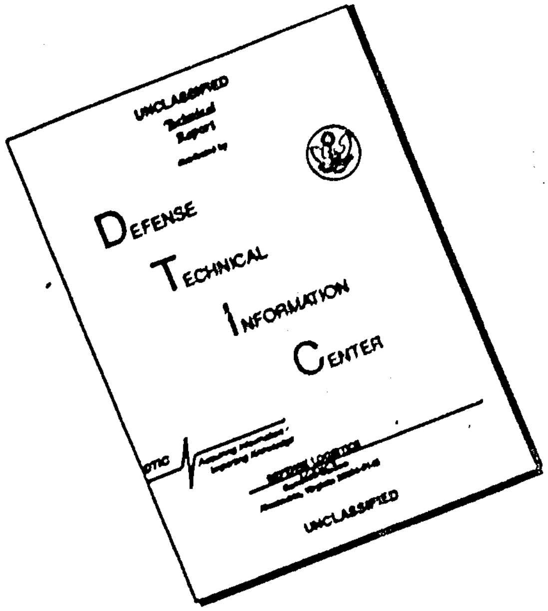
DISCLAIMER NOTICE



491936

BRL

REPORT NO. 478
AUGUST 1944

491936

(6) BALLISTIC COEFFICIENTS OF SMALL ARMS BULLETS OF CURRENT PRODUCTION,

(10) B. G. Karpov.

(11) 1 Aug 44,
(12) 24p.
(14) BRL-478

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

Ballistic Research Laboratory Report No. 478

Ordnance Research Center
Project No. 1931

Karpov/vm
Aberdeen Proving Ground, Md.

1 August, 1944

BALLISTIC COEFFICIENTS OF SMALL ARMS BULLETS OF CURRENT PRODUCTION

Abstract

1. The ballistic coefficients of Caliber 0.50 aircraft ammunition of current production show satisfactory uniformity among different lots of the same manufacturer and among different manufacturers. The uniformity of Caliber 0.30 aircraft ammunition of current production is excellent. The average C_5 of various types of bullets of all manufacturers are:

	Caliber 0.50		Caliber 0.30	
Bullet	Average C_5 all manufacturers	Standard Deviation between manufacturers	Average C_5 all manufacturers	Standard Deviation between manufacturers
API M8	.439	.006		
AP M2	.458	.006	.256	.004
Tracer M1, steel jacket	.446	.012	.301	.005
Incendiary M1	.414	.009	.227	.003
Ball M2	.460	.006	.250	.003

2. The ballistic coefficient is rather sensitive to the bluntness of a bullet's nose, and to the shape of its boattail. For Caliber 0.50 bullets increasing the diameter of nose above its present specification value by .02 inch will decrease the ballistic coefficient by about 3%; for Caliber 0.30 bullets a similar increase in nose diameter will decrease the ballistic coefficient by about 5%. Rounding the edge of the boattail of bullet also appears to decrease its ballistic coefficient.

a. The ballistic coefficient of a tracer is increased by the functioning of the tracer. For Caliber 0.50 tracer the increase in C_5 is about 20%; for Caliber 0.30 tracer this increase is about 35%.

3. The effect on the ballistic coefficient of a Caliber 0.50 bullet, of various errors which occur during range firings has been investigated. The errors considered are in measuring the distances between pick-up devices, in the time measurements, in the wind velocity, in the relative air density, and the effect of yaw. The influence of the first two errors depends markedly on the spacing of the pick-up devices; it is at a minimum when the three pick-up devices, such as two solenoid coils and a target screen, are equally spaced.

Introduction

1. Aircraft ammunition to be employed with fire control equipment must not depart widely from the ballistic data in the computing sights. The ammunition, therefore, must be ballistically uniform, and be loaded to specified velocities. The method of loading ammunition to achieve a specified velocity in a machine gun with machine functioning lies outside of the scope of the present Report which deals only with the ballistic uniformity of the bullets.

2. To keep a check on the ballistic uniformity of currently produced bullets the Ballistic Research Laboratory was requested by the Chief of Ordnance[1] to conduct firings for determination of the ballistic coefficients of small arms bullets of current production intended for aircraft use. For this purpose all manufacturers were required to forward to the Research Center 50 round samples each month of specified types of bullets manufactured by them for this ballistic test.

3. This Report is based on the results of these firings. To date firings have been completed with 187 lots of Caliber 0.50 ammunition received from 11 different manufacturers; 113 lots of Caliber 0.30 ammunition have been fired from 9 manufacturers.

4. In part A of the Report a method used at Aberdeen is described, for determining the ballistic coefficients of aircraft ammunition. Part B contains data on the effect upon a ballistic coefficient of minor changes in a bullet's design. In part C the effects, on the ballistic coefficient of a Caliber 0.50 bullet, of various errors which occur during range firings are discussed.

A. Determination of Ballistic Coefficients.

1. At the Ordnance Research Center, the ballistic coefficients of small arms bullets are usually determined from range firings. These firings are conducted by the Small Arms Branch at their outdoor range at Michaelsville.

These firings consist of measuring the initial velocity of a bullet and its time of flight to a target. Two pick-up devices which might be either solenoid coils, or Aberdeen screens, are placed 100 feet apart with the mid-point 78 feet from the muzzle. A target, an 8x8 foot wire mesh screen, is placed 600 yards from the gun. The times of flight of the bullet between the two pick-up devices, which will be called simply "coils", and the first coil and a target are measured by two R.C.A. Counter Chronographs. The surface wind is recorded for every round. It is measured by a portable anemometer which is located 200 yards from the firing house. Surface air-temperature on the range is also recorded at frequent intervals.

[1] Letter OO 471/31588, 18 Nov. 43.

