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BOUNDARY-LAYER FLOWS WITH LARGE INJECTION RATES

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PREFACE

Recent studies of mass-transfer rates associated with various reentry environments have pointed out the need for understanding the flow-field phenomena related to massive injection into boundary layers. Perhaps the simplest problem of the type that can be examined, without recourse to large-scale computation, concerns the effect of large mass injection on the constant-property laminar boundary layer.

In this Memorandum, the principal goal is to consider the effects of pressure gradient and large mass-injection rates for certain boundary-layer flows, and to explore the nature of the shear layer which may join the inviscid outer potential flow to the inviscid flow dominated by mass transfer near a wall. In addition, the similar problem in natural convection is considered, where temperature difference plays the role of the driving mechanism for flow acceleration.

SUMMARY

The problem of the constant-property laminar boundary-layer flow with large mass-injection rates and favorable pressure gradient is considered. Approximate solutions for the structure of blowing-induced shear layers are obtained for flows that satisfy the requirements of Falkner-Skan similarity. For small pressure gradients ($\beta < 1/2$), the asymptotic structure is shown to consist of a viscous shear layer imbedded in an inviscid flow.

A preliminary analysis of the natural-convection problem with large injection is performed, and the way in which the ideas used in the Falkner-Skan analysis may be extended to this problem is suggested.

CONTENTS

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LIST OF SYMBOLS

C = injection rate at surface, $-f(0)$

f = Falkner-Skan stream function

$\tilde{f}$ = f/C, inner variable

g = $1 - z$

$\tilde{g}_o$ = first term of outer expansion

Pr = Prandtl number

Re^* = Reynolds number, $\frac{U^*L}{\nu}$

r_o = distance between surface and axis of symmetry for axisymmetric flow, or distance between surface and reference plane which is parallel to body force in two-dimensional flow

T = temperature

U = velocity in boundary layer

U^* = reference velocity (Eq. (24) or (25))

U_e = velocity at edge of boundary layer

U_p, V_p = parabolic cylinder functions

u = $df/d\eta$ = U/U_e

$\bar{v}$ = normal velocity in boundary layer, $\sqrt{Pr\ Re^*}\ v$

v^* = inner velocity variable

y^* = inner distance variable

$\bar{y}$ = normal distance in boundary layer, $\sqrt{Pr\ Re^*}\ y$

Z = distance in direction of body force (height)

z = u^2

z_o = first term of inner solution

β = Falkner-Skan pressure-gradient parameter

β_o = volumetric expansion coefficient

η = Falkner-Skan similarity variable

ψ^* = stream function (Eq. (36))

I. INTRODUCTION

In the absence of pressure gradient, the flat-plate boundary layer "blows" off the surface if the injection of mass into the boundary layer is sufficiently high. This "blow-off" point occurs as a singularity in the solutions: it is not possible to find solutions of the boundary-layer equations for blowing rates larger than some critical value.

For flows with favorable pressure gradient, no such critical "blow-off" injection rate seems to exist. For a certain class of such flows, the Falkner-Skan wedge flows, it is possible to obtain solutions to the boundary-layer equations for arbitrarily large blowing rates.*

For more general boundary-layer flows, with large mass injection and favorable pressure gradient, the solution of the equations of motion in the neighborhood of the surface is essentially inviscid, and a simplified Bernoulli equation, obtained by neglecting the component of the kinetic energy associated with the normal velocity component, is an approximate solution of the equations of motion.**

A remarkably simple solution to the problem of the incompressible similar laminar boundary layer with asymptotically*** large injection was proposed by Pretsch[2] in 1944. The Falkner-Skan equation for wedge flows with favorable pressure gradient ($\beta > 0$)

$$f_{\eta\eta\eta} + ff_{\eta\eta} + \beta(1 - f_{\eta}^2) = 0 \qquad (1)$$

was considered subject to the boundary conditions

$$f_{\eta}(0) = 0 \qquad f(0) = -C \qquad f_{\eta}(\infty) \to 1 \qquad (2)$$

---

*A more complete discussion of these points, as well as a discussion of the Falkner-Skan flows, is given in Ref. 1, p. 243.

