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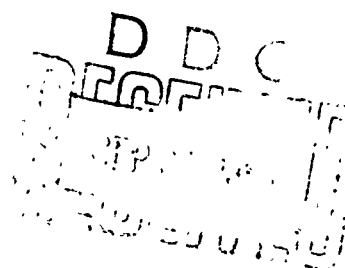
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THESIS

AN ADAPTATION OF A

MARKOV CHAIN MODEL FOR

ANTISUBMARINE WARFARE CARRIER AIRCRAFT

by

George Maurice Lanman

May 1966

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by

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Submitted in partial fulfillment
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ABSTRACT

It is the purpose of this paper to develop a useful mathematical model of ASW aircraft availability. The increasing emphasis of systems studies dictates the use of accurate and representative models of the ASW systems. At present, many studies are using essentially the same models developed during World War II. This paper is an attempt to make use of advanced theory in a more powerful and flexible model and to make the use of the model practical and verifiable.

The writer adapted the time homogeneous bivariate model as developed by F. C. Collins. This is a discrete time Markov process with a stochastic matrix of transition probabilities wherein the maintenance process is modeled as a pulsed input multiple server queue.

The model was programmed in FORTRAN 63 on the CDC 1604 and then modified to allow for variability in the input parameters. Other modifications include an increase in the size of the model to accommodate a 16-aircraft squadron, the largest ASW squadron at present, and an explicit form solution to the maintenance queueing equations.

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TABLE OF SYMBOLS

Symbol	Definition or Meaning
a/c	aircraft
λ	mean repair rate of aircraft
λ_A	mean accident rate
S	the set of all possible outcomes; the probability description space
E	possible outcome(s) or event(s)
$p_{ij}(n, n+1)$	a conditional probability that at time $n+1$ the outcome or state is j given that at time n the state is i
$X_1(t)$	the number of a/c flying at time t, which did not fly the previous cycle
$X_2(t)$	the number of a/c in the maintenance queue at time t
A(t)	the number of a/c desired on station at time t
N(t)	the total number of a/c of type considered at time t
T	the time interval from the launch to recovery at the start of the cycle
$Q(t_o)$	the probability distribution over all possible states at initial time t_o
P(t)	the matrix of transition probabilities at time t

Symbol	Definition or Meaning
$P_{(\alpha, i)(\beta, j)}$	the elements of the P matrix; the probability that $X_1 = \beta$ and $X_2 = j$ at the end of a cycle, given that $X_1 = \alpha$ and $X_2 = i$ at the start of the cycle
γ_{fgh}	the probability given f ready a/c, g are launched, and h enter maintenance
P_γ	probability of entering maintenance just before, during, or immediately after launch
p	the probability of equipment failure during flight requiring maintenance when recovered by the carrier
$\Pi_\alpha(m)$	the probability that of α a/c flying m will enter maintenance upon recovery
D	the number of independent identical maintenance repair stations or "spots"
$P_{ij}(t)$	the probability that i - j a/c are repaired in time interval t

1. INTRODUCTION

The threat to freedom of the seas posed by the vast Soviet sub-marine fleet is perhaps the most thorny problem facing the U. S. Navy today. Two world wars have produced Pyrrhic victories over limited submarine fleets. During the Second World War operations analysis was born into the Navy to aid in the defeat of the German submarine. The classic antisubmarine warfare (ASW) analyses and models developed by Morse [2] and Koopmans [3] are still being used today, over two decades later, in most of the ASW study efforts for the Navy.

These early ASW analyses assumed a given level of search effort available and directly evaluated the probability that an ASW subsystem could detect and/or kill a submarine. This assumption is not only logical to make the problem tractable, but also practical since no immediate changes in ASW force levels could be expected. Moreover, the studies were conducted during the war, not before it started. It is the purpose of this paper to present a probabilistic model to describe the available effort. Such a model can be used to sharpen the estimates of the effectiveness of an ASW subsystem and to study the characteristics of the associated support system.

Naturally, the current study plays an important but limited role in the overall problem of designing an entire ASW system. The difficulties involved in such a specification are legion. First and foremost

is the quantification of the ASW mission in denying the enemy the effective use of his submarines. Currently, the probability of detecting and/or killing submarines is used as the measure of effectiveness of the mission, and it appears that a more encompassing one has not been developed. Second, the specification of an ASW force level to counter a given threat has many inherent subjective elements. These are due to the existing historical bias in predicting the conduct of a future ASW war with an enemy, particularly one who has never before used a large submarine force in its military operations. The reader can imagine why merely defining terms such as "threat" and "effective counter" becomes quite difficult.

Thus, there is a need to investigate the levels of search effort specified. This may require acceptable models to measure the availability of effort, its effectiveness, and determine the logistic support required for any level of available effort. Specifically, the ASW subsystem to be modeled is the carrier-based aircraft, although the model is adaptable to other systems.

The method of investigating the demand for ASW carrier a/c will assume that the desired number of a/c on station is known as an input parameter. The support required to achieve this measure of available effort depends upon maintenance space, manpower, and supply. Generally, we shall consider how an ASW carrier supports this number of a/c on station with the present or proposed number of a/c embarked

on the carrier. The parametric input can be subjected to sensitivity analyses.

The operational commander of the ASW force launches the desired number of a/c on station to screen, search, or actively prosecute a submarine contact. Each a/c is relieved on station. Each such relief requires the launching of another a/c prior to the recovery of the initial a/c. The returning a/c must receive varying degrees of maintenance and requires refueling and rearming. This cycle continues until the mission is completed. Loss of a/c due to accidents, insufficient supply, and lack of repair capability cause deviations in this procedure. Naval operations involve the interaction of many quantities which are random in nature. Not all can be considered in a tractable mathematical model. Some quantities which are important are omitted. One example is the length of each cycle time, which is assumed to be a constant value. Including variables of this nature incurs unnecessary mathematical complication. It is hoped that adequacy of the model can be measured by using fleet data available from the Fleet ASW Data Analysis Program (FADAP).

Collins [5] describes a bivariate Markov model for airborne early warning (AEW) and combat air patrol (CAP) jet a/c operating in an attack carrier force. This model is used to evaluate the probability of maintaining a fixed requirement of a/c on station as a measure of effectiveness of the system. It has subsequently been used in a larger

attack force study for the Navy. The model computes the probabilities of the number of a/c on station and in or awaiting maintenance at any given launch period. The comparable ASW problem differs in the following aspects:

1. Type, range, and speed of a/c;

2. The variable number of a/c required for mission;

3. Attrition due to accidents and supply failures;

4. The greater number of ASW a/c.

It was decided to use the Collins' model with appropriate modification. For immediate reference, the mathematical content of the model will be repeated herein.

In order to incorporate these modifications, it was necessary to spend some time reprogramming on the CDC 1604 digital computer in FORTRAN 63, the CDC version of the IBM FORTRAN IV. The original program was not readily available and was written in an early assembler language. Moreover, the numerical analysis was not sufficiently sharp to handle the larger input values. Also, double precision (two computer words instead of one) arithmetic was required in one subroutine for an accurate explicit solution to the maintenance queueing equations (see Appendix I). This effected a 50% decrease in the computer time required for developing a matrix of transition probabilities.

Following this introduction, section 2 contains a brief description of the operational problems involved and the assumptions made. A brief

description of Markov chains and the mathematical model are presented in section 3. The details for computing the matrix of transition probabilities are given in section 4. General employment of the model follows. The appendices include the solution mentioned on the preceding page, a logical flow diagram of the program, a copy of the program, and some sample results.

2. ASSUMPTIONS

The real-world employment of carrier a/c is cyclic in nature, and the present state of any given a/c (i.e., flying, in or awaiting maintenance) depends largely on what the previous state was. This fact suggests that a Markovian assumption can logically be made for the a/c transition probabilities. In the search phase, a/c may or may not relieve on station; but, in any part of the contact investigation phase, relief on station will be made. To insure full screening and mission coverage, a/c will relieve on station.

The question of resupply during an operation depends primarily on the availability of carrier on-board delivery (COD). This depends on the geographical location and the mission (convoy protection, strike-force protection, hunter-killer operation, etc.). In practice, resupply is not anticipated within a week's period, and around-the-clock operations have continued for two weeks without resupply.

Standard maintenance procedures aboard carriers preclude major maintenance on the flight deck. It will be assumed that sufficient notice is given so that all major 120-hour checks will be completed prior to the operation. This assumption can be modified with an appropriate adjustment in the mean repair rate. The concept of maintenance crews assigned to hangar deck areas ("spots"), as developed by Collins [3], will be used. Each crew will be capable of all types of maintenance

and will operate independently at the identical mean repair rate λ. The number of spots is determined by the average number of such crews available to work continuously around the clock on a watch basis.

The state of each a/c is assumed to be statistically independent of that of others, and the launching and landing transition probabilities will be developed on the basis of independent Bernoulli trials. The parameters can be determined using the maximum likelihood estimators. The range of the number of a/c desired on station at any given cycle will be set by the user. The number to be launched at any time is assumed equally likely within this range. This input parameter is a function of the estimated submarine density (i. e., expected contact rate). The lower limit will be set at the number of a/c desired on station in the search (screening) phase, and the upper limit is set at the maximum practicable number of a/c to be launched during a multiple-contact phase.

Briefly, the assumptions are:

1. a/c will be relieved on station.

2. Any desired length of operation can be set as an input.

3. Major 120-hour checks will be completed prior to the operation.

4. No resupply to the carrier is available.

5. The launch-to-launch cycle for all ASW a/c is four hours.

6. Minor maintenance, refueling, and rearming only can be performed on the flight deck.

7. Each maintenance spot is characterized with an independent exponential repair time with mean repair rate of λ for around-the-clock operations.

8. The number of a/c lost due to attrition is a Poisson random variable for each cycle period with parameter λ_A (a/c accident/flying hours for a/c type).

9. Any a/c lost by accident will not be returned to service due to either (a) physical loss at sea, or (b) insufficient maintenance capability aboard ship and lack of major parts.

10. The number of a/c launched for each cycle is uniformly distributed between the upper and lower limits determined by the user.

3. MODEL DESCRIPTION

3.1 The Theory

A stochastic or random process is a collection of random variables indexed on some set T, $(X(t), t \in T)$. In this case, time is the indexing set, and the Markovian assumption states that the future state of the process depends only on the state at the present time and not on its past history. Due to the cyclic nature of our problem, it is possible to increment time $(T = (0, 1, \ldots))$ using the cycle time from launch to launch as the steps of unit time in a discrete Markov chain. It is assumed that the reader is familiar with the notion of a random variable as a function defined on a sample description space (S) on which the family of events or outcomes (E) of a probability function can be defined [4].

A discrete time Markov chain is described by a sequence of discrete valued random variables and is determined when the one-step transition probabilities of the state variables are specified, i.e., a conditional transition probability of a transition at time n for each pair of i, j = 0, 1, ..., m (m being the number of states in the process) must be given.

$$p_{ij}(n, n+1) = P[X(n+1) = j \mid X(n) = i]$$

If the transition probability functions depend only on the time difference, we have time homogenity

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$$p_{ij}(n + 1, 1) = p_{ij}(0, 1) = p_{ij} \cdot$$

The initial state of the system must be given either as a specific state or randomly as a probability distribution function over the possible states.

The p_{ij} (transition probabilities) are arranged in matrix form and satisfy:

1. $p_{ij} \geq 0$ for $i, j = 0, 1, \ldots, m$;

2. $\sum\limits_{j=0}^{m} p_{ij} = 1$, i.e., the rows of the transition matrix sum to 1 for all i for the states within the description space [4].

3.2 The Model

In order to establish the finite set of states (E) for the model, we shall consider two random variables defined as follows:

$X_1(t)$ = The number of a/c flying at time t not having flown in

        the previous launch-to-launch interval.

$X_2(t)$ = The number of a/c in or awaiting maintenance at time t.

Now, we will consider the vector $X(t) = [X_1(t), X_2(t)]$ as a pair of random variables and thereby have a bivariate stochastic process with the possible states ranging from (0, 0) to (A, N).

$0 \leq X_1(t) \leq A$ = No. of a/c desired on station, and

$0 \leq X_2(t) \leq N$ = No. of a/c of given type aboard carrier.

We will define an operating cycle as an interval unit of time. Process observations of $X(t)$ will be made at successive unit interval launch times. To develop the p_{ij} elements, consider a given time t for launching until A aircraft are flying or until the supply of ready a/c is depleted. Those a/c failing the launch enter the maintenance state at this idealized point in time t (the total launching time required is much less than the total cycle time). At some time T, less than the launch-to-launch unit time interval, the a/c which were relieved on station return and land at the idealized point in time t + T. Some of these a/c will require maintenance and enter the maintenance queue. Those requiring only refueling and preflight inspection will enter a ready status to be tested for the next launch.

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During the unit time interval, maintenance will be performed on those a/c in the not-ready status, and a certain number of aircraft will be repaired according to assumption 7.

In summary, we start the system in some initial state (such as (0, 0) with no a/c flying or in maintenance) or start with a probability distribution $Q(t_o)$ over the states, E, at time t_o. We launch, recover, and repair a/c in the unit interval and repeat the process over each succeeding unit time interval until the end of the operating period. Knowing the transition probabilities within the unit time interval, we can develop the elements of the transition matrix, P, or $\{P_{(\alpha, i), (\beta, j)}\}$. These are the probabilities of going from the state of α a/c flying and i a/c in maintenance to β a/c flying and j a/c in maintenance over the unit time interval.

It was assumed in section 2 that A, the number of a/c to be launched, and N, the total number of a/c on board, are random variables, whereas they have been treated as constants so far in the development. To be analytically correct in including this feature, one should develop the appropriate quadrivariate process. Such a development leads to too large a state space and the author chose to include these effects by using a Monte Carlo simulation technique. That is, at the beginning of each cycle, a random mechanism is used to determine the values on A and N.

The probability of losing an a/c or changing the desired number to be launched is determined from the specified distributions at the beginning

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of each unit interval, and the resulting P matrix containing the

$P_{(\alpha, i), (\beta, j)}$ is then recomputed. The probability distribution $Q(t)$

over the states at any time t may be determined by the appropriate

number of successive iterations of the Q vector times the P matrix, i. e.,

$$Q(t) = P[X_1(t) = \beta, X_2(t) = j] = Q(t - 1) \times P .$$

The probability of maintaining α a/c on station over any given period

of operation may be obtained at any unit time t (i. e., the beginning of

the next cycle) by summing out the appropriate maintenance state proba-

bilities. Thus, $P(\alpha$ a/c are flying at time t) =

$$Pr\ (X_1(t) = \alpha\ ) = \sum_{i=o}^{N} Pr\ (X_1(t) = \alpha, X_2(t) = i\ ) .$$

A mathematical comment appears to be in order. In the case of

fixed A and N, the states of the Markov chain are positive recurrent; and

steady-state probabilities can be found for the entire state space. In the

case of decreasing N due to a/c attrition, this is not true; and (0, 0)

becomes an absorbing state as time (t) goes to infinity. This latter

consideration is not a realistic one for the operational period envisioned.

