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INVESTIGATION OF ROTARY ACTUATION TECHNIQUE

Quarterly Technical Report

RESEARCH LABORATORIES DIVISION

SOUTHFIELD, MICHIGAN

INVESTIGATION OF ROTARY ACTUATION TECHNIQUE

Quarterly Technical Report

3 April 1966

Contract AF 33(615)-3431

Submitted to

Air Force Aeropropulsion Laboratory
Wright-Patterson Air Force Base
Dayton, Ohio

By

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INTRODUCTION

The work covered by this report was accomplished at the Bendix Research Laboratories Division under Contract AF 33(615)-3431, during the period from 3 January 1966 to 3 April 1966. This work is being performed under the sponsorship of the Air Force Aero Propulsion Laboratory, Research and Technology Division, Air Force Systems Command, United States Air Force.

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SECTION 2

PROGRAM DEFINITION

2.1 PROGRAM OBJECTIVE

This program is intended to provide the technology necessary for the integration of hydraulic/mechanical interfaces so that rotary actuators may be utilized for fast response, long endurance applications, such as vehicle flight control surface actuation. This is an exploratory program for the investigation of a unique rotary hydromechanical actuator, The Bendix DYNAVECTOR* Actuator**.

This rotary actuator technology will be demonstrated by the fabrication and life endurance testing of an Engineering Model DYNAVECTOR actuator designed to meet an assumed set of flight test performance requirements as defined in Section 2.3.

2.2 DEVELOPMENT APPROACH

2.2.1 Task Definitions

The program milestone chart, shown in Figure 2-1, defines the program tasks into five major activities:

- Experimental Model Critical Parameters Evaluation

- DYNAVECTOR Actuator Demonstration Model

- Experimental Model Design and Fabrication

- Experimental Model Performance and Life Test

- Program Direction and Reports

Experimental and analytical studies will be conducted in parallel, to achieve the objectives of the program. Early development tests will be conducted on critical components to obtain confirmation of the parameters established for the design of the Engineering Model

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* Trademark of The Bendix Corporation.
** The Bendix Corporation has a patent application pending on this device.

MILESTONE-SCHEDULE

"Investigation of Rotary Actuation Technique"

MAJOR MILESTONES	FORECASTED ACHIEVEMENT DATE	ACTUAL ACHIEVEMENT DATE
E-M CRITICAL PARAMETERS EVALUATION		
1.1 START PARAMETER ANALYSES	1-15-66	1-15-66
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LEGEND
PLANNED MILESTONES △
COMPLETED MILESTONES ▲

Figure 2-1 - Milestone Schedule

DYNAVECTOR Actuator. Primary areas of investigation will include gear strength and wear characteristics, motor performance factors, and seal and bearing characteristics. Analytical studies paralleling the experimental tests will also include a failure mode study to provide maximum assurance that the critical design parameters will be recognized early in the design effort. The results of the component development tests and parameter studies will be utilized to conduct a critical design review, prior to initiation of the fabrication of an Experimental Model of the servo actuator. Upon completion of the fabrication of the Experimental Model, preliminary performance evaluation tests will be performed, using standard MIL-H-5606 hydraulic fluid under room temperature conditions. When the performance characteristics are evaluated and considered satisfactory, the actuator will be tested under the specified life test conditions (Reference Section 2.3.2) using Oronite Fluid 70.

2.2.2   Technical Approach

The DYNAVECTOR actuator is an integral high speed motor and high ratio transmission without high velocity mechanical elements. The major components of the DYNAVECTOR actuator assembly consist of a series of displacement chambers, a unique integral epicyclic transmission, and commutation porting. The transmission and motor use elements common to both, resulting in a much simpler and more reliable design.

The power element is a positive displacement, very low inertia, non-rotating vane motor. Its output is a radial force vector that rotates at high speed and in either direction of rotation. The displacement chambers formed by the vanes and the housing expand and collapse at the same speed as the force vector, but do not rotate. The motor is self-commutating but does not contain a rotating porting plate or spindle. The absence of high velocity members in the motor significantly reduces the inertia, resulting in high acceleration capability.

The unique epicyclic transmission converts the rotating force vector directly into low speed, high torque rotary motion without the use of high speed mechanical input stages. The transmission also has zero backlash without using preloaded members.

The integration of the power element and epicyclic transmission into an integral actuator design results in an ideal servo actuator with a high torque-to-inertia ratio and high constant efficiencies for both small and rated loads.

The operation of the DYNAVECTOR motor is illustrated by Figure 2-2. The basic components are the ring gear, the ground gear and housing, the center output gear, and the unique vanes. The displacement chambers are formed between the ground gear and the ring gear mesh by the vanes. This gear mesh provides displacement motion without rotation because both gears have exactly the same number of teeth. It may be considered as a loose spline but is a true involute gear mesh. The internal portion of the ring gear forms the transmission between the motor and the output shaft and represents the epicyclic transmission.

A force vector is generated by pressurizing three adjacent displacement chambers and venting the remaining three. The vector is made to rotate by pressurizing a vented chamber adjacent to the original
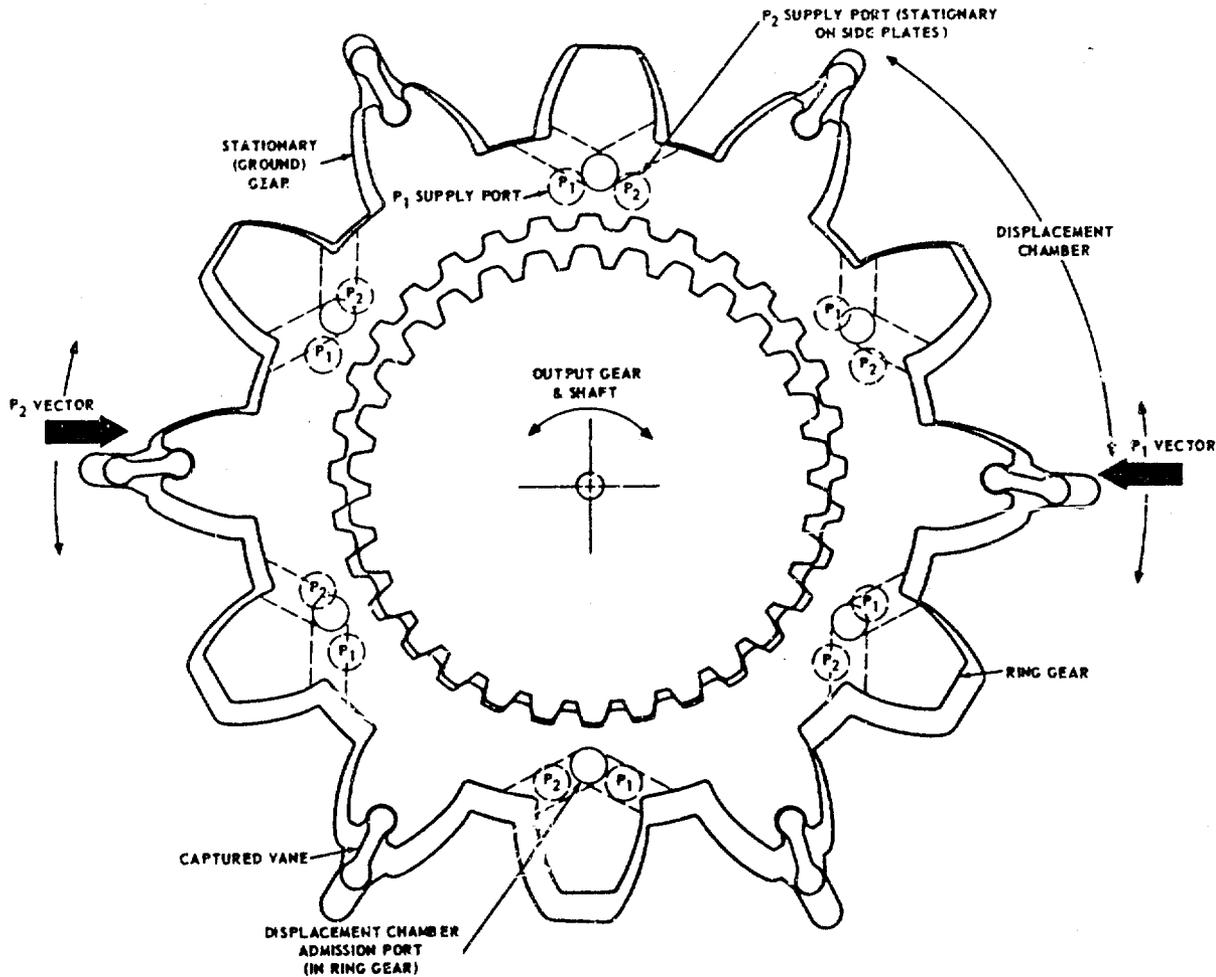
Figure 2-2 - Basic Operation and Design of DYNAVECTOR Actuator

three pressurized chambers while simultaneously venting the diametrically opposite one. If the force vector on the ring gear is located at approximately 90 degrees to the ring and output gear contact point, the ring gear will move, causing the output gear to turn and the contact point to move. If the force vector is also rotated and remains 90 degrees to the contact point, the motion will be continuous and the output shaft will turn continuously but at a much lower speed than the force vector. The ratio will be determined by the difference in number of teeth between the ring gear and the output gear. The gears in Figure 2-2 have 30 and 32 teeth; thus, the reduction ratio is 15:1.

The available differential pressure in the form of two motor port pressures P_1 and P_2 must be commutated to the proper displacement chambers to produce a rotating force vector in phase with the ring gear motion. To ensure that this phase relationship always holds true, the motion or position of the ring gear is used to provide this commutation through a series of ports. Each displacement chamber has a pair of supply ports or a P_1 and P_2 port associated with it. The P_1 ports are all interconnected in the housing and brought out to a single inlet port, as are all the P_2 ports. These ports are in the housing and, therefore, stationary with respect to the displacement chambers. They are also located under the ring gear face, as shown in Figure 2-2, and a port connecting the displacement chamber to the ring gear face is located opposite them.

By locating these P_1 and P_2 ports as shown in Figure 2-2, the ring gear ports will open P_1 ports to half the displacement chambers and P_2 ports to the remaining half. The resulting pressure force on the ring gear from the displacement chambers connected to P_1 is 180 degrees opposite P_2 and 90 degrees from the output gear contact point. Therefore, pressurizing P_1 and venting P_2 produces rotation in one direction, while interchanging pressure and return reverses the motor. This also satisfies the desired relationship between force vector and ring gear position. Because this commutation is created by the displacement member or ring gear itself, it will always rotate in phase with the motor, producing maximum efficiency.

The number of vanes or displacement chambers only determines the very low speed torque ripple present. The number of chambers need not be odd or even since the starting torque is only a function of the force vector angle which varies through an angle equal to the angle included by one displacement chamber.

For the most compact motor, vanes are used to form the displacement chamber. However, most common methods used in fluid motors may be applied. Radial pistons, bellows, flexible diaphragms, etc., could be used to form displacement chambers and generate the required force vector from applied pressures.

One of the primary advantages of the DYNAVECTOR actuator is the potential efficiency, especially outstanding at high ratios. The unique ring gear transmits the load reaction forces at close to one-to-one correspondence to ground and, therefore, is actually an output or high torque member. On the other hand, it is also the dynamic member of the motor, which is the low torque component of the system.

