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Engineering Experiment Station

University of Illinois

Technical Report No. CML - 4

August 1, 1952

The Coalescence of Droplets in a Turbulent Jet

Prepared by: Arnold Kivnick

Approved by: H. F. Johnstone

Contract No. DA-18-064-CML-445
Army Chemical Corps, Department of the Army

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Office of Naval Research, Department of the Navy

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Abstract

An equation for the coalescence of colloidal particles under the influence of velocity gradients has been applied to the problem of coalescence of spray droplets in the turbulent fields surrounding two-fluid atomizers. Two derivations have been made, one which permits the estimation of the probable upper limit of coalescence, and a second which takes into account the effects of dilution of the spray in the jet which reduces the coalescence. The first consideration results in a relationship which may be evaluated easily; the second gives rise to a partial differential equation which is not susceptible to analytical solution. A solution by iteration of the corresponding finite difference equation is discussed in the Appendix.

Introduction

The creation and use of fine sprays are problems currently receiving attention in several fields of scientific endeavor. Process engineers are concerned with systems of mass transport in which the large areas involved in sprays provide a means for increasing overall transfer rates. Engine designers, for similar reasons, pay increasing attention to fuel sprays. The stability of small droplets is of interest in military problems involving smoke screens and the dispersal of toxic materials. Agricultural scientists are similarly concerned with dispersal of insecticides and chemical substances which control or regulate the growth of weeds. In all of these cases, interest in the use of sprays leads to interest in methods of creating them.

The use of any means for producing a fine spray demands that no secondary effect should enlarge the spray particles. If the spray is produced by a two-fluid nozzle, there is little to be gained by operating the nozzle in such a way as to form extremely fine droplets if the turbulence generated by the nozzle causes the droplets to coalesce. Hence the problem of atomization by a two-fluid nozzle involves both the design of the nozzle to produce spray of the desired size, and the prevention of coalescence of the droplets in the turbulent field through which they pass.

Theory

In considering the effect of agitation on the coalescence of colloidal systems whose particles initially are spheres of uniform diameter, Smoluchowski (12) derived the equation

$$n = \frac{4}{3} d^3 \nu^2 \omega \qquad (1)$$

where n is the number of collisions per second per cubic centimeter, ν is the concentration of particles per cubic centimeter, ω is the velocity gradient, centimeters per second per centimeter, and d is the collision diameter of the particles concerned, in centimeters. The collision diameter is defined as the distance between the centers of two spherical particles when a collision occurs. It will be assumed that each time a collision occurs, coalescence takes place. Since each collision results in the disappearance of one particle, n may be replaced by $-\frac{d\nu}{dt}$. Furthermore, as a result of collision, there are formed particles whose collision diameters are greater than d, and such particles are also capable of removing particles from the system by collision. For simplification, it will be assumed that in collisions between unlike particles the collision diameter is equal to the diameter of the larger particle. It follows that the rate of reduction of ν_1 by collision with particles of all sizes is given by the equation

$$-\frac{d\nu_1}{dt} = \frac{4}{3}\omega\left[\nu_1^2 d_1^3 + \nu_1\nu_2 d_2^3 + \nu_1\nu_3 d_3^3 + \ldots\right]$$

$$= \frac{4}{3}\omega\nu_1\left[\nu_1 d_1^3 + \nu_2 d_2^3 + \nu_3 d_3^3 + \ldots\right]$$

where the subscripts refer to the number of particles of the original size which have coalesced to form each particle of the size under consideration. Now

$$\frac{\pi}{6}\left[\nu_1 d_1^3 + \nu_2 d_2^3 + \nu_3 d_3^3\right] = \psi$$

where ψ is the loading, or the volume of the dispersed phase per unit volume of mixture of both phases; therefore

$$-\frac{d\nu_1}{dt} = \frac{8\omega\nu_1\psi}{\pi} \qquad (2)$$

The nature of ω depends upon the hydrodynamics of the system. It will be assumed, first, that the velocities of the dispersed and continuous phases are identical; that is, no slip exists. Tunitzki (13) defines ω as $\dot{u}/L$ where $\dot{u}$ and L are the turbulent velocity and diameter, respectively, of the smallest eddy capable of contributing to the dissipation of energy. Such an eddy is characterized by a Reynolds number of unity if the continuous phase is air:

$$\frac{\dot{u}L\rho}{\mu} = 1; \quad \text{hence} \quad \frac{\rho(\dot{u})^2}{\mu} = \omega$$

where $\frac{\mu}{\rho}$ is the kinematic viscosity of air. Thus

$$-\frac{d\nu_1}{dt} = \frac{8\nu_1\rho}{\pi\mu}\psi(\dot{u})^2 \qquad (3)$$

If a two-fluid nozzle is the means by which the spray is created, the system approximates the hydrodynamics of the free jet, which can be described adequately by a combination of theoretical and empirical equations. It is first necessary to assume that Smoluchowski's equation may be applied without great error in a system of non-uniform particles to the particles whose diameters are within an arbitrarily small range of the mean diameter. Equation (3) will therefore be used without subscript, it being understood that the particles under consideration are those within the size range described above, before any coalescence occurs.

Two ways of treating the problem now appear. In one we may attempt to ascertain an approximate upper limit to the coalescence by a relatively simple calculation. As an alternative, we can estimate more closely the degree of coalescence by means of a system of simultaneous partial differential equations whose solution can be obtained only by means of high-speed computing devices. Both methods will be investigated.

