AD NUMBER

AD485025

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; JUL 1966. Other requests shall be referred to Air Force Weapons Lab., Kirtland AFB, NM.

AUTHORITY

AFWL per DTIC form 55

FIREBALL PHENOMENOLOGY AND CODE DEVELOPMENT

Volume IV

SPUTTER Subroutines for Radiation Transport in Planes

General Atomic Division of General Dynamics Corporation
Special Nuclear Effects Laboratory
San Diego, California

Contract AF 29(601)-6492

TECHNICAL REPORT NO. AFWL-TR-65-143, Vol IV

July 1966

AIR FORCE WEAPONS LABORATORY
Research and Technology Division
Air Force Systems Command
Kirtland Air Force Base
New Mexico

AFWL-TR-65-143, Vol IV

Research and Technology Division
AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base
New Mexico

AFWL-TR-65-143, Vol IV

FIREBALL PHENOMENOLOGY AND CODE DEVELOPMENT

Volume IV

SPUTTER Subroutines for Radiation Transport in Planes

General Atomic Division of General Dynamics Corporation
Special Nuclear Effects Laboratory
San Diego, California

Contract AF 29(601)-6492

TECHNICAL REPORT NO. AFWL-TR-65-143, Vol IV

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of AFWL (WLRT), Kirtland AFB, NM, 87117. Distribution is limited because of the technology discussed in the report.

FOREWORD

This report was prepared by General Atomic Division of General Dynamics Corporation, San Diego, California, under Contract AF 29(601)-6492. The research was performed under Program Element 7.60.06.01.D, Project 5710, Subtask 07.003/005, and was funded by the Defense Atomic Support Agency (DASA).

Inclusive dates of research were 1 June 1963 to 13 July 1965. The report was submitted 15 March 1966 by the Air Force Weapons Laboratory Project Officer, 1Lt F. C. Tompkins III (WLRT). The contractor's report number is GA-6585.

This report is divided into six volumes as follows: Volume I, Summary and the Fireball Models; Volume II, Early Fireball Phenomena in the TIGHTROPE Event;* Volume III, SPUTTER Subroutines for Radiation Transport in Spheres; Volume IV, SPUTTER Subroutines for Radiation Transport in Planes; Volume V, Material Properties; and Volume VI, Extensions of the Physics and Problem Areas.

The SPUTTER subroutines for radiation transport in planes described in Volume IV were developed by Dr. B. E. Freeman and Dr. C. G. Davis, Jr. The cooperation and contributions of Captains Milton Gillespie, William Whittaker, and George Spillman of the Air Force Weapons Laboratory are gratefully acknowledged.

This technical report has been reviewed and is approved.

F. C. TOMPKINS III
1Lt, USAF
Project Officer

RALPH W. PENNINGTON
Lt Colonel, USAF
Chief, Theoretical Branch

WILLIAM H. STEPHENS
Colonel, USAF
Chief, Research Division

* Volume II has been withdrawn , and will not be published.

ABSTRACT

The radiation-transport subroutines of the SPUTTER code for plane slab geometry have been supplemented by alternative formulation based on integration along sampling ray paths through the slab. Angular integrations are performed by the Gaussian quadrature method which determines the ray angles. Options may be exercised to determine the number of angles and the nature of the radiation boundary condition at one boundary of the transport region. The characteristic ray code differs from the current integral method in performing problems having a large number of zones more rapidly and in having more general boundary conditions. For most applications a small number of angles give adequate accuracy. The numerical method used in the ray code is described. In addition, the organization of the code is discussed and subroutines are listed.

CONTENTS

This page intentionally left blank.

SECTION I

INTRODUCTION

The new routines for radiation transport in planes closely parallel the spherical radiation transport subroutines (Volume III) in mathematical formulation and code organization. This parallelism is especially close between the SRADTN (for spheres) and PRADTN (for planes) subroutines in which calculations peripheral to the transport integration are performed. In fact, it is likely that these subroutines can be condensed to form a single subroutine for spheres and planes, although currently they are separate.

The routines reported here are to be considered as alternatives to the routines based on the integral formulation of the transport equation currently in use. By comparison, the current integral version is more accurate in the performance of angular integrations of the intensity, but for problems requiring a large number of zones it requires more computation time. Boundary conditions on the radiation intensity, however, are much more naturally incorporated into the new version.

Conditions suggesting a preference for using the new routines are: (1) a large number of zones and a desire to reduce calculation time and (2) the necessity of specifying a radiation intensity incident on the slab surface which has arbitrary angular and frequency dependence.

The numerical sequences used in solving the transport equation are discussed in Section II. A description of the diffusion approximation used in conjunction with the transport solution is given in Section III and a brief description of the methods of frequency integration is given in Section IV. Section V includes the actual code description in terms of code organization and economics. Section VI includes some initial studies on timing and accuracy in the angular integrations.

SECTION II

NUMERICAL SOLUTION OF THE TRANSPORT EQUATION

The radiation routines described herein contain a formulation based on the numerical solution of the radiation transport equation along a selection of sampling rays through the slab. Relevant averages over the angular distribution are obtained by numerical quadrature, as described in Section 2. 3, and the numerical solution of the transport equation along the photon ray is presented in Section 2. 1. Criteria for selecting the sampling rays are discussed in Section 2. 2. All of the derivations of this section apply to photons of a particular frequency; integration over frequency is discussed in Section IV.

The radiation transport equation in plane geometry that describes the changes in the specific intensity I_ν of photons of frequency ν resulting from pure absorption and emission according to the local thermodynamic equilibrium assumption is

$$\mu \frac{\partial I_\nu}{\partial x} = \sigma'_\nu (B_\nu - I_\nu) \ , \qquad (2.1)$$

where

$$B_\nu = \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/\theta} - 1} \ ,$$

$$\sigma'_\nu = \sigma_\nu (1 - e^{-h\nu/\theta}) \ ,$$

and σ_ν is the pure absorption coefficient. The scattering coefficient is assumed to be negligibly small compared to the absorption coefficient. Additionally, the retardation of the photons is neglected, as is valid when the radiation energy is small and temperatures change slowly. The resulting equation describes the quasi-steady intensity field resulting from the distribution of sources existing at a particular time.

Defining the monochromatic optical depth, τ, as

$$\tau = \frac{1}{\mu} \int_0^x \sigma'_\nu \, dx \ , \qquad (2.2)$$

THIS PAGE IS MISSING IN ORIGINAL DOCUMENT the values

$$B = a_- + b_-\tau, \qquad \tau_{i-1} \leq \tau \leq \tau_{i-\frac{1}{2}} ,$$

where

$$a_- = \frac{B_{i-1}\,\tau_{i-\frac{1}{2}} - B_{i-\frac{1}{2}}\,\tau_{i-1}}{\tau_{i-\frac{1}{2}} - \tau_{i-1}}, \qquad b_- = \frac{B_{i-\frac{1}{2}} - B_{i-1}}{\tau_{i-\frac{1}{2}} - \tau_{i-1}} \tag{2.4}$$

and

$$B = a_+ + b_+\tau, \qquad \tau_{i-\frac{1}{2}} \leq \tau \leq \tau_i ,$$

where

$$a_+ = \frac{B_{i-\frac{1}{2}}\,\tau_i - B_i\tau_{i-\frac{1}{2}}}{\tau_i - \tau_{i-\frac{1}{2}}}, \qquad b_+ = \frac{B_i - B_{i-\frac{1}{2}}}{\tau_i - \tau_{i-\frac{1}{2}}} .$$

For the case of a constant or step-function source, the source function B takes a value dependent on which interface of the zone is affected. If the left interface ($\tau = \tau_{i-1}$) satisfies the criteria for a constant source,

$$B = B_{i-\frac{1}{2}}, \qquad \tau_{i-1} \leq \tau \leq \tau_{i-\frac{1}{2}} .$$

If the right interface ($\tau = \tau_i$) satisfies the criteria,

$$B = B_{i-\frac{1}{2}}, \qquad \tau_{i-\frac{1}{2}} \leq \tau \leq \tau_i .$$

The integral of Eq. (2.3) can be evaluated with the interpolation function of Eq. (2.4) to give for the intensity

$$I_i = \alpha_{i-\frac{1}{2}} + [(I_{i-1} + \gamma_{i-\frac{1}{2}})e^{-\Delta/2} + \beta_{i-\frac{1}{2}}]\,e^{-\Delta/2} , \tag{2.5}$$

where

$$\alpha_{i-\frac{1}{2}} = a_+ + b_+(\tau_i - 1) ,$$

$$\beta_{i-\frac{1}{2}} = a_- - a_+ + (b_- - b_+)\left(\tau_i - \frac{\Delta}{2} - 1\right) ,$$

$$\gamma_{i-\frac{1}{2}} = b_-\,(1 + \Delta - \tau_i) - a_- .$$

In these expressions, $\Delta = \tau_i - \tau_{i-1}$. The coefficients of Eq. (2.5) can be re-expressed by using the definitions of Eq. (2.4):

$$\left.\begin{aligned}\alpha_{i-\frac{1}{2}} &= B_i - \frac{B_i - B_{i-\frac{1}{2}}}{\Delta/2}\ ,\\ \beta_{i-\frac{1}{2}} &= \frac{B_i - B_{i-\frac{1}{2}}}{\Delta/2} - \frac{B_{i-\frac{1}{2}} - B_{i-1}}{\Delta/2}\ ,\\ \gamma_{i-\frac{1}{2}} &= -\left(B_{i-1} - \frac{B_{i-\frac{1}{2}} - B_{i-1}}{\Delta/2}\right).\end{aligned}\right\} \tag{2.6}$$

The terms in Eq. (2.6) may be interpreted as containing combinations of numerical approximations to the values of the source function and the τ derivative of the source function at the boundaries of the interval.

This form of the equation, in fact, can be obtained in another way starting from Eq. (2.3). Two successive integrations by parts transforms the expression for I_i into the following equivalent form:

$$I_i = \left(B - \frac{\partial B}{\partial \tau}\right)_i + \left[I_{i-1} - \left(B - \frac{\partial B}{\partial \tau}\right)_{i-1}\right] e^{-\Delta} + \int_{\tau_{i-1}}^{\tau_i} \frac{\partial^2 B}{\partial \tau^2} e^{-(\tau_1 - \tau)} d\tau\ , \tag{2.7}$$

in terms of values of the source function and the first two derivatives of the source function with respect to τ.

In an optically thin interval, the most important contribution arises from the terms I_{i-1} and B, which represent the transmitted intensity and the emission from the zone. The derivative terms cancel in this approximation; this is perhaps more directly indicated by Eq. (2.3). In the optically thick interval, which is the extreme opposite, only the first two terms evaluated at i are usually of significance. The terms from i - 1 are strongly attenuated and $\partial^2 B/\partial\tau^2$ in the integral is usually small. In the limit, the diffusion approximation results from the term $\partial B/\partial\tau)_i$. Between limits, it is necessary to consider the integral term in Eq. (2.7).

If Δ is not too large, a representative mean value of the exponential in the interval may be taken to give for the integral of Eq. (2.7)

$$\int_{\tau_{i-1}}^{\tau_i} \frac{\partial^2 B}{\partial \tau^2} e^{-(\tau_1 - \tau)} d\tau \cong e^{-\Delta/2}\left[\left.\frac{\partial B}{\partial \tau}\right)_i - \left.\frac{\partial B}{\partial \tau}\right)_{i-1}\right]\ ,$$

and thus the expression for intensity becomes

$$I_i = \left(B - \frac{\partial B}{\partial \tau}\right)_i + \left\{\left[I_{i-1} - \left(B - \frac{\partial B}{\partial \tau}\right)_{i-1}\right] e^{-\Delta/2} + \left[\left(\frac{\partial B}{\partial \tau}\right)_i - \left(\frac{\partial B}{\partial \tau}\right)_{i-1}\right]\right\} e^{-\Delta/2} \ . \tag{2.8}$$

This expression has just the form of Eqs. (2.5) and (2.6) when the difference expressions are identified with the derivatives.

It is clear from the derivation of Eq. (2.5) that the resulting intensity is a positive quantity. With positive values for zone source functions, the linear interpolation expression assures that the integral contribution is always positive. Since the boundary intensity is always a positive quantity, the positivity of all intensities is assured.

In the diffusion approximation limit, only quantities at interface i will survive, and

$$I_i = B_i - \left.\frac{\partial B}{\partial \tau}\right)_i \ ,$$

which can be evaluated as

$$\frac{\partial B}{\partial \tau} = \mu \frac{\partial B}{\partial h} \ , \tag{2.9}$$

where

$$h = \int_0^x \sigma' \, dx \ .$$

The independent variable h depends only on x, so that angular integrations of I_i can be performed explicitly in the diffusion approximation, which takes account of the dependence on angle of Eq. (2.9). A difference approximation can also be based on this expression, assuming that B is linear in h, i.e.,

$$\left.\frac{\partial B}{\partial \tau}\right)_i \cong \frac{B_i - B_{i-\frac{1}{2}}}{h_i - h_{i-\frac{1}{2}}} \mu \ , \tag{2.10}$$

where μ is the cosine of the angle which the ray makes with the slab normal. The corresponding equation for the intensity is Eq. (2.5), in which

$$\alpha_{i-\frac{1}{2}} = B_i - \nu \frac{B_i - B_{i-\frac{1}{2}}}{h_i - h_{i-\frac{1}{2}}} ,$$

$$\beta_{i-\frac{1}{2}} = \mu \frac{B_i - B_{i-\frac{1}{2}}}{h_i - h_{i-\frac{1}{2}}} - \mu \frac{B_{i-\frac{1}{2}} - B_{i-1}}{h_{i-\frac{1}{2}} - h_{i-1}} , \qquad (2.11)$$

$$\gamma_{i-\frac{1}{2}} = - \left(B_{i-1} - \mu \frac{B_{i-\frac{1}{2}} - B_{i-1}}{h_{i-\frac{1}{2}} - h_{i-1}} \right) ;$$

2.1.2. Small-optical-depth Expansion

If the optical depth is very small, the intensity expression in Eq. (2.3) takes a much simpler form,

$$I_i = I_{i-1} + \left[\frac{1}{4} B_i + \frac{1}{4} B_{i-1} + \frac{1}{2} B_{i-\frac{1}{2}} - I_{i-1} \right] \Delta . \qquad (2.12)$$

Although this result is the limiting form of Eqs. (2.5) and (2.6), the terms must cancel through second order in an expansion in Δ before the first surviving term, derived in part from the quadratic terms of the exponentials, is obtained. Consequently, for sufficiently small argument, the finite number of figures used in the exponential will render the result inaccurate. For the exponential from the IBM-7044 system, this restricts the argument to a number greater than $\sim 2 \times 10^{-4}$; but with the lower-accuracy fast exponential (see Section V), the argument must be somewhat larger. Since the relative error approximately equals the argument of the exponential, the criterion for using Eq. (2.12) in the PTRANS subroutine is now set at $\Delta \leq 2 \times 10^{-2}$. With this value, the greatest relative error arising from the expansion and cancellation should be on the order of 1 percent.