2. In computing ballistic coefficients from range firing data, it has been found from experience that for the relatively short ranges, up to 1000 yards for Caliber 0.50 bullets, the use of the 3/2 power law of resistance for all small arms bullets is amply adequate. Although it is known that complete resistance functions for various types of bullets do differ, especially in the neighborhood of sonic velocity, at short ranges the bullet's velocity remains considerably above the velocity of sound, and under these conditions the choice of the appropriate resistance function for a given bullet is not critical.

The advantage of using the 3/2 law in the reduction of range firings described above is that the ballistic coefficient can be expressed by an algebraic formula as a function of the average velocity of the bullet between the coils, which is taken as equal to its instantaneous velocity at the mid-point, and the time of flight of the bullet from this mid-point to the target. As is shown in BRL Report 346, the 3/2 law of resistance agrees with the tabulated G_5 drag function between velocities of 2950 f/s and 1650 f/s. Integration of the equations of motion of the bullet, using the Siacci approximation with the 3/2 law, leads to the following expression for the ballistic coefficient:

$$C_5 = \frac{.059\rho p}{\text{þ}_0^{\frac{1}{2}}\left(1 - \frac{p}{t\,\text{þ}_0}\right)}$$

where ρ is the relative air density in units of .07513 lbs. per cu. ft., p is the Siacci distance along the tangent to the trajectory expressed in units of 1000 feet, and þ_0 is the Siacci pseudo velocity along this tangent expressed in units of 1000 f/s. This ballistic coefficient is legitimately designated as C_5 because, for practical purposes, when the bullet's velocity is between the above limits, the ballistic coefficient computed by above formula is numerically indistinguishable from that computed from the ordinary tabulated G_5 function. The two might differ by about one part in four hundred.

For computation of the ballistic coefficients from range firings, one replaces p by the actual range R, and þ_0 by the measured velocity v. For the instrumentation described above let:

x = distance between coils
R = distance between first coil and the target
t_1 = time of flight of bullet between coils
t = time of flight between first coil and the target,

Then let

$$R' = R - \tfrac{1}{2}x$$
$$t' = t - \tfrac{1}{2}t_1 .$$

Hence, the expression for the ballistic coefficient becomes:

$$(1) \qquad C_5 = \frac{.059\rho\ R'}{v^{\frac{1}{2}}\left(1 - \frac{R'}{t'\ v}\right)}$$

where $v = \frac{x}{t_1}$. If the mid-point of coils is at 78 feet from the muzzle then v is called Instrumental Velocity. After C_5 is found the muzzle velocity is given by

$$(2) \qquad v_o^{\frac{1}{2}} = v^{\frac{1}{2}} + \frac{.059\rho}{C_5}P$$ where P is the distance from the muzzle to the mid-point of the coils.

If wind is present and w be the range wind in units of 1000 f/s and if it be regarded positive if blowing in the direction of the motion of the bullet, then to correct for the effect of wind one replaces, in eq. (1), R' by $R' - wt'$, and v by $v - w$.

3. In addition to the approximations of replacing ρ by R and ρ_o by v, which for short ranges are entirely justifiable, equation (1) involves two more approximations; (a) the average velocity between coils is equal to the instantaneous velocity at the mid-point, and (b) the correction $\frac{1}{2}t_1$ is equal to the time of flight of the bullet from the first coil to the mid-point. It can be easily shown that for the Caliber 0.50 bullet whose ballistic coefficient C_5 is .450, and whose velocity at the first coil is 2800 f/s, the difference between the average velocity between coils $\bar{v}$, and the velocity at mid-point v_m is

$v_m - \bar{v} = 4.3\ x^2$ f/s, where x is the coil separation in units of 1000 feet; also the difference between t_1 and the time of flight to mid-point t_m is approximately

$t_1 - t_m = 7.17\ x^2$ milliseconds. The effect of these approximations on the ballistic coefficient of the above bullet is illustrated in the following table for various coil-separations:

Distance between coils x	Error in the ballistic coefficient C_5 using Eq.(1) Range 300 yards	600 yards
.05	+ .0003	+ .0000
.1	.0014	.0003
.2	.0068	.0014
.4	+ .0382	.0061
.6		.0166
.8		+ .0358

At a range of 600 yards with our usual coil separation of 100 feet the effect of these approximations is negligibly small. But as will be shown later equation (1) can be easily transformed so as to avoid even these approximations.

4. The ballistic coefficients of various Caliber 0.50 and Caliber 0.30 bullets from samples of current production lots are summarized in Tables I, and II. With the exception of API M8 bullets, each entry in Tables I, and II is the average ballistic coefficient based on five rounds fired from a particular lot. For API M8 bullets each entry is based on ten rounds.*

All bullets which could be magnetized to activate solenoid coils were fired through coils. Caliber 0.50 incendiary M1, gilding metal jacketed tracer M1, and until recently, ball M2 could not be sufficiently magnetized and were fired through Aberdeen screens. Of Caliber 0.30 bullets only AP M2 bullets were fired through coils.

As has been shown in BRL Memo Report 240 the ballistic coefficients of bullets as inferred from firings through Aberdeen screens are systematically lower than those found from firings through solenoid coils. For Caliber 0.50 AP M2 bullet this difference, C_5 (solenoids) minus C_5 (screens), was found to be about .02, but it varied from .017 to .025, probably depending on the thickness of tinfoil of the screen. This difference is determined anew at each firing by including a standard AP M2 lot.**

A study of the effect of screens was made only for Caliber 0.50 AP M2 bullet. However, firings with Caliber 0.50 steel jacketed tracer M1, which could be fired through coils and screens, have shown that the effect of screens on this bullet is the same as on the AP M2 bullet. It was assumed, therefore, that the same correction would apply to ball and incendiary bullets.

* The names of the manufacturing plants have been omitted in order not to raise the classification of this report and thus limit its accessibility. Instead, the plants are named by letters A,B,C,..etc. The names of the plants can be supplied upon official request.