**See Ref. 1, p. 347, for the development of these ideas.

***Asymptotic, in this case, refers to blowing rates which are large but can still be considered within the context of boundary-layer theory.

where $C \to \infty$. New variables were introduced,

$$h(\zeta) \equiv f(\eta)/C$$
$$\zeta = \eta/C \tag{3}$$

and a simplified equation was obtained by neglecting terms of order $1/C^2$. The resulting equation, essentially the inviscid version of the Falkner-Skan equation, omits the viscous term $f_{\eta\eta\eta}$, and consequently it is possible to satisfy the two boundary conditions at $\eta = 0$, and the one at $\eta \to \infty$, by a discontinuous solution. Pretsch's solution is valid in the region between the zero streamline, which separates injected fluid from free-stream fluid, and the wall, but it gives no information about the outer flow between the zero streamline and the potential flow.

Several properties of Pretsch's solution are worth noting:

1. The velocity at the bounding streamline is given correctly, in that $f_\eta \equiv U/U_e \to 1$ as $f \to 0$. For nonsimilar incompressible boundary layers, with large injection, this is not necessarily the case. As the bounding streamline is approached from the wall with arbitrary favorable pressure gradient, $U_{inner} \to \sqrt{U_e^2(x) - U_e^2(x = 0)}$. For wedge flows, $U_e(x = 0) = 0$, since $U_e \sim x^n$ and $U_{inner} \to U_e(x)$ on the bounding streamline.

2. For values of $\beta > 1/2$, the bounding streamline is approached from the wall with zero shear stress; for $\beta = 1/2$, with finite shear; and for $\beta < 1/2$, with infinite shear.

In this Memorandum we re-examine the problem posed by Pretsch and indicate how his solution may be matched to a simple viscous solution which is valid in the outer flow, and which is necessary to smooth out the discontinuity in shear resulting when $\beta < 1/2$. This outer solution is shown to be a uniformly valid approximation to the inner flow as well. The blowing rate C enters into the outer solution only as a multiplicative constant. The outer flow consists of a viscous shear layer imbedded in a large inviscid mass-transfer layer only for small values of β.

In addition, we indicate how the inviscid approximation of Pretsch can be applied to free-convection boundary layers with large mass-transfer rates to obtain simple solutions for various flow quantities in terms of a simplified Bernoulli integral.

II. ANALYSIS

We shall find it convenient to work with modified Von Mises variables u(f), f, which are appropriate for similar flows:

$$f_\eta \equiv u$$
$$f \equiv f \qquad (4)$$

In these variables, Eq. (1) becomes

$$u(uu_f)_f + fuu_f + \beta(1 - u^2) = 0 \qquad (5)$$

The boundary conditions become

$$u = 0 \qquad f = -C$$
$$u \to 1 \qquad f \to \infty \qquad (6)$$

Letting $u^2 = z$, we obtain

$$\sqrt{z}(z_{ff}) + fz_f + 2\beta(1 - z) = 0 \qquad (7a)$$
$$z(-C) = 0$$
$$z(\infty) \to 1$$

For the inner region near the wall, a new, independent variable, $\tilde{f} \equiv f/C$, is introduced. The resulting equation and boundary conditions are

$$\frac{1}{C^2}\sqrt{z}(z_{\tilde{f}\tilde{f}}) + \tilde{f}z_{\tilde{f}} + 2\beta(1 - z) = 0 \qquad (7b)$$
$$z(-1) = 0$$
$$z(\infty) \to 1$$

An inner solution that is valid for fixed $\tilde{f}$ and $1/C \to 0$ is considered. The first term of the inner solution, $z_o(\tilde{f})$, is the one proposed by Pretsch and results from the inviscid equation obtained by neglecting the terms of $0\left\{1/C^2\right\}$:

$$\tilde{f} z_{o\tilde{f}} + 2\beta(1 - z_o) = 0 \qquad (8)$$

The no-slip condition is retained:

$$z_o(-1) = 0 \qquad (9)$$

The solution of Eq. (8), subject to the boundary condition of Eq. (9), is

$$z_o = 1 - (-\tilde{f})^{2\beta} \qquad (10)$$

This solution is sketched in Fig. 1.

