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**BRL Report No.** 491

ON JUMP DUE TO BORE CLEARANCE

BY

T. E. STERNE





28 SEPTEMBER 1944

**ABERDEEN PROVING GROUND, MARYLAND**

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28 September 1944

# ON JUMP DUE TO BORE CLEARANCE

## Abstract

It is shown that if there is a yaw $\varepsilon$ at the muzzle because of bore clearance, then there will be a jump $y_0$ given by

$$y_0 = \varepsilon \frac{AN}{md} \frac{K_L}{K_M} \frac{1}{u} \left(\frac{B}{A} - 1\right)$$

where A and B are the axial and transverse moments of inertia, m the mass, d the caliber, $K_L$ and $K_M$ the cross wind force coefficient and moment coefficient, u the muzzle velocity, and N the spin.

It has been observed by Mr. Kent that as a consequence of the preceding relation, the most accurate projectile at close ranges, for any given initial yaw $\varepsilon$, will be one whose design minimizes the ratio $(B^3/K_M)$ and that is fired from a gun whose twist of rifling is just rapid enough to render the projectile barely stable. With any given projectile a twist of rifling, any greater than is necessary to render the projectile barely stable, should result in needless inaccuracy. The stability factor must be, of course, enough larger than unity under standard conditions to insure stability under actual conditions.

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1. If there is an angle $\varepsilon$ between the axis of a projectile and the axis of the bore of a rifled gun at the instant when the projectile clears the muzzle, it is well known* that there results a yaw whose maximum value is

$$\left(2\frac{B}{A} - 1\right)\frac{\varepsilon}{(1-1/s)^{1/2}},$$

provided that $s$ exceeds unity. In the preceding expression B is the projectile's transverse moment of inertia and A its axial moment of inertia, while $\underline{s}$ is the stability factor. The lift forces, arising from the preceding yaw, give rise to a "jump". The amount of the jump will here be rigorously evaluated. A more elementary and less rigorous treatment, yielding the same result, will be given in an appendix.

2. The situation may be considered in the following way. During the flight of any particular round, there is a yawing motion of its axis. This yawing motion consists of a rapid vibration of the instantaneous direction of the axis of the projectile, with respect to a slowly changing average direction. The lift force, arising from the yaw, in turn causes the instantaneous direction of motion of the projectile's center of mass to execute rapid vibrations about a slowly changing average, or "mean" direction of motion. This mean direction of motion is peculiar to each particular round. An actual trajectory somewhat resembles a helix, described about a gradually curving axis; the axis of the helix may be termed the "mean" trajectory, and its direction the "mean" direction of motion.

Just after the projectile has cleared the muzzle, the instantaneous direction of motion of the center of mass is in the process of executing a vibration. At this instant there is therefore a definite angle between the slowly changing mean direction of motion, and the rapidly changing, instantaneous, direction of motion. The angle is in fact the magnitude of the vibration at that instant. The mean direction of motion of any particular round, at the instant of clearing the muzzle, is the ordinary line of departure as determined by a jump-card, not too close, after due allowance for gravitational drop. It determines where the trajectory will intersect a target surface not too close to the gun. On the other hand, the instantaneous direction of motion just after the projectile clears the muzzle is the direction of the bore axis. Since the angle between the bore axis and the line of departure is customarily called "jump", it is clear that the yawing motion arising from bore clearance must produce a jump, whose magnitude is given by the instantaneous value, just after the projectile has cleared the muzzle, of the vibration then affecting the instantaneous direction of motion.

* See, for instance, T. J. Hayes, "Elements of Ordnance", Wiley and Sons, 1938.

