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VERTICAL MOTION OF HIGH ALTITUDE BALLOONS

TECHNICAL REPORT IV

report to

OFFICE OF NAVAL RESEARCH

Arthur D. Little, Inc.

VERTICAL MOTION OF HIGH ALTITUDE BALLOONS

Technical Report IV

Report to

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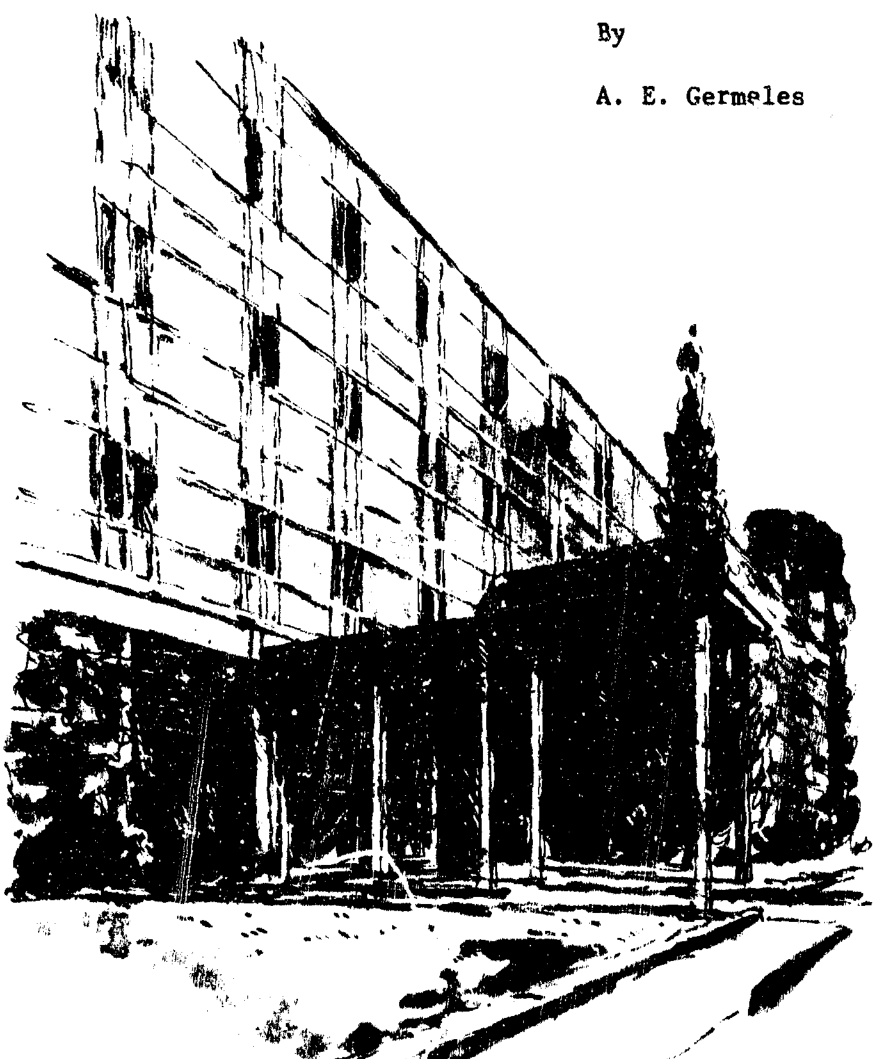
PHYSICS BRANCH
CONTRACT NO. Nonr 3164(00)

By

A. E. Germeles



July 1966

C-62944

Arthur D. Little, Inc.

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I. SUMMARY

A preliminary analysis of the vertical motion of high altitude balloons was presented to the Office of Naval Research on February 27, 1961. That report (Reference 1) was Technical Report I of a series of reports which have been sponsored by ONR contract Nonr-3164(00).

In December 1963, Technical Report II (Reference 2) was published. This report extended the analysis of Reference 1 and presented a computer program which describes balloon motion. Technical Report III which was concerned with the rotational motion of high altitude balloons was also published in December 1963.

Since Technical Report II was published, more information has become available for correlation purposes: the Stratoscope series has continued; the Fort Churchill summer programs have produced flight data for extreme altitudes; and a flight was made by ADL with the help of NCAR which measured helium gas temperatures. During this period the analysis and the computer program have been extended and improved. They are presented in this report, Technical Report IV.

The mathematical formulation of the model is presented in Section II.

In Section III, the computer program is described in great detail. Since this version of the program is more accurate than that presented in Technical Report II, all program users are hereby requested to obtain this newer version. With the permission of ONR, the program is available to any interested party in the subject of high altitude balloon performance.

Finally, in Section IV, the correlation of some computed and actual flights is presented. The correlation is very good. The results from the above mentioned ADL-NCAR flight have increased the confidence in the validity and accuracy of the computer model.

II. FORMULATION OF THE THEORY OF BALLOON DYNAMICS

A. INTRODUCTION

Balloons move upwards because of the force of buoyancy, a simple and well understood force. The upward accelerating force (free lift) is equal to the weight of the displaced air minus the weight of gas (gross list), minus the total weight of the balloon fabric, payload, ballast, etc. So it is a simple matter to write the dynamic equation of vertical motion of a balloon, even though the aerodynamic effects (induced mass and drag) of the surrounding air introduce some complications.

The gross lift, however, is a function of not only the atmospheric conditions, which change as the balloon moves, but also the temperature of the gas, which depends on the net heat received by the balloon. Due to their large surface, balloons are very sensitive to heat transfer, especially to thermal radiation. Thus, a balloon acts somewhat like an engine, converting heat to mechanical energy, and its equation of motion must be coupled to appropriate energy equations. Writing the appropriate energy equations, which will account for all important effects, is not a simple matter.

There is an energy equation for the helium gas. The gas exchanges heat with the fabric through free convection and does work against the atmosphere when it expands. The expansion of the gas increases the lift and, thus, influences the motion of the balloon. This equation of energy becomes more complicated when gas is valved out or exhausted.

There is, also, an equation of energy for the fabric. In addition to exchanging heat with the gas through free convection, the fabric exchanges heat with the air through free and forced convection. Also, the fabric emits thermal radiation and receives thermal radiation from the sun, the earth, and the atmosphere. Because of the importance of these effects, the various radiation fields as well as the radiative parameters of the fabric must be specified accurately.

These and all other important physical processes entering balloon dynamics are described in detail in the following sections and are formulated in a complete system of equations. A few words about the system of units used on the mathematical formulation of the problem are in order here. Length is in feet (ft), time in seconds (sec), mass in pounds (lb), temperature in degrees Rankine (^{o}R), and force in pounds. All other quantities are expressed in terms of these five units. For instance, work and energy, including heat, are in foot-pounds (ft lb). In any case, the units of every quantity are given in the List of Symbols.

B. THE EQUATION OF VERTICAL MOTION

Let w_G, w_F, and w_B be the weights in lbs of the payload, balloon fabric and balloon gas, respectively. The total mass (in slugs) of the balloon system as it moves through the atmosphere is:

$$\frac{1}{g}\left(w_G + w_F + w_B + C_B \rho_A V_B\right)$$

the last term being the apparent additional mass of the system due to the surrounding air. V_B is the volume of the balloon in ft^3, ρ_A is the density of air in lbs/ft^3, and $g = 32.2\ ft/sec^2$. C_B is a constant whose value depends on the shape of the balloon. For a spherical balloon, C_B is equal to 0.5.

Let z denote the altitude of the balloon in ft and y its velocity in ft/sec. Then:

$$y = \dot{z} \tag{1}$$

and the equation of motion of the balloon system is:

$$\left(w_G + w_F + w_B + C_B \rho_A V_B\right)\frac{\dot{y}}{g} = FL - \frac{1}{2g} C_D \rho_A A|y|y \tag{2}$$

where a dot denotes differentiation with respect to time, and the free lift FL is given by:

$$FL = \rho_A V_B - w_B - w_G - w_F \tag{3}$$

The second term of the right hand side of Equation 2 is the drag exerted on the balloon by the surrounding air. C_D is the drag coefficient and A is the effective area of the balloon (in ft^2) in the direction of motion. For a spherical balloon, C_D is equal to about 0.45 for Reynolds numbers in the range of 500 to 2×10^5.

In balloon terminology, the first two terms of the right hand side of Equation 3 are known as gross lift. Using the equation of state, the gross lift can be expressed in terms of quantities which are well known in balloonry. The equation of state (perfect gas law) for both air and gas is:

$$p = \frac{R}{M}\rho T \tag{4}$$

where p is the pressure (in lb/ft^2), ρ is the density in lb/ft^3, T is the temperature in ^oR, M is the molecular weight and R is the universal gas constant in ft lb/lb mol ^oR. The gross lift, GL, can now be expressed in the following form:

$$GL = w_B \left[-1 + \frac{M_A \; p_A(T_A + \theta)}{M_B \; T_A(p_A + \pi)} \right] \tag{5}$$

where the subscripts A and B pertain to air and gas, respectively. The quantity $\theta = T_B - T_A$ is known as the gas superheat (actually, it is a supertemperature), while $\pi = p_B - p_A$ is known as the gas superpressure. Since the percent superheat and superpressure are usually small quantities, Equation 5 can be reduced to the following approximate relation:

$$GL \cong w_B \left[\frac{M_A}{M_B} - 1 + \frac{M_A}{M_B} \left(\frac{\theta}{T_A} - \frac{\pi}{p_A} \right) \right] \tag{5a}$$

For balloons with no superpressure, Equation 5 reduces to:

$$GL = w_B \left(\frac{M_A}{M_B} - 1 + \frac{M_A \theta}{M_B T_A} \right) \tag{5b}$$

When θ is negative, the force $w_B \frac{M_A \theta}{M_B T_A}$ is negative and, in balloon terminology, it is referred to as thermodynamic drag.

C. ENERGY EQUATIONS

For a closed system, the time rate of change of its internal energy, $\dot{U}$, must be equal to the rate of supply of energy (in heat or other form), $\dot{Q}$, minus the rate of work done by the system, $\dot{W}$. In other words:

$$\dot{U} = \dot{Q} - \dot{W} \tag{6}$$

The units in this equation are ft lb/sec. This law of conservation of energy will be applied to the balloon gas system and to the balloon fabric system.

For the balloon gas system:

$$\dot{U} = \frac{d}{dt} \; C_v \; w_B \; T_B \quad = \; C_v \; w_B \; \dot{T}_B + C_v \; T_B \; \dot{w}_B \tag{7}$$

where C_v and T_B are the specific heat at constant volume in ft lb/lb $^{\circ}$R and the temperature in $^{\circ}$R of the gas, respectively. Notice that loss of gas by exhausting or valving is included in the above equation.

The only exchange of heat of the gas is with the balloon fabric through free convection. Let q_6 denote the rate of this heat transfer in ft lb/sec from the gas to the fabric. In addition to this energy, the gas system also loses energy at the rate of $C_v T_B \dot{w}_B$ whenever gas is expelled ($\dot{w}_B$ is negative). Therefore:

$$\dot{Q} = -q_6 + C_v T_B \dot{w}_B \tag{8}$$

If the rate of increase of the balloon gas volume is $\dot{V}_B$, then the rate of work done by the balloon gas on the atmosphere is $p\dot{V}_B$, where p is the atmospheric pressure in psf. Also, the balloon gas does work on the atmosphere, whenever gas is expelled, at the rate of $-p\dot{w}_B/\rho_B$, where ρ_B is the density of the gas in lb/ft^3. Therefore:

$$\dot{W} = p\dot{V}_B - \frac{p}{\rho_B} \dot{w}_B \tag{9}$$

Now, assuming that the pressure in the balloon is equal to the atmospheric pressure and using the equation of state (Equation 4) for the balloon gas, Equation 9 can be written as:

$$\dot{W} = p\dot{V}_B - \frac{R}{M_B} T_B \dot{w}_B \tag{9a}$$

Finally, substituting for $\dot{U}$, $\dot{Q}$ and $\dot{W}$ from Equations 7, 8 and 9a in Equation 6, the following energy equation for the gas in the balloon is obtained:

$$C_v w_B \dot{T}_B = - q_6 - p\dot{V}_B + \frac{R}{M_B} T_B \dot{w}_B \tag{10}$$

For the balloon fabric system:

$$\dot{U} = \frac{d}{dt} \left(C_F w_F T_F\right) = C_F w_F \dot{T}_F \tag{11}$$

where C_F and T_F are the specific heat in ft lb/lb ^oR and the temperature in ^oR of the fabric, respectively. $\dot{W}$ is equal to zero and $\dot{Q}$ is given by:

$$\dot{Q} = q_2 - q_3 + q_4 + q_5 + q_6 + q_7 \tag{12}$$

where all the q's are rates of heat transfer in ft lb/sec, accounting for the effects described below.

q_2: absorption of infrared radiation (from earth) by fabric.

q_3: emission of radiation by fabric.

q_4: forced convection to fabric from air.

q_5: free convection to fabric from air.

q_6: free convection to fabric from balloon gas.

q_7: absorption of solar radiation by fabric.

Thus, the energy equation for the fabric is:

$$C_F \, w_F \, \dot{T}_F = q_2 - q_3 + q_4 + q_5 + q_6 + q_7 \tag{13}$$

Equations 10 and 13 are the two required energy equations. Expressions for the q's will be derived in Sections G and H.

D. THE EQUATION OF EXPANSION OF THE BALLOON GAS

From the equation of state (perfect gas law), the following expression for the rate of expansion of the balloon gas, $\dot{V}_B$, is obtained:

$$\dot{V}_B = \frac{R}{pM_B} \left(w_B \, \dot{T}_B + T_B \, \dot{w}_B \right) - \frac{V_B}{p} \, \dot{p} \tag{14}$$

It should be kept in mind that the pressure of the balloon gas is taken equal to the atmospheric pressure p. However, p conforms to the hydrostatic equation:

$$\dot{p} = - \rho_A \dot{z} \tag{15}$$

Therefore, substituting for $\dot{p}$, Equation 14 becomes

$$\dot{V}_B = \frac{R}{pM_B} \left(w_B \, \dot{T}_B + T_B \, \dot{w}_B \right) + \frac{\rho_A}{p} \, V_B \dot{z} \tag{14a}$$

E. VALVING, EXHAUSTING AND BALLASTING

The pressure in inextensible balloons is not quite the same as the outside atmospheric pressure. There is a slight overpressure which causes the gas to flow out when the valve is open. As the balloon reaches its ceiling and becomes fully inflated, a similar small pressure difference across the appendix of the balloon causes automatic exhausting of gas. When this exhausting is sufficient, it can prevent the balloon from bursting and it causes stabilization of ceiling without high altitude bounce.

Let $\dot{E}$ and $\dot{V}$ be the rate of volumetric gas flow in ft^3/sec due to exhausting and valving, respectively. Both quantities are negative for outflow of gas. Then the rate at which the weight of the balloon gas changes is given by:

$$\dot{w}_B = \rho_B (\dot{E} + \dot{V}) \tag{16}$$

Valving data are usually given as lift lost per unit time (lb/sec), $\dot{L}_V$, which is a positive quantity. Since:

$$\dot{L}_V = -(\rho_A - \rho_B) \dot{V} \tag{17}$$

Equation 16 becomes:

$$\dot{w}_B = \rho_B \dot{E} - \frac{\rho_B}{\rho_A - \rho_B} \dot{L}_V \tag{18}$$

The weight of the payload, w_G, is changed by ballasting. If $\dot{B}$ is the ballasting rate in lb/sec, then:

$$\dot{w}_G = -\dot{B} \tag{19}$$

F. SPECIFICATION OF THE ATMOSPHERE

For a complete specification of the atmosphere only one state variable is required since the other two state variables can be computed from the hydrostatic equation and the equation of state. Usually, the atmosphere is specified by giving a temperature profile with respect to altitude. The pressure and density can then be computed from the hydrostatic equation:

$$\frac{dp}{dz} = -\rho_A \tag{20}$$

and the equation of state:

$$p = \frac{R}{M_A} \rho_A T_A \tag{21}$$

where M_A and T_A are the molecular weight and the temperature of the atmosphere. Eliminating ρ_A from these two equations and integrating once, one obtains:

$$\ln \frac{p}{p_o} = -\frac{M_A}{R} \int_{z_o}^{z} \frac{dz}{T_A} \tag{22}$$

where p_o is the pressure at the initial altitude z_o. It is clear now that, for a complete specification of the atmosphere, the initial pressure p_o (or the initial density) is required in addition to the temperature profile.

Suppose the temperature profile is specified by giving a number of altitudes and corresponding temperature values, z_n and T_n. Then in the interval n and n+1, the temperature can be approximated by the straight line:

$$T_A = s_n z + b_n \tag{23}$$

where:

$$s_n = \frac{T_{n+1} - T_n}{z_{n+1} - z_n} \tag{24}$$

$$b_n = \frac{T_n z_{n+1} - T_{n+1} z_n}{z_{n+1} - z_n} \tag{25}$$

Integrating Equation 22 with T_A as given by Equation 23, the following expressions are obtained. For an isothermal layer, $T_{n+1} = T_n$ and $s_n = 0$:

$$\frac{p}{p_n} = \exp\left[\frac{M_A}{RT_n} (z_n - z)\right] \tag{26}$$

For a nonisothermal layer, $T_{n+1} \neq T_n$ and $s_n \neq 0$:

$$\frac{p}{p_n} = \left(\frac{T_n}{T_A}\right)^{(M_A/s_n R)} \tag{27}$$

The corresponding expressions for the density are:

$$\frac{\rho_A}{\rho_n} = \exp\left[\frac{M_A}{RT_n}(z_n - z)\right] \qquad (28)$$

for an isothermal layer, and:

$$\frac{\rho_A}{\rho_n} = \left(\frac{T_n}{T_A}\right)^{(1 + M_A/s_nR)} \qquad (29)$$

for a nonisothermal layer.

G. HEAT TRANSFER BY CONVECTION

1. Air-Side Forced Convection

Forced and free convection as well as radiation depends on the shape of the heated object. Convection correlations exist in the literature for plates, cylinders and spheres but not for shapes taken by balloons, which can be anywhere from a distorted bubble with a long stem to an onion-like shape at ceiling.

The balloon is considered to be spherical of volume V_B, and all heat transfer calculations are based on this geometry. Whenever the deviation of the actual balloon from the assumed shape is thought to have an important effect on a particular heat transfer mechanism, a correction constant is introduced. These correction constants are evaluated by correlation with actual flights.

The diameter, D, cross-sectional area, A, and surface area, S, of a sphere of volume V_B are given by:

$$D = 1.24V_B^{1/3} \qquad (30)$$

$$A = 1.21V_B^{2/3} \qquad (31)$$

$$S = 4.83V_B^{2/3} \qquad (32)$$

For a sphere in the Reynolds number range of 17 to 70,000, McAdams (Ref. 3) recommends the following correlation for the heat transfer coefficient, h, by forced convection:

$$\frac{hD}{k} = 0.37\ (Re)^{0.6} \tag{33}$$

where k is the conductivity of the surrounding medium and Re is the Reynolds number. He points out that turbulence can increase the above value of h by 40 to 60 percent. The laminar flow past a sphere becomes turbulent when the Reynolds number is about 2.5×10^5.

For all big balloons, the Reynolds number is above 2.5×10^5 for the most part of their vertical flight. In calculating the heat exchange of the balloon fabric with the surrounding air by forced convection, the balloon is assumed to be spherical and Equation 33 is used with a correction constant C_4. The heat transfer is given by:

$$q_4 = 1.44\ C_4 k_A V_B^{1/3}\ (T_A - T_F)(1.24\ V_B^{1/3}\ y \frac{\rho_A}{\mu_A})^{0.6} \tag{34}$$

where k_A is the conductivity of air in ft lb/ft sec ^oR and μ_A is the viscosity of air in lb/ft sec.

2. Free Convection

For free convection from vertical plates and inside vertical cylinders, McAdams recommends the following expression for the heat transfer coefficient, h, when X is in the range 10^9 - 10^{12}.

$$\frac{hL}{k} = 0.13\ X^{1/3} \tag{35}$$

where L is the height of the plate or cylinder. X is the product of the Grashof and Prandtl numbers and it is proportional to the third power of L.

Let us assume that, with an appropriate correction constant, the correlation given by Equation 35 is valid inside or outside a sphere, as long as X based on the diameter of the sphere is within or near the above range.

Taking the balloon as a sphere, it can be shown that, for both the air-side and helium-side of large balloons, X is within or near the above range. Using Equation 35 with correction constants C_5 and C_6, the heat transfers by free convection in the air-side, q_5, and helium-side, q_6, are given by:

$$q_5 = 0.628\ C_5\ k_A\ V_B^{2/3}\ (T_A - T_F)\left(\frac{g\rho_A^2\ |T_A - T_F|}{\mu_A^2\ T_A}\ Pr_A\right)^{1/3} \tag{36}$$

$$q_6 = 0.628\ C_6\ k_H\ V_B^{2/3}\ (T_B - T_F)\left(\frac{g\rho_H^2\ |T_B - T_F|}{\mu_H^2\ T_B}\ Pr_H\right)^{1/3} \tag{37}$$

where Pr_A and Pr_H are the Prandtl numbers of air and helium, respectively. k_H is the conductivity of helium in ftlb/ftsec^{o}R and μ_H is its viscosity in lb/ftsec. It has been assumed that the coefficients of expansion of air and helium are equal to the inverse of their absolute temperature.

3. The Thermal Parameters of Air and Helium

In the following paragraphs a brief description is given of the Prandtl number, viscosity and thermal conductivity of air and helium. The temperature range of interest is 350^{o}R to 550^{o}R.

The Prandtl number for air and helium is essentially constant and equal to 0.7 (Ref. 3).

The viscosity is a function of the temperature only, and, in a given temperature range, it can be taken in the form (Ref. 4):

$$\mu = AT^{n} \tag{38}$$

The constants A and n are determined by making this expression conform to two experimentally measured values of the viscosity at the two extreme temperatures of the range of interest.

From Reference 5, the viscosity of air is 1.333×10^{-4} and 1.827×10^{-4} poise at temperatures -69.4 and 18^{o}C, respectively. Thus, Equation 38 becomes:

$$\mu_A = 1.22 \times 10^{-6}\ T_A^{\ 0.883} \tag{39}$$

where the viscosity is in lb/ft sec and the temperature in ^{o}R.

From Reference 4, the viscosity of helium is equal to 1.587×10^{-4} and 1.967×10^{-4} poise at temperatures -60.9 and 17.6°C, respectively. Therefore:

$$\mu_H = 4.10 \times 10^{-6} \, T_H^{\ 0.682} \tag{40}$$

where the units of μ_H and T_H are lb/ft sec and °R, respectively.

The specific heat at constant pressure is essentially constant for most gases (Ref. 4). Therefore, since the Prandtl number is also constant, the thermal conductivity of air and helium must depend on temperature in the same way as the viscosity, i.e., in the form of Equation 38 with the same n. The only unknown constant A can be determined by making this expression conform to one experimentally measured value of the thermal conductivity at the middle of the temperature range of interest.

From Reference 4, the thermal conductivity of air and helium is 5.80×10^{-5} and 3.52×10^{-4} cal/cm sec °K, respectively, at a temperature of 0°C. Thus:

$$k_A = 4.08 \times 10^{-7} \, T_A^{\ 0.883} \tag{41}$$

$$k_H = 7.63 \times 10^{-6} \, T_H^{\ 0.682} \tag{42}$$

where the thermal conductivity is in ft lb/ft sec °R and the temperature in °R.

H. HEAT TRANSFER BY RADIATION

1. Emission of Radiation by the Balloon Fabric

The fabric emits radiation in the infrared part of the spectrum. The flux emitted by each side of the fabric is $\varepsilon\sigma T_F^4$, where ε is the average emissivity of the fabric in infrared and σ is the Stefan-Boltzmann constant. Since the balloon gas is considered transparent, part of the radiation emitted by the inner side of the fabric is absorbed by the fabric so that the effective emissivity ε_{ef}, of the fabric as a whole is not 2ε but:

$$\varepsilon_{ef} = \varepsilon + \varepsilon - \varepsilon\alpha - \varepsilon R\alpha - \varepsilon R^2\alpha - \varepsilon R^3\alpha - \ldots \tag{43}$$

where α and R are the average absorptivity and reflectivity of the fabric in infrared, respectively. This series can be summed to give the following two alternative results:

$$\varepsilon_{ref} = \varepsilon_r \left[2 - \frac{\alpha_r}{1 - R_r} \right] = \varepsilon_r \left[1 + \frac{\tau_r}{1 - R_r} \right] \quad (44)$$

where the subscript r is used to denote infrared and τ_r is the average transmissivity of the balloon fabric in the infrared spectrum at a reference temperature.

$$\alpha_r + \tau_r + R_r = 1 \quad (45)$$

If the balloon is considered to be spherical, the radiating fabric area is the surface of a sphere of volume V_B as given by Equation 32. Therefore, the effective energy emitted by the fabric is given by:

$$q_3 = 4.83\ \varepsilon_{ref}\ \sigma V_B^{2/3}\ T_F^4 \quad (46)$$

where σ is in ft lb/ft^{2}sec $^oR^4$ so that the units of q_3 are ft lb/sec.