For values of $\tilde{f}$ in the neighborhood of 0, this solution has the property that $\tilde{f}_{\eta\eta} \to \tilde{f}^{2\beta-1}$, and consequently there is a radical difference in the shear profile as β is less than or greater than 1/2, as noted earlier. The outer solution of Eqs. (8) and (9) is $u = 1$ for all values of $f > 0$. The viscous outer solution, which smooths out the shear profile, joins a potential flow at large positive values of f to an inner rotational flow at large negative values of f, and corresponds to a "vorticity-interaction" problem, where the vorticity arises from the total pressure variation along the wall which is convected along the streamlines of the injected fluid.

The solution of Eq. (10) cannot really be continued to $\tilde{f} > 0$. At $\tilde{f} = 0$, z_o reaches the value at infinity. However, the resultant profile has a corner at $z_o = 1$, $\tilde{f} = 0$. This corner is removed by constructing an expansion that is valid near $z = 1$, which is matched to the first term of the inner expansion. The matching takes place as $\tilde{f} \to 0-$ in z_o, and $f \to -\infty$ in the outer expansion. Although the second term in the inner expansion has a singularity as $\tilde{f} \to 0-$, it can be shown that in the region of matching, its contribution is negligible.

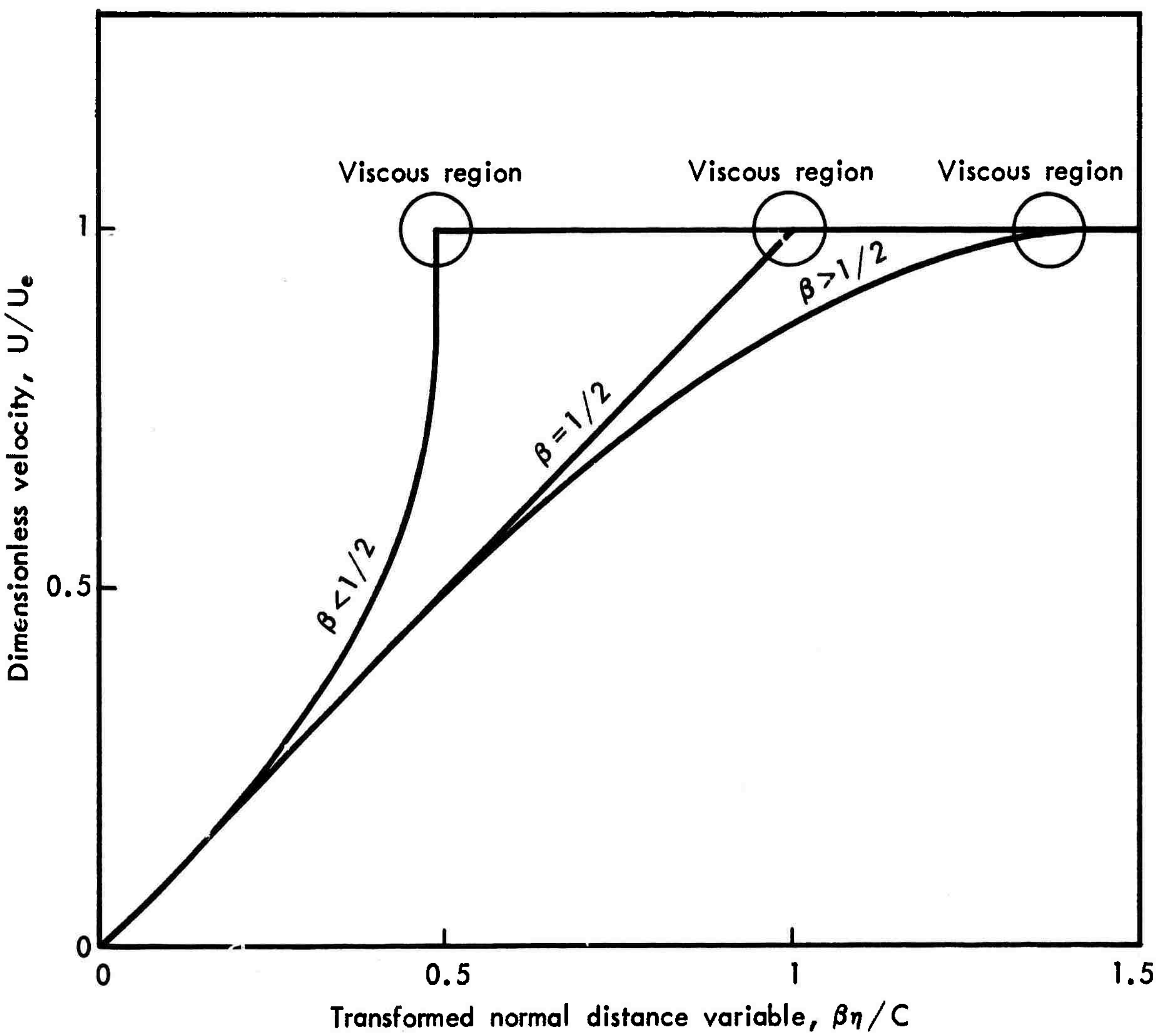

Fig.1—Typical inviscid velocity profiles

Thus a new variable g(f) is introduced:

$$z(f) = 1 - g(f) \qquad (11)$$