**There appears to be little doubt that Aberdeen screens cause the bullet to slow down. For Caliber 0.50 AP M2 bullet when fired at 600 yards range through Aberdeen screens the observed delay in time of flight is about 6 milliseconds. A drop in bullet's velocity, in passing through the screen, of $1\frac{1}{2}$ f/s contributes only 0.5 millisecond to this delay. The remaining portion may be due to an increased yaw of the bullet induced by the screens, or to an increased bluntness of bullet's nose. The increased bluntness may be caused by part of the tinfoil of the screen sticking to bullet's nose. No direct experimental evidence is available to substantiate either of these possibilities. However, studies of the effect of bluntness of bullet's nose on its ballistic coefficient indicate that if twice the thickness of the tinfoil, about .005 inches, is folded around bullet's nose, the bullet will be slowed down by the observed amount.

For Caliber 0.30 bullets a similar correction for the effect of screens has been determined from firings with a standard Caliber 0.30 AP M2 lot.

All ballistic coefficients in Tables I, and II have been reduced to solenoid coils as standard.

All firings were with new or only slightly worn Mann barrels. For Caliber 0.50 ammunition 36" barrels were used; 24" barrels were used for Caliber 0.30 ammunition.

Table I

Summary of the ballistic coefficients (C_5) of Caliber 0.50 bullets of current production:

Ordnance Plant	API M8	AP M2	Tracer M1 steel jacket	Incendiary M1	Ball M2
A		.468			
		.454			
		.467			
B	.432	.459	.441	.403	
	.432	.454	.440	.413	
	.432	.459	.438	.405	
	.435	.452	.434	.408	
	.428	.456	.451	.408	
	.420	.451	.444	.398	
	.422				
	.421				
	.422				
	.424				
C	.439	.452		.415	
	.430			.406	
	.438			.411	
	.440			.410	
	.443			.409	
	.440				
	.444				
	.442				
	.439				
	.442				
	.439				
	.442				
D	.426	.457	.467	.409	
	.439	.450	.455	.405	
	.437	.456	.461	.410	
	.432	.452	.458		
	.438				
	.434				
E		.447			.451
		.448			.455
		.451			
		.450			

Table I (Continued)

Ordnance Plant	API M8	AP M2	Tracer M1 steel jacket	Incendiary M1	Ball M2
F		.453			
		.473			
		.465			
		.462			
		.459			
G	.439	.458		.405	.453
	.429	.458		.406	.449
		.462			.461
		.462			.465
		.459			.458
		.451			.462
H	.454	.453	.446	.424	.463
	.460	.454	.426	.424	.457
	.449	.457	.457	.421	.464
		.460	.450	.420	.455
				.423	.472
				.424	.470
I	.454	.466	.434	.428	
	.460	.462	.441	.423	
	.458	.453	.436	.430	
	.459	.467	.436	.422	
	.460	.474	.442	.424	
	.440	.458	.436	.425	
	.437				
	.442				
	.442				
	.445				
J		.453	.446		
		.459	.475	.399	
		.466		.405	
		.459		.408	
				.418	
K		.452			.461
		.466			.460
		.461			.464
		.456			.454
		.462			.458
Mean	.439	.458	.446	.414	.460
No.of lots	43	48	22	32	19

Table I (Continued)

Gilding metal jacketed Tracer M1

.477
.473
.464
.467
.460
.521
.498

Square base Incendiary M1

.354
.361
.358
.347

Tracer M10

steel jacket	gilding metal
.449	.468
.444	
.437	
.437	
.452	
.450	
.452	
.450	
.456	
.447	
.440	

Table II

Summary of the ballistic coefficients (C_5) of Caliber 0.30 bullets of current production.

Ordnance Plant	AP M2	Tracer M1	Incendiary M1	Ball M2
A	.261	.292		.250
	.260	.292		.252
	.263	.311		.250
	.262	.308		.248
	.259	.305		.251
B	.257		.224	.243
	.253		.232	.248
	.258		.227	.243
	.261		.228	.245
			.229	.248
				.250
C	.259	.288	.224	.244
	.259	.304	.224	.245
	.253	.283	.225	.245
	.265		.227	.247
	.255			
D	.255	.305		.253
	.258	.307		.251
	.255	.305		.254
E	.260			.253
	.260			.253
	.261			.253
	.263			.256
	.261			.252
F	.247	.312		.253
	.247	.308		.250
	.250	.308		.252
	.246	.298		.250
	.257	.305		.251
G	.253	.295		.250
	.258	.293		.250
	.253	.295		.252
	.251	.299		.252
	.250	.298		.251
				.252
				.254
H	.257	.294		.249
	.254	.304		.253
	.255	.299		.257
	.251	.309		.255

Table II (Continued)

Ordnance Plant	AP M2	Tracer M1	Incendiary M1	Ball M2
I				.248 .244 .246 .247
Mean	.256	.301	.227	.250
No. of lots	36	25	9	43

For Caliber 0.50 ammunition, the ballistic coefficients show satisfactory uniformity among different lots of the same manufacturer, and among different manufacturers. The high degree of uniformity is shown by the small standard deviation of the mean C_5's of individual manufacturers about the mean C_5 of all manufacturers:

Bullet	Average C_5 all manufacturers	s.d. between manufacturers	No. of manufacturers
API M8	.439	.006	6
AP M2	.458	.006	11
Tracer M1, steel jacket	.446	.012	5
Incendiary M1	.414	.009	7
Ball M2	.460	.006	4

Tracer ammunition is somewhat more discordant. Probably this is to be expected, since the ballistic coefficient of a functioning tracer is higher than that of a non-functioning tracer. Its value, therefore, is likely to be influenced by the rate of burning, of the tracer mixture, which might be subject to some variations. It has been noted also that tracer bullets show larger variations in bluntness of nose between manufacturers than other bullets, which contributes to a larger dispersion of their ballistic coefficients.