Arthur D. Little, Inc., has developed a computer program for computing the effective multiple absorptivity and emissivity of films. Using measured values of α and R versus wavelength and a given radiation intensity spectrum, $I(\lambda)$, the values of α_r, ε_r and R_r of the film are computed from expressions such as the following:

$$\alpha_r = \frac{\int \alpha(\lambda)\ I(\lambda)\ d\lambda}{\int I(\lambda)\ d\lambda}$$

The effective absorptivity and emissivity of the film for multiple passes of radiation is then computed from Equation 44.

This program is used to calculate the effective absorptivity and emissivity of balloon fabrics in infrared as well as their effective absorptivity in the sun's radiation (0.3 to 3 microns). For infrared the radiation spectrum used is that of a black body at a temperature of about 300°K. For solar radiation, the sun's spectrum outside the earth's atmosphere is used. These values of effective absorptivity and emissivity are used for the entire flight of the balloon.

There are two sources of possible errors in the above described method of computing the effective absorptivity and emissivity of balloon fabrics. Firstly, the spectra of the various radiations are not the same throughout the flight of a balloon: the temperature of the fabric changes, and the spectra of the infrared radiation from the earth and of the solar radiation are altered through the atmosphere. Secondly, the averaging process over wavelength should be carried out on the multiple-pass parameters and not on the one-pass parameters. The errors introduced by our approximate method will be significant when the radiative properties of the fabric change violently with wavelength in the spectrum of interest. Indeed, some fabrics behave in this manner.

2. Infrared Radiation From the Earth

The infrared radiation emitted by the earth is absorbed and emitted again by the atmosphere. To be sure the resulting radiation field varies throughout the atmosphere not only in its total intensity but in its frequency content as well. Water vapor and carbon dioxide are the main constituents of the atmosphere which interact heavily with the infrared radiation of the earth.

There are many papers dealing with the absorbing properties of water vapor and carbon dioxide layers. For instance, Reference 6 gives extensive tables and curves describing these properties. From these data it is possible to construct a theoretical model for the computation of the radiation received by a balloon as it moves through the atmosphere. However, such a model would require an inhibitive amount of computation and it would be almost useless since the concentration and stratification of water vapor and carbon dioxide in the atmosphere is quite unpredictable.

The infrared radiation field in the atmosphere has been measured by many observers. Most important in this field are the works of Gergen (Refs. 7-11). The radiation field versus altitude is measured with black ball flights. The equilibrium temperature of the black ball, T_r, is recorded and given in charts versus altitude.

This type of measurement has been made for many locations. The results show that the profile of T_r varies with geographical and seasonal conditions. Essential changes of the T_r profile for the same location can take place within days. The measurement is carried out during the night, so that the detector is not effected by the sun. It has been estimated that the day radiation field is about $10^{o}F$ higher than the measured night field.

The results of these measurements support the following approximate but simple general rule. At ground, T_r is usually less than the temperature of the air, the deviation being not more than about $10^{o}F$. Then T_r decreases almost linearly with altitude up to tropopause, where σT_r^4 is about 30% of its value at ground. From there to higher altitudes, T_r remains approximately constant.

3. Absorption of Infrared Radiation by the Balloon Fabric

The use of a spherical detector to measure the infrared radiation field is, indeed, a very fortunate coincidence. Since in the theoretical model the balloon is considered to be spherical, i.e., of the same shape as the detector, the radiation flux incident on the balloon is simply σT_r^4 and the energy absorbed by the fabric can be calculated very easily. This would not be the case, if the detector had a different shape. In the following calculation of the infrared radiation absorbed by the fabric, it is assumed that the T_r profile is known. If this profile has not been measured for a particular location and time, then the simple rule, described in the last paragraph of the preceding section, is recommended for its specification.

Since the balloon gas is considered transparent, the absorption of the incident flux σT_r^4 by the fabric is multiple, as in the case of emission described in Section 1. Thus, an effective absorptivity, α_{ref}, and not the one-way absorptivity of the fabric must be used. The calculation of α_{ref} is done as described in Section 1 with a radiation spectrum identical to that of a black body at temperature T_r. With the absorbing surface S as given by Equation 32, the infrared radiation energy absorbed by the fabric is given by:

$$q_2 = 4.83 \ \alpha_{ref} \, \sigma V_B^{2/3} \ T_r^4 \qquad (47)$$

4. Solar Radiation

The total intensity of solar radiation outside the earth's atmosphere is 2.05 cal/cm^{2}min (Ref. 6) or 96 ft lb/ft^{2}sec. The peak of its spectrum is in the visible range of frequencies. A considerable amount of energy is in the infrared and some energy is in the ultraviolet.

As this energy enters the atmosphere, part of it is absorbed by the various atmospheric constituents. Ozone absorbs ultraviolet; ozone, oxygen and water vapor absorb visible; water vapor and carbon dioxide absorb infrared. By the time the solar radiation reaches the earth its total intensity is reduced to 1.44 cal/cm^{2}min, when the azymuth angle of the sun is zero (i.e., the sun is directly overhead), and its frequency spectrum is altered.

An approximate way of describing the attenuation of the solar radiation in the atmosphere is through the optical air mass, m, which is the ratio of the path length of the sun's rays through the atmosphere to the normal path length. The following table is constructed from data given in Reference 6.

Table 1. Variation of Solar Radiation (Relative to Radiation Outside Atmosphere) With Optical Air Mass.

Optical Air Mass (m)	Solar Radiation (T)
0	1.0
1	.7018
2	.5596
3	.4595
4	.3849
5	.3249

For a computer program, it is expedient to have closed form expressions instead of tables of values. The following expression conforms to the values of the above table with an error of less than 3%.

$$T = 1/2 \quad (e^{-0.65m} + e^{-0.095m}) \tag{48}$$

The optical air mass, m, depends on the altitude and on the sun's azymuth angle, δ. When the sun is directly overhead, its azymuth angle is zero. Sea level sunrise or sunset correspond to $\delta = 90^{\circ}$. For $\delta = 0$, m is equal to 0 and 1 for a point outside the atmosphere and at sea level, respectively.

Let m_o be the value of m at sea level. Then, keeping δ constant, the value of m at any altitude, where the pressure is p, is given by:

$$m = m_o \frac{p}{p_o} \tag{49}$$

where p_o is the pressure at sea level.

The variation of m_o with δ is given in Table 16-18 of Reference 6. It can be verified easily that the following expression conforms to the values of this table with an error of less than 0.5%.

$$m_o = \left[1228.6 + (613.8 \cos\delta)^2 \right]^{1/2} - 613.8 \cos\delta, \quad o \leq \delta \leq 90 \tag{50}$$

Thus, for given time (and, therefore, given δ) and given altitude, the optical air mass can be found from Equations 49 and 50. Then the solar radiation (relative to that outside the atmosphere) for that particular time and point in the atmosphere can be computed from Equation 48. Obviously, this procedure is valid for values of δ up to 90°. Sea level sunrise and sunset will occur at $\delta = 90^{\circ}$, and a point at sea level will not receive any solar radiation for δ greater than

90^o. This is not the case for a point higher up in the atmosphere, where sunrise and sunset occur at values of δ greater than 90^o. This point will receive solar radiation for values of δ greater than 90^o. A procedure must be devised for computing the solar radiation in this range of δ.

Consider point A in the atmosphere (Figure 1), which has an altitude z. At this particular time, the sun's rays are along line SA, and δ is larger than 90^o. The atmospheric path travelled by the sun's rays is CA. Line BD, drawn from the earth's center, is perpendicular to line AC. The altitude of point B, z_1, is given by:

$$z_1 = (R + z) \sin \delta - R \tag{51}$$

where R is the radius of the earth.

For point B, the azymuth angle of the sun is equal to 90^o. Thus, the optical air mass for point B, m_1, can be computed from Equations 49 and 50 with $\delta = 90^o$ and $p = p_1$, where p_1 is the pressure at z_1. Then the attenuation of solar radiation at point B is $T(m_1)$ as given by Equation 48.

In travelling the remaining air path BA, the solar radiation is reduced further. Let m_2 be the optical air mass corresponding to altitude z and azymuth angle of the sun equal to $180^o - \delta$, i.e., when the sun is at S'. Any radiation originating at B is reduced to $T(m_1)$ when it arrives at C', and any radiation originating at S' (or C') is reduced to $T(m_2)$ when it arrives at A. Therefore, any radiation originating at B is reduced to $T(m_1)/T(m_2)$ when it arrives at A. Thus, the solar radiation received by point A (relative to that outside the atmosphere) is given by:

$$T = \frac{T^2(m_1)}{T(m_2)} \tag{52}$$

The range of δ covered by this computation is:

$$90^o < \delta \leq 90^o + \cos^{-1} \frac{R}{R + z} \tag{53}$$

For larger values of δ, the solar radiation received by point A is equal to zero.

5. Absorption of Solar Radiation By the Balloon Fabric

The area of a spherical balloon normal to the solar rays, which come from infinity, is equal to the area of a circle, A, as given by Equation 31. This is the effective absorbing area of the fabric for one pass of radiation. However, since the balloon gas is again considered

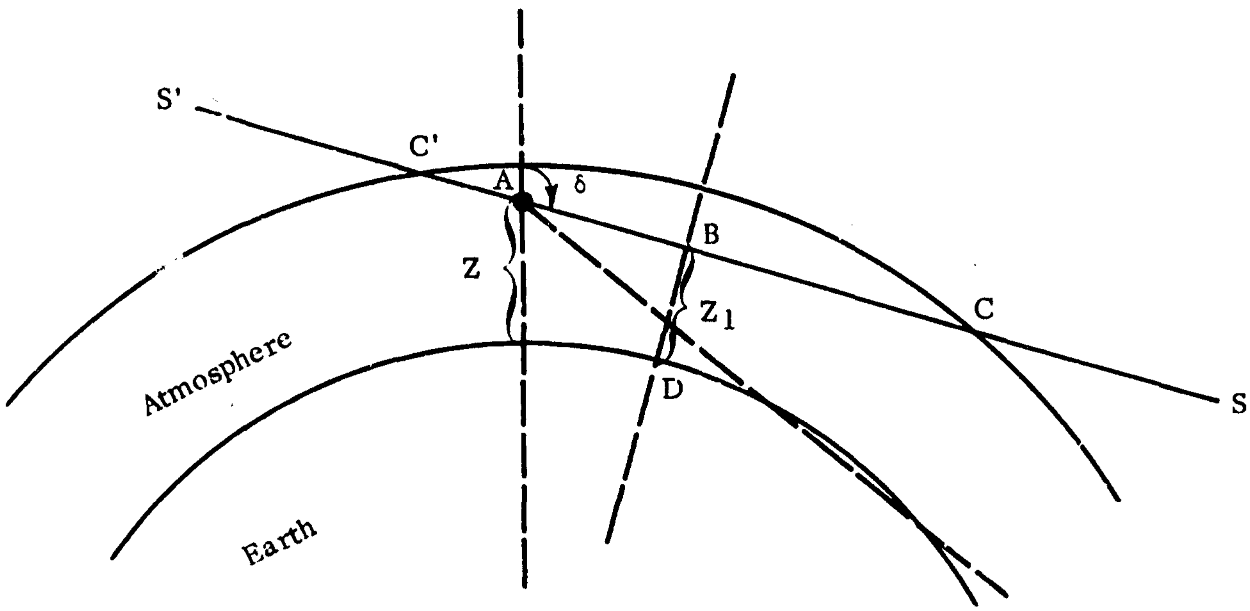


FIGURE 1 CALCULATION OF SOLAR RADIATION FOR AZYMUTH ANGLES GREATER THAN 90°

transparent, there is multiple absorption by the fabric and an effective absorptivity α_{vef} must be used, not the one-way absorptivity of the fabric. The subscript v stands for solar radiation. This absorptivity is computed as described in Section 1 with the solar radiation spectrum.

Thus, the solar energy absorbed by the fabric is given by:

$$q_7 = 116 \quad \alpha_{vef} \; T \, V_B^{\,2/3} \tag{54}$$

The numerical constant in this equation comes from the solar constant outside the atmosphere (96 ft lb/ft^{2}sec) times the numerical constant of the effective area (1.21) as given by Equation 31. T is a function of the optical air mass computed as described in Section 4.

It remains to define the sun's azymuth angle, δ , in a time coordinate system which is related to the flight of the balloon. Let LONG and LAT be the longitude and latitude of the balloon in degrees, respectively. If at time zero, the Greenwich hour angle is GHA, then at any subsequent time t the local hour angle, LHA, is given by:

$$\text{LHA} = \text{GHA} - \text{LONG} + \frac{t}{240} \tag{55}$$

where LHA and GHA are in degrees and t is in seconds. Let DEC be the declination of the sun in degrees. DEC is a slowly varying function of time so that it can be considered constant throughout a balloon flight. Then δ is given by the following equation.

$$\cos\delta = \sin \text{LAT} \sin \text{DEC} + \cos \text{LAT} \cos \text{DEC} \cos \text{LHA} \tag{56}$$

I. SUMMARY OF EQUATIONS. REQUIREMENTS FOR UNIQUE SOLUTION.

In summary, the governing differential equations of the model are:

$$\dot{z} = y \tag{57}$$

$$(w_G + w_F + w_B + C_B \, \rho_A \, V_B) \frac{\dot{y}}{g} = \rho_A \, V_B - w_G - w_F - w_B$$

$$- \frac{1.21}{2g} \, C_D \, \rho_A |y| y \, V_B^{\,2/3} \tag{58}$$

$$C_v \, w_B \, \dot{T}_B = \frac{R}{M_B} \quad T_B \, \dot{w}_B - p\dot{V}_B - q_6 \tag{59}$$

$$C_F \, w_F \, \dot{T}_F = q_2 - q_3 + q_4 + q_5 + q_6 + q_7 \tag{60}$$

$$\dot{V}_B = \frac{R}{pM_B} (w_B \dot{T}_B + T_B \dot{w}_B) + \frac{\rho_A}{p} V_B \dot{z} \tag{61}$$

$$\dot{w}_B = \rho_B \dot{E} - \frac{\rho_B}{\rho_A - \rho_B} \dot{L}_V \tag{62}$$

Notice that in the drag term of Equation 58, the effective area has been taken equal to the cross-sectional area of a spherical balloon of volume V_B.

Thus, there are six first order nonlinear differential equations. The independent variable is, of course, time. Let the six dependent variables be z, y, w_B, V_B, T_B, and T_F. For a unique solution of these variables, all other variables must be known. In detail, the following items must be specified.

1. The launch site (longitude and latitude) and the launch time.

2. The atmospheric temperature (T_A), pressure (p), density (ρ_A), thermal conductivity (k_A) and viscosity (μ_A). The specification of all these properties is accomplished by means of a temperature profile (T_A vs z) and the initial pressure or density.

3. The infrared radiation field in the atmosphere. It is specified by giving the "black ball radiation equilibrium temperature" profile (T_r vs z).

4. The weight (w_F), specific heat (C_F), infrared effective multiple emissivity (ϵ_{ref}) and effective multiple absorptivity for infrared (α_{ref}) and for solar radiation (α_{vef}) of the balloon fabric.

5. The total payload including ballast (w_G), and the ballasting schedule.

6. The valving schedule and exhausting rate. Specification of the latter is very difficult, if not impossible, since, in addition to the size and shape of the appendix, the instantaneous pressure differential across the appendix must be known. In the computer program, exhausting is allowed through a mathematical expedient which will be described in Section III-C.

7. The thermal conductivity (k_H) and viscosity (μ_H) of the gas as functions of the gas temperature (T_B).

8. The Prandtl number for air and for the balloon gas. Also, the specific heat at constant volume of the gas (C_v) and R, M_A and M_B.

9. The five constants C_B C_D, C_4, C_5 and C_6. As it will be shown in Section IV good correlation with actual flights is obtained with the following values for these constants:

$$C_B = 0.5$$

$$C_D = 0.3$$

$$C_4 = 1.5$$

$$C_5 = 1.5$$

$$C_6 = 1.0$$

Furthermore, for a unique solution, initial values of the six dependent variables must be given. For a balloon flight, the initial values of z, y, T_B and T_F can be obtained rather easily with the exception, perhaps, of T_F. As for the initial values w_B and V_B, they can be calculated from the initial value of the free lift, which must be given, and the equation of state with, of course, the gas pressure equal to the atmospheric pressure.

III. THE COMPUTER PROGRAM

A. INTRODUCTION

A computer program has been devised to solve the problem of the vertical flight of a balloon as formulated in Section II. The program integrates numerically the six first order differential equations (Equations 57 to 62) once the necessary input data are given.

The program is coded in the FORTRAN II algebraic language and it has been operated in the IBM 7090 Data Processing Equipment in conjunction with the FORTRAN MONITOR SYSTEM. The computer time depends primarily on the integration time interval, the on and off line printing period and the plotting period. For an integration time interval of 20 seconds, an off-line printing period of 15 minutes and a plotting period of 5 minutes, the times required by the 7090, which does the integration, and by the 1401, which prints the output, are both about one hundreth of the actual flight time.

In the following sections, the program is explained in detail. The FORTRAN listing of the entire program and a general flow chart are given at the end of this report. The symbols used in the program are defined in the List of Symbols. The flow chart is intended to show in one compact picture the general flow of information, sequence of operations and logic of the program.

B. INPUT DATA

The input data contain all the information required for a unique solution (see Section II-I) as well as information pertaining to the integration, and to printing and plotting of output.

All input data are read in the program from the input tape, which gets this information from a deck of cards. No information can be fed to the computer on-line. However, all six sense switches can be used during the course of computation to perform various functions.

Besides the leading title card, which contains alphanumeric information in format 12A6, there are two categories of input cards: control cards and data cards. Control cards must have only an integer number, from 0 to 13, in the first two columns in format I2. Data cards must have the first two columns blank and then, in successive fields of 10, input data in format F10.4. The data are divided in eleven types. Each type must be preceded by a control card.

The content of a complete set of data cards for one flight is shown in Table 2. Each line represents one card. Notice the order of control cards. The control card with the number 1 will make the computer to start computing and, therefore, it must be placed after all the data cards. The control card with the number 2 will make the computer to CALL EXIT instantly and, therefore, it must be the last card in the deck. It is not necessary to have the various types of data in the indicated order. They can be rearranged as long as they carry the correct control card. For instance, data type 4 can be placed after data type 8, and so on. However, within a given type of data, the data must be given in the indicated order. A control card must always be followed by the appropriate data cards.

Type 3 data contain the specific heat of the fabric (C1) in ftlb/lb ^{o}R, the constants of convective heat transfer (C4, C5 and C6) which are equivalent to the constants C_4, C_5 and C_6 of Section II, the infrared effective multiple absorptivity (ABIR) and emissivity (EMIR) of the fabric, and the effective multiple absorptivity for solar radiation (ABUV) of the fabric.

Type 4 data contain the initial weight of the payload including ballast (WGO) in lbs, the weight of fabric (WFO) in lbs, the inflated volume of the balloon (VBM) in ft^3 as specified by its manufacturer, the initial free lift (FLO) in lbs, and the aerodynamic constants for drag (CD) and apparent additional mass (CB) which are equivalent to the constants C_D and C_B of Section II.

Type 5 data contain the initial temperature of the gas (TBO) in ^{o}R, the initial temperature of the fabric (TFO) in ^{o}R, the initial altitude of the balloon above sea level (ZO) in ft, the initial velocity of the balloon (Y20) in ft/sec, the declination of the sun (DEC) in degrees, and the latitude (XLAT) and longitude (XLØNG) of the balloon in degrees.

Type 6 data contain the time interval for the integration (H) in sec, the printing period of output (DPR) in sec, the initial time (XO) in sec, the final time (XT) in sec, the Greenwich mean time (GMTS) in sec at the initial time, the Greenwich hour angle (GHA) in degrees at the initial time, and the plotting period of output (DP) in sec.

Type 7 data contain two arrays of time (TCSI) in sec and corresponding values of the so called "solar radiation factor" (CCSI). They must be in order of increasing TCSI. The number of cards must not exceed 100. Solar radiation for a clear sky is computed automatically. Reduction of the solar radiation caused by transient effects such as clouds are accounted for by the solar radiation factor. When no such effects are considered, the correct and sufficient data of this type are the following two cards:

TCSI(1) = XO	CCSI(1) = 1.0
TCSI(2) = XT	CCSI(2) = 1.0

Table 2. Content and Order of Input Cards for One Flight

Card	2–12	12–22	22–32	32–42	42–52	52–62	62–72	Format
	TITLE							12A6
3								I2
	C1	C4	C5	C6	ABIR	EMIR	ABUV	F10.4
4								I2
	WGO	WFO	VBM	FLO	CD	CB		F10.4
5								I2
	TBO	TFO	ZO	Y2O	DEC	XLAT	XLØNG	F10.4
6								I2
	H	DPR	XO	XT	GMTS	GHA	DP	F10.4
7								I2
	TCSI(1)	CCSI(1)						F10.4
	⋮	⋮						F10.4
	TCSI(NSI)	CCSI(NSI)						F10.4
8								I2
	VO							F10.4
	TV(1)	VV(1)						F10.4
	⋮	⋮						F10.4
	TV(NVI)	VV(NVI)						F10.4
9								I2
	BO							F10.4
	TB(1)	BB(1)						F10.4
	⋮	⋮						F10.4
	TB(NBI)	BB(NBI)						F10.4

Table 2. Content and Order of Input Cards for One Flight (Cont.)

Field								Format
2	12	22	32	42	52	62	72	
10								I2
	RHØO							F10.4
	ZZ(1)	TZ(1)						F10.4
	⋮	⋮						F10.4
	ZZ(NT)	TZ(NT)						F10.4
11								I2
	ZIR(1)	RTIR(1)						F10.4
	⋮	⋮						F10.4
	ZIR(NIR)	RTIR(NIR)						F10.4
12								I2
	ETIME(1)	EALT(1)						F10.4
	⋮	⋮						F10.4
	ETIME(NEX)	EALT(NEX)						F10.4
13								I2
	TIR(1)	CIR(1)						F10.4
	⋮	⋮						F10.4
	TIR(NCIR)	CIR(NCIR)						F10.4
1								I2
2								I2

To account for a transient effect, more cards are needed containing values of CCSI smaller than 1.0 and the corresponding times TCSI.

Type 8 data contain the "manual"* valving rate (VO) in pounds of lost lift per second on the first card. Even when no gas will be valved manually in the course of computation, this card must lead the valving data. The following cards contain two arrays of time (TV) in sec and corresponding values of automatic valving rate (VV) in pounds of lost lift per second. They must be in order of increasing TV. The number of cards must not exceed 100. Also, TV(1) must be equal to or smaller than XO and TV(NVI) must be equal to or greater than XT. The following example shows the meaning of these valving data and the way they are used by the program.

Suppose for a flight XO = 0 and XT = 36,000 sec, and that the valving data for this flight are:

VO = 1.0	
TV(1) = 0	VV(1) = 0.
TV(2) = 18000.	VV(2) = 0.3
TV(3) = 18100.	VV(3) = 0.
TV(4) = 36000.	VV(4) = 0.

These data mean that the "manual" valving rate is 1 and that automatic valving takes place only between times 18000 and 18100 at a constant rate of 0.3. Thus, the total amount of gas valved automatically is equivalent to a loss of 30 lbs of lift.

Type 9 data contain ballasting information. The first card contains the "manual"* ballasting rate (BO) in lbs/sec. Even when no manual ballasting will be done in the course of computation, this card must be first. The following cards contain two arrays of time (TB) in sec and corresponding values of automatic ballasting rate (BB) in lbs/sec. The rules for writing these data as well as their interpretation by the program are similar to those of valving data (type 8).

Type 10 data contain the air density (RHØO) in lbs/ft^3 at ground level on the first card. The following cards contain two arrays of altitude above sea level (ZZ) in ft and the corresponding values of air temperature (TZ) in $^\circ$R. They must be in order of increasing ZZ. The number of cards must not exceed 100. ZZ(1) must be equal to ground level. ZZ(NT) must be equal to or greater than the expected maximum altitude of the balloon. Atmospheric data corresponding to ZZ(NT) are used whenever the balloon goes above ZZ(NT) (see subroutine RHOT).

* See Section III-C1 for an explanation.