Therefore, it is mathematically more feasible to use the former chain in

conjunction with the Monte Carlo technique.

4. DEVELOPMENT OF THE TRANSITION MATRIX

Perhaps the simplest way to view this development is to note the various transition probabilities incorporated in one-unit time cycle defined as follows:

(1) γ_{fgh} = the launching transition probabilities at time t. This is the probability of taking f ready a/c, launching g successfully, and sending h into maintenance. Each a/c to be launched is considered a Bernoulli trial with probability of failure of p_γ, which is estimable and subject to sensitivity analysis. The values of γ_{fgh} are:

   a. 0 if g > A, since only A a/c are desired;

   b. 0 if g + h > f; it is impossible to launch and send into maintenance more a/c than are available;

   c. 0 if g < A, g + h < f; launching continues until A a/c are flying or until all f are used up;

   d. $\binom{f}{g} (1 - p_\gamma)^g (p_\gamma)^{f-g}$ if g < A, g + h = f, standard binomial when all a/c in the ready state are used up but the A a/c are not launched;

   e. $\binom{g+h-1}{h} (1 - p)^g (p)^h$ if g = A, g + h > f, standard negative binomial for g successes in g + h - 1 trials.

(2) $\Pi_\alpha (m)$ = the landing transition probabilities which occur at time t + T. We must consider the probability that if there are a/c flying at time t then m a/c will enter maintenance at recovery time t + T.

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$\Pi_\alpha(m)$ will equal a standard binomial where p = the probability of equipment failure in flight:

$$\Pi_\alpha(m) = \binom{\alpha}{m}(1-p)^{\alpha-m}(p)^m , \quad m = 0, 1, \ldots, \alpha .$$

(3) $p_{ij}(\tau)$ = the maintenance transition probabilities, i. e., the probability of repairing (i - j) a/c in time τ . Two maintenance periods occur: the first starting at time t and ending at time t + T , the second starting at time t + T and ending at the end of the cycle, (t + 1) . Under assumption 7, the pulsed input, multiple exponential server queue is developed with D maintenance "spots" or servers each with identical, independent service rates, λ . For each server, then, the probability of remaining occupied (given the server is busy) in time $\tau = e^{-\lambda\tau}$. The probability of becoming free (i. e., repairing an a/c) = $1 - e^{-\lambda\tau}$. The resulting queueing equations are:

A. $dP_{i,n}(t) / dt = -n\lambda P_{i,n}(t) + (n+1)\lambda P_{i,n+1}(t)$ for $0 \le n < D$;

B. $dP_{i,n}(t) / dt = -D\lambda P_{i,n}(t) + D\lambda P_{i,n+1}(t)$ for $n \ge D$.

Three ranges of i (initial queue state) , j (final queue state) , and D become significant:

a.   When $j \le i \le D$, then not all spots are busy since there are fewer a/c in maintenance than spots. Each spot works independently; therefore, the solution to A is the binomial:

$$p_{ij}(t) = \binom{i}{j}(1 - e^{-\lambda t})^{(i-j)} e^{-\lambda t j} .$$

b. When $D \leq j \leq i$, then all spots are occupied throughout the total

service time, and the closed form solution to B is the Poisson:

$$P_{ij}(t) = \frac{(D\lambda t)^{(i-j)} e^{-D\lambda t}}{(i-j)!} \quad .$$

c. When $j < D < i$, then all spots are busy at the beginning of the

service period, and some spots become idle during the service

period. The explicit form solution of equation A is found using

moment generating function transformation:

$$P_{ij}(t) = \sum_{n=j}^{D-1} \binom{n}{j} \binom{D}{n} \left\{ \left(\frac{D}{D-n}\right)^{(i-D)} e^{-\lambda t n} \right.$$

$$\left. - e^{-\lambda t} \sum_{k=o} \left(\frac{\lambda D t}{k!}\right)^{k} \left(\frac{D}{D-n}\right)^{i-D-k} \right\} \quad .$$

(The derivation of this solution is discussed in Appendix I.)

The figure on the following page will show the relationships of these

transition probabilities within the unit time interval.
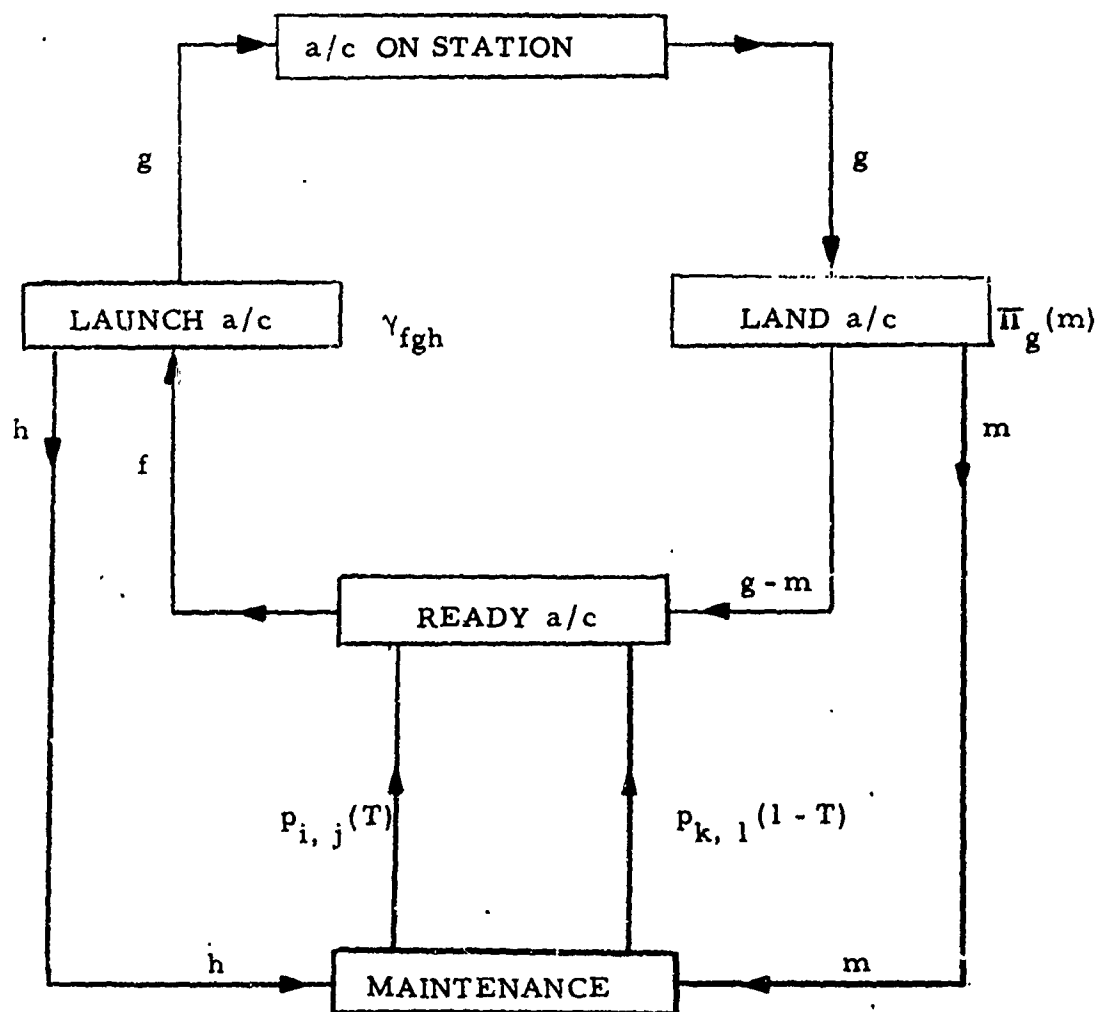
TRANSITION PROBABILITIES WITHIN THE UNIT CYCLE

FIGURE 1

In order to develop each transition probability over the total unit
time interval, we must consider all events taking place within the
interval. Thus, to obtain the probability of going from α a/c flying and
i a/c in maintenance to β a/c flying and j a/c in maintenance, we start
at the state (α, i) at time t. At this time, a/c are launched and some
l a/c failing the launch enter maintenance. These i + l in maintenance
are then serviced until time $t + T$ when some k a/c are still in the
maintenance state. At time $t + T$, of the α a/c previously flying, some
m enter maintenance and $(\alpha - m)$ enter the ready pool. Maintenance is
continued on the $(k + m)$ a/c for the remainder of the cycle $(1 - T)$, until
the end of the unit time interval when j a/c remain in the maintenance
state. In functional form:

$$P_{(\alpha, i), (\beta, j)} = \sum_{l=0}^{N-\alpha-i} \sum_{k=0}^{i+l} \sum_{m=0}^{\alpha} \gamma_{N-\alpha-i, \beta, l}$$

$$\cdot P_{i+l, k}(T) \cdot \Pi_{\alpha}(m) \cdot P_{k+m, j}(1 - T) .$$

5. SUMMARY

Representative values for the mean repair rate and the landing and launching failure rates produced results in agreement with the sensitivity analysis by Collins on these parameters in [5]. For failure probabilities less than .5, and mean repair rate less than 12 hours, the effect of reducing the available maintenance time to 80% of the cycle time was negligible. Optimal loading and cycling policies can be determined for known values of these rates.

The model affords the following checks: (1) the rows of each P matrix are summed as they are computed by the program; and (2) the probability distribution vector (Q J) is summed over the states. Each summation was within 10^{-8} of one in the computer model.

The user may substitute any available distribution over the interval of a/c desired on station. In order to keep A fixed, enter the desired value as both upper and lower limit (A = ALOLIM = LUPLIM). For fixed N, use a very small value for ALAM (such as 10^{-8}). Subroutine KRAN is a uniform generator, using the half open interval (lower limit + 1, upper limit + 2) and a starting number as inputs. KRAN outputs an integer in this interval. Subroutine DRAW was used to provide some intuitive grasp of the results. DRAW was used in binary card form and is not essential to the main program. (The indicated associated statements must be removed, however.)

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The results of reasonable arbitrary parameter values, based on the author's experience, have shown that most of the probabilities concentrate over a few states. Moreover, computation time increases rapidly as a function of N (no. of a/c), see Figure 2. This would indicate that a simple approximation to the model could be developed. One method presently being investigated to reduce computation time is to shrink the probability state space to include only those significant states and, thus, reduce the size of the transition matrix. Alternatively, the eigenvector, eigenvalue representation of the P matrix, might be used.

Originally, it was hoped to utilize the data from the Fleet ASW Data Analysis Program (FADAP) to attempt a verification of the model with its real-world counterpart. The only method available at present for obtaining the necessary data is by direct observation or a program of data collection, as suggested by Collins [5].

Many fruitful areas of investigation exist:

(1) Attrition has been simply modeled by the Poisson method. The two components of attrition, accidents and supply shortage, can be more accurately modeled and used to develop logistic schedules for maintenance and supply. One simple technique is to assume each component is independent and Poisson, and estimate a supply failure rate for AOCP attrition from past data. With these assumptions, the total attrition is Poisson, with the parameter equal to the sum of the accident and supply failure rates.

(2) The model can be modified to make the number of maintenance spots available for any cycle a variable function of time, $D(t)$.

(3) An investigation of the Markovian assumption validity as the cycle times become smaller and smaller.

(4) Development of a continuous time model.

(5) Modification of the model to simulate resupply by COD.

(6) A study of the distribution of submarine contacts to determine the validity of the uniform a/c demand assumption.
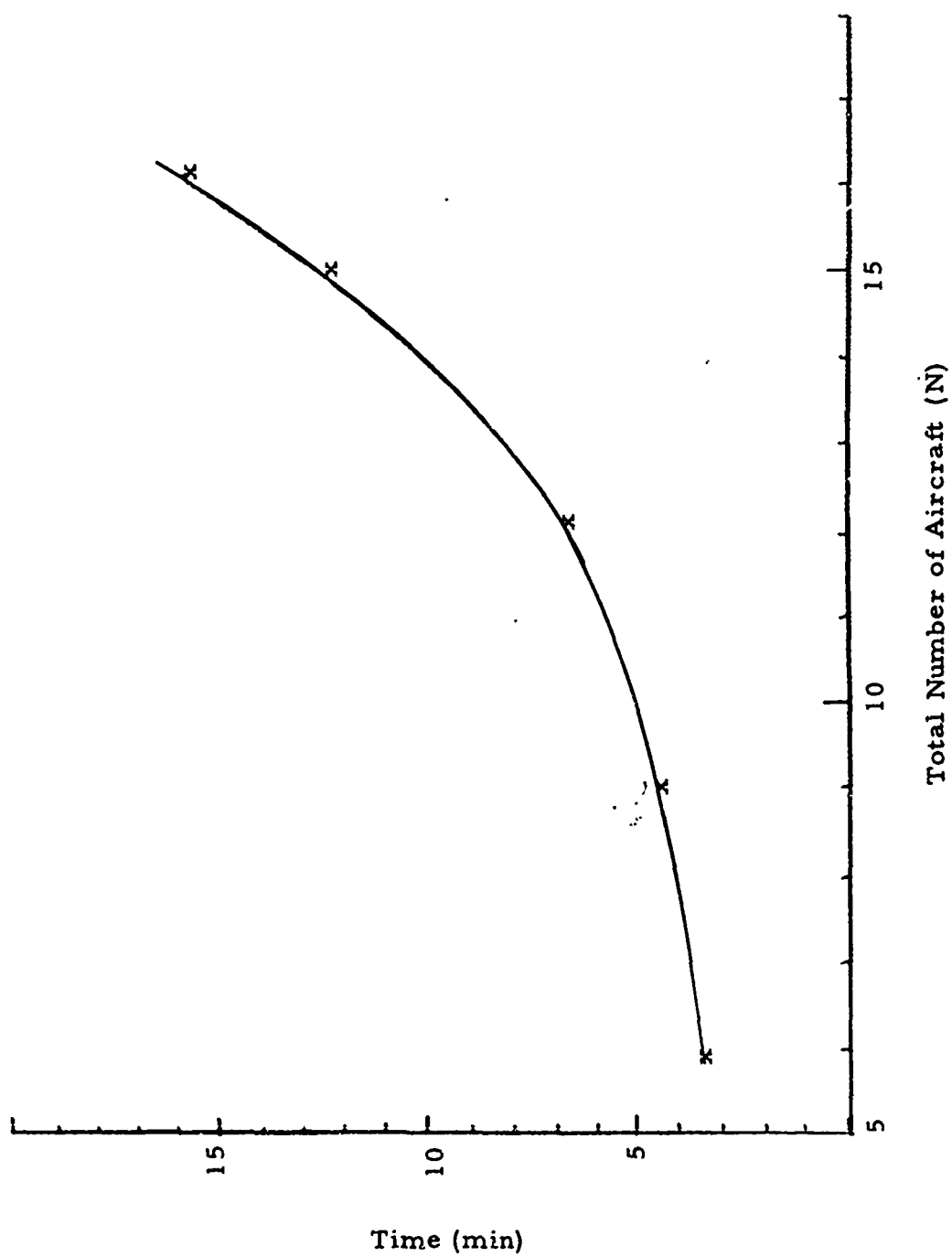
PROGRAM ASSEMBLY AND COMPUTATION TIME
FOR ONE TRANSITION MATRIX (P) AS A FUNCTION
OF THE TOTAL NUMBER OF AIRCRAFT (N)