Caliber 0.30 ammunition shows excellent uniformity among different lots of the same manufacturer and among different manufacturers:

Bullet	Average C_5 all manufacturers	s.d. between manufacturers	No. of manufacturers
AP M2	.256	.004	8
Tracer M1	.301	.005	6
Incendiary M1	.227	.003	2
Ball M2	.250	.003	9

B. The effect on the ballistic coefficients of small arms bullets of small changes in the bullet's contour. The effect of tracer.

In the mass production of bullets, small differences between individual bullets inevitably occur. Within a given plant these differences arise principally from wearing of machinery; between different plants, however, the differences may arise from insufficiently precise specifications. As an example, the diameter of the nose of a bullet is specified in the drawing only by its maximum value. Thus certain Caliber 0.50 bullets of current production were encountered which varied from .08 to .16 inches in nose-diamete The shape of the edge of the boattail is not specified, with a resulting and considerable variation among bullets of the same lot, as well as among different manufacturers.

1. An excellent illustration of small variations in shapes among bullets belonging to the same lot is to be found in the summary of measurements by the Inspection Gauge Suboffice[2] of four types of Caliber 0.50 bullets. About forty bullets were measured of each type. The following summary is taken from this letter.

" NOTES ON .50 CAL. ARMOR PIERCING

Bluntness or diameter at int. of radii varies from .122 to .147 approximately.
Measurement from point to intersection of radii varies from .020 to .031 approximately.
Radius of nose varies from .100 to .130 approximately.
Depth cannelure varies from .007 to .012. Angles on cannelure are irregular but vary generally from 5° to 10°.
Diameter core appearing varies from .070 to .180.
Depth from base to core varies from .028 to .050.
Points are flattened up to .006 and flare at the side up to .004, due to insertion in case by pressure on point.
Radius or chamfer on base varies to such an extent that it is impossible to give a general description regarding condition.

NOTES ON .50 CAL. TRACER CARTRIDGES

Bluntness or diameter at intersection of radii varies from .120 to .150.
Measurement from point to intersection of radii varies from .012 to .029.
Radius of nose varies from .124 to .166.
Depth cannelure varies from .004 to .010.
The chamfer or radius on the base and its point of origin are so irregular that no approximate worthwhile description can be given.
Due to pressure on the point when inserted in the case, these cartridges have a flat on the nose up to .004 and flare out on the ogival radius up to .004.

[2] Letter IGO 413.6/30406, 22 November 1943.

RESTRICTED

NOTES ON .50 CAL. INCENDIARY CARTRIDGES

Bluntness or diameter at intersection of radii varies from .116 to .144.
Measurement from point to intersection of radii varies from .025 to .044.
Radius of nose varies from .078 to .090.
Depth of front cannelure varies from .002 to .010.
Depth of back cannelure varies from .004 to .010.
Boattail radius where it is unpainted averages approx. .170.
Points are flattened up to .006 from pressure on them when inserted into the case.
Radius of chamfer is too variable to give a worthwhile general statement on the condition.
Ogive flares up to .005 at point caused by pressure on the point when inserted into the case. Note - All incendiary cartridges should be inserted in case by pressure on the ogive.

NOTES ON .50 CAL. BALL CARTRIDGES

Bluntness or diameter at intersection of radii variesfrom .118 to .137.
Measurement from point to intersection of radii varies from .019 to .035.
Radius of nose varies from .090 to .122 approximately.
Boattail radius where the cartridge is unpainted averages approximately .160, however, paint on cartridge often produces a step of two to three thousandths.
The chamfer or the radius and its point of origin are so irregular that a general statement of conditions is almost possible.
Diameter core appearing varies from .090 to .210.
Depth from base to core varies from .039 to .053.
The point is flattened up to .005 and there is a flare up to .004 on the ogive due to pressure on the point when inserted into the case. "

These differences were found among bullets belonging to the same lot. A mean value of a given variate might be different for another lot and among different manufacturers.

2. Change in the design of Caliber 0.50 AP M2 bullet. In 1943 the design of the Caliber 0.50 armor piercing bullet was altered: the "French" cannelure of the former AP M2 bullet was replaced by a square cannelure, and the formerly sharp edge of the boattail was slightly rounded, in a new design. In addition, the weight of the bullet was lowered by about 12 grains. The reduction inweight was caused by substituting a manganese molybdenum core for the tungsten chromium core of the former bullet.

These changes were discovered accidentally by the Ballistic Research Laboratory when it was found that the ballistic coefficients of AP M2 bullets from a certain St. Louis lot were 7.5% lower than the ballistic coefficients of AP M2 bullets from certain Frankford Arsenal lots that had been used as standards in all previous firings.

The effect of these changes on the bullet's ballistic coefficient was determined from range firings with bullets specially manufactured for this purpose by the Frankford Arsenal. The results of these firings, through solenoid coils, were as follows:

Caliber 0.50 AP M2

Base	Cannelure	Ballistic coefficient C_5
Round	French	.471
Sharp	Square	.482
Sharp	French	.493
Round	Square	.458

The effect of each change appears to have been as follows:

	Change in the ballistic coefficient
Change from French cannelure to Square	- .012
Change from Sharp to Rounded base	- .023
Change in weight	- .008
Total impairment	- .043

These firings with specially prepared bullets were valuable principally for showing the relative importance of changes in cannelure and base. The total impairment, however, can best be found from a study of all available firings of bullets of standard production.

From firings through Aberdeen screens we have the following data[3]:

Date 1943		C_5			C_5
May 29	AP M2 FA 467	.458	old type	Tracer M1 SL 8044	.462
June 8	SL 7776	.425	new type	" "	.457

The difference, therefore, between the old and the new AP M2 was .028.

The average C_5 of AP M2 bullets of current production is .458 (solenoid coils). The effect of Aberdeen screens on AP M2 was found to be .026[4]. The average C_5 of AP M2 of current production, therefore, if fired through Aberdeen screens should be .458 - .026 = .432. This value of C_5 of the new type AP M2 can be compared with the average C_5 of three standard lots of the old type of AP M2 which were fired through Aberdeen screens[3]:

Lot	C_5
FA 460	.461
FA 484	.458
FA 467	.458
average	.459

[3] BRL Memo Report 194.
[4] BRL Memo Report 240.