Type 11 data contain two arrays of altitude above sea level (ZIR) in ft and the corresponding values of the equilibrium radiation temperature of a black ball (RTIR) in ^{o}R. The rules for writing these data are the same as those for type 10 data. Whenever the balloon goes above ZIR(NIR), infrareddata corresponding to ZIR(NIR) are used (see subroutine YPRIME).

Type 12 data contain two arrays of observed time (ETIME) in sec and corresponding values of observed balloon altitude above sea level (EALT) in ft. These data are used for flight correlation. The only restriction on these data is that they must be in order of increasing ETIME and the number of cards is limited to 400.

Finally, type 13 data contain two arrays of time (TIR) in sec and the corresponding values of the so called, "infrared factor" (CIR). These data pertain to infrared radiation from the earth and the atmosphere. For a predictable atmosphere, infrared radiation is computed by the program from the input data (type 11) of equilibrium radiation temperature of a black ball. The infrared factor accounts for possible transient effects. The rules of writing these data are similar to those of "solar radiation factor" data (type 7).

For an example of a complete set of values of input data see Section III-E.

Multiple Flights. More than one flights can be computed in one run. This is accomplished by placing the decks of input cards for each flight in consecutive order. However, between flights the control card with the number 2 must be replaced by a blank card. If the input data of certain type(s) of a following flight are the same as those of the preceding flight, it is not required to include these data in the deck of the following flight. It should be emphasized that this holds for entire type(s) of data and not for part of the data in a given type.

C. A BRIEF DESCRIPTION OF THE PROGRAM

The program is composed of a MAIN routine and four subroutines (RNGKTA, YPRIME, RHOT and PLOT), which perform special tasks.

1. The MAIN Routine

The MAIN reads the input data from the input tape. When a run is made with more than one flights, the computer will read the input data and compute one flight at a time. The cards containing the input data as well as the control cards must be punched correctly and must be in the proper order, as described in Section III-B. Otherwise, the first time that the MAIN finds an error, it will terminate the entire run after writing a comment in the output tape, which will help one to locate the error in the data.

As a further check that the correct data are used, the MAIN will write in the output tape all the data used for the flight before proceeding to the computation of the flight. The input data printed in the output should be examined carefully to make sure that they are identical to the input data fed in the computer.

A flight will be terminated automatically when:

(a) The entire payload, including the gondola, has been dropped. This could happen by excessive "manual" ballasting (see below). A comment will be written in the output tape indicating that this has happened.

(b) The balloon hits the ground.

(c) The time exceeds the final time of the flight specified in the input data.

Then the program will proceed to the next flight.

A flight can be terminated instantly by turning SENSE SWITCH 6 on. The computer will pause. Turn this switch off and press START to proceed to the next flight.

The MAIN does a considerable amount of computation involved in the reduction of the input data to a suitable form and in the required interpolations of the input data. It, also, computes some of the variables in the desired output form and units, as shown in Section III-E, and stores them in the output tape. A unit of output has two lines of print. The second line, which contains the six heat transfer rates, is obtained only when SENSE SWITCH 2 is on.

The MAIN also stores internally, not in the output tape, the information needed by subroutine PLOT, which is called only once before proceeding to the next flight. The information stored consists of five arrays which contain the time (sec) and the corresponding computed altitude (ft) and temperatures (^{o}R) of the fabric and gas as well as the temperature of air (^{o}R). The dimension of these arrays is 400. For a given flight, after these arrays are filled, no more information is stored.

On-line printing of output is controlled by SENSE SWITCH 3. With this switch on, each time the computer stores output in the output tape it also prints on-line the title of the flight and the following information: Greenwich Mean Time, balloon altitude above sea level (ft), balloon velocity (ft/min), valving rate (in pounds of lost lift per second), percent of total gas valved, ballasting rate (lb/sec), percent of total ballast dropped and percent of total gas exhausted. Other information is also printed on-line, as it is required by the program, and, therefore, the on-line print should always be saved.

Automatic valving and ballasting takes place at rates specified in the input data. Additional valving and ballasting ("manual") can be effected during the course of the computation by SENSE SWITCHES 1 and 4, respectively. When SENSE SWITCH 1 is on, the computer will valve gas in addition to that valved automatically at a rate (VO) which is specified in the input data. The same is true with SENSE SWITCH 4 and ballasting.

The MAIN also performs exhausting of gas in a rather artificial way. The integration is carried out in steps of time intervals equal to H. At the end of each step, the instantaneous volume of the gas, Y(6), is compared with the inflated volume of the balloon, VBM, as specified by its manufacturer. If Y(6) is equal to or smaller than VBM, no gas is exhausted and the computer proceeds to the next step. If Y(6) is greater than VBM, the exhausting rate is set equal to (Y(6)-VMB)/H and the same step is repeated. If again Y(6) comes out greater than VBM, the exhausting rate is increased by (Y(6)-VBM)/H, and so on. Twenty such iterations per integrating step are allowed automatically. If at the end Y(6) is still greater than VBM, the computer will print on-line "ITERATION DOES NOT CONVERGE, etc" and pause. In this position, when START is pressed with SENSE SWITCH 5 on, the computer will enter again the iterative loop for more iterations and so on. On the other hand, when START is pressed with SENSE SWITCH 5 off, the computer will proceed to the following flight. Our experience with this program is that an adequate amount of gas is exhausted with only one iteration per integrating step.

Finally, the MAIN does an energy check, which will be described in Section III-D.

2. Subroutine RNGKTA

The actual integration of the six differential equations is carried out by this subroutine with Gill's fourth order Runge and Kutta method, which is a stable, self starting, accurate numerical integration technique. RNGKTA requires another subroutine, YPRIME, in which the differential equations are stated.

3. Subroutine YPRIME

YPRIME contains the six first order differential equations and evaluates the first derivatives of the dependent variables. It, also, evaluates the various heat transfer rates. Thus, YPRIME is called frequently by MAIN, as well as by RNGKTA, whenever information on the derivatives of the dependent variables or the heat transfer rates is needed.

For the rate of absorption of solar radiation, YPRIME computes the intensity of solar radiation and, when sunrise or sunset occurs at the balloon, it computes the Greenwich Mean Time and writes directly on the output tape the appropriate comment (for instance, GMT = 2.20.13 SUNSET AT BALLOON).

When the balloon goes above the infrared radiation field specified in the input data (type 11), YPRIME will print on-line an appropriate comment and the computer will pause. In this position, if START is pressed, the flight will be continued using the infrared data of the highest altitude for as long as the balloon remains above the specified infrared field. On the other hand, if START is pressed with SENSE SWITCH 6 on, the flight will be terminated.

In computing the convective heat transfer rates due to the atmosphere, YPRIME gets information about the atmosphere from subroutine RHOT.

4. Subroutine RHOT

This subroutine computes the atmospheric temperature and the product of atmospheric temperature and density for given altitude. The computation is done according to formulae developed in Section II-F.

When the balloon goes above the atmospheric temperature profile specified in the input data (type 10), RHOT will print on-line an appropriate comment and the computer will pause. In this position, if START is pressed, the flight will be continued using the atmospheric data of the highest altitude for as long as the balloon remains above the specified atmosphere. On the other hand, if START is pressed with SENSE SWITCH 6 on, the flight will be terminated.

Subroutine RHOT is called frequently by MAIN, as well as by YPRIME, to give information about the atmosphere.

5. Subroutine PLOT

After the completion of the computation of a flight and before proceeding to the next flight, this subroutine is called to store in the output tape information from which the IBM 1401 will produce a plot in graphical form. Subroutine PLOT is furnished to the user in binary card form as the FORTRAN source deck is not available for all the included subroutines.

The grid of the graph is marked with small crosses. The abscissa represents time elapsed from the beginning of the flight in hours, and the ordinate represents altitude above sea level in feet and temperature in ^{o}R. There are two compilations of this subroutine: in one, the range of the ordinate is from 0 to 147,500 feet for altitude and from 300 to 595^{o}R for temperature, and in another from 0 to 100,000 feet for altitude and from 300 to 550^{o}R for temperature.

On the produced graph paper, points for the computed and observed balloon altitude and the computed temperatures of the air, fabric and gas are marked with symbols described in the following key:

$$DEWA = p \dot{V}_B \quad (68)$$

$$DEVE = C_p T_B \dot{w}_B \quad (69)$$

where C_p is the specific heat at constant pressure of the gas.

Notice that Equation 63 relates the rates at which the various forms of energy of the gas-fabric system are exchanged. The interpretation of the various terms is as follows. ETOT is the internal energy. DKE and DPE are the rates of change of kinetic and potential energy, respectively. DDRAG is the rate at which energy is expended to overcome drag. DEWA is the rate of work done (on the atmosphere) when the balloon expands. DEVE is the rate of energy lost when gas is valved and/or exhausted. Finally, the sum of the q terms is the rate of net heat received by the system.

Let the right hand side of Equation 63 be denoted by ER. Integrating this equation from time 0 to t, one obtains:

$$ENET = ETOT - ETOT1 = \int_0^t ERdt \quad (70)$$

where ETOT1 is the value of ETOT at zero time. Let the integral of Equation 70 be approximated by the following first order formula:

$$EP = H \sum_{i=1}^{N} ER_i \quad (71)$$

where ER_i is the value of ER at the end of each time step H.

At the end of each integration by RNGKTA, the MAIN routine evaluates ETOT, and, therefore, ENET from the integrated variables. Similarly, it evaluates the increment in EP, which is $H(ER_i)$, due to each step and keeps track of the total EP. Clearly, ENET and EP, thus evaluated, are not equal for two reasons (if all integrations were exact, ENET and EP would be identical). Firstly, there is an error involved in the integrations done by RNGKTA. Secondly, there is another error involved in the above approximate first order integration of ER. To be sure the second error is larger than the first one. Nevertheless, the ratio:

$$ECHK = \frac{ENET-EP}{ENET} \quad (72)$$

which is evaluated by MAIN and stored in the output tape each time output is stored there, has some bearing on the accuracy of RNGKTA. If ECHK is a small number, this means, at least, that RNGKTA is very accurate.

A - Air Temperature

B - Balloon Gas Temperature

E - Observed Balloon Altitude

F - Balloon Fabric Temperature

L - Overlap of A and B

P - Overlap of A and F

Q - Overlap of B and F

Z - Computed Balloon Altitude

$ - Overlap of E and Z, or of A, B, E, F, Z and Grid.

Since the computer can mark a maximum of sixteen equally spaced points per half hour of abscissa, the uncertainty in time for each marked point can be as large as 1 minute and 53 seconds. Similarly, there is an uncertainty in the ordinate. For instance, with the 0 to 100,000 ft compilation the computer can mark a maximum of six points per 10,000 ft of ordinate and, therefore, the uncertainty for each marked altitude point can be as large as 1,667 ft.

D. ACCURACY OF INTEGRATION

With a program as massive as this program it is highly desireable to have a scheme by which the accuracy of integration can be estimated for given integrating time interval. With such a scheme the integrating time interval can be maximized for a given desired accuracy and, thus, the total computer time per flight can be minimized.

To this end, an energy check is most effective. If Equation 58 times y and Equations 59 and 60 are added and the resulting equation is rearranged, the following energy equation is obtained:

$$\frac{d}{dt}(ETOT) = - DKE + DPE - DDRAG - DEWA + DEVE + q_2 - q_3 + q_4 + q_5 + q_7 \quad (63)$$

where:

$$ETOT = C_v\, w_B\, T_B + C_F\, w_F\, T_F \quad (64)$$

$$DKE = (w_G + w_F + w_B + C_B\, \rho_A\, V_B)\, \frac{\dot{y}}{g}\, y \quad (65)$$

$$DPE = (\rho_A\, V_B - w_G - w_F - w_B)\, y \quad (66)$$

$$DDRAG = \frac{1.21}{2g}\, \rho_A\, C_D\, |y|\, y^2\, V_B^{2/3} \quad (67)$$

$$DEWA = p \dot{V}_B \tag{68}$$

$$DEVE = C_p T_B \dot{w}_B \tag{69}$$

where C_p is the specific heat at constant pressure of the gas.

Notice that Equation 63 relates the rates at which the various forms of energy of the gas-fabric system are exchanged. The interpretation of the various terms is as follows. ETOT is the internal energy. DKE and DPE are the rates of change of kinetic and potential energy, respectively. DDRAG is the rate at which energy is expended to overcome drag. DEWA is the rate of work done (on the atmosphere) when the balloon expands. DEVE is the rate of energy lost when gas is valved and/or exhausted. Finally, the sum of the q terms is the rate of net heat received by the system.

Let the right hand side of Equation 63 be denoted by ER. Integrating this equation from time 0 to t, one obtains:

$$ENET = ETOT - ETOT1 = \int_0^t ERdt \tag{70}$$

where ETOT1 is the value of ETOT at zero time. Let the integral of Equation 70 be approximated by the following first order formula:

$$EP = H \sum_{i=1}^{N} ER_i \tag{71}$$

where ER_i is the value of ER at the end of each time step H.

At the end of each integration by RNGKTA, the MAIN routine evaluates ETOT, and, therefore, ENET from the integrated variables. Similarly, it evaluates the increment in EP, which is $H(ER_i)$, due to each step and keeps track of the total EP. Clearly, ENET and EP, thus evaluated, are not equal for two reasons (if all integrations were exact, ENET and EP would be identical). Firstly, there is an error involved in the integrations done by RNGKTA. Secondly, there is another error involved in the above approximate first order integration of ER. To be sure the second error is larger than the first one. Nevertheless, the ratio:

$$ECHK = \frac{ENET-EP}{ENET} \tag{72}$$

which is evaluated by MAIN and stored in the output tape each time output is stored there, has some bearing on the accuracy of RNGKTA. If ECHK is a small number, this means, at least, that RNGKTA is very accurate.

Many flights have been computed with H equal to 10 and 20 sec. The corresponding values of ECHK are of the order of 0.003 and 0.007.

E. OUTPUTS

As stated in Section III-C, the first part of output is a printout of the input data used for the flight. They are printed in the form shown in Figure 2. Notice that the appropriate control card identification (IT) precedes each type of data. The meaning and units of these data are as described in Section III-B. This printout of data should be checked carefully to make sure that the correct input data have been used by the computer.

The printout of input data is followed by the computed output in the form shown in Figure 3. A unit output contains two lines. The contents of the first line are the values of the variables appearing at the head of each page of output. These variables are:

Greenwich Mean Time (GMT) in hours, minutes and seconds.
Time from beginning of flight (TIME) in sec.
Altitude of balloon from sea level (ALTITUDE) in ft.
Velocity of balloon (VEL) in ft/min.
Atmospheric pressure (PRESS) in mbar.
Atmospheric temperature (TA) in ^{o}R.
Balloon fabric temperature (TF) in ^{o}R.
Balloon gas temperature (TB) in ^{o}R.
Balloon volume (VOLUME) in ft^3.
Weight of payload and balloon fabric (LOAD) in lb.
Weight of balloon gas (GAS WT) in lb.
Free lift (FR LIFT) in lb.
Cumulative ballast dropped (PERB) in percent of initial load (payload plus balloon fabric).
Cumulative gas valved (PERV) in percent of initial gas weight.
Cumulative gas exhausted (PERE) in percent of initial gas weight.
Number of iterations (IRS) required to exhaust an adequate amount of gas during the most recent integrating step.
Energy check (ECHK) as explained in Section III-D.

The second line of unit output contains the six heating rates in ftlb/sec. This line appears in the output only when SENSE SWITCH 2 is on. The heating rates are:

Rate of absorption of infrared radiation by balloon fabric (IRAB).
Rate of emission of infrared radiation by balloon fabric (IREM).
Forced convection heating rate of balloon fabric by air (FCAF).
Natural convection heating rate of balloon fabric by air (NCAF).
Natural convection heating rate of balloon fabric by balloon gas (NCGF).
Rate of absorption of solar radiation by balloon fabric (SLAB).

When sunset or sunrise occurs at the balloon, the Greenwich Mean Time of the occurrence together with the appropriate comment are printed in the output (see Figure 3).

```
BH 3     STRATOSCOPE FLIGHT  S4-2  JULY 23,1965

INPUT DATA

IT= 3
    250.00000        1.50000        1.50000        1.00000        0.69000        0.69000        0.15000

IT= 4
  11120.00000     3320.00000  5505000.00000     1460.00000        0.30000        0.50000

IT= 5
    542.00000      542.00000     1000.00000        0.            19.90000       31.75000      102.50000

IT= 6
     20.00000      900.00000        0.         40080.00000     4920.00000      198.39000      300.00000

IT= 7
      0.             1.00000
  50000.00000        1.00000

IT= 8
      1.00000
      0.            -0.
  34980.00000        0.30000
  35480.00000        0.33000
  35980.00000        0.35000
  36480.00000        0.38000
  36980.00000        0.43000
  37480.00000        0.48000
  37980.00000        0.58000
  38480.00000        0.70000
  38980.00000        0.88000
  39480.00000        1.20000
  39980.00000        1.50000
  41000.00000        1.50000

IT= 9
      1.00000
      0.             0.
     60.00000        1.41670
    240.00000        0.
   2040.00000        1.41670
   2340.00000        0.
  39780.00000        3.00000
  40080.00000       -0.

IT=10
      0.07000
   1000.00000      542.00000
   3400.00000      535.00000
   5025.00000      529.00000
   9700.00000      513.00000
  19400.00000      483.00000
  25000.00000      459.00000
  31800.00000      430.00000
  35950.00000      413.00000
  40800.00000      393.00000
  46500.00000      375.00000
  51000.00000      360.00000
  54550.00000      368.00000
  61600.00000      380.00000
```

FIGURE 2 INPUT DATA PRINTED IN OUTPUT FOR STRATOSCOPE FLIGHT S4-2

68550.00000	384.00000
79100.00000	400.00000
82700.00000	407.00000
100000.00000	423.00000

IT=11

1000.00000	532.00000
51000.00000	392.00000
100000.00000	392.00000

IT=12

0.	1000.00000
780.00000	16000.00000
1380.00000	25000.00000
1980.00000	37000.00000
2580.00000	52000.00000
3180.00000	59000.00000
3780.00000	66500.00000
4380.00000	73000.00000
5100.00000	80000.00000
6780.00000	80000.00000
8580.00000	80000.00000
10300.00000	80000.00000
12100.00000	80000.00000
13900.00000	79500.00000
15700.00000	78500.00000
17500.00000	78000.00000
19300.00000	78000.00000
21180.00000	78000.00000
22980.00000	78000.00000
24780.00000	78000.00000
26580.00000	77500.00000
28380.00000	77500.00000
30180.00000	77500.00000
31980.00000	77500.00000
33780.00000	77000.00000
34980.00000	77000.00000
35580.00000	75000.00000
36180.00000	73500.00000
36780.00000	71000.00000
37380.00000	68000.00000
37980.00000	65000.00000
38580.00000	60000.00000
39180.00000	56000.00000
39780.00000	47000.00000
40080.00000	41500.00000

IT=13

0.	1.00000
41000.00000	1.00000

FIGURE 2 cont'd. INPUT DATA PRINTED IN OUTPUT FOR STRATOSCOPE FLIGHT S4-2

```
SH 3    STRATOSCOPE FLIGHT  S4-2  JULY 23,1965

  GMT       TIME  ALTITUDE     VEL     PRESS    TA      TF      TB      VOLUME     LOAD    GAS WT   FR LIFT  PERB  PERV  PERE  IRS   SCHK

1.22. 0.      0.    1000.       0.    971.5  542.0  542.0  542.0   263646.  14440.  2555.2  1460.00   0.    0.    0.    0    0.
  IRAB= 4.0607E 05   IREM= 4.3747E 05   FCAF=-0.             NCAF=-2.0752E-04  NCGF= 0.             SLAB= 1.4512E 04

1.37. 0.    900.   20267.     994.    479.4  479.3  475.8  444.0   437630.  14185.  2555.2   355.04   1.8  -0.    0.    0    0.005
  IRAB= 4.1457E 05   IREM= 3.6438E 05   FCAF= 1.8161E 04   NCAF= 6.8358E 03   NCGF=-1.4441E 05   SLAB= 2.2862E 04

1.52. 0.   1800.   37628.    1241.    229.7  406.1  411.7  378.3   778084.  14185.  2555.2   451.33   1.8  -0.    0.    0    0.031
  IRAB= 4.0386E 05   IREM= 2.9958E 05   FCAF=-3.0800E 04   NCAF=-1.3244E 04   NCGF=-1.5628E 05   SLAB= 3.0088E 04

2. 7. 0.   2700.   54029.     439.    102.1  366.8  358.1  325.4  1506126.  13760.  2555.2    57.15   4.7  -0.    0.    0    0.036
  IRAB= 3.8252E 05   IREM= 2.6648E 05   FCAF= 2.3008E 04   NCAF= 2.3177E 04   NCGF=-1.5436E 05   SLAB= 3.5645E 03

                    GMT= 2.12.10.       SUNSET AT BALLOON

2.22. 0.   3600.   60972.     462.     72.1  378.9  368.7  335.9  2201481.  13760.  2555.2    41.88   4.7  -0.    0.    0    0.006
  IRAB= 4.9267E 05   IREM= 3.8541E 05   FCAF= 2.7615E 04   NCAF= 2.8856E 04   NCGF=-1.5454E 05   SLAB= 0.

2.37. 0.   4500.   68880.     529.     48.9  384.5  374.4  340.7  3289567.  13760.  2555.2    39.07   4.7  -0.    0.    0    0.006
  IRAB= 6.4394E 05   IREM= 5.3585E 05   FCAF= 2.8813E 04   NCAF= 2.8205E 04   NCGF=-1.5973E 05   SLAB= 0.

2.52. 0.   5400.   75944.     454.     34.9  395.2  382.0  350.0  4742643.  13760.  2555.2    30.72   4.7  -0.    0.    0    0.007
  IRAB= 8.2180E 05   IREM= 7.4108E 05   FCAF= 3.3907E 04   NCAF= 4.0306E 04   NCGF=-1.4854E 05   SLAB= 0.

3. 7. 0.   6300.   79099.    -171.     30.1  400.0  389.2  372.2  5504649.  13760.  2404.9    -4.49   4.7  -0.    5.9   0    0.009
  IRAB= 9.0762E 05   IREM= 8.8196E 05   FCAF= 1.5199E 04   NCAF= 3.0546E 04   NCGF=-6.1278E 04   SLAB= 0.

3.22. 0.   7200.   79066.      44.     30.1  399.9  392.6  386.2  5486201.  13760.  2313.0    60.21   4.7  -0.    9.5   0    0.007
  IRAB= 9.0559E 05   IREM= 9.1147E 05   FCAF= 4.5848E 03   NCAF= 1.8145E 04   NCGF=-1.6076E 04   SLAB= 0.

3.37. 0.   8100.   79354.     -35.     29.7  400.5  393.6  389.0  5504903.  13760.  2273.8   -84.34   4.7  -0.   11.0   1    0.008
  IRAB= 9.0765E 05   IREM= 9.2258E 05   FCAF= 3.7147E 03   NCAF= 1.6639E 04   NCGF=-1.0398E 04   SLAB= 0.

3.52. 0.   9000.   79172.    -176.     30.0  400.1  394.0  392.3  5482115.  13760.  2264.5     8.63   4.7  -0.   11.4   0    0.008
  IRAB= 9.0514E 05   IREM= 9.2337E 05   FCAF= 8.8191E 03   NCAF= 1.4442E 04   NCGF=-2.7169E 03   SLAB= 0.

4. 7. 0.   9900.   79127.      20.     30.0  400.1  393.9  393.5  5476783.  13760.  2259.9    35.15   4.7  -0.   11.6   0    0.007
  IRAB= 9.0455E 05   IREM= 9.2342E 05   FCAF= 2.3524E 03   NCAF= 1.4230E 04   NCGF=-4.4136E 02   SLAB= 0.

4.22. 0.  10800.   79237.      95.     29.9  400.3  394.0  393.2  5494986.  13760.  2257.5    -0.22   4.7  -0.   11.7   0    0.007
  IRAB= 9.0656E 05   IREM= 9.2520E 05   FCAF= 6.1395E 03   NCAF= 1.4695E 04   NCGF=-9.7435E 02   SLAB= 0.

4.37. 0.  11700.   79301.      10.     29.8  400.4  394.0  393.1  5504970.  13760.  2255.7   -21.97   4.7  -0.   11.7   1    0.008
  IRAB= 9.0765E 05   IREM= 9.2591E 05   FCAF= 1.6874E 03   NCAF= 1.5206E 04   NCGF=-1.1455E 03   SLAB= 0.

4.52. 0.  12600.   79266.     -56.     29.8  400.3  393.9  393.5  5500535.  13760.  2255.0    -5.69   4.7  -0.   11.8   0    0.008
  IRAB= 9.0717E 05   IREM= 9.2517E 05   FCAF= 4.5831E 03   NCAF= 1.5085E 04   NCGF=-4.2487E 02   SLAB= 0.

5. 7. 0.  13500.   79227.     -23.     29.9  400.2  393.9  393.9  5495051.  13760.  2254.5    10.69   4.7  -0.   11.8   0    0.008
  IRAB= 9.0656E 05   IREM= 9.2470E 05   FCAF= 2.6264E 03   NCAF= 1.4813E 04   NCGF=-2.0997E 01   SLAB= 0.

5.22. 0.  14400.   79242.      29.     29.9  400.3  393.9  393.9  5497670.  13760.  2254.3     6.39   4.7  -0.   11.8   0    0.007
  IRAB= 9.0685E 05   IREM= 9.2433E 05   FCAF= 3.0754E 03   NCAF= 1.5120E 04   NCGF=-3.1511E 00   SLAB= 0.
```

FIGURE 3   SAMPLE OF PRINTED OUTPUT

Following the printed output, the output contains a plot as shown in Figure 4. The contents of this plot have already been explained in Section III-C5, where subroutine PLOT was described.