Estimate of Upper Limit of Coalescence

The upper limit of coalescence may be obtained by assuming that in Equation (3) the term ψ is constant and ν is affected only by coalescence, and by applying Equation (3) along that path at which $(\dot{u})^2$ has a maximal value. Such conditions exist along the axis of the jet, if the dilution effect of the jet which reduces both ψ and ν is ignored. Rather than consider a system varying with time as Smoluchowski did, let us consider instead one varying with distance along the axis of the jet, and in a steady state at each point. This may be done by expressing u as $\frac{dx}{dt}$. Equation (3) then becomes

$$-u\frac{d\nu}{dx} = \frac{8\nu\rho}{\pi\mu}\psi(\dot{u})^2 \qquad (4)$$

Corrsin (5) has shown that, along the axis of a jet, the turbulent velocity $\acute{u}$ rarely exceeds 0.20 u. Alexander, Baron, and Comings (2) have shown that the axial velocity u of a free jet is related to the axial displacement x by the equation

$$u^2 = \frac{A_o \quad u_o^2}{\pi \, c^2 \, x^2} \tag{5}$$

where A_o is the area of the jet nozzle, u_o is the velocity at the nozzle, and c is a constant whose value may be taken as 0.075. Substituting Equation (5) into (4), and expressing A_o in terms of the diameter of a round jet,

$$- \frac{d \, \nu}{d \, x} = \frac{0.32 \, \nu \, \Psi \, \rho \, D \, u_o}{\pi \, \mu \;\; 2 \, c \, x} \tag{6}$$

Separating the variables, and substituting the numerical value of c,

$$- \frac{d \, \nu}{\nu} = 0.68 \, \frac{\rho}{\mu} \, \Psi \, u_o \, D \, \frac{d \, x}{x} \tag{7}$$

Since the turbulence is quite low in the potential core of the jet, it may be assumed that no coalescence occurs within that region; hence the lower limit of x should be 4D, where experiments show the potential core ends. At this point, the droplet concentration may be taken as ν_o, the original value before coalescence. Then Equation (7) may be integrated to give

$$- \ln \frac{\nu}{\nu_o} = 0.68 \, \frac{\rho}{\mu} \, \Psi \, u_o D \, \ln \left(\frac{x}{4D} \right)$$

Then, recognizing that $\frac{u_o D \rho}{\mu}$ is N_{Re_o}, the Reynolds number for flow through the nozzle, the above equation may be rewritten as follows:

$$\frac{\nu}{\nu_o} = \left(\frac{x}{4D} \right)^{-0.68 \, N_{Re_o} \, \Psi} \tag{8}$$

Finally, in view of the uncertainties discussed later, which are inherent in the foregoing derivation, the numerical constant 0.68 should be replaced by an empirical constant K, which should in no case exceed unity. Thus,

$$\frac{\nu}{\nu_0} = \left(\frac{x}{4D}\right)^{-K N_{Re_0} \psi} \qquad (8a)$$

By the use of Equation (8a) the smallest concentration of droplets likely to exist along the axis of a two-fluid nozzle can be estimated, provided the loading ψ, the condition of airflow at the nozzle as characterized by N_{Re_0}, and the initial concentration of droplets of the mean size, ν_0, are known.

Correction for Dilution in the Jet

A closer approximation to the actual amount of coalescence to be expected must take into account the reduction in ν caused by factors other than coalescence. It is apparent that if the jet entrains air, the concentration of droplets must necessarily fall, and the loading ψ must also be reduced. A differential equation may be set up relating the rate of accumulation of droplets in a small volume to the net rate of transport into the volume and the rate of generation of droplets within the volume:

$$\frac{d\nu}{dt} = \nabla \cdot \mathcal{D} \nabla \nu - \nabla \cdot \overline{\nu V} + R_\nu \qquad (9)$$

The first term in Equation (9) is the rate of accumulation of droplets, the second term the rate of transport by molecular processes, the third term the rate of transport by convective processes, and the fourth term the rate of generation; $\mathcal{D}$ is an effective diffusivity for the droplets and V is the velocity vector. At steady state, the rate of accumulation is zero, and transport by molecular processes is negligible in comparison to convective transport; thus Equation (9) becomes

$$\nabla \cdot \overline{\nu V} - \bar{R}_\nu = 0 \qquad (10)$$

where the bar indicates the time average of the quantity. The first term of Equation (10) represents the removal of droplets from the element of volume by convection, and the second the creation of droplets within the volume.

For the case of axial symmetry, the first term above may be expanded to the form

$$\nabla \cdot \overline{\nu v} = \frac{\partial \overline{\nu u}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\,\overline{\nu v}) \qquad (11)$$

Transport perpendicular to the axis, expressed in the the third term of Equation (11), may now be treated as suggested by Baron and Alexander (4), i.e.,

$$(\overline{\nu v}) = -\frac{C^2 x}{2}\frac{\partial \overline{\nu u}}{\partial r} \qquad (12)$$

The second term of Equation (10) is the rate of generation of droplets. Since coalescence removes droplets, the rate of generation is negative. This term represents the change in concentration of droplets brought about by coalescence alone, and is given by Equation (3), which will be rewritten in the form

$$R_{\nu} = -\frac{8\nu\rho}{\pi\mu}\Psi(\dot{u})^2 \qquad (13)$$

Now substitute Equations (11), (12), and (13) into (10); there results

$$\frac{\partial \overline{\nu u}}{\partial x} = \frac{C^2 x}{2r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{\nu u}}{\partial r}\right) - \frac{8\nu\rho}{\pi\mu}\Psi(\dot{u})^2 \qquad (14)$$

The fluctuating velocity $\dot{u}$ may be related to the mean velocity u at any point whose co-ordinates are x, r, by the expression

$$\frac{\dot{u}}{\bar{u}} = g(x,r)$$

where g(x,r) is an empirical function of x and r. Then

$$(\dot{u})^2 = g^2(\bar{u})^2 \qquad (15)$$

Substituting Equation (15) into (14),

$$\frac{\partial \overline{\nu u}}{\partial x} = \frac{C^2 x}{2r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{\nu u}}{\partial r}\right) - \frac{8\rho}{\pi\mu}\nu\Psi(\bar{u})^2 g^2 \qquad (16)$$

Assume now that $\overline{\nu}\;\bar{u} = \overline{\nu u}$ (17)

and that $\overline{\psi}\;\bar{u} = \overline{\psi u}$

Equation (16) may then be rewritten

$$\frac{\partial\,\overline{\nu u}}{\partial x} = \frac{C^2 x}{2r}\frac{\partial}{\partial r}\left(r\,\frac{\partial\,\overline{\nu u}}{\partial r}\right) - \frac{8\rho}{\pi\mu}\,(\overline{\nu u})(\overline{\psi u})g^2 \tag{18}$$

To determine ν at any values of x and r, Equation (18) must first be solved for $(\overline{\nu u})$ and then divided by the suitable value of $\bar{u}$. This requires a knowledge of $(\overline{\psi u})$, $\bar{u}$ and g^2 as functions of x and r.