FIGURE 2

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APPENDIX I

EXPLICIT SOLUTIONS OF THE

MAINTENANCE QUEUEING EQUATIONS

The queueing equations for the pulsed input queue are essentially

the pure death process given in [1] and [4] as problems and developed

by Collins in [5]. The equations are:

A. $\quad\dfrac{dP_{i,\,n}(t)}{dt} = -n\lambda P_{i,\,n}(t) + (n+1)\lambda P_{i,\,n+1}(t)\quad$ for $0 < n < D$

B. $\quad\dfrac{dP_{i,\,n}(t)}{dt} = -n\lambda P_{i,\,n}(t) + D\lambda P_{i,\,n+1}(t)\quad\quad$ for $n \geq D$

where $P_{ij}(0) = \Delta_{ij}$ and $P_{ij}(t) = 0$ for $i < j$, since no input (arrivals)

occur during the service time.

Equation B is solved directly in closed form:

$$P_{i,\,n}(t) = \frac{(D\lambda t)^{(n-i)}\, e^{-\lambda Dt}}{(n-i)!}\;.$$

Now transforming the first equation (A) using the moment gener-

ating function (MGF), $\quad\mathscr{f}$

$$G(s,\,t) = \sum_{n=0}^{D-1} s^n P_n(t)\;,$$

as outlined in [4] (Chapter 7), and its partial derivatives:

$$(1) \qquad \frac{dG}{dt} = \sum_{n=0}^{D-1} s^n P_n'(t)$$

$$(2) \qquad \frac{dG}{ds} = \sum_{n=0}^{D-1} n s^{n-1} P_n(t)$$

Where $P_n(t)$ denotes the conditional probability $P_{i,\,n}(t)$, by substituting (A) into (1), properly identifying the first summation with (2), and changing the second summation index to $r = n + 1$, we get:

$$\frac{dG}{dt} = - \lambda s \frac{dG}{ds} + \lambda \sum_{r=0}^{D} r s^{r-1} P_r(t), \qquad \text{or}$$

$$(3) \qquad \frac{dG}{dt} = - \lambda (s - 1) \frac{dG}{ds} + \lambda D s^{D-1} P_D(t),$$

since

$$\sum_{r=0}^{D} r s^{r-1} P_r(t) = \frac{dG}{ds} + D s^{D-1} P_D(t).$$

Next, replace the partial differential equation (3) with a system of ordinary differential equations using the Lagrangian auxiliary equations:

$$\frac{dt}{1} = \frac{ds}{\lambda (s - 1)} = - \frac{dz}{\lambda D s^{D-1} P_D(t)}.$$

The solution to the first equation (using the first two differentials) is:

$$\lambda t = \ln (s - 1) + C'$$

and hence

$$s = C_1 e^{\lambda t} + 1$$

or

$$G_1 = e^{-\lambda t} (s - 1) \ .$$

The second equation is: (using first and third differentials)

$$dz = -\lambda D (C_1 e^{\lambda t} + 1)^{D-1} P_D(t) \, dt \ .$$

Using the solution to (B) where $m = i - D$ to replace $P_D(t)$ and

integrating, term wise, the binomial expansion of $(C_1 e^{\lambda t} + 1)^{D-1}$:

$$z = \frac{(\lambda D)^{m+1}}{m!} \sum_{j=0}^{D-1} \binom{D-1}{j} C_1^{j} \int t^{m} e^{-\lambda (D-j)t} \, dt$$

where the integral is evaluated as:

$$- \sum_{k=0}^{m} \frac{t^{k} e^{-\lambda (D-j)t}}{(\lambda (D-j))^{m-k+1}} \, \frac{m!}{k!} + C_2 \ .$$

Thus,

$$C_2 = z + e^{-\lambda Dt} \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^{j} \sum_{k=0}^{m} \frac{(\lambda Dt)^{k}}{k!} \left(\frac{D}{D-j}\right)^{m-k}$$

and the general solution is $\phi (C_1, C_2)$, where ϕ is an arbitrary function

and

$$C_1 = u (s, t, z)$$

and

$$C_2 = v (s, t, z) \ .$$

To get our particular solution, use the boundary conditions for $G(s, t)$:

(1) for $s = 1$,

$$G(1, t) = \sum_{n=0}^{D-1} P_n(t)$$

$$= \Pr[\text{no. in maintenance at } t \text{ is} < D \mid i \text{ at } t = 0]$$

$$G(1, t) = 1 - \sum_{n=0}^{i-D} \frac{e^{-\lambda Dt}(\lambda Dt)^n}{n!} = 1 - \psi_1(t)$$

where

$$u(1, t, z) = C_1 = 0$$

$$v(1, t, z) = C_2 = z + e^{-\lambda Dt} \sum_{k=0}^{m} \frac{(\lambda Dt)^k}{k!}$$

so

$$C_2 = z + \psi_1(t)$$

(2) for $t = 0$,

$$G(s, 0) = \sum_{n=0}^{D-1} s^n P_n(0) = 0, \quad \text{since } i \geq n > D$$

where

$$u(s, 0, z) = (s - 1)$$

$$v(s, 0, z) = C_2 = z + \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j \left(\frac{D}{D-j}\right)^m \quad .$$

Thus,

$$G(s, 0) = z + \sum_{j=0}^{D-1} \binom{D}{j} C^j \left(\frac{D}{D-j}\right)^m - C_2 \quad .$$

Substituting the general value for C_2 above:

$$G(s, t) = \phi(u, v) = \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j e^{-\lambda t j} \left(\frac{D}{D-j}\right)^m$$

$$- \sum_{j=0}^{D-1} \binom{D}{j} (s-1)^j \sum_{k=0}^{m} \frac{(\lambda D t)^k}{k!} e^{-\lambda D t} \left(\frac{D}{D-j}\right)^{m-k}$$

Rearranging terms,

$$G(s, t) = \sum_{n=0}^{D-1} s^n \sum_{j=n}^{D-1} \binom{j}{n} \binom{D}{j} (-1)^j \left[\left(\frac{D}{D-j}\right)^m e^{-\lambda t j} \right.$$

$$\left. - e^{-\lambda D t} \sum_{k=0}^{m} \frac{(\lambda D t)^k}{k!} \left(\frac{D}{D-j}\right)^{m-k} \right]$$

where $P_n(t) = $ the coefficient of s^n.

APPENDIX II

THE LOGICAL FLOW DIAGRAM OF THE COMPUTER PROGRAM

SET INPUTS   (A, N, p , PGAM TIME, ICYCLE, JCYCLE
FLAM, ALAM, AUPLIM, ALOLIM, ETC.)

(805) → DRAW IA   (lower limit ≤ A ≤ upper limit)

IS THIS 1st CYCLE? — (11) yes   no

was 1 a/c lost at t_{-1}?   yes   no

were 2 a/c lost at t_{-1}?   no → IA=IA-1

were 3 a/clost at t_{-1}?   no → IA=IA-2

yes → IA=IA-3

(11) → SET GAMMA(I,J,K)=0

COMPUTE $\pi_\alpha(m)$=PRFM(IAA,MI)

COMPUTE $p_{ij}(t)$ = PTRM(I,IJ,IT)

HAS IA CHANGED?   no   yes

HAS N CHANGED?   yes   no

COMPUTE f_{gh}=GAMMA(IF,IG,IH)

IALAST=IALPH

SET THE TRANSITION MATRIX =0 → (801)

801 → COMPUTE $p_{(\ ,x)(\ ,y)}(t) = PR(I,IBETA,IY)$

$(\ I = IX+(IALPH-1)\times TOTAL\ A/C)$

PRINT THE p_{ij}'s OF THE (IALPH,IX)TH ROW AND ROW SUM

SET THE QJ VECTOR = O

MULTIPLY QI x P matrix

COMPUTE THE (IBETA, IY)TH ELEMENTS OF QJ(J)