When a flight is aborted because of errors in the data, the output contains comments which will help one to locate the errors.

F. OTHER OPERATING INSTRUCTIONS

In accordance with the requirements of the FORTRAN MONITOR SYSTEM (FMS), a few starred cards must be used in the order indicated below when the program is run with FMS.

```
*IDENTIFICATION CARD
*XEQ
 MAIN ROUTINE DECK
 SUBROUTINE RNGKTA DECK
 SUBROUTINE YPRIME DECK
 SUBROUTINE RHOT DECK
 SUBROUTINE PLOT DECK
*DATA
 INPUT DATA DECK
```

The asterisk of the starred cards must be in column 1, and their contents must start from column 7.

The tapes that are used in FMS are A2 (logical 5) for input and A3 (logical 6) for output.

The functions of the six sense switches of FMS, when they are on, have been described in Section III-C. In summary, the functions of these switches are:

SS1: "Manual" valving of gas, in addition to automatic valving, at rate VO.

SS2: Heating rates are printed in output.

SS3: On-line print of output.

SS4: "Manual" ballasting, in addition to automatic ballasting, at rate BO.

SS5: Iterations for exhausting gas will be continued.

SS6: Present flight is discontinued.

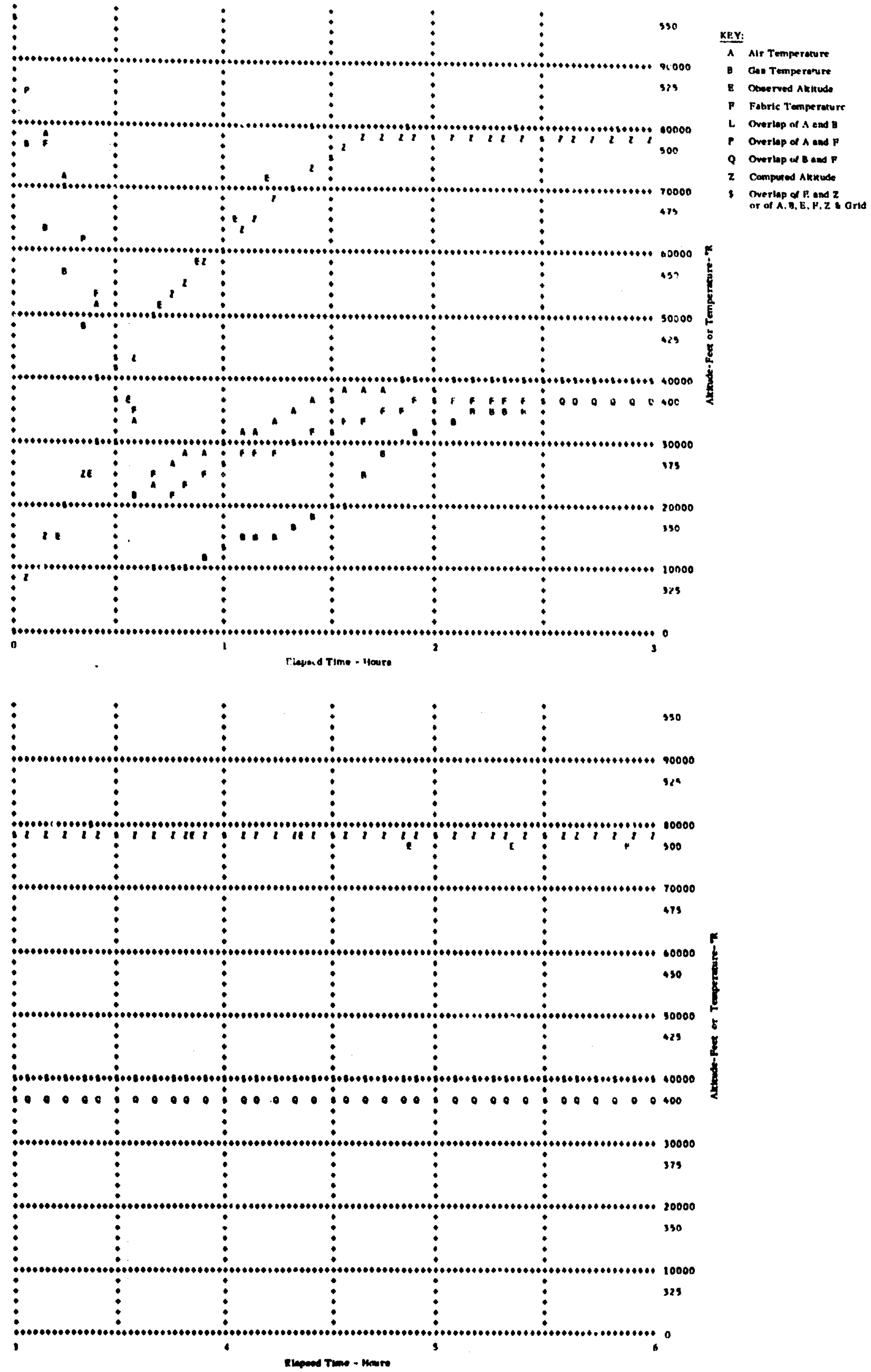


FIGURE 4 PLOTTED OUTPUT FOR STRATOSCOPE FLIGHT S4-2

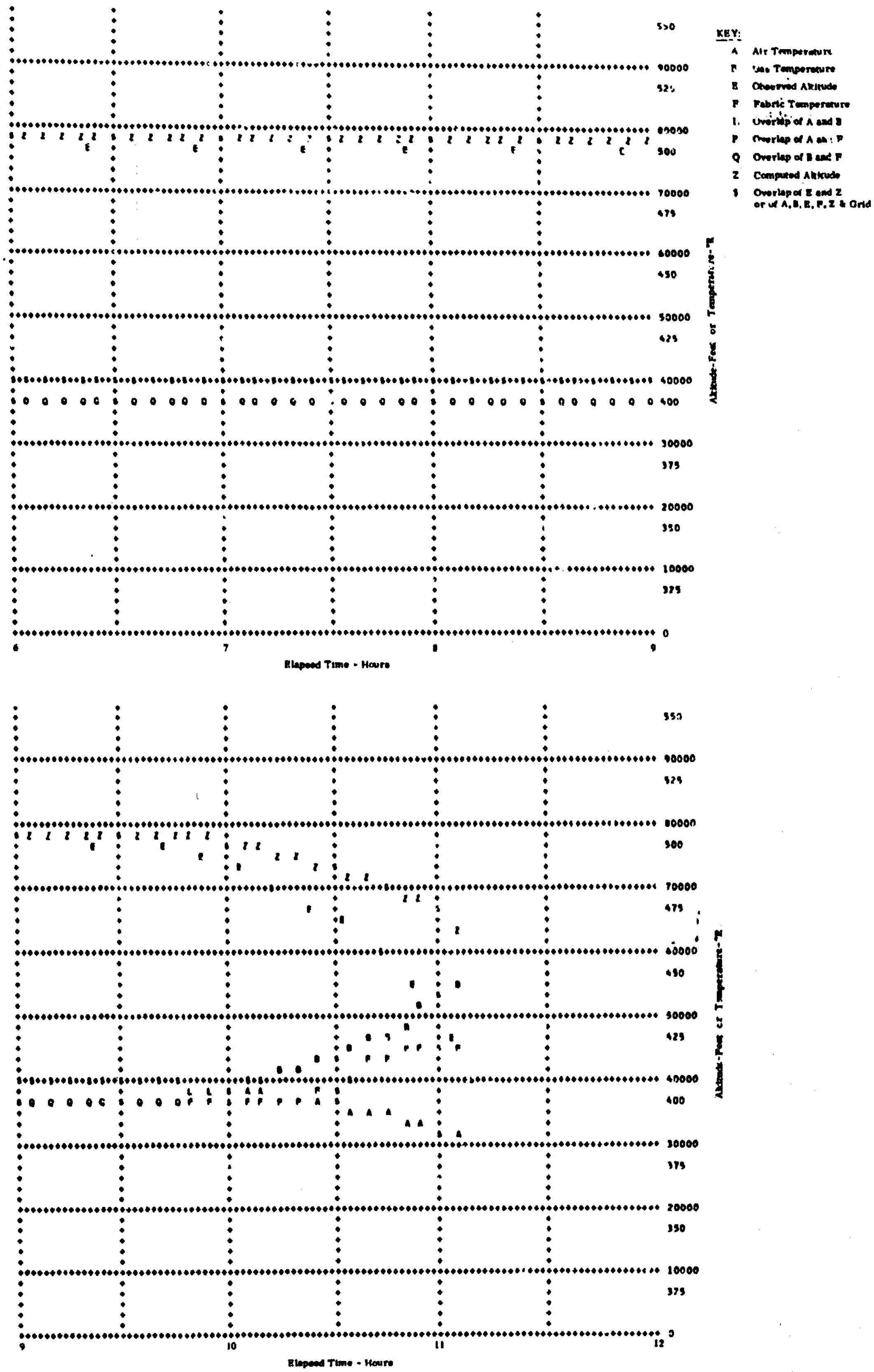


FIGURE 4 cont'd. PLOTTED OUTPUT FOR STRATOSCOPE FLIGHT S4-2

IV. CORRELATION OF COMPUTED AND ACTUAL FLIGHTS

A. A BRIEF DISCUSSION OF THE APPROXIMATIONS IN THE MODEL AND OF THE UNCERTAINTIES IN THE INPUT DATA

To appreciate the correlation of a computed with an actual flight, one should have a good understanding of the mathematical model and of the computer program. Then one would be in a position to know the degree of correlation he should expect. In developing the mathematical model we have made many simplifications, some of them quite gross, of the actual balloon. In addition, the computer program requires a certain amount of specified quantities (input data), and the question arises as to how accurately these data should and could be specified. In this section, we will review briefly the approximations contained in the mathematical model and the factors which, according to the model, play a very important role in balloon performance. Then, in the following section, we will present the correlation of some recent balloon flights with those computed by our program.

In calculating the dynamic parameters of the balloon (drag and apparent additional mass) and all the heat transfer rates (convective and radiant), we have considered the balloon to be a sphere with the same volume as the actual balloon. To compensate for this rough approximation, we have introduced correction factors whose values have been determined by correlation of actual balloon flights with an acceptable degree of accuracy.

Thus, the drag coefficient of the balloon, based on the cross-sectional area of the equivalent sphere, has been taken equal to 0.3, 50% smaller than the drag coefficient of a sphere, since it is expected that a balloon must have less drag than the equivalent sphere. The coefficient of apparent additional mass, based on the mass of the displaced air, has been taken equal to 0.5, which is equal to that of a sphere in potential flow (viscous effects have been neglected). Evidently, this is a reasonable approximation, since this coefficient depends on volume more than it depends on cross-section.

Besides the spherical model approximation, more approximations have been introduced in the calculation of the convective heat transfer rates. For forced convection, it has been assumed that a known formula for spheres in laminar flow is applicable to balloons, even through the Reynolds number of balloons is usually higher than the Reynolds numbers covered by this formula. In fact, due to high Reynolds numbers, the flow of air around a balloon is quite turbulent, which means that the heat transfer is greater than that given by this formula. Thus, we have taken the correction constant equal to 1.5. For free convection, it has been assumed that known formulae which hold for vertical plates and cylinders are applicable to balloons with the diameter of the spherical balloon equal to the height of the plate. The correction constants have been taken equal to 1.5 and 1.0 for the air-side and gas-side, respectively, since it is expected that the air-side must have greater heat transfer than the gas-side.

For the radiant heat transfers, we believe that the spherical model is quite accurate and, therefore, we have not introduced any correction constants explicitly. However, corrections can be introduced through the effective absorptivities and emissivities.

Another basic approximation introduced in the theory is that both the gas and the fabric are characterized by an average temperature. The main support for the validity of this approximation stems from the fact that balloons experience very violent rotations, which tend to mix the gas as well as to equalize the temperature of the fabric and the amount of solar radiation incident on the fabric as a whole. However, any heat transfer associated with the rotations of balloons has not been considered in the mathematical model. Heat transfer as well as other effects associated with the horizontal drift of balloons have also been neglected.

As pointed out earlier, the exhausting process of the model is quite artificial. This may give, sometimes, a bad correlation as the balloon arrives at its ceiling.

There are quite a few parameters which play an important role in balloon performance and which must be specified in the computer program as input data. One of the most important ones is the initial free lift. It has a pronounced effect on the rise of a balloon to its ceiling. The initial free lift is computed by subtracting the weight of the payload from the gross lift (minus weight of fabric) which is measured on ground with a scale. Usually, the initial free lift is not more than about 10% of the gross lift. If there is a 1% error in both the measurement of the payload and the measurement of the gross lift, the uncertainty in free lift can be as large as 20%.

The initial values of temperature of the balloon gas and fabric must be specified as input data. For a balloon which has just been filled with gas and is waiting to be released, specifying these temperatures is not an easy task. In our practice of the computer program we have been taking both of these temperatures approximately equal to the air temperature at ground level. These temperatures have an important effect on balloon performance only during the initial part of the ascent phase.

The effective absorptivity of the balloon for solar radiation and its effective emissivity and absorptivity in infrared are three more crucial factors which are sometimes difficult to specify very accurately. This is so because these parameters are effective for multiple passes of radiation, the balloon gas being considered transparent. The one-pass absorptivity or emissivity of a fabric can be measured very accurately. But, in order to calculate the effective multiple-pass absorptivity or emissivity, the one-pass reflectivity of the fabric must also be known (see Section II.H.1). Accurate measurement of reflectivity is not an easy task.

The black ball equilibrium radiation temperature must also be specified accurately, since a 2.5% uncertainty in this temperature (say, 10°R in a nominal temperature of 400°R) would give an uncertainty of 10% in radiation intensity. Infrared radiation plays an important .e in balloon performance, especially in the absence of solar radiati . as is the case quite often for a major part of the flight of large balloons. In order to specify this temperature accurately, a black ball flight should be made before launching a balloon. If such a flight is not possible and there is no other way of specifying this radiation field, as for instance from black ball measurements made at the launch site or another similar site under similar seasonal conditions, the rule prescribed in Section II.H.2 should be followed in specifying this field. In any case, a change in the radiation temperature of the above order can easily take place overnight.

Air temperature data play a very important role, but usually they can be specified accurately. The same is true for valving and ballasting data.

Finally, it should be pointed out that observed balloon altitude is computed usually from pressure measurements using the hydrostatic law and a standard atmosphere for atmospheric temperature. It can be shown that possible deviations of the atmospheric temperature from that of a standard atmosphere can give changes in altitude of the order of a couple of thousand feet for altitudes above 25,000 ft. or so.

B. FLIGHT CORRELATION

Three recent flights are correlated and discussed in this section. Other flights have been treated in our previous report (Ref. 2). These three flights include a variety of effects: sunset, sunrise, ascent, overnight performance, descent, valving, ballasting, different fabric materials, etc. Fairly accurate and reliable input data can be prescribed for all of them. Keeping in mind the discussion in the preceding section, one can say that the correlation of computed and observed balloon trajectories is reasonable and in fact very good.

The correlation of these and other flights was obtained with the following values of the five correction constants:

CB = 0.5
CD = 0.3
C4 = 1.5
C5 = 1.5
C6 = 1.0

These values are compatible with theoretical considerations as it was pointed out in the preceding se tion. We recommend that these values be used in predicting balloon performance.

1. Stargazer Manned Flight

This balloon was launched from Holloman Air Force Base, New Mexico on December 13, 1962. It was launched at 1100 MST, reached its ceiling of 81,000 ft. in about 90 minutes and dropped to 71,000 ft. after sunset which occurred about 6 hours and 20 minutes after launch.

The input data used for this flight are shown in Figure 5. The fabric of the balloon was GT-12 (mylar and scrim), which has a specific heat of 250 ft lb /lb ^{o}R. Notice the value of 0.69 for ABIR and EMIR and the value of 0.15 for ABUV. These values were estimated by our computer program (see Section II.H.1) and then were corrected somewhat for best flight correlation. Notice, also that the initial gas and fabric temperatures (519^{o}R) are 9^{o}R above the air temperature at ground level. The initial free lift specified in the flight data was 6% of the gross load. To obtain a reasonable correlation in the ascent phase, the initial free lift has been increased to 9.6% of gross load or 650 lb. The infrared radiation field (IT = 11) is specified according to the approximate rule given in Section II.H.2. Ballasting and valving are identical to those of the actual flight. The air temperature data were given by the Holloman base.

The correlation of computed and actual flights is shown in Figure 6. Notice that the agreement is excellent not only in the ascent phase but also during and after sunset. In the computed flight, 14.5% of the initial gas was exhausted by the time the balloon was stabilized at its ceiling. An additional 0.7% was exhausted when, in anticipation of sunset, ballasting took place a little prematurely. Thus, a total of 15.2% of gas was exhausted as compared to 0.9% valved as the balloon was reaching its ceiling. The amount of ballast dropped during sunset was 7.2% of gross load.

2. Thermistor Flight

This flight was sponsored by ONR and was launched by NCAR from Page, Arizona, on October 18, 1964. Arthur D. Little, Inc., requested this flight in order to measure the temperature of the balloon gas as well as the temperature of air at various distances from the balloon. This was accomplished by various thermistors which were placed by Arthur D. Little, Inc., in and near the balloon. The principal objective of this flight was to acquire knowledge about the temperature of the balloon gas and to compare this temperature with gas temperatures computed by our program. Such a comparison is the ultimate check of the validity and accuracy of our model. A description of the flight and the instrumentation will be published in the near future.

A print-out of the input data is shown in Figure 7. The fabric of the balloon was 1.5 mil polyethelene film. Its specific heat is equal to 428 ft lb /lb ^{o}R. The radiative parameters of this fabric (0.17 and 0.10) have been estimated with our computer program (see Section II.H.1). The initial temperature of the balloon gas (519^{o}R) and of the balloon

```
8H 1   STARGAZER MANNED FLIGHT  ,  12-13-63,    1800 GMT

INPUT DATA

IT= 3
    250.00000        1.50000        1.50000        1.00000        0.69000        0.69000        0.15000

IT= 4
   4800.00000     1966.00000  3120000.00000      650.00000        0.30000        0.50000

IT= 5
    519.00000      519.00000     4290.00000        0.            -23.19000       32.91000      105.94000

IT= 6
     10.00000      900.00000        0.          43200.00000    64800.00000       91.36000      300.00000

IT= 7
     -0.             1.00000
  50000.00000        1.00000

IT= 8
      1.00000
      0.             0.
   4500.00000        0.38300
   4607.00000        0.
   6000.00000        0.20800
   6111.00000       -0.
  44700.00000        0.22000
  44916.00000       -0.
  45600.00000        0.22000
  45816.00000       -0.
  48300.00000        0.27500
  48517.00000       -0.
  50400.00000        0.33000
  50490.00000       -0.
  51300.00000        0.30000
  51420.00000       -0.
  52800.00000        0.38000
  52854.00000       -0.
  53640.00000        0.38000
  53698.00000       -0.
  57600.00000        0.55000
  57660.00000       -0.
  59280.00000        0.72000
  59336.00000       -0.
  59820.00000        0.77000
  59875.00000       -0.
  80000.00000        0.

IT= 9
      1.00000
      0.             0.
  19260.00000        1.00000
  19335.00000        0.
  20340.00000        1.00000
  20380.00000        0.
  21600.00000        1.00000
  21675.00000        0.
  22560.00000        1.00000
  22635.00000        0.
  22740.00000        1.00000
  22820.00000        0.


  23460.00000        1.00000
  23540.00000        0.
  24120.00000        1.00000
  24160.00000        0.
  24600.00000        1.00000
  24640.00000        0.
  61500.00000        1.00000
  61580.00000        0.
```

FIGURE 5   INPUT DATA FOR STARGAZER MANNED FLIGHT

61680.00000	1.00000
61955.00000	0.
61800.00000	1.00000
61943.00000	0.
61980.00000	1.00000
61955.00000	0.
62280.00000	1.00000
62320.00000	0.
62340.00000	1.00000
62495.00000	0.
63180.00000	1.00000
64220.00000	0.
65100.00000	1.00000
65105.00000	0.
65760.00000	0.50000
65765.00000	0.
100000.00000	0.