Thus the difference between the old and new types is .027 which agrees with the difference found above from the comparison of two lots. We can write, therefore, the following schedule of relative importance of the changes in the AP M2 bullet:

	Change in the ballistic coefficient
Reduction in weight	- .008
Change from French to Square cannelure	- .007
Change from sharp to rounded base	- .013
Total	- .028

The ballistic coefficient of the old type AP M2 bullet, if fired through solenoid coils, should have been:

	C_5
New AP M2	.458
Impairment	.028
Old AP M2	.486

It would have been desirable to check this result by actual firings with old style AP M2, but none were available. However, the Frankford Arsenal furnished 40 rounds of Caliber 0.50 AP T1E9, the predecessor of, and presumably identical with, AP M2. Twenty rounds of these bullets were fired through solenoid coils with the resulting ballistic coefficient (C_5) of .487, which is in excellent agreement with the expected value, .486.

3. <u>The effect of bluntness of nose on the ballistic coefficient.</u> During firings of bullets of current production it was noticed that their bluntness of nose varied over a considerable range even among bullets from the same box. These variations are illustrated in the following table which was abstracted from the Inspection Gage Suboffice letter referred to above:

Caliber 0.50 Bullet	Av. diam. of nose	s.d.	No. of Bullets	Drawing spec. max. diam.
AP M2	.135"	.005"	40	.120"
Tracer M1	.140	.008	40	.120
Incendiary M1	.135	.009	39	.130
Ball M2	.130	.005	39	.120

It is to be noted that for all four types of bullets the measured average diameter exceeds the values specified in the drawings.

To find the effect of bluntness of nose on ballistic coefficient, special range firings were conducted with Caliber 0.50 and Caliber 0.30 armor piercing bullets.

a. Caliber 0.50 AP M2

Range firings were conducted through solenoid coils at a target at 600 yards with Caliber 0.50 AP M2 bullets, lot FA716. Four groups of bullets with different nose diameters were prepared and fired with the following results:

Group	No. of rds.	Av. nose diameter	C_5
I	8	.12"	.447 ± .005 s.d.
II	10	.14	[illegible] 2
III	7	.21	.339 4
IV	3	.24	.291 9

These results are represented graphically in Fig. 1. From the graph the effect of bluntness of nose on ballistic coefficient, for the range of values which are likely to be encountered in practice, is as follows:

Diameter	C_5
.11"	.451
.12	.448
.13	.443
.14	.438
.15	.430
.16	.422

The average ballistic coefficient of AP M2 bullet of lot FA716 is .451. The average nose diameter, as measured on 20 bullets, is .131" and the standard deviation is .007". From the above table this bullet with a flat nose of .13" diameter has a ballistic coefficient of only .443. The difference between .451 and .443, or about 2%, is due, apparently, to the effect of rounded tip on the bullet's nose. Unfortunately, this rounded tip is usually damaged by the current practice of inserting a bullet into a cartridge by applying pressure on its nose.

For aircraft gunfire, various types of bullets are matched in time of flight. Impairment of the ballistic coefficient of the bullet increases the time of flight and thus affects the match. For the Caliber 0.50 armor piercing bullet M2, and probably also for other Caliber 0.50 bullets, a nose diameter exceeding its maximum specification value of .12" by about .02" could still be tolerated. Larger diameters would seriously affect the match among various bullets. This is illustrated in the following table. The time delay, or the difference in time of flight between bullet with d = .11" and bullet of larger nose diameter, was computed for forward fire, true air speed 300 mi/hr, sea level, 1000 yards future range. The table also gives the increase in the lateral deflection of the bullet, in mils, at 600 yards, plane speed 300 mi/hr, at sea level, and at 16,000 feet altitude.

d	C_5	Time delay at 1000 yds, sea level	Increased deflection at 600 yds sea level	16,000 feet
.11"	.451	.000 seconds	.0 mils	.0 mils
.12	.448	.005	.2	.1
.13	.443	.015	.6	.2
.14	.438	.024	1.0	.5
.15	.430	.041	1.6	.8
.16	.422	.059	2.2	1.2

b. Caliber 0.30 AP M2

Range firings were conducted through solenoid coils at a target at 600 yards with Caliber 0.30 AP M2, lot FA355. Different nose diameters were obtained by filing the bullet's nose. The results are represented graphically in Fig. 2. From the graph the effect on the ballistic coefficient of the bluntness of the bullet's nose, for the range of values of the diameters which are likely to be encountered in practice, are given below:

d	C_5
.06"	.259
.07	.253
.08	.246
.09	.238
.10	.229

For aircraft fire the increase in time of flight to 600 yards, forward fire, true air speed 300 mi/hr, sea level, and the increase in the lateral deflection in sidewise fire at sea level, and at 16,000 feet altitude for bullets with nose diameter larger than .06" are given below:

d	C_5	Increase in time of flight to 600 yds, sea level	Increase in deflection at 300 yds sea level	16,000 feet
.06"	.259	.000 seconds	.0 mils	.0 mils
.07	.253	.013	.6	.4
.08	.246	.030	1.4	.8
.09	.238	.050	2.3	1.3
.10	.229	.077	3.4	1.9

It appears, therefore, that a nose diameter of Caliber 0.30 AP M2 bullets and probably of other Caliber 0.30 bullets, exceeding its specification limit of .06" by .01", or at most by .02", could still be tolerated. Larger diameters would seriously affect the matching among various bullets.

4. The effect of the boattail. Until recently the Remington Ordnance Plant has manufactured square-based Caliber 0.50 Incendiary M1. The average ballistic coefficient of square based bullets from four lots was found to be .355 (Table I). Two more recent lots of Remington incendiaries M1 are boattailed with an average ballistic coefficient of .406. All bullets are of the same weight. Thus the change from a square base to the boattail has increased the ballistic coefficient of Incendiary M1 by about 14%.