where

$$\left(\sqrt{1 - g}\right)g_{ff} + fg_f - 2\beta g = 0 \qquad (12)$$

$$g(\infty) \to 0$$

$$g(-C) = 1$$

This suggests that the first term for the outer expansion of g is

$$g_{outer} = \left(\frac{1}{C}\right)^{2\beta} \tilde{g}_o(f) + \ldots \qquad (13)$$

The resulting equation for $\tilde{g}_o(f)$ is then

$$\tilde{g}_{off} + f\tilde{g}_{of} - 2\beta\tilde{g}_o = 0 \qquad (14)$$

$$\tilde{g}_o(\infty) \to 0$$

$$\tilde{g}_o(f \to -\infty) \to (-f)^{2\beta}$$

The solution to the equation and boundary conditions of Eq. (14) is

$$\tilde{g}_o = A\left[\exp\left(\frac{-f^2}{4}\right)\right]\left[U_p(2\beta + 1/2,\ f) + BV_p(2\beta + 1/2,\ f)\right] \qquad (15)$$

where $U_p(2\beta + 1/2, f)$ and $V_p(2\beta + 1/2, f)$ are the parabolic cylinder functions described in Ref. 3,* and A and B are constants of integration.

---

*In the notation of Ref. 3, $U_p \equiv U$ and $V_p \equiv V$.

We note from Ref. 3 that

$$\pi V(2\beta + 1/2, x) = \Gamma(1 + 2\beta) \sin \pi(2\beta + 1/2) \cdot U_p(2\beta + 1, x) + U_p(2\beta + 1, -x)$$

From the asymptotic expansions for U and V, it is clear that the constant B must be zero, since

$$V_p(2\beta + 1/2, x) \sim \sqrt{\frac{2}{\pi}}\, e^{\frac{x^2}{4}}\, x^{2\beta} \left[1 + 0\left(\frac{1}{x^2}\right) + \dots\right] \quad \text{as } x \to +\infty$$

and

$$U_p(2\beta + 1/2, x) \sim e^{-\frac{x^2}{4}}\, x^{-1-2\beta} \left[1 + 0\left(\frac{1}{x^2}\right) + \dots\right] \quad \text{as } x \to \infty$$

