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AUTHORITY

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Re-order Points

The Calculation of Preferred Risks

Results of testing trial values of shortage cost in a least cost solution.

ALRAND RPT. 21
SPCC MECHANICSBURG, PA.
10 OCTOBER 1960

Re-order Points
(The calculation of preferred risks)

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SECTION I. INTRODUCTION

Safety levels at the Ships Parts Control Center have been set such that the risk of a shortage over the procurement lead time is 15% for all items. Expressed conversely, we have set safety stocks to yield an 85% protection against system-wide stock outs. The motivation for this is a Bureau of Supplies and Accounts policy decision--somewhat obscure in its origin--which set an effectiveness objective of 85% for the SPCC. It is not our purpose here to examine this decision; rather, to investigate what improvements might be made within the limitations of that decision.

It should be noted that the same level of protection was set for every item: 85% across the board. In this way a total system effectiveness of 85% would be attained. However, in various ways, people have expressed a certain amount of dissatisfaction with this. The commanding officer pointed out that the penalties incurred when a "fast mover" ran short were severe. The unfilled requisitions piled up. Desperate supply officers re-submitted unsatisfied requisitions to alternative warehouses. Each unfilled requisition at each warehouse sets off a chain of expediting and follow-up actions. All this is particularly irksome when the shortage happens to be in the "two bit" class. Why all the disruption when for an additional small increase in safety stocks it could be avoided? In sum, should we not provide a higher level of protection for the fast movers and two bit items? Should we not re-balance so that our effectiveness is higher for these? This means a shift of safety stocks to the low cost high demand items.*

We can state the above more formally by saying we want the risk to vary with the particular characteristics of the item. Specifically, we want a smaller risk when (1) the holding costs are small relative to the cost of a shortage; and, (2) the demand rate is high. As a matter of fact, if we start with an objective of minimizing total costs, one arrives at just that kind of a rule:

$$\text{Risk to take over lead time} = \frac{(\text{Annual Unit Holding Cost})(\text{Buy Quantity})}{(\text{Annual Demand})(\text{Shortage Cost})}$$

(For the formal development of this, see ALRAND Report 14, Appendix B and ALRAND Report 9.)

Notice that the expression also considers how often the risk of shortage is run by considering the buy quantity. It can be seen, all other things being equal, the risk decreases when the numerator gets smaller and/or the denominator gets bigger; i.e., when:

1. Unit price decreases. This is so because unit holding cost equals unit price times annual holding cost rate;
2. Annual holding cost rate gets smaller;
3. Buy quantity gets smaller;
4. Annual demand gets larger;
5. Shortage cost gets larger.

*Suggesting a companion piece for the now-famous "Sherwood's Shaft": "Sherwood's Shift"

SECTION II. THE PROBLEM OF SHORTAGE COST

Conceptually, shortage cost seems fairly clear. Whenever a shortage occurs, there are penalties incurred by the ashore system which can be lumped under the title of "Expedite Costs." There may or may not be an additional penalty associated with the degrading of a ship or ships. As a matter of fact, if the on board stock levels were optimally calculated and maintained, the likelihood of a simultaneous shortage ashore and afloat would be very small. In most instances, afloat stocks would be used to replace failed parts; and ashore stocks would simply replace the afloat stock.

We can write a general expression for shortage cost as follows:

Shortage Cost = Expedite Costs + (Essentiality)(Mission Cost)(Probability of Shortage)

Difficulty arises in assigning costs and essentiality weights. Also, in the current method for calculating on board repair parts, shortage probabilities are not estimated. In general, however, side studies show that the probabilities of shortage for allowance list items <u>if carried</u> are very small; so that even though the mission cost might be assigned a high value, the expected afloat shortage cost would be small and could be ignored in practice. An exception to this rule would be an item of high essentiality which is not carried afloat (i.e., a true insurance item).

Conceivably, the expedite costs could be measured and used. A preliminary study made showed that the average expediting costs incurred at the SPCC alone amount to approximately $12 considering the minimum action taken. An extension of this study to cover total system costs may be worthwhile. For the immediate present, however, the costs are not available. Further, the cost figure most probably would be unusable for the current budget year if the costs indicated a change to the financial operating plan for this fiscal year.