$(J=IY+(IBETA-1)\times N)$

SUM THE ELEMENTS OF THE QJ VECTOR

COMPUTE THE MEAN OF A/C LAUNCHED

PLOT QJ(J) VS J

IS THIS LAST CYCLE — yes → STOP

no

SET QI = QJ

REINDEX MATRIX
IALAST =IA
MLAST = MEAN
IALAST1 = IALAST

805

40

APPENDIX III

THE COMPUTER PROGRAM

```
-COOP,,LANMAN,0/49/S/1S/2S/E/45=54,15 ,30000,5.               0001
-BINARY,56.                                                   0001
(RELOCOM.                                                     0002
-FTN,E.                                                       0003
      PROGRAM MARKOV                                          0004
C   THIS PROGRAM IS A NONSTATIONARY BIVARIATE MARKOV CHAIN MODEL OF ASW A/C 0005
C   OPERATIONS.  THE RANDOM VARIABLES ARE THE NUMBER OF A/C FLYING AT THE  0006
C   BEGINNING OF ANY GIVEN LAUNCH CYCLE.  THE MAXIMUM NO. OF A/C ALLOWED IN 0007
C   THE MODEL IS 16(NA).  THE RANGE OF A/C TO BE LAUNCHED AT ANY GIVEN     0008
C   INTERVAL IS 0 TO 6 A/C.  THE FOLLOWING INPUTS ARE REQUIRED.           0009
C          ID= THE NO. OF INDEPENDENT MAINTENANCE SPOTS                    0010
C          NA= TOTAL NO. OF A/C TYPE ON BOARD                              0011
C          TIME=TIME FROM LAUNCH TO RECOVERY/LAUNCH TO LAUNCH CYCLE TIME(HRS)0012
C          FLAM=MEAN REPAIR TIME PER SPOT/LAUNCH TO LAUNCH CYCLE TIME   (HRS)0013
C          PGAM= PROBABILITY OF A/C FAILING LAUNCH(M.L.EST. FROM PAST DATA) 0014
C          P= PROBABILITY OF A/C FAILURE DURING FLIGHT REQUIRING MAINTENANCE 0015
C              AT LANDING (M.L. ESTIMATOR FROM PAST DATA)                  0016
C          QI= THE PROBABILITY DISTRIBUTION VECTOR OVER ALL POSSIBLE STATES 0017
C          (7 X 17 = 119) SUCH THAT THE SUM OF ALL QI(I) = 1.    THIS      0018
C          IS ESTIMATED BY THE USER AND INPUTTED BY USING  A DATA STATEMENT0019
C          ICYCLE = NO. CYCLES DESIRED FOR OPERATION                       0020
C          JCYCLE = LAUNCH TO LAUNCH TIME(HRS)(TOT. TIME=ICYCLE X JCYCLE)  0021
C          ALAM = ACCIDENT RATE FOR TYPE A/C (ACCIDENT/HOURS)              0022
C          ALOLIM = DESIRED LOWER LIMIT ON A                               0022A
C          AUPLIM = DESIRED UPPER LIMIT ON A                               0022E
      COMMON FLAM,TIMF                                        0023
      TYPE DOUBLE FLAM                                        0024
      COMMON PTRM,GAMMA,PR,PRFMA,ID                           0025
      DIMENSION BC(17),A(17),FBC(17)                          0026
      DIMENSION PTRM(17,17,2),GAMMA(17,7,17),PRFMA(7,7)       0027
      DIMENSION PR(119,7,17),QI(119),QJ(119)                  0028
      DIMENSION        FJPLOT(119),JT(12)                     0029
C     ENTER DATA CARDS HERE                                   0030
      DATA((QI(I),I=1,119)=.2,16(.05),102(.0))
      NA=16
      ALAM=.01
      ID = 8
       FLAM=3.0
      PGAM=P=.4
      IYY = 13421773
      TIME = .125
      ICYCLE=20
      JCYCLE=4     .
      ALOLIM=4.
      AUPLIM=6.
C     END OF DATA CARDS                                       0031
      AL=ALOLIM+1.   $ AU=AUPLIM +2.                          0032
      UNITT=1.                                                0033
      N=NA+1                                                  0034
      IAMAX=7                                                 0035
      IALAST=0                                                0036
      D=FLOATF(ID)                                            0037
      NLAST=NEWN=N                                            0038
      KT=1                                                    0039
  809 IA=KRAN(AL,AU,IYY)                                      0040
      IF(KT.EQ.1) 113,115                                     0041
```

```
  115 T1=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM              0042
      IF(T)-TFLC)130,131,132                                     0043
  130 T2=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM              0044
      IF(T1+T2-TFLC) 230,231,131                                 0045
  230 T3=-LOGF(.000000001 + RANF(-1))*2.30258/ALAM              0046
      IF(T1+T2+T3-TFLC) 331,331,231                              0047
  331 NEWN=NLAST-3   $ GO TO113                                  0048
  231 NEWN=NLAST-2   $ GO TO113                                  0049
  132 NEWN=NLAST $ GO TO113                                      0050
  131 NEWN=NLAST-1                                               0051
  113 PRINT 8882,IA,NEWN                                         0052
      IF(NEWN-IA) 15,13,13                                       0053
   15 IA=NEWN                                                    0054
   13 IF(IALAST) 11,12,11                                        0055
   12 CONTINUE                                                   0056
C  FROM THIS NEXT STATEMENT TO NO. 483 IS CONCERNED ONLY WITH THE GRAPH  0057
      DO 482 I=1,12                                              0058
  482 JT(I)=8H                                                   0059
      JT(1)=8HE(A/C) =                                           0060
      JT(3)=8HSPOTS =                                            0061
      JT(5)=8H    T =                                            0062
      JT(7)=8HJ VS QJ                                            0063
      JT(8)=8HVECTOR                                             0064
      JT(9)=8H    N =                                            0065
      JT(11)=8H    A =                                           0066
      DO 483 I=1,119                                             0067
      FI=I                                                       0068
  483 FJPLOT(I)=FI                                               0069
      IALAST=IA                                                  0070
      DO 1235 I=1,17                                             0071
      DO 1235 J=1,IAMAX                                          0072
      DO 1235 K=1,17                                             0073
 1235 GAMMA(I,J,K)=0.0                                           0074
C  AT THIS PT THE LANDING TRANSITION PROBABILITIES ARE COMPUTED  0075
      DO 300 IAA=1,IAMAX                                         0076
      DO 301 MI=1,IAMAX                                          0077
      IF(IAA-MI)31,32,33                                         0078
   31 PRFMA(IAA,MI)=0.                                           0079
      GO TO 301                                                  0080
   32 MM1=MI-1                                                   0081
      PRFMA(IAA,MI)=P**MM1                                       0082
      GO TO 301                                                  0083
   33 IAM1=IAA-1                                                 0084
      MM1=MI-1                                                   0085
      BC(1)=1.0                                                  0086
      PROD=FLOATF(IAA-MI)                                        0087
      DO 50 IP=2,MI                                              0088
      AIP=FLOATF(IP-1)                                           0089
      PROD = PROD +1.0                                           0090
   50 BC(IP)=PROD*BC(IP-1)/AIP                                   0091
      IGO=IAA-MI                                                 0092
      PRFMA(IAA,MI)= (BC(MI)*(1.0-P)**(IGO))*P**MM1             0093
  301 CONTINUE                                                   0094
  300 CONTINUE                                                   0095
C  AT THIS PT THE MAINTENANCE TRANSITION PROBABILITIES ARE COMPUTED  0096
      DO 100 IT=1,2                                              0097
```

```
      IF(IT-1)25,25,26                                      0098
   25 TAU = TIME                                            0099
      GO TO 28                                              0100
   26 TAU = UNITT-TIME                                      0101
   28 DO101 I=1,N                                           0102
      DO 102 IJ=1,N                                         0103
      IF (I-IJ) 14,199,17                                   0104
  199 IF(I-ID) 19,19,1999                                   0105
 1999 PTRM(I,IJ,IT)=EXPF(-FLAM*TAU*D)                       0106
      GO TO 102                                             0107
   14 PTRM(I,IJ,IT)=0.                                      0108
      GO TO 102                                             0109
   19 FJM1=FLOATF(IJ-1)                                     0110
      PTRM(I,IJ,IT)=EXPF(-FLAM*TAU*FJM1)                    0111
      GO TO 102                                             0112
   17 IF(I-ID-1) 1,1,2                                      0113
    1 BC(1)=1.0                                             0114
      PROD=FLOATF(I-IJ)                                     0115
      DO 10 IP =2,IJ                                        0116
      AIP=FLOATF(IP-1)                                      0117
      PROD = PROD + 1.0                                     0118
   10 BC(IP) =PROD*BC(IP-1)/AIP                             0119
      ELT=EXPF(-FLAM*TAU)                                   0120
      PTRM (I,IJ,IT)=BC(IJ)*(1.-ELT)**(I-IJ)*ELT**(IJ-1)    0121
      GO TO 102                                             0122
    2 IF(IJ-1-ID) 22,24,24                                  0123
   22 CONTINUE                                              0124
      CALL PID(I,IJ,IT)                                     0125
      GO TO 102                                             0126
   24 D=FLOATF(ID)                                          0127
      ELDT=EXPF(-D*FLAM*TAU)                                0128
      FACT = 1.0                                            0129
      A(1)=1.0                                              0130
      MM = I-IJ                                             0131
      DO 20 M=2,MM                                          0132
      FACT=FACT+1.0                                         0133
   20 A(M)=A(M-1)*FACT                                      0134
  201 PTRM(I,IJ,IT)=(D*FLAM*TAU)**(I-IJ)*ELDT/A(I-IJ)       0135
  102 CONTINUE                                              0136
  101 CONTINUE                                              0137
  100 CONTINUE                                              0138
      GO TO 120                                             0139
   11 CONTINUE                                              0140
      IF(IA-IALAST) 120,121,120                             0141
  121 IF(NEWN-NLAST)111,117,111                             0142
  117 IALPH=IALASTM1 $ GO TO 801                            0143
  120 CONTINUE                                              0144
C  AT THIS POINT THE LAUNCHING TRANSITION PROBABILITIES ARE COMPUTED  0145
      DO 204 IFF=1,N                                        0146
       IGM = XMINOF (IA,IFF)                                0147
      DO 203 IG=1,IGM                                       0148
      IGM1=IG-1                                             0149
      DO 202 IH=1,N                                         0150
      IHM1=IH-1                                             0151
      BPROD=((1.-PGAM)**IGM1)*(PGAM**IHM1)                  0152
   86 IF(IG-IA) 91,87,84                                    0153
```

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```
     91 IF(IG+IHM1-IFF) 84,82,84                                      0154
     87 IF(IG+IHM1-IFF)85,85,84                                       0155
     84 GAMMA(IFF,IG,IH)=0.                                           0156
        GO TO 202                                                     0157
     82 BC(1)=1.0                                                     0158
        PROD=FLOATF(IFF-IG)                                           0159
        DO 30 IP=2,IG                                                 0160
        AIP=FLOATF(IP-1)                                              0161
        PROD = PROD + 1.0                                             0162
     30 BC(IP)=PROD * BC(IP-1)/AIP                                    0163
        IHM1=IH-1                                                     0164
        TEMP= PGAM**IHM1                                              0165
        TEMP1=(1.-PGAM)**IGM1                                         0166
         BPROD = TEMP*TEMP1                                           0167
        GAMMA(IFF,IG,IH)=BC(IG)*BPROD                                 0168
        GO TO 202                                                     0169
     85 FBC(1)=1.0                                                    0170
        PROD=FLOATF(IGM1-1)                                           0171
        DO 40 IP=2,IH                                                 0172
        AIP = FLOATF(IP-1)                                            0173
        PROD = PROD +1.0                                              0174
     40 FBC(IP)=PROD*FBC(IP-1)/AIP                                    0175
        GAMMA(IFF,IG,IH)=FBC(IH)*BPROD                                0176
    202 CONTINUE                                                      0177
    203 CONTINUE                                                      0178
    204 CONTINUE                                                      0179
C        REMOVE CARDS FROM HERE TO NO 999 IF PRINT OUT NOT DESIRED    0180
        PRINT 9,(((I,IJ,IT,PTRM(I,IJ,IT),IT=1,2),IJ=1,N),I=1,N)      0181
      9 FORMAT (1H1/(2(6H PTRM(I2,1H,I2,1H,I2,3H) = E14.5)))         0182
        PRINT 99,(((IFF,IG,IH,GAMMA(IFF,IG,IH),IFF=1,N),IG=1,IA),IH=1,N)  0183
     99 FORMAT(1H1/(2(7H GAMMA(I2,1H,I2,1H,I2,3H) = E14.5)))         0184
        PRINT 999,((IAA,MI,PRFMA(IAA,MI),IAA=1,IAMAX),MI=1,IAMAX)    0185
    999 FORMAT(1H1/(2(7H PRFMA(I2,1H,I2,3H) = E14.5)))               0186
C  NOW THE TRANSITION MATRIX MUST BE ZEROED                          0187
    111 CONTINUE                                                      0188
        DO 899 J=1,119                                                0189
        DO  899 K=1,7                                                 0190
        DO 899 L=1,17                                                 0191
    899 PR(J,K,L)=0.0                                                 0192
C  START COMPUTING THE ELEMENTS OF EACH ROW, I=IX+ (ALPHA - 1) X TOTAL A/C 0193
        DO 1000 IALPH=1, IALAST                                       0194
    801 CONTINUE                                                      0195
        DO 1100 IX=1,NLAST                                            0196
C COMPUTE THE P ELEMENTS OF THE IAPH,IX ROW AND SUM THE ROW          0197
        TSUM=0.                                                       0198
        I=IX+(IALPH-1)*N                                              0199
        DO 800 IBETA=1,IA                                             0200
        RSUM=0.0                                                      0201
        DO 900 IY=1,NEWN                                              0202
        PR(I,IBETA,IY)=0.                                             0203
        ILIM=NEWN-IALPH-IX+2                                          0204
        PSUM=0.0                                                      0205
        SUM=0.0                                                       0206
        SUML=0.0                                                      0207
        DO 500 IL=1,ILIM                                              0208
        KLIM=IX+IL-1                                                  0209
```

```
      IXPIL = IX +IL - 1                                           0210
      SUMM=0.                                                      0211
      DO 600 MI=1,IALPH                                            0212
      SUMK=0.                                                      0213
    ' DO 700 IK=1,KLIM                                             0214
      IKPMI = IK +MI-1                                             0215
      IF(IXPIL-NEWN) 701,701,700                                   0216
  701 IF(IKPMI-NEWN) 702,702,700                                   0217
  702 GAMH=GAMMA(ILIM,IBETA,IL)                                    0218
      PTRMH1 = PTRM(IXPIL,IK,1)                                    0219
      PRFMAH = PRFMA(IALPH,MI)                                     0220
      PTRMH2 = PTRM(IKPMI,IY,2)                                    0221
      SUM = GAMH * PTRMH1 * PRFMAH * PTRMH2                        0222
      SUMK=SUMK+SUM                                                0223
      PSUM=PSUM+SUM                                                0224
  700 CONTINUE                                                     0225
      SUMM = SUMM + SUMK                                           0226
  600 CONTINUE                                                     0227
      SUML = SUML +SUMM                                            0228
      PSUM2 =SUML                                                  0229
  500 CONTINUE                                                     0230
      RSUM=RSUM+PSUM                                               0231
      PR(I,IBETA,IY)=PSUM                                          0232
  900 CONTINUE                                                     0233
      TSUM=TSUM+RSUM                                               0234
  800 CONTINUE                                                     0235
      PRINT 888 ,     TSUM,IALPH,IX                                0236
  888 FORMAT (                     7H TSUM = E15.8,2I5)            0237
 1100 CONTINUE                                                     0238
 1000 CONTINUE                                                     0239
C     REMOVE  CARD FROM HERE TO 889 IF P MATRIX PRINT OUT NOT DESIRED  0240
      DO 889 J=1,17                                                0241
      DO 889 K=1,7                                                 0242
      DO889 L=1,17                                                 0243
      I=J+(K-1)*N                                                  0244
  889 PRINT 890,(PR(I,LP,L),LP=1,IAMAX),K,J,L                      0245
  890 FORMAT(7E14.5,2HJ=I2,5HK=1,A,2HL=I2)                         0246
      DO 898 I=1,119                                               0247
  898 QJ(I)=0.0                                                    0248
C NOW   MULTIPLY  QI AND P TO GET QJ                               0249
  805 PRINT 807,KT,IALAST,IA                                       0250
  807 FORMAT(1H1,13HQ VECTOR CASE I3/// I5,I5)                     0251
      DO 802 IBETA=1,7                                             0252
      DO 902 IY=1,17                                               0253
CAT THIS POINT CALCULATE THE (IBETA,IY)TH ELEMENT OF THE QJ VECTOR  0254
      J=IY+(IBETA-1)*N                                             0255
      QP1=0.                                                       0256
      QP=0.                                                        0257
      DO 2001 IALPH=1,7                                            0258
      DO 2201 IX=1,17                                              0259
      I=IX+(IALPH-1)*N                                             0260
      QP1=QI(I)*PR(I,IBETA,IY)                                     0261
      QP=QP+QP1                                                    0262
 2201 CONTINUE                                                     0263
 2001 CONTINUE                                                     0264
      QJ(J)=QP                                                     0265
```

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```
      PRINT 8882,IBETA,IY,J,QP                                      0266
 8882 FORMAT(2I4,4H QJ(I3,3H )= E14.8)                              0267
  902 CONTINUE                                                      0268
  802 CONTINUE                                                      0269
C CHECK THE SUM OF THE Q VECTOR                                    0270
      QSUM=0.                                                       0271
      DO 808 J=1,119                                                0272
  808 QSUM=QJ(J)+QSUM                                               0273
      PRINT 8883,QSUM                                               0274
 8883 FORMAT(6H QSUM=  E15.9)                                       0275
      DO 333 I = 18,119                                             0276
      K = (I-1)/17                                                  0277
        FK=FLOATF(K)                                                0278
      FMEAN= FK*QJ(I)+FMEAN                                         0279
  333 CONTINUE                                                      0280
      TFLC=FMEAN*FLOATF(JCYCLE)                                     0281
      PRINT 335,FMEAN                                               0282
  335 FORMAT(   17HMEAN A/C FLYING = E10.4)                        0283
C    STATEMENTS FROM THIS POINT TO THE CALL DRAW STATEMENT REFER TO  GRAPH  0284
      JT(2)=ICODE(FMEAN)                                            0285
      JT(4)=ICODE(D)                                                0286
      FKT=FLOATF(KT)                                                0287
      JT(6)=ICODE(FKT)                                              0288
      FN=FLOATF(NEWN-1)                                             0289
      JT(10)=ICODE(FN)                                              0290
      FIAA=FLOATF(IA-1)                                             0291
      JT(12)=ICODE(FIAA)                                            0292
      CALL DRAW(119,FJPLOT,QJ,0,0,4H      ,JT,0,0,0,0,0,0,8,8,0,LAST   )  0293
      FMEAN = 0.                                                    0294
CNEXT WE MUST MULTIPLY QJ AND P TO GET QK AND SO ON...(QK+...N)    0295
      KT=KT+1                                                       0296
      IF(KT-ICYCLE) 803,803,806                                     0297
  803 DO 804 I=1,119                                                0298
  804 QI(I)=QJ(I)                                                   0299
      IALASTM1=IALAST                                               0300
      IALAST=IA                                                     0301
      NLAST=NEWN                                                    0302
      GO TO  809                                                    0303
  806 STOP 06                                                       0304
      END                                                           0305
      SUBROUTINE PID(I,J,IT)                                        0306
      COMMON FLAM,TIME                                              0307
      COMMON PTRM,GAMMA,PR,PRFMA,ID                                 0308
      TYPE DOUBLE BC,BDC,PROD ,DID3,DID4,DID5,DEXP                  0309
      TYPE DOUBLE DAN,DID1,DID2,SUM,DN,ANM1,FAC,COF,PSUM,PTR,FLAM,TAU,D  0310
      DIMENSION PTRM(17,17,2),BC(11),BDC(11)                        0311
      DIMENSION GAMMA(17,7,17), PRFMA(7,7), PR(119,7,17)            0312
      D=FLOATF(ID)                                                  0313
      IDP1=ID+1                                                     0314
      IF(IT-1)25,25,26                                              0315
   25 TAU = TIME                                                    0316
      GO TO 28                                                      0317
   26 TAU= 1.-TIME                                                  0318
   28 CONTINUE                                                      0319
      IMDP1=I-ID                                                    0320
      PTR=0.0                                                       0321
```