It may be inferred from the preceding general considerations that the amount of the jump will not depend appreciably upon the rate of damping, or even upon whether or not there is damping. The vibrations are described by periodic terms, which may or may not be multiplied by decay factors of the form $\exp(-\lambda t)$, $t$ being the time. There are no terms other than periodic terms (that is, there are no constant or secular terms) provided that the mean direction with respect to which yaw is measured, and the mean direction of motion with respect to which the instantaneous direction of motion is measured, have been properly chosen. The initial values of the vibrations are not influenced appreciably by the rates of damping, which govern merely the rapidity with which the periodic terms, and the associated vibrations, decay. With no damping, or with slow damping, the instantaneous direction of motion will continue to vibrate for a long time about the slowly changing mean direction of motion, while with rapid damping the instantaneous direction of motion rapidly approaches the mean direction of motion as a limit. The initial value, however, of the vibration affecting the direction of motion constitutes in all cases the jump due to bore clearance, and hence cannot depend appreciably upon the damping rates.

## The Yawing Motion

3. The following discussion of the yawing motion is taken largely from BRL Report 345, "The Effect of Yaw upon Aircraft Gunfire Trajectories". Consider a set of right-handed axes moving with the tangent to the mean trajectory of any particular round, so that the axis 01 is the tangent to the mean trajectory drawn in the direction of motion, while 02 is the upward normal and 03 is horizontal and to the right, as viewed from the gun. Denote by $l$, $m^*$, $n$ the direction cosines of the bullet's axis, and by $x$, $y$, $z$ the direction cosines of the velocity vector of the center of mass. One may set $m^* - y = j$, and $n - z = k$. For small angles of yaw, one has very nearly

$$j = \delta \cos \varphi$$

$$k = \delta \sin \varphi$$

where $\delta$ is the angle of yaw, and $\varphi$ is the angle of orientation of the yaw. The angles $\delta$ and $\varphi$ are thus referred to the instantaneous direction of motion of the center of mass, and are appropriate to yaw-card measurements; while $j$ and $k$ are the "rectangular components" (in the direction of the upward normal, and horizontally to the right, respectively) of the yaw. That is to say, $j$ and $k$ are the rectangular coordinates of a point whose plane polar coordinates are $\delta$, $\varphi$.

The discussion of Fowler*, *et al.* (their equation 4.01) shows that if the stability factor $s$ exceeds a number slightly larger than unity, which may perhaps be taken to be about 1.1, and if

* Fowler, Gallop, Lock and Richmond, Phil. Trans. Royal Soc. A, Vol. 221, p. 295, 1920.

the yaw is not larger than ten or fifteen degrees, then j and <u>k</u> are the real and imaginary parts of a complex variable $\eta$ which varies according to the law

$$\eta = K_1 e^{in_1 t - \lambda_1 t} + K_2 e^{in_2 t - \lambda_2 t} \tag{1}$$

where $K_1$ and $K_2$ are two complex constants, whose values are determined by the initial conditions. In this equation

$$\begin{aligned} n_1 &= (AN/2B)(1 + p) \\ n_2 &= (AN/2B)(1 - p) \end{aligned} \tag{2}$$

where

$$p = (1 - 1/s)^{1/2}.$$

Here <u>N</u> is the spin in radians per second reckoned positive if right-handed, and <u>s</u> is the stability factor

$$s = A^2N^2/4B\mu.$$

where $\mu$ is the moment factor

$$\mu = \rho u^2 d^3 K_M$$

in which $\rho$ is the air density, <u>u</u> is the velocity of the bullet, $K_M$ is the overturning moment coefficient, and <u>d</u> is the diameter of the bullet. The damping rates $\lambda_1$ and $\lambda_1$ are given substantially by

$$\lambda_1 = (1/2p)(dp/dt) + \frac{f+\kappa}{2} + \frac{f-\kappa+2\gamma}{2p}$$

and (3)

$$\lambda_2 = (1/2p)(dp/dt) + \frac{f+\kappa}{2} - \frac{f-\kappa+2\gamma}{2p}$$

in which

$$f = \rho\, ud^4 K_H/B$$

$$\kappa = \rho\, ud^2 K_L/m$$

$$\gamma = \rho\, ud^4 K_J/A$$

where <u>m</u> is the mass of the projectile, and $K_H$, $K_L$, and $K_J$ are the dimensionless yawing moment, cross wind force, and Magnus moment coefficients. Although it is now known that the treatment of Fowler, <u>et.al.</u> was logically defective in that some aerodynamic forces and couples that probably occur were not considered,



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nevertheless a recent treatment by J. L. Kelley and E. J. McShane, BRL Report No. 446, has confirmed that equation (1) does hold, with the values of $n_1$ and $n_2$ given by (2), and with values of $\lambda_1$ and $\lambda_2$ believed to agree closely with those given by equations (3).

