALLISTIC DATA FOR CAL. 0.50 API MARK 8 FIRED FORWARD FROM AN AIRPLANE (Cont'd)

V_A = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2870 f.p.s.

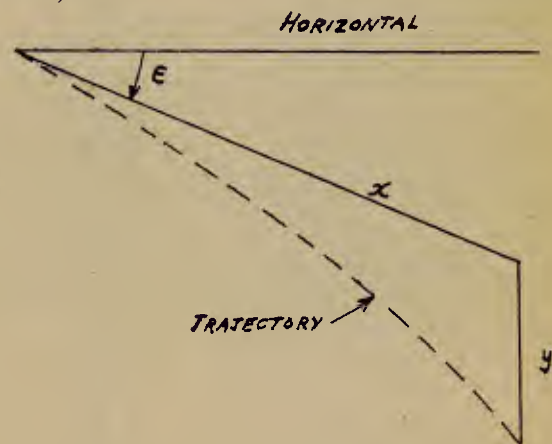
σx ft.	$\sigma^2 y$ ft.	σt_{f_o} sec.	$\sigma^2 \Delta t_f$ sec.
4100	40.1	1.764	
4200	42.7	1.826	
4300	45.4	1.889	
4400	48.3	1.953	
4500	51.3	2.019	.050
4600	54.4	2.086	
4700	57.7	2.154	
4800	61.2	2.224	
4900	64.8	2.295	
5000	68.5	2.368	.076
5100	72.5	2.443	
5200	76.6	2.519	
5300	80.9	2.596	
5400	85.4	2.676	
5500	90.1	2.757	.113
5600	95.1	2.840	
5700	100.2	2.924	
5800	105.6	3.011	
5900	111.3	3.099	
6000	117.2	3.189	.159
6100	123.3	3.281	
6200	129.8	3.374	
6300	136.5	3.468	
6400	143.5	3.564	
6500	150.8	3.660	.202
6600	158.4	3.757	
6700	166.3	3.856	
6800	174.5	3.955	
6900	183.1	4.053	
7000	192.0	4.155	.226
7100	201.1	4.256	
7200	210.7	4.359	
7300	220.5	4.462	
7400	230.7	4.565	
7500	241.3	4.670	.262

Ordnance Research and
Development Center
Project No. 4005

BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4 FIRED FORWARD FROM AN AIRPLANE FROM 20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.)

1. Definitions:

The gun is fired along the line of flight of an airplane flying with speed V_A, at dive angle ε.



x = distance along line of fire.

y = drop from line of fire.

$t_f = t_{f_o} + \Delta t_f \sin \varepsilon$ = time of flight.

t_{f_o} = time of flight for $\varepsilon = 0$.

$\Delta t_f \sin \varepsilon$ = correction for variation of density.

σ = density ratio = density at muzzle ÷ standard density at sea level.

2. Assumptions:

Temperature = 59° F.

Drift = 0.

BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4,
FIRED FORWARD FROM AN AIRPLANE FROM
20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.)

V_A = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2750 f.p.s.

σx Yds.	$\sigma^2 y$ Yds.	σt_{f_0} Sec.	$\sigma^2 \Delta t_f$ Sec.
50	.0	.042	
100	.0	.084	
150	.1	.128	
200	.2	.173	.0001
250	.3	.219	
300	.4	.265	
350	.5	.313	
400	.7	.362	.0005
450	.9	.412	
500	1.1	.464	
550	1.3	.517	
600	1.6	.572	.0018
650	1.9	.628	
700	2.3	.685	
750	2.7	.744	
800	3.1	.805	.0048
850	3.5	.867	
900	4.0	.931	
950	4.6	.997	
1000	5.2	1.066	.0107
1050	5.8	1.136	
1100	6.5	1.208	
1150	7.3	1.282	
1200	8.1	1.359	.0210
1250	9.0	1.439	
1300	9.9	1.521	
1350	10.9	1.605	
1400	12.1	1.692	.0382
1450	13.3	1.782	
1500	14.6	1.876	.0503
1550	16.0	1.973	
1600	17.4	2.072	.0654
1650	19.0	2.175	
1700	20.7	2.282	.0839
1750	22.6	2.393	
1800	24.6	2.507	.1062
1850	26.7	2.625	
1900	28.9	2.747	.1321
1950	31.4	2.873	
[illegible]	34.0	3.004	.1597

BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4,
FIRED FORWARD FROM AN AIRPLANE FROM
20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.) (Cont'd)

V_A = Velocity of Airplane = 600 m.p.h.
Muzzle Velocity = 2750 f.p.s.

σx Yds.	$\sigma^2 y$ Yds.	σt_{f_o} Sec.	$\sigma^2 \Delta t_f$ Sec.
2050	36.8	3.138	
2100	39.8	3.274	.1852
2150	43.0	3.413	
2200	46.4	3.556	.2086
2250	50.0	3.701	
2300	53.9	3.849	.2317
2350	58.0	3.999	
2400	62.4	4.150	.2554
2450	67.0	4.304	
2500	71.8	4.461	.2806
2550	76.9	4.619	
2600	82.3	4.780	.3083
2650	88.0	4.943	
2700	94.0	5.109	.3385
2750	100.3	5.277	
2800	106.8	5.446	.3716
2850	113.7	5.618	
2900	120.9	5.793	.4082
2950	128.4	5.970	
3000	135.3	6.150	.4491

BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4
FIRED FORWARD FROM AN AIRPLANE FROM
20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.)

V_A = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2750 f.p.s.

σx Yds.	$\sigma^2 y$ Yds.	σt_{f_o} Sec.	$\sigma^2 \Delta t_f$ Sec.
50	.0	.047	
100	.0	.096	
150	0.1	.146	
200	0.2	.197	.0001
250	0.3	.250	
300	0.5	.304	
350	0.7	.359	
400	0.9	.416	.0006
450	1.1	.475	
500	1.4	.535	
550	1.7	.597	
600	2.1	.661	.0023
650	2.5	.727	
700	3.0	.794	
750	3.5	.864	
800	4.1	.935	.0061
850	4.7	1.009	
900	5.4	1.085	
950	6.1	1.164	
1000	6.9	1.245	.0136
1050	7.8	1.329	
1100	8.8	1.415	
1150	9.9	1.504	
1200	11.0	1.596	.0268
1250	12.2	1.692	
1300	13.6	1.791	
1350	15.0	1.893	
1400	16.6	1.999	.0488
1450	18.3	2.109	
1500	20.1	2.222	.0640
1550	22.0	2.339	
1600	24.1	2.460	.0821
1650	26.4	2.585	
1700	28.8	2.714	.1021
1750	31.4	2.847	
1800	34.3	2.983	.1214
1850	37.3	3.122	
1900	40.6	3.263	.1391
1950	44.1	3.407	
2000	47.7	3.554	.1565

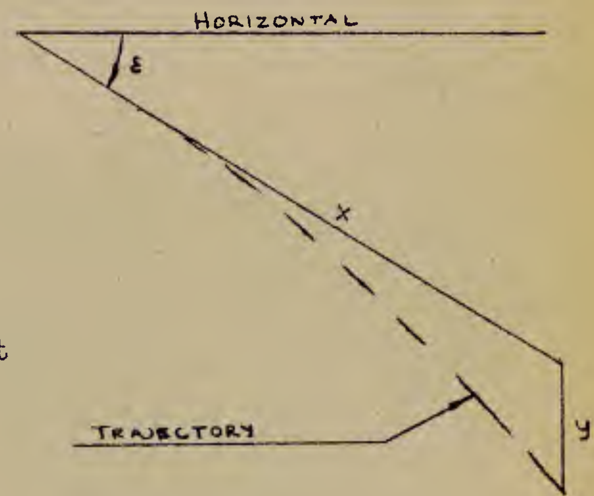
BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4
FIRED FORWARD FROM AN AIRPLANE FROM
20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.) (Cont'd)

V_A = Velocity of Airplane = 300 m.p.h.
Muzzle Velocity = 2750 f.p.s.