For these reasons, it was decided that the only practical approach was to find a value for "shortage cost" which:

a. Kept the present total system safety level stock at its present level; but,

b. Re-balanced safety level among individual items to improve the effectiveness of low-cost fast-moving stock.

The motivation for this is as follows:

Since the sole purpose of maintaining inventory on hand at all is to avoid shortages, we obviously are willing to incur costs to do this. We imply that it is worth the expenditure of dollars to prevent shortages; that there is a trade-off between inventory and shortages. This means that there are shortage costs implied in our current decisions. We simply do not express them explicitly. By somebody's management decision, an effectiveness level of 85% has been set as a goal; conversely, a calculated risk of 15% per item has been set. Therefore we could solve for the shortage cost implicit in our present decisions. For example, if unit price were $10, the buy quantity 10 units and annual demand 100 units; then shortage cost is:

$$\text{Risk} = \frac{\text{Unit Holding Cost} \times \text{Buy Quantity}}{\text{Annual Demand} \times \text{Shortage Cost}}$$

$$15\% = \frac{(\$10 \times 1/10)(10)}{100 \times \text{Shortage Cost}}$$

$$\text{Shortage Cost} = \frac{(\$10 \times 1/10)(15)}{100 \times 15\%}$$

$$= \$1.00$$

If we make the assumption that over the years a reasonable balance has been reached between shortages and inventory, then we can, by trial, find a value of shortage cost which will retain that balance. For this reason, we can substitute the term "balancing cost" for "shortage cost" in order to be precise.

The purpose of the research described herein, then, was to find a proper value for "balancing cost."

SECTION III. DESCRIPTION OF EXPERIMENT

In order to achieve a reasonable balance between confidence and the cost of information, it was decided to perform the experiments on a sample of the stock list (Perpetual Inventory Record). An approximately 10% sample was extracted by selecting out all stock numbers ending in the digit "7." This also ensured randomness and a sample which includes all combinations of item characteristics.

Initially, experiments were performed on all items ending in "57" to reduce the computer computation time until we seemed close to a solution. This sample size was later expanded to include all stock numbers ending in 07, 37, and 67. The final total sample includes 3,200 items.

Since risk is dependent upon the size of the safety level, two basic computations were made. Safety level for each item was computed using (1) the current rules and (2) the variable risk rules. The individual safety levels were summed up for each method; both in units and in dollars.

Trial values were used for balancing cost to find that value which produced a total dollar value of safety level which equalled the total dollar value of safety level computed under the present rules.

The current rules used:

a. Normal distribution describes demand when the average annual demand is 12 or more.

b. The Poisson distribution describes demand when the average annual demand is 11 or less.

c. When the normal distribution is used, a re-order level is computed to produce a 15% risk for all items. Unit price, buy quantity, and frequency of risk are not considered.

d. When the Poisson distribution is used, a re-order level is computed to produce a 1% risk for all items. (Note: This policy was set in order to overcome an unsatisfactory fit of the Poisson distribution in the higher average demand range.) Unit price, buy quantity and frequency of risk are not considered.

The revised rules applied were:

a. Demand was described using three probability distributions as follows:

Demand Rate	Distribution
2 or less per year	Poisson Distribution
Greater than 2 but less than 100	Negative Binomial
Greater than 100	Normal Distribution

b. Risks were decided in accordance with the "variable risk" computation for each item.

c. Re-order levels were computed to take the risk as calculated; except:

(1) A floor was placed under risk so that a risk greater than 50% would never be taken. In other words, the re-order point would never be lower than that necessary to satisfy at least average demand over the lead time.

Preliminary manual testing confirmed a suspicion that a simple single number value of balancing cost would reduce the risk of low cost items, but would sharply increase the risk of the high cost items. An argument could be made that this is what ought to happen. However, it was felt that this kind of result would constitute too radical a change in stocking policies to be acceptable at the present time. The reasons for this are: (1) many people are of the opinion that part essentiality should have a significant effect on ashore back up stock decisions; and, (2) most people are of the opinion that there is a general relationship between essentiality and unit price: in general, the higher the unit price, the greater the relative importance of the item. If one does not make this assumption, stock levels of high cost items would be sharply reduced.

To offset this effect, it is necessary to consider the unit price of the item in the balancing cost. As the unit price goes up, balancing cost must also go up.