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```
      PSUM=0.                                                        0322
      DO 200 NJ = J, ID                                              0323
C  DEVELOP N TAKEN J AT A TIME AND D TAKEN N AT A TIME               0324
      BC(1)=1.0                                                      0325
      PROD=FLOATF(NJ-J)                                              0326
      DO 10 IP=2,J                                                   0327
      AIP=FLOATF(IP-1)                                               0328
      PROD = PROD+1.0                                                0329
   10 BC(IP)=PROD* BC(IP-1)/AIP                                      0330
      BDC(1)=1.0                                                     0331
      PROD=FLOATF(IDP1-NJ)                                           0332
      DO 20 IQ=2,NJ                                                  0333
      AIQ=FLOATF(IQ-1)                                               0334
      PROD=PROD+1.0                                                  0335
   20 BDC(IQ)=PROD*BDC(IQ-1)/AIQ                                     0336
      COF=BC(J)*BDC(NJ)*(-1)**(NJ-J)                                 0337
      ANM1=FLOATF(NJ-1)                                              0338
      DAN=D/(D-ANM1)                                                 0339
      DID4=DEXP(-FLAM*TAU*ANM1)                                      0340
      DID1=(DAN**(I-IDP1))*DID4                                      0341
      SUM=0.                                                         0342
      DN=0.                                                          0343
      DO 201 K=1,IMDP1                                               0344
      FAC=1.                                                         0345
      KM1=K-1                                                        0346
      PROD=0.                                                        0347
      DO 11 IK=1,KM1                                                 0348
      PROD=PROD+1.                                                   0349
   11 FAC=FAC*PROD                                                   0350
      IMIDK=I-ID-K                                                   0351
      SUM=((FLAM*D*TAU)**KM1)*DAN**IMIDK /   FAC                     0352
  201 DN=DN+SUM                                                      0353
      DID3=DEXP(-FLAM*D*TAU)                                         0354
      DID2=DN*DID3                                                   0355
      DID5=DID1-DID2                                                 0356
      PSUM=COF*DID5                                                  0357
  200 PTR =PTR +PSUM                                                 0358
  103 CONTINUE                                                       0359
      PTRM(I,J,IT)=PTR                                               0360
  102 CONTINUE                                                       0361
  101 CONTINUE                                                       0362
      END                                                            0363
      FUNCTION KRAN(A,B,IY)                                          0364
C                                                                    0365
C     THIS ROUTINE RETURNS AN UNIFORMLY DISTRIBUTED RANDOM INTEGER   0366
C                                                                    0367
C     THIS ROUTINE RETURNS A INTEGER RANDOM NUMBER .GE. TO A         0368
C     .LT. B                                                         0369
C     A = BOTTOM LIMIT (INCLUDED) FOR THE RANDOM NUMBER              0370
C     B = TOP LIMIT (NOT INCLUDED) FOR THE RANDOM NUMBER             0371
C     SET IY ONLY ONCE IN MAIN PROGRAM FOR EACH SET OF RANDOM NUMBERS 0372
C     SOME GOOD STARTING VALUES FOR IY FOLLOW                        0373
C     13421773                                                       0374
C     33554433                                                       0375
C     8426219                                                        0376
C     42758321                                                       0377
```

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```
C      56237485                                                    0378
C      62104023                                                    0379
C      ANY OF THESE MAY BE USED                                    0380
C                                                                  0381
C      THIS ROUTINE MAY BE USED IN FORTRAN 60 OR 63                0382
C                                                                  0383
       IY = 3125 * IY                                              0384
       IY = IY - (IY/67108864) * 67108864                          0385
       FY = IY                                                     0386
       KRAN = FY/67108864. * (B-A) + A                             0387
       RETURN                                                      0388
       END                                                         0389
         FINIS                                                     0390
-EXECUTER.                                                         0391
```

Q VECTOR CASE   1

```
   5     5
   1    1  QJ(   1  )=2.64762775E-02
   1    2  QJ(   2  )=3.02020831E-02
   1    3  QJ(   3  )=1.72436823E-02
   1    4  QJ(   4  )=6.68152510E-03
   1    5  QJ(   5  )=2.02525670E-03
   1    6  QJ(   6  )=5.29756823E-04
   1    7  QJ(   7  )=1.30002033E-04
   1    8  QJ(   8  )= .22530060E-05
   1    9  QJ(   9  )=8.58857000E-06
   1   10  QJ(  10  )=2.86285667E-06
   1   11  QJ(  11  )=8.34999861E-07
   1   12  QJ(  12  )=2.08749965E-07
   1   13  QJ(  13  )=4.34895761E-08
   1   14  QJ(  14  )=7.24826268E-09
   1   15  QJ(  15  )=9.06032836E-10
   1   16  QJ(  16  )=7.55027363E-11
   1   17  QJ(  17  )=3.14594735E-12
   2    1  QJ(  18  )=3.02519433E-02
   2    2  QJ(  19  )=3.07381552E-02
   2    3  QJ(  20  )=1.54385405E-02
   2    4  QJ(  21  )=5.18865107E-03
   2    5  QJ(  22  )=1.34374437E-03
   2    6  QJ(  23  )=2.96164302E-04
   2    7  QJ(  24  )=6.07227021E-05
   2    8  QJ(  25  )=1.26203649E-05
   2    9  QJ(  26  )=2.86289503E-06
   2   10  QJ(  27  )=8.35011051E-07
   2   11  QJ(  28  )=2.08752763E-07
   2   12  QJ(  29  )=4.34901589E-08
   2   13  QJ(  30  )=7.24835981E-09
   2   14  QJ(  31  )=9.06044977E-10
   2   15  QJ(  32  )=7.55037481E-11
   2   16  QJ(  33  )=3.14598950E-12
   2   17  QJ(  34  )=              0
   3    1  QJ(  35  )=3.40989043E-02
   3    2  QJ(  36  )=3.07597531F-02
   3    3  QJ(  37  )=1.35265107E-02
   3    4  QJ(  38  )=3.91531781E-03
   3    5  QJ(  39  )=8.57092801E-04
   3    6  QJ(  40  )=1.56625190E-04
   3    7  QJ(  41  )=2.62271034E-05
   3    8  QJ(  42  )=4.44105168E-06
   3    9  QJ(  43  )=8.35125955E-07
   3   10  QJ(  44  )=2.08781489E-07
   3   11  QJ(  45  )=4.34961435E-08
   3   12  QJ(  46  )=7.24935725E-09
   3   13  QJ(  47  )=9.06169656E-10
   3   14  QJ(  48  )=7.55141380E-11
   3   15  QJ(  49  )=3.14542242E-12
   3   16  QJ(  50  )=              0
   3   17  QJ(  51  )=              0
```

```
   4    1  QJ(  52  )=3.79764295E-02
   4    2  QJ(  53  )=3.03224061E-02
   4    3  QJ(  54  )=1.16230913E-02
   4    4  QJ(  55  )=2.87808733E-03
   4    5  QJ(  56  )=5.26888829E-04
   4    6  QJ(  57  )=7.84720454E-05
   4    7  QJ(  58  )=1.04516559E-05
   4    8  QJ(  59  )=1.39145026E-06
   4    9  QJ(  60  )=2.08960624E-07
   4   10  QJ(  61  )=4.35334634E-08
   4   11  QJ(  62  )=7.25557723E-09
   4   12  QJ(  63  )=9.06947154E-10
   4   13  QJ(  64  )=7.55789295E-11
   4   14  QJ(  65  )=3.14912206E-12
   4   15  QJ(  66  )=              0
   4   16  QJ(  67  )=              0
   4   17  QJ(  68  )=              0
   5    1  QJ(  69  )=4.91642835E-01
   5    2  QJ(  70  )=1.43718127E-01
   5    3  QJ(  71  )=2.69531853E-02
   5    4  QJ(  72  )=3.78709962E-03
   5    5  QJ(  73  )=4.23732653E-04
   5    6  QJ(  74  )=3.99381550E-05
   5    7  QJ(  75  )=3.41914062E-06
   5    8  QJ(  76  )=2.98561135E-07
   5    9  QJ(  77  )=3.11088104E-08
   5   10  QJ(  78  )=4.95114755E-09
   5   11  QJ(  79  )=5.92108046E-10
   5   12  QJ(  80  )=4.72886874E-11
   5   13  QJ(  81  )=1.89139810E-12
   5   14  QJ(  82  )=              0
   5   15  QJ(  83  )=              0
   5   16  QJ(  84  )=              0
   5   17  QJ(  85  )=              0
   6    1  QJ(  86  )=              0
   6    2  QJ(  87  )=              0
   6    3  QJ(  88  )=              0
   6    4  QJ(  89  )=              0
   6    5  QJ(  90  )=              0
   6    6  QJ(  91  )=              0
   6    7  QJ(  92  )=              0
   6    8  QJ(  93  )=              0
   6    9  QJ(  94  )=              0
   6   10  QJ(  95  )=              0
   6   11  QJ(  96  )=              0
   6   12  QJ(  97  )=              0
   6   13  QJ(  98  )=              0
   6   14  QJ(  99  )=              0
   6   15  QJ( 100  )=              0
   6   16  QJ( 101  )=              0
   6   17  QJ( 102  )=              0
```

```
7    1 QJ(103 )=              0
7    2 QJ(104 )=              0
7    3 QJ(105 )=              0
7    4 QJ(106 )=              0
7    5 QJ(107 )=              0
7    6 QJ(108 )=              0
7    7 QJ(109 )=              0
7    8 QJ(110 )=              0
7    9 QJ(111 )=              0
7   10 QJ(112 )=              0
7   11 QJ(113 )=              0
7   12 QJ(114 )=              0
7   13 QJ(115 )=              0
7   14 QJ(116 )=              0
7   15 QJ(117 )=              0
7   16 QJ(118 )=              0
7   17 QJ(119 )=              0
QSUM=1.000000000E 00
EAN A/C FLYING =3.1666E 00

GRAPH TITLED
    E(A/C) =3.17E+00SPOTS = 8.00E+00    T = 1.00E+00
    J VS QJ VECTOR        N = 1.60E+01    A = 4.00E+00
HAS BEEN PLOTTED.
```

APPENDIX IV

SAMPLE RESULTS

The following pages present the values of the elements of the proba-

bility distribution vector (QJ) and its graphical plot for five consecutive

iterations, i. e. , $Q \times P^n$ for n = 1, 2, ... , 5. The inputs are those

shown on the first page of Appendix III between statement No. 30 and

No. 31. The printouts of the transition matrices and their computational

elements are omitted. The plot was made using the DRAW subroutine in

the U. S. Naval Postgraduate School computer facility library. Each

vector printout contains the values of all 119 states possible (7 x 17) and

is headed by the past value of A + 1 and the next value of A + 1. The two

indices preceding each element represent $\beta + 1$ and $j + 1$, in the notation

of section 3. For example, in the first row on the next page, the "1 1"

indicates that the probability of being in state (0, 0) after one iteration

is $\doteq$ .026, where the value of A is 4 over the first iteration. Each graph

is labeled with the expected value of a/c flying, the number of mainte-

nance spots available, the vector number (T), total number of a/c avail-

able (N), and the desired number of a/c on station (A). The "E" notation

indicates the power of 10 to multiply by. This sample run demonstrates

the loss in total a/c and variable a/c on station.
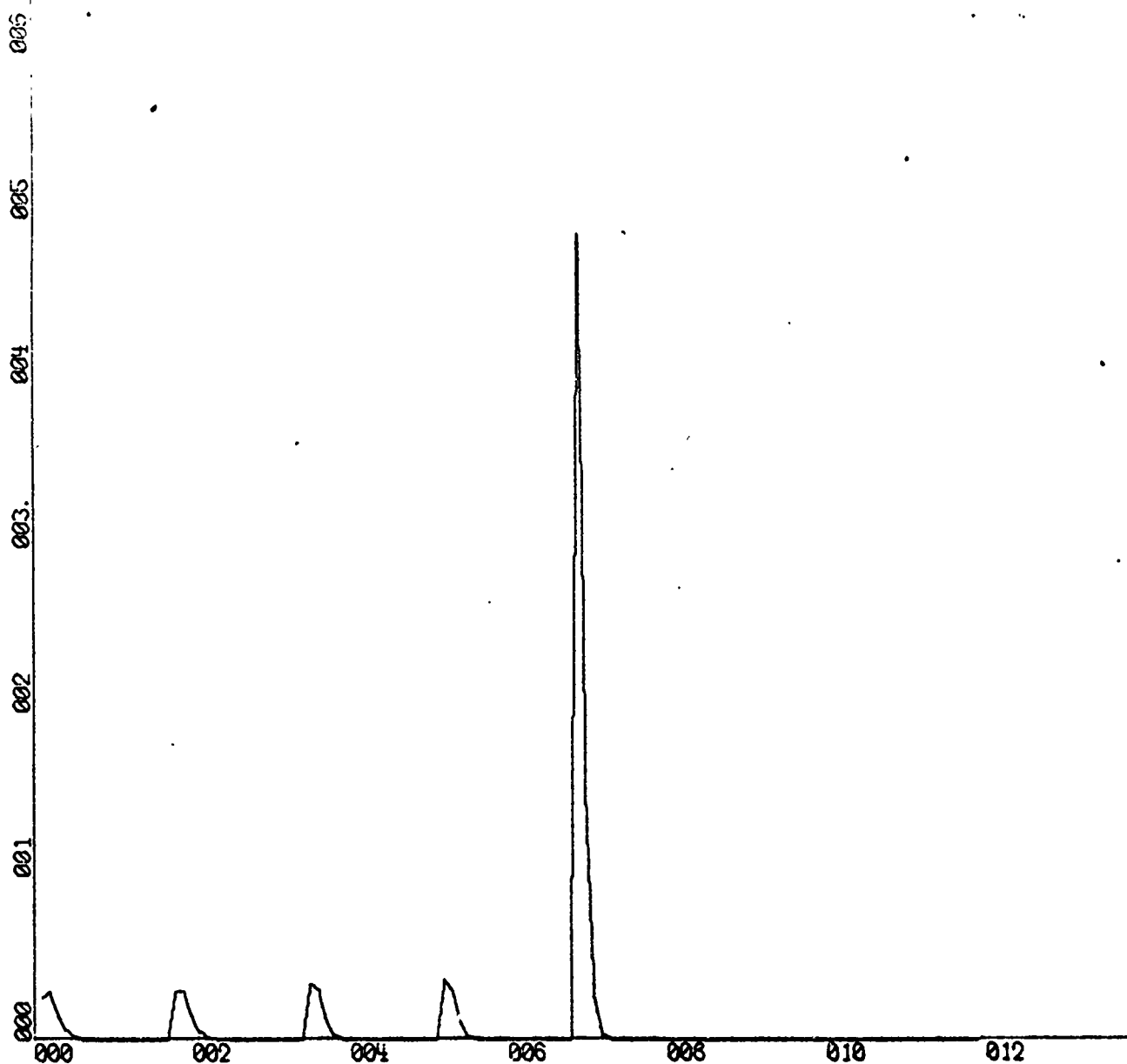
X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =3.17E+00SPOTS = 8.00E+00   T = 1.00E
J VS QJ VECTOR   N = 1.60E+01   A = 4.0

