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EQUATIONS OF MOTION FOR A MODIFIED POINT MASS TRAJECTORY

by

Robert F. Lieske
Mary L. Reiter

March 1966

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U. S. ARMY MATERIEL COMMAND
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

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Robert F. Lieske
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Computing Laboratory

RDT & E Project No. 1P523801A87

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RFLieske/MLReiter/vm
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ABSTRACT

A modified point mass mathematical model which incorp›rates an estimate of the yaw of repose, has been developed to represent the flight of a spin stabilized, dynamically stable, artillery shell. This improved mathematical model has the desirable feature of representing the effects of the significant variables of yaw of repose and axial spin along the trajectory.

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TABLE OF SYMBOLS

Term	Definition	Units
A	Axial moment of inertia	$lb\text{-}ft^2$
AZ	Azimuth of line of fire (clockwise from north)	mil
B	Transverse moment of inertia	$lb\text{-}ft^2$
C_s	Ballistic coefficient for standard mass	lb/in^2
d	Reference diameter of projectile	ft
$\underrightarrow{E}$	Position of projectile with respect to spherical Earth surface	ft
$\underrightarrow{g}$	Acceleration due to gravity	ft/sec^2
g_o	Acceleration due to gravity (surface)	ft/sec^2
$\underrightarrow{H}$	Total angular momentum	$lb\text{-}ft^2\text{-}rad/sec$
$\underrightarrow{I}$	Unit vector in the direction of $\underrightarrow{v}$	———
K_A	Spin damping moment coefficient	———
K_{D_o}	Drag force coefficient	———
K_{D_α}	Yaw drag force coefficient	———
K_F	Magnus force coefficient	———
K_H	Damping moment coefficient	———
K_L	Lift force coefficient	———
K_M	Overturning moment coefficient	———
K_S	Pitching force coefficient	———
K_T	Magnus moment coefficient	———

Term	Definition	Units
l	Lift factor	———
L	Latitude of launch	deg
M	Mach number	———
m	Projectile mass	lb
m_s	Standard projectile mass	lb
N	Axial spin	rad/sec
Q	Yaw drag factor	———
r	Distance between center of Earth and projectile	ft
R	Effective radius of Earth	ft
t	Time	sec
$\vec{u}$	Velocity of projectile with respect to ground	ft/sec
$\vec{v}$	Velocity of the projectile with respect to air	ft/sec
$\vec{w}$	Velocity of the air with respect to ground	ft/sec
$\vec{x}$	Unit vector along the longitudinal axis of the projectile	———
$\vec{X}$	Position of the projectile with respect to a ground-fixed coordinate system	ft
α	Angle of yaw of projectile	rad
$\vec{\alpha}_e$	Approximation for yaw of repose	rad
$\vec{\Lambda}$	Coriolis acceleration due to rotation of the Earth	ft/sec^2
ρ	Air density as a function of E_2 (where E_2 is a component of $\vec{E}$)	lb/ft^3

Term	Definition	Units
Ω	Angular velocity of the Earth	rad/sec
$\dot{}$	First derivative with respect to time	——/sec
$\ddot{}$	Second derivative with respect to time	——/sec^2

INTRODUCTION

The mathematical model for rigid body trajectory simulation, as reported in BRL Report No. 1244, closely matches the results of physical experiments of spin-stabilized projectiles over a spectrum of test conditions. However, its usefulness, as in most rigid body simulations, is hindered by the time required to numerically solve the system of differential equations. This report describes the derivation of a mathematical model which will not be as time consuming to solve as the rigid body system, but which will represent as closely as possible the center of gravity motion of the projectile by utilizing a force system, axial spin and an estimate of the yaw of repose.

BASIC LAWS OF MOTION OF A RIGID BODY

The frame of reference for all vectors to be presented is a ground-fixed, right-handed Cartesian coordinate system with unit vectors $(\vec{1}, \vec{2}, \vec{3})$. Assume the $\vec{2}$ axis to be parallel to the vector $\vec{g}$, and $\vec{g}$ to have the same direction as $\vec{1} \times \vec{3} = -\vec{2}$.

Assume that the body can be considered a solid of revolution. Then spin-stabilized projectiles can be oriented by choosing a unit vector $\vec{x}$ along the axis of symmetry, pointing from tail to nose. N is the magnitude of angular velocity parallel to $\vec{x}$. N is positive if it results from a rotation causing a right-handed screw to advance in the direction of $\vec{x}$.