σx Yds.	$\sigma^2 y$ Yds.	σt_{f_o} Sec.	$\sigma^2 \Delta t_f$ Sec.
2050	51.6	3.703	.1747
2100	55.3	3.854	
2150	60.2	4.008	.1942
2200	64.9	4.165	
2250	69.8	4.323	.2156
2300	75.0	4.482	
2350	80.5	4.644	.2393
2400	86.2	4.809	
2450	92.2	4.976	.2654
2500	98.6	5.145	
2550	105.3	5.317	.2948
2600	112.3	5.491	
2650	119.6	5.667	.3276
2700	127.2	5.846	
2750	135.2	6.027	.3639
2800	143.6	6.211	
2850	152.3	6.398	.4046
2900	161.4	6.587	
2950	170.9	6.779	.4502
3000	180.8	6.974	

Ordnance Research Center
Project No. 4005

BALLISTIC DATA FOR 37MM H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37MM GUN MK. 9



1. Definitions:

The gun is fired along the line of flight of an airplane flying with speed V_A, at dive angle ε.

$x =$ distance along line of fire

$y =$ drop from line of fire

$t_f = t_{f_o} + \Delta t_f \sin \varepsilon =$ time of flight

$t_{f_o} =$ time of flight for $\varepsilon = 0$

$\Delta t_f \sin \varepsilon =$ correction for variation of density

$\sigma =$ density ratio $=$ density $\div$ standard density

2. Assumptions:

Temperature $= 59°F$.

Drift $= 0$

3. Accuracy:

At $(\sigma x) \leqq 2000$ yds., the error in $(\sigma^2 y)$ is $< .5$ yd., the error in $(\sigma t_f) < .02$ secs. Beyond $\sigma x = 2000$ yds., accuracy falls off badly, and the table cannot be considered to represent the facts adequately.

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM. GUN, MK. 9 (CONT'D)

V_A = Velocity of Airplane = 600 m.p.h.
Muzzle Velocity = 2550 f.p.s.

σx Yds.	$\sigma^2 y$ Yds.	σt_{f_o} Sec.	$\sigma^2 \Delta t_f$ Sec.
100	.04	.089	
200	.17	.181	
300	.39	.276	
400	.72	.374	
500	1.15	.477	.0007
600	1.70	.583	
700	2.37	.692	
800	3.18	.806	
900	4.13	.924	
1000	5.24	1.046	.0066
1100	6.51	1.174	
1200	7.96	1.305	
1300	9.60	1.441	
1400	11.45	1.584	
1500	13.54	1.732	.0268
1600	15.86	1.887	
1700	18.45	2.047	
1800	21.34	2.215	
1900	24.52	2.389	
2000	28.06	2.571	.0789
2100	31.96	2.762	
2200	36.27	2.961	
2300	41.04	3.170	
2400	46.28	3.389	
2500	52.06	3.616	.1957
2550	55.16	3.737	
2600	58.43	3.856	
2650	61.85	3.981	
2700	65.43	4.108	.2697
2750	69.20	4.236	
2800	73.16	4.371	
2850	77.31	4.506	
2900	81.66	4.645	.3543

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM. GUN MK. 9 (CONT'D)

V_A = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2550 f.p.s.