Two other factors present in conventional rotary motor plus transmission systems are significantly reduced by the DYNAVECTOR actuator design and operation. The relative velocities between dynamic and static members are very small, because of the small amplitude epicyclic motion. In a DYNAVECTOR actuator, the relative velocity between the ring gear and the housing is only a function of the eccentricity, which is usually less than one-tenth of an inch, times the angular velocity. Whereas, in a conventional motor, there are usually components with a radius more than an inch rotating at the same angular velocity. This also holds true for the transmission which does not have the conventional input gear running at high pitch line velocities. The relative velocities between the meshing teeth correspond to those found in the last stage of a conventional transmission.

The absence of high relative velocities results in:

(1) Friction losses at high motor input speeds are significantly reduced.

(2) Because of low friction losses, high mechanical efficiencies can be obtained.

(3) Wear is greatly reduced, resulting in longer life of the actuator.

The other factor significantly reduced is the actuator or motor inertia. In conventional high speed motors, the motor inertia resulting from significant mass rotating at high angular velocity has always limited the motor acceleration or response capabilities. The small volumes under compression have generally made up for lack of response due to inertia and have placed rotary servos on equal terms

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with piston-cylinder servos having very little inertia. However, the spring rate of the transmission has in some cases presented unwanted decoupling between the load inertia and the motor inertia, resulting in load resonance. The problem is usually solved by stiffening the transmission at the expense of added weight, as it is usually the load-carrying output members that are too weak.

The DYNAVECTOR actuator has no mass rotating at input or force vector speed and only a small reflected inertia, due to the small eccentric rotation of the ring gear, and the low speed output shaft. Therefore, it has an inertia equal to a similar capacity piston-cylinder actuator and a volume under compression equivalent to a conventional similar capacity rotary servo. This combination results in a servo with a response potential many times that obtained by present day systems.

2.3 ACTUATOR DESIGN AND TEST REQUIREMENTS

The design and test requirements for the Experimental Model actuator as specified by the Air Force Aero Propulsion Laboratory under Contract AF 33(615)-3431 are defined as follows:

2.3.1 General Design Requirements

- Actuator Rate   The actuator output rate shall be 30 degrees per second.

- Hinge Moment: The hinge moment shall be 200,000 in-lbs.

- Pressure: The actuator shall operate with pressure received from a 4,000-psi hydraulic system.

- Actuator Travel: The actuator travel shall be ±30 degrees.

- Vibration: The operation of the actuator shall be unaffected by sustained acceleration forces of 10 g's in all directions.

- Temperature: The actuator shall be designed to operate in a temperature range of -65°F to 500°F.

- Fluid: The actuator shall operate with F-50 versilube or its equivalent.

- Leakage: The external leakage from the actuator shall be held to the minimum acceptable for aircraft actuators. Designs utilizing hermetic sealing and/or dynamic seals at low surface speeds should be considered.

- Weight: The weight shall be held to a minimum, consistent with good aircraft design practice.

- Efficiency: An efficiency of 80 percent or better is desired at maximum design conditions. The efficiency shall be defined as the ratio of power output to the power input to the actuator.

- Backlash: The actuator design shall incorporate methods for achieving zero backlash.

- Life: The actuator shall be designed for a life of 3000 hours with a duty cycle similar to that specified in the 1000-hour life test. (Reference Section 2.3.2)

- Lubrication: All parts of the actuator requiring lubrication shall utilize the working fluid for lubrication. An exception to this may be made for those parts which can be dry-film lubricated, providing that such dry film does not need to be re-applied during the life of the actuator.

2.3.2 Experimental Evaluation Requirements

The requirements for the experimental evaluation of the servo actuator Experimental Model consist of three 2.5-hour thermal cycle tests, a 3-hour room ambient test and a 1,000-hour life endurance test consisting of 188 room temperature cycles and 62 cycles at 500°F.

o Thermal Cycle Test: The actuator shall be operated at -65°F for a period of two hours. During this test the actuator shall be cycled as follows:

(1) The output travel shall be ±30 degrees.

(2) The travel frequency shall be 30 ± 5 cpm.

(3) The moment at the actuator shall be 200,000 in-lbs.

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After two hours cold test, with the actuator still operating, the fluid and ambient temperatures shall then be immediately raised to a temperature of 500°F within a 30-minute period, until the actuator has reached a stabilized temperature.

This thermal cycle shall be repeated two additional times.

- Room Ambient Test: Following the thermal cycling of the above test requirements, the actuator shall be cycled for three hours at room ambient temperature per the travel, travel frequency and load torque conditions defined in the thermal cycle test defined above.

- Life Endurance Test: The actuator shall be operated in repetitive cycles as defined below for a total of 1,000 hours operation. Each cycle shall consist of the following:

    (1) Cycle the actuator for 0.5 hour at a rate of 120 ± 5 cpm for a travel of ±3 degrees.

    (2) Cycle the actuator for 3 hours at a rate of 60 ± 5 cpm for a travel of ±15 degrees.

    (3) Cycle the actuator for 0.5 hour at a rate of 30 ± 5 cpm for a travel of ±30 degrees.

    (4) The hinge moment at the actuator during the life tests shall be 200,000 in-lbs.

    (5) During the 1,000-hour life endurance testing, every fourth cycle shall be conducted at a temperature of 500°F.

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SECTION 3

PROGRAM ACCOMPLISHMENTS

The program schedule of tasks and task accomplishments through 3 April 1966 are as shown in Figure 2-1. Program activities have been initiated in the Experimental Model Critical Parameters Evaluation and Experimental Model Analyses. The activity for the fabrication and delivery of the DYNAVECTOR Actuator Demonstration Model has been completed as scheduled.

3.1   DYNAVECTOR DEMONSTRATION MODEL

Two photographs of the plastic DYNAVECTOR actuator demonstration model (Model No. PL-015-U3, Serial No. 1) built and delivered under this contract are shown in Figures 3-1 and 3-2. The assembly drawing for this model is shown in Figure 3-3. The actuator consists of a 6-vane chamber pneumatic power element driving a 15:1 epicyclic transmission.

The major components of this model are the output shaft and gear (5), ring gear (2), ground gear (8), the cover plates (3), captive vanes (1), output shaft bearings (4), and housing (7).

Low pressure pneumatic stall and frequency response tests have been conducted on an equivalent Bendix plastic model design, Model No. PL-015-U1. The unit has produced stall torques up to 60 in-lbs at 50 psig air supply pressure.

Under no-load frequency response tests (reference Figure 3-4) with velocity limits of 30 RPM, at 50 psig supply, the unit demonstrated zero dropoff in amplitude ratio up to 50 cps with the 90-degree phase shift occurring at 36 cps. With a supply of 15 psig, the -3 db poi.' occurred at 18 cps with the 90-degree phase shift at 17 cps.

With a load inertia of 0.40 in-lb-sec^2 and with a velocity limit of 30 RPM, at 60 psig supply, the -3 db point occurred at 11 cps and the 90-degree phase shift occurred at 7.6 cps. Response test results with this inertia load are shown in Figure 3-5.
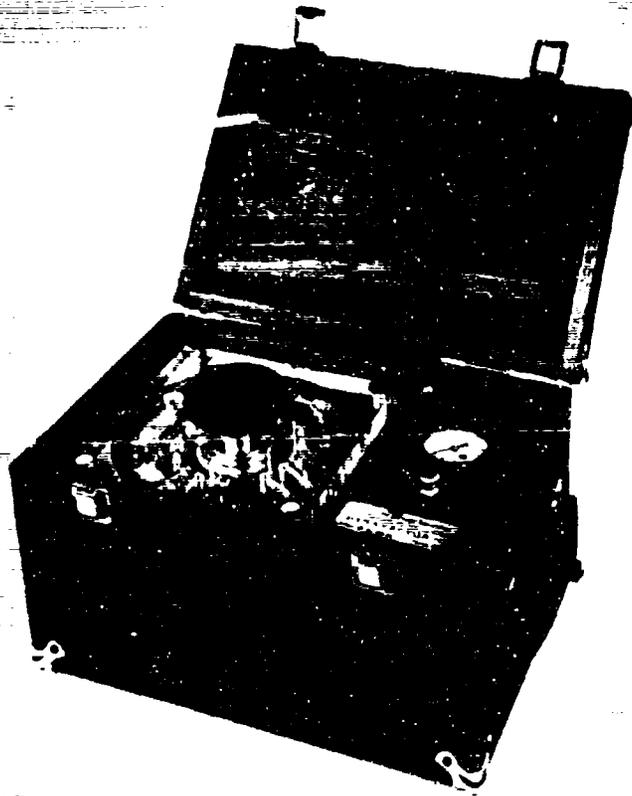
Figure 3-1 - DYNAVECTOR Actuator Demonstration Model PL-015-U3