The following set of simultaneous differential equations of motion for a spin-stabilized projectile is developed in BRL Report 1244.

The equation of motion of the center of mass is:

$$(1.1) \quad \dot{\vec{u}} = -\frac{\rho d^2}{m}(K_{D_o} + \alpha^2 K_{D_\alpha}) v \vec{v} + \frac{\rho d^2}{m} K_L [\vec{v} \times (\vec{x} \times \vec{v})] - \frac{\rho d^3}{m} K_S v \dot{\vec{x}} + \frac{\rho d^3 K_F N}{m} (\vec{x} \times \vec{v}) + \vec{g} + \vec{\Lambda}$$

The total angular momentum of the body can be expressed as the sum of two vectors in the ground-fixed coordinate system:

(a) The angular momentum about $\vec{x}$.

(b) The total angular momentum about an axis perpendicular to $\vec{x}$.

The angular momentum about $\vec{x}$ can be represented by the vector $A N \vec{x}$ and the angular momentum about an axis perpendicular to $\vec{x}$ by the vector $B(\vec{x} \times \dot{\vec{x}})$.

Let $\underset{\rightarrow}{H}$ denote the total angular momentum of the body. The vector representation of $\underset{\rightarrow}{H}$ is:

(1.2) $\underset{\rightarrow}{H} = AN\underset{\rightarrow}{x} + B(\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$

The vector rate of change of angular momentum is the sum of the applied moments.

(1.3) $\dot{\underset{\rightarrow}{H}} = A\dot{N}\underset{\rightarrow}{x} + AN\dot{\underset{\rightarrow}{x}} + B(\underset{\rightarrow}{x} \times \ddot{\underset{\rightarrow}{x}}) =$

$$\rho d^3 K_M v(\underset{\rightarrow}{v} \times \underset{\rightarrow}{x}) - \rho d^4 K_H v(\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$$

$$+ \rho d^4 K_T N[\underset{\rightarrow}{x} \times (\underset{\rightarrow}{x} \times \underset{\rightarrow}{v})] - \rho d^4 K_A N v \underset{\rightarrow}{x}$$

ESTIMATE FOR YAW OF REPOSE

An estimate for the yaw of repose will be derived from the rigid body system of differential equations of motion for the purpose of representing the effects of yaw.

The following conditions are assumed:

(1) The projectile can be represented sufficiently well as a body of revolution.

(2) The projectile is dynamically stable.

(3) Initial yaw is assumed small, i. e., it has no significant effect on the trajectory.

Examination of equation (1. 1) shows the magnitude $|\underset{\rightarrow}{x} \times \underset{\rightarrow}{v}|$ as being present in the lift term and the Magnus term, the lift term being more significant. Examination of equation (1.3) shows its presence also in the corresponding terms of $\dot{\underset{\rightarrow}{H}}$.

If $\underset{\rightarrow}{I} = \underset{\rightarrow}{v}/v$, then $|\underset{\rightarrow}{x} \times \underset{\rightarrow}{I}| = \sin \alpha$. To a first order approximation, this is α ; however, for computational purposes it is a more desirable quantity than α . To get the proper orientation of this quantity, the following vector will be defined:

$$(2.1) \quad \underset{\rightarrow}{\alpha}_e \equiv \underset{\rightarrow}{I} \times (\underset{\rightarrow}{x} \times \underset{\rightarrow}{I}) = \underset{\rightarrow}{x} - \cos\alpha \underset{\rightarrow}{I}$$

Obviously, $\underset{\rightarrow}{\alpha}_e \cdot \underset{\rightarrow}{I} = 0$. $\underset{\rightarrow}{\alpha}_e$ represents vector yaw * directed from $\underset{\rightarrow}{I}$ toward $\underset{\rightarrow}{x}$. The effect of $\underset{\rightarrow}{\alpha}_e$ on the trajectory is generally small under the assumptions stated. Furthermore, it will be assumed that $\dot{\underset{\rightarrow}{\alpha}}_e$ is negligible. This implies $\dot{\alpha}$ is small; moreover, the following approximations are warranted:

$$(2.2) \quad \dot{\underset{\rightarrow}{x}} = \cos\alpha \, \dot{\underset{\rightarrow}{I}}$$

$$(2.3) \quad \ddot{\underset{\rightarrow}{x}} = \cos\alpha \, \ddot{\underset{\rightarrow}{I}}$$