σx Yds.	$\sigma^2 y$ Yds.	σt_{f_o} Sec.	$\sigma^2 \Delta t_f$ Sec.
2950	86.22	4.786	
3000	90.99	4.928	
3050	96.00	5.073	
3100	101.22	5.223	.4235
3150	106.70	5.373	
3200	112.41	5.525	
3250	118.38	5.679	
3300	124.59	5.833	.4878
3350	131.07	5.991	
3400	137.81	6.148	
3450	144.83	6.308	
3500	152.11	6.469	.5538
3550	159.68	6.631	
3600	167.53	6.795	
3650	175.67	6.959	
3700	184.11	7.126	.6254
3750	192.82	7.294	
3800	201.88	7.464	
3850	211.22	7.633	
3900	220.86	7.801	.6987
3950	230.83	7.970	
4000	241.12	8.141	
4050	251.74	8.315	
4100	262.69	8.494	.7800
4150	273.97	8.676	
4200	285.60	8.855	
4250	297.56	9.037	
4300	309.88	9.219	.8704
4350	322.57	9.401	
4400	335.60	9.584	
4450	349.01	9.772	
4500	362.79	9.960	.9703

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM. GUN MK. 9 (CONT'D)

V_A = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2550 f.p.s.

σx Yds.	$\sigma^2 y$ Yds.	σt_{f_o} Sec.	$\sigma^2 \Delta t_f$ Sec.
100	.06	.102	
200	.27	.208	
300	.52	.318	
400	.95	.432	
500	1.53	.550	.0008
600	2.25	.672	
700	3.15	.799	
800	4.21	.930	
900	5.48	1.066	
1000	6.95	1.208	.0080
1100	8.65	1.357	
1200	10.56	1.511	
1300	12.08	1.671	
1400	15.29	1.838	
1500	18.11	2.013	.0332
1600	21.26	2.195	
1700	24.77	2.386	
1800	28.70	2.585	
1900	33.07	2.793	
2000	37.92	3.010	.1000
2100	43.31	3.238	
2200	49.28	3.478	
2300	55.90	3.728	
2400	63.22	3.990	
2500	71.31	4.265	.2356
2550	75.66	4.406	
2600	80.22	4.549	
2650	85.02	4.695	
2700	90.05	4.843	.2937
2750	95.30	4.993	
2800	100.81	5.145	
2850	106.57	5.299	
2900	112.58	5.454	.3448

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM. GUN, MK. 9 (CONT'D)

V_A = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2550 f.p.s.

σx Yds.	$\sigma^2 y$ Yds.	σt_{f_o} Sec.	$\sigma^2 \Delta t_f$ Sec.
2950	118.84	5.611	
3000	125.38	5.769	
3050	132.18	5.925	
3100	139.26	6.085	.3931
3150	146.62	6.250	
3200	154.26	6.413	
3250	162.19	6.578	
3300	170.42	6.745	.4435
3350	178.93	6.910	
3400	187.76	7.077	
3450	196.88	7.247	
3500	206.32	7.419	.5005
3550	216.07	7.592	
3600	226.15	7.765	
3650	236.55	7.939	
3700	247.28	8.115	.5647
3750	258.35	8.293	
3800	269.75	8.472	
3850	281.50	8.652	
3900	293.61	8.834	.6375
3950	306.06	9.018	
4000	318.89	9.203	
4050	332.07	9.388	
4100	345.64	9.574	.7203
4150	359.57	9.762	
4200	373.90	9.953	
4250	388.60	10.146	
4300	403.72	10.339	.3148
4350	419.22	10.532	
4400	435.14	10.726	
4450	451.47	10.922	
4500	468.22	11.122	.9227

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM GUN, MK. 9 (CONT'D)

Temperature Effect on Time of Flight

t_f (at Temp. T) $= t_{f_0} + \sin\varepsilon\, \Delta t_f + (\lambda - 1)\, \delta t_f$

$\lambda = \sqrt{\dfrac{\text{Standard Temp., Abs.}}{\text{Temp., Abs.}}}$

Standard Temperature $= 59^\circ$ F.