Figure 3-2 - DYNAVECTOR Actuator Demonstration Model PL-015-U3
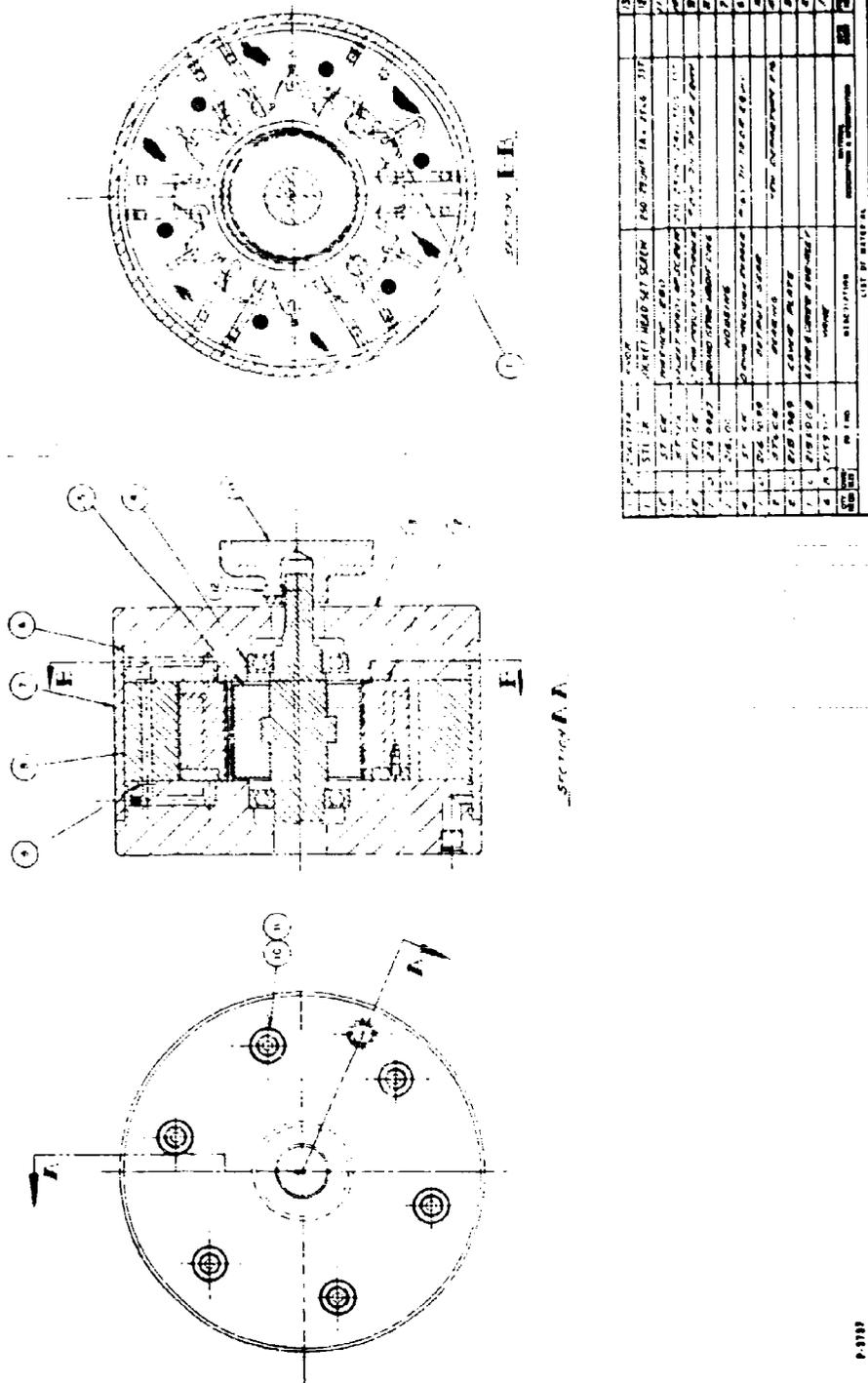
Figure 3-3 - DYNAVECTOR Actuator Model Assembly Drawing Model PL-015-U3, Serial No. 1
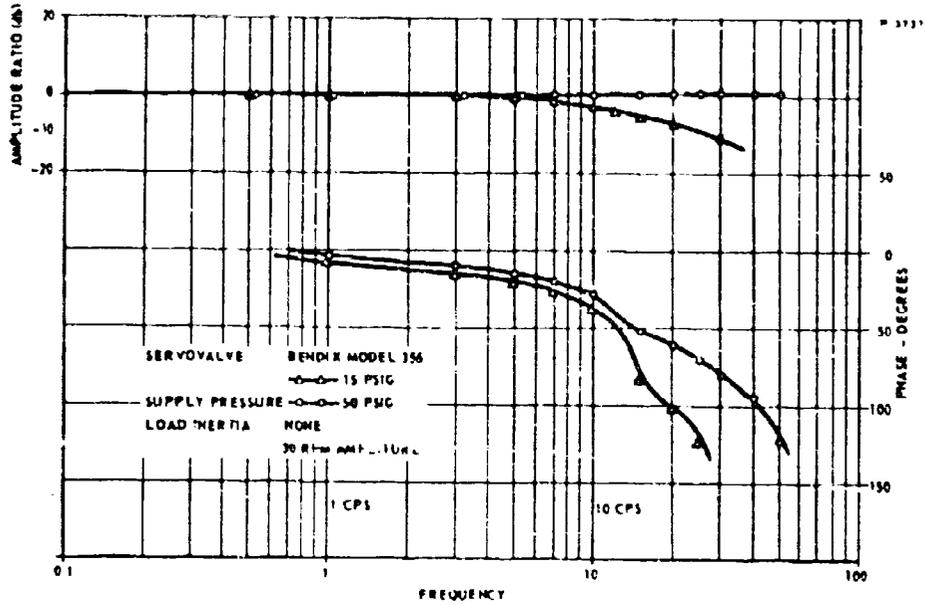
Figure 3-4 - No-Load Frequency Response - DYNAVECTOR
Model PL-015-U1
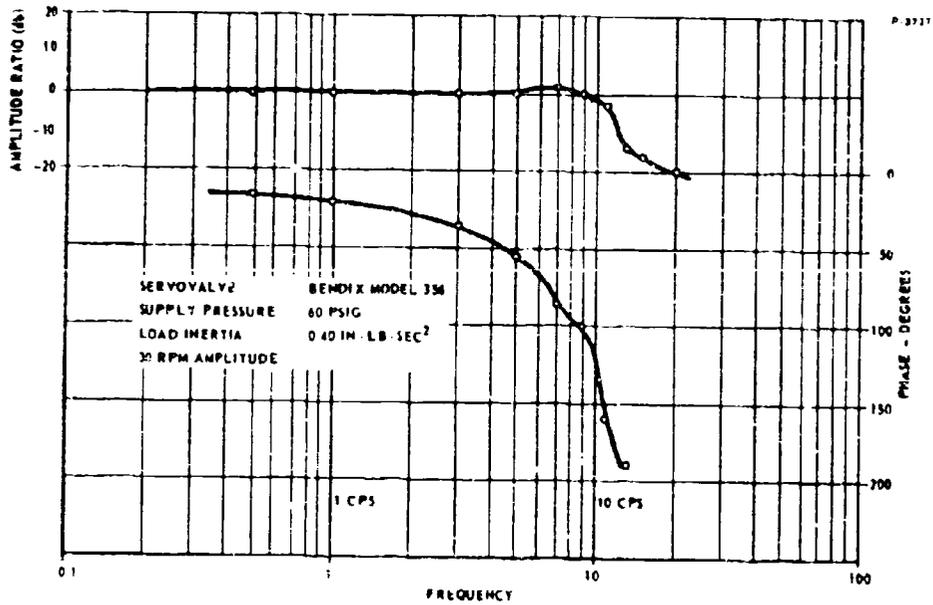
Figure 3-5 - Inertia Load Frequency Response - DYNAVECTOR
Actuator Model PL-015-U1

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3.2 EXPERIMENTAL MODEL CRITICAL PARAMETER EVALUATION

3.2.1 Critical Components and Parameters

The components of the DYNAVECTOR actuator which are considered critical to guarantee success of the actuator in meeting the performance and life endurance requirements of this particular application are defined in Table 3-1. The parameters of these components which are currently being investigated and optimized for this actuator design are also summarized in Table 3-1. The more critical of these parameters will be verified experimentally as discussed in Section 3.2.2, in addition to the analytical study of all parameters listed in Table 3-1.

3.2.2 Critical Parameter Evaluation Plan

The components and/or parameters to be experimentally evaluated are listed in Table 3-2, along with the purpose for the experimental evaluation. Emphasis of the critical parameter study is on motor vane, commutator and ring gear configuration, and epicyclic transmission design because of their effect on the actuator performance. Bearing requirements (i.e., load, DN, lubrication, and life) are state-of-the-art and need only be established to select the proper size bearing. Seal requirements of leakage, pressure, and environment are also state-of-the-art and, therefore, only preliminary analysis and design is required.

The 200,000 in-lb DYNAVECTOR actuator design will be scaled down to a test unit size capable of 20,000 in-lb output torque at the rated speed of 30 degrees per second of the Experimental Model, resulting in a 1.6 rated horsepower output. A scale model will be used to facilitate experimental evaluation by use of existing test equipment and designing the actuator to a configuration readily assembled and disassembled, permitting easy modification of the critical components. This scale model test unit will be designed for 1000 psi hydraulic supply operation. Tests will be conducted up to 3000 psi supply conditions to establish the maximum torque and stress capability of the actuator, to provide design optimization data for the Experimental Model.

Wherever advantageous, component test articles will be fabricated and tested so that a specific performance parameter may be experimentally and analytically isolated from the interaction with other components of the actuator.

Table 3-1 - Critical Components and Parameters

Components	Parameters
Motor	
Vanes	Number, velocities, leakage, configuration, material
Commutator	Area, timing, leakage, configuration
Ring Gear	Mass balance, pressure balance, leakage, clearance, configuration, assembly
Transmission	
Ring Gear	Load capacity, reaction loads, tooth configuration, interference, backlash, material, life, lubrication, stiffness
Bearings	Load capacity, DN, lubrication, wear, life
Output Shaft & Gear	Configuration
Miscellaneous	
Seal	Leakage, life, pressure, material
Structure	Weight, fabricability, pressure
Actuator	Weight, ratio, torque capacity, fabricability, life, performance pressure

Table 3-2 - Parameter Test Evaluation

Component/Parameter	Purpose
Number of Vanes	Minimize torque ripple
Commutation Area	Optimize efficiency and speed
Commutator Timing	Minimize torque ripple and optimize speed
Ring Gear Clearance	Reduce leakage and effect of temperature gradients; improve efficiency
Ring Gear and Output Shaft Load Capacity	Optimize torque/weight ratio and efficiency
Actuator Scale Model	Investigate torque-speed characteristics and overall performance

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Preliminary design of the scale model actuator will reflect the experience of Research Laboratories in the area of DYNA-VECTOR actuator development. The commutator plates and motor configuration will be designed to allow modification of the commutator area and timing and a change in the number of vanes to optimize actuator performance. Steady state torque-speed, pressure-flow and frequency-response characteristics will be used to optimize the power element critical components.

3.3 EXPERIMENTAL MODEL ANALYSIS

Engineering analyses have been initiated on the Experimental Model of the DYNAVECTOR actuator to be fabricated and by which endurance life tests performance will be demonstrated. For this period of activity, these analyses consist of:

(1) Defining actuator design requirements (reference DS-744, Appendix A).

(2) Defining check-out and life test plan (reference PS-371, Appendix B).

(3) Establishing computer program for optimization of DYNAVECTOR gear design (reference Appendix C).

(4) Initiating design of mass balance concepts.

(5) Initiating design of porting and commutation concepts.

The actuator design requirements have been analyzed and are defined in Engineering Specification DS-744 entitled "Preliminary Design Specification for a 200,000 in-lb Hydraulic Actuator" dated 15 March 1966, which is presented as Appendix A. This specification defines the design, environmental and performance requirements of the actuator, based on vehicle flight control surface actuation applications requiring fast response and an endurance operational capability.

The actuator functional check-out and life test requirements have been analyzed and are defined in Engineering Specification PS-371 entitled "Preliminary Check-Out and Life Test Plan: 200,000 in-lbs Hydraulic DYNAVECTOR Actuator", dated 15 March 1966, which is presented as Appendix B. This specification defines the functional check-out and life endurance tests to be imposed on the DYNAVECTOR actuator. The functional check-out tests will be conducted both before

life testing and after actuator refurbishment at the conclusion of the life tests before delivery to Wright-Patterson Air Force Base in August, 1967.

Appendix C, "DYNAVECTOR Gear Design Computer Program", presents the mathematical relationships describing the geometry of involute gears required for the DYNAVECTOR transmission design. These relationships are applied to the problem of optimizing the proportions of non-standard gear designs. In particular, inside and outside radii of internal and external gears, respectively, are found which have the following properties:

(1) The gears operate at maximum efficiency.

(2) The duration of contact between gear teeth is maximum.

(3) The gears are interference-free during operation.

For the optimum gear proportions, expressions have been derived to obtain the thickness at the tips of the gear teeth and to calculate the measurement of the gear over pins or rolls. These mathematical expressions have been programmed for evaluation on the Bendix Research Laboratories' G-20 Digital Computer, by which a series of gear design tables have been derived.

The ring gear design for the Experimental Model will be of a mass balance configuration. All mass eccentricities and dynamic couples will be completely balanced, so as to assure high performance operation. Analytical trade-off studies are being conducted on the four balance configurations as shown in Figures 3-6 through 3-9. The methods of mass balancing may be classified in two categories, depending on the rotor gear mesh configuration, i.e., internal mesh or external mesh. Figure 3-6 shows an internal ring gear mesh balanced by a balance gear. The center of gravity of the balance gear is 180 degrees opposite the center of gravity of the ring gear, thereby producing complete mass balance. The ring gear is driven by the fluid pressure vector, whereas the balance gear is driven by the mesh with the output shaft and therefore produces no work. Proper balance gear design will eliminate tooth interference, yet maintain balance gear mesh with the output shaft gear to prevent the balance gear from falling out of engagement.