From equation (1), it is clear that the motion about the center of mass is the resultant of two fundamental and independent motions, with arbitrarily disposable amplitudes and phases. The first of these is a motion in a circle in the j-k plane, clockwise at the uniform angular rate $n_1$. The center of the circle is the origin, and the radius of the circle diminishes in accordance with the law $e^{-\lambda_1 t}$. The second is a motion in a circle in the j-k plane, clockwise at the uniform angular rate $n_2$. The center of the circle is the origin, and the radius diminishes in accordance with the law $e^{-\lambda_2 t}$. The first motion is always at a faster rate than the second.

## Geometrical Representation of the Yawing Motion

4. The motion about the center of mass has a simple geometrical interpretation, which facilitates the clear conception of the nature of the yawing motion. For the moment, let the damping rates $\lambda$ be ignored. Then by equation (1), j and <u>k</u> are the rectangular coordinates of a point <u>R</u> which is rotating in a clockwise direction at the (algebraic) angular rate $n_1$ in a circular path of radius $a_1$, whose center is a point <u>S</u> that describes a circle of radius $a_2$ clockwise around the origin at an angular rate $n_2$ (algebraically).

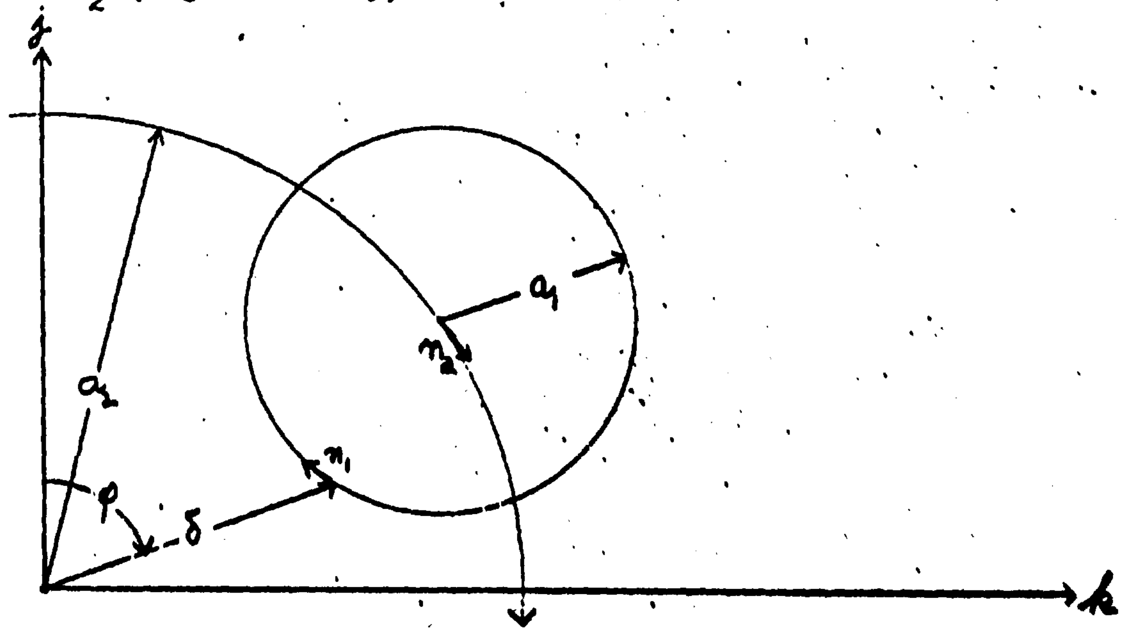


Fig. 1

The clockwise angle, from the j-axis to the radius vector to R, is the angle of orientation, $\varphi$, of the yaw; and the distance of R from the origin is the angle of yaw, $\delta$. It is seen that the motion is merely epicyclic, to the accuracy of Fowler et.al.'s analysis. The part of the motion involving $n_1$, namely the rotation in a circle of radius $a_1$ at a rate $n_1$, may be called the "nutation"; the rotation in a circle of radius $a_2$ at the slower rate $n_2$ may be called "precession"; the resultant of the nutation and precession is the complete yawing motion. The terminology here introduced (which has been used also in BRL Report No. 345) is at variance with that of some ballisticians in the past, who have applied the term "nutation" to the periodic variation of the angle $\delta$ which involves $n_1-n_2$, thus

$$\delta^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(n_1-n_2)t \qquad (4)$$

if the time, t, is measured from a suitable instant. The present terminology appears to be more in keeping, however, with the dynamical situation, and has long been used by astronomers in describing the yawing motion of the earth about its center of mass (the long-period motion being termed "precession", and the superimposed faster motions being termed "nutation").