2.1.3. Boundary Conditions

Integration of the transport equation to obtain intensities is performed through the thickness of a zone, called a "trans" region. At intersections of characteristic rays with the inner and outer surfaces of each layer it is necessary to supply the starting value of the intensity I_{i-1} required in Eq. (2.5). Three classes of boundary conditions occur:

1. The trans region outside boundary coincides with outside zones of the SPUTTER calculation and a prescribed function, I_0, is applied at the left boundary value:

$$I(X_{IA+1}, \mu) = I_0, \quad \mu \leq 0 \text{ or blackbody boundary condition} . \qquad (2.13)$$

2. The right-hand boundary of the slab provides for reflective and transmittal boundary conditions as well as special routines to establish prescribed intensities for angles with $\mu<0$ at the boundary:

$$I(X_{IB}, -\mu) = I(X_{IB}, \mu), \; I(B_{IB}, \mu) = 0 \; ,$$

or (2.14)

$$I(B_{IB}, -\mu) = I_0(\mu, t) \; ,$$

where I_0 is the prescribed negatively directed boundary intensity applied to the outer boundary as a function of angle, frequency, and time. Intensities at up to 50 frequencies and six angles can be accommodated in the table located in the array QINT1(N). The table entries are used as I_0 and are formed in the subroutine QUE4 where they are stored in the QINT1 array. Since this subroutine is appropriate to the thermal interaction application, additional uses may require subroutines tailored to the specific application.

3. All other trans boundaries are bounded by regions in which the diffusion approximation is valid (see Section III). Consequently, the boundary surface intensities on contiguous trans regions inside or outside of a diffusion region are given by the diffusion approximation intensity derived in Section III:

$$I_{i-1} = B_{i-1} - \mu \left.\frac{\partial B}{\partial h}\right)_{i-1} \; . \tag{2.15}$$

2.2. ANGULAR INTEGRATION

Integrals over the polar angle of the intensity are required, as described in Section 2.2.1, to carry the calculation forward in time and to provide edits of informative derived quantities. These are formed by numerical quadrature using the intensities evaluated at a series of discrete values of polar angle by the integrations described in Section 2.2. Since in the plane calculation the value of the polar angle remains fixed along a characteristic ray and enters only parametrically in the equations, it is possible to exercise a choice of polar-angle values in order to optimize the accuracy of the resulting integrals.

The numerical quadrature method used for the PTRANS subroutine is the so-called double Gaussian.[1] In this method the integrals of the radiation quantities (flux, energy, pressure, etc.) are approximated by

$$\int I \, f(\mu) \, d\mu = \sum_{0}^{n} A_m (If)_m \; ,$$

where $(If)_m$ is the known value of the integrand at a chosen value μ_m of the cosine of the polar angle, μ. In the method of Gaussian, not only are the coefficients A_m determined but the values of μ_m are fixed to minimize the difference between the integral and the approximation. The result of this minimization is to relate the μ_m to the zeros of the Legendre polynomial of order n + 1.

For those integrals having the range $-1 \leq \mu \leq 1$ it is frequently advantageous to treat the forward and backward hemispheres separately to allow for the possibility of a discontinuity in I at $\mu = 0$. Such discontinuities or very abrupt changes in the values of the intensity between forward and backward directions may occur in systems which are transparent enough that strong source regions are accessible. In these cases, a better fit to the integrand is obtained by the two approximating functions which permit a discontinuity at $\mu = 0$ than by a single approximating function which imposes a smooth behavior near $\mu = 0$. The method used in PTRANS, based on separate integration regions for $-1 \leq \mu \leq 0$ and $0 \leq \mu \leq 1$, is called the double Gaussian quadrature method. Values of A_m and μ_m are derived by a simple transformation from those for the single integration region. Since the angles for a single integration region are arranged symmetrically about the interval midpoint, for double region integration it is possible to identify pairs of angles $\pm\mu_m$ having the same weight A_m. In Table 2. 1, the values[2] for the $0 \leq \mu \leq 1$ interval are recorded for values of n = 1, 2, 3, 4, 5. The total number of forward and backward angles, 2n + 2, for each n (also equal to the total number of entries in the table of μ_m and A_m for each n) is also listed in the table.

The backward and the corresponding forward ray integrations in the PTRANS subroutine are performed sequentially. Since the same absolute values of μ_m are required for these two calculations, many of the quantities formed in the backward integration pass can be used for the forward pass as well, and hence these quantities are saved to increase calculation efficiency. Contributions of the pair of forward and backward intensities to the weighted sums corresponding to the angular integrals are tallied at the same time that the forward integration pass is being calculated.

Table 2.1

GAUSSIAN WEIGHTS

n = LMDA(37)-1	Total No. of Angles, 2n+2	μ_m = RR(NMU)	A_m	$\mu_m A_m$ = RR(NGS)
1	4	0.2113248	0.5	0.1056624
		0.7886752	0.5	0.3943376
2	6	0.1127017	0.2777778	0.03130603
		0.5	0.4444444	0.2222222
		0.8872983	0.2777778	0.2464718
3	8	0.0694318	0.1739274	0.0120761
		0.3300095	0.3260726	0.1076071
		0.6699905	0.3260726	0.2184655
		0.9305682	0.1739274	0.1618513
4	10	0.0469101	0.1184634	0.00555713
		0.2307653	0.2393143	0.055225436
		0.5	0.2844444	0.1422222
		0.7692347	0.2393143	0.1840889
		0.9530899	0.1184634	0.1129063
5	12	0.0337652	0.0856622	0.00289240
		0.1693953	0.1803808	0.03055566
		0.3806904	0.2339570	0.0890652
		0.6193096	0.2339570	0.1448918
		0.8306047	0.1803808	0.1498251
		0.9662348	0.0856622	0.0827698

SECTION III

THE DIFFUSION APPROXIMATION

The radiation transport equation in the limiting case of an optically thick medium admits of the diffusion approximation in which the expression for the radiation intensity is greatly simplified; only the local properties affect the radiation intensity at the point in question. An expansion of the radiation source function B_ν about the point r permits the intensity $I_\nu(\mu)$ of the radiation field in the direction making an angle, whose cosine is μ, with the linear direction to be formed.

3.1. DIFFERENTIAL FORM OF THE DIFFUSION FLUX

The general solution of the transport equation forms the starting point of the derivation. The integral expression for the intensity applicable to all geometries is

$$I(\tau) = \int_{-\infty}^{\tau} B(\tau')e^{-(\tau-\tau')}\, d\tau' \quad ,$$

where $\tau = \int_0^x \kappa_\nu \rho \, ds$, in which κ_ν is the monochromatic absorption coefficient (in cm^2/g) at frequency ν. By expanding $B(\tau')$ in series about the point τ, i.e.,

$$B(\tau') = B(\tau) + \frac{\partial B}{\partial \tau}(\tau' - \tau) + \frac{1}{2}\frac{\partial^2 B}{\partial \tau^2}(\tau' - \tau)^2 + \cdots \quad ,$$

the intensity becomes

$$I = B - \frac{\partial B}{\partial \tau} + \frac{1}{2}\frac{\partial^2 B}{\partial \tau^2} - \cdots$$

or

$$I = B - \frac{\mu}{\kappa\rho}\frac{\partial B}{\partial x} + \frac{\mu^2}{\kappa\rho}\frac{\partial}{\partial x}\left(\frac{1}{\kappa\rho}\frac{\partial B}{\partial x}\right) - \cdots$$

for plane slab geometry.

The diffusion approximation results from retention of only the first two terms, so that the diffusion intensity is

$$I = B - \frac{\mu}{\kappa\rho}\frac{\partial B}{\partial x} \qquad (3.1)$$

and the monochromatic diffusion flux ϕ_r and radiation energy E_R are

$$\phi_r = 2\pi c \int_{-1}^{1} I\mu \, d\mu = -\frac{4\pi c}{3}\frac{1}{\kappa\rho}\frac{\partial B}{\partial x} \; ,$$

$$E_R = 2\pi \int_{-1}^{1} I \, d\mu = 4\pi B \; . \qquad (3.2)$$

3.2. CRITERIA FOR THE SELECTION OF DIFFUSION REGIONS

The criteria for the validity of the diffusion approximation can be obtained by examination of the above derivation--namely, that the expansion of the source function be justified and that the expansion converge rapidly so that the neglect of all but the leading terms is valid. If the source function is linear in τ' at the point in question and is also linear for a distance of the order of one mean free path on either side of the point, the criteria are satisfied. These criteria are difficult to quantify since they refer to a finite region containing the point in question. If all of the terms (or a large number of them) were checked for rapid convergence, this would imply (making a smoothness assumption) that the diffusion criterion is met. It is not possible with finite differences, however, to form the higher-order local derivatives approximations.

In the SPUTTER subroutine PRADTN, criteria designed to give an indication of both the local and nonlocal behavior have been employed. First, at the zone interface at which the intensity and flux are to be evaluated, the inequality

$$\left|\frac{\partial B}{\partial h}\right| \ll B \qquad (3.3)$$

is required. In this expression $h = \int \kappa\rho \, dx$ is the optical depth normal to the slab; the derivative is approximated by the centered first difference of B between adjacent zones. The resulting expression, of course, contains some nonlocal aspects resulting from the finite difference approximation, which ensures that when neighboring zones are optically thick, no nonlocal source perturbation is close enough to invalidate the diffusion approximation. However, to provide for the cases when a source perturbation is located a fraction of an optical depth from an interface meeting the condition of Eq. (3.3), the diffusion region is constricted. Starting from the closest interfaces outside the diffusion region (where Eq. (3.3) is not satisfied), all of those interfaces lying within a prescribed number of mean free paths are removed from the diffusion region.

The criteria used in SPUTTER are controlled by input numbers. The criterion of Eq. (3.3) uses the input number HCB:

$$|TG| < HCB \times Y2 \quad , \tag{3.4}$$

where TG is the difference approximation to the gradient and Y2 is the source function evaluated at the interface by interpolation. The second criterion uses the input number HVB (in mean free paths). If

$$|Q3(\ ) - Q3(J)| > HVB \quad , \tag{3.5}$$

then the interface with index J which satisfies Eq. (3.4) is removed from the diffusion region. In Eq. (3.5), Q3 is the normal optical depth and I is the index of the nondiffusion interface adjoining the diffusion region.

Although the diffusion calculation is considerably faster than the transport, the establishment of two transport regions separated by the single zone requires still more calculation to set up characteristic rays and perform bookkeeping operations. To avoid the duplicate setup calculations required for an additional transport region, a test is made to eliminate a diffusion region consisting of a single zone.

3.3. DIFFERENCE FORM OF THE DIFFUSION FLUX

The diffusion intensity derived above is

$$I = B - \frac{\mu}{\kappa\rho}\frac{\partial B}{\partial x} \quad .$$

In the group frequency approximation of SPUTTER, the intensity integrated over a frequency interval (ν_j, ν_{j+1}) is required:

$$\int_{\nu_j}^{\nu_{j+1}} I \, d\nu = \int_{\nu_j}^{\nu_{j+1}} B \, d\nu - \frac{\mu}{\rho}\frac{\partial \theta^4}{\partial x} \int_{\nu_j}^{\nu_{j+1}} \frac{\partial B}{\partial \theta^4} \frac{d\nu}{\kappa_\nu} \quad .$$

In terms of the partial Rosseland mean absorption coefficient

$$\kappa_j = \frac{\int_{\nu_j}^{\nu_{j+1}} \frac{\partial B}{\partial \theta^4} d\nu}{\int_{\nu_j}^{\nu_{j+1}} \frac{\partial B}{\partial \theta^4} \frac{d\nu}{\kappa_\nu}} ,$$

the frequency group intensity becomes

$$I_j = \int_{\nu_j}^{\nu_{j+1}} I \, d\nu = \int_{\nu_j}^{\nu_{j+1}} B \, d\nu - \frac{\mu}{\rho} \frac{\partial \theta^4}{\partial x} \frac{\int_{\nu_j}^{\nu_{j+1}} \frac{\partial B}{\partial \theta^4} d\nu}{\kappa_j} . \qquad (3.6)$$

It is desired to evaluate this quantity at each zonal interface in the mesh. Since the known quantities are the zone temperatures and densities, the absorption coefficients κ_j and the integrated source functions X6 $= \int B \, d\nu$ are first evaluated, not at the interfaces but at positions representative of each zone.

The question remains as to how best to approximate the derivatives and interpolate for the coefficients in Eq. (3.6) at the interfaces from the quantities available at zone positions. The answer depends on the temperature and density profile across the interface from which these terms could be calculated directly. Since the profile is not known, we must select a reasonable approximation which will permit the calculation to be carried out. In fact, the appropriate profile depends on the events which have taken place in the calculation and on the energy transport mechanisms of greatest importance in it. As extreme examples, a problem dominated by hydrodynamics might have quantities determined by passage of a strong shock and subsequent linearization in mass coordinates of the pressure behind the shock, whereas a radiation-dominated diffusion problem is characterized by linearity of the radiation potential, which, in turn, depends on the Rosseland opacity. Of course, such detailed information about the progress of a problem is generally unavailable, so, at best, an approximation based on over-all accuracy is needed.

Since the terms under consideration are the radiation diffusion equations, the interpolation is performed in a way to give greatest accuracy when the diffusion terms are most important--namely, when the profile is being determined entirely by radiation diffusion. It is also desirable to reduce the number of coefficients requiring interpolation. This can be done by noting the identity

$$\frac{\partial}{\partial_r}\int_{\nu_j}^{\nu_{j+1}} B\, d\nu = \frac{\partial\theta^4}{\partial x}\int_{\nu_j}^{\nu_{j+1}} \frac{\partial B}{\partial\theta^4}\, d\nu$$

and by forming the variable $\tau = \int \rho \kappa_j \, dx$. In terms of these quantities, the intensity can be written as

$$I_j = \int_{\nu_j}^{\nu_{j+1}} B\, d\nu - \mu \frac{\partial \int_{\nu_j}^{\nu_{j+1}} B\, d\nu}{\partial \tau} .$$

SECTION IV

FREQUENCY INTEGRATION

Equations derived in Sections II and III which are applicable to a particular frequency of the radiation field are of limited usefulness in the SPUTTER calculations. Although in principle a calculation at a particular frequency might be valuable for comparison with high-resolution spectroscopy, in practice no such data have been available. Of much more use are intensities averaged over a wide frequency band. These quantities can be compared with data from wide-band measurements and, most important of all, can be summed for use in the energy integration in the SPUTTER code. The quantities to be summed are the frequency-integrated radial flux component, the radiation energy density, and the radiation pressure. For performing interaction calculations, it is also valuable to form other components of the radiation flux.