Figure 3-7 shows an internal mesh balance configuration with the pressurized vane chambers between ring gear A and ring gear B. Both gears, therefore, produce work and because their center of
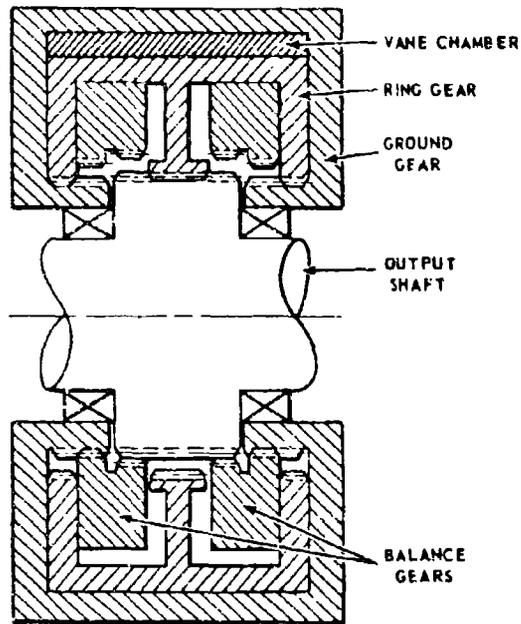
Figure 3-6 - Internal Mesh Balance Configuration (Idling Balance Gear)
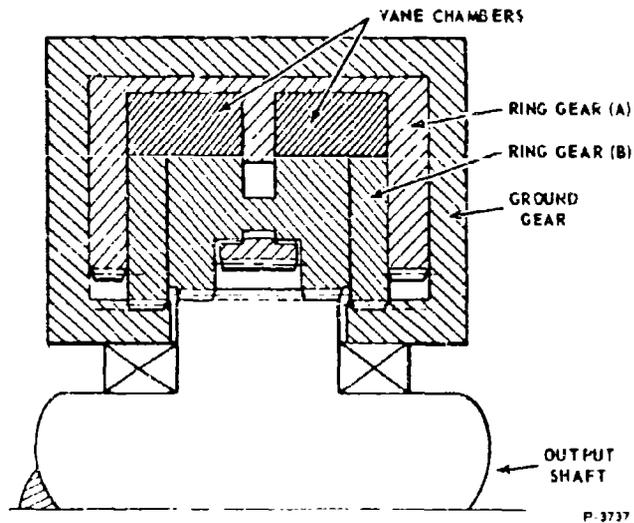
Figure 3-7 - Internal Mesh Balance Configuration (Both Gears Working)
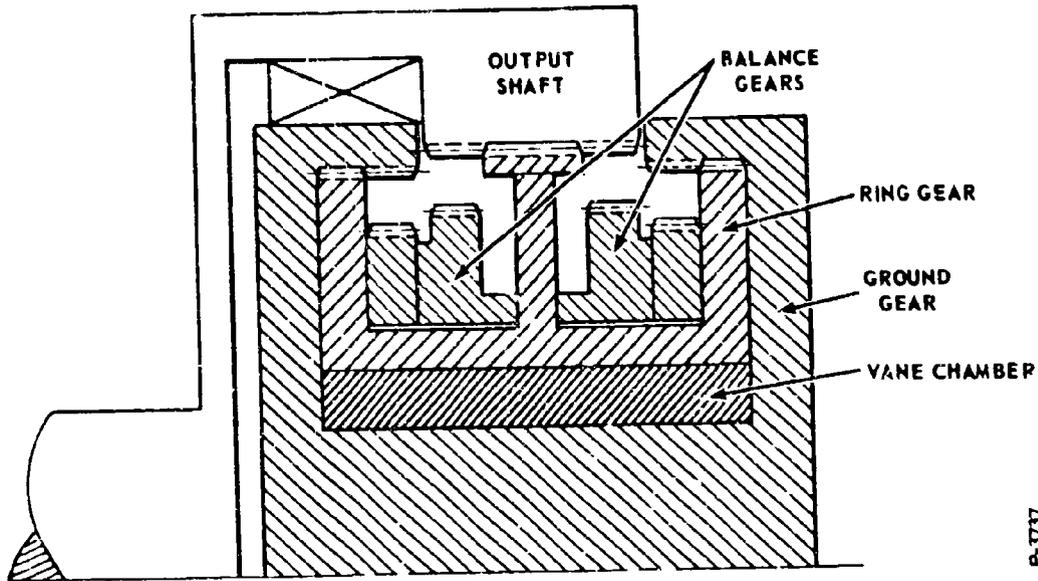
Figure 3-8 - External Mesh Balance Configuration
(Idling Balance Gears)
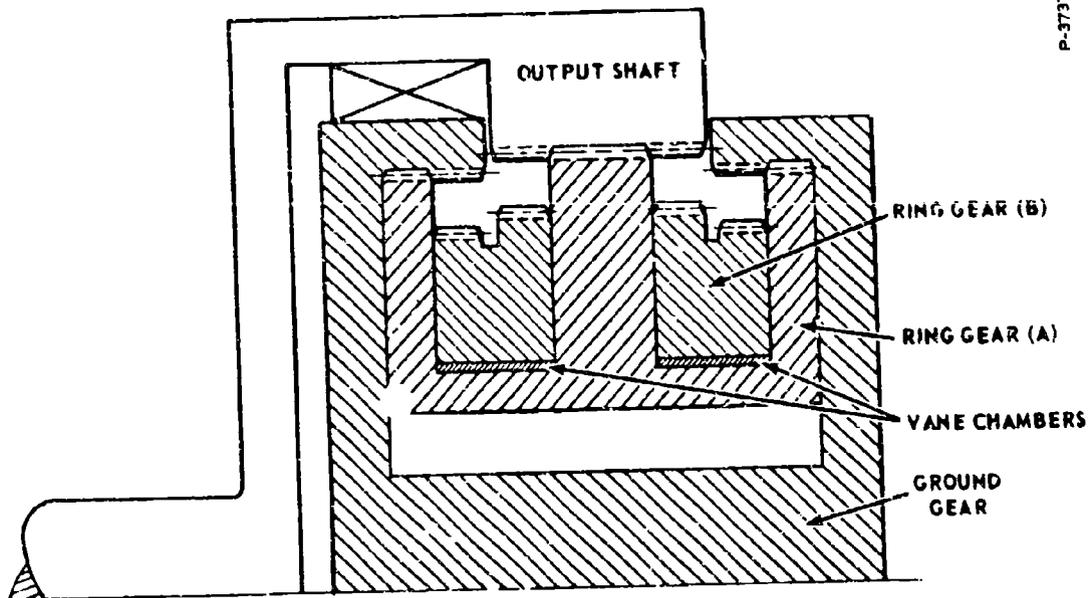
Figure 3-9 - External Mesh Balance Configuration
(Both Gears Working)

gravities are 180 degrees apart the system is balanced. A double set of vanes is required as shown for this configuration.

Figure 3-8 shows an external mesh balance configuration with a single ring gear producing work and balance gears providing balance but no work.

Figure 3-9 shows an external mesh balance configuration with two ring gears, both of which are driven by the pressure vector and are therefore producing work. A double set of vanes is required as shown for this configuration.

As indicated in the discussion of the critical parameters in Section 3.2, optimization of the commutation technique, i.e., timing and flow passage sizing, is considered to be critical to the actuator performance. Design layouts have been initiated to investigate various types of porting arrangements, compatible with the motion relationship between the side faces of the ring and ground gear.

APPENDIX A

PRELIMINARY DESIGN SPECIFICATION
FOR A 200,000 IN-LB
HYDRAULIC ACTUATOR

PROJECT NO.	THE BENDIX CORPORATION RESEARCH LABORATORIES DIVISION SOUTHFIELD, MICHIGAN	CODE IDENT.	SPECIFICATION NO.	REV.
2837-3110		11272	DS - 744	

ENGINEERING SPECIFICATION

TITLE Preliminary Design Specification for a 200,000 In-Lb Hydraulic Actuator	DATE March 15, 1966

1.0 DESCRIPTION

This specification defines the design, environmental and performance requirements for a rotary hydraulic actuator. The actuator shall be capable of driving a spring rate type of load and shall be designed to demonstrate fast response and an operational endurance capability required for vehicle flight control surface actuation applications.

2.0 DESIGN REQUIREMENTS

2.1 Design Strength

The following safety factors shall be applied to the stress analysis of the unit:

(a) Yield Strength   = 1.3 x Design Yield Strength
(b) Ultimate Strength = 1.5 x Design Ultimate Strength
                        (Reference MIL-A-8860)

2.2 Installation and Volume Dimensions

The installation and volume dimensions for the actuator are open and undefined. However, materials of the lightest possible weight consistent with the service and strength requirements, shall be used so as to assure compliance with the weight design goal defined in 2.3.

2.3 Weight

The actuator shall have a weight design goal of 150 pounds.

2.4 Marking of Ports

All ports shall be durably and legibly marked as to function. Decals shall not be used. Single letters, such as "P" for pressure, will not be acceptable.

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ENGINEERING SPECIFICATION

TITLE Preliminary Design Specification for a 200,000 In-Lb Hydraulic Actuator	DATE March 15, 1966

3.0 ENVIRONMENTAL REQUIREMENTS

3.1 Supply Pressure

The maximum available supply received from the hydraulic system shall be 4,000 psig.

3.2 Fluid

The actuator shall operate with F-50 Versilube, Oronite 70 or an equivalent.

3.3 Temperature

The actuator shall be designed to operate in an air ambient and fluid temperature range of -65°F to 500°F. For a cold start thermal transient, the actuator shall be designed to operate with an air ambient of -65°F and fluid temperature rise from -65°F to +500°F in 0.5 hours.

3.4 Lubrication

All parts of the actuator requiring lubrication shall utilize the working fluid for lubrication. An exception to this may be made for those parts which can be dry-film lubricated, providing that such dry film does not need to be re-applied during the life of the actuator.

3.5 Vibration

The operation of the actuator shall be unaffected by sustained acceleration forces of 10 g's in all directions.

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4.0 PERFORMANCE REQUIREMENTS

4.1 Leakage

The external leakage from the actuator shall be held to the minimum acceptable for aircraft actuators. Dynamic seal designs utilizing seals at low surface speeds should be employed wherever possible. The leakage design goal for actuator dynamic performance shall be a maximum of 12 drops per hour (0.60 cc). The shaft seal leakage design goal for actuator static performance shall be 2 drops per hour (0.10 cc).

External leakage through static gasket seals, other than a slight wetting insufficient to form a drop, is unacceptable.

4.2 Actuator Rate

The actuator output rate capability shall be at least 30 degrees per second at any torque load specified in Paragraph 4.3.

4.3 Actuator Torque Load

The actuator torque load shall be a spring rate type load of 6,670 in-lb per degree with a maximum torque of 200,000 in-lb at 30 degrees travel.

Inertia and friction type loads are zero.

4.4 Actuator Travel

The actuator travel shall be ± 30 degrees maximum.

4.5 Efficiency

An efficiency of 80 percent or better is desired at maximum design conditions. The efficiency shall be defined as the ratio of power output to the power input to the actuator.

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4.6 Backlash

The actuator design shall incorporate methods for achieving zero backlash.

4.7 Life

The actuator shall be designed for a life of 3,000 hours with a duty cycle similar to that specified in the 1,000 hour life test (Reference PS-371).

4.8 Cyclic Torque-Speed Requirements

The actuator shall be designed to meet the following cyclic torque-speed requirements (Reference Appendix A).