5. The effect of tracer functioning on the ballistic coefficient. Caliber 0.50 Tracer M1 bullets are square-based projectiles. Inferred from the ballistic coefficient of the square based incendiary M1 (619 grains) the ballistic coefficient of non-functioning tracer (641 grains) should be .368. However, the ballistic coefficient of a functioning tracer is .446 (Table I). Thus the effect on the ballistic coefficient of Caliber 0.50 Tracer M1 with a functioning tracer is to increase the ballistic coefficient of an otherwise rather poor bullet by about 21%.

Certain experimental Caliber 0.50 bullet has a boattail and a tracer. Its ballistic coefficient is .441, and its weight is 613 grains. Without the functioning tracer its ballistic coefficient should be .406 instead of .441, or an increase of about 9%.

For Caliber 0.30 Tracer M1 bullet the increase in the ballistic coefficient because of a functioning tracer is even greater. The contours of Caliber 0.30 Tracer M1 and AP M2 bullets are very similar. Their ballistic coefficients and weights are:

Caliber 0.30	C_5	Weight
AP M2	.256	165 grains
Tracer M1	.301	142

Thus at 142 grains the ballistic coefficient of a non-functioning tracer bullet should have been $.256 \times \frac{142}{165} = .220$, whereas the ballistic coefficient of a functioning tracer is .301, or an increase of 37%.

The 20mm projectile AP T9E5 with the tracer has its ballistic coefficient increased due to a functioning tracer only by about 3%. The data are:

	C_5	Weight
20mm AP T9E5 with tracer	.413	2000 grains
AP T9E5 with inerted tracer	.401	2000

RESTRICTED

It has been found that the effect of burning tracer on the ballistics of the bullet can be approximately represented by an empirical formula

$$C - C' = kC'\left(\frac{d}{D}\right)^2 \frac{1}{D^2}$$

where C and C' are the ballistic coefficients of functioning and non-functioning tracer respectively, d is the diameter of tracer hole, D is the diameter of the projectile, and k is a constant. If D is expressed in inches, and the ballistic coefficient is referred to the type 5 drag function, or the 3/2 power law, the constant k, adjusted to represent the observations, is found to be 0.102. Thus the effect on the ballistic coefficient C_5 of tracer functioning is

$$C_5 - C'_5 = 0.102C'_5\left(\frac{d}{D}\right)^2 \frac{1}{D^2} .$$

The following table illustrates the application of this formula to the tracer bullets discussed above:

Projectiles	C'_5	d	D	C_5 comp.	C_5 obs.
Caliber 0.30 Tracer M1	.220	.17"	.30"	.302	.301
0.50 Tracer M1	.368	.36	.50	.449	.446
0.50 Experimental	.406	.22	.50	.442	.441
20mm AP T9E5	.401	.30	.79	.414	.413

In spite of excellent agreement between observed and computed ballistic coefficients the above formula should only be used to obtain qualitative estimate of the increase in the ballistic coefficient due to a functioning tracer and not as a substitute to actual firings.

C. The effect on the ballistic coefficient C_5 of various errors in the instrumentation.

As was previously mentioned, equation (1), part A, which is used for the computation of ballistic coefficients from range firing data, contains two approximations: the average velocity between coils is assumed to be equal to the instantaneous velocity at mid-point, and the time of flight of the bullet from first coil to mid-point is taken as equal to one-half of the time of flight between coils. It has been shown that the effect of these approximations on the ballistic coefficient, when the separation of the coils is 100 feet and a target screen is at 600 yards, (our usual arrangement) is negligibly small.

1. Equation (1), however, can be rewritten in a form which avoids both of the above approximations. Let x_1 and x_2 be the distances, in units of 1000 feet, between the first and second coils, and between the second coil and a target respectively; let t_1 and t_2 be the corresponding times of flight. Let $R = x_1 + x_2$ be the range, and $t = t_1 + t_2$. We may consider two cases depending upon the nature of the hook-up of the timing devices: a) if we reckon coils from the muzzle as 1, 2, and 3 (target) then the first electronic counter may be connected to coils 1 and 2, and the second counter to 1 and 3; in another case b) the connection would be 1 - 2, 2 - 3. As rewritten, equation (1) assumes the following forms:

Case a)

$$C_5 = .059\rho R^{\frac{1}{2}} \frac{\left[(R-x_1)x_1 t_1 t(t-t_1)\right]^{\frac{1}{2}}}{x_1 t - R t_1}$$

Case b)

$$C_5 = .059\rho R^{\frac{1}{2}} \frac{\left[x_1 x_2 t_1 t_2 \ (t_1+t_2)\right]^{\frac{1}{2}}}{x_1 t_2 - x_2 t_1} .$$

The two equations are identical but each is expressed in terms of quantities directly measured during firings. To correct for wind one replaces R by $R-wt$, x_1 by x_1-wt_1, and x_2 by x_2-wt_2 where w, the tail wind, is in units of 1000 f/s.

2. Errors in C_5 arising from errors in x and R. The following errors were computed for a caliber 0.50 bullet with a ballistic coefficient .450 and a velocity at the first coil of 2800 f/s. The instrumentation corresponds to case (a) above.

Table IV

Error in C_5 due to errors in x, and R

Distance between coils x	Error in the ballistic coefficient due to a one-inch error in x		due to a one-foot error in R	
Range	600 yds	300 yds	600 yds	300 yds
.05	-.0051	-.0109	+.0018	+.0075
.1	.0026	.0058	.0019	.0079
.2	.0014	.0033	.0020	.0090
.4	.0008	.0023	.0022	.0126
.6	.0006	.0026	.0026	.0208
.8	.00056	-.0058	.0031	+.0617
1.0	-.00056		+.0038	

Table IV shows the accuracy with which x and R should be measured to attain a desired accuracy in the ballistic coefficient.