Basically, the quantity which is required for each of the above applications is the frequency-group intensity I_{ij},

$$I_{ij} = \int_{\nu_j}^{\nu_{j+1}} I_i \, d\nu \quad . \tag{4.1}$$

Then, for example, this quantity can be integrated over angles to form ϕ_{ij}, the contribution to the flux at position i of frequency group j:

$$\phi_{ij} = \int_{-1}^{1} (I_{ij}^{+} - I_{ij}^{-}) \, \mu \, d\mu \quad ,$$

and thus the total radiant flux at position i is

$$\phi_i = \sum_j \phi_{ij} \quad .$$

Equation (2.8) gives the expression for the frequency-dependent intensity to be used in Eq. (4.1). The frequency integration of Eq. (2.8) has been reported recently,[4] but the current SPUTTER code does not include the transmission functions. The first two terms of Eq. (2.8) which form the diffusion limit can be integrated, as in Section 3.3, to give

$$I_{ij} = B_{ij} - \frac{\mu}{\sigma_{R_j}} \frac{\partial B_{ij}}{\partial x} \quad \text{(diffusion limit)} , \tag{4.2}$$

in which the first term

$$B_{ij} = \int_{\nu_j}^{\nu_{j+1}} B_i(\nu)\, d\nu$$

is the frequency-group Planck function and the second term contains the frequency-group Rosseland mean absorption coefficient $\sigma_{R_j} = \rho\, \kappa_j$. In this form, Eq. (4.2) correctly gives the frequency-group intensity for the optically thick limiting case. The remaining B_i and $\partial B/\partial_T)_i$ terms of Eq. (2.8) are formed in the same way. Thus,

$$I_{ij} = B_{ij} - \left(\frac{\mu}{\sigma_R} \frac{\partial B}{\partial x}\right)_{ij} + \left[\left(\frac{\mu}{\sigma_R} \frac{\partial B}{\partial x}\right)_{ij} - \left(\frac{\mu}{\sigma_R} \frac{\partial B}{\partial x}\right)_{i-1,\,j}\right] \overline{e^{-\Delta/2}}$$

$$+ \left[I_{i-1,\,j} - B_{i-1,\,j} + \left(\frac{\mu}{\sigma_R} \frac{\partial B}{\partial x}\right)_{i-1,\,j}\right] \overline{e^{-\Delta}} . \tag{4.3}$$

In Eq. (4.3), mean values of the exponentials have been extracted from the frequency integrals and the outstanding problem is to specify their values. Two options are available; they differ in the absorption coefficient used to calculate the optical depth. The first is

$$\overline{e^{-\Delta}} = e^{-\sigma_R \delta} \tag{4.4}$$

and the second is

$$\overline{e^{-\Delta}} = e^{-\sigma_P \delta} ,$$

where

$$\sigma_P = \frac{\int_{\nu_j}^{\nu_{j+1}} \sigma_\nu B_\nu \, d\nu}{B_{ij}}$$

and

$$\delta = x_i - x_{i-1} .$$

For small optical depth, the correct result makes use of the Planck mean absorption coefficient. From Eq. (2.12) the frequency integration then gives

$$I_{ij} = I_{i-1,j} + \left[\frac{1}{4}B_{ij} + \frac{1}{4}B_{i-1,j} + \frac{1}{2}B_{i-\frac{1}{2},j} - I_{i-1,j}\right]\sigma_P\delta \; . \qquad (4.5)$$

The above prescriptions for frequency-group means are far from satisfying and call for further work. Considerable economies can be made through reductions in the number of frequency groups if a more accurate means of averaging within groups can be found. Presently used choices of frequency groups appear to give a reasonably accurate result, however, as indicated by comparisons between calculations with the nominal number of frequency groups and calculations with a very large number of frequency groups. (It is expected that a unique correct result will be obtained as the number of frequency groups is increased, irrespective of the choice of the weighting function in the frequency-group-average absorption coefficient.) Consequently, a very few frequency groups should be adequate if a suitable averaging procedure were developed.

Even with a crude averaging scheme, considerable improvement in accuracy results from choice of frequency-group boundaries so as to reduce the variation of the absorption coefficient within the group.

Work on the absorption coefficient for air indicates that approximately 20 groups, carefully selected as to their locations, afford quite adequate resolution. Enough information is known about air to make this selection appear quite reasonable. Air absorption coefficient tapes (DIANE)* have been prepared for 18, 20, and 90 groups. The 90-group tape is used to check on the frequency integrations at selected times. The proper averages to use are difficult to decide on at this time. There are provisions for reading into storage from the DIANE tapes both the Rosseland and Planck averages, which are used at present in the thick or thin limits, respectively.

---

*See Section VI of Volume V.

SECTION V

SUBROUTINE ORGANIZATION AND ECONOMICS

The present plane transport subroutines were written with the idea of removing unnecessary calculations from inside the frequency loop and characteristic ray integrations. These improvements required an increase in storage for the subroutines to attain a decrease in calculational time. The reorganized subroutines will be discussed in two sections, corresponding to the two major subroutines: (1) the radiation subroutine (PRADTN) in which most of the preliminary setup and the diffusion calculation is completed and (2) the transport subroutine (PTRANS) in which the intensity calculation and angular integrations are performed. The subroutines which execute the opacity interpolations (KAPPA), Planck function (PLNKUT), and fast exponential (FREXP) will be discussed in Section 5.4. The input numbers and the output edits will be presented in Sections 5. 5 and 5. 6.

5. 1. THE PRADTN SUBROUTINE

In PRADTN, the high-frequency groups are merged, a source region is established, boundary sources and derivatives are calculated, regions for transport and diffusion are formed, diffusion fluxes are calculated, frequency integration is performed, and the radiation time-step control is evaluated. Each of these activities in PRADTN will be discussed in subsequent paragraphs.

5. 1. 1. Merge Frequency Groups

Frequency groups that are too far out on the Planck tail for a "maximum" temperature in the mesh are merged. The criterion used is as follows: If the lower frequency boundary $h\nu_1$ of the group in question ($h\nu_1$, $h\nu_2$) is greater than ten times the maximum temperature (THMAX) in the mesh, this group will be merged with the next lower group. Merging will continue until over half the groups have been merged; at this point, either the calculation is terminated or a second DIANE tape is called. On merging, Rosseland and Planck averages are formed by using the following equation for $dB/d\theta^4$ and the appropriate sums:

$$\frac{dB_\nu}{d\theta^4} \cong \frac{0.0384974}{\theta^4}\left[\left(\frac{h\nu_2^4}{1 - e^{-h\nu_2/\theta}}\right)e^{-h\nu_2/\theta} - \left(\frac{h\nu_1^4}{1 - e^{-h\nu_1/\theta}}\right)e^{-h\nu_1/\theta}\right] ,$$

$$\sum b_j\theta^4,\ \sum \frac{dB_\nu}{d\theta^4},\ \sum b_j\theta^4\kappa_P,\ \text{and} \sum \frac{dB_\nu}{d\theta^4\kappa_R} . \tag{5.1}$$

The Planck weighting functions (b_j) are obtained from PLNKUT, as described later. On completing the merging, the merged opacities are formed:

$$\overline{\kappa_R} = \sum dB_\nu/d\theta^4 \Big/ \sum dB_\nu/(d\theta^4 \times \kappa_R) \quad \text{(CAPAR)} ,$$

$$\overline{\kappa_P} = \sum b_j\theta^4\kappa_P \Big/ \sum b_j\theta^4 \quad \text{(CAPAC)} . \tag{5.2}$$

5.1.2. Set Up Sources and Derivatives

The frequency-dependent sources must be established at the interfaces from the zonal quantities $b_j\theta^4_{i+\frac{1}{2}}$ (X6(i)) and $\tau_{i+\frac{1}{2}}$ (H3(i)). The difference equations used were given in Section 2.1. Before the calculation of the Planck function (b_j) is made, i.e., before calling PLNKUT, a test is made to see if u_1 (i.e., the reduced frequency $h\nu_1/\theta) \geq 19$; if so, $b_j = 0$ (i.e., the source X6(i) = 0.0). If $u_1 < 19.0$ and $u_2 \leq 0.01$, then $b_j = 0$ also, assuming that for $\theta^4 < 10^5$, the small b_j ($b_j \sim 10^{-5}$) will produce a negligibly small source contribution. An index (ICX) is set equal to the last zone that contains a source. This source index is used to limit the transport calculation to the region containing sources. While setting up the sources and derivatives, tests are made on their discontinuous nature to use either a linear or constant form in the intensity integrations. The initial check is on the minimum optical depth of adjacent zones to ensure that both are transparent (less than 0.3). If this condition holds and if both the sources and optical depths are changing rapidly in x (change greater than a factor of two), the derivative at that interface (TC(i)) is set equal to zero. The zero source derivative is used in PTRANS, as a test, to set up the constant source terms. For the intensity integration, special boundary sources and derivatives are also established at the edge of the source region (I = ICX) and at the outside of the mesh (I = IM) (see Section 2.1.3).

5.1.3. Determine Diffusion Region

The principal criterion for defining a diffusion region is that the first derivative of the source function (TG) be small compared to the source (Y2) (see Section 3.2). When the zone is found to be diffusion, the boundary is tagged by setting (X3 = -1.). Before incorporating this interface into a diffusion region, the possible influence from sources on either side is considered and a further test is made. From the last diffusion boundary, a test is made for an optical depth in succeeding zones to the left. If more than HVB optical depths appear in the next zone, then this zone is calculated by transport and removed from the diffusion region (set X4(i) = -1.0). HVB is an input number, which is usually around 5. When x = 0 is reached after testing each zone, zones out to the right of the present transport region are tested in the same manner. The above test buffers the transport region with an (HVB) mean-free-path-thick diffusion boundary. If the zone boundary stays diffusion, i.e., X3(i) = -1.0 and X4(i) = 0.0, a diffusion flux is calculated from the source gradients, as described in Section 3.1. The regions where X3(i) = 0. or X3(i) = -1. and X4(i) = -1. have been established as transport regions because they did not meet the diffusion criteria or they reverted to transport regions by the optical-depth test described above. This transport region is then identified by setting the left boundary to IAX and the right boundary to IBX. More than one trans region may be set up in PRADTN, and if so, a PTRANS calculation will be made for each region. No one-zone diffusion region is allowed and the region outside the sources (I > ICX) is always considered a transport region.

5.1.4. Time-step Control and Monofrequency Calculation

These two aspects of the new code are related since the "grey" absorption coefficients from the DIANE tape are used to estimate a radiation time step as well as to form the monofrequency time-dependent calculation. In the multifrequency calculation, after all groups have been processed, an additional call for KAPPA is made to read in the grey absorption coefficients. These averages were obtained by integrating the frequency-dependent absorption coefficients for both Planck (κ_P) and Rosseland (κ_R) in the DIANE code. The actual time step for radiation transfer is then obtained from the formula

$$\Delta t_R = (0.5 + 1.5\ H3(i)^2)/(ac\kappa_R \theta^3) \times CV(i) \quad , \tag{5.3}$$

where CV(i) is the specific heat and $ac = 4.12 \times 10^{12}$. The mass point in question is also checked to ensure that it will not gain or lose more than half its original energy:

$$\Delta t_R = 0.5 \times CV(i) \times \theta(i) \times G(i)/|ER(i)| \quad , \tag{5.4}$$

where ER(i) is the divergence of the flux and G(i) is the mass in the zone. The minimum of these values is compared to the hydro time step (Courant) and if smaller,

$$\text{NRAD} = \text{FIX(DTH2/DTRMIN)} \quad \text{and} \quad \text{DTR} = \text{DTH2/NRAD} \qquad (5.5)$$

is set to cycle NRAD times through the radiation routine.

The monofrequency calculation also uses the grey absorption coefficients from the DIANE tape. If KMAX = 0.0 and S15 = 1.0, the frequency-averaged opacities are bypassed on the tape and only the grey absorption coefficients are read into storage. For succeeding cycles, S15 is set equal to zero and the interpolations for κ_R and κ_P are performed in KAPPA using the stored opacities originally read into KAPPA's common storage. When the problem is restarted it is therefore necessary to reload S15 equal to one. If the DIANE tape is not designated (the tape unit assigned must be stored in AMASNO(J+17), where J is the material number), then the KAP routine is called (KAP8 for air) and used for the monofrequency calculation.

5.2. THE PTRANS SUBROUTINE

The subroutine PTRANS is called by PRADTN to carry out the intensity integration between IAX and IBX, saving various quantities on the inward pass that will be used on the outward pass as well as the angular integration of the flux between rays ($\int_{-1}^{1} I\mu\, d\mu$). After the intensity transport along a typical ray in the outward direction (iA → iB) is done, the flux is calculated while the inward pass of the intensity calculation is being completed. The angular integration is based on a linear interpolation of the intensities between rays. The logic in PTRANS is described in detail in the following sections.

5.2.1. Selection of Angles

At present, only five sets of Gaussian angles and weights are stored in the subroutine. These can be selected by setting an input number (LMDA(37), the number of angles with μ>0) to the desired n+1. The selection from storage is made from the following indices

$$\text{NY} = \text{LMDA}(37) - 1$$

$$\text{NMU} = (\text{NY}-1) \times (\text{NY}+2) + 1$$

$$\text{NGS} = \text{NMU}+\text{NY} + 1 \ ,$$

NMU selects the cosine of the angle (μ_m); NGS selects the relation ($\mu_m A_m$), the cosine of the angle times the Gaussian weights for the flux formulation.

5.2.2. Intensity Integration along Characteristic Rays

The integration using Eq. (2.11) starts at the left boundary with the appropriate boundary condition and proceeds outward, storing the exponentials $e^{-\Delta\tau}$ in (H4(i)), the derivatives $\mu\ \partial B/\partial h)_{i+\frac{1}{2}}$ in X8(i), and the calculated intensities in sum X3(i). The more general boundary conditions are established (see Section 2.1) and the stored quantities are now used except for the change of sign of $\mu\ \partial B/\partial h)_{i+\frac{1}{2}}$ in Eq. (2.1) to calculate the intensities I(F2), on the outward pass.

The regions where constant sources, and therefore zero boundary derivatives, should be used in the intensity integrations were established in PRADTN by setting TG(i) equal to zero. In the integration along a particular ray, a test is made on TG(i) at each interface; if zero, the source terms Y2(i - 1) and Y2(i) are set equal to X6(i - $\frac{1}{2}$) respectively (see Fig. 2.1).

As discussed in Section 2.1.1, the accuracy of the exponential term and the effect of truncating errors mean that the general formula will not reduce in the limit of small optical depths to the transparent case. To correct this situation, a test is made on τ_i (the half optical depth τ is stored in H2(i)), and if $\tau < 10^{-2}$ a switch is made to the limiting form of the transport equation (Eq. (2.12) developed in Section II).

5.2.3. Angular Integration

The only integral over angle formed in PTRANS, at present, is the flux; ($\int I\mu\ d\mu$) the formula for energy ($\int I d\mu$) is included for possible use later. These integrals are formed on the outward pass from the intensity (sum X3(i)) stored on the inward pass and the intensity being calculated (F2). The difference forms of the equations are

$$X2(i) = \sum (F2 - \text{sum } X3(i)) \times \mu_m A_m ,$$

$$ER(i) = \sum (F2 + \text{sum } X3(i)) \times A_m .$$

5.3. DIFFERENCES WITH INTEGRAL FORMULATION

The principal difference in the subroutines is in replacing the integration of angle done explicitly in the integral formulation by a sampling scheme of a double Gaussian nature. It is expected that accuracy can be achieved with a minimum number of rays (presumably less than n = 6, see Section 2.2). This result is in logical argeement with the use of the S4 approximation in the neutron-transport work. The advantage, therefore, will appear in problems with many zones, since the integral method will increase as the number of zones squared whereas the present method will only increase linearly with zones. Furthermore, the present method makes it possible to have special boundary conditions depending on angle (see Section 2.1.3).

5.4. AUXILIARY SUBROUTINES

In addition to the two new basic subroutines PRADTN and PTRANS, some changes have been made in the auxiliary subroutines EXP, PLNKUT, and KAPPA. These changes include (1) a fast exponential (FREXP), (2) a two-argument Planck function, and (3) the use of the average opacities from KAPPA (θ and ρ interpolation) for the monofrequency calculation as well as for the Planck opacities.

The new fast exponential routine FREXP uses table lookup and interpolation rather than the normal expansion methods. The routine is written in machine language but uses the library routine EXP(X) for positive X or X > -10. An over-all gain in speed of a few percent was achieved in one comparison SPUTTER calculation.

The PLNKUT routine, with its associated tables PLNKTT, has been corrected and made more efficient by using a two-argument call which now calculates from either the analytic form or from the tables the difference in

$$b(u_1, u_2) = \frac{1}{B} \int_{\nu_1}^{\nu_2} \frac{h\nu^3}{c^2} \frac{1}{e^{-h\nu/kT} - 1} d\nu = b_j \quad . \tag{5.6}$$

The accuracy is improved since now not only differences of nearly equal numbers are subtracted.