SECTION IV. TEST RESULTS

An initial trial was made using a balancing cost of ($10 + Unit Price). For a sample consisting of all items ending in 57, the sum of the safety levels was increased significantly. Whereas the current rules called for a safety level of $193.011.23, the revised rules produced a safety level of $285,588.14. An examination of individual items showed that all the variable risks were small, even for the high priced items. The net effect was to raise all safety levels. For example, for FSN HR5930-642-9367 with a unit price of $1,610 and an annual demand rate of 1, a risk of .055 is called for. The risks taken on the small unit cost items were very near zero.

The simplest way to allow balancing cost to increase with unit price, but at a diminished rate, is to raise unit price to some fractional exponent. Raising unit price to the $\frac{1}{2}$ power produced the results shown in Chart 1. Trial values of $25, $50 and $75 were taken as alternative values for the fixed element of balancing cost, so that:

$$\text{Balancing Cost} = \text{Fixed Cost} + (\text{Unit Price})^{\frac{1}{2}}$$

As expected, inventory rises as balancing cost rises; but at a diminishing rate. A value of balancing cost higher than expected would be required to produce the same size safety level as presently maintained. Before proceeding with these higher values it was decided to investigate further.

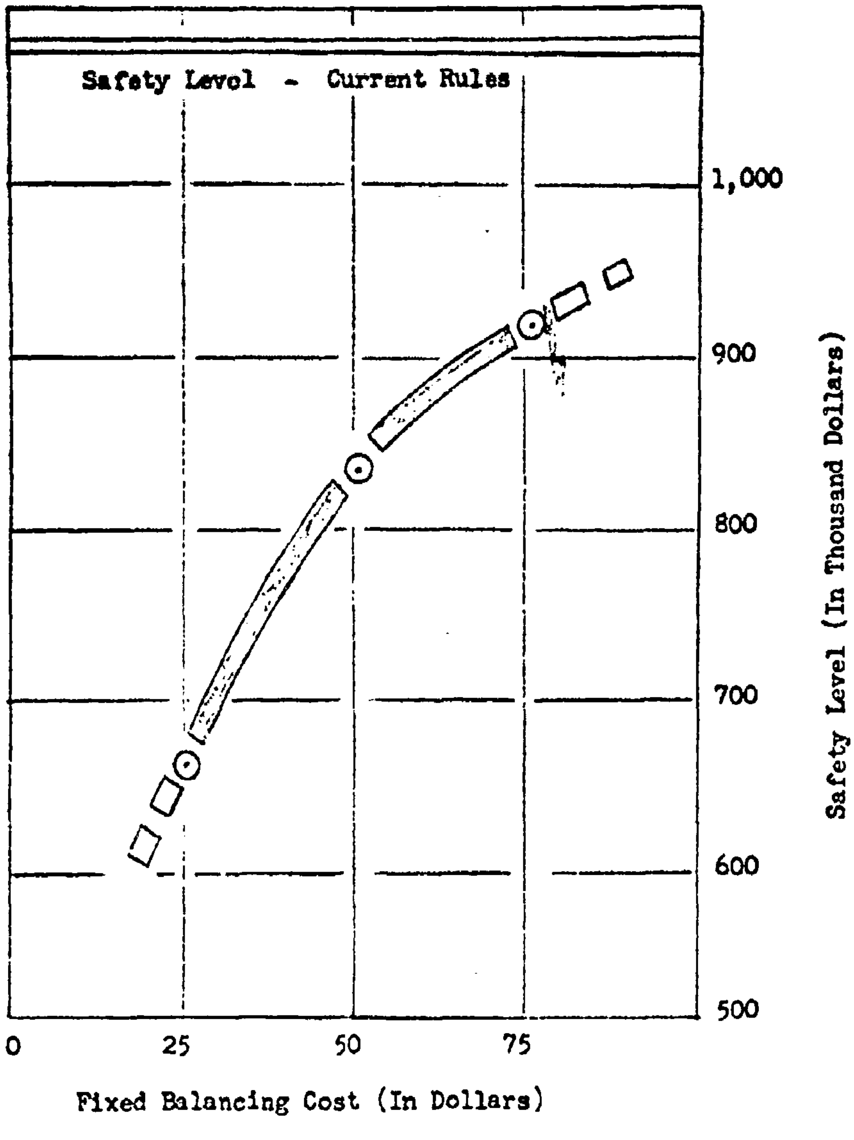

Chart 1. Safety Levels in Dollars

The depth of stocks maintained was examined. By "depth" is meant here the number of repair parts maintained in safety level inventory. The changes here were quite startling. For each trial value of balancing cost, the number of repair parts was over double the number maintained under current rules. The results are shown in Chart 2. Again, the number of parts increases with the increased size of balancing cost but at a slow rate.