The asymptotic behavior as $x \to -\infty$

$$U_p(2\beta + 1, x) \sim \frac{\pi\sqrt{\frac{2}{\pi}}\, e^{\frac{x^2}{4}}\, (-x)^{2\beta}}{\Gamma(1 + 2\beta)} \left[1 + 0\left(\frac{1}{x^2}\right) + \dots\right]$$

fixes the constant A to be

$$\frac{\Gamma(1 + 2\beta)}{2\pi}$$

The one-term outer solution is thus

$$g_o(f) \sim \left(\frac{1}{c}\right)^{2\beta} \frac{\Gamma(1 + 2\beta)}{2\pi}\, e^{-\frac{f^2}{4}}\, U_p(2\beta + 1/2, f) \tag{16}$$

The shear function corresponding to this outer flow is

$$2f_{\eta\eta} = \frac{dg(f)}{df} = \left(\frac{1}{c}\right)^{2\beta} \frac{\Gamma(1 + 2\beta)}{\sqrt{2\pi}} U_p(2\beta - 1/2, f) \cdot e^{-\frac{f^2}{4}} \tag{17}$$

where the recurrence relation

$$U_p'(2\beta + 1/2, x) - (1/2)xU_p(2\beta + 1/2, x) + U_p(2\beta - 1/2, x) = 0$$

has been used.

Since the inner solution, Eq. (10), is contained within the outer solution, Eq. (16), then

$$g_o(C, f) = \left(\frac{1}{C}\right)^{2\beta} \frac{\Gamma(1 + 2\beta)}{\sqrt{2\pi}} e^{-\frac{f^2}{4}} U_p(2\beta + 1/2, f) \qquad (18)$$

is a uniformly valid one-term expansion.

Figure 2 indicates the shear-function variation for various values of the pressure-gradient parameter β. The shear function

$$(C)^{2\beta} \frac{dg_o(f)}{df}$$

is equal to

$$(C)^{2\beta}\, 2f_{\eta\eta}(\eta)$$

in the usual Falkner-Skan variables.

We note that for $\beta < 1/2$ (in this case $\beta = 0.1$), a genuine shear layer appears, as suggested by Pretsch's inviscid solution,[2] but for larger values of the pressure-gradient parameter, the shear is a maximum at the wall, and the boundary layer remains on the wall rather than being "blown off" into a shear layer.