The subroutine KAPPA, which calls in the group-averaged absorption coefficients from the DIANE tape and performs a bilinear log interpolation in temperature and density, has been modified to obtain the grey absorption coefficients as well as the Planck averages. At present, the format of the DIANE (absorption coefficient) tape includes a BCD record for tape identification, the Rosseland and Planck averages for a selected set of temperatures and densities from 0.25 ev to 50 ev and from 10 normal to 10^{-6} normal, and the actual integration, $\int \kappa_\nu \, d\nu$, for the grey case. The grey or frequency-integrated averages are also used for an estimate of the time steps in PRADTN. KAPPA reads in first the tape name, the number of frequency groups, and the size of the records. If the sentinel for multifrequency is set to KMAX = 1, then the first frequency group, $h\nu_1$, and its absorption coefficients are read into storage. The interpolations in log θ_i and log ρ_i are performed and a return to PRADTN is made. If KMAX = 0, then KAPPA skips over the frequency-dependent absorption coefficients and reads into storage the grey averages. A signal, S15 = 0, is subsequently set, and for further cycles the interpolations are made on the stored quantities; the tape is not called again.

5.5. INPUT NUMBERS

The input quantities used in the radiation-transport subroutines and their functions are listed in Table 5.1. The entries in it are as follows: column 1 is the storage location number used for entering the quantity into storage with the CARDS subroutine, column 2 lists the FORTRAN name of the stored quantity, column 3 gives the range of admissible values of the input quantity, column 4 describes its function and identifies special values it may assume, and column 6 records a set of values of the quantities which might be typical of those for a normal problem. Included is a set of values for the input quantities selected for solving typical problems.

5.6. EDITS

The editing of such frequency-dependent quantities as H3, the optical depth (Rosseland), X6, the source ($b_j\theta^4$), X2, the flux (in ergs/4/3 π sec) X2/DHNU, the flux divided by the frequency group, THETA, the temperature (ev), and EI, the energy (ERGS/G) versus radius is accomplished by setting S12 to the desired number of cycles between prints. These multi-frequency edits have been used to evaluate the criteria for the subroutines as well as for diagnostics during the calculations.

A list of sample editing for a particular frequency group is given on page 27. The HNU is in electron volts. The quantities found useful to display for each frequency group and for a characteristic ray are listed on page 28. The format statements, in the listings appended, have been revised for the debug print from those used on page 28.

Table 5.1

PLANE RADIATION INPUT QUANTITIES

Card No.	Quantity	Range of Values	Description	Typical Value
37	LMDA(37)	2, 3, 4, 5, 6	Number of angles with $\mu > 0$.	2
44	KMAX	----	$\neq 0$, performs multigroup frequency approximation; $= 0$, performs single-group frequency approximation.	0
81	HVB	≥ 0	Number of optical mean free paths by which a transport region is extended at the expense of each adjacent diffusion region. (See Section 3.2.)	5
83	HCB	≥ 0	Criterion to define a diffusion region in terms of relative gradient of the source function. Diffusion regions are eliminated if 0. (See Section 3.2.)	0.1
87	CB	≥ 0	Criterion to combine frequency groups. If the lower frequency of the group is more than CB times the temperature of the hottest zone, that group is combined with the adjacent group of lower frequency. A half-integer value presents termination of the problem when half or more of the groups have been combined. (See Section 5.1.1.)	10.5
88	GA	≥ 0	One of two criteria for choice of linear or stepwise constant source within a zone. (See Section 5.1.2.)	0.333
90	GL	Neg., 0, $\frac{1}{2}$, pos. integer	Indicator for radiation boundary condition at IB. GL = negative, total reflection; GL = 0, intensity for $\mu < 0$ is zero; GL = $\frac{1}{2}$, blackbody intensity based on temperature located in THETA(IB) for $\mu < 0$; GL = positive integer, intensity for $\mu < 0$ obtained from source routine. GL must equal number of frequency groups. (See Section 2.1.3.)	0
121	AC	≥ 0	One of two criteria for choice of linear or stepwise constant source. Minimum value for using a linear source. (See Section 5.1.2.)	0.3
127	ACO3T4	----	Transport debug edit criterion. Edit occurs if $\neq 0$ and < cycle number.	9
147	S12	----	Number of cycles between multifrequency edits.	10
150	S15	0, 1	Trigger controlling call of DIANE tape. Must have value $\neq 0$ on starts or restarts.	1
8466	TELM(25)	≥ 0	Constant multiplying the radiation time step. Can be used to modify the stability criterion.	1
8478	TELM(37)	≥ 0	Maximum permissible fractional energy in any zone due to radiation. Time step may be reduced to meet this requirement.	0.05
8858	SOLID(10)	----	Thick-thin criterion. If 0, Planck mean is used to form H2; otherwise, Rosseland mean. (See Section 4.)	1

Table 5.1

SAMPLE EDIT FOR A CHARACTERISTIC DIRECTION FOR PTRANS

FREQUENCY BAND FROM 17.8000 TO 20.1000

JJ = 0 IAX = 1 IBX = 31 ICX = 31 ICY = 23

I	X	X6	H2	H3	SUMX3	SUMX4	X2
1	-7.5994889-08	0.	2.4339197+02	2.4339197+02	0.	0.	-0.
2	9.9999999+00	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
3	1.0010000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
4	1.0020000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
5	1.0030000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
6	1.0040000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
7	1.0050000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
8	1.0060000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
9	1.0070000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
10	1.0080000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
11	1.0090000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
12	1.0100000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
13	1.0110000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
14	1.0120000+01	0.	2.4339192-01	2.4339192-01	0.	0.	-0.
15	1.0130000+01	0.	2.4339192-01	2.4339192-01	2.4917989-37	0.	-0.
16	1.0140000+01	0.	2.4339192-01	2.4339192-01	8.0013965-33	0.	-4.4464560-35
17	1.0150000+01	0.	2.4339192-01	2.4339192-01	2.5603224-28	0.	-1.4277981-30
18	1.0160000+01	0.	2.4339191-01	2.4339191-01	8.2503306-24	0.	-4.5847912-26
19	1.0170000+01	0.	2.4339191-01	2.4339191-01	2.6492568-19	0.	-1.4722185-21
20	1.0180000+01	0.	2.4339191-01	2.4339191-01	8.5070054-15	0.	-4.7274279-17
21	1.0190000+01	0.	2.4339191-01	2.4339191-01	2.7316771-10	0.	-1.5180203-12
22	1.0199993+01	0.	2.4314869-01	2.4314869-01	8.7716646-06	0.	-4.8745017-08
23	1.0209539+01	4.9129211-04	1.1762804-01	1.1762804-01	2.7876050-01	0.	-1.5491000-03
24	1.0216361+01	8.7425114+00	2.7366146-03	2.7366146-03	1.2072228+01	5.4142551+00	-3.6999020-02
25	1.0219025+01	3.0955258+01	1.7085025-03	1.7085025-03	1.2082294+01	6.1653042+00	-3.2881306-02
26	1.0219959+01	3.2643747+01	1.6747771-03	1.6747771-03	1.0801617+01	7.7731507+00	-1.6829408-02
27	1.0220899+01	3.2132795+01	1.6698545-03	1.6698545-03	9.2048018+00	9.4688165+00	1.4671562-03
28	1.0221667+01	3.1355411+01	1.6399733-03	1.6399733-03	7.5149186+00	1.1024201+01	1.9501436-02
29	1.0222843+01	3.3702363+01	1.4826168-03	1.4826168-03	5.7588749+00	1.2424946+01	3.7044026-02
30	1.0223844+01	4.5108789+01	1.0629491-03	1.0629491-03	3.8479576+00	1.3815454+01	5.5390377-02
31	1.0224930+01	6.0067696+01	7.6399395-04	7.6399395-04	1.8744850+00	1.5234913+01	7.4245233-02
32	1.0226205+01	0.	0.	0.	0.	1.6622066+01	9.2370485-02

Table 5.2

SAMPLE MULTIFREQUENCY EDIT FOR PTRANS

CYCLE = 63. TIME = 0.305089-07 HNU FROM 17.8000 TO 20.1000

I	R	H3	X6	X2	X2/DHNU	RHO	THETA	EI
1	-7.81073952-08	2.28637884+02	0.	-0.	-0.	0.	2.49999813-02	2.20139597+09
2	9.99999988+00	2.28637835-01	0.	-1.08559455+07	-4.71997637+06	0.	2.49999925-02	2.20139750+09
3	1.00099999+01	2.28637835-01	0.	-1.81788317+07	-7.90386000+06	0.	2.49999918-02	2.20139741+09
4	1.00199999+01	2.28637831-01	0.	-3.04437485+07	-1.32364126+07	0.	2.49999922-02	2.20139747+09
5	1.00299999+01	2.28637831-01	0.	-5.09894950+07	-2.21693460+07	0.	2.49999922-02	2.20139747+09
6	1.00399998+01	2.28637833-01	0.	-8.54158400+07	-3.71373225+07	0.	2.49999927-02	2.20139754+09
7	1.00499998+01	2.28637829-01	0.	-1.43122512+08	-6.22271800+07	0.	2.49999934-02	2.20139763+09
8	1.00599998+01	2.28637831-01	0.	-2.39907566+08	-1.04307639+08	0.	2.49999929-02	2.20139757+09
9	1.00699998+01	2.28637833-01	0.	-4.02371492+08	-1.74944216+08	0.	2.49999927-02	2.20139754+09
10	1.00799998+01	2.28637833-01	0.	-6.75427088+08	-2.93663956+08	0.	2.49999936-02	2.20139766+09
11	1.00899998+01	2.28637833-01	0.	-1.13520550+09	-4.93567620+08	0.	2.49999939-02	2.20139770+09
12	1.00999998+01	2.28637833-01	0.	-1.91150912+09	-8.31090936+08	0.	2.49999939-02	2.20139770+09
13	1.01099998+01	2.28637831-01	0.	-3.22769322+09	-1.40325796+09	0.	2.49999943-02	2.20139776+09
14	1.01199998+01	2.28637831-01	0.	-5.47133190+09	-2.37884000+09	0.	2.49999948-02	2.20139782+09
15	1.01299998+01	2.28637831-01	0.	-9.32924749+09	-4.05619462+09	0.	2.49999946-02	2.20139779+09
16	1.01399997+01	2.28637831-01	0.	-1.60407853+10	-6.97425459+09	0.	2.49999950-02	2.20139786+09
17	1.01499997+01	2.28637829-01	0.	-2.79074358+10	-1.21336680+10	0.	2.49999953-02	2.20139789+09
18	1.01599997+01	2.28637829-01	0.	-4.93469158+10	-2.14551811+10	0.	2.49999962-02	2.20139802+09
19	1.01699997+01	2.28637626-01	0.	-8.91642235+10	-3.87670543+10	0.	2.49999964-02	2.20139808+09
20	1.01799997+01	2.28637641-01	0.	-1.65617304+11	-7.20075244+10	0.	2.49999994-02	2.20139837+09
21	1.01899997+01	2.28602695-01	0.	-3.18095679+11	-1.38302472+11	0.	2.50133509-02	2.20266403+09
22	1.01999871+01	2.19749393-01	0.	-6.36769354+11	-2.76856246+11	0.	9.64961499-02	9.86790720+09
23	1.02093679+01	8.22925232-02	4.91292107-04	-1.42616981+12	-6.20073837+11	0.	1.22273718+00	4.73666470+11
24	1.02157248+01	2.33086449-03	8.66041410+00	-8.76098961+11	-3.80912599+11	0.	3.36150387+00	2.19259465+12
25	1.02179935+01	1.76119059-03	3.10979107+01	-6.73073799+11	-2.92640784+11	0.	4.34233952+00	2.96451965+12
26	1.02189515+01	1.65541358-03	3.27618031+01	.72651330+11	-1.18544058+11	0.	4.39482421+00	2.99672643+12
27	1.02198687+01	1.63309816-03	3.22924628+01	1.52772108+11	6.64226570+10	0.	4.37915915+00	3.00061631+12
28	1.02208086+01	1.65783847-03	3.15173852+01	5.69157353+11	2.47459723+11	0.	4.35496664+00	2.99155453+12
29	1.02218024+01	1.51176269-03	3.37328744+01	9.87196968+11	4.29216072+11	0.	4.62670870+00	3.11306739+12
30	1.02228336+01	1.05740126-03	4.51018038+01	1.40871439+12	6.12484530+11	0.	4.74113518+00	3.45431513+12
31	1.02239041+01	7.48561593-04	6.00003300+01	1.80893424+12	7.86493161+11	0.	5.10125035+00	4.14972179+12
32	1.02251422+01	6.19309597-01	0.	2.16810211+12	9.42653104+11	0.	0.	0.

SECTION VI

TIMING STUDIES AND ACCURACY IN ANGULAR INTEGRATION

6.1. TIMING CALLS

The comparison in calculation time was obtained by using timing calls at selected locations in the logic of the code. To use the timing calls, it was necessary to establish a fiducial time from the system clock and then print the location of the time call, the time, and the difference in time between calls for each call. The subroutine that carries out these steps is CLOCK.

In the calculations described above, the subroutine CLOCK was called at the following locations in PRADTN and PTRANS:

PRADTN

13.105 - Before frequency loop
13.140 - After call KAPPA on merge
13.701 - After call KAPPA on main frequency loop
13.151 - After calculating general sources
13.180 - Before calling PTRANS
13.292 - After EDIT (normal) end of frequency loop
13.286 - After last frequency start time step
-13.239 - End of cycle (return to main program)

PTRANS

14.708 - Before debug print

The following calculation was timed in units of 1/60 sec for the above breakdown in computing time.

The calculation described here did not use the TG criteria (see Section 5.1.2) nor the special boundary conditions (see Section 5.2.4). Three ray passes were completed for each frequency group for a total of six angles, forward and back, and with 32 active zones. The total time for 21 frequency groups was ~14.9 sec. The breakdown in time for a single frequency group (in units of 1/60 sec) are the following:

Call KAPPA for absorption coefficient	14.
After sources	1.
Characteristic ray passes (3)	3.
After call PTRANS with EDIT	23.
Average time required	~41/hν
The start and merge of KAPPA is	~32
Total time with EDIT	~869 (1/60 sec)
Total time without EDIT	~515 (1/60 sec)

6.2. ACCURACY IN ANGULAR INTEGRATIONS

Comparisons have been made between calculations for two sets of angles for a radiation shock problem. The problem consists of a hot (5 ev) shock moving into cold low-density air. The effect on the flux versus linear zones for a set of four and eight angles is given in Fig. 6.1. This result indicates that as few as six and probably even four angles would be sufficient for reasonable accuracy.