IT=10

0.06694	
4290.00000	509.85000
6571.39996	506.15000
39498.00000	389.07000
52696.00000	370.35000
56000.00000	365.67000
59304.00000	365.67000
72529.00000	383.67000
105660.00000	419.67000

IT=11

4290.00000	500.00000
39498.00000	392.00000
100000.00000	392.00000

IT=12

200.00000	5000.00000
1400.00000	23500.00000
2600.00000	40000.00000
3800.00000	57500.00000
5000.00000	73500.00000
6200.00000	80500.00000
7400.00000	81000.00000
8600.00000	81000.00000
9800.00000	81000.00000
11000.00000	81000.00000
12200.00000	81000.00000
13400.00000	81000.00000
14600.00000	81000.00000
15800.00000	81000.00000
17000.00000	81000.00000
18200.00000	81000.00000
19400.00000	80500.00000
20600.00000	80400.00000
21800.00000	80000.00000
23000.00000	79500.00000
24200.00000	78000.00000
25400.00000	76500.00000
26600.00000	74500.00000
27800.00000	73000.00000
29000.00000	72000.00000
30200.00000	71700.00000
31400.00000	71300.00000
32600.00000	71000.00000

IT=13

-0.	1.00000
50000.00000	1.00000

FIGURE 5 cont'd. INPUT DATA FOR STARGAZER MANNED FLIGHT

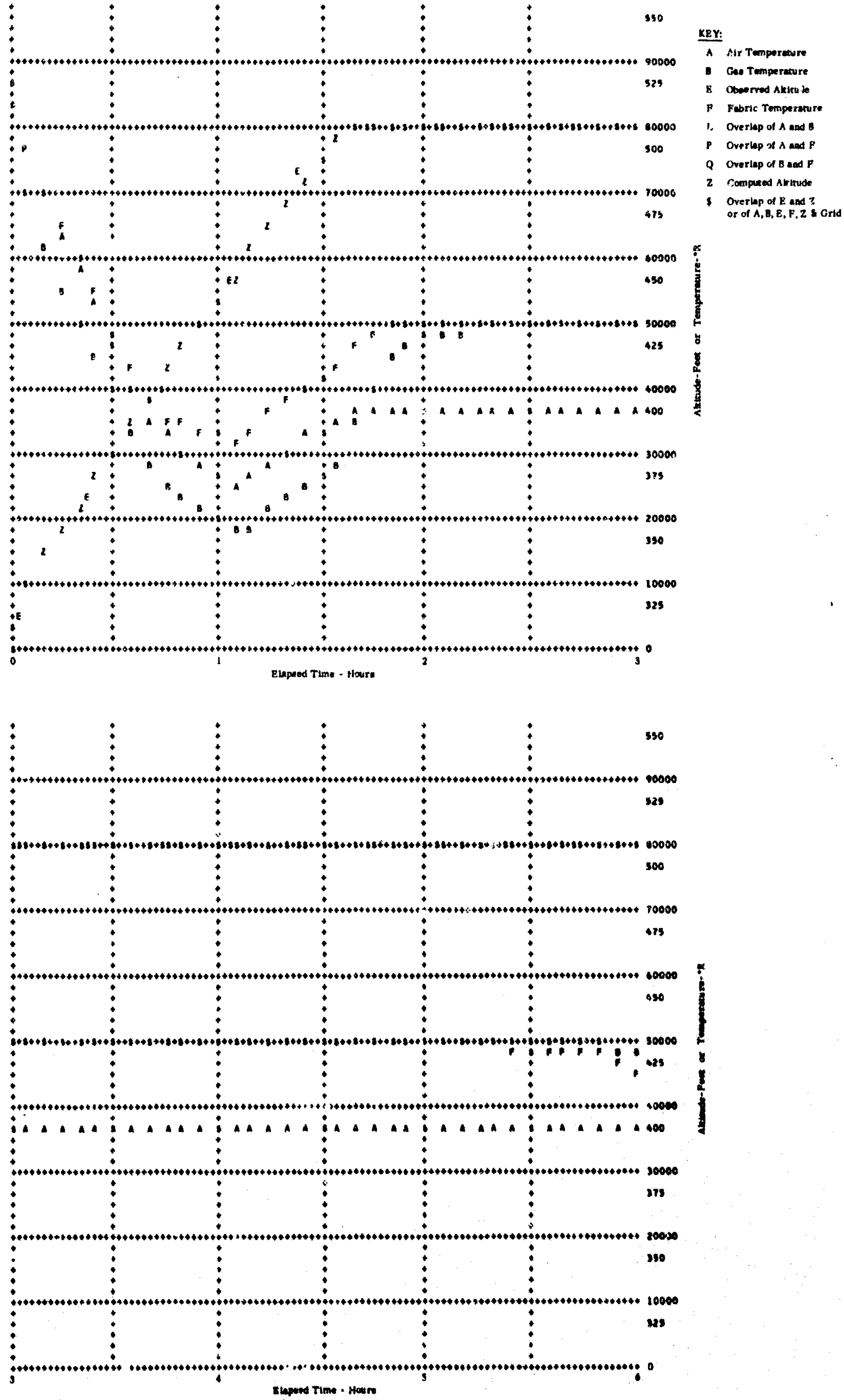
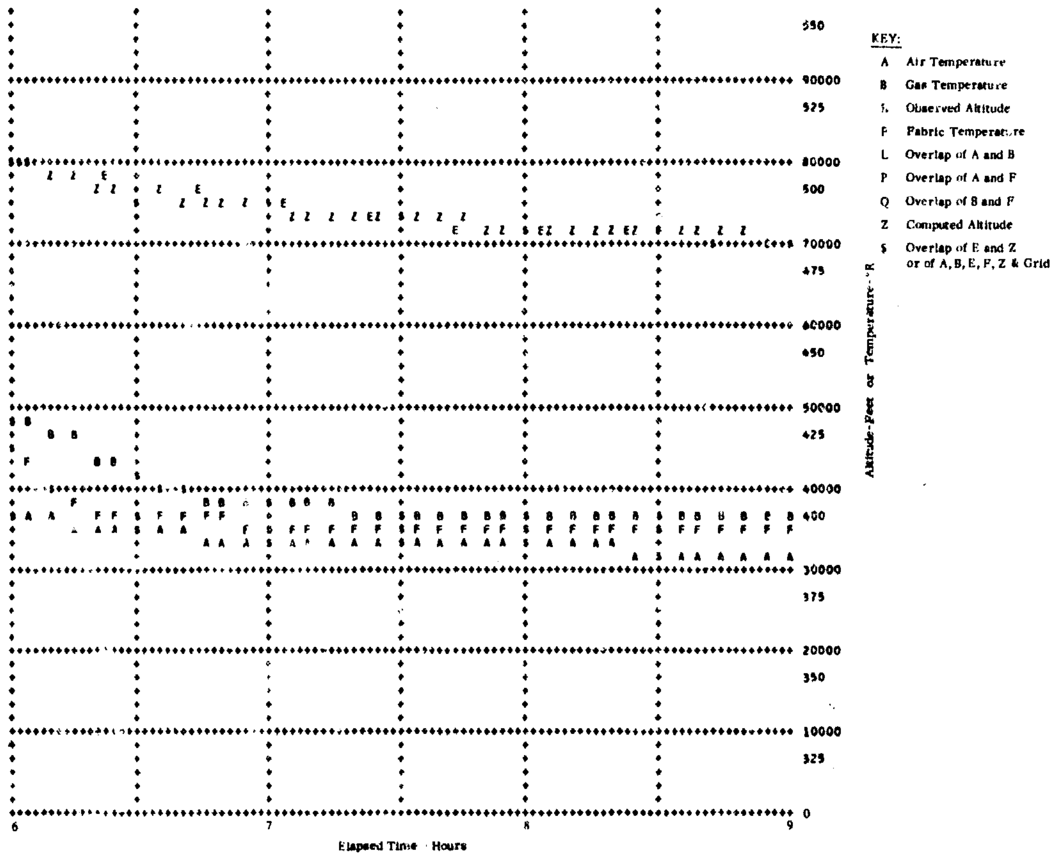


FIGURE 6 PLOTTED OUTPUT FOR STARGAZER MANNED FLIGHT



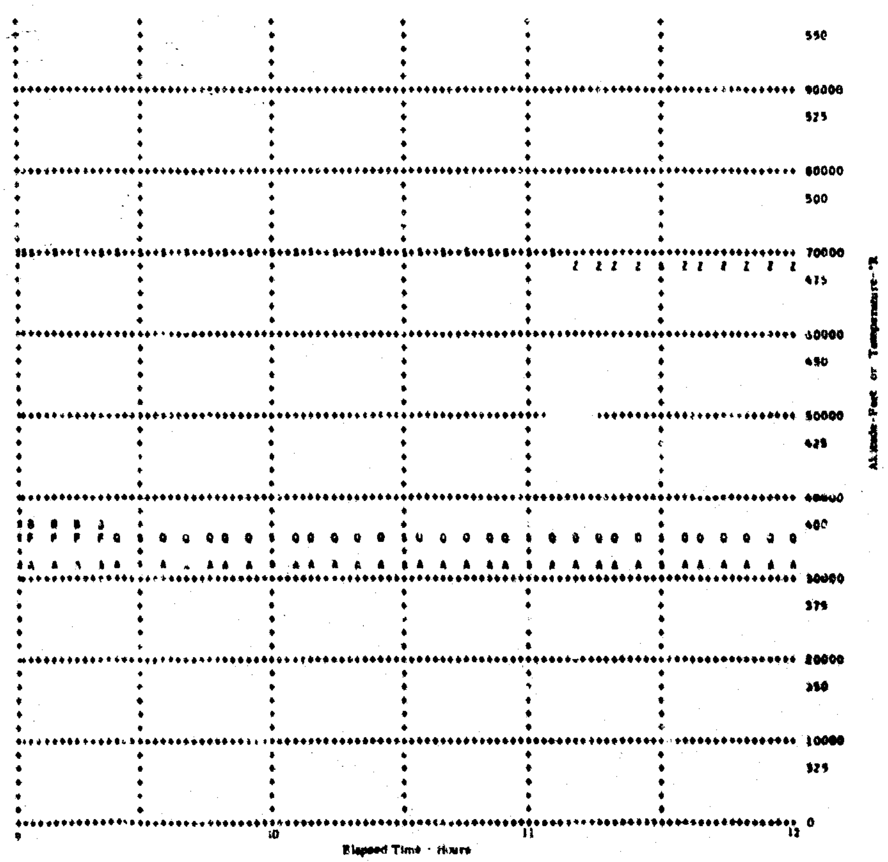


FIGURE 6 cont'd. PLOTTED OUTPUT FOR STARGAZER MANNED FLIGHT

```
8H 1   THERMISTOR FLIGHT   OCTOBER 18,1964

INPUT DATA

IT= 3
     428.00000        1.50000        1.50000        1.00000        0.17000        0.17000        0.10000
IT= 4
     325.00000      189.00000   250000.00000       10.00000        0.30000        0.50000
IT= 5
     519.00000      525.00000     4300.00000       0.            -9.80000       37.00000      111.45000
IT= 6
      10.00000      150.00000       0.            6600.00000    57840.00000       73.72000      300.00000
IT= 7
       0.             1.00000
   20000.00000        1.00000

IT= 8
       1.00000
       0.             0.
   20000.00000        0.

IT= 9
       1.00000
       0.             0.
     900.00000        0.60000
     910.00000       -0.
    4020.00000        0.60000
    4030.00000       -0.
    4440.00000        0.80000
    4450.00000       -0.
   20000.00000       -0.

IT=10
       0.06520
    4300.00000      525.00000
   10680.00000      510.00000
   18935.00000      472.98000
   21107.00000      466.81000
   22193.00000      459.92000
   24553.00000      454.04000
   25774.00000      447.52000
   26594.00000      445.07000
   27568.00000      439.42000
   28436.00000      435.52000
   29260.00000      432.77000
   30181.00000      429.31000
   31131.00000      424.52000
   31882.00000      420.93000
   32651.00000      415.64000
   33519.00000      411.77000
   34331.00000      407.81000
   35166.00000      404.45000
   36028.00000      400.61000
   36916.00000      396.65000
   37742.00000      394.05000
   38691.00000      389.86000
   39574.00000      386.19000


   40386.00000      383.44000
   41226.00000      382.13000
   41880.00000      384.32000
   42676.00000      387.65000
   43506.00000      385.07000
   44362.00000      380.94000
   45117.00000      377.17000
   46027.00000      372.88000
   47107.00000      369.29000
   48098.00000      368.41000
   48540.00000      373.03000
   49309.00000      377.17000
```

FIGURE 7   INPUT DATA FOR THERMISTOR FLIGHT

49948.00000	376.04000
50774.00000	373.77000
51633.00000	374.51000
52167.00000	372.90000
53081.00000	372.37000
53848.00000	369.61000
54943.00000	368.71000
55679.00000	375.97000
56335.00000	372.12000
57222.00000	376.45000
58199.00000	372.41000
59194.00000	371.44000
59977.00000	375.13000
60657.00000	374.32000
61360.00000	374.93000
62389.00000	375.10000
63008.00000	382.57000
63815.00000	379.86000
64651.00000	379.98000
65168.00000	377.78000
65877.00000	377.60000
66614.00000	379.85000
67381.00000	380.41000
68382.00000	379.27000
69221.00000	380.78000
69923.00000	385.80000
70892.00000	387.55000
71761.00000	386.58000
72667.00000	386.38000
73560.00000	386.86000
74439.00000	387.34000
75535.00000	387.55000
76137.00000	389.15000
77142.00000	390.31000
78065.00000	390.65000
78753.00000	393.33000
79177.00000	391.99000
80055.00000	395.81000
80587.00000	395.70000
81454.00000	398.03000
81779.00000	399.02000
82532.00000	399.79000
83134.00000	396.57000
83933.00000	398.38000
99999.00000	414.00000

IT=11

4300.00000	515.00000
41300.00000	382.00000
99999.00000	382.00000

IT=12

0.	4300.00000
685.00000	9073.00000
1408.00000	14125.00000
2115.00000	21106.00000
2790.00000	31131.00000
3510.00000	41226.00000
4290.00000	51633.00000
5010.00000	61360.00000
5670.00000	70892.00000
5970.00000	75535.00000
6270.00000	79177.00000
6510.00000	81779.00000
6630.00000	83134.00000
11100.00000	83933.00000
14370.00000	83933.00000

IT=13

0.	1.00000
20000.00000	1.00000

FIGURE 7 cont'd. INPUT DATA FOR THERMISTOR FLIGHT fabric (525^oR) were deduced from the measurements of some thermistors. Notice that the gas is colder than the fabric, the latter being at ground air temperature. The air temperature data are actual temperature measurements made by a thermistor hanging below the gondola. The infrared radiation data are specified according to the approximate rule given in Section II.H.2. The observed altitude data (IT = 12) were computed from measurements of both atmospheric temperature and pressure using the hydrostatic law. Therefore, we believe that they are very accurate.

The correlation of computed and actual flights is excellent as shown in Figure 8. Unfortunately, the thermistor measuring air temperature ceased to function properly after the balloon reached its ceiling and, therefore, we cannot continue the correlation to the end of the ceiling phase. Four hours and twenty minutes after launch, the cut-down command was given and the flight was terminated. The payload was brought to ground by parachute.

The correlation of computed and measured temperatures is shown in Figure 9. Several thermistors were suspended inside the balloon along its centerline. Only the upper three functioned properly throughout the entire flight. They were at a fixed distance of 7, 19 and 31 feet from the top of the balloon. The relation of the temperature of each thermistor to the average temperatures of the gas and fabric depends, of course, on the position of the thermistor with respect to the volume of the balloon. The position of each thermistor during the flight can be deduced roughly from the sketch on the lower right of Figure 9, in which the balloon is represented grossly by a sphere. The diameter of the balloon was about 26, 46, and 80 feet at ground, tropopause and ceiling, respectively. Thus, at ground, the lower thermistor was buried in the loose fabric.

With the aid of this sketch, a careful examination of Figure 9 shows that the correlation of computed average temperatures and measured temperatures is very good. The maximum deviation, at some parts of the flight, is less than 10^oR. During most of the flight, the deviation is much less than 10^oR.

We think that this flight has given considerable support to the validity of our analysis, and that it has verified our long held view that the fabric of balloons experiences rather low temperatures through tropopause. The temperature of the top thermistor must be closely coupled to the fabric temperature. As shown in Figure 9, the fabric of this balloon was at a temperature of about 380^oR (-78^oF) at tropopause.

The small oscillations in the computed temperatures at tropopause are due to corresponding oscillations in the input air temperature data.

3. Stratoscope Flight S4-2

Stratoscope II Flight S-4 (Photo) was launched from Palestine, Texas, on July 23, 1965, at 122 GMT. It reached its ceiling of 80,000 ft

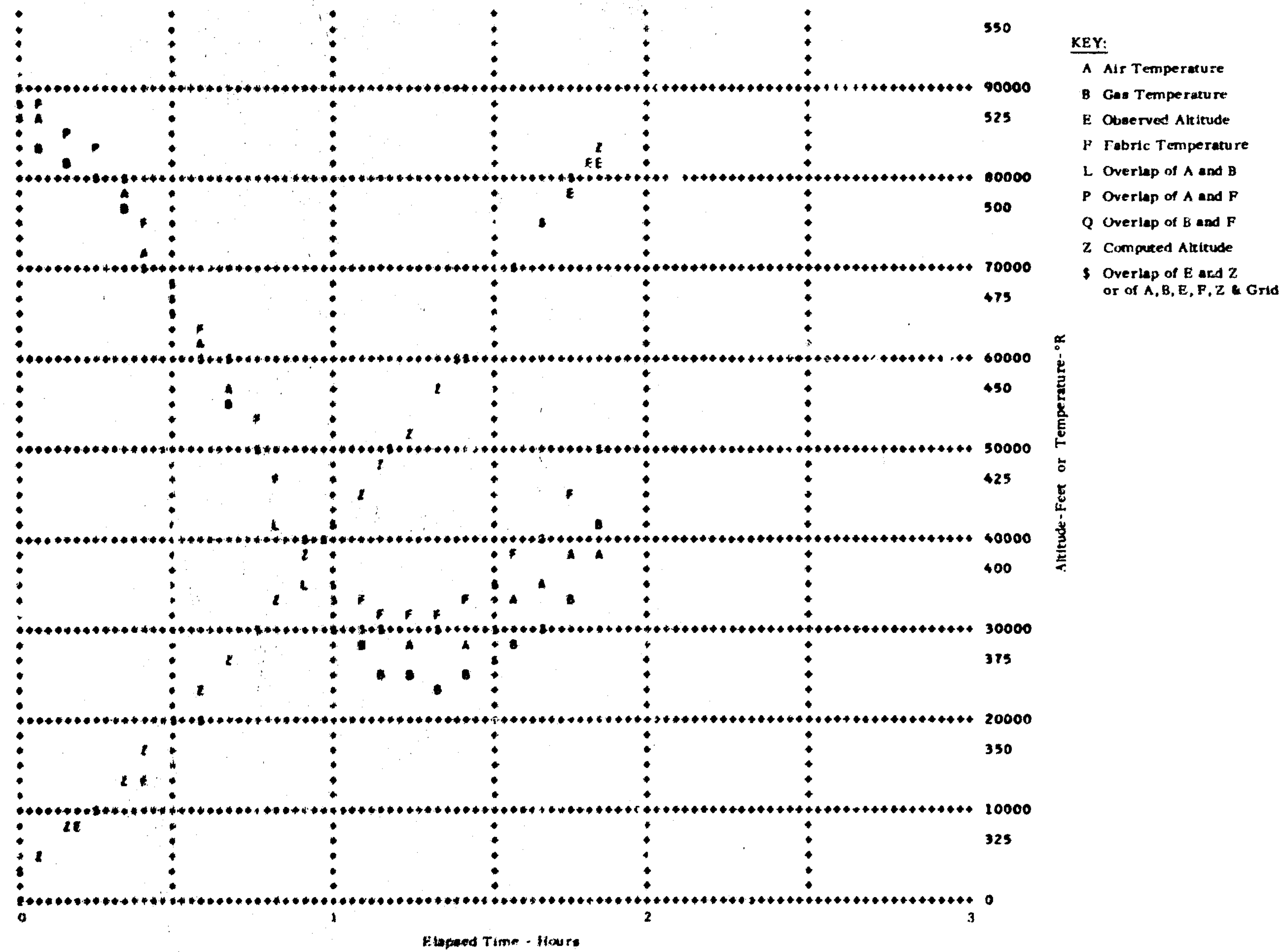


FIGURE 8 PLOTTED OUTPUT FOR THERMISTOR FLIGHT

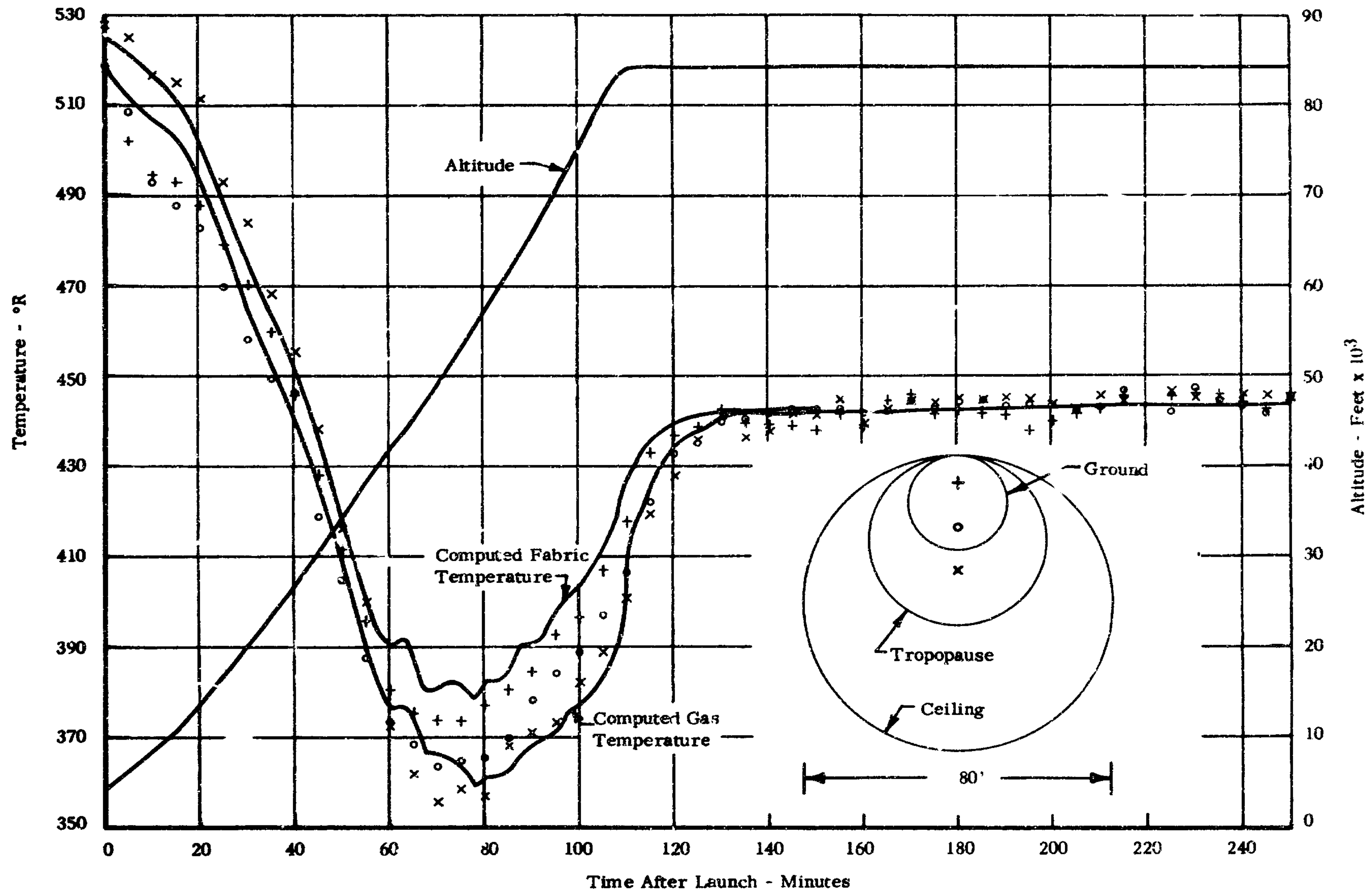


FIGURE 9 TEMPERATURE CORRELATION OF THERMISTOR FLIGHT. SKETCH AT LOWER RIGHT SHOWS THE POSITION OF THE THREE THERMISTORS WITH RESPECT TO BALLOON VOLUME.

after one hour and twenty-five minutes, approximately 35 min after balloon sunset. It dropped about 3,000 ft during the night. It was brought down the next day during sunrise.

The input data are shown in Figure 2. The fabric of the balloon was GT-12 (mylar and scrim). Notice that the fabric properties are identical to those of the Starg zer Manned Flight. The initial free lift (1,460 lb) is exactly the value specified in the flight data. The initial balloon gas and fabric temperatures ($542^{o}R$) are taken equal to air temperature at ground level. The air temperature data are not measured values for Palestine. They were computed from the table of Reference 12 pertaining to the July subtropical atmosphere ($30^{o}N$). The infrared data have been computed according to the approximate rule given in Section II-H.2.

The correlation of actual and computed flights is shown in Figure 4. Considering the fact that standard instead of actual atmospheric data have been used, the correlation is fairly good during the ascent and early ceiling phases. From this correlation and from the excellent correlation of the Stargazer Manned Flight, it appears that the following values for the properties of the GT-12 (mylar and scrim) fabric are acceptable:

$$C1 = 250 \text{ ftlb/lb}^{o}R$$

$$ABIR = 0.69$$

$$EMIR = 0.69$$

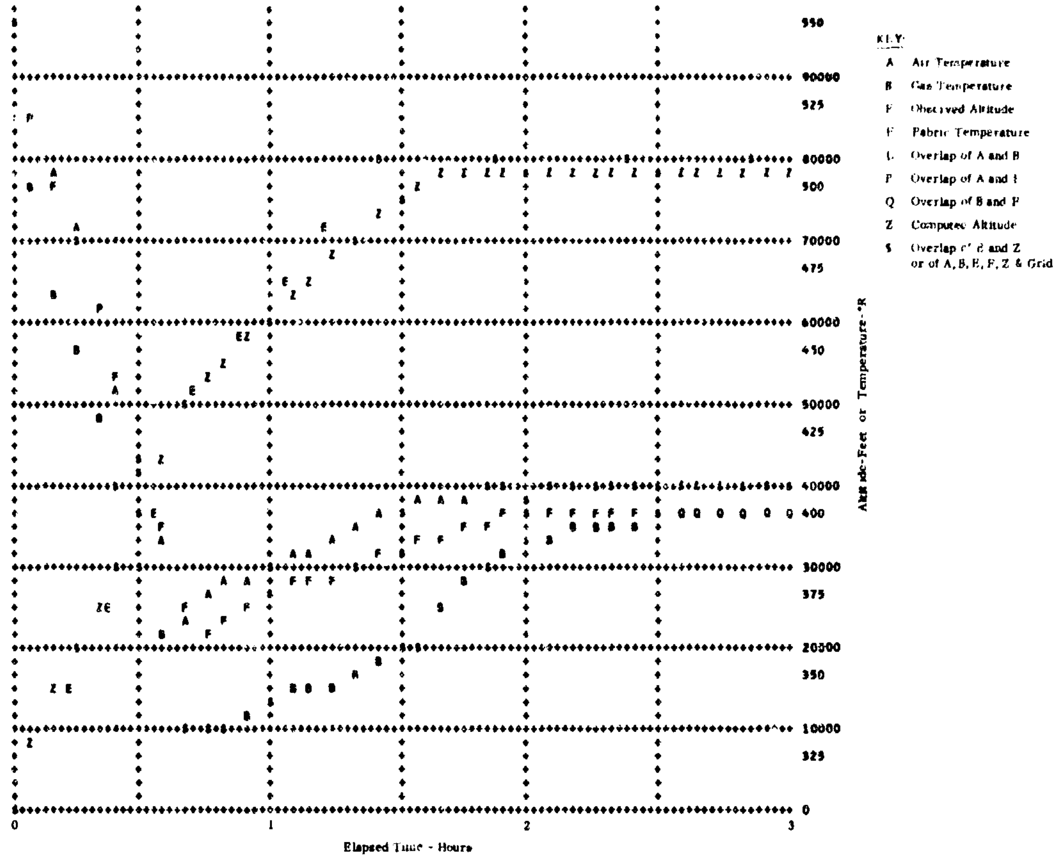
$$ABUV = 0.15$$

A better correlation during the latter part of the ceiling phase can be obtained by reducing the infrared radiation during the night. This can be achieved through the infrared radiation factor data (IT = 13). Leaving all the other data as they are and changing the infrared radiation factor data as follows:

IT = 13

0.0	1.0
7200.0	1.0
36000.0	0.9
41000.0	1.0

we obtain the correlation shown in Figure 10. The above data mean that from 2 to 10 hours after launch the infrared radiation incident on the balloon is reduced by 10%. This means that the black ball radiation temperature is about $10^{o}R$ less than that given in the input data (IT = 11).