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```
Q VECTOR CASE   2

    5     5
 1    1 QJ(  1 )=9.12184624E-06      4    1 QJ( 52 )=7.23035242E-03
 1    2 QJ(  2 )=7.94491028E-06      4    2 QJ( 53 )=4.45693238E-03
 1    3 QJ(  3 )=3.41380533E-06      4    3 QJ( 54 )=1.28968676E-03
 1    4 QJ(  4 )=9.83913729E-07      4    4 QJ( 55 )=2.35500532E-04
 1    5 QJ(  5 )=2.20530359E-07      4    5 QJ( 56 )=3.11156232E-05
 1    6 QJ(  6 )=4.28567319E-08      4    6 QJ( 57 )=3.30021596E-06
 1    7 QJ(  7 )=7.97994342E-09      4    7 QJ( 58 )=3.15123146E-07
 1    8 QJ(  8 )=1.56348438E-09      4    8 QJ( 59 )=3.13674353E-08
 1    9 QJ(  9 )=3.46529220E-10      4    9 QJ( 60 )=3.81044171E-09
 1   10 QJ( 10 )=1.01221242E-10      4   10 QJ( 61 )=6.98663591E-10
 1   11 QJ( 11 )=2.58173003E-11      4   11 QJ( 62 )=1.02568738E-10
 1   12 QJ( 12 )=5.63385201E-12      4   12 QJ( 63 )=1.13092255E-11
 1   13 QJ( 13 )=1.02295616E-12      4   13 QJ( 64 )=8.32962386E-13
 1   14 QJ( 14 )=1.48426307E-13      4   14 QJ( 65 )=3.07550593E-14
 1   15 QJ( 15 )=1.61419113E-14      4   15 QJ( 66 )=          0
 1   16 QJ( 16 )=1.17030587E-15      4   16 QJ( 67 )=          0
 1   17 QJ( 17 )=4.24536876E-17      4   17 QJ( 68 )=          0
 2    1 QJ( 18 )=1.69105396E-04      5    1 QJ( 69 )=7.70076632E-01
 2    2 QJ( 19 )=1.31058642E-04      5    2 QJ( 70 )=1.88159529E-01
 2    3 QJ( 20 )=4.93425071E-05      5    3 QJ( 71 )=2.32480412E-02
 2    4 QJ( 21 )=1.22416819E-05      5    4 QJ( 72 )=1.93788734E-03
 2    5 QJ( 22 )=2.31695297E-06      5    5 QJ( 73 )=1.22762068E-04
 2    6 QJ( 23 )=3.73462197E-07      5    6 QJ( 74 )=6.39420794E-06
 2    7 QJ( 24 )=5.70661929E-08      5    7 QJ( 75 )=3.02738808E-07
 2    8 QJ( 25 )=9.22154646E-09      5    8 QJ( 76 )=1.56312395E-08
 2    9 QJ( 26 )=1.72702939E-09      5    9 QJ( 77 )=1.13519205E-09
 2   10 QJ( 27 )=4.41481881E-10      5   10 QJ( 78 )=1.50627061E-10
 2   11 QJ( 28 )=9.65869676E-11      5   11 QJ( 79 )=1.51200491E-11
 2   12 QJ( 29 )=1.75883018E-11      5   12 QJ( 80 )=1.02093434E-12
 2   13 QJ( 30 )=2.56023495E-12      5   13 QJ( 81 )=3.47880865E-14
 2   14 QJ( 31 )=2.79434001E-13      5   14 QJ( 82 )=          0
 2   15 QJ( 32 )=2.03393035E-14      5   15 QJ( 83 )=          0
 2   16 QJ( 33 )=7.41007441E-16      5   16 QJ( 84 )=          0
 2   17 QJ( 34 )=          0         5   17 QJ( 85 )=          0
 3    1 QJ( 35 )=1.42525319E-03      6    1 QJ( 86 )=          0
 3    2 QJ( 36 )=9.83757697E-04      6    2 QJ( 87 )=          0
 3    3 QJ( 37 )=3.24331350E-04      6    3 QJ( 88 )=          0
 3    4 QJ( 38 )=6.90234010E-05      6    4 QJ( 89 )=          0
 3    5 QJ( 39 )=1.09403657E-05      6    5 QJ( 90 )=          0
 3    6 QJ( 40 )=1.44032446E-06      6    6 QJ( 91 )=          0
 3    7 QJ( 41 )=1.76353392E-07      6    7 QJ( 92 )=          0
 3    8 QJ( 42 )=2.28131763E-08      6    8 QJ( 93 )=          0
 3    9 QJ( 43 )=3.51078290E-09      6    9 QJ( 94 )=          0
 3   10 QJ( 44 )=7.70559333E-10      6   10 QJ( 95 )=          0
 3   11 QJ( 45 )=1.40851171E-10      6   11 QJ( 96 )=          0
 3   12 QJ( 46 )=2.05928494E-11      6   12 QJ( 97 )=          0
 3   13 QJ( 47 )=2.25869291E-12      6   13 QJ( 98 )=          0
 3   14 QJ( 48 )=1.65302266E-13      6   14 QJ( 99 )=          0
 3   15 QJ( 49 )=6.05799194E-15      6   15 QJ(100 )=          0
 3   16 QJ( 50 )=          0         6   16 QJ(101 )=          0
 3   17 QJ( 51 )=          0         6   17 QJ(102 )=          0
```

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```
7    1  QJ(103 )=              0
7    2  QJ(104 )=              0
7    3  QJ(105 )=              0
7    4  QJ(106 )=              0
7    5  QJ(107 )=              0
7    6  QJ(108 )=              0
7    7  QJ(109 )=              0
7    8  QJ(110 )=              0
7    9  QJ(111 )=              0
7   10  QJ(112 )=              0
7   11  QJ(113 )=              0
7   12  QJ(114 )=              0
7   13  QJ(115 )=              0
7   14  QJ(116 )=              0
7   15  QJ(117 )=              0
7   16  QJ(118 )=              0
7   17  QJ(119 )=              0
QSUM=1.000000000E 00
EAN A/C FLYING =3.9799E 00

GRAPH TITLED
    E(A/C) =3.98E+00SPOTS = 8.00E+00     T = 2.00E+00
    J VS QJ VECTOR        N = 1.60E+01     A = 4.00E+00
HAS BEEN PLOTTED.
```
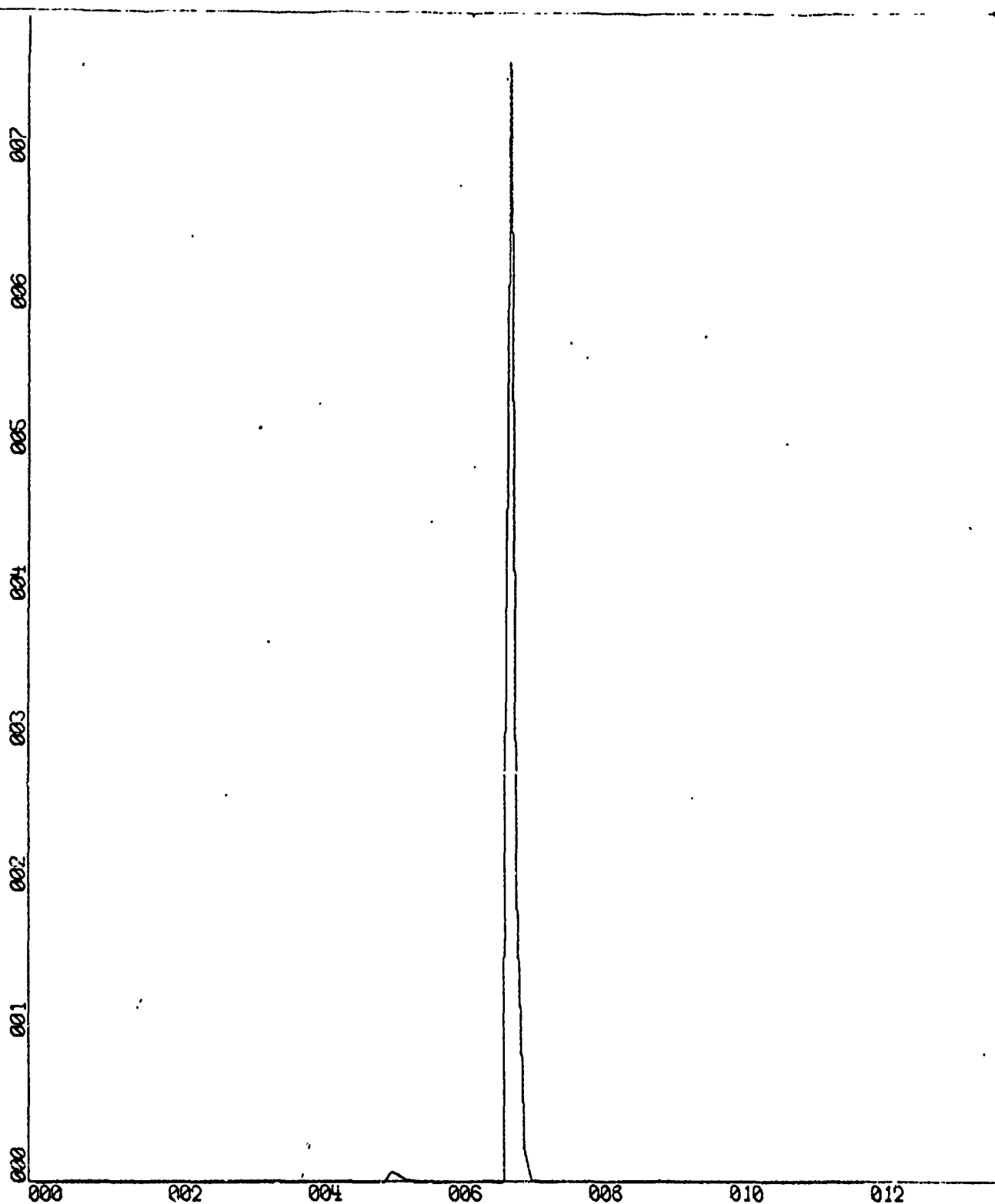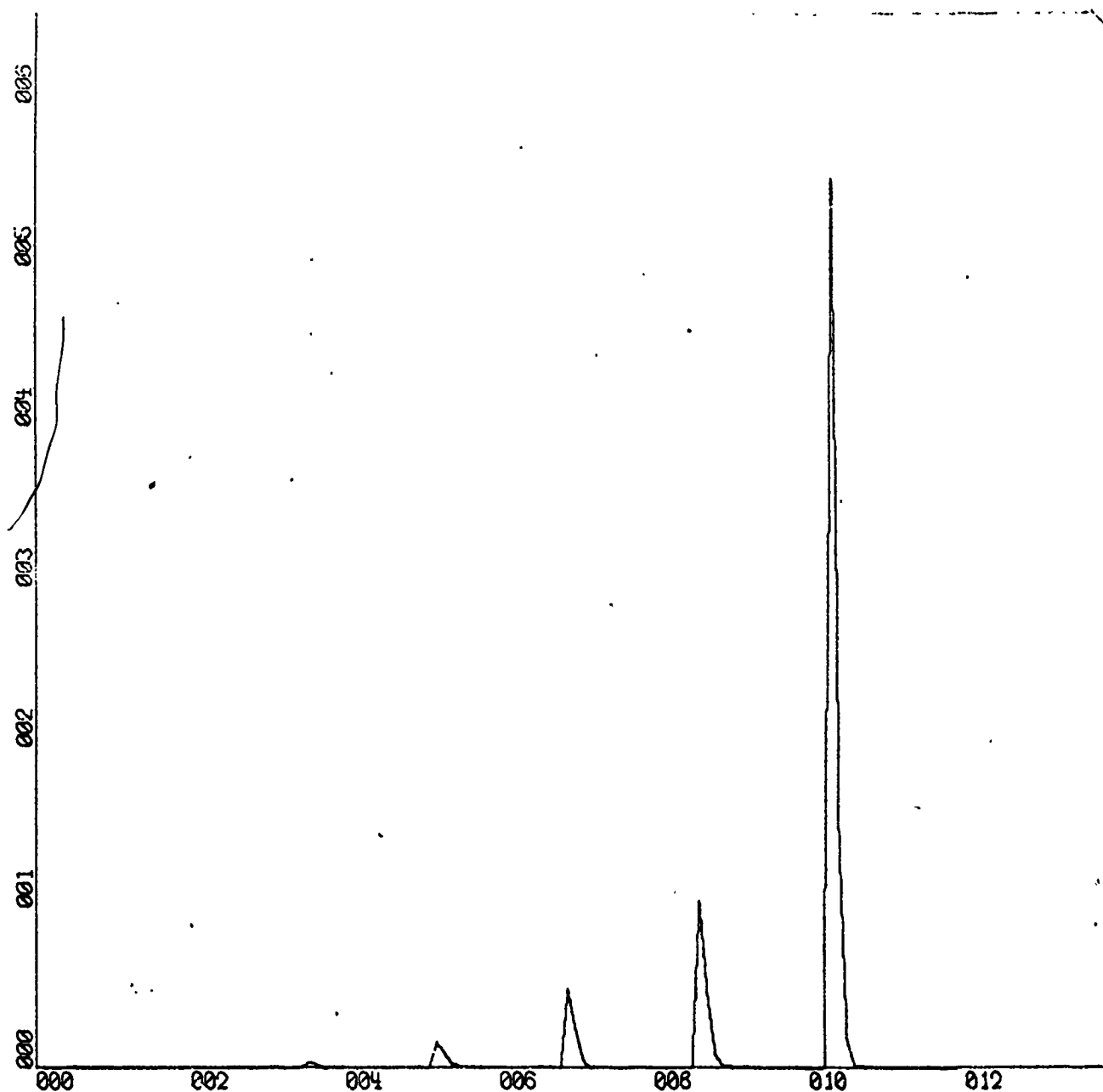
X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.
E(A/C) =3.98E+00SPOTS = 8.00E+00   T = 2.00E
J VS QJ VECTOR   N = 1.60E+01   A = 4.0(