The separation of $\dot{\underset{\rightarrow}{H}}$ into components parallel and perpendicular to $\underset{\rightarrow}{x}$ yields:

$$(2.4) \quad A\dot{N} = -\rho d^4 K_A N v$$

$$(2.5) \quad AN\dot{\underset{\rightarrow}{x}} + B(\underset{\rightarrow}{x} \times \ddot{\underset{\rightarrow}{x}}) = \rho d^3 K_M v (\underset{\rightarrow}{v} \times \underset{\rightarrow}{x})$$
$$- \rho d^4 K_H v (\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}}) + \rho d^4 K_T N [\underset{\rightarrow}{x} \times (\underset{\rightarrow}{x} \times \underset{\rightarrow}{v})]$$

To determine $\underset{\rightarrow}{\alpha}_e$, first replace $\underset{\rightarrow}{x}$ and its derivatives in (1.1) and (2.5) using equations (2.1), (2.2) and (2.3).

* NOTE: $|\underset{\rightarrow}{\alpha}_e| \neq \alpha$. However, as mentioned earlier, in magnitude $|\underset{\rightarrow}{\alpha}_e|$ is a first-order approximation for yaw. $\underset{\rightarrow}{\alpha}_e$ is in the plane determined by $\underset{\rightarrow}{x}$ and $\underset{\rightarrow}{I}$, in the direction from $\underset{\rightarrow}{I}$ toward $\underset{\rightarrow}{x}$. Hence, $\underset{\rightarrow}{a}_e$ will be referred to as vector yaw.

The resulting equations are:

$$(2.6)\quad \dot{\underset{\to}{u}} = \frac{-\rho d^2(K_{D_o} + \alpha^2 K_{D_\alpha})v^2 \underset{\to}{I}}{m} + \frac{\rho d^2 K_L v^2 \underset{\to}{\alpha}_e}{m} - \frac{\rho d^3 K_S v \cos\alpha \dot{\underset{\to}{I}}}{m}$$

$$+ \frac{\rho d^3 K_F N v (\underset{\to}{\alpha}_e \times \underset{\to}{I})}{m} + \underset{\to}{g} + \underset{\to}{\Lambda}$$

$$(2.7)\quad AN \cos\alpha \dot{\underset{\to}{I}} + B\cos\alpha (\underset{\to}{\alpha}_e \times \ddot{\underset{\to}{I}}) + B\cos^2\alpha (\underset{\to}{I} \times \ddot{\underset{\to}{I}})$$

$$= \rho d^3 K_M v^2 (\underset{\to}{I} \times \underset{\to}{\alpha}_e) + \rho d^4 K_T N v [\cos\alpha (\underset{\to}{\alpha}_e + \cos\alpha \underset{\to}{I}) - \underset{\to}{I}]$$

$$- \rho d^4 K_H v \cos\alpha [(\underset{\to}{\alpha}_e + \cos\alpha \underset{\to}{I}) \times \dot{\underset{\to}{I}}]$$

Cross multiplication by $\underset{\to}{I}$ of both sides of equations (2.6) and (2.7) and, with $\underset{\to}{\Lambda}$ negligible in comparison to $\underset{\to}{g}$, the simultaneous solution of the resulting equations gives the following for $\underset{\to}{\alpha}_e$.

$$(2.8)\quad \underset{\to}{\alpha}_e = \{m\rho d^4 K_T N v \cos\alpha [\underset{\to}{I} \times (\dot{\underset{\to}{u}} - \underset{\to}{g} + \frac{\rho d^3 K_S v \cos\alpha \dot{\underset{\to}{I}}}{m})]$$

$$+ \rho d^2 K_L v^2 [-AN\cos\alpha (\underset{\to}{I} \times \dot{\underset{\to}{I}}) + B\cos^2\alpha [\underset{\to}{I} \times (\ddot{\underset{\to}{I}} \times \underset{\to}{I})]$$

$$+ \rho d^4 K_H v \cos^2\alpha \dot{\underset{\to}{I}}]\} \Big/ \{\rho^2 d^7 K_F K_T N^2 v^2 \cos\alpha$$

$$+ \rho d^2 K_L v^2 [\rho d^3 K_M v^2 + B\cos\alpha \underset{\to}{I} \cdot \ddot{\underset{\to}{I}}]\}$$