Equation (18) was obtained by means of a balance over the number of droplets within an arbitrarily small size range. A similar balance can be performed with respect to the total amount of the dispersed phase: that is, the rate of accumulation of the dispersed phase in a given small volume element must be equal to the net rate of transport of that phase through the surface of the element by both molecular and convective transport plus the rate of creation of dispersed phase within the element. The mathematical statement of the balance is

$$\frac{\partial\psi}{\partial t} = \nabla\cdot\mathcal{D}'\nabla\psi - \nabla\cdot\overline{\psi V} + R_{\psi} \tag{19}$$

The first term in Equation (19) is the rate of accumulation of dispersed phase, which is zero in a steady-state system; the second term is the rate of transport by molecular processes, which should be negligible in comparison with the rate of transport by convective processes, the third term; the fourth term, the rate of creation of dispersed phase, is zero. Equation (19) therefore becomes

$$\nabla\cdot\overline{\psi V} = 0 \tag{20}$$

If Equation (20) is expanded for the case of axial symmetry, and the assumption embodied in Equation (12) is applied, there results

$$\frac{\partial\,\overline{\psi u}}{\partial x} - \frac{C^2 x}{2\,r}\frac{\partial}{\partial r}\left(r\,\frac{\partial\,\overline{\psi u}}{\partial r}\right) = 0 \tag{21}$$

Equation (21) is linear in $\overline{\Psi u}$; Baron and Alexander (4) obtained the following equation as a solution at all values of r and x:

$$(\overline{\Psi u})_{x,r} = \int_0^{2\pi}\int_0^{D/2} \frac{(\overline{\Psi u})_{0,s}}{\pi C^2 x^2} \exp\left[-\frac{r^2+s^2-2rs\cos\gamma}{C^2 x^2}\right] s\,ds\,d\gamma \qquad (22)$$

In Equation (22), s and γ refer to a system of polar co-ordinates in the plane of the nozzle from which the jet of air and dispersed phase issue, D is the diameter of the nozzle, and C is a constant whose value has been found empirically to be 0.087 (9). At values of x such that $x/D > 20$, Equation (22) simplifies to the form

$$(\overline{\Psi u})_{x,r} = \frac{(\overline{\Psi u})_{0,0}}{4C^2(x/D)^2} \exp\left[-\left(\frac{r}{Cx}\right)^2\right] \qquad (23)$$

By similar reasoning applied to conservation of momentum, there results

$$(\overline{u^2})_{x,r} = \int_0^{2\pi}\int_0^{D/2} \frac{(\overline{u^2})_{0,s}}{\pi c^2 x^2} \exp\left[-\frac{r^2+s^2-2rs\cos\gamma}{c^2 x^2}\right] s\,ds\,d\gamma \qquad (24)$$

in which the value for the constant c has been found to be 0.075 (2). At values of $x > 20D$, Equation (24) simplifies to the form

$$(\overline{u^2})_{x,r} = \frac{(\overline{u^2})_{0,0}}{4c^2(x/D)^2} \exp\left[-\left(\frac{r}{cx}\right)^2\right] \qquad (25)$$

By the same assumption as was involved in Equation (17), $\bar{u}$ may be obtained by taking the square root of the result of Equations (24) or (25).

Finally, Corrsin (5) has measured the intensity of turbulence at various points in the field of a free jet. From his data values of g^2 have been computed as a function of x and r. Figures 1-3 show g^2 as a function of r, at several values of x; Figures 4 and 5 show g^2 as a function of x at several values of r.

Thus ν can be determined as a function of x and r by the solution of Equation (18) combined with Equations (22) and (24) and the data of Figures 1-5. It has not been possible to obtain such a solution analytically, and a numerical or graphical approach is indicated.

The methods investigated thus far are integration by means of the Differential Analyzer (8), by punched-card computation (11), and by high-speed digital computer (3). The two last mentioned would require that Equation (18) be set up as a finite-difference equation. This has been done by Rose and Schilk (11) for this case; their analysis is presented in the Appendix. There is no certainty that a solution can be obtained, for it is by no means obvious that the difference equations converge rapidly enough to give a solution within a reasonable number of iterative computations on punched-card computing equipment. Alt (3) has indicated that solution by means of a high-speed digital computer is possible. According to Hunt (8), solution by means of the Differential Analyzer is not possible.

Evaluation of Errors

In the course of the foregoing derivations, many assumptions have been made, some of which obviously are not completely valid. It is therefore necessary to investigate the effects upon the resulting solution brought about by these assumptions.

The equation of Smoluchowski which forms the basis of this analysis is subject to the limitation that the particles concerned be of such a size that the fluid medium behaves as a continuum. In dealing with particles in the range of 10 microns, an error of about 1 per cent is made by ignoring the particulate nature of the fluid; this error increases markedly as the particle size decreases, and decreases as the particle size increases. The analysis is therefore limited to particles greater than 10 microns in diameter.

The assumption of spherical particles obviously collapses entirely if the particles are fibrous or needle-like. If the particles involved are liquid droplets, the error introduced by the assumption of sphericity should not be great, since small liquid droplets are often spherical or nearly so. There is also question about the assumption that coalescence is assured each time collision between particles occurs. There is some doubt that this is the case for either solid particles or liquid droplets. On the other hand, it is impossible that coalescence occur without collision. Thus the assumption made predicts the maximum possible rate of coalescence, and reduces the probability that the actual coalescence should exceed the predicted value. If the ratio of coalescence to collision is less than unity, K in Equation (8a) should be less than 0.68.

The assumption that, in collision between unlike particles, the collision diameter is equal to the diameter of the larger particle, is obviously not correct. The error is small in the case of particles not widely different in size, and increases with the difference in particle size. The collision diameter so assumed is always too large, and the effect is, again, to increase the estimated degree of coalescence. However, there should be relatively few particles so large that the predicted collision diameter is seriously in error. Thus the number of collisions involving those particles should be small. In any case, the net effect is to bring about an overly large calculated coalescence rate, which is safe from the viewpoint of the upper limit calculation. So far as Equation (8a) is concerned, this would make the true value of K less than 0.68.