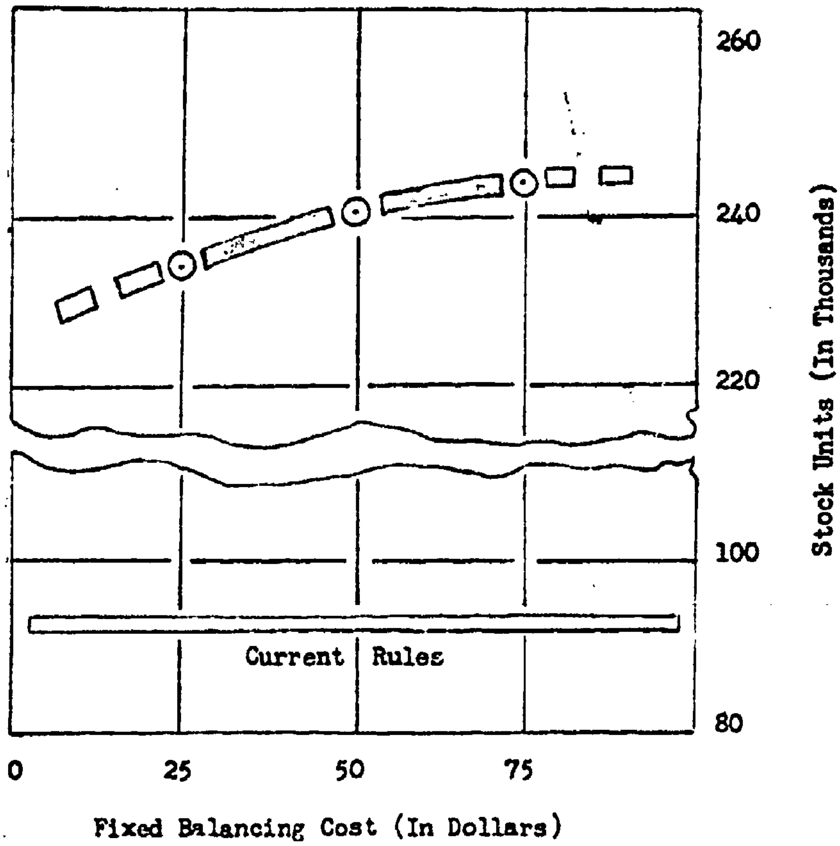

Chart 2. Safety Level In Units

With the number of repair parts represented in safety level increased to approximately 250%, one would expect a significant increase in the protection against stock-outs. This in fact happens when the fixed element of balancing cost is greater than $25:

Value of Balancing Cost	Estimated Protection against Shortages
$25 + $C^{\frac{1}{2}}$	84.4 %
50 + $C^{\frac{1}{2}}$	89.8
75 + $C^{\frac{1}{2}}$	92.3

The distribution of the protection rates is of interest. These are shown in Charts 3-5 for the various values of balancing cost. The changes to "effectiveness" brought about by shifting or rebalancing safety levels is clearly shown. In each case, the effectiveness is raised over the present 85% for most of the stock list. By decreasing the effectiveness of a smaller number of items, the effectiveness of a larger number of items is raised. When a high enough value of balancing cost (greater than $25 + $C^{\frac{1}{2}}$) is used, the average effectiveness is raised above the present level. And this is done with a smaller dollar value of safety level inventory than currently used.

The "humps" at the tail ends of the distributions are caused by the restriction against allowing items to fall below 50% protection. When the rules said to take a risk over 50%, the overriding feature automatically lowered the risk back to 50%. If this feature were not incorporated, the distributions would tail off gradually to 0%. For those in the 0% category, the rules would be saying, "Don't protect this item. Order it only when a requisition is received." In other words, it's a non-stock item.