4.8.1 Cycle frequency 2.0 cps: Cycle amplitude 6 degrees peak to peak against spring rate load of 6,670 in-lb per degree (load variation zero to $\pm$ 20,000 in-lb).

4.8.2 Cycle frequency 1.0 cps: Cycle amplitude 30 degrees peak to peak against spring rate load of 6,670 in-lb per degree (load variation zero to $\pm$ 100,000 in-lb).

4.8.3 Cycle frequency 0.5 cps: Cycle amplitude 60 degrees peak to peak against spring rate load of 6,670 in-lb per degree (load variation zero to $\pm$ 200,000 in-lb).

APPENDIX A

Project 2837-3110   DS - 744

200,000 in-lb. Actuator
Cyclic Torque-Speed Requirements

Legend:

—·—2.0 cps, ± 3°

— — —1.0 cps, ± 15°

———0.5 cps, ± 30°

Output Torque (in-lbs x 10³)

Output Speed (degrees per second)

A-5
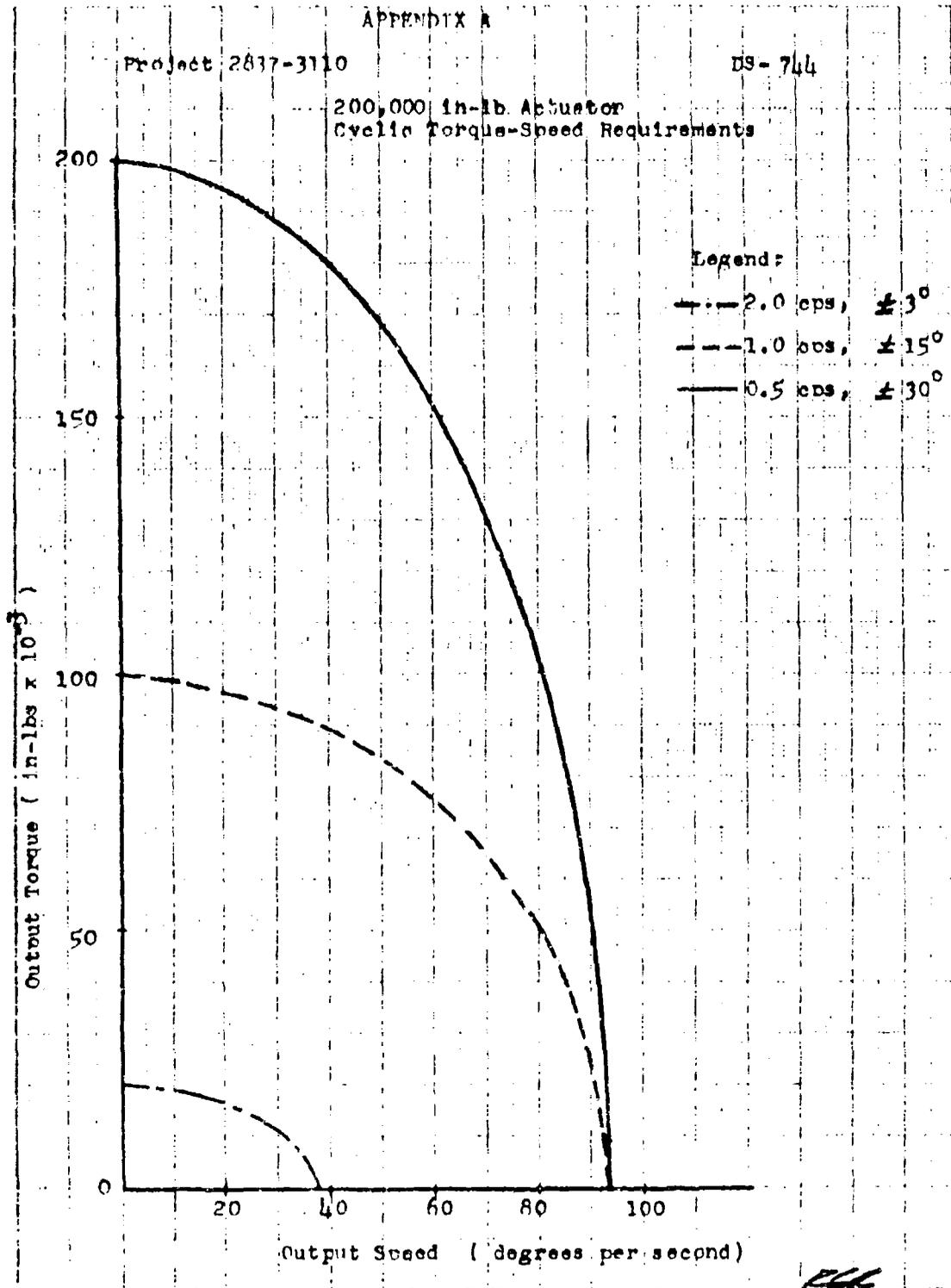
APPENDIX B

PRELIMINARY CHECK-OUT AND LIFE TEST PLAN:
200,000 IN-LB HYDRAULIC DYNAVECTOR ACTUATOR

ENGINEERING SPECIFICATION

TITLE Preliminary Check-Out and Life Test Plan: 200,000 In-Lb Hydraulic DYNAVECTOR Actuator	DATE March 15, 1966

1.0 PURPOSE OF TEST PLAN

This preliminary test plan defines the functional check-out tests and life endurance tests to be imposed on a rotary hydraulic 200,000 in-lb torque capacity DYNAVECTOR. The functional check-out tests will be conducted both prior to life testing and after actuator refurbishment at the conclusion of the life test before delivery to Wright-Patterson Air Force Base under Contract AF 33(615)-3431 in August 1967.

2.0 LOG BOOK DOCUMENTATION

An equipment log shall be compiled for the DYNAVECTOR Actuator and shall contain the following items:

2.1 **Title Page:** Log Book Rotary Hydraulic DYNAVECTOR Model _____. Serial Number _____, BRLD Project 2837-1110, Wright-Patterson Air Force Base Contract AF 33(615)-3431

2.2 **Table of Contents**

2.3 **Section I, Equipment Complement**

Specify the following as applicable documents:

(a) DYNAVECTOR Assembly Drawing _____.
(b) Check-Out Test Fixture Schematic and Equipment Lists.
(c) Life Test Fixture Schematic and Equipment Lists.
(d) DS-744 Preliminary Design Specification.

2.4 **Section II, Inspection Reports**

All inspection reports pertaining to the actuator assembly deliverable hardware are to be compiled in this section. If any part initially rejected by inspection is used in the actuator assembly, a copy of the report defining part disposition and statement of subsequent acceptance (rework, waiver, etc.) is to be incorporated in Section II.

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2.5 Section III, Calibration Data

State test equipment used and calibration history of said equipment.

2.6 Section IV, Functional Check-Out Test Data

A compilation of all check-out data prior to life tests and observations of test articles and test stand functioning shall be entered in this Section.

2.7 Section V, Life Test Data

A compilation of all life test data accumulated and observations of test articles and test stand functioning shall be entered in this Section.

2.8 Section VI, Refurbished Assembly Functional Check-Out Test Data

A compilation of all check-out data recorded on the refurbished unit prior to shipment to the customer and observations of said unit and test stand functioning shall be entered in this Section.

2.9 Section VII, Engineering Release Notices, Change Notices, and E.I. Forms

Copies of all drawing release notices, change notices and engineering instruction forms of deliverable hardware are to be included in this Section.

2.10 Section VIII, ...tional Check-Out Operating Events

Record each time deliverable hardware is tested during check-out tests prior to life tests and define adjustments or modifications required to operate deliverable hardware and/or test equipment (repairs, rework, inspections and maintenance) Test data is to be recorded in Section IV.

2.11 Section IX, Life Test Operating Events

Record sequence of life tests and define adjustments or modifications required to operate deliverable hardware and/or test equipment (repairs, rework, inspection, and maintenance). Test data is to be recorded in Section V.

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TITLE	Preliminary Check-Out and Life Test Plan: 200,000 In-Lb Hydraulic DYNAVECTOR Actuator	DATE March 15, 1966

2.12 <u>Section X, Refurbished Assembly Functional Check-Out Operating Events</u>

Record each time deliverable hardware is tested and define adjustments or modifications required to operate deliverable hardware and/or test equipment (repairs, rework, inspection, and maintenance). Test data is to be recorded in Section VI.

2.13 <u>Section XI, Failure Reports</u>

A copy of BRLD failure data report BC-RLD-47 shall be filed in this section in the event of catastrophic or degradation failure of deliverable hardware during any deliverable hardware tests.

3.0 ENVIRONMENTAL CONDITIONS

All functional check-out tests will be conducted at room temperature conditions with MIL-O-5606 oil.

Life endurance tests will be conducted with Oronite High Temperature Hydraulic Fluid 70 at the temperature conditions (-65°F to $+500^\circ$F) as defined in the Life Test Requirements Section 4.2.

4.0 TEST REQUIREMENTS

A torsion tube load fixture with a spring rate characteristic of 6670 in-lbs per degree with a ± 30 degree travel capability will be used as the torque load mechanism for the applicable tests defined below.

4.1 <u>Functional Check-Out Tests</u>

All tests are to be conducted under room temperature conditions using MIL-O-5606 fluid. Tests 4.1.1 and 4.1.2 are to be performed in the proper sequence as indicated. Tests 4.1.3 through 4.1.8 may be performed in any sequence as is convenient.

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4.1.1 Examination of Actuator

The actuator shall be examined to determine conformance with the applicable assembly drawings and all requirements of design specification DS-744 and this specification.

4.1.2 Break-In Run

The break-in run shall be for a duration of one (1) hour minimum. Shaft torque loading may be constant or vary sinusoidally so as to impose a peak actuator pressure differential of 25% maximum required pressure differential to produce 200,000 in-lb torque. Shaft speed may be constant at a minimum of 30 degrees per second or vary sinusoidally from zero to 30 degrees per second maximum.

After the break-in run, the actuator shall be disassembled and examined. If all parts are in acceptable condition, the actuator shall be reassembled and tests continued per 4.1.3. If working parts require replacement, the actuator shall be reassembled using the replacement parts, and the break-in run and subsequent disassembly, examination, and reassembly repeated.

4.1.3 Proof Test

A proof pressure of 1.5 times the maximum required actuator supply pressure necessary for the actuator to meet the performance requirements defined in DS-744 shall be simultaneously applied to both pressure ports at least two successive times and held two (2) minutes for each pressure application. The actuator shall be operated per 4.1.2 between application of the proof test pressure.

During application of the proof test pressure there shall be no evidence of external leakage, other than a slight wetting insufficient to form a drop, excessive distortion, or permanent set.

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TITLE Preliminary Check-Out and Life Test Plan: 200,000 In-Lb Hydraulic DYNAVECTOR Actuator	DATE March 15, 1966

4.1.4 No-Load Breakout

Record pressure differential and supply pressure at instant of actuator output shaft rotation for both directions of rotation with zero torque load applied.

4.1.5 No-Load Speed

Record actuator output shaft speed and hydraulic supply flow rate for actuator pressure differentials of 10%, 25%, 50%, 75% and 100% maximum required pressure differential with zero torque load applied. Record for both directions of rotation.

4.1.6 Stall Torque

Assemble torsion tube load mechanism to actuator output shaft. Record stall torque and leakage consumption at actuator pressure differentials of 10%, 25%, 50%, 75% and 100% maximum required pressure differential to produce 200,000 in-lbs torque. Repeat for opposite torque direction.