One may call $a_1$ the "amplitude" of the nutation, and $a_2$ the amplitude of the precession. It is now clear that the occurrence of damping rates $\lambda$ does not alter the geometrical construction that has been given, but merely involves the gradual variation of the two amplitudes. The amplitude of the nutation is thus some constant multiplied by $e^{-\lambda_1 t}$,

while the amplitude of the precession is some other constant multiplied by $e^{-\lambda_2 t}$; and the general, resultant, motion may therefore be described as being damped epicyclic so long as s is not so small, or the amplitudes so large, as to invalidate the theory on which equation (1) rests.

### The Vibration of the Direction of Motion of the Center of Mass

5. Associated with the yawing motion about the center of mass there is a motion of the center of mass itself about the mean trajectory. The lift force acting on the center of mass in the O2 direction is

$$\rho u^2 d^2 K_L \delta$$

so that the acceleration in the O2 direction due to lift is

$$\rho u^2 d^2 K_L \delta/m.$$

This acceleration may also be written, however, as

$$(d/dt)\ yu = \dot{y}u + y\dot{u}$$

$$= \dot{y}u$$

very nearly, since $y$ and $\dot{u}$ are each of the first order of small quantities so that the term $y\dot{u}$ is of the second order, and is as small as other terms ignored in the treatments of Fowler et al., and of Kelly and McShane. Hence

$$\dot{y} = \rho\, ud^2 K_L\ j/m$$

$$= \varkappa j.$$

By the same argument,

$$\dot{z} = \varkappa k.$$

Hence it follows that the O2 and O3 direction cosines, y and z, of the vector velocity of the center of mass are the real and imaginary parts, respectively, of the time integral of $\varkappa\eta$, namely

$$y + iz = \frac{\varkappa K_1}{in_1 - \lambda_1}\, e^{in_1 t - \lambda_1 t} + \frac{\varkappa K_2}{in_2 - \lambda_2}\, e^{in_2 t - \lambda_2 t} \qquad (5)$$

The $\lambda$'s are very small compared with the n's, and thus it is clear that in general $y$ and z perform vibrations having the same periods as the vibrations in j and k, with amplitudes equal to the amplitudes of the j and k vibrations multiplied by $\varkappa/n_1$ for the nutation and $\varkappa/n_2$ for the precession, and with phases that are smaller by 90° (at any time) than the phases of the j, k vibrations. These results follow also from equation (3.215) of Fowler et. al. They are accurate if the inclination of the mean trajectory changes only slowly with the time.