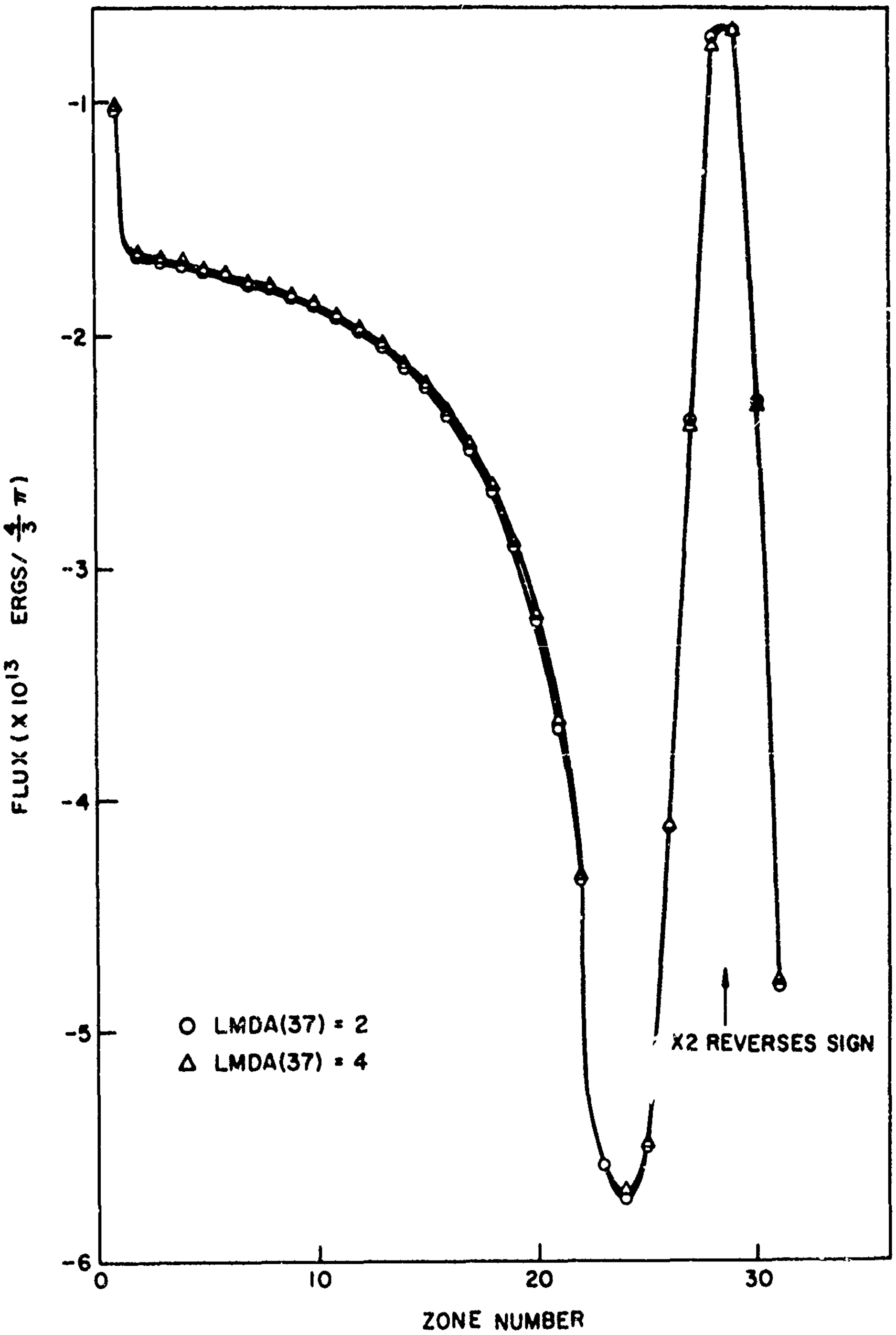

Fig. 6.1--Flux vs linear zones for six and ten angles

REFERENCE

1. Margenall, H., and G. Murphy, The Mathematics of Physics and Chemistry, D. Van Nostrand Co. Inc., New York, 1943.

Appendix A

PRADTN

```
$IBFTC PRADTN  FULIST,DECK,REF                                           PRAD0000
      SUBROUTINE  RADTN                                                  PRAD0010
CCMPILED OCTOBER 7, 1965  WBL
C     PLANE CHARACTERISTIC TRACE WITH DOUBLE GAUSSIAN INTEGRATION        PRAD0030
C**********************************************************************PRAD0040
C*                    S P U T T E R   C O M M O N                     **PRAD0050
C                                                                      *PRAD0060
      COMMON    LMDA(37), NR     , NSMLR , IA     , IB     , ICA    , ICB    ,PRAD0070
     1   KMAX   , BLANK1, BLANK2, BLANK3, IAP1   , IBP1   , ICAP1  , ICBP1  ,PRAD0080
     2   II     , IG     , NRAD   , BLANK4, IAM1   , IBM1   , ICAM1  , ICBM1  ,PRAD0090
     3   IIP1   , IGM1   , IALPHA, BLANK5, TH     , TMAX   , BLANK6, DELPRT,PRAD0100
     4   FREQ   , CNTMAX, AR     , ASMLR , PUSHA  , PUSHB  , BOILA  , BOILB  ,PRAD0110
     5   CVA    , CVB    , SLUG   , ALPHA , HVA    , HVB    , HCA    , HCB    ,PRAD0120
     6   EMINA  , EMINB  , CA     , CB     , GA     , GB     , GL     , GR     ,PRAD0130
     7   RHOL   , RHOR   , EPIO   , EPSI   , RIA    , RIB    , RDIA   , RDIB   ,PRAD0140
     8   RPIA   , RPIB   , RPDIA  , RPDIB  , TPRINT, TA     , TB     , TC      PRAD0150
      COMMON      TD     , TE     , DTH2   , DTH2P  , DTH1   , DTRMIN, DTMAX  ,PRAD0160
     1   DTMAX1, DTMAX2, DTMAX3, DTR    , SWITCH, CO     , CMIN   , DELTA  ,PRAD0170
     2   GAMA   , WCRIT  , SIGMAQ, AC     , ACO3T4, CNVRT  , SUMRA  , SUMRB  ,PRAD0180
     3   ROIA   , ROIAM1, ROIB   , ROIBP1, GMS    , S1     , S2     , S3     ,PRAD0190
     4   S4     , S5     , S6     , S7     , S8     , S9     , S10    , S11    ,PRAD0200
     5   S12    , S13    , S14    , S15    , S16    , S17    , S18    , S19    ,PRAD0210
     6   S20    , EO     , FO     , TAU    , ZERO   , R     (152), DELTAR(152),PRAD0220
     7 ASQ    (152), RD     (152), VD     (152), RDD    (152), SMLR   (152),PRAD0230
     8 DELR   ( 37), P      (152), P1     (152), PB     (152), PB1    (152) PRAD0240
      COMMON           P2     (152), SV     (152), RHO    (152), THETA  (152),PRAD0250
     1 W      (152), E      (152), EI     (152), EK     (152), A      (152),PRAD0260
     2 V      (152), G      (152), D      (152), C      (152), X2     (152),PRAD0270
     3 X3     (152), X4     (152), X5     (152), X6     (152), X7     (152),PRAD0280
     4 SMLA   (152), SMLB   (152), SMLC   (152), SMLD   (152), SMLE   (152),PRAD0290
     5 EC     (152), ER     (152), SMLG   (152), SMLH   (152), BIGA   (152),PRAD0300
     6 BIGB   (152), CV     (152), BC     (152), BR     (152), CHIC   (152),PRAD0310
     7 CHIR   (152), CAPAC  (152), CAPAR  (152), CRTC   (152), CRTR   (152),PRAD0320
     8 CRTPC  (152), GOFR   (152), FEW    (152), CAR    (152), OKLM   ( 37) PRAD0330
      COMMON           TELM   ( 37), EKLM   ( 37), ELM    ( 37), FCLM   ( 37),PRAD0340
     1 FRLM   ( 37), WLM    ( 37), QLM    ( 37), AMASNO( 37), CHRNO  ( 37),PRAD0350
     2 ZP1    ( 37), ZP2    ( 37), SOLID  ( 37), ECHCK  ( 37), RK     (104),PRAD0360
     3 RL     ( 37), RHOK   (104), ROK    (104), THETAK(104), TEMP   ( 16),PRAD0370
     4 HEAD   ( 12), MAXL          , MAXLM                                PRAD0380
C*                                                                    **PRAD0390
C**********************************************************************PRAD0400
      DIMENSION  Q3(1),TG(1),H2(1),Q1(1),X8(1),SUMX3(1),SUMX4(1)          PRAD0410
      DIMENSION  H4(1),Y2(1),H(1),SUMX2(1),Q2(1)                          PRAD0420
      DIMENSION  Q37(1), Q38(1), H3(1)                                    PRAD0430
      COMMON /LINDLY/ HNU,SGNL,IHNU,NHNU,HNUP,NT,IM,IN,DHNU,THICK,NY     PRAD0440
      COMMON /CNTRL/ SCYCLE, JMULT                                        PRAD0450
      COMMON /DAVIS/  ICX, ICY                                            PRAD0460
      COMMON  /TQ/  QINT1(300), QINT2(300), TITLE(12)                     PRAD0470
C                                                                         PRAD0480
      EQUIVALENCE(SMLA,H4),(SMLD,Y2)                                      PRAD0490
      EQUIVALENCE  (BC,TG),(BIGB,H),(CRTR,SUMX2),(CHIC,SUMX3)             PRAD0500
      EQUIVALENCE  (SMLH,X8),(CAR,Q37),(CHIR,Q38),(SMLC,H3)               PRAD0510
      EQUIVALENCE  (ACO3T4,TRDBG),(S12,EDITMF)                            PRAD0520
      EQUIVALENCE (EC,Q1),(X7,H2),(BIGA,SUMX4),(GOFR,Q3)                  PRAD0530
C                                                                         PRAD0540
```

```
C******************************************************************************PRAD0550
C                                                                            *PRAD0560
C                               EDITMF  SAME AS     S12                      *PRAD0570
C                               H       SAME AS    BIGB                      *PRAD0580
C      H2 SAME AS X7                                                          PRAD0590
C                               H3      SAME AS    SMLC                      *PRAD0600
C                               H4      SAME AS    SMLA                      *PRAD0610
C                               Q1      SAME AS      EC                      *PRAD0620
C                               Q3      SAME AS    GOFR                      *PRAD0630
C                               Q37     SAME AS     CAR                      *PRAD0640
C                               Q38     SAME AS    CHIR                      *PRAD0650
C                               SUMX2   SAME AS    CRTR                      *PRAD0660
C                               SUMX3   SAME AS    CHIC                      *PRAD0670
C                               SUMX4   SAME AS    BIGA                      *PRAD0680
C                               TG      SAME AS      BC                      *PRAD0690
C                               TRDBG   SAME AS AC03T4                       *PRAD0700
C                               X8      SAME AS    SMLH                      *PRAD0710
C                               Y2      SAME AS    SMLD                      *PRAD0720
C                                                                            *PRAD0730
C******************************************************************************PRAD0740
C                                                                             PRAD0750
C                                                                             PRAD0760
C      FEX, FM, SUMRHO, CSQD, XSQD, Y, YSQD, Q2, NOT USED                     PRAD0770
C                                                                             PRAD0780
C******************************************************************************PRAD0790
C                                                                            *PRAD0800
C                               P L A N E S   O N L Y                        *PRAD0810
C                                                                            *PRAD0820
C******************************************************************************PRAD0830
C                                                                            *PRAD0840
C                          B O U N D A R Y   C O N D I T I O N S              PRAD0850
C                                                                            *PRAD0860
C******************************************************************************PRAD0870
C                                                                             PRAD0880
C               L.H. BLACKBODY IF SOLID ON LEFT                               PRAD0890
C                                                                             PRAD0900
C               NO VAPOR ZONE HAS FLUX OUT FROM SOLID                         PRAD0910
C                                                                             PRAD0920
      NTIMES=BOILB                                                            PRAD0930
      IM=IBM1                                                                 PRAD0940
      IN=IA                                                                   PRAD0950
      IF(ZP1(26).EQ.0.) GO TO 15                                              PRAD0960
C     SAVE STUFF FROM EIONX FOR NONEQ AND RESET IN OR IM
      IF (PUSHA .LT. 0.0)  GO TO 100
      IM = NR - 1
      WSZ2=BC(IM+1)
      WSZ3=BR(IM+1)
      WSZ4 = CRTC(IM+1)
      WSZ5=RHO(IM+1)
      GO TO 15
  100 IN = NR
      WSZ2 = BC(IN-1)
      WSZ3 = BR(IN-1)
      WSZ4 = CRTC(IN-1)
      WSZ5 = RHO(IN-1)
```

```
   15 CONTINUE                                                          PRAD0990
      IMP1=IM+1                                                         PRAD1000
      INM1=IN-1                                                         PRAD1010
      CALL CVCHK (K000FX)                                               PRAD1020
      IF (IMP1-IN) 190,190,125                                          PRAD1030
C                                                                       PRAD1040
C                                  NO VAPOR ZONES                       PRAD1050
C                                                                       PRAD1060
  190 X2(IMP1) = 1.0283E12 * A(IM) * (THETA(IM)**4 - THETA(IMP1)**4)    PRAD1070
      ER(IM)=-X2(IMP1)                                                  PRAD1080
      GO TO 1300                                                        PRAD1090
  125 IR=IN                                                             PRAD1100
      THTAMX=.025                                                       PRAD1110
      IF (IALPHA-1) 130,140,130                                         PRAD1120
  130 S1 = 13.0130                                                      PRAD1130
      CALL UNCLE                                                        PRAD1140
  140 DO 180 I=IN,IM                                                    PRAD1150
      X3(I)=0.                                                          PRAD1160
      X4(I)=0.                                                          PRAD1170
      X5(I)=0.                                                          PRAD1180
      X6(I)=0.                                                          PRAD1190
      CRTR(I)=0.                                                        PRAD1200
C                                                                       PRAD1210
C                    SET UP FOR KAPPA INTERPOLATION                     PRAD1220
C                                                                       PRAD1230
      Q1(I)=THETA(I)**4                                                 PRAD1240
      Q37(I)=ALOG(THETA(I))                                             PRAD1250
      Q38(I)=ALOG(SV(I))                                                PRAD1260
C                                                                       PRAD1270
C     FIND IR, RIGHTMOST ZONE WITH THETA GREATER THAN 0.05 EV           PRAD1280
C                                                                       PRAD1290
      IF (THETA(I)-THTAMX) 160,160,150                                  PRAD1300
  150 THTAMX=THETA(I)                                                   PRAD1310
  160 IF (THETA(I)-0.05) [illegible]80,170                              PRAD1320
  170 IR=I                                                              PRAD1330
  180 CONTINUE                                                          PRAD1340
      IF (THTAMX .LT. THETA(IB))  THTAMX = THETA(IB)                    PRAD1350
C                                                                       PRAD1360
C                                                                       PRAD1370
C***********************************************************************PRAD1380
C                                                                      *PRAD1390
C                    B E G I N   F R E Q U E N C Y   L O O P           *PRAD1400
C                                                                      *PRAD1410
C***********************************************************************PRAD1420
  200 HNUP = 3.E3                                                       PRAD1430
C                                                                       PRAD1440
C                    SET UP MAX FREQ BOUNDARY                           PRAD1450
C                                                                       PRAD1460
      HNUP4 = 8.1E13                                                    PRAD1470
      IHNU=1                                                            PRAD1480
      DO 210 I=IN,IMP1                                                  PRAD1490
  210 SUMX2(I)=0.0                                                      PRAD1500
      IF (KMAX.EQ.0) GO TO 280                                          PRAD1510
C                                                                       PRAD1520
C     THIS CODING WONT WORK IF HNU NOT EVALUATED                        PRAD1530
```

```
C                                                                       PRAD1540
  220 CALL KAPPA(IN,IM)                                                 PRAD1550
      HNU4=HNU**4                                                       PRAD1560
      DHNUP = DHNU                                                      9/29/65
      DHNU = HNUP - HNU                                                 PRAD1570
C                                                                       PRAD1580
C     MERGE GROUPS WITH HNU MORE THAN CB  TIMES LARGEST THETA           PRAD1590
C                                                                       PRAD1600
      IF (THTAMX- HNU/CB) 240,300,300                                   PRAD1610
C                                                                       PRAD1620
C               REJECT TAPE IF MORE THAN HALF OF GROUPS MERGE           PRAD1630
C                                                                       PRAD1640
  240 IF (IHNU+IHNU-NHNU) 260,250,250                                   PRAD1650
  250 IF (AMOD(CB,1.) .EQ. 0.5)  GO TO 260                              PRAD1660
      S1=13.0250                                                        PRAD1670
      CALL UNCLE                                                        PRAD1680
  260 DO 270 I=IN,IM                                                    PRAD1690
      BETA=HNU/THETA(I)                                                 PRAD1700
      BETAP=HNUP/THETA(I)                                               PRAD1710
      DFB=PLNKUT(BETA,BETAP)                                            PRAD1720
      IF (DFB.EQ.0.) GO TO 270                                          PRAD1730
      TEMP(1)=DFB*Q1(I)                                                 PRAD1740
      EMB1=EXP(-BETA)                                                   PRAD1750
      EMB2=EXP(-BETAP)                                                  PRAD1760
      TEMP(2)=DFB+0.0384974/Q1(I)*(HNU4/(1.0-EMB1)                      PRAD1770
     1*EMB1-HNUP4/(1.0-EMB2)*EMB2)                                      PRAD1780
C                                                                       PRAD1790
C               FORM NUMERATORS AND DENOMINATORS OF MERGED KAPPAS       PRAD1800
C                                                                       PRAD1810
      X6(I)=X6(I)+TEMP(1)                                               PRAD1820
      X4(I)=X4(I)+TEMP(2)                                               PRAD1830
      X5(I)=X5(I)+CAPAC(I)*TEMP(1)                                      PRAD1840
      X3(I)=X3(I)+TEMP(2)/CAPAR(I)                                      PRAD1850
  270 CONTINUE                                                          PRAD1860
      IF (GL .LT. 1. .OR. IHNU .EQ. 1) GO TO 275                        9/29/65
C     MERGE FREQUENCY-DEPENDENT EXTERNAL INPUT INTENSITIES              PRAD1880
      NMU = LMDA(37)                                                    PRAD1890
      IQNT = NMU * (IHNU - 2)                                           9/29/65
      DO  272  I = 1, NMU                                               PRAD1910
      IQNT1 = IQNT + I                                                  PRAD1920
      IQNT2 = IQNT1 + NMU                                               PRAD1930
      IF(IHNU.GT.2) DHNUP=1.                                            9/29/65
  272 QINT1(IQNT2) = QINT1(IQNT2)*DHNU + QINT1(IQNT1)*DHNUP             9/29/65
  275 HNUP=HNU                                                          PRAD1950
      IHNU=IHNU+1                                                       PRAD1960
      HNUP4=HNU4                                                        PRAD1970
      IF (THTAMX- HNU/CB) 220,310,310                                   PRAD1980
C                                                                       PRAD1990
C               MONOFREQUENCY CALCULATION                               PRAD2000
C                                                                       PRAD2010
  280 NHNU=1                                                            PRAD2020
      CALL KAPPA (IN,IM)                                                PRAD2030
      DO 290 I=IN,IM                                                    PRAD2040
      X5(I)=1.                                                          PRAD2050
  290 X6(I)=Q1(I)                                                       PRAD2060
```

```
      DFB=1.0                                                           PRAD2070
      HNU = .001                                                        PRAD2080
      ICX=IR                                                            PRAD2090
      IF (GL .GT. 0.0)  ICX = IM
      ICY=IN                                                            PRAD2100
      GO TO 480                                                         PRAD2110
  300 IF (IHNU-1) 550,370,260                                           PRAD2120
C                                                                       PRAD2130
C                     FORM MERGED KAPPAS                                PRAD2140
C                                                                       PRAD2150