4.1.7 Cyclic Torque - Speed Tests

The following cyclic torque-speed conditions shall be imposed on the actuator output shaft for a minimum of 8 hours test time. External leakage, actuator flow consumption and actuator efficiency are to be recorded in addition to load torque and speed:

(a) Cycle the actuator for 0.25 hours at a cycle frequency of 2.0 cps. Cycle amplitude shall be six (6) degrees peak to peak against a spring rate load of 6670 in-lbs per degree (load variation zero to $\pm$ 20,000 in-lbs).

(b) Cycle the actuator for 1.5 hours at a cycle frequency of 1.0 cps. Cycle amplitude shall be 30 degrees peak to peak against a spring rate load of 6670 in-lbs per degree (load variation zero to $\pm$ 100,000 in-lbs).

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TITLE Preliminary Check-Out and Life Test Plan: 200,000 In-Lb Hydraulic DYNAVECTOR Actuator	DATE March 15, 1966

(c) Cycle the actuator for 0.25 hours at a cycle frequency of 0.5 cps. Cycle amplitude shall be 60 degrees peak to peak against a spring rate load of 6670 in-lbs per degree (load variation zero to $\pm$ 200,000 in-lbs).

4.1.8 Frequency Response Tests

Frequency response tests shall be conducted under no-load conditions (actuator decoupled from load mechanism) and against a torque load (actuator driving load mechanism spring rate and inertia about a zero deflection null).

During all tests the actuator output shaft speed shall be velocity limited to 30 degrees per second. Tests shall be conducted to establish the frequency to effect a 90 degree phase shift and amplitude decay of 3 db.

4.2 Life Endurance Tests

The requirements for the life endurance evaluation of the actuator consist of three 2.5 hour thermal cycle tests, a three hour room ambient test and a 1000 hour life endurance test consisting of 188 room temperature cycles and 62 cycles at $500^{\circ}F$. All tests are to be conducted at the temperature conditions as specified below using Oronite HTHF 70.

The test procedure for the life endurance tests is as follows:

(a) Stabilize the test unit at $-65^{\circ}F$.

(b) Maintain ambient temperature at $-65^{\circ}F$ with hydraulic fluid at $100 \pm 20^{\circ}F$. Cycle the actuator for two hours at a frequency of 0.5 cps, amplitude of 60 degrees peak to peak against a spring rate load of 6670 in-lbs per degree (load variation zero to $\pm$ 200,000 in-lbs).

(c) While still operating as defined in (b) above raise the ambient and fluid temperature to $500^{\circ}F$ within thirty (30) minutes and continue test until actuator has reached a stabilized temperature.

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TITLE Preliminary Check-Out and Life Test Plan: 200,000 In-Lb Hydraulic DYNAVECTOR Actuator	DATE March 15, 1966

(d) Repeat paragraphs (a) through (c) two additional times.

(e) At the conclusion of third thermal cycle, stabilize the actuator at room ambient temperature and cycle for three hours at a frequency of 0.5 cps, amplitude of 60 degrees peak to peak against a spring rate load of 6670 in-lbs per degree (load variation zero to $\pm$ 200,000 in-lbs).

(f) The actuator shall then be operated in repetitive cycles for a total of 1000 hours of operation against a spring rate load of 6670 in-lbs per degree. Each cycle shall consist of the following:

    1. Cycle the actuator for 0.5 hours at a cycle frequency of 2.0 cps. Cycle amplitude shall be six degrees peak to peak. Load variation shall be zero to $\pm$ 20,000 in-lbs.

    2. Cycle the actuator for 3.0 hours at a cycle frequency of 1.0 cps. Cycle amplitude shall be 30 degrees peak to peak. Load variation shall be zero to $\pm$ 100,000 in-lbs.

    3. Cycle the actuator for 0.5 hours at a cycle frequency of 0.5 cps. Cycle amplitude shall be 60 degrees peak to peak. Load variation shall be zero to $\pm$ 200,000 in-lbs.

    The above tests shall be performed at ambient temperature and 100 $\pm$ 20°F fluid temperature for three repetitive cycles followed by a fourth cycle performed at 500°F ambient and fluid temperature to the completion of a total of 1000 hours of operation.

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REVISIONS

APPENDIX C

DYNAVECTOR GEAR DESIGN
COMPUTER PROGRAM

APPENDIX C

DYNAVECTOR GEAR DESIGN COMPUTER PROGRAM

1.0 SUMMARY

This appendix presents the mathematical relationships describing the geometry of involute gears required for the DYNAVECTOR transmission design. These relationships are applied to the problem of optimizing the proportions of non-standard gear designs. In particular, inside and outside radii of internal and external gears, respectively, are found which have the following properties:

(i) The gears operate at maximum efficiency.

(ii) The duration of contact between gear teeth is maximum.

(iii) The gears are interference-free during operation.

For the optimum gear proportions, expressions are derived to obtain the thickness at the tips of the gear teeth and to calculate the measurement of the gear over pins or rolls.

The mathematical expressions have been programmed for numerical evaluation on the Bendix Research Laboratories Division G-20 Digital Computer. This appendix contains a discussion of the numerical methods employed in computing the optimum gear proportions.

2.0 INTRODUCTION

2.1 General

In the designing of internal-external involute gear pairs, certain physical characteristics are sought. The engineer requires that the gears be efficient, have a relatively high contact ratio and be interference free. In addition, dimensions such as base radii, pitch radii and center distance, are specified. The problem becomes to find inside and outside radii which will satisfy the design requirements. No particular difficulty is encountered for conventional gear specifications. However, when non-standard designs are required as in the DYNAVECTOR design, the requisite gear proportions must be obtained from a systematic search program.

Having the proportions of the gears, the exact tooth form can be determined. Load considerations lead to the conclusion that the thickness of the tips of the internal and external teeth should be equal. As the teeth come into and go out of mesh, the equal tooth thicknesses will tend to balance the stresses between the internal tooth and the external tooth.

Finally, when the exact proportions of the involute tooth profiles are established, the quantity "measurement over rolls" can be determined. During the machining of the gears, the measurement over rolls assures proper tooth spacing and a proper involute tooth form. Two pins of known dimensions are placed, diametrically, between gear teeth and the distance from one pin to the other is measured. This is the measurement over rolls which must be maintained throughout the circumference of the gear.

In the remaining paragraphs in this section, the design characteristics will be discussed in greater detail. Section 3 presents the basic elements of involute trigonometry and of involute gear geometry. Formulas for various and pertinent properties of involute gears are derived in this section. Section 4 presents a discussion of the numerical evaluation of involute gear geometry as an aid to the engineering design of such gears.

2.2   Gear Design Objectives

2.2.1 Efficiency

In general, the efficiency of gears is expressed as a ratio of the work input to the driven gear and the work output of the driving gear over the arc of contact of a pair of gear teeth. The efficiency can also be expressed in terms of the proportions of the internal and external gears. Factors such as the relative number of teeth the length of the arc of approach (i.e., the arc from the point where the teeth come into contact to the pitch point of the gears' and the length of the arc of recess (i.e., the arc length from the pitch point to the last point of contact) affect the efficiency.

Clearly, power transmission is directly related to efficiency and, therefore, high levels of efficiency are desirable. Since relationships between gear geometry and efficiency can be established, it is possible to select gear proportions with efficiency as a design parameter. In addition, it is possible to maximize the efficiency of involute gears by an appropriate choice of gear proportions.

2.2.2 Contact Ratio

The contact ratio, or duration of contact, of gears is the ratio between the length of the arc through which mating teeth come into and

go out of contact and the arc length between successive teeth on the gear. The former is called "the arc of action," and its length consists of an arc of approach and an arc of recess.

As a function of gear proportions, the contact ratio ranges from a lower bound of zero to an upper bound which depends upon the radii at which the gear teeth come to a point. In practice, the upper bound for the contact ratio depends upon tooth interference conditions. If the gear teeth are to be free of interference between the profile of one tooth and the tip of the other tooth, then the contact ratio is limited to gear proportions which satisfy non-interference conditions.

Since the contact ratio involves the arc of approach and the arc of recess, and since gear efficiency is also related to these quantities, the condition that the gear operation be free of interference may be imposed in such a manner as to maximize both the contact ratio and the gear efficiency. It is well to note that these two quantities maximized together do not necessarily represent individual maxima. Thus, it may be possible to achieve a higher contact ratio at the expense of lower efficiency.

2.2.3 Tooth Interference

Tooth interference is the condition where the teeth cannot pass freely out mesh as the gears are in operation. Figure 1 shows typical involute gear tooth orientations at various angles of rotation. The outside radius, R_0, of the external gear tooth and the inside radius, R_i, of the internal gear tooth are marked, as is the center distance, C.

Tooth interference occurs as a function of the combined magnitudes of R_0, R_i and C. Given an inside radius and a center distance, interference will occur if the outside radius is too large. Similarly, given an outside radius and a center distance, interference will occur if the inside radius is too small.

During the operation of the gears the involute profiles of the teeth are in contact only along the arc of action. As the external tooth moves down the profile of the internal tooth, its corner traces a trochoidal path. This path exists, because the centers of rotation of the two gears are displaced. Analysis of the interference characteristics of gears involves a comparison between the simultaneous paths of the tips of the internal and external teeth. When this comparison is made for all combinations of inside and outside radii ranging from the base radii to the radii at which the teeth come to a point, the optimum gear proportions can be obtained which satisfy the design requirements.
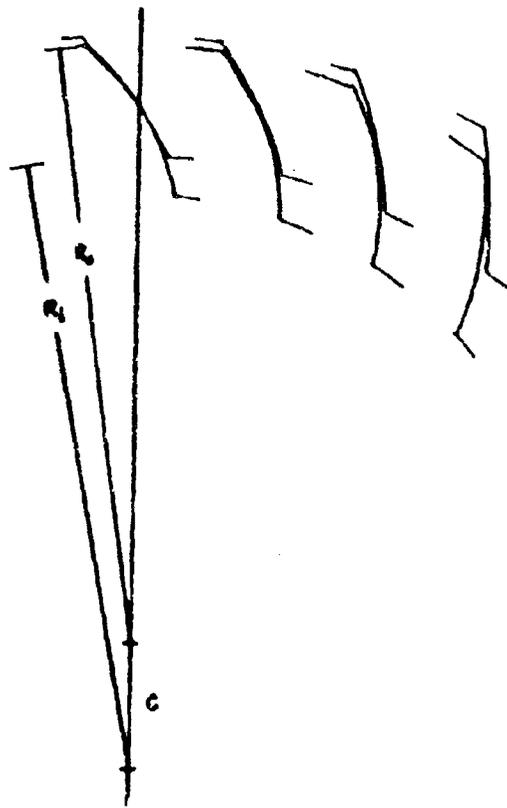
Figure 1 - Involute Gear Tooth Orientation

3.0 ANALYSIS

3.1 General

This section presents the mathematical relationships describing the geometry of involute gears. The equations for the contact ratio and tip interference will be developed. The maximum efficiency condition for gear operation will be established. The three relationships will be used to obtain the inside radius of the internal gear and the outside radius of the external gear for the highest possible contact ratio.

The arc tooth thickness geometry will be derived. The tooth thickness required for the teeth to mesh tightly at a given center distance, and the thickness at the tips of the internal and external teeth are combined in a single relationship. The relationship is constructed under the condition that the internal and external tooth tip thickness is the same.