For further substitution into (2.8) the following are required:

(2.9) $\vec{I} = \vec{v} \,/\, v$

(2.10) $\dot{\vec{I}} = [\dot{\vec{v}} - (\dot{\vec{v}} \cdot \vec{I})\, \vec{I}] \,/\, v = [\, \vec{I} \times (\dot{\vec{v}} \times \vec{I})\,] \,/ v$

(2.11) $$\ddot{\vec{I}} = -\,2\,(\dot{\vec{v}} \cdot \vec{I})\, \dot{\vec{v}} \,/\, v^2 + 3(\dot{\vec{v}} \cdot \vec{I})^2\, \vec{I} / v^2$$

$$+\, \ddot{\vec{v}} \,/\, v - (\,\dot{\vec{v}} \cdot \dot{\vec{v}}\,)\vec{I} \,/\, v^2 - (\ddot{\vec{v}} \cdot \vec{I})\, \vec{I} / v$$

(2.12) $$\ddot{\vec{v}} \,/\, v - (\ddot{\vec{v}} \cdot \vec{I})\, \vec{I} \,/\, v = [\vec{I} \times (\,\ddot{\vec{v}} \times \vec{I})] \,/\, v$$

(2.12) will be considered negligible since $\ddot{\vec{v}}$ can be approximated by $\ddot{\vec{u}}$, and $\ddot{\vec{u}}$ is essentially parallel to $\vec{I}$.

With this assumption,

(2.13) $$\vec{c}_e = \{-\,AK_L N \cos a\ (\vec{v} \times \dot{\vec{v}}) + md^2 K_T N \cos a\ [\vec{v} \times (\dot{\vec{u}} - \vec{g}$$

$$+\, \rho d^3 K_S \cos a\ \dot{\vec{v}} \,/\, m)\,] - K_L [\,\dot{\vec{v}} - (\,\dot{\vec{v}} \cdot \vec{I})\, \vec{I}\,]$$

$$[\,2\,B \cos^2 a\ (\dot{\vec{v}} \cdot \vec{I}) - \rho d^4 K_H \cos^2 a\ \ v^2]\ \} \Big/$$

$$\{\rho d^3 K_L K_M v^4 + \rho d^5 K_F K_T N^2 \cos a\ v^2$$

$$+\, K_L B \cos a\ [\,(\dot{\vec{v}} \cdot \vec{I})^2 - (\dot{\vec{v}} \cdot \dot{\vec{v}})\,]\ \}$$

In the denominator of equation (2.13) the v^4 term predominates ; therefore, $K_L B \cos a [\ (\dot{\vec{v}} \cdot \vec{I})^2 - (\dot{\vec{v}} \cdot \dot{\vec{v}})\,]$ will be considered negligible. The ratio of this term to the remaining terms in the denominator is of the order of (g^2/v^4). In the numerator, $[\,\dot{\vec{v}} - (\dot{\vec{v}} \cdot \vec{I})\, \vec{I}\,]$ is the component of $\dot{\vec{v}}$ perpendicular to the projectile flight in the air coordinate system.

For most spin stabilized trajectories $\left|\dot{\underset{\rightarrow}{v}} - (\dot{\underset{\rightarrow}{v}} \cdot \underset{\rightarrow}{I})\underset{\rightarrow}{I}\right|$ is no more than the magnitude of $\underset{\rightarrow}{g}$; hence, consider

$$(2.14) \quad \left| K_L \; [\dot{\underset{\rightarrow}{v}} - (\dot{\underset{\rightarrow}{v}} \cdot \underset{\rightarrow}{I})\underset{\rightarrow}{I}\,][\,2\,B \cos^2 \alpha\,(\dot{\underset{\rightarrow}{v}} \cdot \underset{\rightarrow}{I}) - \rho d^4 K_H \cos^2 \alpha\, v^2\,] \right| \approx$$

$$\left| K_L\, \underset{\rightarrow}{g} \;\; [\,2\,B \cos^2 \alpha\,(\dot{\underset{\rightarrow}{v}} \cdot \underset{\rightarrow}{I}) - \rho d^4 K_H \cos^2 \alpha\, v^2\,] \right|$$