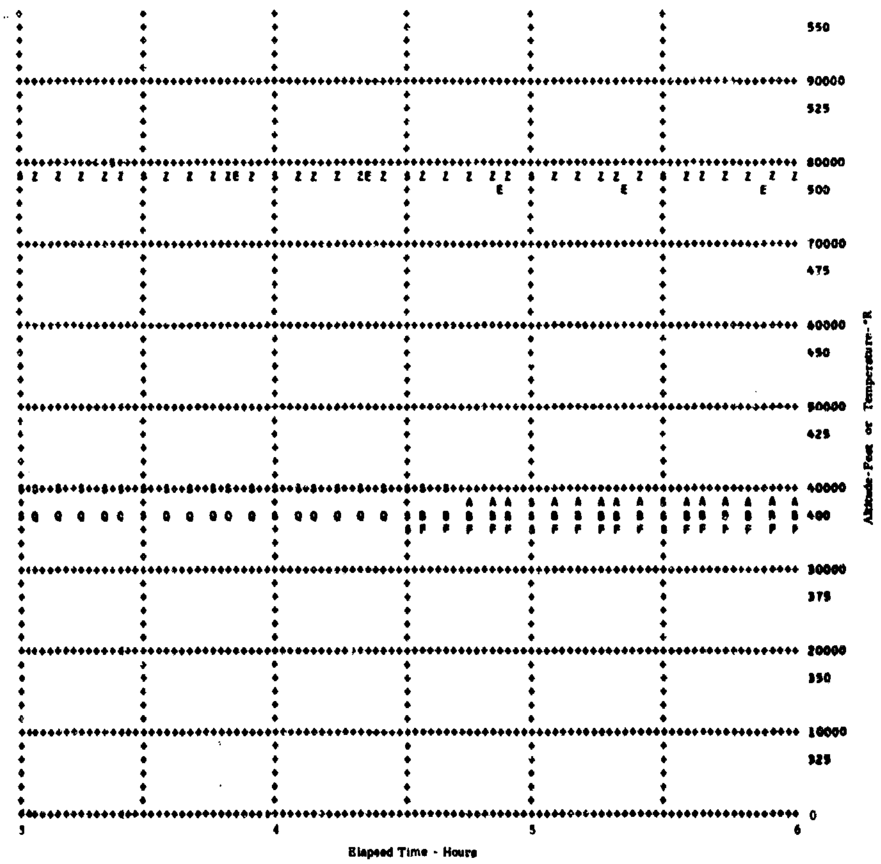
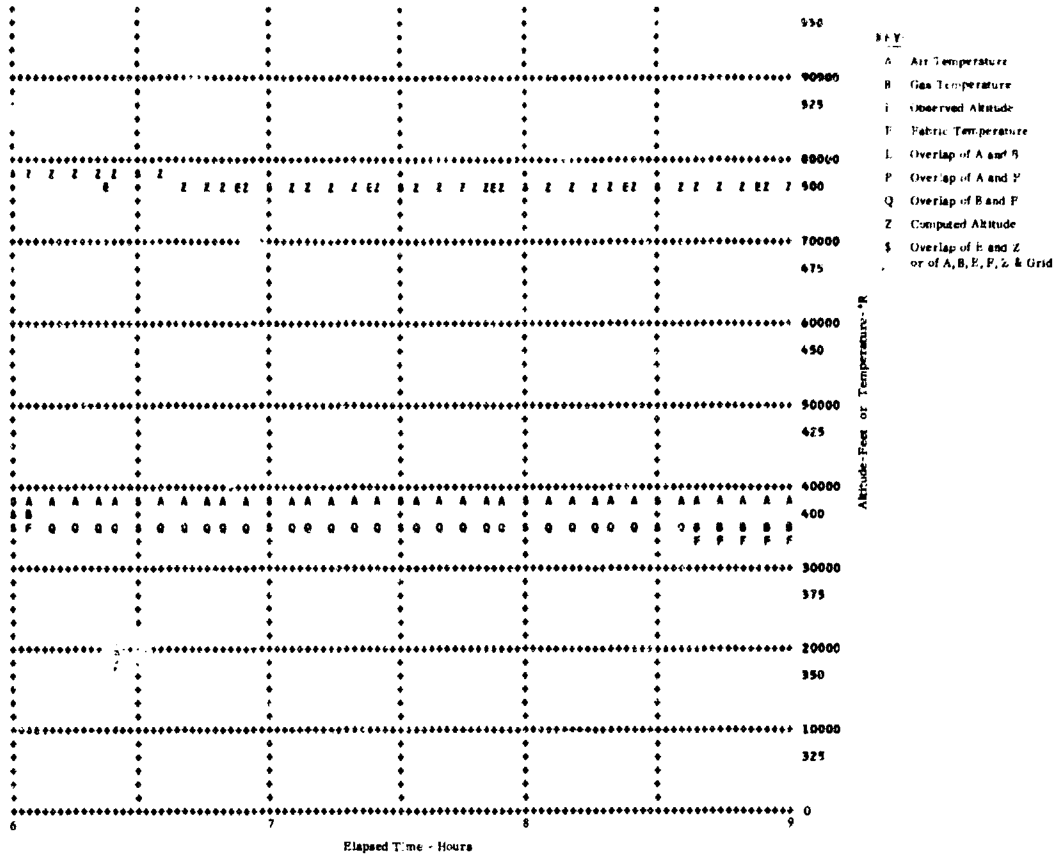


FIGURE 10 PLOTTED OUTPUT FOR STRATOSCOPE FLIGHT S4-2 WITH REDUCED (by 10%) INFRARED RADIATION DURING NIGHT



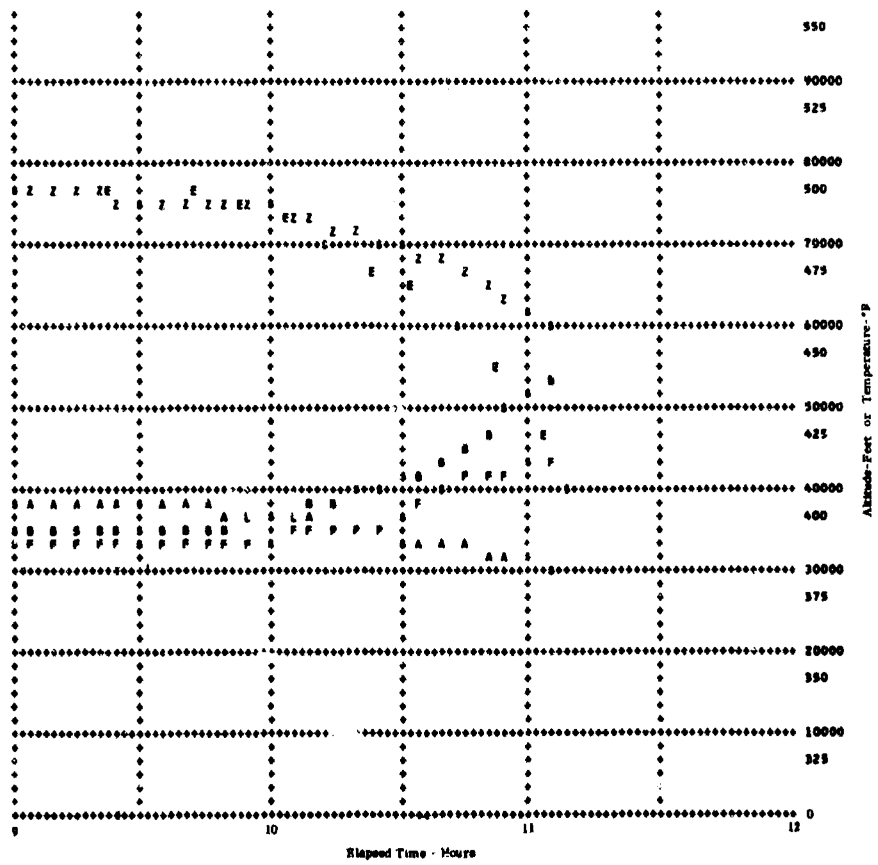


FIGURE 10 cont'd. PLOTTED OUTPUT FOR STRATOSCOPE FLIGHT S4-2 WITH REDUCED (by 10%) INFRARED RADIATION DURING NIGHT

Such a change in the infrared radiation field from day to night has, in fact, been noticed (see Section II-H.2). Notice that the correlation during the night is now much better. Evidently, reduction of infrared radiation caused the actual balloon to drop 3,000 ft during the night.

The correlation of the last part of the desent phase is not good. This must be due to the fact that there is some uncertainty in the time of closing of the helium valve.

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LIST OF SYMBOLS

Algebraic	Fortran	Description	Units
A		Effective area of balloon in direction of motion, cross-sectional area of spherical balloon.	ft^2
b_n	TZB(I)	See Equation 25.	^oR
$\dot{B}$	B	Ballasting rate.	lb/sec
C_B	CB	Apparent additional mass coefficient of balloon.	
C_D	CD	Drag coefficient of balloon.	
C_F	C1	Specific heat of balloon fabric.	$ftlb/lb^oR$
C_P	CP	Specific heat of balloon gas at constant pressure.	$ftlb/lb^oR$
C_V	CV	Specific heat of balloon gas at constant volume.	$ftlb/lb^oR$
C_4	C4	Correction constant for forced convection heat transfer from air to balloon fabric.	
C_5	C5	Correction constant for free convection heat transfer from air to balloon fabric.	
C_6	C6	Correction constant for free convection heat transfer from balloon gas to balloon fabric.	
D		Diameter of spherical balloon.	ft
DDRAG	DDRAG	Rate of energy spent to overcome drag.	ftlb/sec
DEC	DEC	Declination of sun.	deg
DEVE	DEVE	Rate of energy lost when gas is valved and/or exhausted.	ftlb/sec
DEWA	DEWA	Rate of work done by balloon gas on atmosphere when balloon expands.	ftlb/sec
DKE	DKE	Rate of change of kinetic energy of balloon.	ftlb/sec
DPE	DPE	Rate of change of potential energy of balloon.	ftlb/sec
$\dot{E}$	E	Rate of volumetric gas flow due to exhausting.	ft^3/sec
ECHK	ECHK	Energy check ratio of integration (see Section III-D).	
ENET	ENET	Difference of internal energy of gas and fabric from that at beginning of flight.	ftlb
EP	EP	Same as ENET, but found from integration (see Section III-D).	ftlb
ER		Rate of change of mechanical and thermal energy of balloon.	ftlb/sec

Algebraic	Fortran	Description	Units
ER_i		Value of ER at the end of each step of integration.	ftlb/sec
ETOT	ETOT	Internal energy of balloon gas and fabric.	ftlb
ETOT1	ETOT1	Value of ETOT at beginning of flight.	ftlb
FL	FL	Free lift.	lb
g	G	Acceleration of gravity (= 32.2).	ft/sec^2
GHA	GHA	Greenwich hour angle at beginning of flight.	deg
GL	GL	Gross lift.	lb
h		Coefficient of heat transfer.	$ftlb/ft^2sec^oR$
k		Thermal conductivity.	$ftlb/ftsec^oR$
k_A	CAIR	Thermal conductivity of air.	$ftlb/ftsec^oR$
k_H	CHE	Thermal conductivity of balloon gas.	$ftlb/ftsec^oR$
$\dot{L}_V$	V	Lost lift per unit time due to valving.	lb/sec
LAT	XLAT	Latitude of balloon.	deg
LHA		Local hour angle.	deg
LONG	XLØNG	Longitude of balloon.	deg
m	AIRM	Air mass.	
m_o		Air mass at sea level.	
m_1	AIRM1	Air mass for point B in Figure 1.	
m_2	AIRM2	Air mass corresponding to the supplement of actual azymuth angle of sun.	
M_A	XMA	Molecular weight of air.	
M_B	XMB	Molecular weight of balloon gas.	
n		Index.	
p	P	Atmospheric pressure.	lb/ft^2
p_n		Atmospheric pressure at altitude z_n.	lb/ft^2
p_o	PZER	Atmospheric pressure at ground level.	lb/ft^2
p_1	PZ1	Atmospheric pressure at point B in Figure 1.	lb/ft^2
Pr_A		Prandtl number of air.	
Pr_H		Prandtl number of balloon gas.	
q_2	DEH2	Rate of absorption of infrared radiation by balloon fabric.	ftlb/sec
q_3	DEH3	Rate of emission of infrared radiation by balloon fabric.	ftlb/sec

Algebraic	Fortran	Description	Units
q_4	DEH4	Rate of heating of balloon fabric by air through forced convection.	ftlb/sec
q_5	DEH5	Rate of heating of balloon fabric by air through free convection.	ftlb/sec
q_6	DEH6	Rate of heating of balloon fabric by balloon gas through free convection.	ftlb/sec
q_7	DEH7	Rate of absorption of solar radiation by balloon fabric.	ftlb/sec
Q		Energy supplied to a system.	ftlb
R	R	Gas constant per mol (= 1,545), or average reflectivity of balloon fabric, or	ftlb/lbmol ^oR
	RETH	average radius of earth (= 20,903,520).	ft
R_r		Infrared average reflectivity of balloon fabric.	
Re		Reynolds number.	
s_n	TZA(I)	See Equation 24.	$^oR/ft$
S		Surface area of spherical balloon.	ft^2
t	X	Time from beginning of flight.	sec
T	TRANS	Intensity of solar radiation relative to that outside atmosphere.	
T	TA	Temperature of air.	^oR
T_B	Y(4)	Temperature of balloon gas.	^oR
T_F	Y(3)	Temperature of balloon fabric.	^oR
T_n	TZ(I)	Temperature of air at altitude Z_n.	^oR
T_r		Black ball equilibrium radiation temperature.	^oR
U		Internal energy of a system.	ftlb
$\dot{V}$		Rate of volumetric gas flow due to valving.	ft^3/sec
V_B	Y(6)	Volume of balloon.	ft^3
w_B	Y(5)	Weight of balloon gas.	lb
w_G		Weight of payload.	lb
w_F	WF	Weight of balloon fabric.	lb
W		Work done by a system.	ftlb
X		Product of Grashof and Prandtl numbers.	

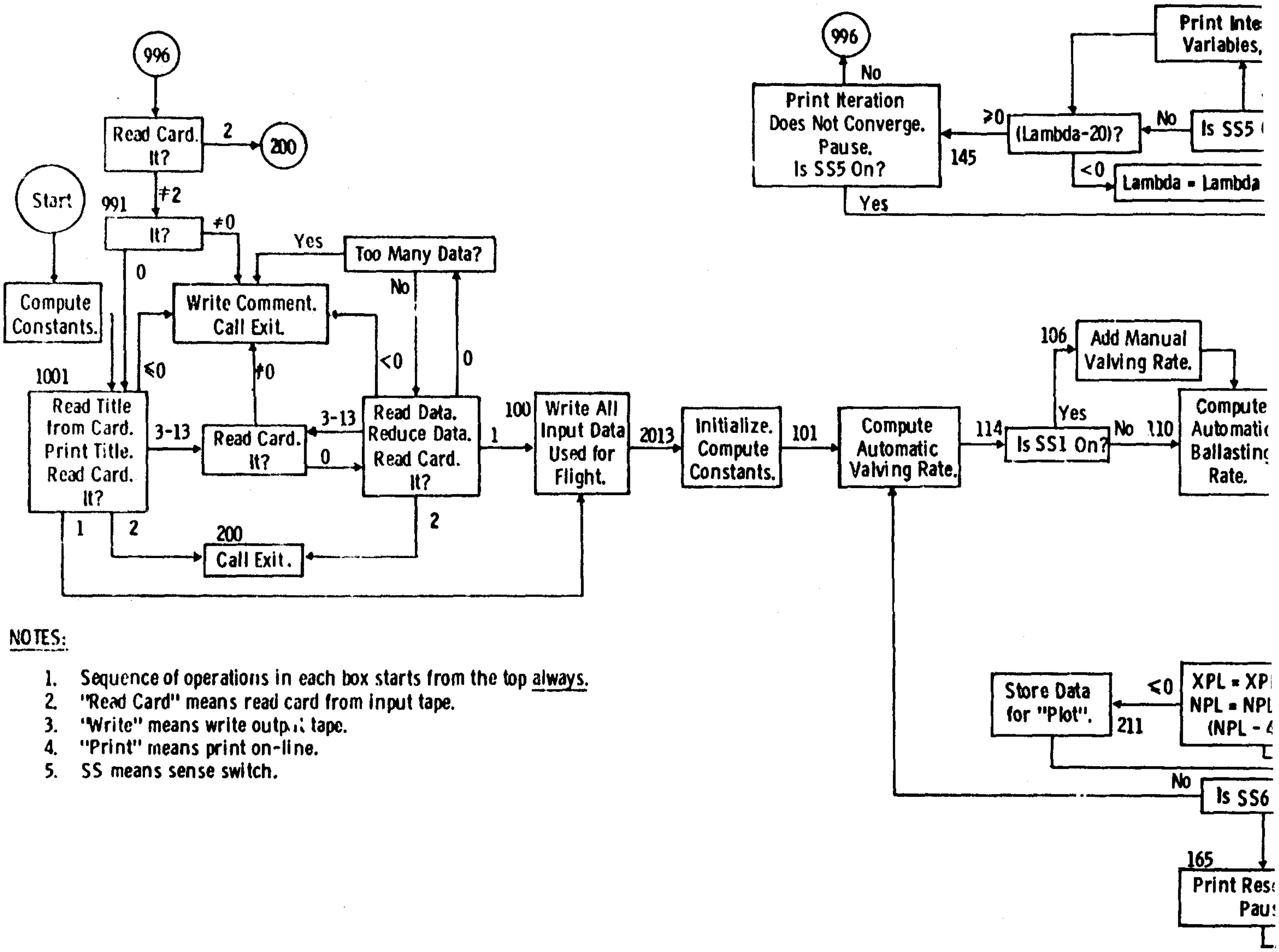
Algebraic	Fortran	Description	Units
$\dot{y}$	Y(2), Γ(1)	Vertical velocity of balloon.	ft/sec
z	Y(1)	Altitude of balloon above sea level.	ft
z_n	ZZ(I)	Altitudes at which atmospheric input data are given.	ft
z_o		Altitude of ground above sea level.	ft
z_1	ZZ1	Altitude of point B in Figure 1.	ft
α		Average absorptivity of balloon fabric.	
α_r		Infrared average absorptivity of balloon fabric.	
α_{ref}	ABIR	Effective infrared absorptivity of balloon fabric for multiple absorption.	
α_{vef}	ABUV	Effective ultraviolet absorptivity of balloon fabric for multiple absorption (solar radiation).	
δ		Azymuth angle of sun.	deg
ε		Average emissivity of balloon fabric.	
ε_{ef}		Effective average emissivity of balloon fabric for multiple emission.	
ε_r		Infrared average emissivity of balloon fabric.	
ε_{ref}	EMIR	Effective infrared emissivity of balloon fabric for multiple emission.	
μ_A	VISA	Viscosity of air.	lb/ftsec
μ_H	VISH	Viscosity of balloon gas.	lb/ftsec
ρ_A	RHØ	Density of air.	lb/ft^3
ρ_B	RHØH	Density of balloon gas.	lb/ft^3
ρ_n		Density of air at altitude z_n.	lb/ft^3
σ	SBØLZ	Stefan - Boltzmann constant (= 3.6995×10^{-10}).	$ftlb/ft^2 sec\,^oR^4$
τ_r		Infrared average transmissivity of balloon fabric.	

Algebraic	Fortran	Description	Units
	AH	Hours part of Greenwich Mean Time.	hours
	AM	Minutes part of Greenwich Mean Time.	min
	AS	Seconds part of Greenwich Mean Time.	sec
	BO	Manual ballasting rate.	lb/sec
	BB(I)	Array of specified automatic ballasting rates	lb/sec
	CAM	Cosine of azymuth angle of sun.	
	CCIR	Infrared radiation factor.	
	CCSI(I)	Array of specified solar radiation factors.	
	CLATD	Cosine of LAT times cosine of DEC.	
	CIR(I)	Array of specified infrared radiation factors.	
	CSI	Solar radiation factor.	
	DCIRDT(I)	Array of computed slopes of infrared radiation factor.	sec^{-1}
	DIRDZ(I)	Array of computed slopes of FIR(I).	$ftlb/ft^3 sec$
	DP	Plotting time interval.	sec
	DPR	Printing time interval.	sec
	DSIDT(I)	Array of computed slopes of solar radiation factor.	sec^{-1}
	EALT(I)	Array of observed balloon altitudes above sea level corresponding to times ETIME(I).	ft
	ETIME(I)	Array of times.	sec
	FCAF	Forced convection heating rate of balloon fabric by air.	ftlb/sec
	F(2)	Vertical acceleration of balloon.	ft/sec^2
	F(3)	Time derivative of balloon fabric temperature.	$^oR/sec$
	F(4)	Time derivative of balloon gas temperature.	$^oR/sec$
	F(5)	Time derivative of balloon gas weight.	lb/sec
	F(6)	Time derivative of balloon volume.	ft^3/sec
	FIR(I)	Array of computed atmospheric radiation fluxes.	$ftlb/ft^2 sec$
	FLO	Initial free lift.	lb
	FLUXIR	Infrared radiation flux.	$ftlb/ft^2 sec$

Algebraic	Fortran	Description	Units
	FSØL	Value of solar constant outside earth's atmosphere (= 96).	$ftlb/ft^2 sec$
	GMT	Greenwich Mean Time.	hr:min:sec
	GMTS	Greenwich Mean Time at beginning of flight.	sec
	H	Time interval for integration.	sec
	IRAB	Rate of absorption of infrared radiation by balloon fabric.	ftlb/sec
	IREM	Rate of emission of infrared radiation by balloon fabric.	
	IRS, LAMBDA	Number of iterations in exhausting gas.	
	LOAD	Weight of payload and balloon fabric.	lb
	NCAF	Natural convection heating rate of balloon fabric by air.	ftlb/sec
	NCGF	Natural convection heating rate of balloon fabric by balloon gas.	ftlb/sec
	PERB	Cumulative ballast droped in percent of initial load (payload plus balloon fabric).	
	PERE	Cumulative gas exhausted in percent of initial balloon gas weight.	
	PERV	Cumulative gas valved in percent of initial balloon gas weight.	
	PHI	One-third power of balloon volume.	ft
	PM, PRESS	Atmospheric pressure.	mbar
	RAD	Conversion factor from degrees to radians $\left(= \frac{\pi}{180}\right)$.	rad/deg
	RHØO	Density of air at ground level.	lb/ft^3
	RT	Product of density and temperature of air	$lb\,^{o}R/ft^3$
	RTIR(I)	Array of specified black ball equilibrium radiation temperatures.	^{o}R
	RTZ(I)	Array of products of density and temperature of air computed from atmospheric data.	$lb\,^{o}R/ft^3$
	SLAB	Rate of absorption of solar radiation by balloon fabric.	ftlb/sec
	SLATD	Sine of LAT times sine of DEC.	