  310 DO 350 I=IN,IM                                                    PRAD2160
      IF (X6(I)) 320,350,330                                            PRAD2170
  320 S1=13.0320                                                        PRAD2180
      CALL UNCLE                                                        PRAD2190
  330 CAPAR(I)=X4(I)/X3(I)                                              PRAD2200
  340 CAPAC(I)=X5(I)/X6(I)                                              PRAD2210
  350 CONTINUE                                                          PRAD2220
      HNUP = 3.E3                                                       PRAD2230
      HNUP4 = 8.1E13                                                    PRAD2240
      DHNU = HNUP - HNU                                                 PRAD2250
      IHNU=IHNU-1                                                       PRAD2260
      IF(GL.LT.1.) GO TO 370                                            9/29/65
      DO 355 I=1,NMU                                                    9/29/65
      IQNT2 = IQNT + I + NMU                                            9/29/65
  355 QINT1(IQNT2) = QINT1(IQNT2)/DHNU                                  9/29/65
      GO TO 370                                                         PRAD2270
C                                                                       PRAD2280
C                     TYPICAL GROUP CALCULATION OF SOURCES              PRAD2290
C                                                                       PRAD2300
  360 CALL KAPPA(IN,IM)                                                 PRAD2310
      DHNU=HNUP-HNU                                                     PRAD2320
      HNU4=HNU**4                                                       PRAD2330
  370 IF (GL-1.) 390,380,380                                            PRAD2340
  380 IF (HNU.NE.RDK(IHNU+52)) GO TO 490                                PRAD2350
      IF (GL.NE.FLOAT(NHNU)) GO TO 490                                  PRAD2360
C A L C U L A T E   I C X ,   I C Y                                     PRAD2370
  390 ICX=IN                                                            PRAD2380
      ICY = IN                                                          PRAD2390
      IF (GL .LE. 0.)  GO TO 395                                        PRAD2400
      ICX = IM                                                          PRAD2410
      DO  392  I = IN, IM                                               PRAD2420
      DFB = PLNKUT(HNU / THETA(I), HNUP / THETA(I))                     PRAD2430
  392 X6(I)=DFB*Q1(I)                                                   PRAD2440
      GO TO 480                                                         PRAD2450
  395 DO 470 I=IN,IR                                                    PRAD2460
      BETA=HNU/THETA(I)                                                 PRAD2470
C                                                                       PRAD2480
C                     AVOID CALCULATION OF DFB LESS THAN 1E-5           PRAD2490
C                                                                       PRAD2500
      IF (BETA-19.0) 400,410,410                                        PRAD2510
  400 BETAP=HNUP/THETA(I)                                               PRAD2520
C     EMB2=EXP(-BETAP)                                                  PRAD2530
      IF (BETAP-0.01) 410,410,460                                       PRAD2540
  410 IF (ICX-IR) 430,420,420                                           PRAD2550
  420 ICX=I-'                                                           PRAD2560
```

```
  430 IF (I-ICY) 440,440,450
  440 ICY=ICY+1
C
C                 ICX IS INDEX OF LAST ZONE WITH SIGNIFICANT SOURCE
C
C                 ICY IS INDEX OF FIRST ZONE WITH SIGNIFICANT SOURCE
C
  450 X6(I)=0.0
      X5(I)=0.0
      GO TO 470
C
C                 FORM SOURCES X6 AND X5
C
  460 DFB=PLNKUT(BETA,BETAP)
      X6(I)=DFB*Q1(I)
C     TEMP(2)=0.0384974/Q1(I)*(HNU4/(EXP(BETA)-1.0)
C    1     -HNUP4/(1.0-EMB2)*EMB2)
C     X5(I)=DFB+TEMP(2)
      ICX=IR
  470 CONTINUE
  480 IF (INM1) 490,520,500
C
C                 SET BLACKBODY CONDITION FOR IA GREATER THAN 1
C
  490 S1=13.0490
      CALL UNCLE
  500 DFB = PLNKUT (HNU/THETA(INM1),HNUP/THETA(INM1))
      X6(INM1) = DFB * THETA(INM1)**4
C     SET BLACKBODY CONDITION IF DESIRED FOR IMP1
  520 IF (GL.NE.0.5) GO TO 530
      DFB = PLNKUT(HNU / THETA(IMP1), HNUP / THETA(IMP1))
      X6(IMP1) = DFB * THETA(IMP1)**4
  530 Q31=0.0
C
C                 FORM ROSSELAND AND PLANCK OPTICAL DEPTHS
C
      DO 590 I=IN,IM
      IF (CAPAR(I)) 550,550,540
  540 IF (CAPAC(I)) 550,550,560
  550 S1=13.0550
      CALL UNCLE
C
C                 CHOOSE ALL ROSSELAND IF SOLID 10 IS POSITIVE
C
  560 IF (SOLID(10).EQ.0.) GO TO 570
      H(I)=CAPAR(I)/SV(I)
      GO TO 580
  570 H(I)=CAPAC(I)/SV(I)
  580 H2(I)=H(I)*DELTAR(I)
      IF (ALPHA .GT. 1.)  GO TO 586
      H3(I) = CAPAR(I) * G(I)
      GO TO 588
CAVEAT.  ASYNCHRONISMS IN SV AND DELTAR LEAD TO ERRONEOUS FLUCTUATIONS
C     IN H3.  THIS CAN BE FIXED BY SUBSTITUTING G IN PLANES, BUT SPHERES
C     WILL STILL HAVE THIS TROUBLE.
```

```
  586 H3(I)=CAPAR(I)/SV(I)*DELTAR(I)
  588 Q31=Q31+H3(I)
      Q3(I+1)=Q31
      H(I)=0.5*H(I)
      H2(I)=0.5*H2(I)
      H3(I)=0.5*H3(I)
C
C                ZERO DIFFUSUIN INDICATORS AND X2
C
      X2(I)=0.0
      X3(I)=0.0
      X4(I)=0.0
  590 RHC(I)=0.0
      X2(IMP1)=0.0
      X3(IMP1)=0.0
      X4(IMP1)=0.0
      IF (ICY .GT. ICX)  GO TO 990
C
C                        STEP-LINEAR CRITERION AT ICY
C                UNCONDITIONAL STEP AS BOUNDARY CONDITION IF ICY = IN
C
      IF (ICY-IN) 675,600,610
  600 Y2(IN)=X6(IN)
      TG(IN)=0.0
      GO TO 620
  610 TEMP(1)=H3(ICY-1)+H3(ICY)
      TG(ICY)=X6(ICY)/TEMP(1)
      Y2(ICY)=TG(ICY)*H3(ICY-1)
C
C     FORM Y2 AND TG  SET X3=-1 IF A DIFFUSION CRITERION MET USING HCB
C
  620 ICXM1=ICX-1
      IF (ICY .GT. ICXM1)  GO TO 672
      DO 670 I=ICY,ICXM1
      TEMP(1)=H3(I+1)+H3(I)
      IF (AMAX1(X6(I), X6(I+1)) .LE. 0.)  GO TO 640
      IF (AMIN1(H3(I), H3(I+1)) .GT. AC)  GO TO 650
      IF (ABS((H3(I)-H3(I+1))/TEMP(1)) .GT. GA)  GO TO 640
      IF (ABS((X6(I)-X6(I+1))/(X6(I)+X6(I+1)))-GA) 650,650,640
  640 TG(I+1)=0.0
      GO TO 670
  650 TG(I+1)=(X6(I+1)-X6(I))/TEMP(1)
C
C                TG(I+1)=(Q1(I+1)-Q1(I))/TEMP(1)*(X5(I+1)+X5(I))/2.0
C
      Y2(I+1)=(X6(I+1)*H3(I)+X6(I)*H3(I+1))/TEMP(1)
      IF (ABS(TG(I+1))-HCB*Y2(I+1)) 660,670,670
  660 X3(I+1)=-1.0
  670 CONTINUE
C
C                RADIATION BOUNDARY CONDITION AT ICX
C                 (VACUUM IF ICX = IM AND GL NOT 1/2)
C
  672 IF (ICX-IM) 680,690,675
  675 S1 = 13.0675
```

```
      CALL UNCLE                                                        PRAD3670
  680 TEMP(1)=H3(ICX+1)+H3(ICX)                                         PRAD3680
      TG(ICX+1)=-X6(ICX)/TEMP(1)                                        PRAD3690
      Y2(ICX+1)=-TG(ICX+1)*H3(ICX+1)                                    PRAD3700
      GO TO 700                                                         PRAD3710
  690 IF (GL .EQ. 0.5)  GO TO 700                                       PRAD3720
      Y2(IMP1)=X6(ICX)                                                  PRAD3730
      TG(IMP1)=0.0                                                      PRAD3740
C                                                                       PRAD3750
C EXTEND TRANSPORT REGION BOUNDARIES, IF NEEDED, TO PROVIDE HVB MEAN    PRAD3760
C FREE PATHS                                                            PRAD3770
C                                                                       PRAD3780
  700 I=IN+1                                                            PRAD3790
  710 IF (X3(I)) 720,740,730                                            PRAD3800
  720 I=I+1                                                             PRAD3810
      IF (I-ICX-1) 710,740,820                                          PRAD3820
  730 S1=13.0730                                                        PRAD3830
      CALL UNCLE                                                        PRAD3840
  740 J=I-1                                                             PRAD3850
  750 IF (Q3(I)-Q3(J)-HVB) 760,760,770                                  PRAD3860
  760 X4(J)=-1.0                                                        PRAD3870
      J=J-1                                                             PRAD3880
      IF (J-IN) 770,750,750                                             PRAD3890
  770 I=I+1                                                             PRAD3900
      IF (I-ICX-1) 780,780,820                                          PRAD3910
  780 IF (X3(I)) 790,770,730                                            PRAD3920
  790 J=I                                                               PRAD3930
  800 IF (Q3(J)-Q3(I-1)-HVB) 810,810,720                                PRAD3940
  810 X4(J)=-1.0                                                        PRAD3950
      J=J+1                                                             PRAD3960
      IF (J-ICX-1) 800,720,720                                          PRAD3970
  820 I=IN+1                                                            PRAD3980
C                                                                       PRAD3990
C                   TEST TO FORM TRANSPORT REGIONS
C                                                                       PRAD4010
  830 IAX=IN                                                            PRAD4020
  840 IF (X3(I)) 850,860,730                                            PRAD4030
  850 IF (X4(I)) 860,870,730                                            PRAD4040
C                                                                       PRAD4050
C                   REMOVE ONE ZONE DIFFUSION REGION
C                                                                       PRAD4070
  860 I=I+1                                                             PRAD4080
      IF (I-ICX-1) 840,950,950                                          PRAD4090
  870 I=I+1                                                             PRAD4100
      IF (I-ICX-1) 890,950,950                                          PRAD4110
  880 IF (X3(I)) 890,840,730                                            PRAD4120
  890 IF (X4(I)) 840,900,730                                            PRAD4130
  900 IP=I-3                                                            PRAD4140
      GO TO 960                                                         PRAD4150
  910 IF (X3(I)) 920,940,730                                            PRAD4160
  920 IF (X4(I)) 940,930,730                                            PRAD4170
C     FORM X2 FOR DIFFUSION ZONES IN ORDER
  930 X2(I) = -1.37E12 * TG(I)                                          PRAD4190
      I=I+1                                                             PRAD4200
      IF (I-ICX-1) 910,980,980                                          PRAD4210
```

```
C                                                                       PRAD4220
C                DO TRANSPORT TO IM IN REGION OF NO SOURCE
C                                                                       PRAD4240
  940 IAX=I                                                             PRAD4250
      GO TO 860                                                         PRAD4260
  950 IBX=IM                                                            PRAD4270
  960 CALL TRANS(IAX,IBX)                                               PRAD4280
      IF (IBX-IM) 970,990,990                                           PRAD4290
  970 I=IBX+2                                                           PRAD4300
      GC TC 930                                                         PRAD4310
  980 IF (I .GT. IM)  GO TO 990
      IAX=I
      GO TO 950                                                         PRAD4330
C                                                                       PRAD4340
C                OPTIONAL EDIT OF X2 ETC.                               PRAD4350
C                                                                       PRAD4360
  990 IF (EDITMF) 1020,1020,1000                                        PRAD4370
 1000 IARG1=SOLID(18)+0.001                                             PRAD4380
      IARG2=EDITMF+0.001                                                PRAD4390
      IF (MOD(IARG1,IARG2)) 1020,1010,1020                              PRAD4400
 1010 WRITE (3) HNU, IN, IM, IMP1, SOLID(18), TH, DHNU                  PRAD4410
C     IN SPHERICAL VERSION, REDIT IS GIVEN RHO.                         PRAD4420
C     REPLACED HERE BY CAPAR.                                           PRAD4430
      WRITE (3) (C(I), I=IN,IMP1), (H3(I), I=IN,IM), (X6(I), I=IN,IMP1),PRAD4440
     1(X2(I),I=IN,IMP1),                                                PRAD4450
     2 (CAPAR(I), I=IN,IMP1), (THETA(I), I=IN,IMP1), (EI(I), I=IN,IM)   PRAD4460
      XX=-2.0                                                           PRAD4470
      WRITE (3) XX,XX,XX,XX,XX,XX,XX                                    PRAD4480
      BACKSPACE 3                                                       PRAD4490
      JMULT=1                                                           PRAD4500
 1020 DC 1030 I=IN,IMP1                                                 PRAD4510
      SUMX2(I)=SUMX2(I)+X2(I)                                           PRAD4520
 1030 CCNTINUE                                                          PRAD4530
C                                                                       PRAD4540
C     ADVANCE FREQ, STORE EMERGENT FLUX, TEST FOR COMPLETION OF GROUPS  PRAD4550
C                                                                       PRAD4560
      HNUP=HNU                                                          PRAD4570
      HNUP4=HNU4                                                        PRAD4580
      IHNU = IHNU + 1                                                   PRAD4590
      IF (IHNU-NHNU) 1040,1040,1060                                     PRAD4600
 1040 CALL OVCHK (KOOOFX)                                               PRAD4610
      GO TO (1050,360), KOOOFX                                          PRAD4620
C***********************************************************************PRAD4630
C                                                                      *PRAD4640
C                   E N D   F R E Q U E N C Y   L O O P                *PRAD4650
C                                                                      *PRAD4660
C***********************************************************************PRAD4670
 1050 S1 = 13.1050                                                      PRAD4680
      CALL UNCLE                                                        PRAD4690
 1060 SUMX2(INM1) = 0.0                                                 PRAD4700
      DO 1070 I=INM1,IMP1                                               PRAD4710
      X2(I) = SUMX2(I)                                                  PRAD4720
 1070 ER(I) = SUMX2(I) - SUMX2(I+1)                                     PRAD4730
C                                                                       PRAD4740
C           FORM MONOFREQUENCY QUANTITIES AND FIND MIN TIME STEP        PRAD4750
```

```
C
      WSB = 0.0
      DO  1075  I = 1, MAXLM
 1075 WSB = WSB + ELM(I)
      DTR1=1.E10
      DTR2=1.E10
      IF (KMAX.EQ.0) GO TO 1080
      CALL KAPPA(IN,IM)
 1080 DO 1230 I=IN,IM
C
C               IF ROSS IS ZERO EXIT
C
      IF (CAPAR(I)) 1090,1090,1100
 1090 S1=13.1090
      CALL UNCLE
 1100 TEMP(3)=CAPAR(I)
      IF (SOLID(10)) 1110,1120,1110
 1110 TEMP(1)=CAPAR(I)
      GO TO 1130
 1120 TEMP(1)=SQRT(CAPAR(I)*CAPAC(I))
 1130 IF (TEMP(1)) 1090,1090,1140
 1140 H(I)=0.5*TEMP(1)/SV(I)
      H3(I) = H(I) * DELTAR(I)
      IF (.001-THETA(I)) 1160,1230,1230
 1160 IF (H3(I).GT.0.1) GO TO 1170
      IF (ER(I).EQ.0.) GO TO 1170
      WSBB = E(I) * G(I)
      IF (TELM(37) .EQ. 0.0)  GO TO 1170
      IF (WSBB - TELM(37) * WSB)  1170, 1165, 1165
 1165 TEMP(2)=.5*CV(I)*THETA(I)*G(I)/ABS(ER(I))
      GO TO 1180
 1170 TEMP(2)=(.5+1.5*H3(I)**2)*CV(I)/(4.1132E12*TEMP(3)*THETA(I)**3)
      TEMP(2)=TEMP(2)*TELM(25)
C***********************************************************************
C                                                                      *
C             F I N D   M I N I M U M   T I M E   S T E P              *
C                                                                      *
C***********************************************************************
 1180 IF (TEMP(2)) 1230,1230,1190
 1190 CONTINUE
      IF (TEMP(2)-DTR1) 1200,1210,1210
 1200 DTR2=DTR1
      IMN2=IMN1
      DTR1=TEMP(2)
      IMN1=I
      GO TO 1230
 1210 IF (TEMP(2)-DTR2) 1220,1230,1230
 1220 DTR2=TEMP(2)
      IMN2=I
 1230 CONTINUE
      DTRMIN=DTR1
      EO=IMN1
C
C               PRINT MINIMUM TIME STEPS BETWEEN EDITS
C
```

```
      IF (DTR1-TELM(26)) 1240,1250,1250
 1240 TELM(26)=DTR1
      TELM(27)=IMN1
      TELM(28)=DTR2
      TELM(29)=IMN2
      TELM(30)=SOLID(18)+1.0
 1250 CONTINUE
C
C          DETERMINE IF RADIATION OR HYDRO WILL SUBCYCLE
C
      IF (DTRMIN-DTR) 1280,1300,1260
 1260 BLANK3=TH+AMIN1(DTRMIN,GR*DTH2)
      IF (S17) 1300,1270,1300
 1270 S9 = 1.0
      GO TO 1300
C*********************************************************************
C                                                                    *
C                    R E D U C E   T I M E   S T E P                 *
C                                                                    *
C*********************************************************************
 1280 NRAD=ZP1(18)/DTRMIN+1.0
      DTR=ZP1(18)/FLOAT(NRAD)
      IF (NRAD-NTIMES) 1300,1300,1290
 1290 S1=13.1290
      CALL UNCLE
C     ZERO OUT STRAY QUANTITIES FROM PREVIOUS CYCLES
 1300 DO  1310  I = IMP1, IG
      CAPAR(I) = 0.
      CAPAC(I) = 0.
      X2(I+1) = 0.
      X3(I+1) = 0.
      X4(I+1) = 0.
      SUMX2(I+1) = 0.
      SUMX3(I+1) = 0.
      SUMX4(I+1) = 0.
 1310 ER(I+1) = 0.
      IF (ZP1(26) .EQ. 0.0)  GO TO 1400
C     RESTORE GOODIES FOR NONEQ
      .  (PUSHA .LT. 0.0)  GO TO 1350
      BC( IMP1)=WSZ2
      BR(IMP1)=WSZ3
      CPTR(IMP1) = WSZ4
      RHO( IMP1)=WSZ5
      GO TO 1400
 1350 BC(INM1) = WSZ2
      BR(INM1) = WSZ3
      CRTR(INM1) = WSZ4
      RHC(INM1) = WSZ5
 1400 RETURN
      END
```