Q VECTOR CASE 3

```
  5    7
1   1 QJ(  1 )=2.83568314E-05        4   1 QJ( 52 )=1.66499369E-02
1   2 QJ(  2 )=2.15851385E-05        4   2 QJ( 53 )=9.01023249E-03
1   3 QJ(  3 )=7.94343376E-06        4   3 QJ( 54 )=2.23842146E-03
1   4 QJ(  4 )=1.91420746E-06        4   4 QJ( 55 )=3.40006373E-04
1   5 QJ(  5 )=3.49125464E-07        4   5 QJ( 56 )=3.57140995E-05
1   6 QJ(  6 )=5.37405166E-08        4   6 QJ( 57 )=2.82593562E-06
1   7 QJ(  7 )=7.77939446E-09        4   7 QJ( 58 )=1.85894467E-07
1   8 QJ(  8 )=1.18756605E-09        4   8 QJ( 59 )=1.19355508E-08
1   9 QJ(  9 )=2.11283841E-10        4   9 QJ( 60 )=9.51529557E-10
1  10 QJ( 10 )=5.18992660E-11        4  10 QJ( 61 )=1.28538146E-10
1  11 QJ( 11 )=1.08528620E-11        4  11 QJ( 62 )=1.28940846E-11
1  12 QJ( 12 )=1.87739309E-12        4  12 QJ( 63 )=8.53320924E-13
1  13 QJ( 13 )=2.57762883E-13        4  13 QJ( 64 )=2.79262670E-14
1  14 QJ( 14 )=2.63176291E-14        4  14 QJ( 65 )=            0
1  15 QJ( 15 )=1.77499822E-15        4  15 QJ( 66 )=            0
1  16 QJ( 16 )=5.92695364E-17        4  16 QJ( 67 )=            0
1  17 QJ( 17 )=            0         4  17 QJ( 68 )=            0
2   1 QJ( 18 )=4.81711691E-04        5   1 QJ( 69 )=4.98253597E-02
2   2 QJ( 19 )=3.26173533E-04        5   2 QJ( 70 )=2.42738832E-02
2   3 QJ( 20 )=1.04905695E-04        5   3 QJ( 71 )=5.35895703E-03
2   4 QJ( 21 )=2.16142330E-05        5   4 QJ( 72 )=7.10282973E-04
2   5 QJ( 22 )=3.28306882E-06        5   5 QJ( 73 )=6.33588530E-05
2   6 QJ( 23 )=4.09033643E-07        5   6 QJ( 74 )=4.08232868E-06
2   7 QJ( 24 )=4.68083562E-08        5   7 QJ( 75 )=2.05024689E-07
2   8 QJ( 25 )=5.62586373E-09        5   8 QJ( 76 )=9.26759069E-09
2   9 QJ( 26 )=8.10258169E-10        5   9 QJ( 77 )=5.04461339E-10
2  10 QJ( 27 )=1.69317736E-10        5  10 QJ( 78 )=5.13281492E-11
2  11 QJ( 28 )=2.92764116E-11        5  11 QJ( 79 )=3.44456222E-12
2  12 QJ( 29 )=4.01871703E-12        5  12 QJ( 80 )=1.14275058E-13
2  13 QJ( 30 )=4.10297009E-13        5  13 QJ( 81 )=            0
2  14 QJ( 31 )=2.76746676E-14        5  14 QJ( 82 )=            0
2  15 QJ( 32 )=9.24174352E-16        5  15 QJ( 83 )=            0
2  16 QJ( 33 )=            0         5  16 QJ( 84 )=            0
2  17 QJ( 34 )=            0         5  17 QJ( 85 )=            0
3   1 QJ( 35 )=3.67589958E-03        6   1 QJ( 86 )=1.04300023E-01
3   2 QJ( 36 )=2.22029313E-03        6   2 QJ( 87 )=4.52895010E-02
3   3 QJ( 37 )=6.25702594E-04        6   3 QJ( 88 )=8.79453771E-03
3   4 QJ( 38 )=1.10292585E-04        6   4 QJ( 89 )=1.00690538E-03
3   5 QJ( 39 )=1.38900387E-05        6   5 QJ( 90 )=7.55512291E-05
3   6 QJ( 40 )=1.38024736E-06        6   6 QJ( 91 )=3.92418576E-06
3   7 QJ( 41 )=1.21057358E-07        6   7 QJ( 92 )=1.47699319E-07
3   8 QJ( 42 )=1.09262020E-08        6   8 QJ( 93 )=4.43630448E-09
3   9 QJ( 43 )=1.21362937E-09        6   9 QJ( 94 )=1.43899935E-10
3  10 QJ( 44 )=2.09057619E-10        6  10 QJ( 95 )=9.80396467E-12
3  11 QJ( 45 )=2.86055733E-11        6  11 QJ( 96 )=3.30144045E-13
3  12 QJ( 46 )=2.91366712E-12        6  12 QJ( 97 )=            0
3  13 QJ( 47 )=1.96282619E-13        6  13 QJ( 98 )=            0
3  14 QJ( 48 )=6.55549374E-15        6  14 QJ( 99 )=            0
3  15 QJ( 49 )=            0         6  15 QJ(100 )=            0
3  16 QJ( 50 )=            0         6  16 QJ(101 )=            0
3  17 QJ( 51 )=            0         6  17 QJ(102 )=            0
```

```
   7    1  QJ(103  )=5.55639840E-01
   7    2  QJ(104  )=1.48858751E-01
   7    3  QJ(105  )=1.84227232E-02
   7    4  QJ(106  )=1.37794578E-03
   7    5  QJ(107  )=6.83380035E-05
   7    6  QJ(108  )=2.32262953E-06
   7    7  QJ(109  )=5.43332378E-08
   7    8  QJ(110  )=8.80653967E-10
   7    9  QJ(111  )=1.18536160E-11
   7   10  QJ(112  )=3.82356527E-13
   7   11  QJ(113  )=            0
   7   12  QJ(114  )=            0
   7   13  QJ(115  )=            0
   7   14  QJ(116  )=            0
  .7   15  QJ(117  )=            0
   7   16  QJ(118  )=            0
   7   17  QJ(119  )=            0
QSUM=1.000000000E 00
EAN A/C FLYING =5.5636E 00

GRAPH TITLED
    E(A/C) =5.56E+00SPOTS = 8.00E+00     T = 3.00E+00
    J VS QJ VECTOR         N = 1.50E+01     A = 6.00E+00
HAS BEEN PLOTTED.
```

X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =5.56E+00SPOTS = 8.00E+00   T = 3.00E