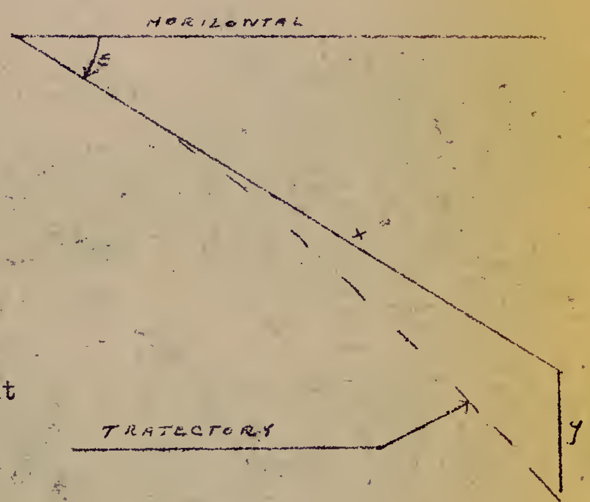
[Thus the λ corresponding to 0° F. is $\sqrt{\dfrac{518.4}{459.4}} = 1.064$]

$\sigma\, \delta t_f$
(Sec.)

σx (Yds.) \ V_A (m.p.h.)	300	600
500	-.01	.00
1000	-.08	-.02
1500	-.22	-.11
2000	-.51	-.28
2500	-.87	-.62
2700	-.82	-.78
2900	-.60	-.90
3100	-.29	-.85
3300	.07	-.63
3500	.45	-.32
3700	.85	.03
3900	1.26	.42
4100	1.68	.80
4300	2.12	1.22
4500	2.57	1.64

Ordnance Research Center
Project No. 4005

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1.



1. **Definitions:**

The gun is fired along the line of flight of an airplane flying with speed V_A, at dive angle ϵ.

x = distance along line of fire

y = drop from line of fire

$t_f = t_{f_0} + \Delta t_f \sin \epsilon$ = time of flight

t_{f_0} = time of flight for $\epsilon = 0$

$\Delta t_f \sin \epsilon$ = correction for variation of density

σ = density ratio = density at muzzle ÷ standard density at sea level

2. **Assumptions:**

Temperature = 59°F.

Drift = 0

3. **Accuracy:**

Through $\sigma x = 4500$ yds., σt_{f_0} is correct to 0.02 secs. and $\sigma^2 y$ to 1 yd. Beyond $\sigma x = 4500$ yds., the accuracy falls off so that by $\sigma x = 6000$ yds., σt_{f_0} is in error by about 0.07 secs. and $\sigma^2 y$ by 3.5 yds.

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1, (CONT'D)

V_A = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 1930 f.p.s.

σx yds.	$\sigma^2 y$ yds.	σt_{f_0} sec.	$\sigma^2 \Delta t_f$ sec.
100	0.1	0.106	
200	0.2	0.214	
300	0.6	0.324	
400	1.0	0.435	
500	1.6	0.548	.0003
600	2.3	0.661	
700	3.1	0.776	
800	4.1	0.892	
900	5.3	1.010	
1000	6.6	1.128	.0023
1100	8.0	1.249	
1200	9.6	1.371	
1300	11.4	1.495	
1400	13.3	1.622	
1500	15.4	1.748	.0083
1600	17.7	1.878	
1700	20.2	2.008	
1800	22.8	2.140	
1900	25.6	2.274	
2000	28.7	2.411	.0208
2100	31.9	2.548	
2200	35.3	2.687	
2300	39.0	2.827	
2400	42.8	2.971	
2500	46.9	3.117	.0439
2600	51.2	3.262	
2700	55.8	3.412	
2800	60.6	3.563	
2900	65.6	3.716	
3000	70.9	3.871	.0806

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

V_A = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 1950 f.p.s.