This page intentionally left blank.

Appendix B

PTRANS

This page intentionally left blank.

```
$IBFTC PTRANS  FULIST,DECK,REF                                          PTRA0000
      SUBROUTINE TRANS(N,M)                                             PTRA0010
C     CCMPILED JULY 1, 1965   WBL                                       PTRA0020
C     PLANE CHARACTERISTIC TRACE WITH DOUBLE GAUSSIAN INTEGRATION       PTRA0030
C     DECK DBLGAU REQUIRED FOR INTEGRATION COEFFS.                      PTRA0040
C***********************************************************************PTRA0050
C*                   S P U T T E R   C O M M O N                      **PTRA0060
C                                                                      *PTRA0070
      COMMON   LMDA(37), NR     , NSMLR , IA     , IB     , ICA    , ICB    ,PTRA0080
     1   KMAX  , BLANK1, BLANK2, BLANK3, IAP1   , IBP1   , ICAP1  , ICBP1  ,PTRA0090
     2   II    , IG    , NRAD  , BLANK4, IAM1   , IBM1   , ICAM1  , ICBM1  ,PTRA0100
     3   IIP1  , IGM1  , IALPHA, BLANK5, TH     , TMAX   , BLANK6, DELPRT ,PTRA0110
     4   FREQ  , CNTMAX, AR    , ASMLR , PUSHA  , PUSHB  , BOILA  , BOILB  ,PTRA0120
     5   CVA   , CVB   , SLUG  , ALPHA , HVA    , HVB    , HCA    , HCB    ,PTRA0130
     6   EMINA , EMINB , CA    , CB    , GA     , GB     , GL     , GR     ,PTRA0140
     7   RHCL  , RHOR  , EPIO  , EPSI  , RIA    , RIB    , RDIA   , RDIB   ,PTRA0150
     8   RPIA  , RPIB  , RPDIA , RPDIB , TPRINT, TA      , TB     , TC     ,PTRA0160
      COMMON      TD   , TE    , DTH2  , DTH2P , DTH1   , DTRMIN, DTMAX  ,PTRA0170
     1   DTMAX1, DTMAX2, DTMAX3, DTR   , SWITCH, CO      , CMIN   , DELTA  ,PTRA0180
     2   GAMA  , WCRIT , SIGMAQ, AC    , ACO3T4, CNVRT  , SUMRA  , SUMRB  ,PTRA0190
     3   ROIA  , ROIAM1, ROIB  , ROIBP1, GMS    , S1     , S2     , S3     ,PTRA0200
     4   S4    , S5    , S6    , S7    , S8     , S9     , S10    , S11    ,PTRA0210
     5   S12   , S13   , S14   , S15   , S16    , S17    , S18    , S19    ,PTRA0220
     6   S20   , EO    , FO    , TAU   , ZERO   , R    (152), DELTAR(152),PTRA0230
     7  ASQ   (152), RD    (152), VO    (152), RDD   (152), SMLR  (152),PTRA0240
     8  DELR  ( 37), P     (152), P1    (152), PB    (152), PB1   (152) PTRA0250
      COMMON          P2    (152), SV    (152), RHO   (152), THETA (152),PTRA0260
     1  W     (152), E     (152), EI    (152), EK    (152), A     (152),PTRA0270
     2  V     (152), G     (152), D     (152), C     (152), X2    (152),PTRA0280
     3  X3    (152), X4    (152), X5    (152), X6    (152), X7    (152),PTRA0290
     4  SMLA  (152), SMLB  (152), SMLC  (152), SMLD  (152), SMLE  (152),PTRA0300
     5  EC    (152), ER    (152), SMLQ  (152), SMLH  (152), BIGA  (152),PTRA0310
     6  BIGB  (152), CV    (152), BC    (152), BR    (152), CHIC  (152),PTRA0320
     7  CHIR  (152), CAPAC (152), CAPAR (152), CRTC  (152), CRTR  (152),PTRA0330
     8  CRTPC (152), GOFR  (152), FEW   (152), CAR   (152), OKLM  ( 37) PTRA0340
      COMMON          TELM  ( 37), EKLM  ( 37), ELM   ( 37), FCLM  ( 37),PTRA0350
     1  FRLM  ( 37), WLM   ( 37), QLM   ( 37), AMASNO( 37), CHRNO ( 37),PTRA0360
     2  ZP1   ( 37), ZP2   ( 37), SOLID ( 37), ECHCK ( 37), RK    (104),PTRA0370
     3  RL    ( 37), RHOK  (104), RDK   (104), THETAK(104), TEMP  ( 16),PTRA0380
     4  HEAD  ( 12), MAXL       , MAXLM                                 PTRA0390
C*                                                                    **PTRA0400
C***********************************************************************PTRA0410
      DIMENSION  Q3(1),TG(1),H2(1),Q1(1),X8(1),SUMX3(1),SUMX4(1)        PTRA0420
      DIMENSION  H4(1),Y2(1),H(1),SUMX2(1),Q2(1)                        PTRA0430
      DIMENSION  Q37(1), Q38(1), H3(1)                                  PTRA0440
      COMMON /LINDLY/ HNU,SGNL,IHNU,NHNU,HNUP,NT,IM,IN,DHNU,THICK,NY    PTRA0450
      COMMON /CNTRL/ SCYCLE, JMULT                                      PTRA0460
      COMMON /DAVIS/  ICX, ICY                                          PTRA0470
      COMMON  /TQ/  QINT1(300), QINT2(300), TITLE(12)                   PTRA0480
      EQUIVALENCE  (SMLA,H4),(SMLD,Y2)                                  PTRA0490
      EQUIVALENCE  (BC,TG),(BIGB,H),(CRTR,SUMX2),(CHIC,SUMX3)           PTRA0500
      EQUIVALENCE  (SMLH,X8),(CAR,Q37),(CHIR,Q38),(SMLC,H3)             PTRA0510
      EQUIVALENCE  (ACO3T4,TRDBG),(S12,EDITMF)                          PTRA0520
      EQUIVALENCE  (EC,Q1),(X7 ,H2),(BIGA,SUMX4),(GOFR,Q3)              PTRA0530
C***********************************************************************PTRA0540
```

```
      DIMENSION  RR(40)                                                 PTRA0550
      DATA  RR/2.113248E-01,7.886752E-01,1.056624E-01,3.943376E-01,     PTRA0560
     1         1.127017E-01,5.000000E-01,8.872983E-01,3.130600E-02,     PTRA0570
     2         2.222222E-01,2.464718E-01,6.943180E-02,3.300095E-01,     PTRA0580
     3         6.699905E-01,9.305682E-01,1.207610E-02,1.076071E-01,     PTRA0590
     4         2.184655E-01,1.618513E-01,4.691010E-02,2.307653E-01,     PTRA0600
     5         5.000000E-01,7.692347E-01,9.530899E-01,5.557100E-03,     PTRA0610
     6         5.522540E-02,1.422222E-01,1.840889E-01,1.129063E-01,     PTRA0620
     7         3.376520E-02,1.693953E-01,3.806903E-01,6.193096E-01,     PTRA0630
     8         8.306047E-01,9.662348E-01,2.892400E-03,3.055570E-02,     PTRA0640
     9         8.906520E-02,1.448918E-01,1.498251E-01,8.276980E-02/     PTRA0650
C                                                                      *PTRA0660
C                           EDITMF SAME AS     S12                     *PTRA0670
C                           H      SAME AS    BIGB                     *PTRA0680
C                           H2     SAME AS     X7                       PTRA0690
C                           H3     SAME AS    SMLC                     *PTRA0700
C                           H4     SAME AS    SMLA                     *PTRA0710
C                           Q1     SAME AS      EC                     *PTRA0720
C                           Q3     SAME AS    GOFR                     *PTRA0730
C                           Q37    SAME AS     CAR                     *PTRA0740
C                           Q38    SAME AS    CHIR                     *PTRA0750
C                           SUMX2  SAME AS    CRTR                     *PTRA0760
C                           SUMX3  SAME AS    CHIC                     *PTRA0770
C                           SUMX4  SAME AS    BIGA                     *PTRA0780
C                           IG     SAME AS      BC                     *PTRA0790
C                           IRDBG  SAME AS AC03T4                      *PTRA0800
C                           X8     SAME AS    SMLH                     *PTRA0810
C                           Y2     SAME AS    SMLD                     *PTRA0820
C                                                                      *PTRA0830
C***********************************************************************PTRA0840
C                                                                       PTRA0850
C                                                                       PTRA0860
C     FEX, FM, SUMRHO, CSQD, XSQD, Y, YSQD, Q2, NOT USED                PTRA0870
C                                                                       PTRA0880
C                                                                       PTRA0890
C***********************************************************************PTRA0900
C                                                                      *PTRA0910
C                           P L A N E S   O N L Y                      *PTRA0920
C                                                                      *PTRA0930
C***********************************************************************PTRA0940
      IAX=N                                                             PTRA0950
      IBX=M                                                             PTRA0960
      IN=IA                                                             PTRA0970
      INM1=IN-1                                                         PTRA0980
      IMP1 = IM + 1                                                     PTRA0990
      CALL DVCHK(KOOOFX)                                                PTRA1000
      GO TO (100,110), KOOOFX                                           PTRA1010
  100 S1=14.0100                                                        PTRA1020
      CALL UNCLE                                                        PTRA1030
  110 IBXP1=IBX+1                                                       PTRA1040
      IALPHA=ALPHA                                                      PTRA1050
C                                                                       PTRA1060
C               ERROR IF NOT PLANE                                      PTRA1070
C                                                                       PTRA1080
      GO TO (130,120,120), IALPHA                                       PTRA1090
```

```
  120 S1=14.0120
      CALL UNCLE
  130 NY = LMDA(37) - 1
      NMU = (NY - 1) * (NY + 2) + 1
      NGS = NMU + NY + 1
      JJ = 0
C
C                DO POSITIVE ANGLES FIRST
C
  140 I=IAX
      F2=0.0
C
C     IF IAX=IN TRANSFER TO 150 TO SET SPECIAL BOUNDARY CONDITIONS
C
      IF (IAX-IN) 360,150,180
C
C                CALCULATE BOUNDARY SOURCE INTENSITY
C
  150 IF (INM1) 160,310,170
C
C                SET BLACKBODY CONDITION FOR PUSHER
C
  160 S1=14.0160
      CALL UNCLE
  170 F2=X6(INM1)
      GO TO 310
C
C                DIFFUSION BOUNDARY CONDITION AT IAX
C
  180 IF (TG(I) .EQ. 0.)  Y2(I) = X6(I-1)