Algebraic	Fortran	Description	Units
	TAA(I)	Array of computed air temperatures stored for subroutine PLOT.	^oR
	TB(I)	Array of specified times for automatic ballasting.	sec
	TBO	Initial temperature of balloon gas.	^oR
	TCSI(I)	Array of specified times for solar radiation factor.	sec
	TFO	Initial temperature of balloon fabric.	^oR
	TIR(I)	Array of specified times for infrared radiation factor.	sec
	TV(I)	Array of specified times for automatic valving.	sec
	VALD	Cumulative weight of valved gas.	lb
	VBM	Inflated volume of balloon.	ft^3
	VEL	Vertical velocity of balloon.	ft/min
	VO	Manual valving rate (in pounds of lost lift per second).	lb/sec
	VV(I)	Array of specified automatic valving rates (in pounds of lost lift per second).	lb/sec
	WBO	Initial weight of balloon gas.	lbs
	WFO	Weight of balloon fabric.	lb
	WG	Weight of payload and balloon fabric.	lb
	WGO	Initial weight of payload.	lb
	WT	Initial weight of payload and balloon fabric.	lb
	XLAH	Local hour angle.	rad
	XP	Printing time.	sec
	XPL	Plotting time.	sec
	XT	Specified final time of flight.	sec
	XO	Specified initial time of flight.	sec
	XXX(I)	Array of times stored for subroutine PLOT.	sec
	Y1(I)	Array of computed balloon altitudes stored for subroutine PLOT.	ft
	Y20	Specified initial vertical velocity of balloon.	ft/sec
	Y3(I)	Array of computed temperatures of balloon fabric stored for subroutine PLOT.	^oR

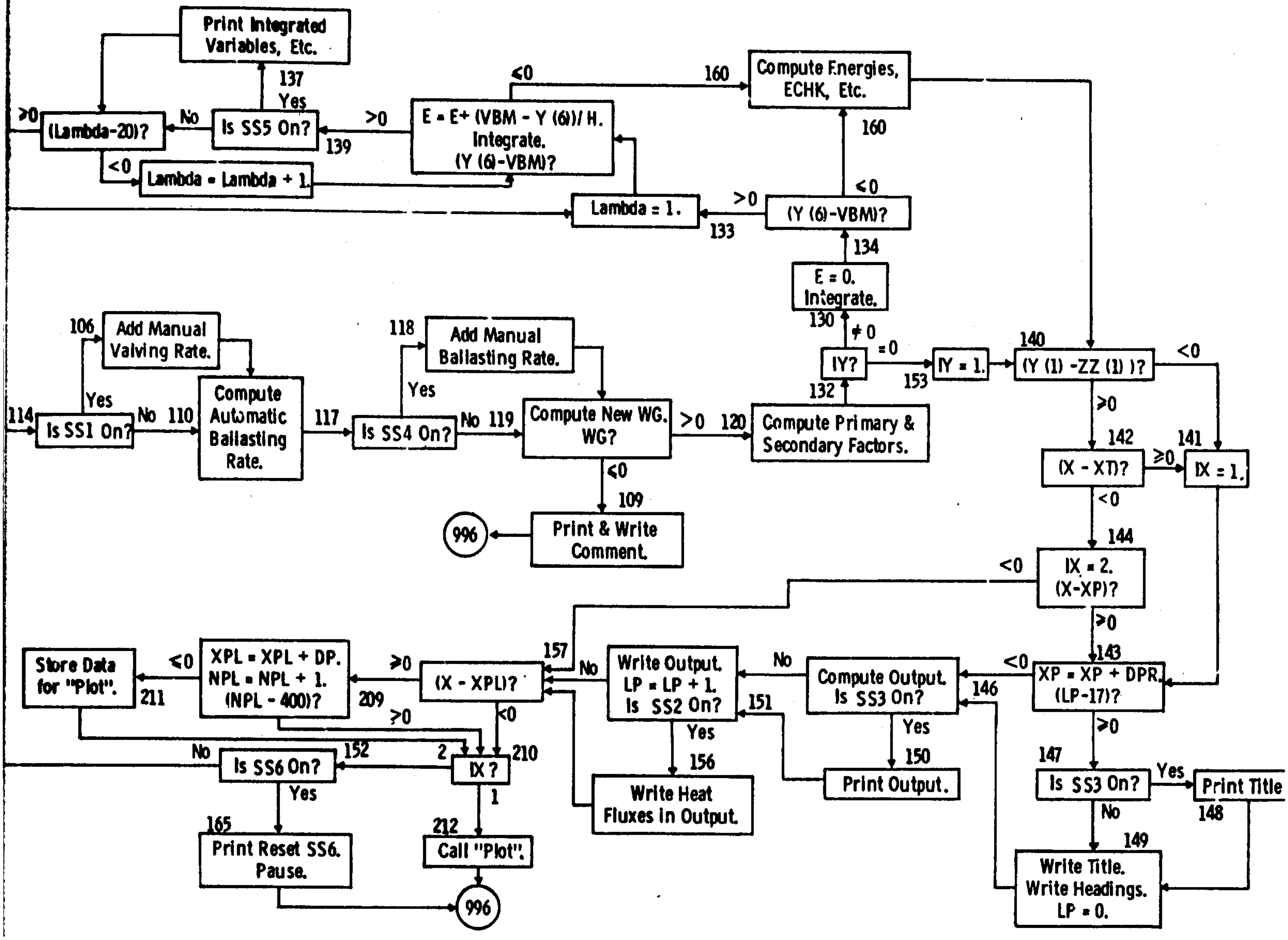
Algebraic	Fortran	Description	Units
	Y4(I)	Array of computed temperatures of balloon gas stored for subroutine PLOT.	°R
	ZIR(I)	Array of altitudes above sea level at which RTIR(I) is given.	ft
	ZO	Specified initial altitude of balloon above sea level.	ft



NOTES:

1. Sequence of operations in each box starts from the top always.
2. "Read Card" means read card from input tape.
3. "Write" means write output tape.
4. "Print" means print on-line.
5. SS means sense switch.