For artillery shells, this term is generally bounded by

$$(2.15) \quad K_L \;|g|\; \left| 2\,B \; [illegible] \; (10g) \right| = \left| 20 K_L\, B g_o{}^2 \right|$$

Now

$$(2.16) \quad \left| -\,AK_L N\,(\underset{\rightarrow}{v} \times \dot{\underset{\rightarrow}{u}}) + md^2 K_T N\,[\,\underset{\rightarrow}{v} \times (\dot{\underset{\rightarrow}{u}} - \underset{\rightarrow}{g})\,] \right| \approx$$

$$N v \left| \{-\,AK_L + md^2 K_T\,\}\, K_L\, \frac{\rho d^2}{m}\, v^2 + AK_L\, g_o \sin \phi \right|$$

where $\cos \phi = \dfrac{\underset{\rightarrow}{g} \cdot \underset{\rightarrow}{v}}{g \;\; v}$

For artillery shells, it is usually true that

$$(2.17) \quad \left| \{-\,AK_L + md^2 K_T\,\}\, K_L\, \frac{\rho d^2}{m}\, v^2 \right| << \left| AK_L\, g_o \sin \phi \right|$$

Comparison of (2.15) and (2.17) shows that the ratio of (2.14) to (2.16) is less than the order of $(20g_o / Nv)$. For high spin, (2.14) will be considered negligible.

Let

(2.18) $\underset{\rightarrow}{v} = \underset{\rightarrow}{u} - \underset{\rightarrow}{w}$

(2.19) $\underset{\rightarrow}{\dot{v}} = \underset{\rightarrow}{\dot{u}} - \underset{\rightarrow}{\dot{w}}$

If the effects of the pitching force (K_S term) and $\underset{\rightarrow}{\dot{w}}$ are assumed insignificant, and cos α can be approximated by 1, equation (2.13) is reduced to

(2.20 $$\underset{\rightarrow}{\alpha}_e = \frac{-AK_L N (\underset{\rightarrow}{v} \times \underset{\rightarrow}{\dot{u}}) + md^2 K_T N \; [\underset{\rightarrow}{v} \times (\underset{\rightarrow}{\dot{u}} - \underset{\rightarrow}{g})]}{\rho d^3 K_L K_M v^4 + \rho d^5 K_F K_T N^2 v^2}$$

UTILIZATION

The primary goal of the development of $\underset{\rightarrow}{\alpha}_e$ was the acquisition of a mathematical model which would incorporate the effects of yaw, but would not require the computing time of a complete rigid body simulation. The following representation was devised to incorporate $\underset{\rightarrow}{\alpha}_e$ in a modified point-mass mathematical model. This representation includes auxiliary equations necessary for the numerical solution of the differential equation of motion of the center of mass.

The equation of motion of the center of mass is:

$$(3.1)^* \quad \dot{\underset{\rightarrow}{u}} = -\frac{\rho m_s}{144\, C_s m} \{K_{D_o} + K_{D_\alpha} [Q \alpha_e]^2\} v \underset{\rightarrow}{v}$$

$$+ \frac{\rho d^2}{m} K_L v^2 l \underset{\rightarrow}{\alpha}_e + \underset{\rightarrow}{g} + \underset{\rightarrow}{\Lambda}$$

$$+ \frac{\rho d^3}{m} K_F N Q (\underset{\rightarrow}{\alpha}_e \times \underset{\rightarrow}{v})$$

where: C_s = ballistic coefficient for standard mass

l = lift factor

m = projectile mass

m_s = standard projectile mass

Q = yaw drag factor

*Note: If Q and l are set to zero (0) in equation (3.1), this system reduces to the classical point mass equations of motion.