      TEMP(1)=H2(I-1)/RR(NMU)
      X8(I)=TG(I)*RR(NMU)
      IF (TEMP(1)-1.E-2) 190,190,200
  190 F2X = Y2(I-1) - TG(I-1) * RR(NMU)
      F2 = ((Y2(I) + Y2(I-1)) * 0.5 + X6(I-1) - F2X-F2X) * TEMP(1) + F2X
      GO TO 250
  200 H4(I-1)=FREXP(-TEMP(1))
      F2=Y2(I)-X8(I)+(X8(I)-RR(NMU)*TG(I-1))*H4(I-1)
      GO TO 250
  210 IF (TG(I-1) .EQ. 0.)  Y2(I-1) = X6(I-1)
      X8(I-1)=TG(I-1)*RR(NMU)
C
C                REGULAR INTEGRATION STEP FOR F2, POSITIVE MU
C
  220 IF (TG(I) .EQ. 0.)  Y2(I) = X6(I-1)
      X8(I)=TG(I)*RR(NMU)
      TEMP(1)=H2(I-1)/RR(NMU)
      IF (TEMP(1)-1.E-2) 230,230,240
  230 F2 = ((Y2(I) + Y2(I-1)) * 0.5 + X6(I-1) - F2 - F2) * TEMP(1) + F2
      GO TO 250
  240 H4(I-1)=FREXP(-TEMP(1))
      F2=Y2(I)-X8(I)+((F2-Y2(I-1)+X8(I-1))*H4(I-1)
     1+X8(I)-X8(I-1))[illegible](I-1)
  250 IF (F2.LT.0.0) GO TO 280
  260 SUMX3(I)=F2
```

```
      IF (TG(I) .EQ. 0.)  Y2(I) = X6(I)
      I=I+1
      IF (I-IBXP1) 270,270,320
  270 IF (I-ICX-1) 220,220,290
C
C                 NEGATIVE F2 ERROR
C
  280 S1=14.0280
      CALL UNCLE
C
C                 NO SOURCE IN ZONE GREATER THAN ICX
C
  290 IF (F2.EQ.0.0) GO TO 260
      TEMP(1)=H2(I-1)/RR(NMU)
      H4(I-1)=FREXP(-TEMP(1)-TEMP(1))
      F2=F2*H4(I-1)
      GO TO 260
  300 IF (F2.EQ.0.0) GO TO 310
      TEMP(1)=H2(I-1)/RR(NMU)
      H4(I-1)=FREXP(-TEMP(1)-TEMP(1))
      F2=F2*H4(I-1)
  310 SUMX3(I) = F2
      I=I+1
      IF (I-ICY) 300,300,210
C
C                 DO NEGATIVE ANGLES SECOND
C
  320 I=IBXP1
      IF (IBX-IM) 370,330,360
  330 IF (GL) 480,520,340
C     GL = 1/2 MEANS BLACKBODY CONDITION SET AT IMP1
C     GL = POSITIVE INTEGER MEANS INTENSITIES FROM QINT1 TABLE AT IMP1
C     GL = 0 MEANS VACUUM AT IMP1
C     GL NEGATIVE MEANS REFLECTIVE CONDITION AT IMP1
  340 IF (GL.NE.0.5) GO TO 350
      F2 = X6(IMP1)
      GO TO 480
  350 IQNT = JJ + 1 + (NY + 1) * (IHNU - 1)
      F2 = QINT1(IQNT) / 68.5 * DHNU
      GO TO 480
C
C                 ERROR IF INDEX EXCEEDS NORMAL RANGE
C
  360 S1=14.0360
      CALL UNCLE
C
C                 DIFFUSION BOUNDARY CONDITION AT IBXP1
C
  370 IF (TG(I) .EQ. 0.)  Y2(I) = X6(I)
      TEMP(1)=H2(I)/RR(NMU)
      IF (TEMP(1)-1.E-2) 380,380,390
  380 F2X = Y2(I+1) + TG(I+1) * RR(NMU)
      F2 = ((Y2(I) + Y2(I+1)) * 0.5 + X6(I) - F2X - F2X) * TEMP(1) + F2X
      GO TO 430
  390 H4(I)=FREXP(-TEMP(1))
```

```
      F2=Y2(I)+X8(I)+(RR(NMU)*TG(I+1)-X8(I))*H4(I)
      GO TO 430
  399 IF (TG(I+1) .EQ. 0.)  Y2(I+1) = X6(I)
C
C                   REGULAR INTEGRATION STEP FOR F2, NEGATIVE MU
C
  400 IF (TG(I) .EQ. 0.)  Y2(I) = X6(I)
      TEMP(1)=H2(I)/RR(NMU)
      IF (TEMP(1)-1.E-2) 410,410,420
  410 F2 = ((Y2(I) + Y2(I+1)) * 0.5 + X6(I) - F2 - F2) * TEMP(1) + F2
      GO TO 430
  420 F2=Y2(I)+X8(I)+((F2-Y2(I+1)-X8(I+1))*H4(I)+X8(I+1)-X8(I))*H4(I)
  430 IF (F2.LT.0.) GO TO 460
  440 SUMX4(I)=F2
C
C                   FORM CONTRIBUTION TO X2
C
      X2(I)=X2(I)-(F2-SUMX3(I))*RR(NGS)
C
C     RHO(I)=RHO(I)+(F2+SUMX3(I))*RR(NGS)/RR(NMU)
C
      IF (TG(I) .EQ. 0.)  Y2(I) = X6(I-1)
      I=I-1
      IF (I-IAX) 530,450,450
  450 IF (I-ICY) 500,400,400
  460 S1=14.0460
      CALL UNCLE
C
C                   NO SOURCE IN ZONE LESS THAN ICY
C
  470 IF (F2.EQ.0.0) GO TO 480
      TEMP(1)=H2(I)/RR(NMU)
      H4(I)=FREXP(-TEMP(1)-TEMP(1))
      F2=F2*H4(I)
  480 SUMX4(I)=F2
  490 X2(I)=X2(I)-(F2-SUMX3(I))*RR(NGS)
      I=I-1
      IF (I-1-ICX) 399,470,470
C
C                   NO SOURCE IN ZONE LESS THAN ICY
C
  500 IF (F2.EQ.0.0) GO TO 510
      TEMP(1)=H2(I)/RR(NMU)
      H4(I)=FREXP(-TEMP(1)-TEMP(1))
      F2=F2*H4(I)
  510 SUMX4(I)=F2
      X2(I)=X2(I)-(F2-SUMX3(I))*RR(NGS)
      I=I-1
      IF (I-IAX) 530,500,500
  520 F2=0.0
      GC TO 480
  530 CONTINUE
      IF (TRDBG .EQ. 0.0 .OR. TRDBG .GE. SOLID(18))  GO TO 539
C     STORE INTENSITIES FOR DEBUG PRINT
      XX = -0.5
```

```
      JJJ = JJ + 1
      WRITE (3)  XX, IAX, JJJ, IBXP1, SOLID(18), TH, RR(NMU)
      WRITE (3)  (SUMX3(I), SUMX4(I), I = IAX, IBXP1)
      XX = -2.
      WRITE (3) (XX, I = 1, 7)
      BACKSPACE 3
  539 DHNU=HNUP-HNU
  540 JJ = JJ + 1
      NMU = NMU + 1
      NGS = NGS + 1
      IF (JJ-NY) 140,140,550
  550 DO 560 I=IAX,IBXP1
  560 X2(I) = X2(I)* 2.052E12
      RETURN
      END
```

DISTRIBUTION

Copy no.

HEADQUARTERS USAF

Copy no.	
1	Hq USAF (AFSRT, Maj Dunn), Wash, DC 20330
	MAJOR AIR COMMANDS
1	AFSC (SCT, Maj R. E. Linkous), Andrews AFB, Wash, DC 20331
1	AUL, Maxwell AFB, Ala 36112
	AFSC ORGANIZATIONS
1	RTD (RTTW, Maj Kritzer), Bolling AFB, Wash, DC 20332
1	BSD (BSYDV), Norton AFB, Calif 92409
2	AF Avionics Laboratory (AVR(L), Mr. Don Nordstrom), Wright-Patterson AFB, Ohio 45433
1	AF Flight Dynamics Laboratory (FDTR, Mr. Adams), Wright-Patterson AFB, Ohio 45433
1	RADC (RALS, Capt James Klaus), Griffiss AFB, NY 13442
	KIRTLAND AFB ORGANIZATIONS
1	AFSWC (SWEH), Kirtland AFB, NM 87117
	AFWL, Kirtland AFB, NM 87117
15	(WLIL)
1	(WLRTH, Capt W. A. Whittaker)
1	(WLRT, Col Pennington)
2	(WLRTH, Capt Spillman)
1	(WLRTH, Lt Tompkins)
	OTHER AIR FORCE AGENCIES
	Director, USAF Project RAND, via: Air Force Liaison Office, The RAND Corporation, 1700 Main Street, Santa Monica, Calif 90406
1	(RAND Physics Div, Dr. Harold Brode)
1	(RAND Library)
	Hq OAR, Bldg T-D, Wash, DC 20333
1	(RROSP, Lt Col V. G. Radovich)
1	(RROP, Marked for Contract AF 29(601)-5695, ARPA Order No. 313-62, Project 8814)
	ARMY ACTIVITIES
1	Redstone Scientific Information Center, US Army Missile Command, ATTN: Army Nike X Project Officer, Henry Solomonson, Redstone Arsenal, Ala 35809
	Director of Research and Development, US Army Missile Command, Ala
1	(AM SMI-RFE, Dr. W. Davis)
1	(Mr. William D. McKnight)

DISTRIBUTION (ccnt'd)

Copy no.	
1	Army Research Office Physical Sciences Division, Office Chief, Research and Development, ATTN: Dr. Robert A. Watson, Wash, DC 20310
	NAVY ACTIVITIES
1	Director, Special Projects, Department of the Navy, ATTN: Technical Division, Wash, DC 20360
	OTHER DOD ACTIVITIES
	Director, Defense Atomic Support Agency, Wash, DC 20301
1	(Document Library Branch)
1	(Radiation Division, Col Howard C. Rose)
20	DDC (TIAAS), Cameron Station, Alexandria, Va 22314
1	Joint Advance Study Group, J-5 Plans and Policy Directorate, Joint Chiefs of Staff, Requirements and Development Division, ATTN: Special Projects Branch, Rm 2D982, The Pentagon, Wash, DC 20301
	ODD, R&E, The Pentagon, Wash, DC 20301
1	(Dr. Sherwin, Rm 3E1030)
1	(Mr. Ernest Wood, Rm 3D1051)
	AEC ACTIVITIES
1	Sandia Corporation, ATTN: Dr. Nick Beauchamp, Box 5800, Sandia Base, NM 87115
1	Sandia Corporation (Tech Library), P. O. Box 969, Livermore, Calif 94551
1	University of California Lawrence Radiation Laboratory, ATTN: Director's Office (Library), P. O. Box 808, Livermore, Calif 94551
	Director, Los Alamos Scientific Laboratory, P. O. Box 1663, Los Alamos, NM 87554
1	(Helen Redman, Report Library)
1	(Dr. Herman Heirlin, J-15)
1	(C. G. Davis, J-15)
	OTHER
1	Institute for Defense Analysis, ATTN: Dr. William Culver, 1666 Connecticut Ave NW, Wash, DC 20330 THRU: ARPA
1	Westinghouse Electric Corp, Electronics Div, ATTN: Mr. Charles Bell, Project Comet, 5561, Baltimore, Md THRU: RADC (RALS, Capt J. Klaus), Griffiss AFB, NY 13441
2	AVCO Corp, RAD Div, ATTN: Dr. R. E. Schlier, 201 Lowell Street, Wilmington, Mass Contract AF 29(601)-5674
2	General Atomic Division of General Dynamics Corp, ATTN: Dr. K. D. Pyatt, Box 608, San Diego 12, Calif Contract AF 29(601)-6390
1	Official Record Copy (Lt Tompkins, WLRTH)

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) General Atomic Division of General Dynamics Corporation San Diego, California	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

FIREBALL PHENOMENOLOGY AND CODE DEVELOPMENT; Vol IV, SPUTTER Subroutines for Radiation Transport in Planes

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

1 June 1963-13 July 1965

5. AUTHOR(S) (Last name, first name, initial)

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
July 1966	62	1

8a. CONTRACT OR GRANT NO. AF 29(601)-6492	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 5710	AFWL-TR-65-143, Vol IV
c. Subtask No. 07.003/005	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	GA-6585

10. AVAILABILITY/LIMITATION NOTICES This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of AFWL (WLRT), Kirtland AFB, NM, 87117. Distribution is limited because of the technology discussed in the report.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY AFWL (WLRT) Kirtland AFB, NM 87117

13. ABSTRACT

The radiation-transport subroutines of the SPUTTER code for plane slab geometry have been supplemented by alternative formulation based on integration along sampling ray paths through the slab. Angular integrations are performed by the Gaussian quadrature method which determines the ray angles. Options may be exercised to determine the number of angles and the nature of the radiation boundary condition at one boundary of the transport region. The characteristic ray code differs from the current integral method in performing problems having a large number of zones more rapidly and in having more general boundary conditions. For most applications a small number of angles give adequate accuracy. The numerical method used in the ray code is described. In addition, the organization of the code is discussed and subroutines are listed.

DD FORM 1 JAN 64 1473

UNCLASSIFIED
Security Classification

UNCLASSIFIED

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Numerical routines for radiation transport Infinite plane systems Fireball diffusion flux Radiation transport						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS. Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

UNCLASSIFIED

Security Classification