Tunitzki's relationships regarding the term ω represent the most serious possible error. Both the definition of ω and the choice of a limiting Reynolds number of unity are without substantial experimental confirmation. This may introduce an error of an order of magnitude, in either direction.

Finally, the application of the Smoluchowski equation to a system of particles non-uniform from the start ignores the effects of coalescence among particles smaller than the mean diameter. If these particles coalesce with each other, they eventually form particles of the mean diameter, and bring about a decrease in the total rate of removal of particles with that mean diameter. If they coalesce with particles larger than the mean, they tend to increase the diameters of those particles, and hence to increase the rate of removal of particles of the mean diameter. It is likely that these effects counterbalance each other entirely, or that the net effect is so small as to be negligible.

In summary, the assumption regarding the relative frequency of coalescence and collision, and the assumption regarding the collision diameter of unlike particles have the effect of increasing the estimated degree of coalescence above the value to be expected, while the uncertainties inherent in the value of ω can either increase or decrease the estimated value. It appears reasonable, however, to suppose that all these possible errors may be encompassed by assuming that K in Equation (8a) may vary from 0.1 to 1.0. In the determination of the upper limit of coalescence, the assumptions make the system as a whole behave as the narrow pencil of flow down the axis, and the flowing nature of the jet does not alter the loading or the droplet concentration within the pencil. The effect of this choice of a model assures that the coalescence predicted will exceed that which occurs, and that the upper limit predicted will be greater than the probable value.

The correction for the dilution of the jet eliminates the inaccuracy of the overly simplified model previously described. The errors inherent in the derivation lie in the extension of Corrsin's data to yield sufficiently detailed intensity-of-turbulence data, in the averaging procedures by means of which the ν- and ψ-fluxes transported by the turbulent components are ignored, leaving only the fluxes transported by mean components of u, ν, and ψ, and in the rate of coalescence term as expressed in Equation (13). It is supposed that although the actual amount of coalescence which occurred at any part of the flow field may be subject to errors as large as plus or minus 50 per cent, the relative amounts of coalescence at various parts of the jet should be reasonably close to predicted values. Thus an experimental program should require far fewer observations than would be required in the absence of a guiding relationship such as that here obtained.

Discussion

No experimental program has accompanied this work, and it is not possible at this time to obtain a quantitative evaluation of the relationships obtained. There are, however, experimental fragments which are in qualitative agreement with the preceding equations.

It has been observed beyond reasonable doubt that coalescence does occur in two-fluid atomization systems. Donnelly (6) has shown, by means of high-speed motion pictures, that collision and consequent coalescence between single droplets in a spray are frequent occurrences. Nukiyama and Tanasawa (10), in their equation relating mean droplet diameter to the conditions of operation of two-fluid atomizing nozzles, show that the particle size increases as the 3/2 power of the ratio of volumetric liquid flow rate to the volumetric gas flow rate. This is consistent with the predictions here obtained; one might therefore postulate that the increase in particle size with loading is a phenomenon related to coalescence.

Feild (7) studied the charge on sprays created in a two-fluid atomizer, using cylindrical probes of two different diameters, connected to an electronic voltmeter, and making traverses of the flow field at several different axial positions. He integrated the total charge at each of the axial positions, and found that there was an apparent loss of charge as the traverse moved away from the nozzle. This might be explained as follows: The process of atomization was accompanied by a separation of charges; thus some of the droplets retained a positive charge, some a negative charge. The electronic voltmeter was sensitive to a net charge of one sign only. If all the droplets in existence close to the nozzle were conserved, then there should have been no loss of charge, and successive integrated profiles should have indicated no loss of charge. If coalescence occurred, it is probable that some droplets bearing opposite charges united, and that neutralization took place. In that case, the resultant droplets must have had a net charge smaller with respect to the electronic voltmeter than did the original droplets before coalescence. Hence the loss of charge is consistent with the occurrence of coalescence. Alexander (1), in similar experiments, confirmed the observations noted above.

Equation (8a) can be used to estimate the maximum coalescence likely in a given application. It might be assumed, for example, that little or no coalescence would occur farther than thirty nozzle diameters downstream of the nozzle. Thus 30D might be substituted for x. The Reynolds number would be determined by the flow conditions. Then a particular loading, ψ, would be needed in order to prevent the ratio $\frac{\nu}{\nu_0}$ from falling below the desired value. Or, if the value of the loading were predetermined, it would be possible to compute the smallest value of the ratio $\frac{\nu}{\nu_0}$ likely to exist.

For example, suppose that it was necessary to estimate the upper limit of coalescence for a case in which a liquid, predominantly water, was atomized in a two-fluid nozzle by air at 80°F. flowing at 500 ft./sec. through a 1/2 inch nozzle. The gas flow rate establishes the mean droplet size originally formed; that mean size need not be known for the purposes of this calculation. By using Equation (8a), it is possible to compute the fraction of the droplets of the original mean diameter which remained unaffected by coalescence at a distance of 30 diameters from the nozzle, as a function of ψ. The Reynolds number for the flow conditions above is 127,000; assume that K is 0.5. A weight ratio, water to air, of 1.0 corresponds to a loading, ψ, of 1.18×10^{-3}. Table I gives ν/ν_0 as a function of ψ.

Table I

Upper Limit of Coalescence; ν/ν_0

as a function of ψ at $x/D = 30$

ψ	ν/ν_0
1.18×10^{-3}	1.2×10^{-65}
1×10^{-4}	2.8×10^{-5}
1×10^{-5}	0.28
1×10^{-6}	0.88

The solution of Equations (18), (22) and (24) should permit one to make a close estimate of the actual amount of coalescence likely to occur under any given circumstances. Because of the complexities of the solution, the value of the results must be weighed against the expense entailed. Such an evaluation is beyond the scope of this report.