σx yds.	$\sigma^2 y$ yds.	σt_{f_o} sec.	$\sigma^2 \Delta t_f$ sec.
3100	76.4	4.028	
3200	82.3	4.186	
3300	88.4	4.348	
3400	94.7	4.513	
3500	101.4	4.680	.1364
3600	108.4	4.847	
3700	115.6	5.018	
3800	123.2	5.191	
3900	131.2	5.367	
4000	139.4	5.545	.2189
4100	148.0	5.726	
4200	157.0	5.908	
4300	166.3	6.097	
4400	176.0	6.285	
4500	186.1	6.476	.3344
4600	196.6	6.672	
4700	207.5	6.867	
4800	218.8	7.066	
4900	230.5	7.270	
5000	242.7	7.477	.4905
5050	249.0	7.582	
5100	255.4	7.686	
5150	261.9	7.791	
5200	268.5	7.897	.5661
5250	275.3	8.005	
5300	282.2	8.113	
5350	289.2	8.221	
5400	296.3	8.331	.6502
5450	303.6	8.442	
5500	310.9	8.553	

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

V_A = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 1950 f.p.s.

σx yds.	$\sigma^2 y$ yds.	σt_{f_0} sec.	$\sigma^2 \Delta t_f$ sec.
5550	318.5	8.666	
5600	326.1	8.779	.7429
5650	333.9	8.892	
5700	341.9	9.005	
5750	349.9	9.120	
5800	358.2	9.182	.8450
5850	366.5	9.352	
5900	375.0	9.470	
5950	383.7	9.590	
6000	392.5	9.710	.9571

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M55, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

V_A = Velocity of Airplane = 300 m.p.h.
Muzzle Velocity = 1950 f.p.s.

σx yds.	$\sigma^2 y$ yds.	σt_{f_0} sec.	$\sigma^2 \Delta t_f$ sec.
100	0.1	0.126	
200	0.3	0.253	
300	0.8	0.384	
400	1.4	0.516	
500	2.2	0.647	.0005
600	3.2	0.784	
700	4.4	0.919	
800	5.8	1.057	
900	7.4	1.199	
1000	9.2	1.340	.0031
1100	11.2	1.484	
1200	13.5	1.630	
1300	16.0	1.777	
1400	18.7	1.929	
1500	21.7	2.079	.0112
1600	24.9	2.234	
1700	28.4	2.391	
1800	32.2	2.548	
1900	36.2	2.708	
2000	40.5	2.871	.0272
2100	45.1	3.036	
2200	49.9	3.203	
2300	55.1	3.372	
2400	60.6	3.544	
2500	66.4	3.718	.0559
2600	72.6	3.895	
2700	79.1	4.075	
2800	85.9	4.255	
2900	93.1	4.441	
3000	100.7	4.630	.1033

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

V_A = Velocity of Airplane = 300 m.p.h.
Muzzle Velocity = 1950 f.p.s.

σx yds.	$\sigma^2 y$ yds.	σt_{f_0} sec.	$\sigma^2 \Delta t_f$ sec.
3100	108.6	4.819	
3200	116.9	5.012	
3300	125.7	5.208	
3400	134.8	5.404	
3500	144.4	5.606	.1747
3600	154.5	5.812	
3700	165.0	6.019	
3800	175.9	6.228	
3900	187.4	6.442	
4000	199.3	6.660	.2776
4100	211.7	6.879	
4200	224.7	7.103	
4300	238.2	7.329	
4400	252.2	7.557	
4500	266.9	7.789	.4173
4600	282.1	8.027	
4700	298.0	8.265	
4800	314.4	8.509	
4900	331.5	8.755	
5000	349.3	9.007	.6021
5050	358.4	9.133	
5100	367.7	9.260	
5150	377.2	9.388	
5200	386.8	9.516	.6897
5250	396.7	9.646	
5300	406.7	9.777	
5350	416.9	9.909	
5400	427.3	10.042	.7839
5450	437.9	10.175	
5500	448.7	10.308	

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

V_A = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 1950 f.p.s.

σx yds.	$\sigma^2 y$ yds.	σt_{f_o} sec.	$\sigma^2 \Delta t_f$ sec.
5550	459.6	10.445	
5600	470.8	10.581	.8853
5650	482.2	10.718	
5700	493.7	10.854	
5750	505.5	10.994	
5800	517.5	11.134	.9820
5850	529.7	11.274	
5900	542.1	11.414	
5950	554.7	11.554	
6000	567.5	11.696	1.0571