### The Effect of Yaw in the Bore

6. Due to bore clearance, a projectile will have some initial yaw, $\varepsilon$, at the instant it clears the muzzle. To fix matters, suppose that the orientation angle of this yaw is 270°. The rate of increase of $\delta$ at this instant is zero, but the rate of increase of the orientation angle $\varphi$ at this instant is the spin, N. Thus the initial conditions are

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$$\delta = \varepsilon \qquad \varphi = 270°,$$

$$\dot{\delta} = 0, \qquad \dot{\varphi} = N.$$

The initial situation must therefore be as shown in the diagram below,

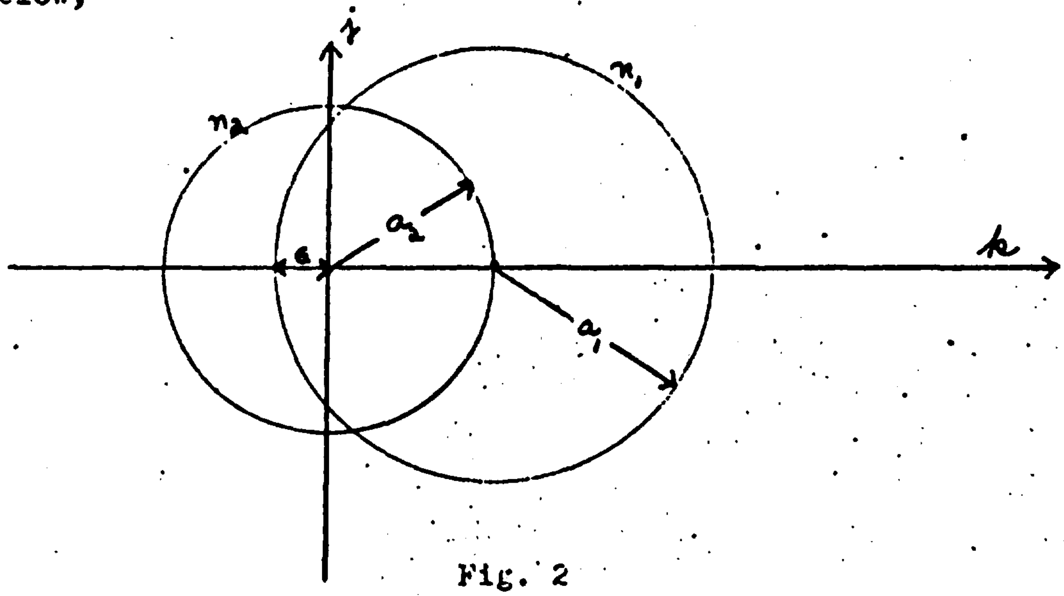


Fig. 2

and it follows from the initial conditions that at the instant of clearing the muzzle,

$$a_1 - a_2 = \varepsilon$$

$$n_1 a_1 - n_2 a_2 = \varepsilon N$$

whence
$$a_1 = \varepsilon \frac{N-n_2}{n_1-n_2}$$

$$a_2 = \varepsilon \frac{N-n_1}{n_1-n_2}$$

with a phase 90° for the precessional motion and a phase 270° for the nutational motion, phases being measured, like the orientation angle $\varphi$, clockwise from the j-axis. Hence the maximum yaw, $a_1 + a_2$, is given by

$$a_1 + a_2 = \varepsilon \frac{2N-n_1-n_2}{n_1-n_2}$$

$$= \varepsilon\left(2\cdot\frac{B}{A} - 1\right)\frac{1}{(1-1/s)^{1/2}}, \qquad (6)$$

a well known relation, by equations (2).