J VS QJ VECTOR   N = 1.50E+01   A = 6.00

59

Q VECTOR CASE 4

```
 7    5'
 1   1 QJ(  1 )=4.37066460E-04       4   1 QJ( 52 )=8.03893389E-02
 1   2 QJ(  2 )=2.66337542E-04       4   2 QJ( 53 )=3.58842827E-02
 1   3 QJ(  3 )=7.61707863E-05       4   3 QJ( 54 )=7.24458728E-03
 1   4 QJ(  4 )=1.37453763E-05       4   4 QJ( 55 )=8.75756513E-04
 1   5 QJ(  5 )=1.79617995E-06       4   5 QJ( 56 )=7.09643835E-05
 1   6 QJ(  6 )=1.89112432E-07       4   6 QJ( 57 )=4.12757545E-06
 1   7 QJ(  7 )=1.81098891E-08       4   7 QJ( 58 )=1.85478755E-07
 1   8 QJ(  8 )=1.84326720E-09       4   8 QJ( 59 )=7.45697706E-09
 1   9 QJ(  9 )=2.33815659E-10       4   9 QJ( 60 )=3.67973791E-10
 1  10 QJ( 10 )=4.51123736E-11       4  10 QJ( 61 )=3.56621692E-11
 1  11 QJ( 11 )=7.18762832E-12       4  11 QJ( 62 )=2.27683580E-12
 1  12 QJ( 12 )=9.07642678E-13       4  12 QJ( 63 )=7.17866372E-14
 1  13 QJ( 13 )=8.51451859E-14       4  13 QJ( 64 )=            0
 1  14 QJ( 14 )=5.27326607E-15       4  14 QJ( 65 )=            0
 1  15 QJ( 15 )=1.61678538E-16       4  15 QJ( 66 )=            0
 1  16 QJ( 16 )=            0        4  16 QJ( 67 )=            0
 1  1  QJ( 17 )=            0        4  17 QJ( 68 )=            0
 2   1 QJ( 18 )=5.18126447E-03       5   1 QJ( 69 )=6.25520845E-01
 2   2 QJ( 19 )=2.86433504E-03       5   2 QJ( 70 )=1.72496943E-01
 2   3 QJ( 20 )=7.33497609E-04       5   3 QJ( 71 )=2.20943200E-02
 2   4 QJ( 21 )=1.16340651E-04       5   4 QJ( 72 )=1.73104557E-03
 2   5 QJ( 22 )=1.30125431E-05       5   5 QJ( 73 )=9.18785067E-05
 2   6 QJ( 23 )=1.13029797E-06       5   6 QJ( 74 )=3.46552930E-06
 2   7 QJ( 24 )=8.54466567E-08       5   7 QJ( 75 )=9.61462191E-08
 2   8 QJ( 25 )=6.65517506E-09       5   8 QJ( 76 )=2.13537750E-09
 2   9 QJ( 26 )=6.57969504E-10       5   9 QJ( 77 )=5.31168734E-11
 2  10 QJ( 27 )=1.05656138E-10       5  10 QJ( 78 )=3.22108252E-12
 2  11 QJ( 28 )=1.34342964E-11       5  11 QJ( 79 )=9.66869152E-14
 2  12 QJ( 29 )=1.26766173E-12       5  12 QJ( 80 )=            0
 2  13 QJ( 30 )=7.88829950E-14       5  13 QJ( 81 )=            0
 2  14 QJ( 31 )=2.42720088E-15       5  14 QJ( 82 )=            0
 2  15 QJ( 32 )=            0        5  15 QJ( 83 )=            0
 2  16 QJ( 33 )=            0        5  16 QJ( 84 )=            0
 2  17 QJ( 34 )=            0        5  17 QJ( 85 )=            0
 3   1 QJ( 35 )=2.69210521E-02       6   1 QJ( 86 )=            0
 3   2 QJ( 36 )=1.34302284E-02       6   2 QJ( 87 )=            0
 3   3 QJ( 37 )=3.06604116E-03       6   3 QJ( 88 )=            0
 3   4 QJ( 38 )=4.26067113E-04       6   4 QJ( 89 )=            0
 3   5 QJ( 39 )=4.06841633E-05       6   5 QJ( 90 )=            0
 3   6 QJ( 40 )=2.90134684E-06       6   6 QJ( 91 )=            0
 3   7 QJ( 41 )=1.70443704E-07       6   7 QJ( 92 )=            0
 3   8 QJ( 42 )=9.76651595E-09       6   8 QJ( 93 )=            0
 3   9 QJ( 43 )=7.11954817E-10       6   9 QJ( 94 )=            0
 3  10 QJ( 44 )=9.16366982E-11       6  10 QJ( 95 )=            0
 3  11 QJ( 45 )=8.74738102E-12       6  11 QJ( 96 )=            0
 3  12 QJ( 46 )=5.50237415E-13       6  12 QJ( 97 )=            0
 3  13 QJ( 47 )=1.70994033E-14       6  13 QJ( 98 )=            0
 3  14 QJ( 48 )=            0        6  14 QJ( 99 )=            0
 3  15 QJ( 49 )=            0        6  15 QJ(100 )=            0
 3  16 QJ( 50 )=            0        6  16 QJ(101 )=            0
 3  17 QJ( 51 )=            0        6  17 QJ(102 )=            0
```

```
7    1  QJ(103 )=               0
7    2  QJ(104 )=               0
7    3  QJ(105 )=               0
7    4  QJ(106 )=               0
7    5  QJ(107 )=               0
7    6  QJ(108 )=               0
7    7  QJ(109 )=               0
7    8  QJ(110 )=              -0
7    9  QJ(111 )=               0
7   10  QJ(112 )=               0
7   11  QJ(113 )=               0
7   12  QJ(114 )=               0
7   13  QJ(115 )=               0
7   14  QJ(116 )=               0
7   15  QJ(117 )=               0
7   16  QJ(118 )=               0
7   17  QJ(119 )=               0
QSUM=1.000000000E 00
EAN A/C FLYING =3.7578E 00

GRAPH TITLED
     E(A/C) =3.76E+00SPOTS = 8.00E+00     T = 4.00E+00
     J VS QJ VECTOR          N = 1.40E+01     A = 4.00E+00
HAS BEEN PLOTTED.
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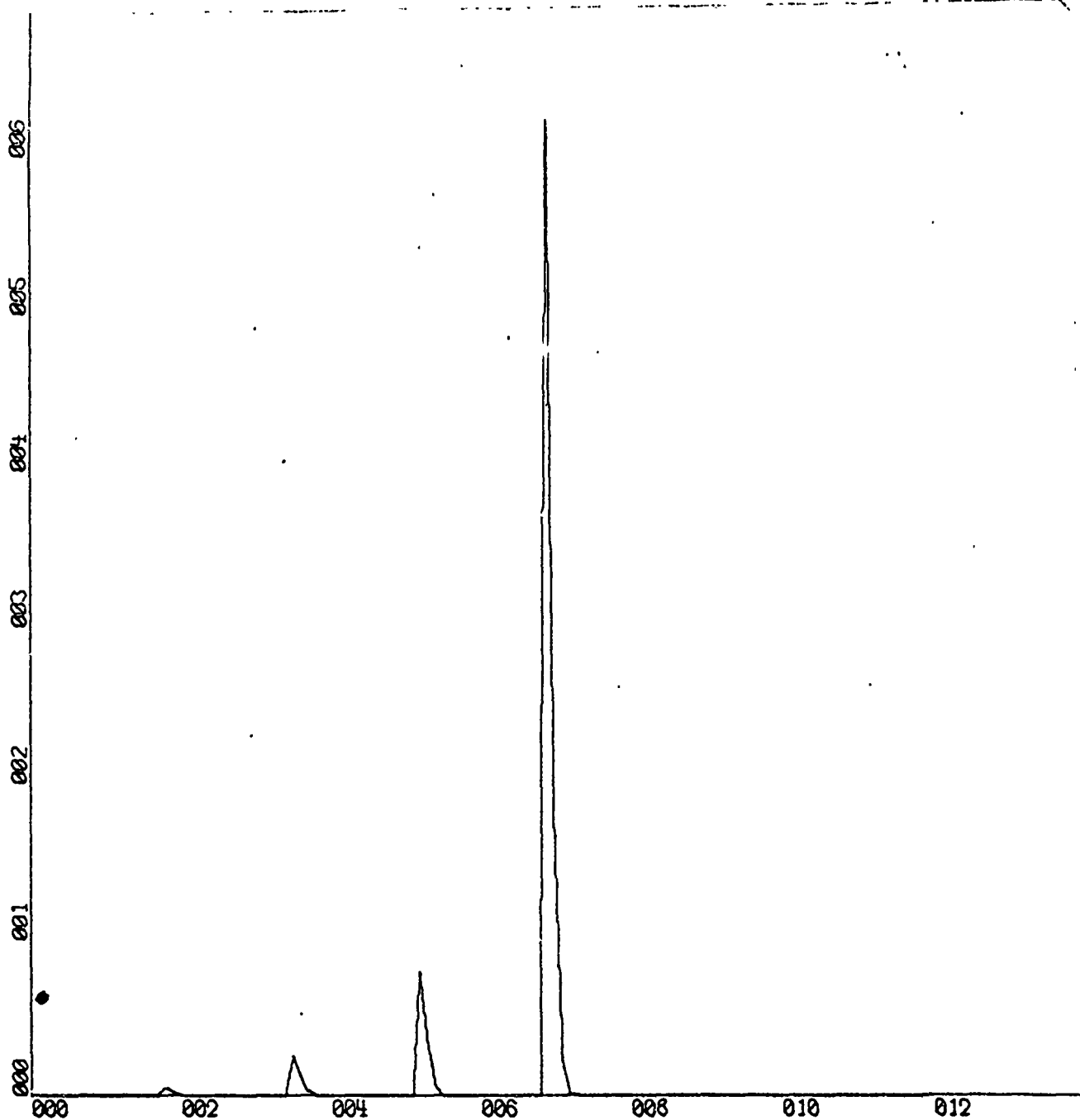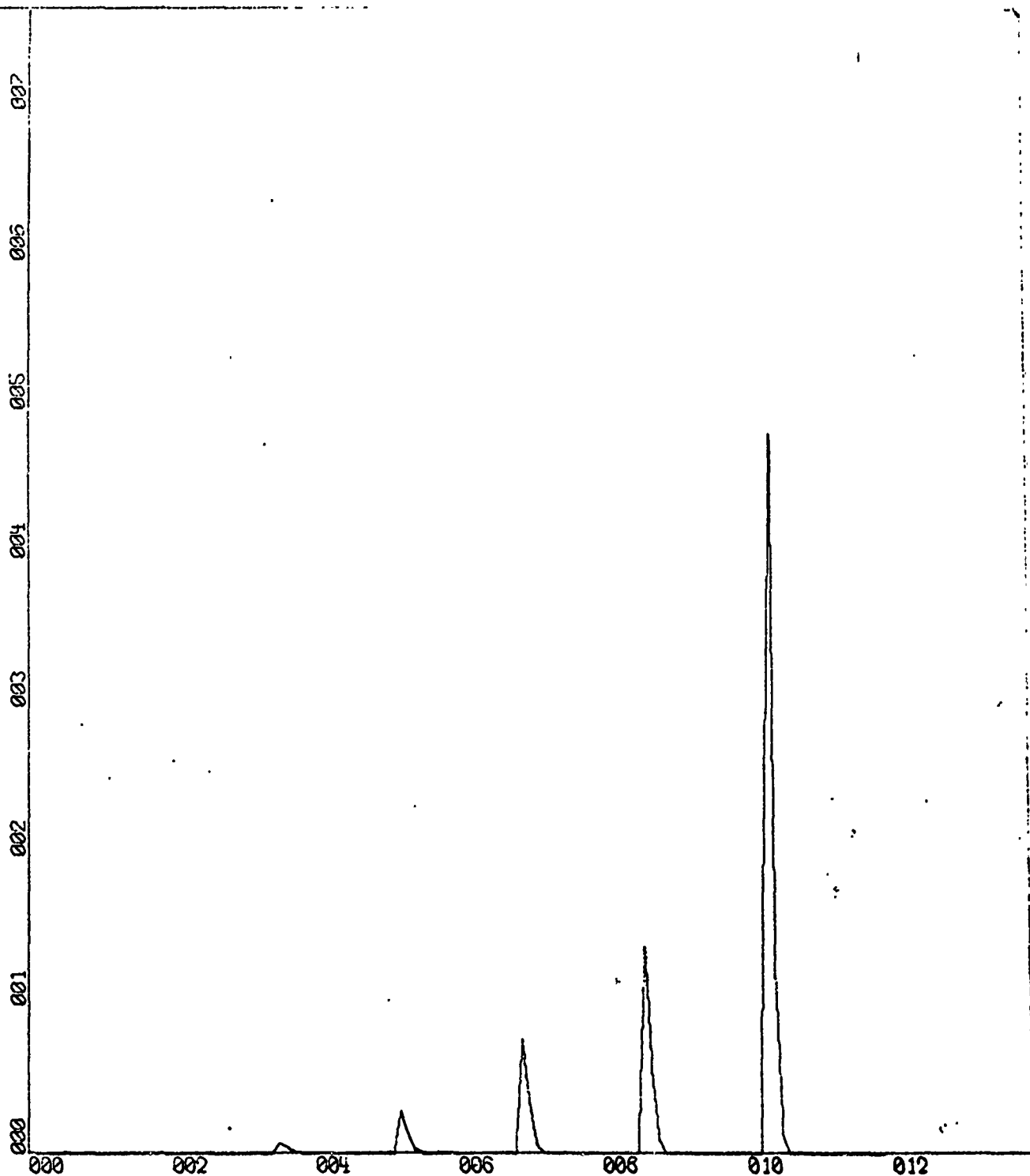
X-SCALE = 2.00E+01 UNITS/INCH.
Y-SCALE = 1.00E-01 UNITS/INCH.

E(A/C) =3.76E+00SPOTS = 8.00E+00   T = 4.00E-

J VS QJ VECTOR   N = 1.40E+01   A = 4.00

Q VECTOR CASE 5

```
5    7
1   1  QJ(   1 )=7.15520995E-05        4   1  QJ( 52 )=2.93940831E-02
1   2  QJ(   2 )=4.87189674E-05        4   2  QJ( 53 )=1.43984862E-02
1   3  QJ(   3 )=1.57753676E-05        4   3  QJ( 54 )=3.19845745E-03
1   4  QJ(   4 )=3.27727099E-06        4   4  QJ( 55 )=4.26970762E-04
1   5  QJ(   5 )=5.02868635E-07        4   5  QJ( 56 )=3.84133994E-05
1   6  QJ(   6 )=6.34199800E-08        4   6  QJ( 57 )=2.50141306E-06
1   7  QJ(   7 )=7.35880233E-09        4   7  QJ( 58 )=1.27347959E-07
1   8  QJ(   8 )=8.96925196E-10        4   8  QJ( 59 )=5.85531451E-09
1   9  QJ(   9 )=1.30710966E-10        4   9  QJ( 60 )=3.24180085E-10
1  10  QJ(  10 )=2.75401174E-11        4  10  QJ( 61 )=3.33199523E-11
1  11  QJ(  11 )=4.80498022E-12        4  11  QJ( 62 )=2.26209155E-12
1  12  QJ(  12 )=6.66122816E-13        4  12  QJ( 63 )=7.60530690E-14
1  13  QJ(  13 )=6.87539584E-14        4  13  QJ( 64 )=            0
1  14  QJ(  14 )=4.69377133E-15        4  14  QJ( 65 )=            0
1  15  QJ(  15 )=1.58863224E-16        4  15  QJ( 66 )=            0
1  16  QJ(  16 )=            0         4  16  QJ( 67 )=            0
1  17  QJ(  17 )=            0         4  17  QJ( 68 )=            0
2   1  QJ(  18 )=1.08771615E-03        5   1  QJ( 69 )=7.70471329E-02
2   2  QJ(  19 )=6.60751603E-04        5   2  QJ( 70 )=3.36780394E-02
2   3  QJ(  20 )=1.87506066E-04        5   3  QJ( 71 )=6.58809654E-03
2   4  QJ(  21 )=3.33939735E-05        5   4  QJ( 72 )=7.60547395E-04
2   5  QJ(  22 )=4.25508484E-06        5   5  QJ( 73 )=5.76075719E-05
2   6  QJ(  23 )=4.27752034E-07        5   6  QJ( 74 )=3.02551600E-06
2   7  QJ(  24 )=3.81603116E-08        5   7  QJ( 75 )=1.15424789E-07
2   8  QJ(  25 )=3.50828898E-09        5   8  QJ( 76 )=3.52590714E-09
2   9  QJ(  26 )=3.95927530E-10        5   9  QJ( 77 )=1.16438952E-10
2  10  QJ(  27 )=6.89194802E-11        5  10  QJ( 78 )=8.00922104E-12
2  11  QJ(  28 )=9.53886847E-12        5  11  QJ( 79 )=2.72622038E-13
2  12  QJ(  29 )=9.83824310E-13        5  12  QJ( 80 )=            0
2  13  QJ(  30 )=6.71883385E-14        5  13  QJ( 81 )=            0
2  14  QJ(  31 )=2.27773386E-15        5  14  QJ( 82 )=            0
2  15  QJ(  32 )=            0         5  15  QJ( 83 )=            0
2  16  QJ(  33 )=            0         5  16  QJ( 84 )=            0
2  17  QJ(  34 )=            0         5  17  QJ( 85 )=            0
3   1  QJ(  35 )=7.37121582E-03        6   1  QJ( 86 )=1.39125275E-01
3   2  QJ(  36 )=4.01163961E-03        6   2  QJ( 87 )=5.35756492E-02
3   3  QJ(  37 )=1.00338550E-03        6   3  QJ( 88 )=9.10073278E-03
3   4  QJ(  38 )=1.53687580E-04        6   4  QJ( 89 )=8.95105006E-04
3   5  QJ(  39 )=1.63163199E-05        6   5  QJ( 90 )=5.62254445E-05
3   6  QJ(  40 )=1.30932633E-06        6   6  QJ( 91 )=2.34682483E-06
3   7  QJ(  41 )=8.77478713E-08        6   7  QJ( 92 )=6.58960130E-08
3   8  QJ(  42 )=5.76387628E-09        6   8  QJ( 93 )=1.26580622E-09
3   9  QJ(  43 )=4.69636023E-10        6   9  QJ( 94 )=1.99736156E-11
3  10  QJ(  44 )=6.43465918E-11        6  10  QJ( 95 )=6.91079850E-13
3  11  QJ(  45 )=6.56029037E-12        6  11  QJ( 96 )=            0
3  12  QJ(  46 )=4.42292681E-13        6  12  QJ( 97 )=            0
3  13  QJ(  47 )=1.47863511E-14        6  13  QJ( 98 )=            0
3  14  QJ(  48 )=            0         6  14  QJ( 99 )=            0
3  15  QJ(  49 )=            0         6  15  QJ(100 )=            0
3  16  QJ(  50 )=            0         6  16  QJ(101 )=            0
3  17  QJ(  51 )=            0         6  17  QJ(102 )=            0
```

```
7    1  QJ(103 )=4.84130317E-01
7    2  QJ(104 )=1.18774563E-01
7    3  QJ(105 )=1.31776649E-02
7    4  QJ(106 )=8.59940961E-04
7    5  QJ(107 )=3.59275288E-05
7    6  QJ(108 )=9.79278393E-07
7    7  QJ(109 )=1.69313196E-08
7    8  QJ(110 )=1.69139063E-10
7    9  QJ(111 )=7.45214538E-13
7   10  QJ(112 )=           0
7   11  QJ(113 )=.          0
7   12  QJ(114 )=           0
7   13  QJ(115 )=           0
7   14  QJ(116 )=           0
7   15  QJ(117 )=           0
7   16  QJ(118 )=           0
7   17  QJ(119 )=           0
QSUM=1.000000000E 00
EAN A/C FLYING =5.3577E 00

GRAPH TITLED
    E(A/C) =5.36E+00SPOTS = 8.00E+00    T = 5.00E+00
    J VS QJ VECTOR         N = 1.40E+01    A = 6.00E+00
HAS BEEN PLOTTED.
```

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Y-SCALE = 1.00E-01 UNITS/INCH.
E(A/C) =5.36E+00SPOTS = 8.00E+00   T = 5.00E·
J VS QJ VECTOR   N = `1.40E+01   A = 6.00

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4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Master's Thesis

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13. ABSTRACT

It is the purpose of this paper to develop a useful mathematical model of ASW aircraft availability. The increasing emphasis of systems studies dictates the use of accurate and representative models of the ASW systems. At present, many studies are using essentially the same models developed during World War II. This paper is an attempt to make use of advanced theory in a more powerful and flexible model and to make the use of the model practical and verifiable.

The writer adapted the time homogeneous bivariate model as developed by F. C. Collins. This is a discrete time Markov process with a stochastic matrix of transition probabilities wherein the maintenance process is modeled as a pulsed input multiple server queue.

The model was programmed in FORTRAN 63 on the CDC 1604 and then modified to allow for variability in the input parameters. Other modifications include an increase in the size of the model to accommodate a 16-aircraft squadron, the largest ASW squadron at present, and an explicit form solution to the maintenance queueing equations.

DD FORM 1 JAN 64 **1473**

14. KEY WORDS		LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
ASW Aircraft							
Computer Markov Model							
Availability							

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