FIGURE 11 GENERAL FL



GURE 11 GENERAL FLOW CHART

```
C     ONR-ADL COMPUTED VERTICAL TRAJECTORY OF HIGH ALTITUDE BALLOONS
C     COMPILATION 8H
C     SENSE SWITCH CONTROLS          DOWN                   UP
C     SENSE SWITCH   1          VALVE GAS (VO)         NORMAL
C     SENSE SWITCH   2          PRINT DETAIL            NORMAL
C     SENSE SWITCH   3          PRINT ON-LINE          NORMAL
C     SENSE SWITCH   4         DROP BALLAST (BO)       NORMAL
C     SENSE SWITCH   5        CONTINUE ITERATIONS      STOP AFTER 20
C     SENSE SWITCH   6          ABORT FLIGHT           NORMAL
    5 FORMAT ( I2,3F10.5)
   10 FORMAT ( I2, 7E10.3)
   15 FORMAT (77H1 YOU HAVE NOW THROWN AWAY ALL OF THE BALLAST - - INCLU
     1DING THE GONDOLA./1H0/1H0)
   20 FORMAT ( I2, 7F10.4)
   25 FORMAT (28H ITERATION DOES NOT CONVERGE/54H  SENSE SWITCH 5 DOWN A
     1ND START TO CONTINUE ITERATIONS/40H  SENSE SWITCH 5 UP AND START T
     2O END RUN/1H0/1H0)
   30 FORMAT ( I2, 2F10.4)
   31 FORMAT(I2,2F10.4)
   35 FORMAT ( 7H0 TIME=F10.5,4H  Z=F10.2,7H  ZDOT=F10.5,5H  TF=F10.4,
     15H  TB=F10.4,5H  WB=F10.4,5H  VB=E15.8/ 7H0  VBM=E15.8,3H E=E15.8)
   40 FORMAT ( 7H1 TIME=F10.5,8H  TV(1)=F10.5)
   45 FORMAT(15H INPUT IS WRONG/1H0/1H0)
   50 FORMAT (31H CARD AFTER TITLE CARD IS WRONG/ 1H0)
   55 FORMAT(7H1 TIME=F10.5,8H  TB(1)=F10.5)
   60 FORMAT (9H NO TYPE I2,5H DATA/1H0)
   65 FORMAT ( 7H1 TIME=F10.5,9H TCSI(1)=F10.5)
   70 FORMAT (15H TOO MANY TYPE I2,5H DATA/1H0)
   75 FORMAT (20H1 WBDOT IS POSITIVE F10.5)
   80 FORMAT (20H IT AFTER DATA TYPE I2,12H IS NEGATIVE/1H0)
   81 FORMAT(7H1 TIME=F10.5,8H TIR(1)=F10.5)
   90 FORMAT (16H0 TOO MANY TYPE I2,13H DATA CARDS. /1H0 /1H0)
   91 FORMAT(5H0GMT=F3.0,F3.0,F3.0,10H ALTITUDE=F6.0,10H VELOCITY=F6.0,
     110H VAL RATE=F5.2,13H PERCENT VAL=F6.2,10H BAL RATE=F5.2,
     213H PERCENT BAL=F6.2,13H PERCENT EXD=F6.2/1H0/1H0)
  993 FORMAT(1H1,51H CARD AFTER CARD TYPE 1 OF PREVIOUS PROBLEM IS WRONG
     1/1H0)
  994 FORMAT ( 1H1, 12A6 )
  995 FORMAT ( 12A6 )
  975 FORMAT ( I2, 5F10.5)
  976 FORMAT(1H0,3X,3HGMT,6X,4HTIME,2X,8HALTITUDE,4X,3HVEL,4X,5HPRESS,3X
     1,2HTA,5X,2HTF,5X,2HTB,5X,6HVOLUME,4X,4HLOAD,3X,6HGAS WT,2X,7HFR LI
     2FT,2X,4HPERB,2X,4HPERV,2X,4HPERE 2X,3HIRS,3X,4HECHK)
  977 FORMAT    (1H0,3F3.0,1XF7.0,2XF7.0,2XF6.0,2XF6.1,2XF5.1,2XF5.1,2XF
     15.1,2XF8.0,2XF6.0,2XF6.1,2XF7.2,2XF4.1,2XF4.1,2XF4.1,2XI2,2XF6.3)
 5001 FORMAT(4X,5HIRAB=1PE11.4,3X,5HIREM=1PE11.4,3X,5HFCAF=1PE11.4,3X,5H
     1NCAF=1PE11.4,3X,5HNCGF=1PE11.4,3X,5HSLAB=1PE11.4)
 2000 FORMAT(1H0,11H INPUT DATA)
 2001 FORMAT(1H0,4H IT=I2)
 2002 FORMAT(7F16.5)
 2005 FORMAT (2F16.5)
 2006 FORMAT (F16.5)
      DIMENSION SCALE(60)
      DIMENSION  ZZ(100), TZ(100), RTZ(100),  TZA(100), TZB(100)
      DIMENSION  TCSI(100),CCSI(100),   TV(100),   VV(100)
```

```
      DIMENSION   TB(100),    BB(100)    ,   XX(100)
      DIMENSION  Y(100), F(100), Q(100)
      DIMENSION  TITLE (12)
      DIMENSION RTIR(100)
      DIMENSION ZIR(100),FIR(100),DIRDZ(100),DSIDT(100)
      DIMENSION ZUV(100),FUV(100),DUVDZ(100)
      DIMENSION Y1(400),Y3(400),Y4(400),XXX(400),TAA(400)
      DIMENSION ETIME(400),EALT(400)
      DIMENSION TIR(100),CIR(100),DCIRDT(100)
      COMMON X, Y, F, Q
      COMMON NT, ZZ, TZ, RTZ, TZA, TZB
      COMMON C1, C2, C3, C4, C5, C6, C7
      COMMON TA, RT, P, RHO, PHI, WG, OMEGA, CSI, E, V
      COMMON XMA, XMB, G, CP, CV, R, CD, VRM ,WF
      COMMON DEH2,DEH3,DEH4 DEH5,DEH6,DEH7
      COMMON WFH,DWFH,WFO
      COMMON CB
      COMMON ZIR,FIR,DIRDZ,NIR,ZUV,FUV,DUVDZ,NUV,DSIDT
      COMMON ABIR,EMIR,ABUV
      COMMON M,Y1,Y3,Y4,TAA,XXX
      COMMON DP,YT,XO,ETIME,EALT
      COMMON CCIR
      COMMON  SLATD,CLATD,RAD,GHA,XLONG,PZER,FSOL,SBOLZ,AH,AM,AS,GMTS
      FSOL=96.
      SBOLZ=3.6995E-10
      XMB= 4.
      XMA= 28.89
      CP=972.69
      CV=586.73
      R=1545.
      G = 32.2
      RAD=3.14159/180.
 1001 READ INPUT TAPE 5, 995,(TITLE(I), I=1,12)
      DO 990 MZY=1,400
      Y1(MZY)=0.
      Y3(MZY)=0.
      Y4(MZY)=0.
      XXX(MZY)=0.
      TAA(MZY)=0.
  990 CONTINUE
      PRINT 994, (TITLE(I), I=1,12)
  999 READ INPUT TAPE 5, 10, IT
      IF(IT) 998,998,1000
  998 WRITE OUTPUT TAPE 6,994,(TITLE(I),I=1,12)
      WRITE OUTPUT TAPE 6,50
      CALL EXIT
 1000 GOTO(100,200,300,400,500,950,700,800,900,600,960,     980,1400),IT
  997 WRITE OUTPUT TAPE 6,994,(TITLE(I),I=1,12)
      WRITE OUTPUT TAPE 6,80,IP
      CALL EXIT
  970 WRITE OUTPUT TAPE 6,994,(TITLE(I),I=1,12)
      WRITE OUTPUT TAPE 6,60,IP
      CALL EXIT
  971 WRITE OUTPUT TAPE 6,994,(TITLE(I),I=1,12)
      WRITE OUTPUT TAPE 6,70,IP
```

```
      CALL EXIT
  200 CALL EXIT
  100 WRITE OUTPUT TAPE 6,994,(TITLE(I),I=1,12)
      WRITE OUTPUT TAPE 6,2000
      I=3
      WRITE OUTPUT TAPE 6,2001,I
      WRITE OUTPUT TAPE 6,2002,C1,C4,C5,C6,ABIR,EMIR,ABUV
      I=4
      WRITE OUTPUT TAPE 6,2001,I
      WRITE OUTPUT TAPE 6,2002,WGO,WFO,VBM,FLO,CD,CB
      I=5
      WRITE OUTPUT TAPE 6,2001,I
      WRITE OUTPUT TAPE 6,2002,TBO,TFO,ZO, Y2O,DEC,XLAT,XLONG
      I=6
      WRITE OUTPUT TAPE 6,2001,I
      WRITE OUTPUT TAPE 6,2002,H,DFR,XO,XT,GMTS,GHA,DP
      I=7
      WRITE OUTPUT TAPE 6,2001,I
      DO 2004 I=1,NSI
 2004 WRITE OUTPUT TAPE 6,2005,TCSI(I),CCSI(I)
      I=8
      WRITE OUTPUT TAPE 6,2001,I
      WRITE OUTPUT TAPE 6,2006,VO
      DO 2007 I=1,NVI
 2007 WRITE OUTPUT TAPE 6,2005,TV(I),VV(I)
      I=9
      WRITE OUTPUT TAPE 6,2001,I
      WRITE OUTPUT TAPE 6,2006,BO
      DO 2008 I=1,NBI
 2008 WRITE OUTPUT TAPE 6,2005,TB(I),BB(I)
      I=10
      WRITE OUTPUT TAPE 6,2001,I
      WRITE OUTPUT TAPE 6,2006,RHOO
      DO 2009 I=1,NT
 2009 WRITE OUTPUT TAPE 6,2005,ZZ(I),TZ(I)
      I=11
      WRITE OUTPUT TAPE 6,2001,I
      DO 2010 I=1,NIR
 2010 WRITE OUTPUT TAPE 6,2005,ZIR(I),RTIR(I)
      I=12
      WRITE OUTPUT TAPE 6,2001,I
      DO 2011 I=1,NEX
 2011 WRITE OUTPUT TAPE 6,2005,ETIME(I),EALT(I)
      I=13
      WRITE OUTPUT TAPE 6,2001,I
      DO 2012 I=1,NCIR
 2012 WRITE OUTPUT TAPE 6,2005,TIR(I),CIR(I)
 2013 CLATD=COSF(XLAT*RAD)*COSF(DEC*RAD)
      SLATD=SINF(XLAT*RAD)*SINF(DEC*RAD)
      Y(1)=ZZ(1)
      CALL RHOT (Y(1),TEMP,RT)
      PZER=RT*R/XMA
      FL=FLO
      Y(1) = ZO
```

```
      Y(2)=Y20
      Y(3) = TFO
      Y(4) = TBO
      X =  XO
      XP= XO
      XPL=XO
      WF=WFO
      WG=WGO+WFO
      VALD=0
      WT=WG
      LP= 25
      LPC =1
      IY=0
      NPL=0
      CALL RHOT (Y(1), TEMP, RT)
      RHO  = RT / TEMP
      P=RT*R/XMA
      TA   = TEMP
      Y(5)=(FL+WG)/(XMA*Y(4)/(XMB*TA)-1.)
      WBO=Y(5)
      Y(6)=R*Y(5)*Y(4)/(P*XMB)
      CSI=CCSI(1)
      CCIR=CIR(1)
      PHI=Y(6)**(1./3.)
C              ---- INITIALIZING TOTAL ENERGY ----E1----
      CALL YPRIME
      DKE=(WG+Y(5)+.5*RHO*Y(6))*Y(2)*F(2)/G
      DPE=  (RHO*Y(6)-WG-Y(5))*Y(2)
      DDRAG=CD*.60375*PHI*PHI*RHO*Y(2)*ABSF(Y(2))*Y(2)/G
      DEWA=P*F(6)
      DEVE=Y(4,*CP*F(5)
      ETOT=CV*Y(5)*Y(4)+C1*WFO*Y(3)
      ETOT1=ETOT
      ENET=.0
      EP=0.0
      ECHK=0.
  101 LAMBDA=0
      IF (X -TV(1)) 102,103,103
  102 WRITE OUTPUT TAPE 6,40,X,TV(1)
      WRITE OUTPUT TAPE 6,45
      GO TO 996
  103 DO 104 I=2,100
      IF (TV(I)+ 0.5* H - X) 104,104,105
  104 CONTINUE
  107 WRITE OUTPUT TAPE 6,108
  108 FORMAT (28H1  X IS GREATER THAN TV(100)/1HO/1HO)
      GO TO 996
  105 V= VV(I-1)
  114 IF (SENSE SWITCH 1) 106, 110
  106 V= V+ VO
  110 IF(X-TB(1)) 111,112,112
  111 WRITE OUTPUT TAPE 6,55,X,TB(1)
      WRITE OUTPUT TAPE 6,45
      GO TO 996
```

```
112 DO 113 I= 2, 100
    IF ( X - TB(I) -0.5*H ) 116, 113, 113
113 CONTINUE
    WRITE OUTPUT TAPE 6,125
125 FORMAT (28H1  X IS GREATER THAN TB(100)/1H0/1H0)
    GO TO 996
116 B = BB (I-1)
117 IF (SENSE SWITCH 4) 118, 119
118 B = B+B0
119 WG = WG -B*H
    IF (WG) 109, 109, 120
109 WRITE OUTPUT TAPE 6,15,WG
    PRINT 15,WG
    GO TO 996
120 IF ( X - TCSI(1) ) 121, 122, 122
121 WRITE OUTPUT TAPE 6,65,X,TCSI(1)
    WRITE OUTPUT TAPE 6,45
    GO TO 996
122 DO 123 I=2,NSI
    IF(X-TCSI(I  )+.5*H)124,123,123
123 CONTINUE
124 CSI=DSIDT(I-1)*(X-TCSI(I-1))+CCSI(I-1)
    IF(X-TIR(1))126,127,127
126 WRITE OUTPUT TAPE 6,81,X,TIR(1)
    WRITE OUTPUT TAPE 6,45
    GOTO 996
127 DO 128 I=2,NCIR
    IF(X-TIR(I)+.5*H)129,128,128
128 CONTINUE
129 CCIR=CIR(I-1)+DCIRDT(I-1)*(X-TIR(I-1))
132 IF(IY)130,153,130
153 IY=1
    GOTO140
130 E = 0.
    XS = X
    DO 131 I=1,6
131 Y(I+10) = Y(I)
    CALL RNGKTA (H, 6, 1, 1)
134 IF(Y(6)-VBM)160,160,133
160 PHI = Y(6)**(1./3.)
C              ---- INCREMENTING TOTAL ENERGY ----E2----
    CALL YPRIME
    DKE=(WG+Y(5)+.5*RHO*Y(6))*Y(2)*F(2)/G
    DPE=  (RHO*Y(6)-WG-Y(5))*Y(2)
    DDRAG=CD*.60375*PHI*PHI*RHO*Y(2)*ABSF(Y(2))*Y(2)/G
    DEWA=P*F(6)
    DEVE=Y(4)*CP*F(5)
    EP=EP+(-DKE+DPE-DDRAG-DEWA+DEVE+DEH2-DEH3+DEH4+DEH5+DEH7)*H
    ETOT=CV*Y(5)*Y(4)+C1*WF0*Y(3)
    ENET=ETOT-ETOT1
    ECHK=(ENET-EP)/ENET
    FL=Y(5)*(XMA*Y(4)/(XMB*TA)-1.)-WG
    VALD=VALD+H*V/((XMA*Y(4))/(XMB*TA)-1.)
    GO TO 140
```

```
138 GO TO 996
133 DO 135 LAMBDA=1, 20
    E = E + ( VBM - Y(6)) / H
    X = XS
    DO 136 I=1,6
136 Y(I) =  Y(I+10)
    CALL RNGKTA ( H, 6, 1, 1)
    IF(Y(6)-VBM)160,160,139
139 IF (SENSE SWITCH 5) 137, 135
137 PRINT 35 , X, ( Y(I),I=1,6), VBM, E
135 CONTINUE
145 PRINT 25
    PAUSE
    IF(SENSE SWITCH 5) 133, 138
140 IF(Y(1)-ZZ(1))141,142,142
141 IX = 1
    GO TO 143
142 IF ( X -XT) 144, 141, 141
144 IX=2
    IF(X-XP)157,143,143
143 XP = XP + DPR
    IF(LP-17)146,147,147
147 IF (SENSE SWITCH 3) 148, 149
148 PRINT 994, (TITLE (I), I=1,12)
149 WRITE OUTPUT TAPE 6, 994, (TITLE(I), I=1,12)
    WRITE OUTPUT TAPE 6, 976
    LPC = LPC+1
    LP  = 0
146 VR = V - RHO*E
    AH=INTF((GMTS+X)/3600.)
    AM=INTF((GMTS+X-3600.*AH)/60.)
    AS=GMTS+X-60.*AM-3600.*AH
    AM=ABSF(AM)
    AS=ABSF(AS)
    PERB=100.*(WT-WG)/WT
    PERV=100.*VALO/WBO
    PERE=100.*(WBO-Y(5))/(WBO)-PERV
    VEL=60.*Y(2)
    PM=0.478801*P
    IF ( SENSE SWITCH 3) 150, 151
150 PRINT 91,AH,AM,AS,Y(1),VEL,V,PERV,R,PERB,PERE
151 WRITE OUTPUT TAPE 6,977,AH,AM,AS,X,Y(1),VEL,PM,TA,Y(3),Y(4),Y(6),W
   1G,Y(5),FL,PERB,PERV,PERE,LAMBDA,ECHK
    LP= LP+1
155 IF(SENSE SWITCH 2)156,157
156 WRITE OUTPUT TAPE 6,5001,DEH2,DEH3,DEH4,DEH5,DEH6,DEH7
157 CONTINUE
    IF(X-XPL) 210,209,209
209 XPL=XPL+DP
     NPL=NPL+1
    IF(NPL-400) 211,211,210
211 N=NPL
    Y1(N)=Y(1)
    Y3(N)=Y(3)
    Y4(N)=Y(4)
```

```
      XXX(N)=X
      TAA(N)=TA
  210 GOTO (212,152),IX
  212 CALL PLOT
      GOTO 996
  152 IF (SENSE SWITCH 6) 165, 101
  165 PRINT 166
  166 FORMAT (22HO RESET SENSE SWITCH 6/1HO/1HO)
      PAUSE
  996 READ INPUT TAPE 5,10,IT
      IF(IT-2) 991,200,991
  991 IF(IT)  992,1001,992
  992 WRITE OUTPUT TAPE 6,993
      CALL EXIT
  300 READ INPUT TAPE 5,20,IT,(XX(I),I=1,7)
      IP=3
      IF(IT) 997,302,970
  302 C1 = XX(1)
      C4=XX(2)
      C5=XX(3)
      C6=XX(4)
      ABIR=XX(5)
      EMIR=XX(6)
      ABUV=XX(7)
  304 READ INPUT TAPE 5,20,IT
      IF(IT) 997,971,1000
  400 READ INPUT TAPE 5, 20, IT, ( XX(I), I=1,7 )
      IP=4
      IF(IT) 997,402,970
  402 WGO=XX(1)
      WFO=XX (2)
      VBM=XX(3)
      FLO=XX(4)
      CD=XX(5)
      CB=XX(6)
  404 READ INPUT TAPE 5, 20, IT
      IF(IT) 997,971,1000
  500 READ INPUT TAPE 5,20,IT,(XX(I),I=1,7)
      IP=5
      IF(IT) 997,502,970
  502 TBO=XX(1)
      TFO=XX(2)
      ZO=XX(3)
      Y20=XX(4)
      DEC=XX(5)
      XLAT=XX(6)
      XLONG=XX(7)
  504 READ INPUT TAPE 5, 20, IT
      IF(IT) 997,971,1000
  950 READ INPUT TAPE 5,20,IT,(XX(I),I=1,7)
      IP=6
      IF(IT) 997,952,970
  952 H = XX(1)
      DPR=XX(2)
      XO=XX(3)
```

```
      XT=XX(4)
      GMTS=XX(5)
      GHA=XX(6)
      DP=XX(7)
  954 READ INPUT TAPE 5, 975, IT
      IF(IT) 997,971,1000
  700 IP=7
      DO 701 I=1,100
      READ INPUT TAPE 5, 30, IT, TCSI(I), CCSI(I)
      IF(IT)997,701,704
  701 CONTINUE
      READ INPUT TAPE 5, 30, IT
      IF(IT) 997,971,706
  706 I=I 1
  704 NSI=I-1
      DO 705 J=2,NSI
      DSIDT(J-1)=(CCSI(J)-CCSI(J-1))/(TCSI(J)-TCSI(J-1))
  705 CONTINUE
      GOTO 1000
  800 READ INPUT TAPE 5, 30, IT, XX(1)
      IP=8
      IF(IT)997,802,970
  802 VO = XX(1)
      DO 801 I= 1,100
      READ INPUT TAPE 5, 30, IT, TV(I), VV(I)
      IF(IT) 997,801,804
  801 CONTINUE
      READ INPUT TAPE 5, 30, IT
      IF(IT) 997,971,805
  805 I=I+1
  804 NVI=I-1
      GO TO 1000
  900 READ INPUT TAPE 5, 30, IT, XX(1)
      IP=9
      IF(IT; 997,902,970
  902 BO = XX(1)
      DO 901 I=1,100
      READ INPUT TAPE 5, 30, IT, TB(I), BB(I)
      IF(IT) 997,901,905
  901 CONTINUE
      READ INPUT TAPE 5, 30, IT
      IF(IT) 997,971,904
  904 I=I+1
  905 NBI=I-1
      GO TO 1000
  600 READ INPUT TAPE 5, 30, IT, XX(1)
      IP=10
      IF(IT) 997,602,970
  602 RHOO= XX(1)
      DO 603  I=1,100
      READ INPUT TAPE 5, 30, IT, ZZ(I), TZ(I)
      IF (IT) 997, 603, 608
  603 CONTINUE
      READ INPUT TAPE 5, 30, IT
```

```
      IF(IT) 997,971,609
  609 I = I+1
  608 NT = I-1
      RTZ(1) = RHO0 * TZ(1)
      DO 604 I=2, NT
      IF ( TZ(I) -TZ(I-1)) 605, 606, 605
  605 TZA(I-1) = ( TZ(I) - TZ(I-1))/(ZZ(I) -ZZ(I-1))
      TZB(I-1) =(ZZ(I) *TZ(I-1) - ZZ(I-1) * TZ(I)) / (ZZ(I) - ZZ(I-1))
      RTZ(I) = RTZ(I-1) *(TZ(I-1)/TZ(I))**(   XMA/( R     *TZA(I-1)) )
      GO TO 604
  606 TZA( I-1) = 0.0
      TZB( I-1) = TZ(I)
      RTZ(I) = RTZ(I-1) *EXPF(   XMA*(ZZ(I-1)-ZZ(I))/(R    *TZ(I-1)))
  604 CONTINUE
      GO TO 1000
  960 IP=11
      DO 961 I=1,100
      READ INPUT TAPE 5,30,IT,ZIR(I),RTIR(I)
      FIR(I)=SBOLZ*RTIR(I)**4.0
      IF(IT)997,961,962
  961 CONTINUE
      READ INPUT TAPE 5,30,IT
      IF(IT) 997,971,964
  964 I=I+1
  962 NIR=I-1
      DO 963 J=2,NIR
      DIRDZ(J-1)= (FIR(J)-FIR(J-1))/(ZIR(J)-ZIR(J-1))
  963 CONTINUE
      GOTO 1000
  980 IP=12
      DO 984 MZY=1,400
      ETIME(MZY)=0.
      EALT(MZY)=0.
  984 CONTINUE
      DO 981 I=1,400
      READ INPUT TAPE 5,31,IT,ETIME(I),EALT(I)
      IF(IT)997,981,982
  981 CONTINUE
      READ INPUT TAPE 5,30,IT
      IF(IT) 997,971,983
  983 I=I+1
  982 NEX=I-1
      GO TO 1000
 1400 IP=13
      DO 1401 I=1,100
      READ INPUT TAPE 5,30,IT,TIR(I), CIR(I)
      IF(IT)997,1401,1402
 1401 CONTINUE
      READ INPUT TAPE 5,30,IT
      IF(IT) 997,971,1403
 1403 I=I+1
 1402 NCIR=I-1
      DO1404 I=2,NCIR
      DCIRDT(I-1)=(CIR(I)-CIR(I-1))/(TIR(I)-TIR(I-1))
 1404 CONTINUE
      GO TO 1000
      END
```

```
      SUBROUTINE RNGKTA(H1,N1,N2,N3)
      COMMON X,Y,F,Q
      DIMENSION Y(100),F(100), Q(100)
      IF(N3-1)2,1,2
    1 H=H1
      HH=.5*H
      N=N1
      M=N2
      DO 3 I=1,N
    3 Q(I)=0.0
    2 DO 4 J=1,M
      CALL YPRIME
      DO 5 I=1,N
      S=F(I)*H
      T=.5*(S-2.*Q(I))
      Y(I)=Y(I)+T
    5 Q(I)=Q(I)+3.*T-.5*S
      X=X+HH
      CALL YPRIME
      DO 6 I=1,N
      S=F(I)*H
      T=.29289322*(S-Q(I))
      Y(I)=Y(I)+T
    6 Q(I)=Q(I)+3.*T -.29289322*S
      CALL YPRIME
      DO 7 I=1,N
      S=F(I)*H
      T=1.7071067*(S-Q(I))
      Y(I)=Y(I)+T
    7 Q(I)=Q(I)+3.*T-1.707106*S
      X=X+HH
      CALL YPRIME
      DO 8 I=1,N
      S=F(I)*H
      T=(S-2.*Q(I))/6.
      Y(I)=Y(I)+T
    8 Q(I)=Q(I)+3.*T-,5*S
    4 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE  YPRIME
C     COMPILATION 8D
   10 FORMAT(14H DIVIDE CHECK ,I1)
      DIMENSION SCALE(60)
      DIMENSION  ZZ(100), TZ(100), RTZ(100),  TZA(100), TZB(100)
      DIMENSION  TCSI(100),CCSI(100),    TV(100),    VV(100)
      DIMENSION   TB(100),    BB(100)    ,  XX(100)
      DIMENSION  Y(100), F(100), Q(100)
      DIMENSION  TITLE (12)
      DIMENSION ZIR(100),FIR(100),DIRDZ(100),DSIDT(100)
      DIMENSION ZUV(100),FUV(100),DUVDZ(100)
      DIMENSION Y1(400),Y3(400),Y4(400),XXX(400),TAA(400)
      DIMENSION ETIME(400),EALT(400)
      COMMON X, Y, F, Q
      COMMON NT, ZZ, TZ, RTZ, TZA, TZB
      COMMON C1, C2, C3, C4, C5, C6, C7
      COMMON TA, RT, P, RHO, PHI, WG, OMEGA, CSI, E, V
      COMMON XMA, XMB, G, CP, CV, R, CD, VBM ,WF
      COMMON DEH2,DEH3,DEH4,DEH5,DEH6,DEH7
      COMMON WFH,DWFH,WFO
      COMMON CB
      COMMON ZIR,FIR,DIRDZ,NIR,ZUV,FUV,DUVDZ,NUV,DSIDT
      COMMON ABIR,EMIR,ABUV
      COMMON M,Y1,Y3,Y4,TAA,XXX
      COMMON DP,XT,XO,ETIME,EALT
      COMMON CCIR
      COMMON  SLATD,CLATD,RAD,GHA,XLONG,PZER,FSOL,SBOLZ,AH,AM,AS,GMTS
      CALL RHOT ( Y(1), TA, RT )
      P=RT*R/XMA
      RHO = RT / TA
      PHI=Y(6)**(1.0/3.0)
      DO 181 I=2,NIR
      IF(Y(1)-ZIR(I  ))182,181,181
  181 CONTINUE
      GO TO (184,183),IXY
  184 PRINT 20,X,Y(1)
   20 FORMAT(1H0,9H AT TIME=F7.0,75H SEC THE BALLOON WENT ABOVE THE INFR
     1ARED FIELD SPECIFIED IN THE INPUT DATA./18H BALLOON ALTITUDE=F7.0,
     26H FEET./123H START TO CONTINUE FLIGHT USING INFRARED DATA OF UPPE
     3R POINT OF SPECIFIED FIELD FOR AS LONG AS BALLOON REMAINS ABOVE FI
     4ELD./51H SENSE SWITCH 6 DOWN AND START TO TERMINATE FLIGHT.)
      PAUSE
      IXY=2
      GO TO 183
  182 IXY=1
  183 FLUXIR= DIRDZ(I-1)*(Y(1)-ZIR(I-1))+FIR(I-1)
      DEH2=CCIR*ABIR*4.83*PHI*PHI*FLUXIR
      DEH3=EMIR*4.83*PHI*PHI*SBOLZ*(Y(3)**4.)
  186 XLAH=RAD*(GHA-XLONG+X/240.)
      CAM= SLATD + CLATD* COSF(XLAH)
      IF (CAM) 100,101,101
  101 AIRM= (P/PZER)*((1228.6 +376750.44*CAM*CAM)**.5 -613.8*CAM)
      TRANS=.5*(EXPF(-.65*AIRM)+EXPF(-.095*AIRM))
      FLUXUV= FSOL*TRANS
```

```
      GOTO 110
  100 RETH =  20903520.
      CAL=RETH/(RETH+Y(1))
      CAMM=-1.*(1.-CAL*CAL)**.5
      IF (CAMM-CAM)102,103,103
  102 ZZ1=(RETH+Y(1))*(1.-CAM*CAM)**.5-RETH
      CALL RHOT(ZZ1,TTT,RTT)
      PZ1=RTT*R/XMA
      AIRM1=35.1*PZ1/PZER
      AIRM2=(P/PZER)*((1228.6+376750.44*CAM*CAM)**.5+613.8*CAM)
      TRAN1=.5*(EXPF(-.65*AIRM1)+EXPF(-.095*AIRM1))
      TRAN2=.5*(EXPF(-.65*AIRM2)+EXPF(-.095*AIRM2))
      FLUXUV=FSOL* TRAN1*TRAN1/TRAN2
      GOTO 110
  103 FLUXUV=0.
  110 CONTINUE
      DEHH7=DEH7
      DEH7=CSI*ABUV*1.208*PHI*PHI*FLUXUV
      IF(DEHH7)111,111,115
  111 IF(DEH7)119,119,114
  114 IF(X)119,119,120
  120 INM=1
      GO TO 121
  112 WRITE OUTPUT TAPE 6,113,AH,AM,AS
  113 FORMAT (1H0,25X,4HGMT=3F3.0,5X,18HSUNRISE AT BALLOON)
      GO TO 119
  115 IF(DEH7)118,118,119
  118 INM=2
  121 AH=INTF((GMTS+X)/3600.)
      AM=INTF((GMTS+X-3600.*AH)/60.)
      AS=GMTS+X-60.*AM-3600.*AH
      AM=ABSF(AM)
      AS=ABSF(AS)
      GO TO(112,116),INM
  116 WRITE OUTPUT TAPE 6,117,AH,AM,AS
  117 FORMAT (1H0,25X,4HGMT=3F3.0,5X,17HSUNSET AT BALLOON)
  119 VISA=1.096E-05*(TA/460.)**.883
      CAIR=260.1*VISA
      VISH=1.211E-05*(Y(4)/460.)**.682
      CHE=1443.*VISH
      RHOH=(XMB*P)/(R*Y(4))
      FNUS=.37*((1.24*RHO*PHI*ABSF(Y(2))/VISA)**.6)
      DEH4=3.895*C4*CAIR*PHI*FNUS*(TA-Y(3))
      XNUSA=.1612*PHI*((RHO*RHO*G*.7*ABSF(TA-Y(3)))/(VISA*VISA*TA))**
     1(1.0/3.0)
      DEH5=3.895*C5*CAIR*PHI*XNUSA*(TA-Y(3))
      XNUSH=.1612*PHI*((RHOH*RHOH*G*.7*ABSF(Y(4)-Y(3)))/(VISH*VISH*Y(4))
     1)**(1.0/3.0)
      DEH6=3.895*C6*CHE*PHI*XNUSH*(Y(4)-Y(3))
      F(1) = Y(2)
      F(2)=((RHO*Y(6)-WG-Y(5))*G-.6038*CD*PHI*PHI*RHO*Y(2)*ABSF(Y(2)))/
     1(WG+Y(5)+CB*RHO*Y(6))
      F(3)=(DEH2-DEH3+DEH4+DEH5+DEH6+DEH7)/(C1*WF)
      F(4)=(-DEH6-RHO*Y(6)*Y(2))/(Y(5)*(CV+R/XMB))
      F(5)=RHOH*E-RHOH*V/(RHO-RHOH)
      F(6)=((F(4)/Y(4))+(F(5)/Y(5))+(XMA*Y(2)/(R*TA)))*Y(6)
      RETURN
      END
```

```
      SUBROUTINE RHOT ( Z, T, RT)
C     COMPILATION 8C
      DIMENSION SCALE(60)
      DIMENSION  ZZ(100), TZ(100), RTZ(100),  TZA(100), TZB(100)
      DIMENSION  TCSI(100),CCSI(100),    TV(100),    VV(100)
      DIMENSION   TB(100),    BB(100)    ,  XX(100)
      DIMENSION  Y(100), F(100), Q(100)
      DIMENSION  TITLE (12)
      DIMENSION ZIR(100),FIR(100),DIRDZ(100),DSIDT(100)
      DIMENSION ZUV(100),FUV(100),DUVDZ(100)
      DIMENSION Y1(400),Y3(400),Y4(400),XXX(400),TAA(400)
      DIMENSION ETIME(400),EALT(400)
      COMMON X, Y, F, Q
      COMMON NT, ZZ, TZ, RTZ, TZA, TZB
      COMMON C1, C2, C3, C4, C5, C6, C7
      COMMON TA, RT, P, RHO, PHI, WG, OMEGA, CSI, E, V
      COMMON XMA, XMB, G, CP, CV, R, CD, VBM ,WF
      COMMON DEH2,DEH3,DEH4,DEH5,DEH6,DEH7
      COMMON WFH,DWFH,WFO
      COMMON CB
      COMMON ZIR,FIR,DIRDZ,NIR,ZUV,FUV,DUVDZ,NUV,DSIDT
      COMMON ABIR,EMIR,ABUV
      COMMON M,Y1,Y3,Y4,TAA,XXX
      COMMON DP,XT,XO,ETIME,EALT
      IF ( Z- ZZ(1)) 100,200, 200
  100 T = TZ(1)
      RT = RTZ(1)
      RETURN
  200 NM1 = NT-1
      DO 201 J=1, NM1
      IF ( Z - ZZ( J+1)) 300, 300, 201
  201 CONTINUE
      GO TO (202,203),IYX
  202 PRINT 20,X,Y(1)
   20 FORMAT(1H0,9H AT TIME=F7.0,63H SEC THE BALLOON WENT ABOVE ATMOSPHE
     1RE SPECIFIED IN INPUT DATA./18H BALLOON ALTITUDE=F7.0,6H FEET./128
     2H START TO CONTINUE FLIGHT USING ATMOSPHERIC DATA OF UPPER POINT O
     3F SPECIFIED ATMOSPHERE FOR AS LONG AS BALLOON REMAINS ABOVE IT./51
     4H SENSE SWITCH 6 DOWN AND START TO TERMINATE FLIGHT.)
      IYX=2
      PAUSE
      GO TO 203
  300 IYX=1
  203 IF ( TZA(J)) 301, 302, 301
  301 T =  TZA( J) * Z + TZB (J)
      RT=RTZ(J)*(TZ(J)/T)**(XMA/(R*TZA(J)))
      RETURN
  302 T = TZB (J)
      RT=RTZ(J)*EXPF(XMA*(ZZ(J)-Z)/(R*T))
      RETURN
      END
```

```
      SUBROUTINE PLUT   /

      SUBROUTINE PLUT
C     COMPILATION 7D
 3000 FORMAT(14H GRAPH PRINTED)
      DIMENSION SCALE(60)
      DIMENSION ZZ(100),TZ(100),RTZ(100),TZA(100),TZB(100)
      DIMENSION TCSI(100),CCSI(100),TV(100),VV(100)
      DIMENSION TB(100),BB(100),XX(100)
      DIMENSION Y(100),F(100),Q(100)
      DIMENSION TITLE (12)
      DIMENSION ZIR(100),FIR(100),DIRDZ(100),DSIDT(100)
      DIMENSION ZUV(100),FUV(100),DUVDZ(100)
      DIMENSION Y1(400),Y3(400),Y4(400),XXX(400),TAA(400)
      DIMENSION ETIME(400),EALT(400)
      COMMON X,Y,F,Q
      COMMON NT,ZZ,TZ,RTZ,TZA,TZB
      COMMON C1,C2,C3,C4,C5,C6,C7
      COMMON TA,RT,P,RHO,PHI,WG,OMEGA,CSI,E,V
      COMMON XMA,XMB,G,CP,CV,R,CD,VBM,WF
      COMMON DEH2,DEH3,DEH4,DEH5,DEH6,DEH7
      COMMON WFH,DWFH,WFO
      COMMON CB
      COMMON ZIR,FIR,DIRDZ,NIR,ZUV,FUV,DUVDZ,NUV,DSIDT
      COMMON ABIR,EMIR,ABUV
      COMMON M,Y1,Y3,Y4,TAA,XXX
      COMMON DP,XT,XO,ETIME,EALT
      SCALE(5)=6H 325
      SCALE(7)=6H 10000
      SCALE(11)=6H 350
      SCALE(13)=6H 20000
      SCALE(17)=6H 375
      SCALE(19)=6H 30000
      SCALE(23)=6H 400
      SCALE(25)=6H 40000
      SCALE(29)=6H 425
      SCALE(31)=6H 50000
      SCALE(35)=6H 450
      SCALE(37)=6H 60000
      SCALE(41)=6H 475
      SCALE(43)=6H 70000
      SCALE(47)=6H 500
      SCALE(49)=6H 80000
      SCALE(53)=6H 525
      SCALE(55)=6H 90000
      SCALE(59)=6H 550
   16 M=1
      MT=XINTF((XT-XO)/10800.)+1
    7 MM=M+1
      GO TO (4,12,13,14,15,16,17),MM
   12 SCALE(1)=6H 0-3
      GO TO 9
   13 SCALE(1)=6H 3-6
      GO TO 9
   14 SCALE(1)=6H 6-9
      GO TO 9
   15 SCALE(1)=6H 9-12
      GO TO 9
```

```
      SUBROUTINE PLOT

   16 SCALE(1)=6H 12-15
      GO TO 9
   17 SCALE(1)=6H 15-18
    9 XM=FLOATF(M)
      XXM=(XM-1.)*10800.
      XXXM=XM*10800.
      CALL LIMITS (XXM,XXXM,0.,100000.)
      CALL GRID (XXM,1800.,0.,10000.)
      K=XFIXF((10800./DP)*(XM-1.))+1
      L=XFIXF((10800./DP)*(XM))+1
      DO90J=K,L
   90 CALL POINTS (XXX(J),Y1(J),35)
   10 CALL LIMITS (XXM,XXXM,300.,550.)
      DO100J=K,L
  100 CALL POINTS (XXX(J),Y3(J),15)
   11 CALL LIMITS (XXM,XXXM,300.,550.)
      DO110J=K,L
  110 CALL POINTS (XXX(J),Y4(J),11)
      CALL LIMITS (XXM,XXXM,300.,550.)
      DO120J=K,L
  120 CALL POINTS (XXX(J),TAA(J),10)
      CALL LIMITS (XXM,XXXM,0.,100000.)
      DO130J=1,400
  130 CALL POINTS (ETIME(J),EALT(J),14)
      CALL GRAPH (SCALE)
    4 PRINT 3000
      M=M+1
      IF(M-7)131,19,19
  131 IF(M-MT)7,7,19
   19 RETURN
      END(1,0,0,0,1,0,0,0,0,0,0,0,0,0,0)
```

```
      SUBROUTINE PLOT
C     COMPILATION FOR 147500 FT
 3000 FORMAT(14H GRAPH PRINTED)
      DIMENSION SCALE(60)
      DIMENSION  ZZ(100), TZ(100), RTZ(100),  TZA(100), TZB(100)
      DIMENSION  TCSI(100),CCSI(100),    TV(100),    VV(100)
      DIMENSION   TB(100),    BB(100)    ,   XX(100)
      DIMENSION  Y(100), F(100), Q(100)
      DIMENSION  TITLE (12)
      DIMENSION ZIR(100),FIR(100),DIRDZ(100),DSIDT(100)
      DIMENSION ZUV(100),FUV(100),DUVDZ(100)
      DIMENSION Y1(400),Y3(400),Y4(400),XXX(400),TAA(400)
      DIMENSION ETIME(400),EALT(400)
      COMMON X, Y, F, Q
      COMMON NT, ZZ, TZ, RTZ, TZA, TZB
      COMMON C1, C2, C3, C4, C5, C6, C7
      COMMON TA, RT, P, RHO, PHI, WG, OMEGA, CSI, E, V
      COMMON XMA, XMB, G, CP, CV, R, CD, VBM ,WF
      COMMON DEH2,DEH3,DEH4,DEH5,DEH6,DEH7
      COMMON WFH,DWFH,WFO
      COMMON CB
      COMMON ZIR,FIR,DIRDZ,NIR,ZUV,FUV,DUVDZ,NUV,DSIDT
      COMMON ABIR,EMIR,ABUV
      COMMON M,Y1,Y3,Y4,TAA,XXX
      COMMON DP,XT,XO,ETIME,EALT
      SCALE(8)=6H 340
      SCALE(9)=6H 20000
      SCALE(16)=6H 380
      SCALE(17)=6H 40000
      SCALE(24)=6H 420
      SCALE(25)=6H 60000
      SCALE(32)=6H 460
      SCALE(33)=6H 80000
      SCALE(40)=6H 500
      SCALE(41)=6H100000
      SCALE(48)=6H 540
      SCALE(49)=6H120000
      SCALE(56)=6H 580
      SCALE(57)=6H140000
   18 M=1
      MT=XINTF((XT-XO)/10800.)+1
    7 MM=M+1
      GO TO (4,12,13,14,15,16,17),MM
   12 SCALE(1)=6H 0 - 3
      GO TO 9
   13 SCALE(1)=6H 3 - 6
      GO TO 9
   14 SCALE(1)=6H 6 - 9
      GO TO 9
   15 SCALE(1)=6H 9-12
      GO TO 9
   16 SCALE(1)=6H 12-15
      GO TO 9
   17 SCALE(1)=6H 15-18
```

```
    9 XM=FLOATF(M)
      XXM= (XM-1.)*10800.
      XXXM=XM*10800.
      CALL LIMITS(XXM,XXXM,0.,147500.)
      CALL GRID (XXM,1800.,0.,20000.)
      K=XFIXF((10800./DP)*(XM-1.))+1
      L=XFIXF((10800./DP)*(XM))+1
      DO 90 J=K,L
   90 CALL POINTS (XXX(J),Y1(J),35)
   10 CALL LIMITS (XXM,XXXM,300.,595.)
      DO 100 J=K,L
  100 CALL POINTS (XXX(J),Y3(J),15)
   11 CALL LIMITS (XXM,XXXM,300.,595.)
      DO 110 J=K,L
  110 CALL POINTS (XXX(J),Y4(J),11)
      CALL LIMITS (XXM,XXXM,300.,595.)
      DO 120 J=K,L
  120 CALL POINTS (XXX(J),TAA(J),10)
      CALL LIMITS (XXM,XXXM,0.,147500.)
      DO 130 J=1,400
  130 CALL POINTS (ETIME(J),EALT(J),14)
      CALL GRAPH (SCALE)
    4 PRINT 3000
      M=M+1
      IF(M-7)131,19,19
  131 IF(M-MT)7,7,19
   19 RETURN
      END
```