Finally, the equations for the diameter of a gear "over rolls"
will be derived for internal and external gears. The measurement
over rolls is a quantity used during the machining of the gears to assure
the proper tooth configuration and dimension.

3.2   Involute Gear Geometry

3.2.1   The Involute Function

The involute curve is generated from a base circle according to
the following construction The path traced by the end of a perfectly
flexible line drawn tangent to a circle is called the "involute of the
circle." The geometry is illustrated in Figure 2.
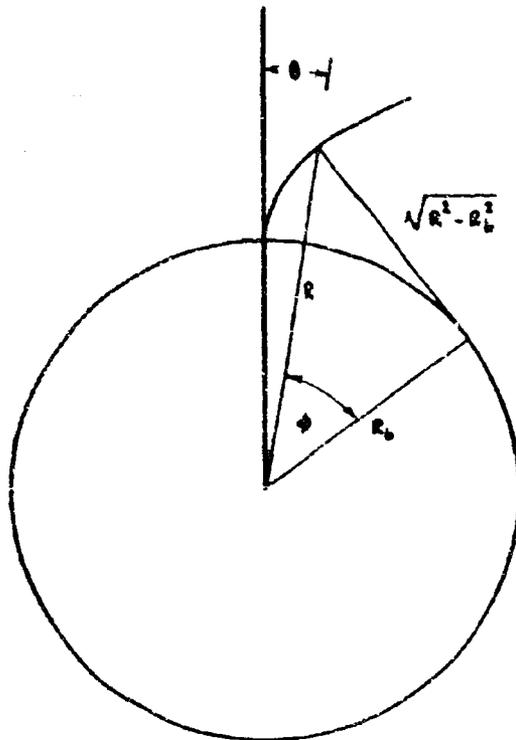
Figure 2 - Involute Geometry

Referring to Figure 2:

$$\theta = \tan \phi - \phi = \text{inv } \phi \tag{1}$$

Note that the arc ϕ is the length of the circumference on the circle over the angle $\theta + \phi$. If the tangent function is expressed as a ratio, this is seen immediately.

$$\theta + \phi = \frac{\sqrt{R^2 - R_b^2}}{R_b} \tag{2}$$

or

$$R_b \left[\theta + \phi\right] = \sqrt{R^2 - R_b^2} \tag{3}$$

It follows that the radius of curvature of the involute curve at any point is the length of the generating line, $R_b \left[\theta + \phi\right]$, at that point.

3.2.2 The Trochoid Function

A trochoid is the path of a point in a moving plane as it progresses along a fixed curve. For example, the trochoid of a circle as it rolls on a straight line is illustrated in Figure 3. The point p on the radius of the circle traces the trochoid path shown. With respect to gear teeth, trochoid action occurs between mating teeth as they progress along a path of contact. Two cases will be considered:

The trochoid of the corner of an external gear tooth at the root of an internal gear tooth is described by the geometry in Figure 4(a).
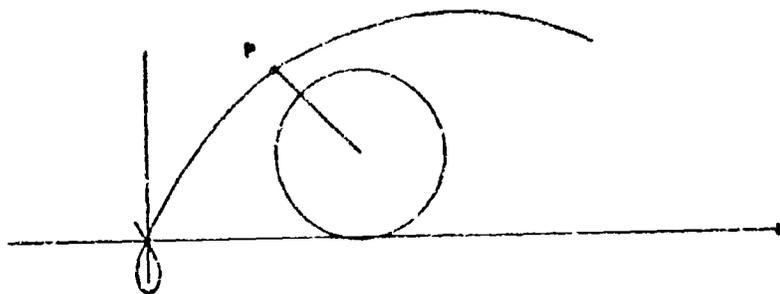
Figure 3 - Trochoid of a Circle
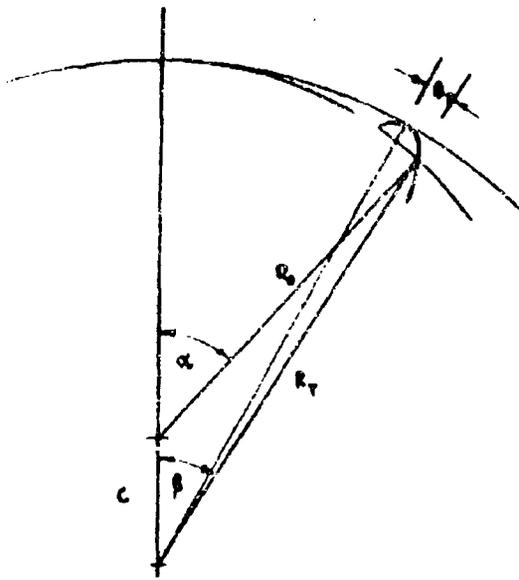
Figure 4(a) - Trochoid Geometry   Figure 4(b) - Trochoid Geometry
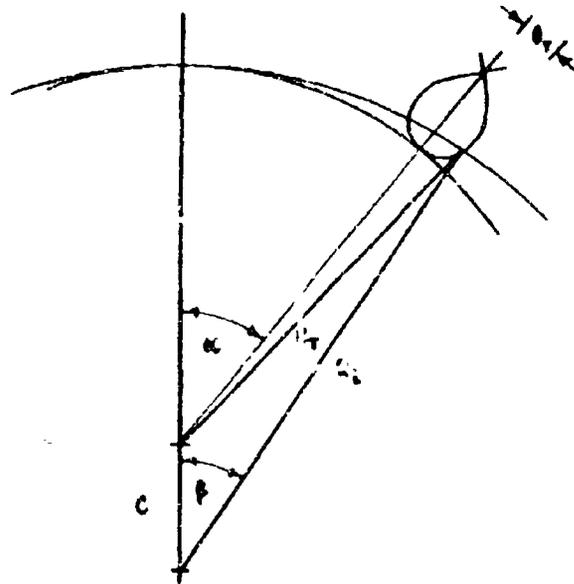
Let

    R_o = outside radius of external gear

    C = center distance

    α = rotation angle of external gear

    β = rotation angle of internal gear

    R_T = radius to trochoid on the internal gear

    θ_T = vectorial angle of trochoid

From Figure 4(a) the geometrical conditions are:

$$r_T = \sqrt{R_o^2 + C^2 + 2 R_o C \cos \alpha} \tag{4}$$

and

$$\theta_T = \text{Arcsin} \left\{ \frac{R_o}{R_T} \sin \alpha \right\} - \beta \tag{5}$$

where

$$\beta = \frac{R_{po}}{R_{pi}} \cdot \alpha$$

R_{po}, R_{pi} = pitch radii of external and internal gears, respectively.

The trochoid of the corner of the internal gear tooth at the root of the external tooth is described by the geometry shown in Figure 4(b). Let R_i denote the inside radius of the internal gear.

$$R_T = \sqrt{R_i^2 + C^2 + 2 R_i C \cos \beta} \qquad (6)$$

and

$$\theta_T = \text{Arcsin} \left\{ \frac{R_i}{R_T} \sin \beta \right\} - \alpha \qquad (7)$$

where

$$\alpha = \frac{R_{pi}}{R_{po}} \cdot \beta$$

3.3 The Contact Ratio

The duration of contact between internal and external gear teeth is defined on the line of action. Formally, the contact ratio is given by the length of the path of contact between its intersection with the inside radius of the internal gear and its intersection with the outside radius of the external gear divided by the base pitch of the gears. The geometry for obtaining the contact ratio is shown in Figure 5.

Let

R_{b1} = base radius, external gear

R_{b2} = base radius, internal gear

C = center distance

R_{p1} = pitch radius, external gear

R_{p2} = pitch radius, internal gear

R_o = outside radius, external gear
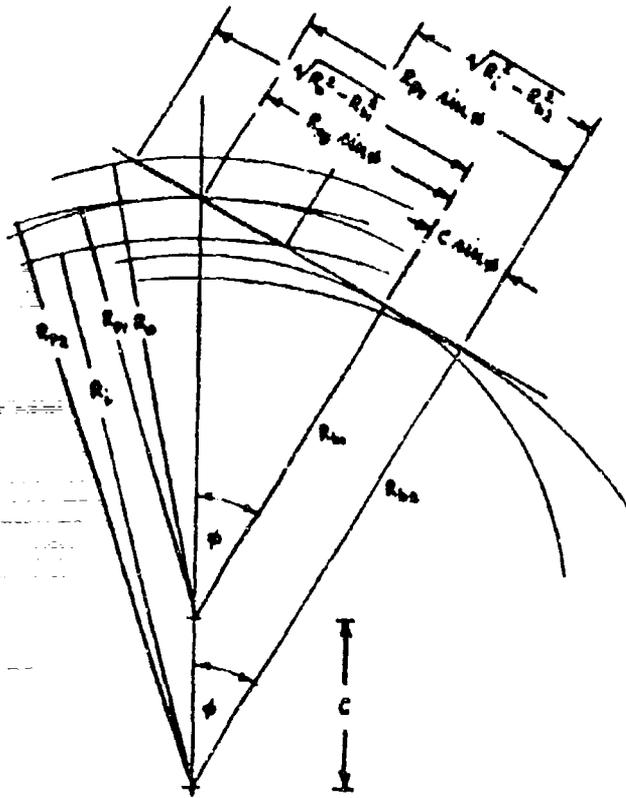
Figure 5 - Contact Ratio Geometry

R_i = inside radius, internal gear

ϕ = operating pressure angle at the pitch line

N_1 = number of teeth, external gear

N_2 = number of teeth, internal gear

The contact ratio, m_p, is given by

$$m_p = \frac{C \sin \phi + \sqrt{R_o^2 - R_{b1}^2} - \sqrt{R_i^2 - R_{b2}^2}}{\dfrac{2 \pi R_{b1}}{N_1}}$$

$$= \frac{C \sin \phi + \sqrt{R_o^2 - R_{b1}^2} - \sqrt{R_i^2 - R_{b2}^2}}{\dfrac{2 \pi R_{b2}}{N_2}} \tag{8}$$

Substitution of equation (3) into the above yields a relation between the contact ratio and the involute functions at the radii, R_o and R_i. Thus,

$$m_p = \frac{N_1}{2 \pi} \left[\frac{C}{R_{b1}} \sin \phi + (\text{inv}\, \phi_o + \phi_o) - \frac{R_{b2}}{R_{b1}} (\text{inv}\, \phi_i + \phi_i) \right]$$

$$= \frac{N_2}{2 \pi} \left[\frac{C}{R_{b2}} \sin \phi + \frac{R_{b1}}{R_{b2}} (\text{inv}\, \phi_o + \phi_o) - (\text{inv}\, \phi_i + \phi_i) \right] \tag{9}$$

3.4   Efficiency of Gears

The maximum efficiency of internal and external involute gear is taken to be the condition: the R_o - intercept and the R_i - intercept on the line of action are equidistant from the pitch point. From Figure 5 it is seen that this condition is met when

$$\sqrt{R_o^2 - R_{b1}^2} + \sqrt{R_i^2 - R_{b2}^2} = (R_{p2} + R_{p1}) \sin \phi \tag{10}$$

In terms of the involute functions the maximum efficiency occurs for the condition

$$R_{b1} (\text{inv}\, \phi_o + \phi_o) + R_{b2} (\text{inv}\, \phi_i + \phi_i) = (R_{p2} + R_{p1}) \sin \phi \tag{11}$$

Substitution of equation (11) into equation (9) yields an expression involving the contact ratio as a function of the involute value at the tip of the external gear tooth. The result is:

$$m_p = \frac{N_1}{2\pi}\left[\left(\frac{C - R_{p2} - R_{p1}}{R_{b1}}\right)\sin\phi + 2\,(\text{inv}\,\phi_o + \phi_o)\right] \quad (12)$$

3.5 Tip Interference

Tip interference occurs when the trochoid of the path of the corner of the external gear tooth intersects the involute profile of the internal gear tooth. If the trochoid is outside of the tooth form, then no tip interference will exist.