Summary

The coalescence of droplets in sprays from two-fluid nozzles has been attacked by analogy with the coalescence of colloidal particles under the influence of velocity gradients, as set forth by Smoluchowski. Two attacks have been employed: the first considers only a thin pencil of spray along the axis of the nozzle, and assumes that the jet is not diluted by mixing with air. This method results in an upper limit to the amount of coalescence likely, and gives rise to an equation which may be readily employed. The second method takes into account the dilution effect of the jet, and affords a measure of the number of particles within a given size range existing at any point in the field of the jet. The latter result is presented in the form of a partial differential equation whose solution by analytical means has not been possible. Solution by iteration of a finite difference equation is discussed in detail in the Appendix.

The limits of accuracy of both methods are discussed. Experimental data in qualitative agreement with this treatment are described.

Acknowledgment

This study was performed as part of a program in fluid mixing under Contract No. DA-18-064-CML-445, Army Chemical Corps. The program has been and is now being conducted under the title "The Mixing of Fluid Streams", under Contract No. N6-ori-71 T.O. XI, with the Office of Naval Research and currently the Flight Research Laboratory.

The author acknowledges with thanks the valued suggestions of Dr. Thomas Baron, formerly on the faculty of the University of Illinois, and the permission, granted by Prof. Arthur Rose of the Pennsylvania State College, to use the report by Rose and Schilk as an appendix to this report.

Table of Nomenclature

a	h_x
b	h_r
A_o	Area of a round nozzle
C	Jet spreading coefficient for droplets, whose numerical value is 0.087
c	Jet spreading coefficient for momentum, whose numerical value is 0.075
D	Diameter of a round nozzle
$\mathcal{D}$	Diffusion coefficient for droplets of a given size
$\mathcal{D}'$	Diffusion coefficient for the total discontinuous phase
d	Diameter of particles in Smoluchowski's equation for particle coalescence
g	A function of x and r by means of which $\overset{\circ}{u}$ and u are related
h	Distance between two points on a grid
h_x	Distance between two points on a grid in the axial direction
h_r	Distance between two points on a grid in the radial direction
K	A constant
L	A length characteristic of the smallest eddy by means of which energy may be dissipated
N_{Re_o}	The Reynolds number for flow through the nozzle
n	The rate of collision per unit volume; the number of collisions per second per cubic centimeter
R_ν	The rate of generation of droplets of a given size per cubic centimeter
R_ψ	The rate of generation of discontinuous phase per cubic centimeter

r, x	The radial and axial co-ordinates, respectively, in an axially symmetrical co-ordinate system
r', x'	Normalized radial and axial co-ordinates in the axially symmetrical co-ordinate system, equal to r/D and x/D
s, γ	A system of polar co-ordinates in the plane of the nozzle
t	Time, seconds
u	Axially directed velocity, centimeters per second
$\mathring{u}$	Fluctuating component of axially directed velocity; the root-mean-square value of the velocity fluctuation
V	The velocity vector
μ	The viscosity of air, grams per centimeter per second
ν	The concentration of droplets of a given size, in droplets per cubic centimeter
ρ	The density of air, grams per cubic centimeter
ψ	The loading, volume of the dispersed phase per unit volume of mixture of both phases
ω	A velocity gradient, in $\sec.^{-1}$
∇	The vector operator, $\frac{d(\ )}{dx} + \frac{1\,d\,(r\ )}{r\,dr} + \frac{1\,d\,(\ )}{r\,d\theta}$

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Appendix

REPORT ON PRELIMINARY INVESTIGATION OF SOLUTION OF ILLINOIS JET COALESCENCE EQUATION

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The method of iteration has been carefully studied as a possible approach to the solution of the jet coalescence differential equation

$$\frac{\partial \overline{\nu u}}{\partial x} = \frac{C^2 x}{2r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial \overline{\nu u}}{\partial r}\right) - \frac{8\rho}{\pi\mu}(\overline{\nu u})(\overline{\Psi u})g^2 \qquad \text{(A-1)}$$

This iteration method involves first the estimation of a series of first trial values of the dependent variable, $\overline{\nu u}$, and then improvement of these by means of successive iterations with the difference equation corresponding to the differential equation.

If $\xi = (\overline{\nu u})$, and $f(x,r) = (\overline{\Psi u})g^2$, then Equation (A-1), expressed in terms of difference quotients, takes the following forms: Using the average first difference quotients, and $h_x = h_r$ (see nomenclature)

when $r \neq 0$

$$\xi(x,r) = \frac{C^2x\left[\xi(x,r+h)+\xi(x,r-h)\right] - \frac{C^2xh}{2r}\left[\xi(x,r+h)-\xi(x,r-h)\right] + h\left[\xi(x-h,r)-\xi(x+h,r)\right]}{2C^2x + \frac{16h^2\rho}{\pi\mu}f(x,r)} \qquad \text{(A-2)}$$

when $r = 0$

$$\xi(x,r) = \frac{2C^2x\left[\xi(x,r+h)+\xi(x,r-h)\right] + h\left[\xi(x-h,r)-\xi(x+h,r)\right]}{4C^2x + \frac{16h^2\rho}{\pi\mu}f(x,r)} \qquad \text{(A-3)}$$

when $r = 0$

$$\xi(x,r) = \frac{2C^2x\left[\xi(x,r+b)+\xi(x,r-b)\right] + \frac{b^2}{a}\left[\xi(x-a,r)-\xi(x+a,r)\right]}{4C^2x + \frac{16b^2\rho}{\pi\mu} f(x,r)} \tag{A-9}$$

In order that the difference equation can be used for solution to the problem, a grid with initial trial values of the dependent variable, ξ, must be constructed. The iterative method of solution merely involves repeated use of the difference equations for one point after another in the grid, and repetition of this procedure, until successive values of $\xi(x,r)$ for a given point are as nearly identical as is desired. For the present problem, a grid for integral values of $x = 0$ to 40 and $r = 0$ to 5 was necessary because of the boundary conditions (see Fig. 6). This grid was first prepared for $h_x = h_r = 1.0$, obtaining the first trial values of $\xi(x,r)$ by linear interpolation from the boundary values (see Table II). Values of f(x,r) were also calculated for the points of this grid (see Table III). Using Equations (A-2) and (A-3) and $h = 1.0$, a series of pilot calculations were made at various points of the grid. This involved two iterations and use of Equations (A-2) or (A-3), a total of 115 times to obtain 115 new values of ξ. It was found that some of the resulting values of $\xi(x,r)$ were greater than 1.0, or were negative. In an attempt to avoid this lack of convergence, the size of h was reduced. Additional first trial values of $\xi(x,r)$ were obtained for fractional values of r and x, by linear interpolation from previous values, and more pilot calculations were then made. Reduction in h greatly increases the number of points on the grid, and the amount of computation required.