From the general relation that was stated in paragraph 5, it follows that the initial phase of the precessional component of the center of mass must be zero degrees (90°-90°), and of the nutational component 180°(270°-90°). Hence $\underline{z}$ must initially be zero. For $\underline{y}$ the initial value is clearly

$$y_o = (\kappa a_2/n_2) - (\kappa a_1/n_1)$$

$$= \kappa\varepsilon \, \frac{N-n_1-n_2}{n_1 n_2}$$

or

$$y_o = \kappa\varepsilon \, \frac{4BS}{AN}\left(\frac{B}{A} - 1\right) \tag{7}$$

by equations (2). Replacing $\kappa$ by its expression in paragraph 3, one finds

$$y_o = \varepsilon\rho \, \frac{4BS}{ANm} \, d^2 \, K_L u \left(\frac{B}{A} - 1\right) \tag{8}$$

and if one replaces $\underline{s}$ by its expression given in paragraph 3 one obtains, finally,

$$y_o = \varepsilon \, \frac{AN}{md} \, \frac{K_L}{K_M} \, \frac{1}{u} \left(\frac{B}{A} - 1\right) . \tag{9}$$

It will be noticed that if the initial yaw, $\varepsilon$, due to bore clearance is to the left, then the initial vibration of the direction of motion (characterized by a zero value of $\underline{z}$, in the $\underline{k}$-direction, and by the value of $\underline{y}$, in the j-direction, that is given by equation (9)) is in the direction of the upward normal. The instantaneous value of the vibration of the direction of motion is thus given by (9), in the same units as $\varepsilon$. It follows from the discussion of paragraph 2 that the yaw $\varepsilon$ to the left has produced a jump, in the direction of the downward normal, given by equation (9). In general, the orientation angle of the jump will always be 90° less than that of the initial yaw $\varepsilon$, at the muzzle, and whatever the orientations may be, equation (9) always furnishes the relation between the magnitude of the initial yaw, $\varepsilon$, and the resulting jump.

## Projectile Design

7. It has been pointed out by Mr. R. H. Kent that in consequence of equation (9) the smallest jump of any particular projectile will result from a given initial yaw $\varepsilon$ if the twist of the rifling is as small as possible, consistently with a value of $\underline{s}$ only slightly larger than unity. A greater twist

than this will result in unnecessarily inaccurate fire, at least at close ranges. Further, the best design of projectile for accuracy at close ranges should be one having a small ratio $(B^3/K_M)$, and the rifling should then have a twist just sufficiently rapid to render the projectile stable. This can be seen from equation (9). If $B$ and $K_M$ are varied, while $N$ is varied simultaneously in such a manner as to keep the stability factor $\underline{s}$ constant and slightly larger than unity, then $N$ must be made to vary like the square root of $BK_M$. Hence, since $B/A$ is large compared to unity, the jump for any given $\varepsilon$ will vary like $(B^3/K_M)^{1/2}$, which should be made small by a suitable design.

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## APPENDIX

### Elementary Evaluation of the Jump Due to Bore Clearance

8. Denote the angle of yaw by $\delta$, the angle of orientation of the yaw by $\varphi$, and the initial angle of yaw just after clearing the muzzle by $\varepsilon$. Any particular state of yaw may be represented by a representative point, whose polar coordinates are $\delta$ and $\varphi$, in a $\delta,\varphi$ plane. In the present elementary treatment, damping of the yaw will be neglected. Then it is known that if the stability factor s exceeds unity, the motion of the representative point in the $\delta$, $\varphi$ plane is damped epicyclic. The representative point rotates in a clockwise direction (see Figure 1) in a circle of arbitrarily disposable radius $a_1$ at an angular rate

$$n_1 = \frac{AN}{2B}(1 + p)$$

radians per second, while the center of this circle rotates in a clockwise direction at an angular rate

$$n_2 = \frac{AN}{2B}(1-p)$$

in a circle whose arbitrarily disposable radius is $a_2$ and whose center is the origin. The meaning of p is

$$p = (1 - 1/s)^{1/2} ,$$

and N is the clockwise spin of the projectile in radians per second, while A and B are the axial and transverse moments of inertia.