The axial spin is:

$$(3.2)\quad N = -\int_0^t \frac{\rho d^4}{A} K_A Nv$$

The approximation for the yaw of repose is:

$$(3.3)\quad \underset{\rightarrow}{\alpha}_e = (\alpha_b - \alpha_a)\ (\underset{\rightarrow}{v} \times \dot{\underset{\rightarrow}{u}}) - \alpha_b\ (\underset{\rightarrow}{v} \times \underset{\rightarrow}{g})$$

$$\text{where: } \alpha_a = \frac{AK_L N}{\rho d^3 K_L K_M v^4 + \rho d^5 K_F K_T N^2 v^2}$$

$$\alpha_b = \frac{m\ K_T N}{\rho d\ K_L K_M v^4 + \rho d^3 K_F K_T N^2 v^2}$$

The velocity of the projectile with respect to air is:

$$(3.4)\quad \underset{\rightarrow}{v} = \underset{\rightarrow}{u} - \underset{\rightarrow}{w}$$

The position of the projectile with respect to ground-fixed Cartesian coordinate system is:

$$(3.5)\quad X = \int_0^t \underset{\rightarrow}{u}\ dt$$

The approximation for the position of the projectile with respect to spherical Earth surface is:

$$(3.6)\qquad \underset{\rightarrow}{E} = \begin{bmatrix} X_1 \\ X_2 + R - (R^2 - X_1^{\,2})^{1/2} \\ X_3 \end{bmatrix}$$

where R = effective radius of Earth.

The approximation of the force of gravity is:

$$(3.7) \qquad \underset{\rightarrow}{g} = -g_o \frac{R^2}{r^3} \begin{bmatrix} X_1 \\ X_2 + R \\ X_3 \end{bmatrix}$$

$$r = [X_1^2 + (X_2 + R)^2 + X_3^2]^{1/2}$$

where g_o = value of gravity at point of launch

r = distance between center of Earth and projectile

The Coriolis acceleration due to rotation of the Earth is :

$$(3.8) \qquad \underset{\rightarrow}{\Lambda} = \begin{bmatrix} -\lambda_1 u_2 - \lambda_2 u_3 \\ \lambda_1 u_1 + \lambda_3 u_3 \\ \lambda_2 u_1 - \lambda_3 u_2 \end{bmatrix}$$

For the northern hemisphere, the λ's are defined by the following equations. [For the southern hemisphere replace L by (- L).]

$\lambda_1 = 2 \ \Omega \cos L \sin AZ$

$\lambda_2 = 2 \ \Omega \sin L$

$\lambda_3 = 2 \ \Omega \cos L \cos AZ$

where: Ω = Angular velocity of the Earth (radians/sec)

L = Latitude of launch point

AZ = Azimuth of fire measured clockwise from north

The orientation of yaw (Ψ) is the angle between the plane containing both $\underset{\rightarrow}{v}$ and $\underset{\rightarrow}{a}_e$ and a vertical plane containing $\underset{\rightarrow}{v}$. It is measured clockwise from the vertical plane. If desired, Ψ is given by the expression:

$$(3.9) \qquad \Psi = \tan^{-1} \left[\frac{(v_1 a_{e_3} - v_3 a_{e_1})\, v}{(v_1 a_{e_2} - v_2 a_{e_1})\, v_1 - (v_2 a_{e_3} - v_3 a_{e_2})\, v_3} \right]$$

The dimensionless aerodynamic coefficients are functions of many dimensionless power products, including the dimensionless shape parameters, Reynolds number and Mach number. Aerodynamic coefficients are defined with reference to a specific set of shape parameters and may be expressed as functions of Mach number.

CONCLUSIONS

A modified, point-mass mathematical model has been developed which incorporates an estimate of the yaw of repose. This improved mathematical model has the desirable feature of representing the effects of the significant variables of yaw of repose and axial spin along the trajectory. By the incorporation of this yaw of repose, the factors (ballistic coefficient, yaw drag factor and lift factor) used to match empirical results have been found to vary little from theoretically determined values over the spectrum of conditions.

In comparisons between the rigid body mathematical model, the most complete representation available, and point-mass representation, the modified point-mass model accounted for better than 90 percent of the discrepancies in the time-dependent variables range, height and deflection. These comparisons were made for spin-stabilized artillery rounds currently being used by the U. S. Army. The modified point-mass model solution required only approximately twice the computation time of the point-mass solution while the rigid body solution requires about one hundred to one thousand times that of the point-mass solution.

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13. ABSTRACT

A modified point mass mathematical model, which incorporates an estimate of the yaw of repose, has been developed to represent the flight of a spin stablized, dynamically stable, artillery shell. This improved mathematical model has the desirable feature of representing the effects of the significant variables of yaw of repose and axial spin along the trajectory.

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