The following values of h, x, and r were used for the successive series of trial calculations, with the objective of finding an approach that would lead to convergence (see Table IV).

These pilot calculations showed that the largest value of h which could be used in Equations (A-2) and (A-3) was $h = 0.1$. This would involve calculation of $\xi(x,r)$ for approximately 12,000 points for

Using forward first difference quotients and $h_x = h_r$

when $r \neq 0$

$$\xi(x,r) = \frac{2h\xi(x+h,r) - C^2x\left[\xi(x,r+h) + \xi(x,r-h)\right] - \frac{C^2xh}{r}\xi(x,r+h)}{2h - 2C^2x - \frac{C^2xh}{r} - \frac{16h^2\rho}{\pi\mu}f(x,r)} \qquad \text{(A-4)}$$

when $r = 0$

$$\xi(x,r) = \frac{2h\xi(x+h,r) - 2C^2x\left[\xi(x,r+h) + \xi(x,r-h)\right]}{2h - 4C^2x - \frac{16h^2\rho}{\pi\mu}f(x,r)} \qquad \text{(A-5)}$$

Using backward first difference quotients and $h_x = h_r$

when $r \neq 0$

$$\xi(x,r) = \frac{2h\xi(x-h,r) + C^2x\left[\xi(x,r+h) + \xi(x,r-h)\right] - \frac{C^2xh}{r}\xi(x,r-h)}{2h + 2C^2x - \frac{C^2xh}{r} + \frac{16h^2\rho}{\pi\mu}f(x,r)} \qquad \text{(A-6)}$$

when $r = 0$

$$\xi(x,r) = \frac{2h\xi(x-h,r) + 2C^2x[\xi(x,r+h) + \xi(x,r-h)}{2h + 4C^2x \quad \frac{16h^2\rho}{\pi\mu}f(x,r)} \qquad \text{(A-7)}$$

Using average first difference quotients, $h_x \neq h_r$, $h_x = a$,
$h_r = b$
when $r \neq 0$

$$\xi(x,r) = \frac{C^2x\left[\xi(x,r+b) + \xi(x,r-b)\right] + \frac{C^2xb}{2r}\left[\xi(x,r+b) - \xi(x,r-b)\right] + \frac{b^2}{a}\left[\xi(x-a,r) - \xi(x+a,r)\right]}{2C^2x + \frac{16b^2\rho}{\pi\mu}f(x,r)} \qquad \text{(A-8)}$$

each iteration. This is a prohibitive volume of computation.

Use of Equations (A-4) and (A-5) and (A-6) and (A-7) resulted in reasonable values of $\xi(x,r)$ for h = 1.0 in most portions of the grid, but resulted in nonconvergence near certain of the boundaries.

Trial calculations were next performed with h_x = 1.0 and h_r = 0.1. It was found that this method resulted in convergent values of $\xi(x,r)$ throughout the grid, as far as the process was carried.

A tentative IBM program plan has therefore been worked out on this basis, and cost estimates prepared accordingly.

As far as can be determined, Equation (A-1) is a type of partial differential equation whose numerical solution has not been attempted previously. There is, therefore, no guarantee that the proposed iterative computation will give convergence and a solution within a reasonable number of iterations. Apparently, the solution of this equation is itself a research problem of considerable importance in applied mathematics.

TABLE II

ABBREVIATED TABLE OF FIRST TRIAL VALUES OF $\xi(\chi, r)$

χ / r	0	1	2	3	4	5
1	.9750	0	0	0	0	0
5	.8750	.2500	0	0	0	0
10	.7500	.2813	0	0	0	0
15	.6250	.3125	0	0	0	0
20	.5000	.3077	.1154	0	0	0
25	.3750	.2613	.1477	.0341	0	0
30	.2500	.1859	.1218	.0577	0	0
35	.1250	.0979	.0707	.0435	.0163	0
39	.0250	.0201	.0152	.0103	.0054	.0005

Note: The complete table (127 values of ξ) has been prepared for all intermediate integral values of χ and r , and also for certain additional intermediate fractional values.

TABLE III

ABBREVIATED TABLE OF FIRST TRIAL VALUES OF $f(\chi, r) \times 10^4$

$$(\overline{\psi u})_0 = 1 \times 10^{-2}$$

χ / r	0	1	2	3	4	5
1	0	0	0	0	0	0
5	1.515	0.053	0	0	0	0
10	1.427	0.234	0	0	0	0
15	0.832	0.541	0	0	0	0
20	0.470	0.388	0.153	0	0	0
25	0.302	0.271	0.155	0.054	0	0
30	0.210	0.196	0.137	0.068	0	0
35	0.154	0.149	0.116	0.071	0.033	0
39	0.124	0.122	0.101	0.068	0.038	0.017

Note: The complete table (127 values) has been prepared for all intermediate integral values of χ and r , and also for certain additional intermediate fractional values.