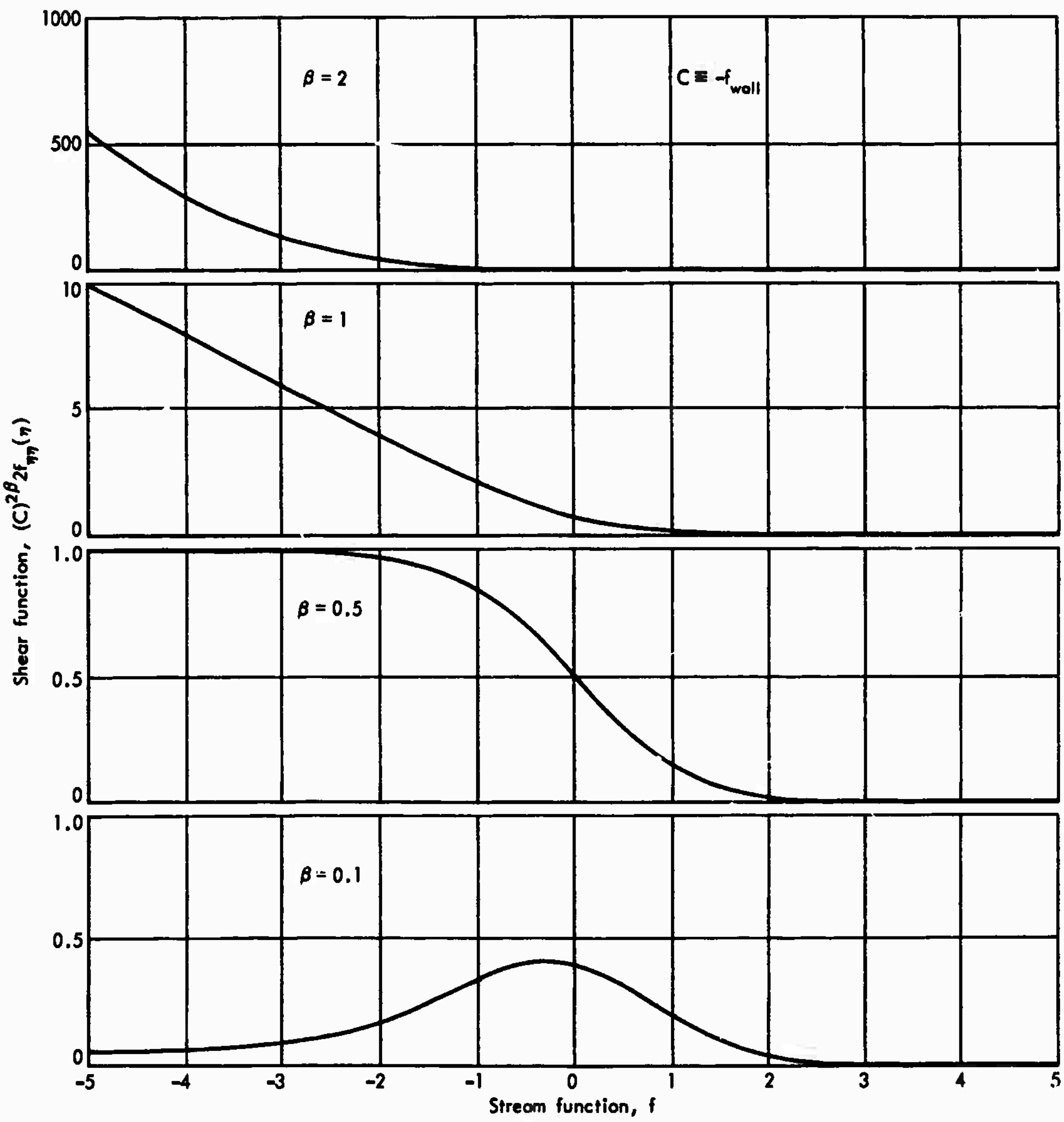

Fig. 2—Shear variation in wedge flows with large injection rates

III. FREE-CONVECTION BOUNDARY LAYERS WITH INJECTION

The inviscid approximation may be applied to free-convection boundary layers if injection rates are sufficiently large.

(For convenience, we will use the notation and coordinate system of Ref. 4.)

For a free-convection boundary layer on a two-dimensional or axisymmetric body, the equations of motion in dimensional form are

$$uu_x + vu_y = \nu u_{yy} + \beta_o g(T - T_\infty)\sqrt{1 - r_o'^2} \tag{19}$$

$$uT_x + vT_y = \frac{\nu}{Pr} T_{yy} \tag{20}$$

$$(r_o^j u)_x + (r_o^j v)_y = 0 \tag{21}$$

where $j = 0$ for two-dimensional flow or 1 for axisymmetric flow. The appropriate reduced variables for a free-convection boundary layer are

$$\bar{y} = \sqrt{Pr\ Re^*}\ y \tag{22}$$

$$\bar{v} = \sqrt{Pr\ Re^*}\ v \tag{23}$$

where

$$Re^* = \frac{U^* L}{\nu}$$

and U^* is some reference velocity,

$$U^* = \sqrt{\beta_o g \Delta T L} \qquad \text{for } Pr \sim O(1) \tag{24}$$

$$U^* = \sqrt{\frac{\beta g \Delta T L}{Pr}} \qquad \text{for } \frac{1}{Pr} \to 0 \tag{25}$$