3. Error in C_5 due to an erroneous value of the wind. An error in measuring the wind causes an error in C_5 of $-.0006\delta w$, where δw = w(true)- w(observed) is in miles per hour. The error in C due to an error in measuring the wind is very nearly independent of wind-velocity, range and coil separation.

4. The effect of timing errors on the ballistic coefficient. Since electronic counters were installed at the Small Arms range at Michaelsville, they are being used exclusively in connection with range firings for determination of ballistic coefficients of small arms bullets. At the beginning of each firing program, it is our practice to check the two counters against each other. This is done by measuring the time of flight of the same bullet on both counters simultaneously. Small systematic differences between readings of the two counters occasionally appear. These differences increase with the range and the length of the measured time interval, and probably are due in part at least to slight differences in the frequencies of the two counters. At 600 yards range these discrepancies seldom exceed .03 milliseconds. It is believed that barring gross malfunctioning of the counters, which can be easily detected, the systematic and random errors of the counters themselves are probably small.

However, in range firings, counters are connected to some sort of pick-up devices. Errors, which are much larger than the intrinsic errors of counters themselves, are known to occur in such counter pick-up systems. An example of such an error is the time lag between the instant the bullet passes through the solenoid coil and counter response. Combination of different types of pick-up devices, such as solenoid coils and wire mesh target screens, for which time lags are different, introduce errors into the measured time intervals which, in certain circumstances, may become very serious.

The following results were obtained in measuring the differential time lag between solenoid coils and the target screen. The coils were 18 inches in diameter with 240 turns of wire. The target consisted of two fine wire mesh screens mounted on a wooden frame and separated from each other by strips of 1/4 inch ply wood.

Caliber 0.50	No. rds.	Differential time lag coil-target screen milliseconds mean	s.d.	Distance 1st coil - target
AP M2	5	.164	.015	150 feet
API M8	4	.405	.005	110
Tracer M10	4	.468	.010	110

The standard deviation is that of a single round.

Whether the differences in the differential time lags of different bullets are real is not known. The first difference, for AP M2, was determined in October 1943, the other two in June 1944. Thus the times of flight of all firings using solenoid coils, preceding the June 1944 firings, were corrected by .15 milliseconds. However, if the true value of the differential lag is .4 milliseconds, all former ballistic coefficients were too low by .001, which is not serious. The differential lag between a coil and a target screen is known to increase with an increasing number of turns of wire in the coil. Thus for a coil with 500 turns, the differential lag between the coil and the same wire mesh target as above is about .8 milliseconds. It appears, advantageous, therefore, to use coils with as few turns of wire as is practicable.

The effect of timing errors on the ballistic coefficient of a Caliber 0.50 bullet, whose ballistic coefficient is .450 and whose velocity at the first coil is 2800 f/s, has been computed, with various coils separations, for the hook-up arrangements of case (a) and case (b). For case (a), let δt_1 and δt be the errors, in milliseconds, of counters connected to coils 1-2, and 1-3 (target) respectively. For case (b), let δt_1 and δt_2 be the errors, in milliseconds, for the counters 1-2, and 2-3 respectively. The effects of these errors are given in Table V.

Table V

Error in the ballistic coefficient C_5 of a Caliber 0.50 bullet due to timing errors.

Case (a): hook up 1-2, 1-3.

Distance between coils	Range 600 yards δC_{t_1}	Range 600 yards δC_t	Range 300 yards δC_{t_1}	Range 300 yards δC_t
.05	+ .1695δt_1	- .0038δt	+ .3622δt_1	- .0181δt
.1	.0866	.0039	.1909	.0192
.2	.0453	.0041	.1074	.0219
.4	.0251	.0046	.0728	.0304
.6	.0189	.0054	.0780	.0502
.8	.0164	.0064	+ .1700	- .1494
1.0	.0159	.0079		
1.2	+ .0171	- .0105		

Case (b): hook up 1-2, 2-3

Distance between coils	Range 600 yards δC_{t_1}	Range 600 yards δC_{t_2}	Range 300 yards δC_{t_1}	Range 300 yards δC_{t_2}
.05	+ .1658δt_1	- .0038δt_2	+ .3441δt_1	- .0181δt_2
.1	.0827	.0039	.1717	.0192
.2	.0412	.0041	.0855	.0219
.4	.0204	.0046	.0424	.0304
.6	.0135	.0054	.0280	.0502
.8	.0100	.0064	+ .0209	- .1494
1.0	.0080	.0079		
1.2	+ .0066	- .0105		

For both types of hook-up the influence of the timing errors is least when the pick-up devices are spaced equidistantly. Moreover, the hook-up corresponding to case (b) is more advantageous than that of case (a).

Table V shows that if relative to the range, the coils' separation is small, the more accurate of the two counters should be connected to the circuit 1-2. The table also shows that as far as the timing errors are concerned, firings at 300 yards with equally spaced coils are as accurate as firings at 600 yards with coils separated by 100 feet. The values in the table should be nearly correct for all types of Caliber 0.50 bullets.

5. Error in C_5 due to error in the relative air density. The ballistic coefficient is directly proportional to the relative air density. Thus the error in C is

$$\delta C = C \frac{\delta \rho}{\rho}$$ where $\delta\rho$ is the error in the relative air density. Air density depends on barometric pressure, air temperature, and to some extent on relative humidity. Whereas the barometric pressure is essentially constant over any small area of level ground, the temperature may vary over a considerable range depending upon local conditions. Thus on the Proving Ground the meteorological station is located near the bay, whereas the Michaelsville firing range is some 4 miles inland. Differences of surface air temperature as large as 10° F have occasionally been observed between these two stations. It is important, therefore, to observe the surface air temperature at the firing range in order to avoid a possible air-density error in the ballistic coefficient.