TABLE IV

Equations Used	Range of h	Range of r	x	Maximum Iterations	Total New Values of ξ
A-2, A-3	$h = 1.0$	$r = 0, 1$	$x = 1\text{-}39$	2	105
A-4, A-5, A-6, A-7	$h = 1.0$	$r = 0, 1$	$x = 1\text{-}39$	1	78
A-2, A-3	$h = 0.5$	$r = 0, 0.5$	$x = 0.5\text{-}6.0$	1	24
A-2, A-3	$h = 0.2$	$r = 0, 0.2$	$x = 0.2\text{-}3.0$	1	28
A-2, A-3	$h = 0.1$	$r = 0, 1.0$ $r = 0, 0.1, 0.2$	$x = 6.0\text{-}7.0$ $x = 0.1\text{-}1.5$	2 1	42 45
A-8, A-9	$h_x = 1.0$ $h_r = 0.1$	$r = 0, 1, 2, 3, 4, 5$	$x = 1\text{-}39$	1	128

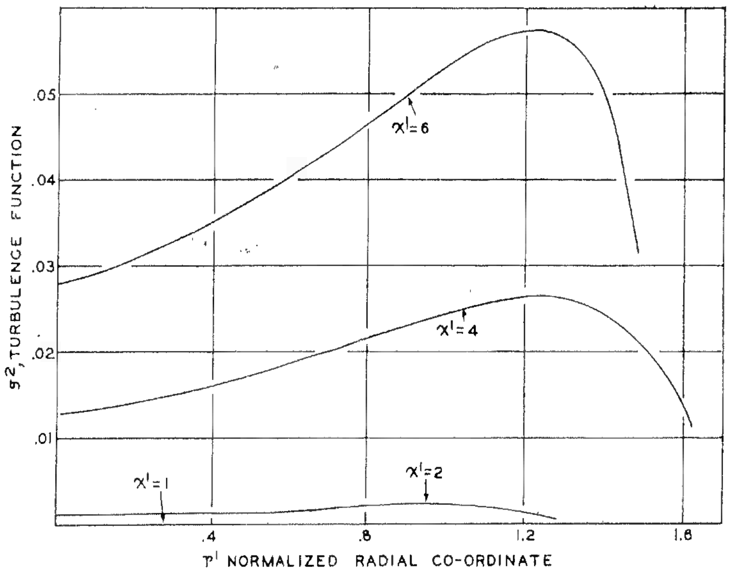


Figure 1.

Turbulence Function vs. Normalized Radial Co-ordinate

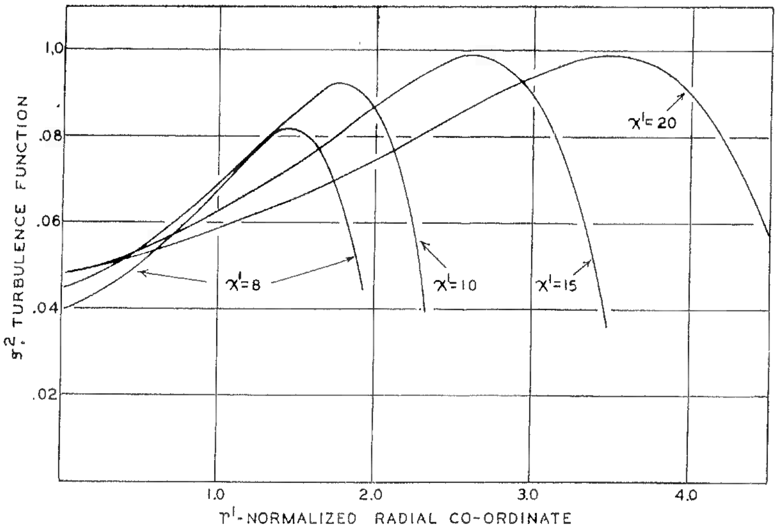


Figure 2.

Turbulence Function vs. Normalized Radial Co-ordinate

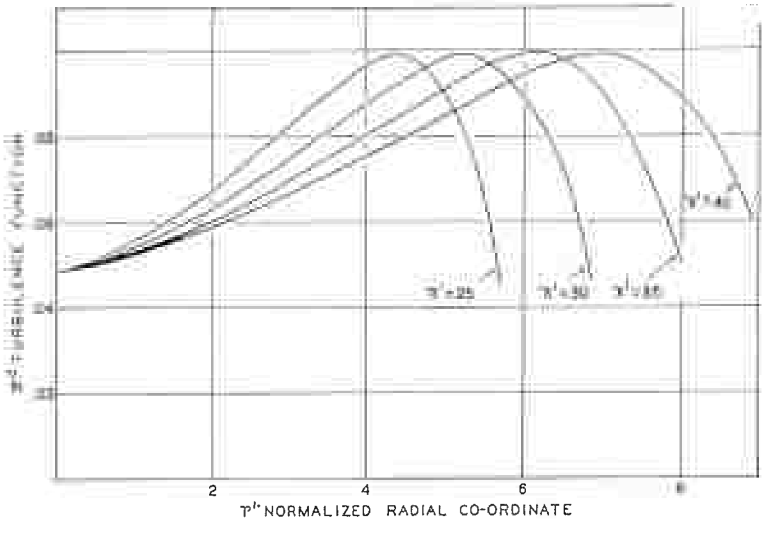


Figure 3.