In these reduced variables, the equations of motion become

$$uu_x + \bar{v}u_{\bar{y}} = Pr\, u_{\bar{y}\bar{y}} + \beta_o g(T - T_\infty)\sqrt{1 - (r_o')^2} \qquad (26)$$

$$uT_x + \bar{v}T_{\bar{y}} = T_{\bar{y}\bar{y}} \qquad (27)$$

$$(r_o^j u)_x + (r_o^j \bar{v})_{\bar{y}} = 0 \qquad (28)$$

The boundary conditions for asymptotically large blowing are

$$u(x, 0) = 0 \qquad u(x, \infty) = 0$$

$$\bar{v}(x, 0) = Cv_w(x) \qquad (29)$$

$$T(x, 0) = T_w(x) \qquad T(x, \infty) = T_\infty$$

where $C \to \infty$, and $v_w(x) \sim 0(1)$.

Our discussion centers around mass-injection rates which are much larger than $1/\sqrt{Pr\, Re^*}$ but are still sufficiently small for the boundary-layer equations to be valid.

New variables are introduced:

$$v^* = \frac{\bar{v}}{C} \qquad (30)$$

$$y^* = \frac{\bar{y}}{C} \qquad (31)$$

and we consider solutions to Eqs. (26) to (28), subject to the boundary conditions of Eqs. (29) valid for $C \to \infty$, which are of the form

$$u = u(x, y^*, C) = u_o(x, y^*) + \dots$$

$$v = v^*(x, y^*, C) = Cv_o^*(x, y^*) + \ldots$$

$$T = T^*(x, y^*, C) = T_o^*(x, y^*) + \ldots$$

The equations for $u_o^*(x, y^*)$, $v_o^*(x, y^*)$, $T_o^*(x, y^*)$ become

$$u_o^* \frac{\partial u_o^*}{\partial x} + v_o^* \frac{\partial u_o^*}{\partial y^*} = \beta_o g(T - T_\infty)\sqrt{1 - (r_o')^2} \qquad (32)$$

$$u_o^* \frac{\partial T_o^*}{\partial x} + v_o^* \frac{\partial T_o^*}{\partial y^*} = 0 \qquad (33)$$

$$\frac{\partial(r_o^j u_o)}{\partial x} + \frac{\partial(r_o^j v_o^*)}{\partial y^*} = 0 \qquad (34)$$

with boundary conditions

$$v_o^*(x, 0) = v_w(x)$$

$$u_o^*(x, 0) = 0 \qquad u_o^*(x, \infty) = 0 \qquad (35)$$

$$T_o^*(x, 0) = T_w^*(x) \qquad T_o^*(x, \infty) = T_\infty$$

The solution of these equations and boundary conditions can be obtained easily by the introduction of Von Mises variables x, ψ^* as independent variables.

Introducing

$$\frac{\partial\psi^*}{\partial x} = -v^* r_o^j, \qquad \frac{\partial\psi^*}{\partial y^*} = u_o^* r_o^j \qquad (36)$$

and noting that

$$\frac{\partial}{\partial y^*} = u_o^* r_o^j \frac{\partial}{\partial \psi^*}, \qquad \frac{\partial}{\partial x} = \frac{\partial}{\partial x} - v_o^* r_o^j \frac{\partial}{\partial \psi^*}$$

results in

$$\frac{\partial}{\partial x}\left[\frac{u_o^{*2}}{2}(x, \psi^*)\right] = \beta_o g(T_o^*(x, \psi^*) - T_\infty)\sqrt{1 - r_o'^2} \qquad (37)$$

$$\frac{\partial}{\partial x}\left[T_o^*(x, \psi^*)\right] = 0 \qquad (38)$$

It is convenient to use the variable x^*, which marks the position on the body where the ψ^* streamline crosses the body surface

$$dx^* = -\frac{d\psi^*}{r_o^j(x^*) v_w(x^*)} \qquad (39)$$

In the x, x^* coordinates,

$$\frac{u_o^*}{2}(x, x^*) = \int_{x^*}^{x} \beta_o g(T(x^*) - T_\infty)\sqrt{1 - r_o'^2(\eta)}\, d\eta \qquad (40)$$