In terms of angles, the angle to the tip of the internal tooth is given by the involute function at the inside radius, R_1, that is,

$$y = \text{inv}\,\omega_1 \quad (13)$$

Referring to Figure 4(a), the trochoid of the path of the tip of the external gear tooth is given by

$$\theta_T = \text{Arcsin}\left\{\frac{R_o}{R_1}\sin\alpha\right\} - \beta \quad (14)$$

where

$$\alpha = \text{arccos}\left\{\frac{R_1^2 - C^2 - R_o^2}{2\,C\,R_o}\right\}$$

$$\beta = \frac{N_1}{N_2}\cdot\alpha$$

The angle θ_T is measured with respect to the axis of symmetry of the trochoid. The axis of symmetry is at an angle θ_o, with respect to the line of centers, given by

$$\theta_o = \frac{N_1}{N_2}\left[\text{inv}\,\phi_o - \text{inv}\,\phi\right] + \text{inv}\,\phi \quad (15)$$

Define

$$x = \theta_o - \theta_T \quad (16)$$

It follows that no tip interference occurs for any gear proportions R_0 and R_1 such that $x \geq y$.

3.6 Tooth Thickness

The arc tooth thickness of an involute gear tooth is a function of the radius of the arc and is a function of the thickness at some other radius. Figure 6 illustrates the geometry for an external gear tooth. Referring to Figure 6, the involute tooth form, generated from base radius R_b, of an external tooth is shown.

Let the arc tooth thickness T_1 at radius R_1 be known.

For all positions on the involute curve,

$$\phi = \arccos \frac{R_b}{R} \tag{17}$$

yields values for the arguments ϕ_1 and ϕ_2 in particular. At R_2 the arc tooth thickness is given by

$$T_2 = R_2 \left[\frac{T_1}{R_1} + 2 \text{ inv } \phi_1 - 2 \text{ inv } \phi_2 \right] \tag{18}$$
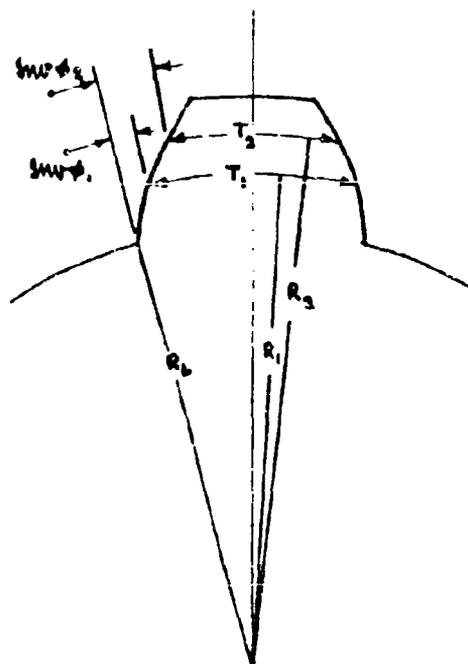
Figure 6 - Tooth Thickness

For an internal gear tooth similar analysis leads to the result

$$T_2 = R_2 \left[\frac{T_1}{R_1} + 2 \text{ inv } \phi_2 - 2 \text{ inv } \phi_1 \right] \qquad (19)$$

Given the proportions of an internal and external gear pair, it is desired to find the arc tooth thickness at the tips of the internal and external teeth which is equal for the two teeth and which permits the teeth to mesh tightly at the pitch point. In order that the teeth mesh tightly at the pitch point the arc tooth thickness must be such that

$$T_{p1} + T_{p2} = \frac{2\pi}{N_1} R_{p1} = \frac{2\pi}{N_2} R_{p2} \qquad (20)$$

where

T_{p1} = arc tooth thickness of the external gear tooth at the pitch radius, R_{p1}

T_{p2} = arc tooth thickness of the internal gear tooth at the pitch radius, R_{p2}

N_1, N_2 = number of teeth in external and internal gears, respectively

From equation (18) the arc tooth thickness at the tip of the external tooth is

$$T_o = R_o \left[\frac{T_{p1}}{R_{p1}} + 2 (\text{inv } \phi - \text{inv } \phi_o) \right] \qquad (21)$$

and from equation (19) the arc tooth thickness at the tip of the internal tooth is

$$T_i = R_i \left[\frac{T_{p2}}{R_{p2}} + 2 (\text{inv } \phi_i - \text{inv } \phi) \right] \qquad (22)$$

where

ϕ = operating pressure angle at R_{p1} and at R_{p2}.

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Setting $T_0 = T_1 = T$ and solving the resulting expression simultaneously with equation (20) for T_{p1}, substitution of this value into equation (21) gives the requisite tooth tip thickness. After some manipulation it is found that

$$T = \frac{R_0}{R_{p1}} \left\{ \frac{\frac{2\pi}{N_i} \cdot \frac{R_{p3}}{R_{p2}} R_1 + 2 R_0 \, \text{inv} \, \varphi_0 + 2 R_1 \, \text{inv} \, \phi_1 - 2(R_0 + R_1) \, \text{inv} \, \varphi}{\frac{R_0}{R_{p1}} + \frac{R_1}{R_{p2}}} \right\} + 2 R_0 (\text{inv} \, \varphi - \text{inv} \, \phi_0) \qquad (23)$$

3.7   Measurement Over Rolls

3.7.1 Internal Gear

In order to obtain the measurement over rolls for an internal, it is necessary to determine the position of the roll in the tooth form. The geometry of this case is shown in Figure 7.

Figure 7 - Measurement of Rolls

Setting $T_o = T_1 = T$ and solving the resulting expression simultaneously with equation (20) for T_{p1}, substitution of this value into equation (21) gives the requisite tooth tip thickness. After some manipulation it is found that

$$T = \frac{R_o}{R_{p1}} \left\{ \frac{\dfrac{2\pi}{N_1} \cdot \dfrac{R_{p3}}{R_{p2}} R_1 + 2 R_o \, \text{inv}\, \phi_o + 2 R_1 \, \text{inv}\, \phi_1 - 2(R_o + R_1) \, \text{inv}\, \phi}{\dfrac{R_o}{R_{p1}} + \dfrac{R_1}{R_{p2}}} \right\} + 2 R_o (\text{inv}\, \phi - \text{inv}\, \phi_o) \tag{23}$$

3.7   Measurement Over Rolls

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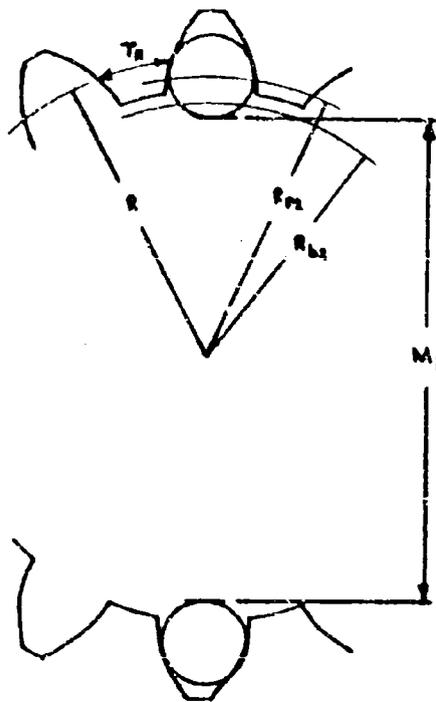
Figure 7 - Measurement of Rolls

Let

W = radius of the roll

N_2 = number of teeth in internal gear

The roll rests in the tooth form at a point such that

$$\text{inv } \phi_R = \frac{\pi}{N_2} + \text{inv } \phi - \frac{T_{p2}}{2 R_{p2}} - \frac{W}{R_{b2}} \tag{24}$$

From the geometry of involutes,

$$R = \frac{R_{b2}}{\cos \phi_R} \tag{25}$$

When the number of teeth, N_2, is even, the measurement over rolls is given by

$$M_i = 2 (R - W) \tag{26}$$

and when N_2 is odd, the measurement is

$$M_i = 2 \left(R \cos \frac{\pi}{2 N_2} - W \right) \tag{27}$$

3.7.2 External Gear

The analysis for the external tooth form is similar to that for the internal tooth. The roll rests in the tooth form at a point such that

$$\text{inv } \phi_R = \frac{T_{p1}}{2 R_{p1}} + \text{inv } \phi + \frac{W}{R_{b1}} - \frac{\pi}{N_1} \tag{28}$$

It follows that

$$R = \frac{R_{b1}}{\cos \phi_R} \tag{29}$$

When the number of teeth, N_1, is even, the measurement over rolls is

$$M_o = 2 (R + W) \tag{30}$$

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and when N_1 is odd, the measurement is

$$M_o = 2 \left(R \cos \frac{\pi}{2 N_1} + W \right)$$

4.0 NUMERICAL EVALUATION

The relationships derived in Section 3 have been programmed for numerical evaluation on the Research Laboratory's G-20 Digital Computer. The computer routine searches the domain of admissible gear proportions, imposing the conditions that maximum operating efficiency and minimum interference exist, and arrives at a single pair of radii representing the optimum gear proportions. The base radii, the number of teeth on each gear, the center distance and the operating pressure angle are given, and serve as inputs to the computer program.

The numerical method for obtaining optimum gear proportions is iterative. An initial value for the contact ratio is selected and the equation

$$m_p = \frac{N_1}{2 \pi} \left[\left(\frac{C - R_{p2} - R_{p1}}{R_{b1}} \right) \sin \phi + 2 (\text{inv} \, \phi_o + \phi_o) \right]$$

is applied to obtain the outside radius of the external gear which yields the maximum efficiency for the given contact ratio. The inside radius for exactly zero interference is obtained. This is the radius R_i which satisfies the equation

$$\frac{N_1}{N_2} \left[\text{inv} \, \phi_o - \text{inv} \, \phi \right] + \text{inv} \, \phi + \frac{N_1}{N_2} \alpha - \arcsin \left(\frac{R_o}{R_i} \sin \alpha \right) - \text{inv} \, \phi_1 = 0$$

where

$$\alpha = \arccos \left(\frac{R_i^2 - C^2 - R_o^2}{2 C R_o} \right)$$

The contact ratio m_p^*, given by

$$m_p^* = \frac{N_1}{2\pi}\left[\frac{C}{R_{b1}}\sin\phi + (\text{inv }\phi_o + \phi_o) \cdot \frac{R_{b2}}{R_{b1}}(\text{inv }\phi_i + \phi_i)\right]$$

is computed. If the relative error

$$\epsilon = \frac{\left|m_p - m_p^*\right|}{m_p^*}$$

is sufficiently small, R_o, R_i are retained as the optimum gear proportions. If $\epsilon > 10^{-4}$, then the new value m_p^* replaces m_p and the equations are evaluated again.

Having the radii R_o and R_i, equation (23) is evaluated directly to obtain the tip thickness, T. Finally, the measurement over rolls is calculated by choosing the appropriate formulas in Sections 3.7.1 and 3.7.2.