6. Effect of Yaw on the ballistic coefficient C_5. For yawing bullets the air resistance is greater than for non-yawing bullets, and hence the time of flight to a given range is correspondingly greater. Since in range firings the ballistic coefficient of the bullet is inferred from its time of flight together with its initial velocity, a longer time of flight will lead to a lower ballistic coefficient.

The effect of yaw on the ballistic coefficient inferred from time of flight firings can be computed by the formulae contained in BRL Report 345. If the first maximum yaw is δ_m, and the first minimum is zero at the muzzle, then the effect of yaw on the ballistic coefficient of Caliber 0.50 bullet as inferred from range firings at 300 yards and 600 yards is as follows:

The effect of yaw on C_5
Caliber 0.50 bullet

δ_m°	300 yards	600 yards
0	.450	.450
2	.448	.449
4	.441	.446
6	.431	.441
8	.417	.433
10	.400	.424

The first maximum yaw of Caliber 0.50 occurs at about 14 feet from the muzzle. But it is best located by locating the position of the second minimum. Our practice is to place four sheets of photographic paper, which is the best paper for measuring yaws of small arms bullets, two feet apart at, say, 26, 28, 30, and 32 feet from the muzzle respectively. One measures the major axis of an elliptical perforation caused by the yawing bullet and plots it against the position of the card. The minimum yaw can be accurately located by this rough method. The first maximum then is half-way between this point and the muzzle.

The average maximum yaws which were observed for Cal. 0.50 AP M2, and API M8 bullets were of the order of 3 - 4 degrees. The effect of this yaw on the ballistic coefficient at range firings at 600 yards is less than one percent, hence it was not corrected for.

To make the first maximum yaw as small as possible it is important that firings be conducted in new or only slightly worn barrels.

B. G. Karpov

B. G. Karpov

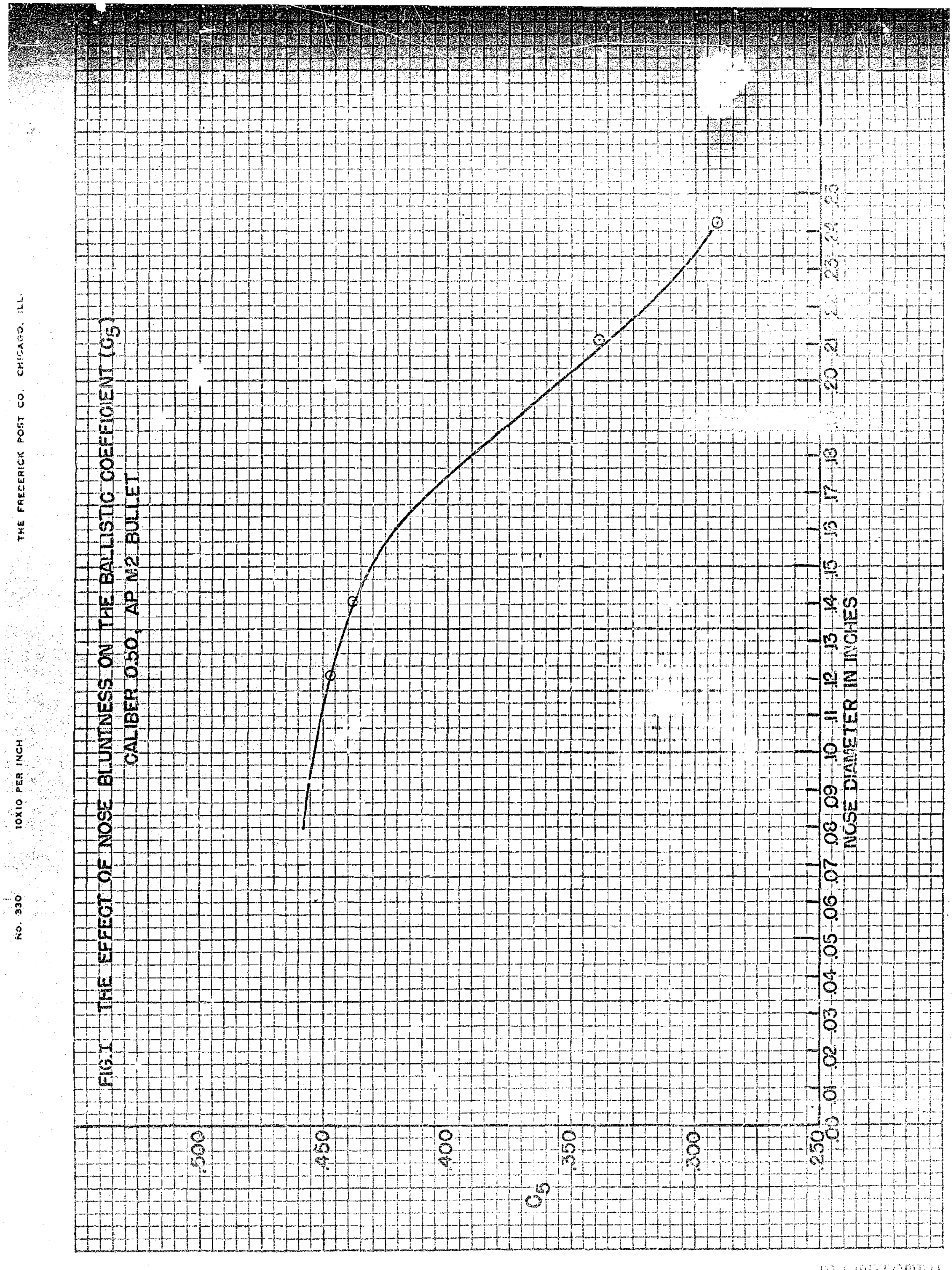


FIG. I THE EFFECT OF NOSE BLUNTNESS ON THE BALLISTIC COEFFICIENT (C5)
CALIBER 0.50, AP M2 BULLET

FIG. II THE EFFECT OF NOSE BLUNTNESS ON THE BALLISTIC COEFFICIENT (C_5)
CALIBER .030, AP M2 BULLET