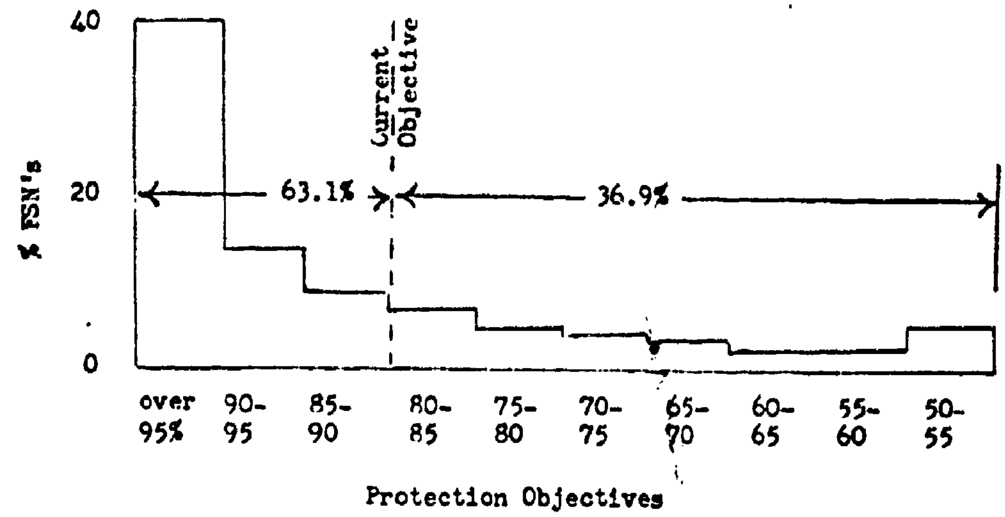

Chart 3. Balancing Cost = \$25 + $C^{\frac{1}{2}}$

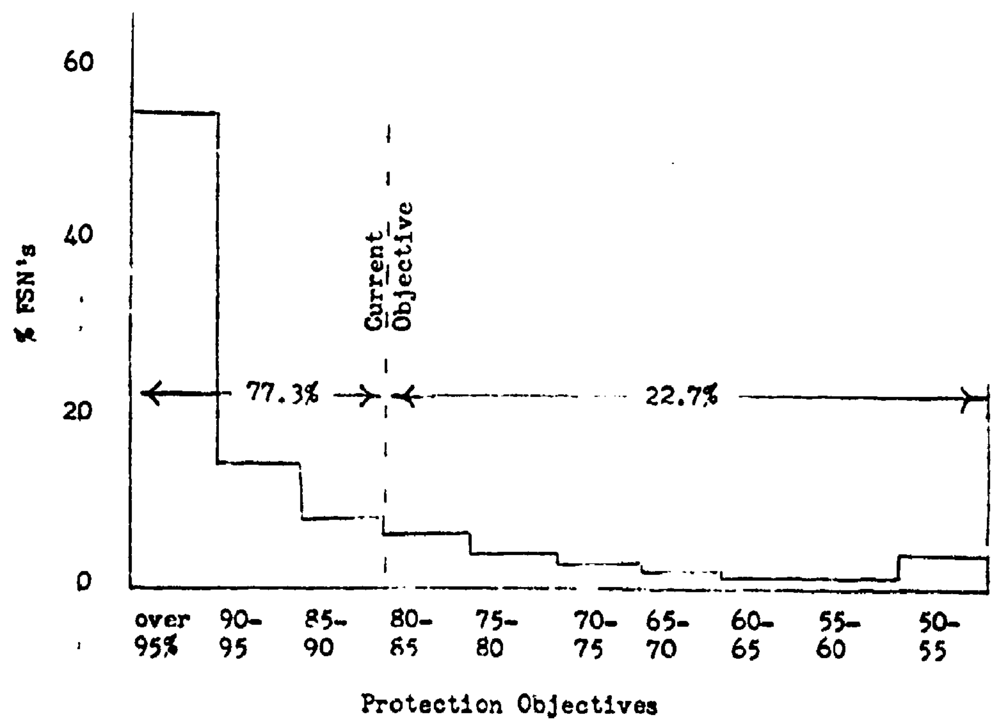

Chart 4. Balancing Cost = \$50 + $C^{\frac{1}{2}}$

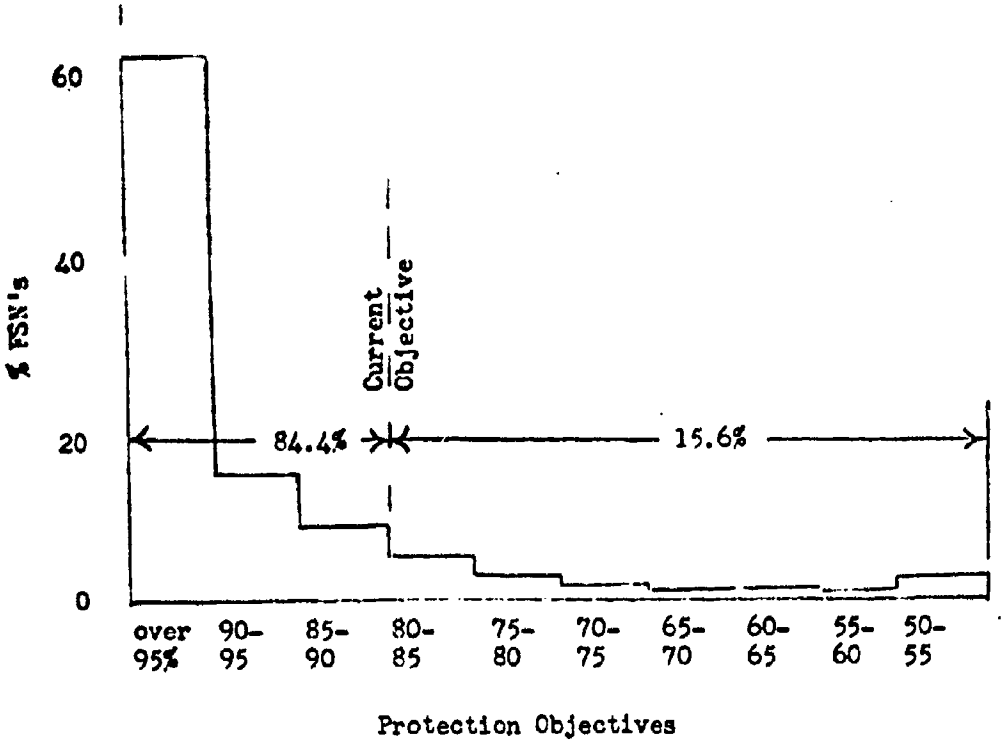

Chart 5. Balancing Cost = $75 + C^{\frac{1}{2}}$

It's of some interest to see the relationship between average effectiveness and safety level. Chart 6 shows this. It indicates the trade-off between safety level in dollars and effectiveness. The data show that it costs increasingly more in terms of inventory to increase effectiveness as 100% is approached.