$$T_o^*(x, x^*) = T_w^*(x^*) \qquad (41)$$

where the no-slip condition at the wall for u^* has been applied. The solution for $u_o^*(x, y^*)$ is given parametrically in terms of x, x^*, where y^* is determined from

$$r_o^j(x) y^*(x, x^*) = r_o^j y_o^*(x) - \int_0^{x^*} \frac{r_o^j(\xi) v_w(\xi)}{u_o(x, \xi)}\, d\xi \qquad (42)$$

where

$$r_o^j y_o^*(x) = \int_o^x \frac{r_o(x^*) v_w(x^*)\, dx^*}{u_o(x, x^*)} \qquad (43)$$

Note that $y_o(x) = Cy_o^*(x)$ is the location of the dividing streamline. For $y^* > y_o^*$, $u_o = 0$ and $T_o^* = T_\infty$. The dividing streamline is a surface of discontinuity in both temperature and velocity if the temperature at $x = 0$ is not T_∞. If the temperature at $x = 0$ is T_∞, then there may be a discontinuity in either shear or temperature gradient at $y_o^*(x)$. We observe that

$$\int_{x^*}^{x} \sqrt{1 - (r_o')^2}\, d\eta = Z(x) - Z(x^*) \qquad (44)$$

where $Z(x) - Z(x^*)$ is the difference in height of the points on the body surface, x and x^*.

Equation (40) is a simplified Bernoulli equation which can be written as

$$\frac{u_o^2}{2}(x, x^*) = \beta g\left[T_o^*(x^*) - T_\infty\right]\left[Z(x) - Z(x^*)\right] \qquad (45)$$

For a vertical flat plate, to give an example, $Z(x) = x$. Then

$$\frac{u_o^2}{2}(x, x^*) = \beta_o g\left[T_o^*(x^*) - T_\infty\right]L \cdot (x - x^*) \qquad (46)$$

and

$$Y_o^*(x) = \int_o^x \frac{v_w(x^*)\, dx^*}{u_o^*(x, x^*)} \qquad (47)$$

If $T(x^*) = T_w$, constant, and $v_w(x^*) = v_w$, constant, then

$$Y_o^*(x) = \frac{v_w}{2 \cdot (\beta_o g)^{1/2}(T_w - T_\infty)} \frac{\sqrt{x}}{2} \qquad (48)$$

Using the same ideas which we have introduced in the calculation of the viscous correction for forced flow, it should be possible to obtain the viscous correction which smooths out the discontinuity at $Y_o^*(x)$ for certain wall-temperature distributions. However, it appears that the problem of constant wall temperature would require a full discussion of a thermally driven free shear layer at $y = y_o$.

Since it is not at all clear that the natural-convection problem with large injection rates has technical applications or significance, we have included the inviscid analysis only to indicate how the usual ideas of boundary-layer theory for both forced and natural flow can be simplified if problems involving larger mass-transfer rates are considered.

The principal simplification results from the observation that the flow in the neighborhood of the boundary is inviscid, so that the boundary layer, which occurs at the wall for small mass-transfer rates, can now become a shear layer. For the forced-flow case, the skin-friction parameter is determined completely by the inviscid solution, while for the natural-flow case, the heat transfer to the boundary is zero.

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10. ABSTRACT

A consideration of the problem of the constant-property laminar boundary-layer flow with large mass-injection rates and favorable pressure gradient. Approximate solutions for the structure of blowing-induced shear layers are obtained for flows that satisfy the requirements of Falkner-Skan similarity. For small pressure gradients ($\beta<\frac{1}{2}$), the asymptotic structure is shown to consist of a viscous shear layer imbedded in an inviscid flow. A preliminary analysis of the natural-convection problem with large injection is performed, and the way in which the ideas used in the Falkner-Skan analysis may be extended to this problem is suggested.

11. KEY WORDS

Boundary layer
Mathematics
Mass transfer