Turbulence Function vs. Normalized Radial Co-ordinate

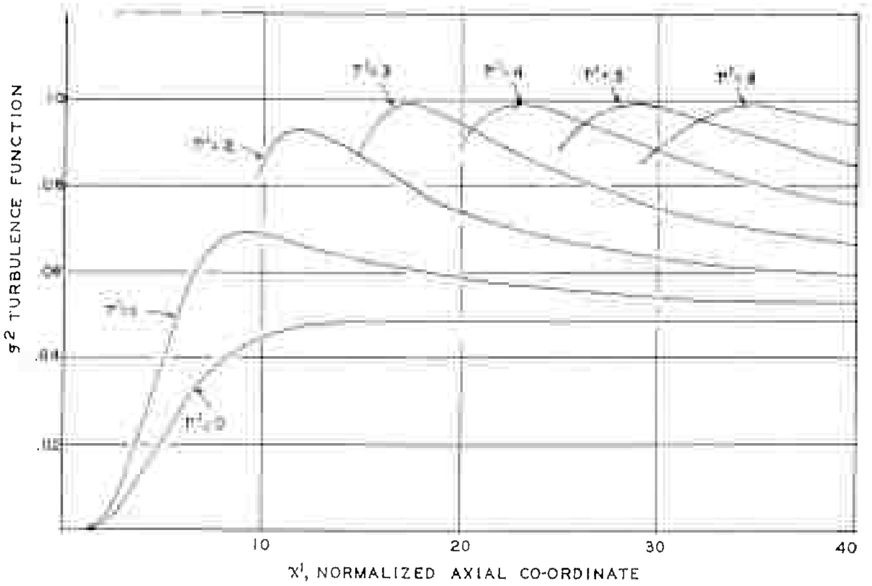


Figure 4. Turbulence Function vs. Normalized Axial Co-ordinate

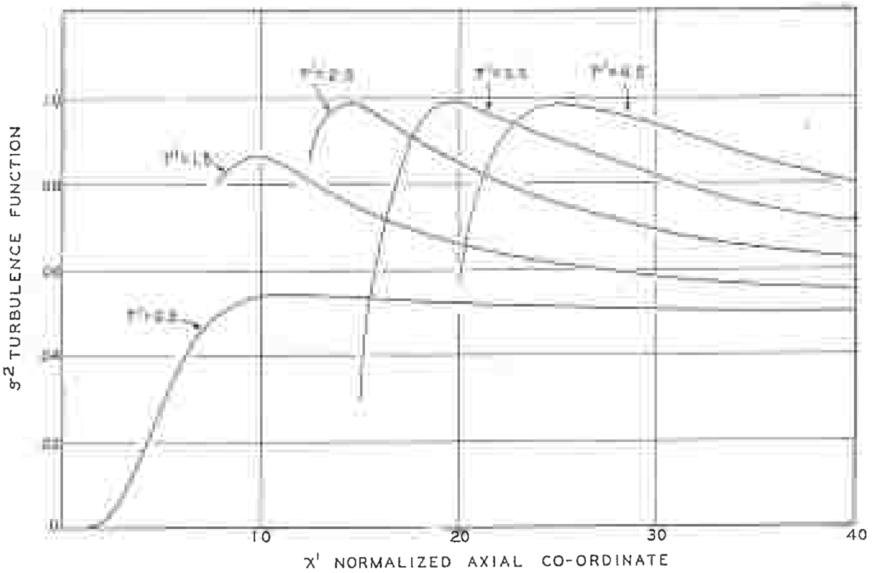
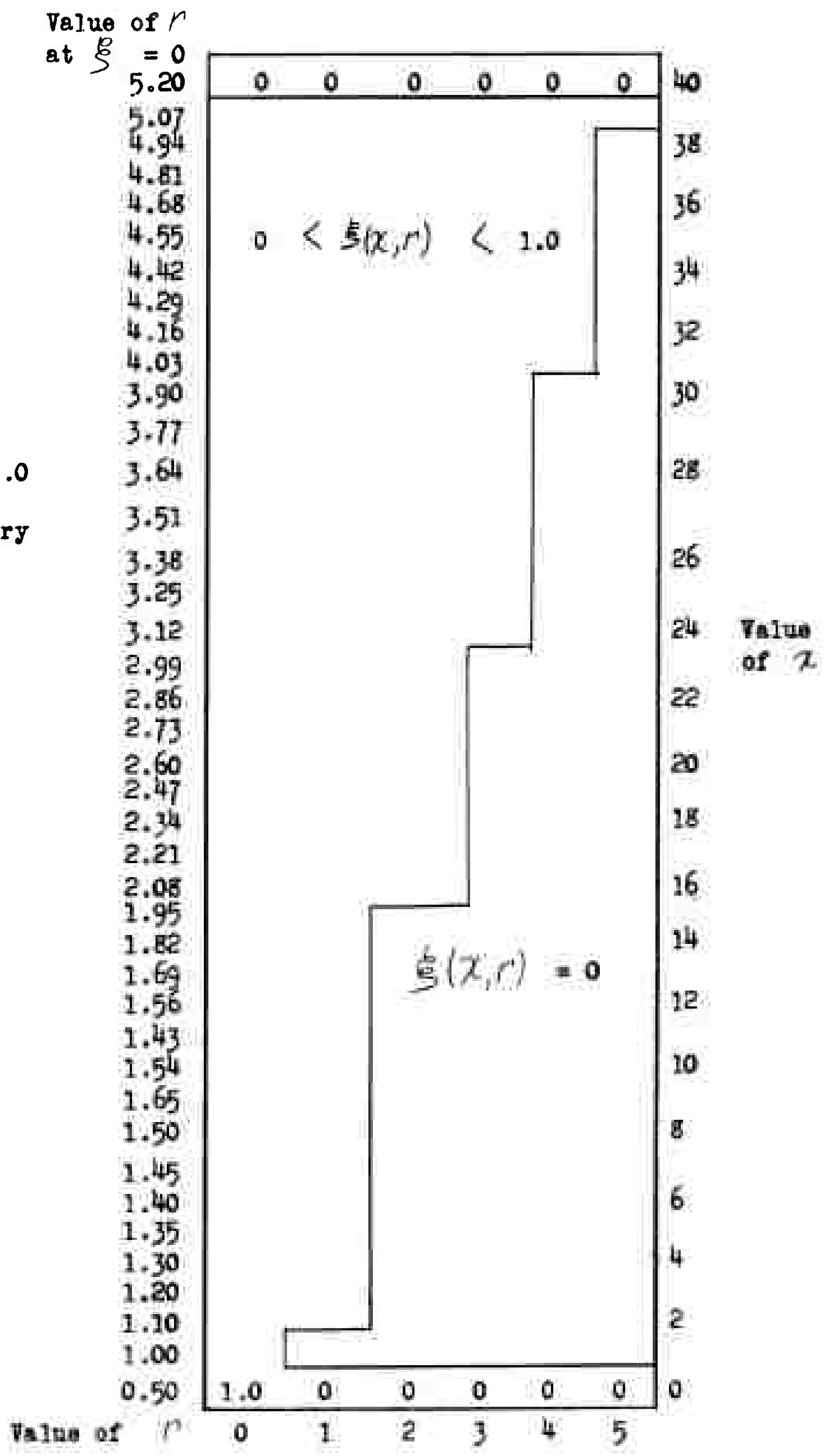


Figure 5. Turbulence Function vs. Normalized Axial Co-ordinate



χ - r grid

$h_\chi = h_r = 1.0$

with boundary values of $\xi(\chi, r)$

FIGURE 6

Boundary Conditions