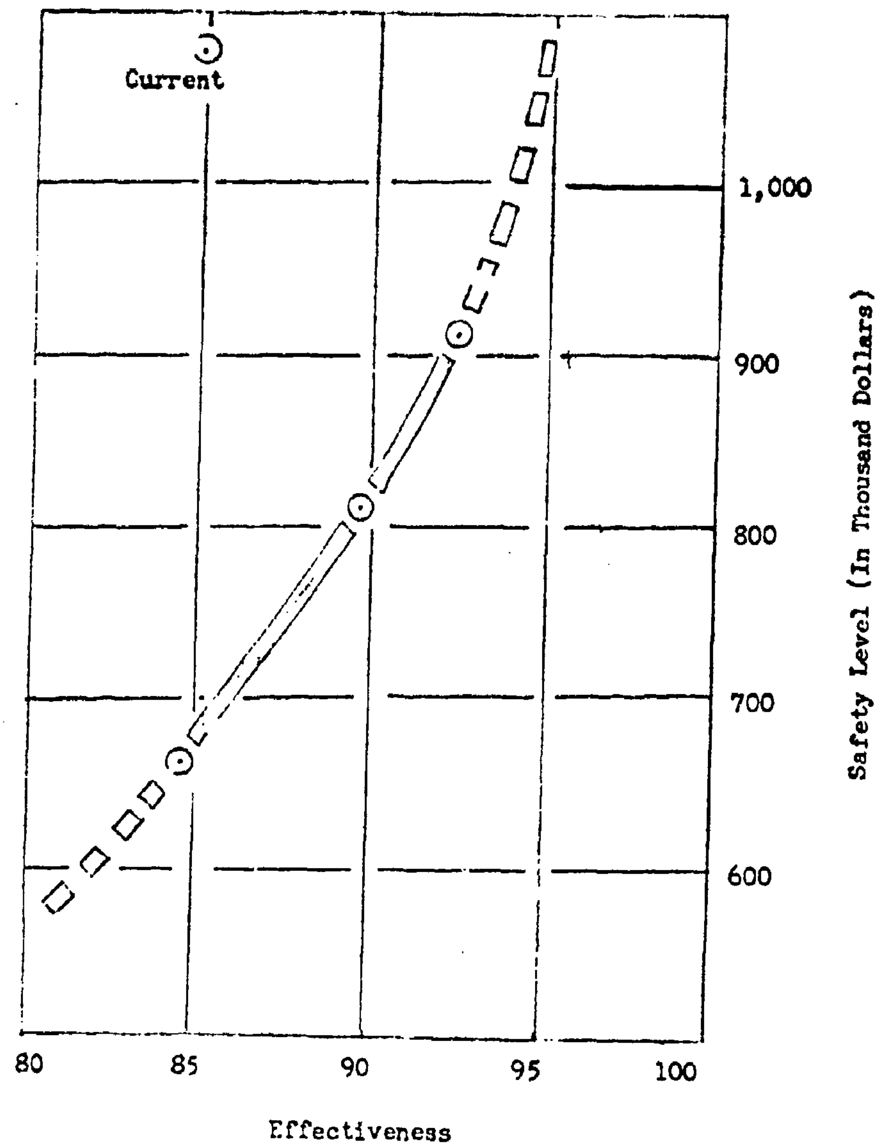

Chart 6. Effectiveness vs. Safety Level in Dollars for Sample

SECTION V. EVALUATION

Commodity Manager Preferences

Safety level preferences were obtained on 75 items selected by supervisory personnel and commodity managers. The sampling included stock control personnel of all branches and combinations of price, demand rates, variation in demand and essentiality insofar as it was known by individuals. The evaluators were asked to state a preference from among all the alternatives after a review of the consolidated stock status report. An evaluation sheet was made out on each item. The results in summary:

Preference	Number of Items
Variable Risk VSL	45
Current (Fixed Risk) VSL	9
Neither	10

11 items were "disqualified" from the sample for the reasons shown:

Obsolescent; not to be re-ordered	6
Newly provisioned item with insufficient background for appraisal	1
Trends up or down taken into account	4

To dispense with the last-named first: obsolescent items pose no problem to the variable risk concept. They should be coded as terminal items with an associated "no buy" rule and excluded from the re-order point--re-order quantity review.

The newly-provisioned item points up the present awkward position the commodity manager is currently in. For a proposal to bring these items into the fold, see AIRAND Reports 12, 14 and 20.

With reference to trends, these are automatically taken into account by the exponential smoothing and tracking techniques. (See ALRAND Reports 15 and 18 for details.) However, if a commodity manager believes more drastic action is required, he can either (1) change the smoothing constant upwards, (2) re-estimate the MAD, or do both (1) and (2).

Returning to the summary and the 9 items for which the current (lower) safety level was preferred: all 9 were "fast-moving" low unit price items (under $10 unit price, over 20 demands per year.) In each case the opinion was expressed that "demand was relatively constant" and the higher VSL afforded by the variable risk scheme was "unnecessary."

The reasons for rejecting both the current and proposed in favor of some other quantity were:

1 item - Low unit price, high annual demand. Analyst felt that relatively constant demand eliminated need for a VSL.

5 items - Low unit price, low annual demand. Two analysts favored lower VSL on the grounds no additional investment was warranted. Three analysts favored a higher VSL to eliminate "constant reviewing."

3 items - Low demand, high unit price. In each case the average demand over the lead time was 1; the VSL under both methods was zero. A re-order point of one (1) was felt to be low on "high essentiality" items.

1 item - Analyst feels all slow movers should have the highest levels.