Suppose that the angle of orientation of the initial yaw $\varepsilon$ is 270°. Then the initial conditions are

$$\delta = \varepsilon , \qquad \varphi = 270°$$

$$\dot{\delta} = 0, \qquad \dot{\varphi} = N.$$

From the initial conditions it follows that $a_1$ and $a_2$ must satisfy the conditions

$$a_1 - a_2 = \varepsilon$$

$$n_1a_1 - n_2a_2 = \varepsilon N$$

and that the initial phase of the $n_1$ motion must be 270° while that of the $n_2$ motion must be 90°. The situation must be as shown in Figure 2, and it follows that

$$a_1 = \varepsilon \frac{N-n_2}{n_1-n_2},$$

$$a_2 = \varepsilon \frac{N-n_1}{n_1-n_2}.$$

Let $K_L$ be the cross wind force coefficient, $\rho$ the air density, $\underline{d}$ the diameter of the projectile, $\underline{m}$ its mass, and $\underline{u}$ its velocity and let

$$\varkappa = \rho u d^2 K_L/m. \tag{10}$$

Let X, Y, Z be a set of perpendicular axes such that the X axis is the bore axis, that the Y axis is in the vertical plane through the bore axis, and pointing upward, and that the Z axis is horizontal and to the right as viewed from the gun. Then the components of acceleration in the Y and Z directions due to lift are

$$\ddot{Y} = \varkappa u \left[a_1 \cos(n_1 t + 270°) + a_2 \cos(n_2 t + 90°)\right]$$

$$\ddot{Z} = \varkappa u \left[a_1 \sin(n_1 t + 270°) + a_2 \sin(n_2 t + 90°)\right].$$

Ignoring the slow variation in $\underline{u}$ due to drag, one finds by integration that

$$\dot{Y} = \varkappa u \left[\frac{a_1}{n_1} \sin(n_1 t + 270°) + \frac{a_2}{n_2} \sin(n_2 t + 90°) + \frac{a_1}{n_1} - \frac{a_2}{n_2}\right]$$

$$\dot{Z} = \varkappa u \left[-\frac{a_1}{n_1} \cos(n_1 t + 270°) - \frac{a_2}{n_2} \cos(n_2 t + 90°)\right]$$

where the constants of integration have been chosen so as to render $\dot{Y}$ and $\dot{Z}$ zero at the muzzle, when the time, t, is zero. It is seen that $\dot{Z}$, the component of velocity in the Z direction, contains only periodic terms so that its average value is zero. Hence there is no jump, due to bore clearance, to the left or right. On the other hand $\dot{Y}$ contains, in addition to purely



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periodic terms, the constant terms

$$\kappa u \left(\frac{a_1}{n_1} - \frac{a_2}{n_2}\right)$$

which constitute its average value. Hence there is a jump, due to bore clearance, in the Y-direction, of angular amount

$$\kappa\left(\frac{a_1}{n_1} - \frac{a_2}{n_2}\right).$$

From the expressions for $a_1$, $a_2$, $n_1$ and $n_2$ that have already been given it follows that the jump in the Y-direction is

$$-\kappa\varepsilon \frac{4Bs}{AN}\left(\frac{B}{A} - 1\right). \tag{7}$$

In the preceding expression, $\kappa$ may be replaced by its expression (10) and $\underline{s}$ by its expression

$$s = A^2N^2/4B\rho u^2 d^3 K_M$$

where $K_M$ is the moment coefficient. If this is done, the jump in the Y-direction,

$$-\varepsilon \frac{AN}{md} \frac{K_L}{K_M} \frac{1}{u} \left(\frac{B}{A} - 1\right)$$

is obtained, in agreement with equation (9) that was derived more rigorously in the body of this report. The negative sign means that the jump is in the direction of the downward normal, if the initial yaw $\varepsilon$ is directed to the left.



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ABSTRACT:

Explanation is presented of the formula used in the calculation of jump which is due to bore clearance. Jump is estimated as result of equating axial and transverse moments of inertia, mass, caliber, cross-wind force coefficient and moment coefficient, muzzle velocity, and spin into a single expression. It was observed that the most accurate projectile will be one whose design minimizes the ratio ($B^3/K_M$) (bore to moment coefficient) and is fixed from a gun whose twist of rifling is just rapid enough to render the projectile barely stable.

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