Of the 45 items which were accepted under the variable risk method, \$75 + $C^{\frac{1}{2}}$ was the value of balancing cost most often preferred. However, as the unit price rose, the risk which the analyst was willing to accept also rose. So that for items with a high unit cost, the most preferred value for balancing cost was \$25 + $C^{\frac{1}{2}}$.

Errors

Nothing has been said of errors in the data. There are several sources. The sample itself is an approximately 3% sample of the active inventory. Also, for each item a sample of eight quarters of demand were used. So that there are two sources of sampling error. Both of these could be reduced by additional sampling; however, in our opinion, the increased accuracy will not warrant the additional cost of information.

In calculating safety levels under the current methodology, it was assumed that the rules were strictly applied in each instance. However, as can be seen from the evaluations by commodity managers, this is not always the case. Aside from individual judgments which influence safety level, there are others which introduce uncertainty into the calculations; e.g., "J" items have an arbitrary "stockage objective" set.

An attempt was made to determine the actual current size of safety level. However, the data available are not considered accurate or reliable enough for this purpose.

As a consequence, the test results cannot be interpreted without some allowance for error. Also with regard to interpretation, it must be kept in mind that the levels of protection and risk were based upon a single hypothetical warehouse. Errors in distribution are not accounted for.

Meeting the Objective

The objective was to decrease stock outs on low priced and fast moving items. The "variable risk" rule, which considers all costs simultaneously, accomplishes this objective. The rule is quite flexible; so that within reasonable limits a variety of policies can be carried out. The calculation of "shortage cost" is merely one of a variety which might be used. Essentiality, for example, could be introduced precisely; rather than in the rather crude way suggested. This is not recommended, however, until (1) it is better known how essentiality ought to be introduced into the ashore decision--if at all; (2) a more decisive rating system is devised.

A different balance of safety stock could be struck. For example, if more safety level were desired for the high priced stock, the unit price can be raised to a higher power; say $(\text{unit price})^{3/4}$ rather than $(\text{unit price})^{1/2}$.

Total system safety level can be increased by increasing the size of the constant (fixed) cost, or the power to which unit price is raised; or both. Conversely, safety level can be decreased by decreasing either or both of these numbers.

If we specify that risk is to range from .1% to 50%, we could find a more complicated function to do this; and avoid simply decreasing any calculated risk over 50% back to 50%. One function of shortage cost which will do this more efficiently is:

$$4\sqrt{\text{Unit Price}} + \sqrt{1 + \text{Unit Price}}$$

If increased effectiveness, as currently defined and measured by the Bureau of Supplies and Accounts, were the sole objective, then effectiveness could be significantly increased with a much lower level of safety stock. Effectiveness, as now defined and measured, is indifferent to unit price or essentiality. Dropping the unit price consideration from the balancing cost, we could greatly reduce inventory at no loss of effectiveness.

Conceptual Changes

Traditionally the Navy has operated with emphasis on protection. Stock has generally been managed by quantity only. We divided our inventories into "fractions" which were based upon quantity demand rates. Thus attention was focused on management by quantity and away from management by dollar value.

The revised rule, in considering demand quantity and unit price, considers not simply protection; but protection per dollar. All other things being equal, ten times as much protection can be bought at $10 than at $100. The analogy is loading up on salt and pepper and playing the canned ham close to the vest.

We also turn the page in "Poor Richard's Almanac." The urge to earn a penny by saving a penny is very strong within us. We watch the ups and downs of the 25¢ seller like a hawk. And also the $100 line. But do the savings in the two bit stuff offset the shortage and management costs? A 10% saving on 25¢ is 2¢ or 3¢. A 10% saving on $100 is 400 times greater.

Decisions Required

Command decisions are required on two basic questions:

1. Does the rule better satisfy management objectives?

2. What should be the trade-off between inventory and effectiveness?

The second question is answered by the selection of the balancing cost figure. To assist in this selection, the alternatives and expected results are summarized in Chart 7.

Expected Change to:	Value of Balancing Cost		
	$\$25 + C^{\frac{1}{2}}$	$\$50 + C^{\frac{1}{2}}$	$\$75 + C^{\frac{1}{2}}$
Safety Level in Dollars	- 39%	- 23%	- 15%
Safety Level in Units	+ 152%	+ 159%	+ 162%
Protection	None	+ 5%	+ 7%

Chart 7. Summary of Expected Results